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ABSTRACT

A Time-Frequency Analysis of the Coherences of the US Business Cycle and the European Business Cycle*

The search for and dating of a possible european business cycle, has been inconclusive. At this stage, there is no consensus on the existence of such a cycle, or of its periodicity and amplitude, or of the relationship of individual member countries to that cycle. Yet cyclical convergence is the key consideration for countries which have to decide whether they wish to be members of a currency union such as the euro. The confusion over whether and to what degree the UK is converging on the cycles of its European partners, or whether its cycle is more in line with the US, is a classic example of the difficulties caused by this lack of consensus. We argue that different countries will vary in the components and characteristics that make up their output cycles, as well as vary in the state of their cycle at any point of time. We show how to decompose a business cycle in a time-frequency framework. This then allows us to decompose movements in output, both at the European level and in member countries, into their component cycles and allows those component cycles to vary in importance and cyclical characteristics over time. It also allows us to determine if the nonconclusive results so far have appeared because member countries have some cycles in common, but diverge (i.e. have nothing in common) at other frequencies.

JEL Classification: C22, C29, C49, F43 and O49
Keywords: business cycle, coherence, growth rates and time-frequency analysis

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1 Introduction: a Literature Review

The search for and dating of a possible European business cycle, has been inconclusive. At this stage, there is no consensus on the existence of such a cycle, or of its periodicity and amplitude, or of the convergence of individual member countries on that cycle if it exists. Yet cyclical convergence is the key consideration for countries which have to decide whether to join, or remain in, a common currency union such as the Euro (Treasury, 2003).

On the theoretical front, the neoclassical growth model developed by Solow (1956) and others argues that every economy approaches its *own* steady-state income level determined by that economy's aggregate parameters such as the discount rate, intertemporal elasticity of factor substitution, depreciation rate, capital share, and population growth rate. Once at the steady-state, the economy grows at a constant rate. Thus, to the extent that the determinants of the steady-state are similar across economies, convergence is expected. But if the determinants are different, they won't. For example Sala-I-Martin (1996) shows wide convergence of regions within countries. Mankiw et al. (1992) find evidence of absolute convergence for a sample of OECD countries at similar level of development over the years 1960-1985. But they reject the convergence hypothesis in a wider sample of 75 economies where structures and the degree of uncertainty varies a good deal more. Baumol (1986), Dowrick and Nguyen (1989), Wolff (1991), Barro (1991), Barro and Sala-I-Martin (1991; 1992) reach similar results. The absence of absolute convergence has motivated some economists to develop growth models in which income disparity among economies can persist indefinitely. The endogenous growth models were pioneered by Romer (1986) and Lucas (1988) in which investment embodies spill over effects, are a case in point. Convergence disappears when the assumption of diminishing returns is dropped because an economy can now grow without limits. Thus, Quah (1993) finds divergence in a sample of 118 countries. It is worth noting that all of these studies reach their conclusions by examining the cross-sectional relationship between the growth rate of output per capita over some time period and the initial level of per capita output. Evans and Karras (1996) have shown that even this model of convergence can only be valid under very strict assumptions. Specifically, the economies must have identical first-order autoregressive dynamic structures; and all the permanent cross-economy differences must be explicitly controlled for. Evidently we need to be extremely careful about how the statistical model is set up, if we wish to get clear and reliable results.

As far as Europe is concerned, Artis and Zhang (1997), Frankel and Rose (1998), and Razzak (1998) argue that if exchange rates are successfully pegged, business cycles are likely to converge. This is bolstered by growing trade and financial links. They find a shift of the ERM countries away from the US business cycle, and towards the German business cycle. They also find that the UK is distinguished from the other ERM countries. In contrast, Inklaar and de Haan (2000) do not find any evidence for an European business cycle, while Agresti and Mojon (2001) and Luginbuhl and Koopman (2003) find large similarities between the European business cycles. Meanwhile, Frankel and Rose (1998) and Prasad (1999) argue that the intensity of trade alone will lead to correlated business cycles, especially within EMU. Fichtner (2003), on the other hand, finds that convergence of European business cycles is not driven by trade links, but by technology spillovers. Different again, Chauvet and Potter (2001) find that the US business cycle was in line with the G7 from the mid 70s to the mid 80s, but then started to diverge. Kiani and Bidarkota (2003), Stock and Watson (2003), and McAdam (2003) find divergence as well which is caused by structural breaks. Hughes Hallett and Piscitelli (2002) find that large (stable) and well intregrated economies are likely to diverge whilst smaller less well integrated economies will converge. Furthermore, Kontolemis and Samiei (2000) argue that the UK has been more volatile than the European counterparts. The reason for this is the different role of monetary policy in these countries. They find that if the UK would adopt a monetary policy which is more in line with the European one the business cycle would perhaps converge towards the European one. This result is also confirmed by Suardi (2001), although his results are qualified by differences in the transmission mechanisms working in Europe. As a consequence, he still finds the largest differences are between the ERM on the one side and the UK and Sweden on the other. Nevertheless he suggests that the transmission mechanisms may converge as financial structures converge. The important point Suardi makes is that the link between countries is not constant: the link between cycles may change as new policies and regimes are introduced. In that vein, Stock and Watson (2002) not only find that the US business cycle has changed. They emphasise that the nature of the change is important. The business cycle may change because of changes in the conditional mean of the process, or because of changes in the innovation variance. They test changes in the conditional mean using the Kalman filter. We follow their idea in this paper by searching for shifts in the underlying structure using a Kalman filter.

If there are structural breaks which change the conditional mean of the process then that may explain the lack of consensus over whether and to what degree the UK is converging on her European partners, or whether her business cycle is more in line with the US. Moreover, although some have found evidence of a common European cycle[1], it has not been possible to estimate it precisely and the estimates appear rather unstable. That suggests the underlying problem may be that different countries vary in the components and characteristics that make up their cycles, as well as in the state of their cycle at any point in time. That would mean that any European cycle would have a different composition at different times, both in terms of which under-lying component cycles are actually present, and in the relative weights that each cyclical frequency should have.

We therefore argue that a time varying spectral approach is the right way to analyse the emerging European business cycle. Such an approach allows us to decompose movements in output into their component cycles and allows those cycles to vary in importance and cyclical characteristics over time. It will also allow us to determine if the inconclusive results of the past appeared because member countries have some cycles in common, but diverge at other frequencies. Or whether they have converged (with some cycles in common) at certain times, but have diverged at the same frequencies at other times. In either case no clear cut results could be expected from using conventional methods. Conventional methods would assume constant cycles with unvarying cyclical characteristics over the sample; and also that the relative importance of the component cycles within any country, and the importance of each country's cycles within the union, was constant at all times.Such assumptions are unlikely to be valid in a period of economic change – and as countries adapt and converge as they integrate to form a single European economy in particular.

Our approach is to adapt a time-frequency analysis from the natural sciences. This will allow us to perform the cyclical decompositions we want, and also allow the cycles and their contribution to the overall pattern of output to vary over time. Time-frequency analysis has been an active area of research in other disciplines (Claasen and Mecklenbräuker, 1980a; 1980b; 1980c; Cohen, 1989). Recent examples appear in Boashash (2003), Grochenig (2001), and Papandreou-Suppappola (2002). Conventional Fourier analysis cannot do the same job

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[1] See HM Treasury (2003) for a wide ranging review. Papers by Bayoumi and Eichengreen (1993), Levy and Dezhbakhash (2003), give a idea of the range of different results that have been found.

because it assumes all cycles to have constant weights. Nor can wavelet analysis because that relies on having *preassigned* frequencies over time.

Our solution is to seek a representation which is a two variable distribution whose domain is the two dimensional time-frequency space. We then examine the two marginal distributions: the time constant cross-section will show the frequencies present (and their importance) at each time t; and the constant frequency cross-section, in which the importance of each frequency varies over time. To do this we start from a time-varying spectrum for each country separately estimated by using the Kalman filter[2]. The information from this filtering is then used to derive the time-varying coherence between national cycles. Since the change of the coherence contains information on a change on the structural changes <u>and</u> changes in the underlying spectra, we can follow the impact of economic links over time.

This paper is organised as follows: the next section provides a short introduction in the techniques used in this paper. The following section gives firstly the country specific results. In the next part of that section we present the coherences and we finish with the conclusions.

2 A Short Technical Introduction to Time Frequency Analysis and Coherence

2.1 Introduction

Evans and Karras (1996), Bernard and Durlauf (1991), estimate the following the data-generating process:

$$\Delta\left(y_{nt}-\overline{y}_{t}\right)=\delta_{n}+\rho_{n}\left(y_{n,t-1}-\overline{y}_{t-1}\right)+\sum_{i=1}^{p}\varphi_{n,i}\Delta\left(y_{n,t-i}-\overline{y}_{t-i}\right)+u_{n,t} \tag{2.1}$$

where y_n is the log of the real per capita output of economy n during period t; $\overline{y}_t \equiv \sum_{n=1}^{N} y_{n,t}/N$ and N is the number of economies analysed; δ_n and φ_ns are parameters. The

[2] The Kalman filter was also used by Chauvet and Potter (2001) in order to detect structural breaks.

parameter of concern is ρ_n. This parameter is negative if the economies converge and zero if they diverge. There are several problems with that approach:

1) Although Evans and Karras (1996) point out correctly that a neoclassical assumption is that the growth rates of the economies analysed have to follow the same AR-process, they do not really test this with this approach. What they test is cointegration of one country to the average of countries. That is there may be convergence towards the average, but to no country in particular. On the other hand, even if growth rates do not follow the same AR-process that does not exclude convergence as we will show below.

2) The estimated relationship is time invariant. Structural changes of the relationship are not included in this approach. This is particularly important since those changes may disappear by taking the average of countries or neglection of structural breaks my lead to inconsistent estimates.

3) They estimate a long-run convergence. The parameter ρ shows convergence in the long run, i.e. a (common) trend relationship. Their approach cannot say anything about a possible short run convergence or a possible convergence of some frequencies in the short run and/or the long run and how that may change over time.

In this paper, we tackle these problems. That is, we first of all estimate the data generating process of each growth rate individually. That gives us already an idea whether the economies follow the same data generating process, which is a requirement of the neoclassical approach. Given this estimation in the time domain, we can then transfer these results in the frequency domain. Since we estimated the data generating process in a time varying manner, we end up with a time frequency analysis of the spectrum. We are able to observe over time, how the data generating process has changed and which frequencies are affected. As we will see, some processes change completely in nature, i.e. they change from an AR(1) process to an AR(2) and back again.

Once we have done this, we estimate the one to one relationship of two economies. By transferring the time domain results into the frequency domain, we can see how the relationship of these two economies changed in terms of individual frequencies. That is we are able to investigate whether convergence took place over time and if so for which frequencies. As one expects, the result will be ambiguous. Some frequencies will converge (but not completely, maybe) others will not. As a measure of the relationship between two economies, we use the coherence. Also, by analysing a one-to-one relationship we avoid the problem of multicollinearity between several countries. Last but not least, we can decompose the coherence in order to see, whether the change of the coherence is caused by a change of the relationship between the two variables or the change of the data generating process itself or both and to what degree. With a time-invariant method that cannot be done[3]. The following section outlines these ideas. The non-interested reader may just jump to section 3 to see the empirical results.

2.2 Outline of the Method Used

Generally speaking, a spectrum shows the decomposition of the variance of a sample of data across different frequencies. The power spectrum itself shows the relative importance of different cycles in creating movements in that data, and hence describes the cyclical properties of a particular time series. It is assumed that the fluctuations of the underlying process are produced by a large number of elementary cycles of different frequencies. Furthermore, it is also usually assumed that the contribution of each cycle is constant throughout the sample. However, as Chauvet and Potter (2001) showed for the US, business cycles cannot be assumed to be constant (e.g. due to structural breaks). Hence, the spectrum would no longer be constant over time due to the changing distribution of weights associated with each of the elementary cycles[4]. This problem is particularly relevant in the presence of unit roots. However, that problem poses no difficulty in this study since we calculated our growth rates as the log-difference of the real GDP and none of the raw data tested contained more than one unit root[5].This is a quite common result, as recent research by Rapach (2002) and Gerdtham and Löthgren (2002) for example, suggests. Hence, using the first differences circumvents the unit root problem. Nevertheless, recent research in time-frequency analysis has opened up ways to estimate time-varying spectra directly even in the case of non-stationary processes as Matz and Hlawatsch (2003) have shown. We follow that general approach here.

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[3] One may ask why we are not using band-pass filtering since it is supposed to show frequency changes over time as well. We decided not to use band pass filtering because, as Benati (2000) shows, band-pass filtering can cause spurious results. It also disturbs stylised facts of the business cycle.
[4] This problem is for example discussed in greater detail in: Matz and Hlawatsch (2003).
[5] Test results are available from the authors on request.

A second advantage of our general approach is that time frequency analysis, like wavelet analysis, does not assume a constant cyclical behaviour. On the contrary, the importance of cycles may change over time. This is a key point in this study since we are particularly interested in determining whether the contributions of each cycle have changed, and if so to what extent. That is not to imply that the time varying behaviour of the spectrum is caused by unit roots only. We are also interested in structural breaks which cause a change in lag structure. Although Perron (1989) showed that structural breaks may cause a time series to appear to contain a unit root, empirical evidence by Rapach (2002), Gerdtham and Löthgren (2002) and Pappel and Prodan (2003) suggest that we have both: a unit root and structural breaks. In this paper we can cater for both. By taking first differences we tackle the problem of a unit root; and by estimating the relationship using the Kalman filter, we can tackle the problem of structural breaks.

In order to outline the approach used in this paper, suppose we are interested in the relationship between the variables, $\{Y_t\}$ and $\{X_t\}$ say, where $\{Y_t\}$ is the US growth rate and $\{X_t\}$ is a European growth rate. We assume that they are related in the following way[6]:

$$V(L)_t Y_t = A(L)_t X_t + u_t \tag{2.2}$$

where A(L) and V(L) are filters, and L is the lag operator such that $LY_t = Y_{t-1}$. Notice that the lag structure, A(L), is time-varying. That means we need to use a state space model (we have used a Kalman filter) to estimate the implied lag structure. The rationale is that, using estimated coefficients from (2.2) vastly simplifies estimation of the spectra; and since we have observations of those coefficients for each point in time, we can generate a time-varying spectrum from them. That allows us to overcome two problems. First, one of the disadvantages of a direct estimation approach is the large number of observations that would be necessary to carry out the necessary frequency analysis. This may be a particular problem in the case of structural breaks, since the subsamples could be too small for estimating the spectra directly. Second, using an indirect estimation allows us to estimate the spectra, and associated cross-spectral components, in a time varying manner. Hughes Hallett and Richter (2002; 2003a; 2003b; 2004) show the time-varying cross spectrum ($f_{YX}(\omega)_t$) is then equal to

[6] where the moduli of the eigenvalues of A(L) are less than one in absolute value.

$$f_{YX}(\omega)_t = \left|A(\omega)_t\right| f_{XX}(\omega)_t \quad (2.3)$$

where A(ω) is the Fourier transform of the weights $\{a_j\}_{j=-\infty}^{\infty}$ in (2.1). The last expression in (2.3), $f_{XX}(\omega)_t$, is the spectrum of the exogenous variable. This spectrum may be time varying as well.

In this paper we are particularly interested in the coherence and, more importantly, in the decomposition of the change of the coherence over time. So, we need to establish the link between the coherence and the gain. The spectrum of the dependent variable is defined as (Jenkins and Watts, 1968; Laven and Shi, 1993; Nerlove et al., 1995; Wolters, 1980):

$$f_{YY}(\omega)_t = \left|A(\omega)\right|_t f_{XX}(\omega)_t + f_{VV}(\omega)_t \quad (2.4)$$

where $f_{VV}(\omega)_t$ is the time varying residual spectrum and $f_{YY}(\omega)_t$ is the time varying spectrum of the endogenous variable. $\left|A(\omega)\right|_t^2$ is sometimes called the gain, which may be time varying as well.

Given knowledge of $f_{YY}(\omega)_t$, $\left|A(\omega)\right|^2$, and $f_{XX}(\omega)_t$, one can calculate the coherence as

$$K_{YX,t}^2 = \frac{1}{\left\{1 + f_{VV}(\omega)_t \Big/ \left(\left|A(\omega)\right|_t f_{XX}(\omega)_t\right)\right\}} \quad (2.5)$$

The coherence is nothing else than the R^2 of the time domain. The coherence measures, for each frequency, the degree of fit between X and Y: or the R^2 between each of the corresponding cyclical components in X and Y.

In this paper, we are concerned about structural changes of the coherence. The question is, in which cyclical components do structural breaks appear? We define structural changes as changes that occur as a result of changes in the underlying relationship of two variables. That is to say: changes in the lag structure or the regression coefficients. In order to identify those changes, we reformulate the coherence. Solving (2.4) for $f_{VV}(\omega)$ and substituting it into (2.5) yields:

$$
\begin{aligned}
K^2_{XY,t} &= \frac{1}{\left\{1+\left(f_{YY}(\omega)_t - \left|A(\omega)\right|_t f_{XX}(\omega)_t\right)\Big/\left(\left|A(\omega)\right|_t f_{XX}(\omega)_t\right)\right\}} \\
&= \frac{1}{\left\{1+\dfrac{f_{YY}(\omega)_t}{\left|A(\omega)\right|_t f_{XX}(\omega)_t} - \dfrac{\left|A(\omega)\right|_t f_{XX}(\omega)_t}{\left|A(\omega)\right|_t f_{XX}(\omega)_t}\right\}} \\
&= \frac{1}{\dfrac{f_{YY}(\omega)_t}{\left|A(\omega)\right|_t f_{XX}(\omega)_t}} \\
&= \left|A(\omega)\right|_t \frac{f_{XX}(\omega)_t}{f_{YY}(\omega)_t}
\end{aligned}
\tag{2.6}
$$

Finally, defining

$$\frac{f_{XX}(\omega)_t}{f_{YY}(\omega)_t} = f_{DD}(\omega)_t \tag{2.7}$$

we get

$$K^2_{YX,t} \equiv \left|A(\omega)\right|_t f_{DD}(\omega)_t \tag{2.8}$$

The last equation (2.8) allows us to analyse structural changes in the coherence between X and Y. We define

$\Delta K^2_{XY,t} = K^2_{XY,t} - K^2_{XY,t-1}$, $\Delta\left|A(\omega)\right|^2_t = \left|A(\omega)\right|^2_t - \left|A(\omega)\right|^2_{t-1}$, and $\Delta f_{DD}(\omega)_t = f_{DD}(\omega)_t - f_{DD}(\omega)_{t-1}$.

Given (2.8) we can write the changes in the coherence as:

$$\Delta K^2_{XY,t} = \Delta\left|A(\omega)\right|_t \Delta f_{DD}(\omega)_t \tag{2.9}$$

In this paper, we allow all the terms in (2.9) to change if they need to. To get to that point, we need an empirical representation of (2.9). We use an approach which we applied in Hughes Hallett and Richter (2002; 2003a; 2003b; 2004). It consists of the following idea: in the first step, we estimate an AR-model for each variable separately. This model can be converted into a time-varying spectrum of the individual variable, since the AR model is estimated using the Kalman filter. Appendix 2 shows how. In a second step, we estimate (again using a Kalman filter) the relationship between two variables. The time-varying coefficients then give us the time-varying gain, as defined above. Thirdly, we calculate the time-varying coherence using equations (2.4) - (2.9).

3 Single Spectra

3.1 Data

In this section we study the spectra, and elements of the cross-spectra, of output growth in 4 countries over the past 25 years: namely in the US, UK, Eurozone, and Germany. We use quarterly, seasonally adjusted data for real GDP in all four countries. The data comes from real GDP levels, as published in the *OECD Economic Indicators.* For all countries, we took the log and differenced it once in order to generate growth rates. The resulting series was then fitted to an AR-model as described below, and then tested for stationarity and statistical significance. Our sample starts in 1980Q1 and finishes in 2003Q1.

3.2 USA

We first estimated the US growth rate as an time varying AR(9) process (Table 1). From this AR(9) we calculated the spectrum as described above. The estimation results suggest an AR(9) model, which is stationary with non-autocorrelated residuals. Among all alternatives, this model has the lowest AIC (Akaike) value which justifies our use of a lag length of 9. We obtained the following spectrum estimate:

- Figure 1 about here –

From the above figure it can be seen that the US growth rate is characterised by two peaks representing the longer term cycles: a frequency of $\omega = 0.1$ implies a cycle length of 62 quarters; a frequency of $\omega = 0.5$, a period = 12 quarters. This suggests a strong persistence in the shocks. These cycles, however, lose their importance towards the end of the sample.

Shorter cycles ($\omega = 2$, with a period = 3.1 quarters; $\omega = 3$, with a period = 2 quarters) are also important, but their weight is always 5 to 10 times less than that of the longer cycles. This result is also in line with Levy and Dezhbakhsh (2003). The following figure therefore shows the frequencies of 0.5 and 0.1 and their development over time:

- Figure 2 about here –

From figure 2 we can see how the importance of these frequencies have changed over time. Basically, their relative importance has halved in the 20 years between 1982 and 2002[7]. That brings the advantage of our approach into a clear perspective: a time-invariant estimation of the spectrum would have been highly misleading, since it would have implied on constant (average) spectrum for this data. And, given the large number of structural breaks (at least 4 in this case), each sub-sample would have been too small to create a reliable estimate of the spectrum in each sub-period. Moreover, our approach clearly shows the importance of changing coefficients: in the underlying dynamic process. They lead to changing cyclical behaviour in the growth rate. This point has not been investigated before. The previous literature only allowed a change in the conditional mean of the process. But here we see how the cycle itself is changing over time. We therefore have the same result as Stock and Watson (2002), but with the crucial difference that we can attribute the smaller variance to a smaller long term cycle. That result may seem in counterintuitive. But it is not: as Stock and Watson (2002) point out, the period between 1984 to 2002 was characterised by market liberalisation. Higher market flexibility in turn implies a higher importance of short cycles. What is more important, we can also see how this result occurred. In this case, it is because the importance the long cycles decreased. This result is significant because the opposite could equally well have been possible. Long cycles could have remained constant while short cycles gain importance. That would result in a higher overall variance. But reducing the long cycles, as here, implies a smaller total variance, which is precisely what Blanchard and Simon (2001), Chauvet and Potter (2001), and Stock and Watson (2002) find, but cannot explain.

Moreover this figure also highlights the importance of being able to pick out the structural breaks. Most notably, we can identify the main structural break in 1989Q4 where the US business cycle changes its shape. However, there are also lesser structural breaks in 1984Q1, 1993Q2, and 2001Q1. Taken together, each of these break points represents a recession and a recovery period for the US economy.

The aim of this paper is to find out whether the UK business cycle has changed in the same way, or whether it has become more like the Eurozone spectrum which does not have that feature (as we will show below). Before we can answer that question, we have to present the UK, Eurozone, and German spectra. These spectra will show how the individual cycles

[7] This result is also confirmed by Stock and Watson (2002).

have changed in those economies. Then we can examine the coherences from their cross-spectra in order to judge whether any of these cycles have been converging.

3.3 UK

Table 2 presents the regression results of the British growth rate. We estimated the British growth rate as an AR(8) model. This is sufficient since the Box-Pierce test shows that the residuals are not correlated with each other and the AIC statistic reaches its minimum at that point. The model therefore suggests a stationary AR(8) process. The spectrum for Britain is shown in figure 3:

- Figure 3 about here –

Again the British growth rate spectrum shows two peaks. One is for long cycles at a frequency of 0.1, or 62 quarters. This represents the long-term trend. However, there is another peak at a frequency of 1.9, or a cyclical length of 3.3 periods. When compared to the US, these two frequency representations appear to be rather similar. However, the UK's long cycle is a little weaker, and the short cycles a little stronger than those in the US. Against that, the UK's short cycles diminished in importance in the 1990s, whereas there were near zero in the US but increasing. Then there is a clear structural break in the UK's spectrum in 1989Q4-1992Q4 (the period of the UK's membership of the EMS) which, although present, was not so marked in the US case. These results are consistent with the results obtained Levy and Dezhbakhsh (2003).

In order to identify regime changes, we again look at the cyclical behaviour at frequencies of 0.1 and 0.8 (figure 4). As in the case of the US, the UK's long cycles lose power over time.

- Figure 4 about here –

However, the pattern is not as clear as before. In the first part of the sample, the spectrum is very volatile. It settles down in the end of the 80s and in the EMS period, only to become more volatile again in the 1990s – even if less volatile than that at the beginning of the sample. Therefore, although we can observe a similar pattern to the changes of the US, there are some individual characteristics. We review the significance of those variations below. And it is also clear that, in contrast to the US, the shorter cycle (ω = 0.8) has very much less weight. For the UK, the important cycles will be the long ones (10 – 15 years).

3.4 The Eurozone

For the Eurozone growth rate, we estimated an AR(4) model. The statistics given in Table 3 suggest non-autocorrelated residuals. Furthermore, the heteroscedasticity tests of the residuals suggest no heteroscedastic behaviour. Among other AR specifications, this model had the lowest AIC value. The spectrum that results from this regression is then shown in the following figure.

- Figure 5 about here –

This spectrum shows, once again, that long cycles are more important than short cycles. At least that was true in the 1990s. But the degree of long cycle dominance was a good deal less in the 1980s and a good deal less than that seen in the UK or US at any time in this period. Nevertheless, growth in the Eurozone shows a smooth long cycle post-1992, with some short-term instability in the 1989-94 period. Comparing our result with Levy and Dezhbakhsh (2003), it looks like this result is mainly driven by the French and German spectra (given that the UK is not included in this sample). For this reason we will examine the German spectrum separately in the next section.

Figure 6 shows the changes in the Eurozone long cycles in more detail. It suggests that the Eurozone has operated under three different regimes since 1981. All three apply to the long cycle/trend position of the economy. The first regime goes up to 1989Q1. Volatility in this regime is relatively low.

- Figure 6 about here –

The power of the long cycle increases, but with it the degree of volatility. This regime lasts until 1994Q2. After that the trend lost in importance, but growth itself becomes very stable and rather low compared to the OECD average. It is this stability (towards the end of the sample) that distinguishes the Eurozone growth path from the UK. However, in the Euro case the low frequencies, 0.1 to 0.3, are the most important, while in the US the most important frequencies are those in the range 0.1 to 0.5. Nevertheless, and in contrast to the UK, the US frequency becomes relatively stable towards the end of the sample. In that regard, the US and the Eurozone are more similar. But in all three cases, the trend loses its importance towards the end of the sample.

3.5 The German Spectrum

In order to see if the similarities between the British growth spectrum and that of the Eurozone, such as they are, are in any way carried over into the spectrum of the Eurozone's largest member we also estimated a spectrum for Germany's output growth based on an AR(8) model with dummy variables for outliers. The results are given in Table 4 in appendix A. The residuals are white noise, and the heteroscedasticity tests do not indicate outliers. Over time, the spectrum develops as follows:

- Figure 7 about here –

At the beginning of the sample, the German spectrum does not show a particular peak, although different frequencies do have a different impact on the business cycle. One should not be misled however: the variance is only 0.07. Therefore an increase of the spectrum from 0.01 to 0.015 is a 50% increase. Then, after reunification in 1991Q3, the German spectrum shows several peaks: 0.1, 0.8, 1.6, and 2.4, where – disturbingly - the highest frequency is the most important one (at least at the beginning of the sample). This shape is quite different to all the other countries. Interestingly, also, the trend is not the most important cycle in Germany. However, the regression results are very volatile with respect to the estimated coefficients. That explains large changes in the underlying frequencies. A look at the most important frequency ($\omega = 2.4$) will highlight this point. We compare that to the most important long cycle ($\omega = 0.8$).

- Figure 8 about here –

The importance of the high frequency component is fairly constant at the start of the sample, where it clearly dominates the long cycle to a certain degree. This regime however is interrupted in 1991Q3 when both cycles experience a huge jump in importance. Since this is happening 1 year after reunification, this jump reflects the increase in volatility which the German economy suffered at that time. However, it evidently affected short run volatility more than it affected long run volatility. Then, although there is another small increase after 1991, the importance of this frequency declines after 1991. And at the end of the sample, the power of these frequencies was most of the way back to its starting point. This suggests that it took the German economy 12 years to "digest" reunification.

Whatever the explanation, the German spectrum is very different from its counterparts. Shorter cycles (2.4 quarters = half a year) are more important in Germany than for all other countries. Although, longer cycles do gain a little in weight, the even spread of power over all frequencies is the distinguishing feature of the German spectrum.

Since the individual data generating processes are quite different, it is an important issue to determine whether there has been any genuine convergence in between them. The following section will show the results.

4 Cross-Spectral Analysis

4.1 The Coherence between the British Growth Rate and the US Growth Rate

In this section, we turn our attention to the coherence between the British and US growth rates at different frequencies and cycle lengths. We are specifically interested in determing which British cycles are coincident with, and might therefore be caused by changes in US growth rates. The results, using new estimates of the time varying distributed lag model

$$V(L)_t Y_t = A(L)_t X_t + \varepsilon_t \tag{3.1}$$

are given in Table 5 of appendix 1, where Y = UK growth rate and X = the US growth rate. Then we apply (A.4) and (A.9) to get the gain and coherence. The final result is shown in figure 9. Across the frequency range there are two clear peaks: one at a frequency of 0.1 to 0.3, and a lesser one at 2.1 to 2.5. There appears to be some kind of relationship in between those two peaks as well, at $\omega = 0.9$. Nevertheless, the relationship between Britain and the US is strongest at the long run trend.

- Figure 9 about here –

Up to 34% of the UK cycle can be "explained" by that common trend. And apart from occasional peaks in 1992 and 2002, that relationship has been growing in strength through the 1990s. However, it has been far from stable as the following figure shows:

- Figure 10 about here –

Apart from the very strong peak in 1983Q1 (and again in 1994Q3 and 2003Q1), the 1980s were characterised by a relatively low impact of the US growth rate on the UK growth rate. This period also exhibits a large volatility of about 5%. This period ends in 1994Q3. After that the coherence reaches a higher value of 25% and then successively fluctuates between 18% and 30%. Finally, the coherence reaches another high, at 28%, in 2002Q3.

As a result, the UK growth rate does depend on the US growth rate. But over time this link has decreased in importance by about 10% along the line of peaks; but it has increased by about 30% (ie from 0.1 to 0.13) at other frequencies. Does this relatively weak and rather volatile coherence therefore mean that the link between the UK growth rate and the EMU growth rate has become stronger or more stable? We investigate that possibility next.

4.2 The Coherence between the UK Growth Rate and the Eurozone

Table 6 shows the regression results for this case. In terms of (3.1), Y = the UK growth rate and X Eurozone growth. In this regression, the first, third and sixth lags of the UK growth rate are significant and are included. On the EMU side, the current growth rate and the second, fourth, and sixth lags. The residuals are not autocorrelated and the AIC criterion was the lowest one among other alternatives.

- Figure 11 about here –

In contrast to the previous section, the coherence between the Eurozone and UK growth rate shows several peaks. The strongest coherence appears to be at the following frequencies: 0.1, 0.6-0.7, and 2.1 to 2.9. Nevertheless the strongest link to the Eurozone is at a frequency of 0.1 after 1991, and at 0.6-0.7 throughout. Both represent long run trends – but that at 0.1 gains in importance during the 1990s. Nevertheless, as in the previous case, these links too are very volatile. The following figure highlights that volatility:

- Figure 12 about here –

It shows that the coherence is relatively low in the beginning of the sample. It then jumps up to 68% in 1990Q3 and stays at around that level up to 1993Q4 when it falls to 40%. These events highlight the impact of the ERM crisis and Britain was certainly hit by those events and the introduction of inflation targeting afterwards. However, the coherence fluctuates at around 50% until the end of the sample. Over time therefore, the EMU business cycle gained some weight. But its impact is far from being constant; and is now far lower than it was when Britain was a member of the ERM system.

Notice also that the 0.6-0.7 frequency band shows no such tendency to increase and then decrease in power, while those in the 2.1–2.9 range are gradually decreasing. This suggests that, while there has been an increase and then a decrease in the coherence between the UK and Eurozone cycles at the long end of the spectrum – with the result that this coherence is now only slightly above where it was in the 1980s, there is no such increase at shorter cycles. In fact, at a frequency of 0.6 (corresponding to cycle lengths of 11 quarters or 3 years) there has been no change at all. And at frequencies of 2.1-2.9 (cycle lengths of less than one year), this coherence has been falling.

Second, it is important to note that while the strength of the coherence between the British and Eurozone growth cycles appears to have become stronger than the corresponding coherences between the UK and US growth patterns, this happens only at specific frequencies and only in certain time periods. For example, the UK-EU coherence coefficient measures 0.4- 0.5 at a frequency of 0.1 or 0.6 in the 1990s. But it measures only 0.1 or 0.2 at any other frequency or at any other time (the frequency band 2.1-2.9 apart). That contrasts with a coherence value of 0.1 or 0.2 at a frequency of 0.1, in the US-UK case. These results therefore imply that there may have been a general increase in coherence between all three countries (the US, UK, and the Eurozone) at the long end of the spectrum, but a *decrease* at the short cycle to business cycle end. In other words, what we are picking up here is the integration effect of increasing globalisation; and that, once we have stripped that out, there is rather little (even a declining level of) coherence left at the business cycle end of the spectrum. But to confirm that hypothesis, we need to look at the US-EU coherence diagram more closely.

4.3 The Coherence between the Eurozone and US Growth Rates

In this section, we investigate the relationship between the US growth rate and the EMU growth rate, i.e we analyse the dependency of the EMU growth rate from the US rate.

The final result of the Kalman filter is shown in Table 7. The EMU growth rate depends on its first lag and the third and ninth US lag. All coefficients are significant. The residuals are not autocorrelated. The following figure shows the resulting coherence:

- Figure 13 about here –

The strongest link between the EMU and the US is, as hypothesised, at to the long run trend (frequency of 0.1). However, the short term also matters: frequnecies of 2.1 and 2.9 respectively. In the beginning of the sample the highest long run coherence is about 53% in 1984Q1: higher than that between th EU and UK or between the US and UK. But towards the end of the sample it decreases to 30%: lower than that between the UK and EU, but similar to that between US and UK. This pattern is also true for the other two frequencies.

There is one frequency that gains a little weight over the sample: $\omega = 0.8$. However, at the end of the sample this frequency starts to lose weight as well. As a result, the impact of the US growth rate on the Eurozone is probably decreasing, but to a different extent at different frequencies. Nevertheless, there is a link even if a rather volatile one. The key point perhaps is that the general coherence level for the UK-EU cycles is 0.2-0.3, while it is around 0.2 for the US-EU cycles and 0.1 for the UK-US case. Only when we get to the peak coherences for the long cycles/trends, representing the effects of globalisation, do these figures rise to around 0.4-0.5, 0.3 and 0.15 respectively. Since the UK pairs are very little different from the US-EU pair, these results suggest that:

a) the Eurozone and the US may actually be at least as similar to each other as either is to the UK (or, at least; no more dissimilar) and

b) the globalisation effects dominate, and it is here that the UK starts to differ from either the US or the EU. And since the common monetary policy will be directed at short run/business cycle developments, where Britain is relatively different to the others, this might cause problems.

c) At the long cycle end, where she is more similar to her EU partners, Britain has greater freedom of action through fiscal policy and structural reform.

It therefore appears, from the British perspective, that EMU could well bring common policies where they are not needed, but no such policies where they would be appropriate.

- Figure 14 about here –

4.4 *The Coherence between the German Growth Rate and the Eurozone Growth Rates*

In this section we estimate how much the German growth rate is depending on the EMU growth rate. Figure 15 shows that the German growth rate is closely allied with (or depends on) Eurozone growth at all frequencies. In fact, in this model Germany leads (see Table 7). This dependence is slightly stronger for the longer cycles (ω = 0.1 to 1.5) than for the short cycles, but the difference is marginal. The trend/long cycle components in Germany account for 50%-60% of the corresponding EU cycles, but that weakens after 1999 and in the shorter cycles. Moreover, up to 1993, the coherences are remarkably uniform across frequencies and time periods, but vary more across frequency bands (but not over time) thereafter.

- Figure 15 about here –

- Figure 16 about here –

In summary, these results show a coherence pattern that is *quite* different to the UK. Moreover the coherence level is much higher than that for the UK or US with the Eurozone, and at all frequencies. Hence, to the extent that Germany is representative of the Eurozone, to ask if the UK economy is more coherent or better correlated with the Eurozone or the US, is probably to ask the wrong question. The important question will be how well the UK fits in with the Eurozone countries. It may be that the UK fits passably well with the Eurozone average, but not with any of the countries which actually influence policy or for whom policy is designed – assuring of course some differentiation between the national policy mixes (the monetary policies themselves will, but for transmission differences, all be the same). That appears to be the case with respect to Germany at least.

5 Conclusion

This paper has done four things. First we have presented a technique by which business cycles can be decomposed into their component cycles and compared; and we have shown how that can be done such that the cycles themselves, and their relative importance, can vary over time. As a result, we have shown how individual data generating processes have varied. Therefore one neoclassical assumption for common growth patterns is not fulfilled. Second we have shown how to extend this univariate analysis in order to determine the coherence between the components cycles in different economies, and to allow that coherence measure to vary over time.

Third we have shown how to apply these methods to answer the question, is there an emerging European business cycle, and how well have existing and candidate countries converged at different cycles and different periods of time? As expected there is some communality between the four economies studied, but only at certain frequencies. There has also been considerable change in the degree of that similarity between cycles, but that too is different in different places. The US and UK have more in common in the long run cycles, as do the US and Eurozone to a much smaller extent. But the UK-Eurozone association is mostly at shorter frequencies and appears to be fairly unstable. In addition, Germany is now quite different from the UK and US at most frequencies, and also from the Euro average. That would make it difficult the Eurozone has to establish and implement one set of policies, for all, for both short-term stabilisation and longer term growth.

For the UK herself, the results suggest that she may have started to converge on the Eurozone at some frequencies, but it is unclear whether she has converged across the board. Euro policies could therefore be adequate for short run purposes. But American style policies would be needed in the long term. Secondly cyclical convergence, to the extent it exists, may fall short of what currency union requires because the UK's coherence with the Eurozone is still a lot weaker than that of her partners. This would pose problems if EU policy does not react to the variability in national conditions, or if it fails to adapt to those who are outliers.

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Appendix 1: Statistical Results

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLUSGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	84	Degrees of Freedom	79
Centered R^2	0.280419	R Bar^2	0.243984
Uncentered R^2	0.733540	T * R^2	61.617
Mean of Dependent Variable	0.0078742505	Std Error of Dependent Variable	0.0060746196
Standard Error of Estimate	0.0052818316	Sum of Squared Residuals	0.0022039219
Akaike Information Criterion:	0.013	Ljung-Box Test: Q*(9) =	18.1554
Variable	Coeff	Std Error	T-Stat
Constant	0.002094651	0.001842617	1.13678
DLUSGDP	0.317320029	0.093212957	3.40425
DLUSGDP{2}	0.261475759	0.089633099	2.91718
DLUSGDP{5}	-0.183476687	0.080909547	-2.26768
DLUSGDP{9}	0.158385410	0.066891841	2.36778

Table 1: AR(9) Model of the US-Growth Rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLUKGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	84	Degrees of Freedom	80
Centered R^2	0.181512	R Bar^2	0.150819
Uncentered R^2	0.650454	T * R^2	54.638
Mean of Dependent Variable	0.0065511928	Std Error of Dependent Variable	0.0056900162
Standard Error of Estimate	0.0052434073	Sum of Squared Residuals	0.0021994656
Akaike Information Criterion:	0.0034	Ljung-Box Test: Q*(9) =	20.1425
Variable	Coeff	Std Error	T-Stat
Constant	0.003200857	0.002849429	1.12333
DLUKGDP{1}	0.259626752	0.142599397	1.82067
DLUKGDP{3}	0.205911012	0.112975619	1.82261
DLUKGDP{8}	-0.163455252	0.097871277	-1.67010

Table 2: AR(8) Model of the British Growth Rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLEMUGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	84	Degrees of Freedom	80
Centered R^2	0.115800	R Bar^2	0.082643
Uncentered R^2	0.614569	T * R^2	51.624
Mean of Dependent Variable	0.0054066836	Std Error of Dependent Variable	0.0047814030
Standard Error of Estimate	0.0045795691	Sum of Squared Residuals	0.0016777962
Akaike Information Criterion:	0.0021	Ljung-Box Test: Q*(9) =	9.1900
Variable	Coeff	Std Error	T-Stat
Constant	0.0024725489	0.0009518128	2.59773
DLEMUGDP{1}	0.2538116923	0.0804268193	3.15581
DLEMUGDP{3}	0.1256808757	0.0753019714	1.66903
DLEMUGDP{4}	0.1035850687	0.0753650637	1.37444

Table 3: AR(4) model for the EMU growth rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLDEGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	84	Degrees of Freedom	78
Centered R^2	0.538616	R Bar^2	0.509040
Uncentered R^2	0.620220	T * R^2	52.099
Mean of Dependent Variable	0.0047084286	Std Error of Dependent Variable	0.0102184548
Standard Error of Estimate	0.0071599224	Sum of Squared Residuals	0.0039986302
Akaike Information Criterion:	0.059	Ljung-Box Test: Q*(9) =	15.5264
Variable	Coeff	Std Error	T-Stat
Constant	0.004354976	0.001338928	3.25259
DLDEGDP{1}	-0.077886003	0.012494944	-6.01957
DLDEGDP{2}	-0.016310333	0.010626309	-1.53490
DLDEGDP{8}	0.112140346	0.062297315	1.80008
DUM1	0.017173279	0.002783186	6.17037
DUM2	0.011415027	0.002905507	3.92876

Table 4: AR(8) Model of the German Growth Rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLUKGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	84	Degrees of Freedom	79
Centered R^2	0.211619	R Bar^2	0.171701
Uncentered R^2	0.663312	T * R^2	55.718
Mean of Dependent Variable	0.0065511928	Std Error of Dependent Variable	0.0056900162
Standard Error of Estimate	0.0051785370	Sum of Squared Residuals	0.0021185624
Akaike Information Criterion:	0.97437	Ljung-Box Test: Q*(9) =	11.745640
Variable	Coeff	Std Error	T-Stat
Constant	0.0014411862	0.0027718438	0.2703376036
DLUSGDP	0.1312917493	0.0849136479	2.3906725924
DLUSGDP{3}	0.1459814634	0.0777120131	3.5287246801
DLUKGDP{1}	0.1830814576	0.0945088924	3.7527050961
DLUKGDP{3}	0.1368813449	0.0817911962	2.8007696025

Table 5: Kalman Filter Results for the British and the US Growth Rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLUKGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	86	Degrees of Freedom	78
Centered R^2	0.388282	R Bar^2	0.333384
Uncentered R^2	0.730057	T * R^2	62.785
Mean of Dependent Variable	0.0063954183	Std Error of Dependent Variable	0.0057170779
Standard Error of Estimate	0.0046677956	Sum of Squared Residuals	0.0016994886
Akaike Information Criterion:	1.27435	Ljung-Box Test: Q*(9) =	12.90781
Variable	Coeff	Std Error	T-Stat
Constant	0.004798806	0.001873759	2.56106
DLUKGDP{1}	0.198166012	0.113345282	1.74834
DLUKGDP{3}	0.199830075	0.094618389	2.11196
DLUKGDP{6}	0.209456749	0.106274026	1.97091
DLEMUGDP	0.215020717	0.105711310	2.03404
DLEMUGDP{2}	-0.283454458	0.123200795	-2.30075
DLEMUGDP{4}	-0.237641915	0.107687136	-2.20678
DLEMUGDP{6}	-0.227269649	0.114917724	-1.97767

Table 6: Kalman Filter Results between the UK and the EMU Growth rate

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLEMUGDP	Quarterly Data From	1981:04 To 2003:01
Usable Observations	86	Degrees of Freedom	82
Centered R^2	0.258098	R Bar^2	0.230955
Uncentered R^2	0.673010	T * R^2	57.879
Mean of Dependent Variable	0.0053297353	Std Error of Dependent Variable	0.0047592005
Standard Error of Estimate	0.0041735908	Sum of Squared Residuals	0.0014283466
Akaike Information Criterion:	1.15038	Ljung-Box Test: Q*(9) =	3.222994
Variable	Coeff	Std Error	T-Stat
Constant	0.0015821493	0.0006632325	2.38551
DLEMUGDP{1}	0.1886366732	0.1085764202	1.73736
DLUSGDP{3}	0.1557923328	0.0725317143	2.14792
DLUSGDP{9}	0.1773760091	0.0818392050	2.16737

Table 7: Kalman Filter Results between the EMU Growth Rate and the US Growth Rate

Appendix 2: Calculation of the Coherences and Gains

For the ease of exposition we start off with the time invariant case. The time varying case will then be straight forward. We need to calculate the sequence of coefficients $\{a_j\}$ from (2.1), in order to obtain the lag structure of the underlying economic model. In order to do this, we start from a general linear model of distributed lags, where we keep the lag structure constant in this appendix for ease of exposition:

$$V(L)Y_t = U(L)X_t + \varepsilon_t \tag{A.1}$$

where

$V(L) = \sum_{s=0}^{p} v_s L^s$, $v_0 = 1$, and $U(L) = \sum_{r=0}^{q} u_r L^r$ define the models coefficients[1].

Thus, as long as all eigenvalues of the characteristic equation of V(L) lie within the unit circle, as they will do if Y_t is stationary, we can write

$$Y_t = \frac{U(L)}{V(L)} X_t + \frac{1}{V(L)} \varepsilon_t \tag{A.2}$$

In order to derive the gain on variable X_t, we need to focus on the first ratio in eq. (A.2). Let

$$\frac{U(L)}{V(L)} = W(L) \tag{A.3}$$

where $W(L) = \sum_{j=0}^{k} w_j L^j$ is the weighting function from (A.3). The sequence $\{w_j;\ j = 0, 1, ..., k\}$ therefore defines the model's impact, on Y_t, which results from a change in the explanatory variables j periods ago. In particular, w_0 is the instantaneous reaction coefficient. In order to achieve a sensible economic interpretation, it is required that if $X_t = X_{t-1} = ... = X$, i.e. in equilibrium, then the dependant variable Y should also be constant. Hence, it is required that

$$\sum_{j=0}^{k} w_j = W(1) = \frac{U(1)}{V(1)} = w \tag{A.4}$$

where $0 < |w| < \infty$.

[1] p and q were determined according to their significance, using the Akaike criterion. Once p, q are determined, k follows directly from (A.2).

In order to calculate w_j in terms of u_r and v_s, we can make use of the following relationship (Hendry, 1995; Laven and Shi, 1993):

$$\left(\sum_{j=0}^{k} w_j L^j\right)\left(\sum_{s=0}^{p} v_s L^s\right) = \sum_{r=0}^{q} u_r L^r \tag{A.5}$$

Equating the powers of L on the two sides of (A.5), and noting that $v_0 = 1$, we get the following recursive equations:

$w_0 = u_0$;

$w_1 + w_0 v_1 = u_1$;

$w_2 + w_1 v_1 + w_0 v_2 = u_2 \ldots$

Solving for the unknown coefficients w_j, we have

$w_0 = u_0$;

$w_1 = u_1 - u_0 v_1$;

$w_2 = u_2 - \left(u_1 - u_0 v_1\right) v_1 - u_0 v_2$; and so on.

Given the lag structure in (A.1), we are now able to generate the gain according to:

$$a(\omega) = \left| \sqrt{\sum_{j=0}^{k} w_j z \sum_{j=0}^{k} w_j z^{-1}} \right| \tag{A.6}$$

where $z = e^{-i\omega}$. The coherence between Y_t and X_t, at each frequency and each t, now follows from (2.7) and (2.9)

We now have established of how to calculate the gains and coherences, given estimated coefficients. We now explain how to filter the observations to get the time varying elements into those calculations. We start off with the general form:

$$V(L)_t Y_t = U(L)_t X_t + \varepsilon_t \tag{A.7}$$

We use the Kalman filter to estimate (A.7) directly. In particular, we estimate the following state space model:

$$Y_t = D_t X_t + \varepsilon_{1,t} \tag{A.8}$$

where (A.8) is the measurement equation; and X_t is a matrix that contains the explanatory variables and their lags, as well as the lags of the dependent variable. Finally, we note

$$D_t = D_{t-1} + \varepsilon_{2,t} \tag{A.9}$$

with $\varepsilon_{a,t}$ ~ i.i.d. (0, $\sigma^2_{\varepsilon_a}$) for a = 1, 2, as the state equation. Using an estimate of (A.8) at each point in time as our value for W(L) at each t, we are now able to calculate the gain for each point in time, as

$$\left(a(\omega)\right)_t = \left|\sqrt{\sum_{j=0}^{k} w_j z \sum_{j=0}^{k} w_j z^{-1}}\right|_t \tag{A.10}$$

using the recursion formula above (A.6) to determine the w_j coefficients for that value of t.
The coherence estimates, for that value of t, now follows from (2.7) and (2.10).

Appendix 3: Figures

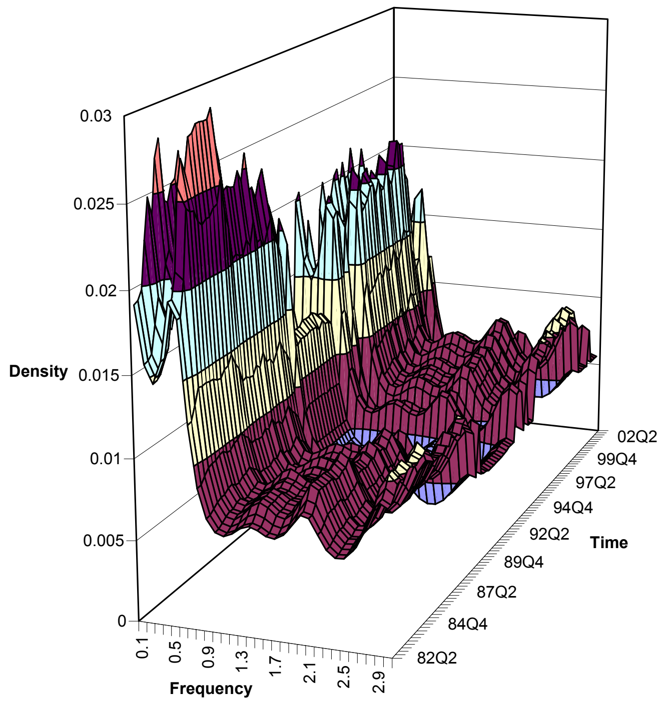

Figure 1: Spectrum of the US Growth Rate

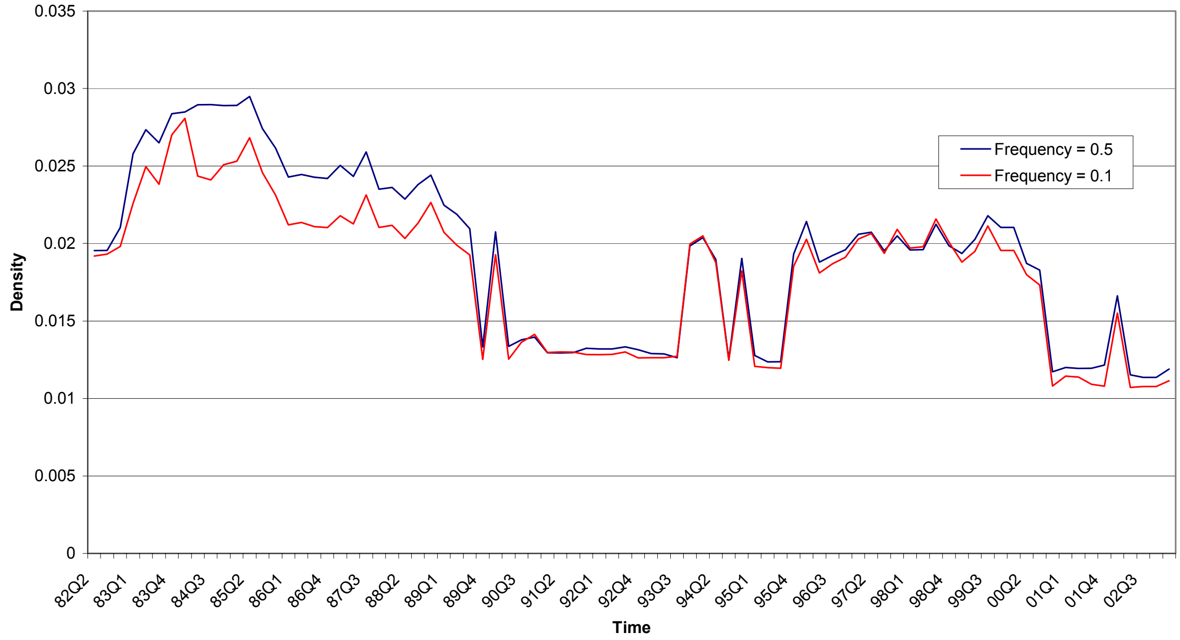

Figure 2: Selected US Frequencies over Time

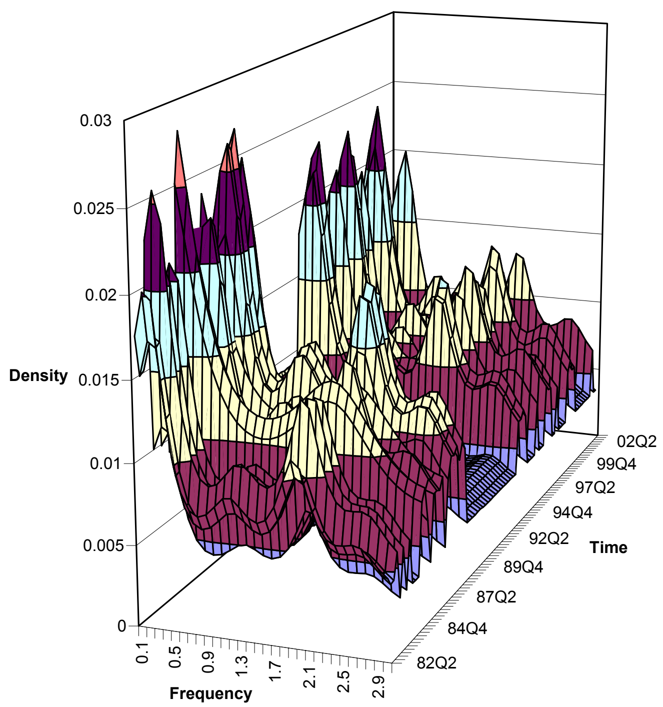

Figure 3: Spectrum of the British Growth Rate

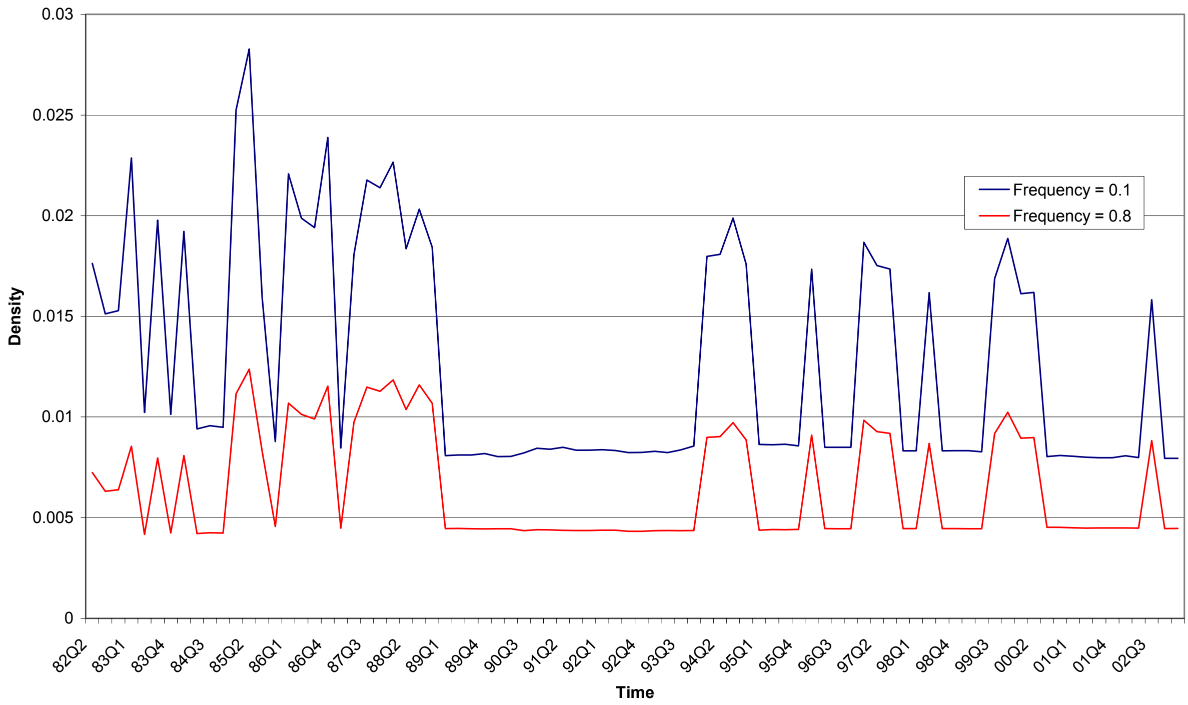

Figure 4: Selected UK Frequencies over Time

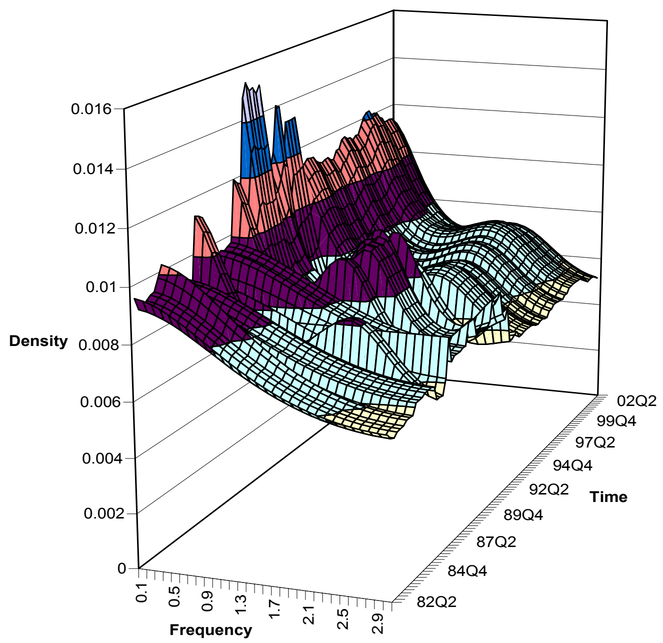

Figure 5: Spectrum of the EMU Growth Rate

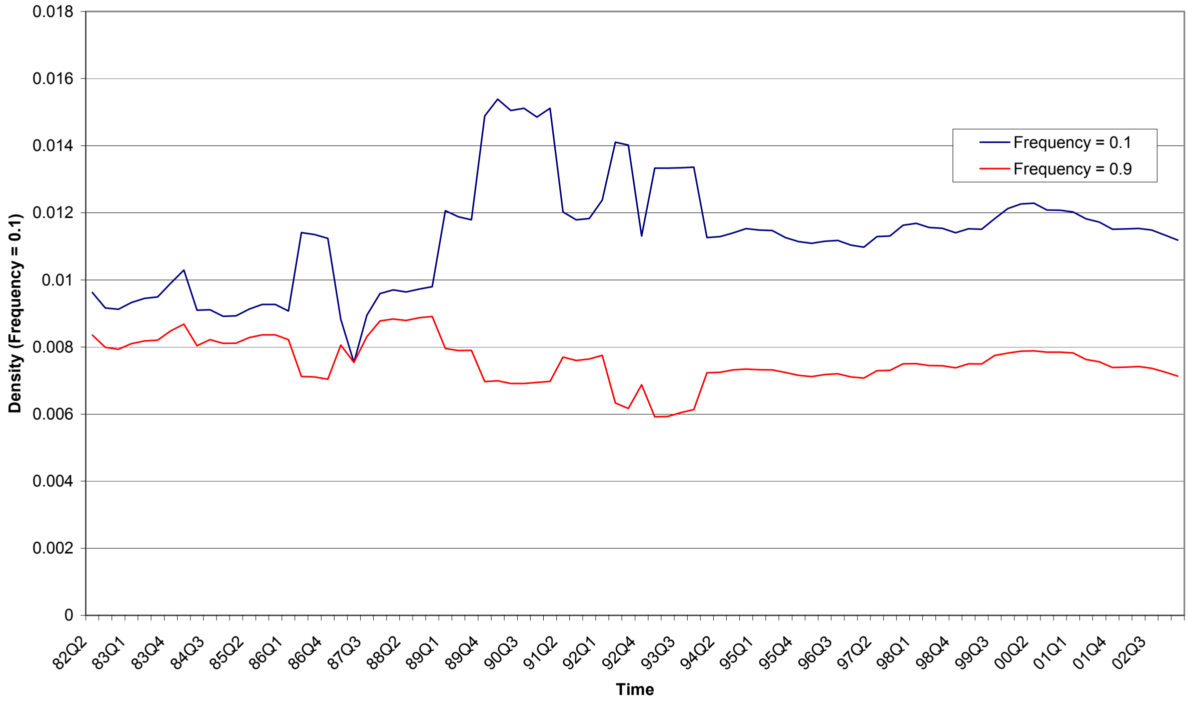

Figure 6: Selected EMU Frequencies over Time

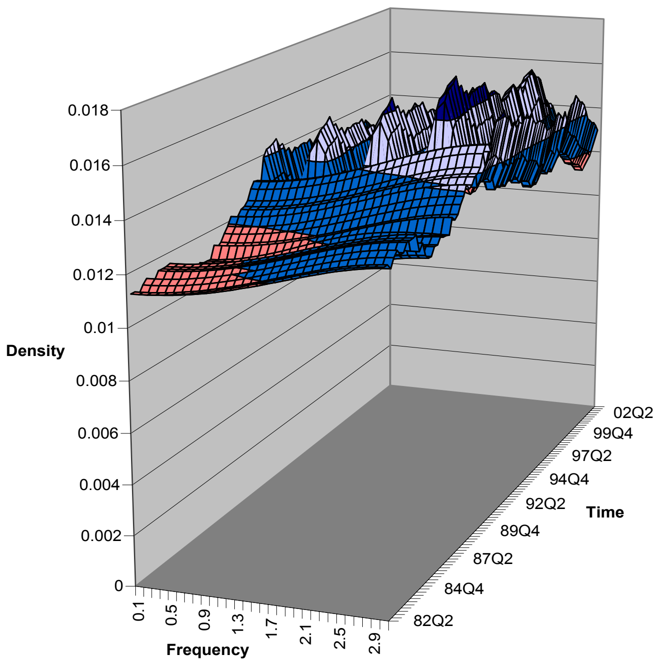

Figure 7: The Spectrum of the German Growth Rate

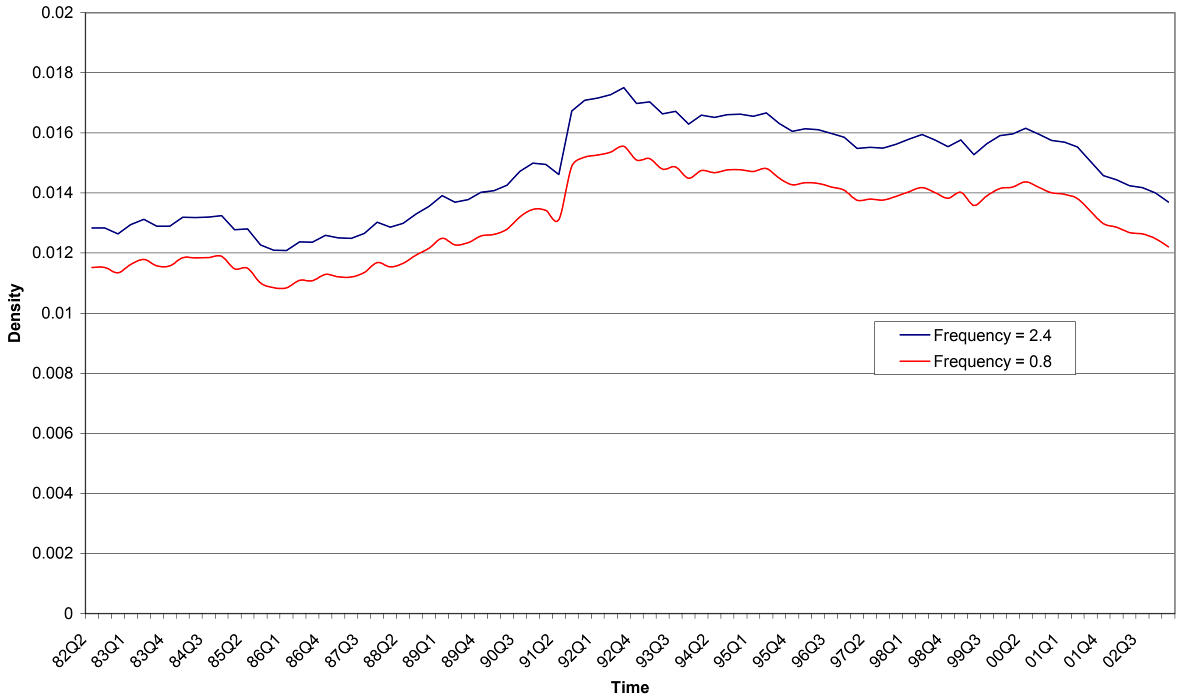

Figure 8: Selected German Frequencies over Time

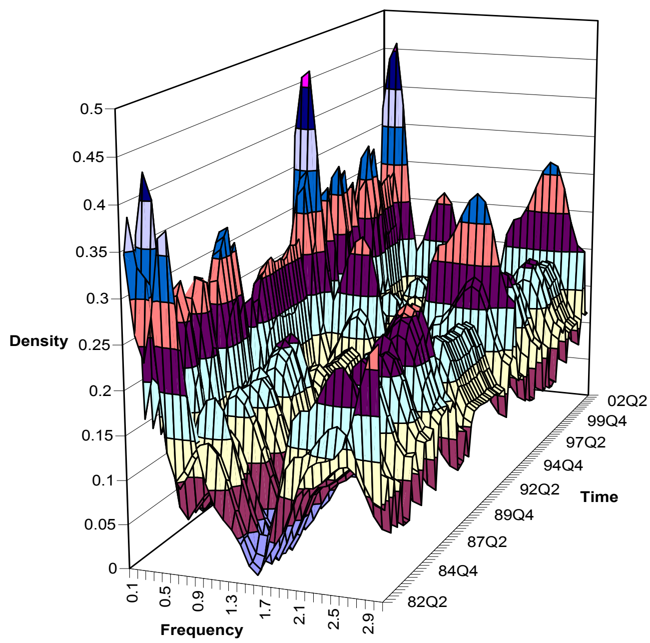

Figure 9: Coherence between the British and the US Growth Rate

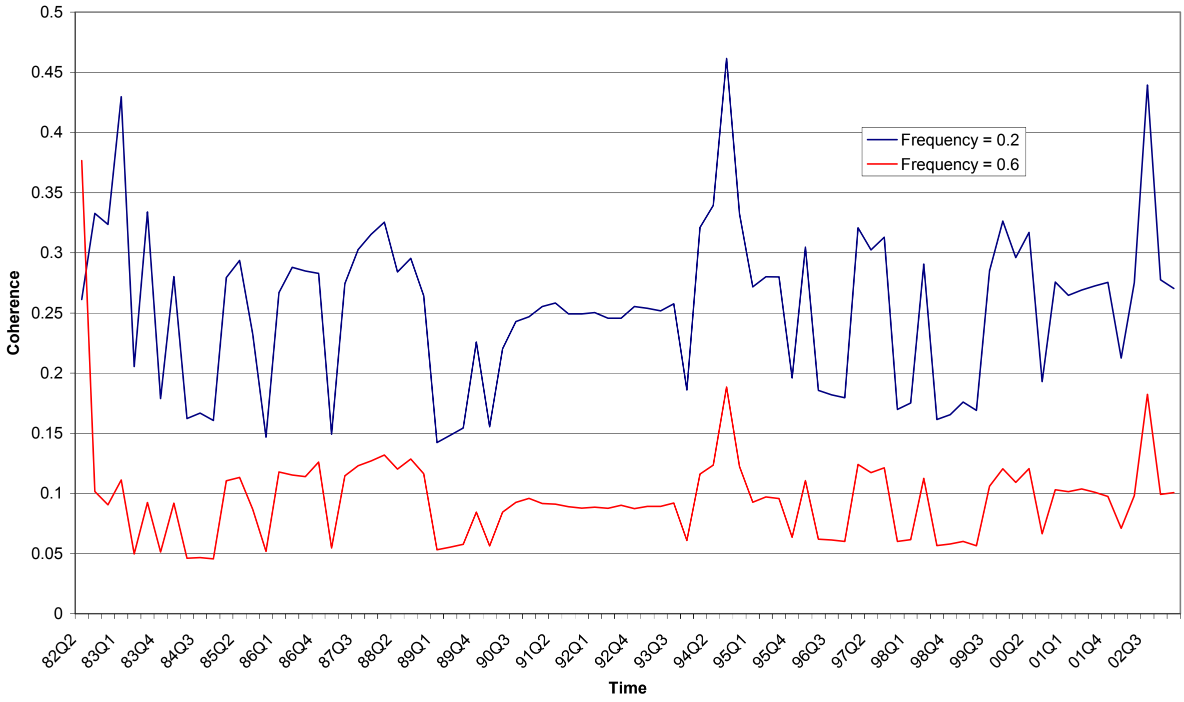

Figure 10: Selected Frequencies between the UK Growth Rate and the US Growth Rate

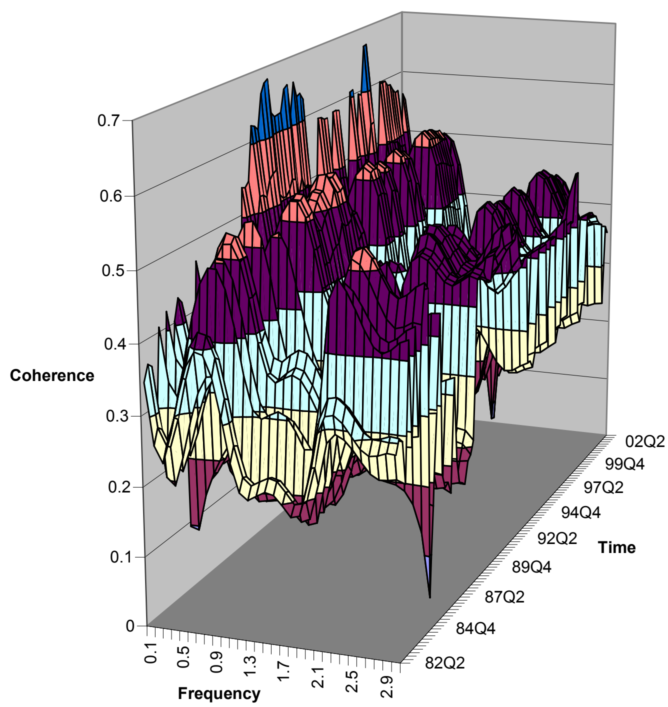

Figure 11: Coherence between the British Growth Rate and the EMU Growth Rate

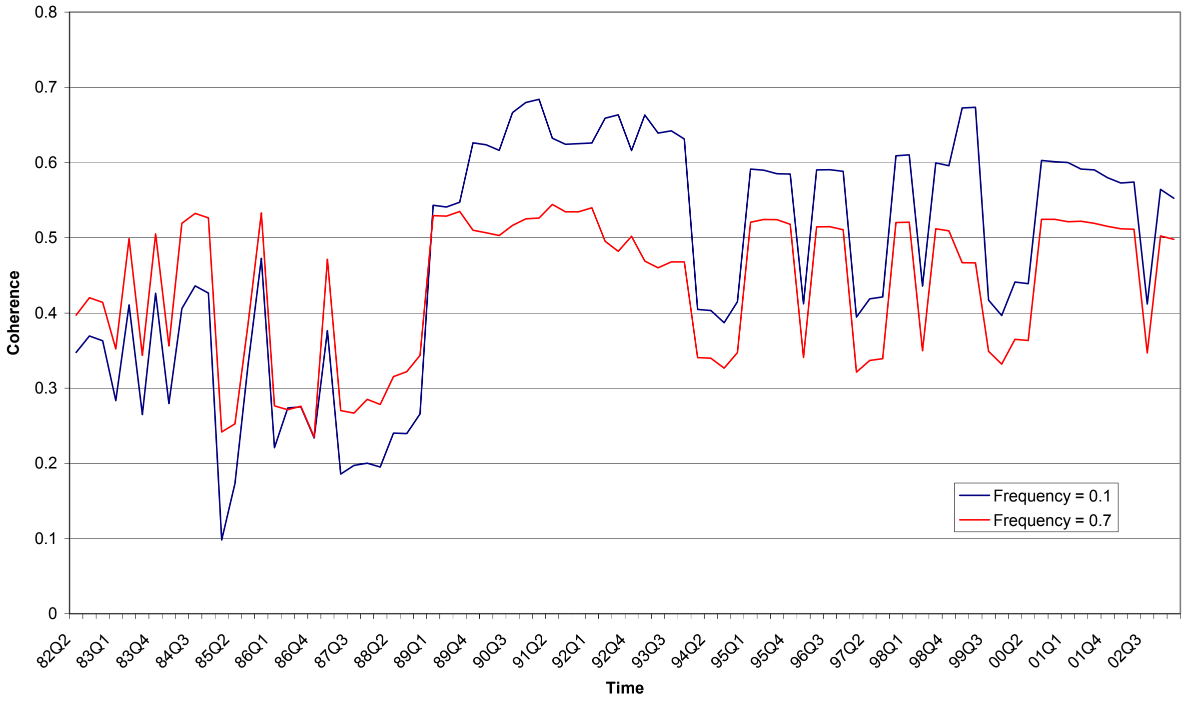

Figure 12: Selected Coherences between the UK Growth Rate and the EMU Growth Rate

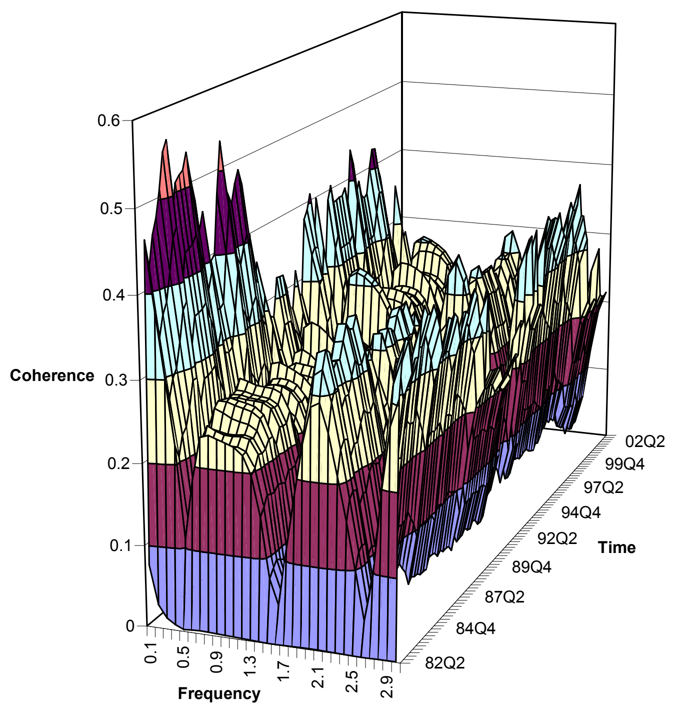

Figure 13: The Coherence between the EMU and US Growth Rate

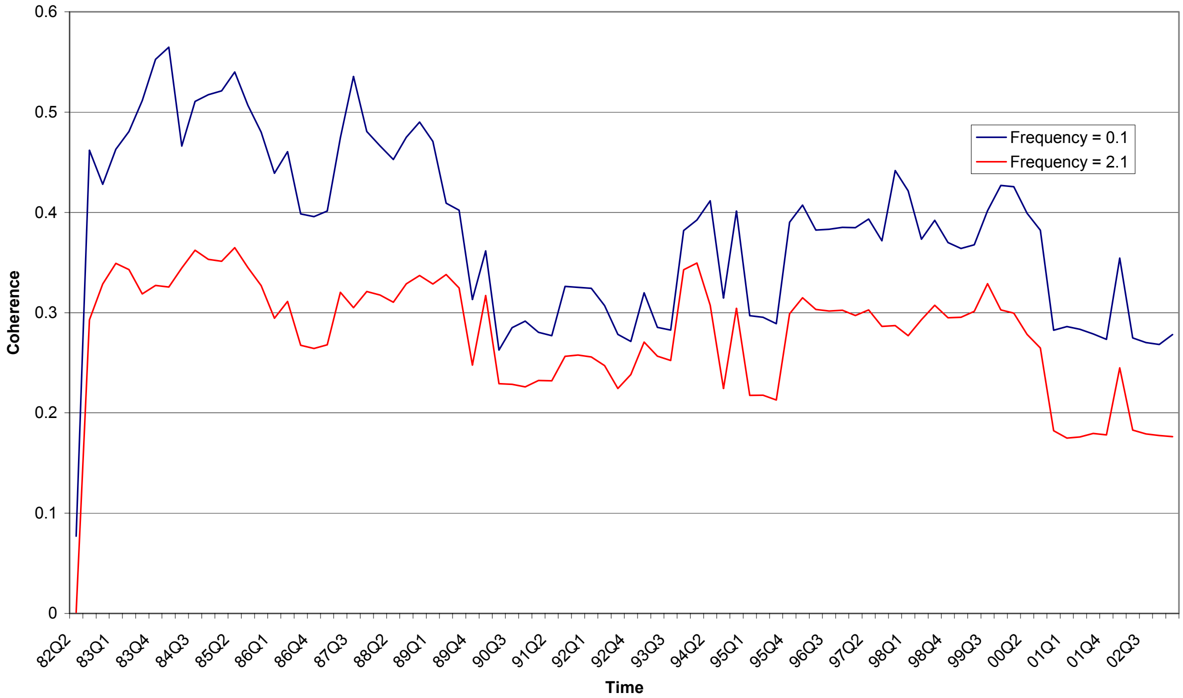

Figure 14: Selected Coherences between the EMU Growth Rate and the US Growth Rate

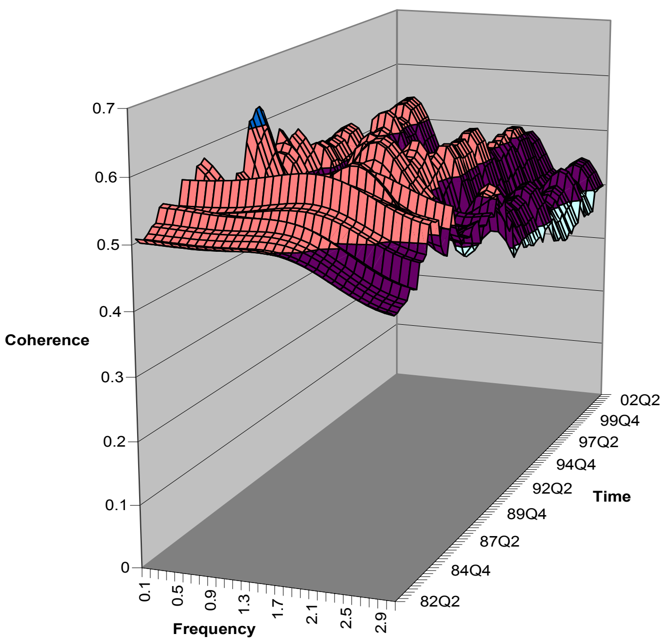

Figure 15: Coherence between the German and the EMU Growth Rate.

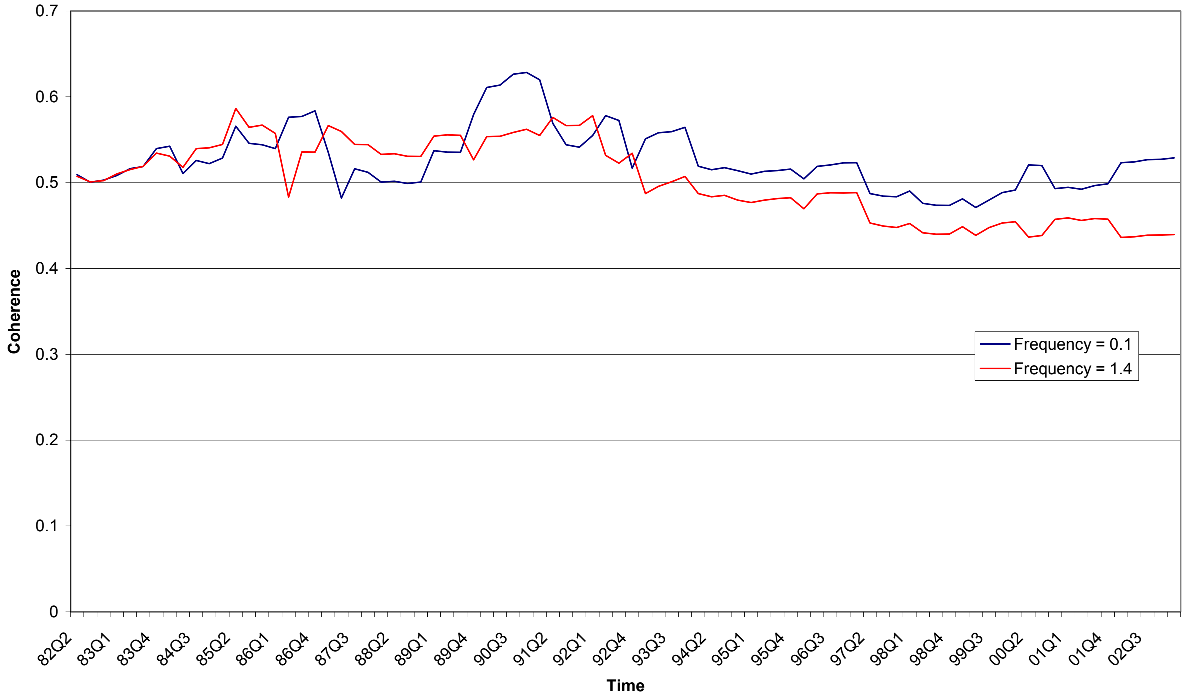

Figure 16: Selected Coherences between the German Growth Rate and the EMU Growth Rate