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Implementing Efficient Allocations in a Model of Financial Intermediation*

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ABSTRACT

In a finite-trader version of the Diamond-Dybvig (1983) model, the symmetric, ex-ante efficient allocation is implementable by a direct mechanism (i.e., each trader announces the type of his own ex-post preference) in which truthful revelation is the strictly dominant strategy for each trader. When the model is modified by formalizing the sequential-service constraint (cf. Wallace, 1988), the truth-telling equilibrium implements the symmetric, ex-ante efficient allocation with respect to iterated elimination of strictly dominated strategies.

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1. Introduction

This paper concerns the welfare analysis of maturity transformation in financial structure. Maturity transformation is the financing of an intermediary's assets by liabilities (demand deposits at a bank, in particular) that are callable, and that some traders do call in equilibrium, before the assets themselves mature. Bryant (1980) shows that such a portfolio structure is a means of insuring the depositors against unobservable risks, and he also identifies a multiplicity-of-equilibrium problem. He implicitly represents a bank as a rule or "allocation mechanism" that specifies the outcome, in each state of nature, of each possible profile of traders' decisions regarding whether or not to exercise the call options on their deposits. This rule constitutes a framework for strategic interaction among the traders. Bryant observes that maturity transformation is necessary in order to implement the symmetric, ex-ante efficient, allocation as a Bayesian Nash equilibrium. He shows also that some mechanisms that do implement that efficient allocation—notably the mechanism that most faithfully reflects the features of a bank-deposit contract in the context of his model—also can possess other equilibriums that are strictly Pareto dominated by the "intended" equilibrium.

Diamond and Dybvig (1983) address a related set of issues to Bryant. They study a model that brings the role of aggregate risk into sharp focus. They prove four main results.

1. The phenomenon of Pareto-ranked bank-deposit-contract equilibriums can occur even in an environment where there is no aggregate risk.

2. However there is an allocation mechanism, suggested by historical banking regimes that have permitted suspension of convertability of deposits when a "run" occurs, that implements the symmetric, ex-ante optimal allocation in strictly dominant strategies. This is intuitively a particularly compelling notion of implementation that implies, among other things, that the Bayesian Nash equilibrium is unique. Obviously, then, there cannot be multiple, Pareto-ranked equilibriums.

3. In some environments with aggrate risk, a deposit scheme with suspension of payments cannot implement an ex-ante efficient allocation.

4. However, despite the presence of aggregate risk, it is possible to implement the symmetric, ex-ante efficient allocation in Bayesian Nash equilibrium. If deposit insurance is feasible, then it can provide one means to do so. Diamond and Dybvig's analysis does not establish whether or not there is any allocation mechanism that implements the symmetric, efficient

allocation as its unique Bayesian Nash equilibrium. Wallace (1988) provides a formalization of the sequential-service constraint to which previous researchers had appealed informally. He proves the following result that bears on Diamond and Dybvig's fourth point.

5. If the provision of deposit insurance is genuinely regarded as a feature of the over-all allocation mechanism, and if it is this over-all mechanism to which the sequential-service constraint applies, then deposit insurance is not feasible to provide.

Taken together, the last three of these results raise the possibility that existence of multiple, Pareto-ranked equilibriums might be an unavoidable problem for any mechanism that implements the symmetric, ex-ante efficient allocation as a Bayesian Nash equilibrium in an environment with aggregate risk. Suppose that that were indeed the situation, and that one believed that traders' strategic interactions were much more likely to proceed according to the Pareto-dominated equilibrium than according to the efficient one. If there were another mechanism that had a unique, "mediocre," equilibrium that was situated strictly between the other two according to the Pareto relation, then one might be inclined to choose that mechanism and to tolerate the inefficiency of its equilibrium rather than to incur the substantial risk of doing even worse, in order to have any chance of attaining efficiency. To the contrary, if there were a mechanism possesing a unique equilibrium, and if the symmetric efficient allocation were the outcome that would result from that equilibrium being played, then one would reject without hesitation the mechanism with the mediocre equilibrium if one were convinced that the Bayesian Nash equilibriums of both mechanisms would actually be played.

This tension between efficiency of outcome allocations and stability in the sense of uniqueness of Bayesian Nash equilibrium (and of characterization of equilibrium in terms of strategic dominance) is the specific topic of this paper. We review some basic concepts of implementation theory in section 3, and in section 4 we use this implementation framework to present a version of the Diamond-Dybvig environment with aggregate risk. (The environment that we study differs from Diamond and Dybvig's in having only finitely many traders and, in section 4, not having a sequential-service constraint. We consider this finite-trader version both to introduce aggregate risk in a natural and explicit way, and also, in section 5, to clarify the formulation of the sequential-service constraint.) A naturally-defined mechanism makes it a dominant strategy for each trader to communicate his type truthfully, and via this dominant-strategy equilibrium it implements the symmetric, ex-ante efficient allocation. That is in sharp contrast to Diamond and Dybvig's deposit-with-suspension mechanism, since for our mechanism the distinction between environments with and without aggregate risk is immaterial. Finally, in section 5 we consider the analogous allocation mechanism in environments with aggregate risk and also a sequential-service constraint. We show that, under the assumption that traders' utility functions exhibit non-increasing absolute risk aversion, for traders truthfully to communicate their types remains the unique strategy profile that survives iterated elimination of strictly dominated strategies. Thus, again, the mechanism has a unique Bayesian Nash equilibrium that possesses an intuitively compelling stability property, and the outcome of that equilibrium being played is the symmetric, ex-ante efficient, equilibrium.[1]

2. An informal theory of implementing the efficient allocation

Consider a population of I agents each of whom is endowed with one unit of a (divisible) good. The good can be transformed into a consumption good available at either date 0 or date 1. For each unit of the good, the transformation leads to $R > 1$ units of consumption at date 1. But if the transformation process is terminated at date 0, only one unit of consumption good can be obtained. At the beginning of their lives, each agent is uncertain about how long they are going to live. With some probability, an agent will become "short-lived," in which case she values the date 0 consumption only. A "long-lived agent," on the other hand, cares about the sum of date 0 and date 1 consumption. The agents each learn about their individual lifetimes at the beginning of date 0. A state of nature specifies which agents are "short-lived" and which are "long-lived."

Given the transformation technology, a feasible consumption path for each agent is to consume her endowment if she turns out to be "short-lived," and to enjoy R units of consumption at date 1 if she is "long-lived." But, the population can do better by pooling their resources in order to insure against the event of becoming "short-lived." To see what the efficient insurance arrangement looks like, consider the optimization problem of a social planner whose objective is to maximize the sum of agents utilities. Here we consider ex post utility-maximization in each state of nature. Later in the paper, we will show that the solution achieves ex ante efficiency as well.

To simplify, suppose that the utility of a typical short-lived agent is a piecewise linear function of consumption (at date 0). (See Figure 1.) In particular, the marginal utility for consumption levels beyond a threshold point, c^*, is much smaller than the marginal utility for consumption levels below

[1] An antecedent result in this spirit is due to De Nicoló (1995), who constructs a mechanism having a unique Bayesian equilibrium and implementing an allocation arbitrarily close to being efficient in Wallace's (1988) sequential-service version of the Diamond-Dybvig model.

c^*. Since one unit of date 0 good can be transformed into $R > 1$ unit of date 1 consumption, the corresponding indirect utility of endowment for a long-lived agent, who will consume after transforming it, lies above that of a short-lived agent but with threshold point $R^{-1}c^*$.

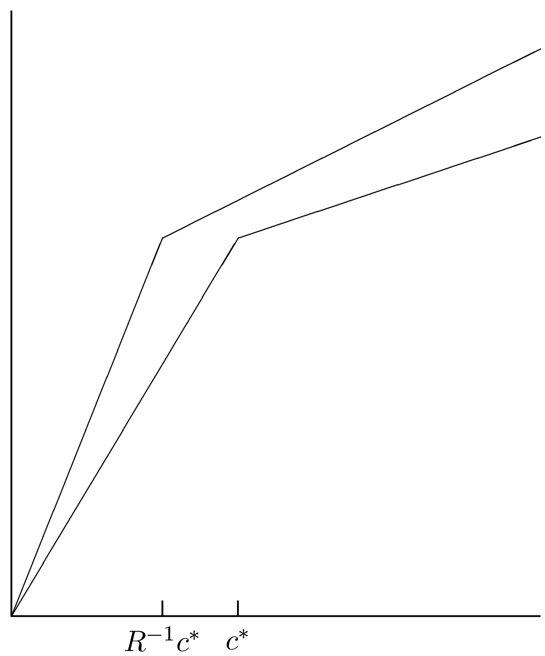

Figure 1

A. Full-information case

First, suppose that the realization of types is observed by the planner. In a given state of nature, the social planner can count the number of long-lived agents and then based on this distribute the aggregate endowment. Given the specification of the preferences, ex post efficiency requires the following way of allocating the resources. The first unit of the resources should go to the long-lived agents because they have the highest marginal utility. In fact, the long-lived agents should each receive an amount up till their threshold $R^{-1}c^*$, at which point the remaining resources should be used to serve the short-lived agents conditional on that each of them gets at most c^*. After this, if there are still resources left, they should be divided equally among the long-lived agents as their extra consumption. The short-lived agents never consume more than c^*.

B. Case with unobservable types.

If agent's types are private information, then the planner has to allocate the resources based on agents' reported types. Ideally, in this case, one would want to implement the full-information (ex post) efficient allocation. To accomplish this, the social planner can announce the following arrangement. At date 0, each agent is going to tell the planner whether they are short-lived or long-lived. The reported messages are then going to be regarded as the "true" state of nature, and, based on this, each agent's consumption is determined by the algorithm in the case of perfect information. Resources assigned to long-lived agents are invested *by* the social planner up till date 1. Also assume that resources received by "short-lived" agents must be consumed at date 0. (This assumption will be dropped in the formal analysis below.)

The above arrangement may have a problem supporting the full-information efficient allocation if the agents do not truthfully communicate with the planner about their types.. But, it is easy to see that under this arrangement, the agents have no incentive to lie about their types no matter what the other agents' reports are. First, short-lived agents never want to claim to be long-lived because the resource kept up till date 1 has no value to them. If an agent turns out to be long-lived, then by reporting truthfully, she receives $R^{-1}c^*$ at date 0 which will grow into c^* by date 1. If she claims to be short-lived, then at most she receives c^* which must be consumed at date 0. Given the specification of utility function, the agent prefers to report her true type, regardless of the other agents' actions. Thus, the (ex post) efficient allocation can be supported as a strictly dominate strategy equilibrium. Since the ex post efficient allocation is implementable, it is necessarily ex ante efficient.

In the formal analysis below, we will extend the above arguments to the case with smooth concave utility function and sequential service.

(Note that in the above algorithm (with or without private information), how much an agent consumes depends not only on her realized (or claimed) type, but also on the type profile of the entire population. In particular, the date 0 consumption of a short-lived depends the number of long-lived agents and thus varies across the states of nature. The reason that the arrangement of Diamond and Dybvig (1983) posses a "run" equilibrium is directly related to that an agent consumption bundle depends only on her reported type. In the absence of a suspension scheme, a claimed short-lived agent always gets the consumption level specified by the social optimal allocation, no matter what reports the other agents send to the bank. In a sense, the arrangement in Diamond and Dybvig is an ill-defined or incomplete mechanism.)

One property of the mechanism in our paper is that the consumption of a short-lived agent is a nondecreasing function of the number of long-lived agents $((d)/(d\eta)(\Gamma(\eta))/(I-\eta) \geq 0$, see the proof of lemma 3). This property seems to correspond a partial suspension scheme: the more people claim to be short-lived, the less consumption each and every one of the short-lived agents (not just the late-arrived) receives.

3. Intermediation as an allocation mechanism

One way of viewing a financial intermediary is as a trading club. People want to join such a club because features of the environment (including the informational features that engender problems of "adverse selection" and "moral hazard") make arms-length transactions infeasible or unsatisfactory. Instead, a trading club operates according to a charter that specifies which trades are to be made as function of information provided by members according to an explicitly defined protocol of communication and negotiation.

A. The environment of a Bayesian allocation mechanism

Consider a formal representation of an environment where such an intermediary would have a rationale. Let $\mathbf{I} = \{1, 2, \ldots, I\}$ be a set of traders who live in a risky environment. The possible *states* of this environment are the sample points of a probability space $(- \ , \mathcal{B}, \Pr)$. There is a measurable space of *ex-post allocations* which will be denoted by $(\mathbf{A}, \mathcal{A})$. A *state-contingent allocation* is simply a $\mathcal{B}$-measurable function from - to $\mathbf{A}$. Denote the set of such $\mathcal{B}$-measurable functions by $\mathbf{A}^{-}$. If $\vec{a} \in \mathbf{A}^{-}$ and $\omega \in -$, then $\vec{a}(\omega)$ is the ex-post allocation that the state-contingent allocation $\vec{a}$ specifies for state ω. (Henceforth a state-contingent allocation or an ex-post allocation will often be called simply an allocation, when it is clear from context which type of entity is being discussed.)

There is a set $\mathbf{F} \subseteq \mathbf{A}^{-}$ of *feasible state-contingent allocations.* (That is, $\mathbf{F}$ is a set of $\mathcal{B}$-measurable functions $f\colon - \ \to \mathbf{A}$.) The specification of F is supposed to reflect both individual restrictions such as nonnegativity of consumption and also aggregate restrictions such as materials balance.

This model will incorporate the *Harsanyi doctrine* that all traders are Bayesian utility maximizers, and that moreover Pr characterizes the prior beliefs common to all traders at "birth." Typically the model is used to understand the traders' behavior in "adulthood" after they have revised their beliefs in light of experience. The experience of trader i is called his *type*, and is represented as a sub σ-algebra $\mathcal{E}_i$ of $\mathcal{B}$. When a trader's type is described in terms of a $\mathcal{B}$-measurable random variable which the trader is assumed observe, $\mathcal{E}_i$ will be taken to be the smallest σ-algebra with respect to which the random variable observed by i is measurable. (Typically $\mathcal{E}_i$ is strictly smaller than $\mathcal{B}$ itself.)

In addition, assume that there is a sub σ-algebra $\mathcal{E}_0$ of $\mathcal{B}$ that represents information that is directly usable for allocation. That is, an allocation can be made contingent on this information without traders having to reveal it.

Each trader i has a state-dependent utility function $u_i\colon \mathbf{A} \times \Omega \to \mathbb{R}$, and maximizes the expectation of this function conditional on his type. Denote this conditional expectation by the function by $U_i\colon \mathbf{A}^{\Omega} \times \Omega \to \mathbb{R}$, which is defined by[2]

$$U_i(\vec{a}, \omega^*) = \mathrm{E}[u_i(\vec{a}(\omega), \omega)|\mathcal{E}_i](\omega^*). \tag{1}$$

B. Specification of an allocation mechanism

An allocation mechanism is specified in terms of two structures, a communication protocol and an allocation rule. The allocation rule is a function which determines an ex-post allocation on the basis of the data generated by traders' use of the communication protocol.

A communication protocol is described formally in terms of a finite *message space* M. Each trader i chooses, on the basis of his type, a message $m_i \in M$ to send. As a function of the state of the environment, then, trader i's message is an $\mathcal{E}_i$-measurable function $\mu_i\colon \Omega \to M$. This function μ_i will be called i's *communication strategy.* When each trader follows his communication strategy in state ω, a profile $\mu(\omega) = (\mu_1(\omega), \ldots, \mu_I(\omega))$ is generated which can be used as an informational basis for allocation.

Thus the *allocation rule* of the mechanism is a measurable function

$$\alpha\colon \Omega \times M^I \to \mathbf{A} \tag{2}$$

such that

$$\forall m \in M^{\mathbf{I}} \;\; \alpha(\omega, m) \text{ is } \mathcal{E}_0\text{-measurable} \quad \text{and} \quad \forall \mu \; \exists f \in \mathbf{F} \; \forall \omega \;\; f(\omega) = \alpha(\omega, \mu(\omega)). \tag{3}$$

[2] See Breiman (1968) or another graduate-level textbook of probability theory for the definition of expectation conditional on a σ-algebra. In the following definition, a trader's conditional expected utility is written for notational convenience as depending on the entire ex-post allocation. Actually, in the model to be studied here, a trader i's own consumption will be the only aspect of the allocation that matters for the determination of i's utility.

(The domain of quantification of μ is the set of all communication-strategy profiles. The function α must be restricted in the way specified by (3), in order to guarantee that the mechanism will always determine a feasible allocation regardless of which communication strategies traders choose.) An allocation rule α and a communication-strategy profile μ together determine an allocation $\alpha \circ \mu \in \mathbf{F}$.[3]

An equilibrium (specifically a *Bayesian Nash equilibrium*) of the allocation mechanism (M, α) is a communication-strategy profile μ^* such that, for any trader i and any profile μ that i can obtain by unilaterally changing his communication strategy while others' strategies remain the same, $U_i(\alpha \circ \mu, \omega) \leq U_i(\alpha \circ \mu^*, \omega)$ almost surely.

If μ^* is an equilibrium of (M, α) and $\vec{a} = \alpha \circ \mu^*$, then $\vec{a}$ will be called an *implementable allocation* of (M, α). Let $\mathcal{I}$ denote the set of every state-contingent allocation such that there exists a mechanism that implements it. An allocation $\vec{a} \in \mathcal{I}$ is *efficient* if

$$\forall\, \vec{c} \in \mathcal{I} \qquad \Big\{ \exists\, i \in \mathbf{I}\ \big\{ \mathrm{E}[u_i(\vec{c}(\omega), \omega)] < \mathrm{E}[u_i(\vec{a}(\omega), \omega)] \big\} \\ \text{or} \quad \forall\, i \in \mathbf{I}\ \big\{ \mathrm{E}[u_i(\vec{c}(\omega), \omega)] = \mathrm{E}[u_i(\vec{a}(\omega), \omega)] \big\} \Big\}. \tag{4}$$

This definition conforms to the usual definition of ex-ante Pareto efficiency (cf. Myerson, 1991).

Consider the allocation that, in each state of nature, maximizes the sum of traders' utilities. Typically this allocation will not be implementable, so it cannot be efficient. However, this maximizing allocation is necessarily efficient if it is implementable.

Lemma 1 *Suppose that $\vec{a}$ is implementable and satisfies*

$$\sum_{i \in \mathbf{I}} E[u_i(\vec{a}(\omega), \omega)] = \max_{\vec{c} \in \mathbf{F}} \sum_{i \in \mathbf{I}} E[u_i(\vec{c}(\omega), \omega)]. \tag{5}$$

Then $\vec{a}$ is efficient.

Proof: It follows immediately from (5) that $\sum_{i \in \mathbf{I}} \mathrm{E}[u_i(\vec{a}(\omega), \omega)] \geq \sum_{i \in \mathbf{I}} \mathrm{E}[u_i(\vec{c}(\omega), \omega)]$ for every $\vec{c} \in \mathcal{I}$. This means that if $\mathrm{E}[u_i(\vec{a}(\omega), \omega)] \leq \mathrm{E}[u_i(\vec{c}(\omega), \omega)]$ for every $i \in \mathbf{I}$, then $\mathrm{E}[u_i(\vec{a}(\omega), \omega)] = \mathrm{E}[u_i(\vec{c}(\omega), \omega)]$ for every $i \in \mathbf{I}$. That is, the condition (4) defining efficiency must hold. ■

Lemma 1 has the following, immediate corollary.

[3] We use the notation $\alpha \circ \mu$ to denote the state-contintent allocation that takes each state ω to the ex-post allocation $\alpha(\omega, \mu(\omega))$.

Lemma 2 *Suppose that $\vec{a}$ is implementable and satisfies*

$$\forall \omega \in \Omega \quad \sum_{i \in \mathbf{I}} u_i(\vec{a}(\omega), \omega) = \max_{a \in F(\omega)} \sum_{i \in \mathbf{I}} u_i(a, \omega). \tag{6}$$

Then $\vec{a}$ is efficient.

4. Banking—A schematic model

Bryant (1980) and Diamond and Dybvig (1983) introduce models of banking which Jacklin (1987) simplifies further to study capital-structure issues.[4] Now we formulate a finite-trader version of Jacklin's maturity-transformation model.

A. An environment where a maturity-transforming intermediary has a role

Define Ω and Pr by

$$\Omega = \{0,1\}^{\mathbf{I}} \quad \text{and} \quad \forall \omega \;\; \Pr(\omega) = 2^{-I}, \tag{7}$$

and define $\mathcal{E}_0$ and $\mathcal{E}_i$ by

$$\mathcal{E}_0 = \{\emptyset, \Omega\} \quad \text{and} \quad \forall i \in \mathbf{I} \;\; \mathcal{E}_i = \{\emptyset, \Omega, \{\omega | \omega_i = 0\}, \{\omega | \omega_i = 1\}\}. \tag{8}$$

Suppose that there is an aggregate endowment of one unit of a good per person, which can be transformed into a consumption good available at either date 0 or date 1. The transformation is simply storage until date 0, but whatever is not consumed at date 0 is augmented by a gross factor of $R > 1$ at date 1. Thus feasible ex-post allocations are the elements of the set

$$\mathbf{A} = \left\{ a \colon \mathbf{I} \to \mathbb{R}^2_+ \;\middle|\; \sum_{i \in \mathbf{I}} [a_0(i) + R^{-1} a_1(i)] \leq I \right\}. \tag{9}$$

and

$$\mathbf{F} = \mathbf{A}^{\Omega}. \tag{10}$$

A trader's utility from allocation a in state ω is given by a function $v \colon \mathbb{R}_+ \to \mathbb{R}$ of a consumption aggregate which includes consumption at both dates if i is of type 1, but which consists

[4] Jacklin deliberately neglects the *sequential service constraint*, which Diamond and Dybvig discuss informally and which Wallace (1988) formalizes and analyzes. Wallace emphasizes that a serious treatment of this constraint shows the institutional arrangement of deposit insurance as modelled by Diamond and Dybvig to be infeasible.

of consumption at date 0 alone if i is of type 0.[5] That is,

$$\forall i \quad \forall \omega \quad u_i(a,\omega) = v(a_0(i) + \omega_i a_1(i)). \tag{11}$$

Assume that[6]

$v(0) = 0$;

v is strictly increasing, continuously twice differentiable and strictly concave;

v satisfies the Inada conditions $\lim_{\gamma \to 0} v'(\gamma) = \infty$ and $\lim_{\gamma \to \infty} v'(\gamma) = 0$;

$\forall \gamma \;\; \gamma v''(\gamma)/v'(\gamma) \leq -1$ (Relative risk aversion ≥ 1 everywhere). (12)

Consider the problem of choosing $\vec{a} \in \mathbf{F}$ to maximize the sum of traders' expected utilities, if the allocation could be made measurable in the traders' types (that is, the "fully-informed utilitarian social planner's problem"). By strict concavity of v, Jensen's inequality, and the fact that $R > 1$ while consumption goods at the two dates are perfect substitutes for type-1 traders, the following conditions should hold. In each state ω, all type-0 traders should receive identical consumption bundles $(c_0(\omega), 0)$ and all type-1 traders should receive identical consumption bundles $(0, c_1(\omega))$. Letting $\theta(\omega) = \sum_{i \in I} \omega_i$ as in the preceding example, each ex-post allocation $a = \vec{a}(\omega)$ should satisfy the following two equations (a first-order condition and a feasibility condition derived from (9), respectively).

$$v'(c_0(\omega)) = Rv'(c_1(\omega)) \tag{13}$$

and

$$[I - \theta(\omega)]c_0(\omega) + R^{-1}\theta(\omega)c_1(\omega) = I. \tag{14}$$

These two equations determine $\vec{a}(\omega)$ uniquely. It is evident that $c_0(\omega)$ and $c_1(\omega)$ depend on ω only through $\theta(\omega)$. The following lemma explains the significance of the assumption regarding relative risk aversion in (12).

[5] This formulation follows Diamond and Dybvig. Jacklin also considers a utility formulation in which both types of trader receive positive marginal utility from consumption of each date, but in which type-0 traders discount consumption at date 1 more heavily than type-1 traders do.

[6] Phil Dybvig has pointed out to us that the assumption that $v(0) = 0$ is inconsistent with the assumption that relative risk aversion is greater than 1 everywhere. Fortunately, this specification of $v(0)$ was adoped solely for notational convenience. The assumption that $v(0) = 0$ is related to the proof of Lemma 7 only in terms of the existence of function $\phi : R_+ \to R_+$ (the middle of p. 17 below (53)). Without restricting the domain of ϕ to be R_+, one can claim that there exists a function $\phi : S \to R_+$, where $S \subseteq R$ is the image of v, such that $\phi' < 0, \phi'' > 0$, and for all c, $v'(c) = \phi(v(c))$. In fact, the function $\phi(s) \equiv v'(v^{-1}(s))$ will work. This ϕ is decreasing because v^{-1} is increasing and $v'' < 0$. By assumption (44), $\phi'' = (d)/(dc)\,[(v''(c))/(v'(c))] \cdot (dc)/(ds)$, where $c = v^{-1}(s)$, is positive.

Lemma 3 *Suppose that v satisfies the assumptions (12), including that $\forall\gamma\ \gamma v''(\gamma)/v'(\gamma) \leq -1$. (Relative risk aversion ≥ 1 everywhere.) Then the allocation $\vec{a}$ defined from (13) and (14) by*

$$[\vec{a}(\omega)]_i = ((1-\omega_i)\, c_0(\omega),\ \omega_i\, c_1(\omega)) \tag{15}$$

is efficient. The consumption level $c_1(\omega)$ of type-one traders is a nondecreasing function of $\theta(\omega)$. More generally, let η be a real variable taking values in $(0, I)$ and consider the problem of maximizing

$$(I-\eta)v\left(\frac{\gamma}{I-\eta}\right) + \eta v\left(\frac{R(I-\gamma)}{\eta}\right). \tag{16}$$

The solution, parametrized by η, is a function $\Gamma(\eta)$ that satisfies

$$\frac{d}{d\eta}\frac{R(I-\Gamma(\eta))}{\eta} \geq 0. \tag{17}$$

Proof: By (13) and (14) and the concavity of v, $\vec{a}$ satisfies the optimality condition (6) of lemma 2. Therefore $\vec{a}$ is efficient by lemma 2.

To see that the general monotonicity assertion (17) implies the more specific assertion regarding c_1, note that if $0 < \theta(\omega) = \eta < I$, then $c_0(\omega) = \Gamma(\eta)/(I-\eta)$ and $c_1(\omega) = R(I-\Gamma(\eta))/\eta)$ by (13) and (14). This equivalence can be extended to $\theta(\omega) \in \{0, I\}$, in view of the Inada conditions on v. (That is, defining $\Gamma(0) = I$ and $\Gamma(I) = 0$ extends the definition of Γ on $(0, I)$ continuously.)

Corresponding to (13), the first-order condition for (16) is

$$v'\left(\frac{\Gamma(\eta)}{I-\eta}\right) - Rv'\left(\frac{R(I-\Gamma(\eta))}{\eta}\right) = 0. \tag{18}$$

Taking the derivative of (18) with respect to η yields

$$\left[\Gamma'(\eta) + \frac{\Gamma(\eta)}{I-\eta}\right]\frac{v''(\Gamma(\eta)/(I-\eta))}{I-\eta} + R^2\left[\Gamma'(\eta) + \frac{I-\Gamma(\eta)}{\eta}\right]\frac{v''(R(I-\Gamma(\eta))/\eta)}{\eta} = 0. \tag{19}$$

Now consider the derivative in (17).

$$\frac{d}{d\eta}\frac{R(I-\Gamma(\eta))}{\eta} = \frac{-R}{\eta}\left[\Gamma'(\eta) + \frac{I-\Gamma(\eta)}{\eta}\right]. \tag{20}$$

In order to prove the lemma by establishing (17), then, it must be shown that the bracketed expression in (20) is negative. This expression is identical to one of the two bracketed expressions in (19), and (19) shows that either those two expressions are both zero or else they have opposite signs. Thus the inequality

$$\Gamma'(\eta) + \frac{I - \Gamma(\eta)}{\eta} \leq 0, \tag{21}$$

which proves the lemma, is equivalent to

$$\frac{I - \Gamma(\eta)}{\eta} \leq \frac{\Gamma(\eta)}{I - \eta} . \tag{22}$$

Inequality (22) follows from the assumption that $\forall \gamma \; \gamma v''(\gamma)/v'(\gamma) \leq -1$. To see this, note that the assumption implies that

$$\frac{\partial}{\partial r}[rv'(rs)] \leq 0. \tag{23}$$

This inequality and equation (18) imply that

$$v'\left(\frac{I - \Gamma(\eta)}{\eta}\right) \geq v'\left(\frac{\Gamma(\eta)}{I - \eta}\right), \tag{24}$$

which implies (22) by the concavity of v. ∎

B. A mechanism with a unique, efficient equilibrium

Next we will show that conditions (13) and (14) imply that the efficient allocation can be implemented by a truth-telling equilibrium of an allocation mechanism The mechanism here possesses a property that the mechanism in that other model lacks: that truth-telling is the *strictly dominant strategy* for each trader. By definition, this condition means that whether a trader is of type 0 or of type 1, he receives a higher utility level from revealing his type truthfully than from misrepresenting it—regardless of what reports other traders give. It follows (cf. Myerson, 1991) that the truth-telling equilibrium is the unique Bayesian Nash equilibrium of the mechanism. Therefore no alternative, inefficient, "run" equilibrium of this mechanism can exist.

After having characterized the allocation that maximizes ex-ante expected utility (which has been done already by deriving conditions (13) and (14)), the mechanism possessing the dominant-strategy property is defined by depending on the truthfulness of traders' reports, and using them as the basis for assigning traders the ex-post consumption bundles determined by that allocation.

Recall that, ordinarily, such a straightforward approach would be unsuccessful because truth-telling would not be a trader's equilibrium strategy. However, because of the particular form (11) of the state-contingent utility function and special features of the efficient, symmetric allocation, the approach does work in this case.

Theorem 1 *Let $M = \{0, 1\}$ be the set of signals for each trader. Define $x\colon M \times \{0, \ldots, I\} \to \mathbb{R}$ by the conditions (analogous to (13) and (14)) that*

$$v'(x(0, \eta)) = Rv'(x(1, \eta)) \tag{25}$$

and

$$[I - \eta]x(0, \eta) + R^{-1}\eta x(1, \eta) = I. \tag{26}$$

Define $\alpha\colon$- $\times M^I \to \mathbf{A}$ by

$$[\alpha(\omega, m)]_i = \left((1 - m_i)\, x(m_i, \sum_{j \in \mathbf{I}} m_j),\ m_i\, x(m_i, \sum_{j \in \mathbf{I}} m_j) \right). \tag{27}$$

The truthful communication strategy $\hat{\mu}_i(\omega) = \omega_i$ is the strictly dominant strategy for each trader i. The mechanism thus implements the efficient, symmetric allocation in strictly dominant strategies, and consequently the profile of truthful communication strategies is its unique Bayesian Nash equilibrium

Proof: If $\eta = \theta(\omega)$, then conditions (25) and (26) on $(x(0, \eta), x(1, \eta))$ are identical to conditions (13) and (14) on $(c_0(\omega), c_1(\omega))$. Lemma 3 therefore implies that the mechanism implements the efficient, symmetric allocation if the profile of truthful communication strategies is a Bayesian Nash equilibrium.

By Myerson (1991), a profile of strictly dominant strategies for a mechanism is the unique Bayesian Nash equilibrium of the mechanism. Therefore, to prove the lemma, it is sufficient to show that truthful communication is the strictly dominant strategy for each trader. To verify this, consider separately each of the two possible values of ω_i. If $\omega_i = 0$, then by (25) and (27), i will receive a positive amount of consumption at date 0 if he sends message 0, but will receive 0 consumption at date 0 if he sends message 1. Because he has utility only for consumption at date 0 (by the definition (11) of his utility function), and because his utility is strictly increasing in the amount of this good that he consumes (by (11) and (12)), he strictly prefers to send message 0 rather than message 1 in state ω.

Now consider the alternative case that $\omega_i = 1$. The strict concavity of v assumed in (12), together with (25), implies that

$$x(1, 0 + \sum_{j \neq i} \mu_j(\omega)) > x(0, 0 + \sum_{j \neq i} \mu_j(\omega)) \tag{28}$$

regardless of which communication strategies μ_j the other traders use. By (17) of lemma 3 and the fundamental theorem of calculus,

$$x(1, 1 + \sum_{j \neq i} \mu_j(\omega)) \geq x(1, 0 + \sum_{j \neq i} \mu_j(\omega)). \tag{29}$$

Therefore, given the functional form of i's utility function (equations (11) and (12)) and the specification of the mechanism (equation (27)), inequalities (28) and (29) together imply that trader i must strictly prefer to send message 1 rather than message 0. ∎

5. Banking in an environment with sequential service

The schematic model of banking studied above abstracts from an important feature of an actual bank: that traders do not all contact the bank at the same time, and that the bank must deal promptly with traders who contact it early. The bank therefore is constrained from making its treatment of those traders contingent on information yet to be provided by later traders, especially if the early traders wish to make withdrawals. This feature plays an important role in Diamond and Dybvig's (1983) intuitive discussion of their model, and it is formalized by Wallace (1988) who derives further consequences from it. In view of the striking discepancy between theorem 1 and Diamond and Dybvig's analyisis, and of the closer analogy between the theorem and Jacklin's (1987) analysis that also abstracts from the sequential-service constraint, it is a salient question whether or not theorem 1 can be extended to an environment with sequential service. Now we investigate this question and find an answer that is more or less in the affirmative. Specifically, if v satisfies non-increasing absolute risk aversion as well as the conditions specified in (12), then the profile of truthful communication strategies is the unique profile that survives iterated elimination of strictly dominated strategies. It follows that, as in theorem 1, this is the unique Bayesian Nash equilibrium of the natural mechanism that implements the efficient allocation.

In the present formalization of the sequential-service constraint, every trader contacts the bank at some time during date 0, these "arrival times" for different traders are stochastic and independently distributed, and each trader's arrival time is in his own information set. This last detail is crucial, for it implies that a trader who arrives very late can be almost certain that he is the last trader to arrive. Conditional on being last, truthful communication is the trader's unique utility-maximizing action. That is, any strategy that involves some untruthful communication by a trader when he arrives very late can be eliminated as being dominated by the strategy that agrees with it except at very late times, but that specifies truthful communication at those times. This result can then be "bootstrapped" to apply to communication at earlier times as well. One should note that, in Wallace's formalization of sequential service, a trader's time of arrival is not in his own information set. Under Wallace's assumption, it seems that iterated elimination of strictly dominated strategies may not lead necessarily to truthful communication.

A. Formalization of sequential service

Modelling sequential service requires that the maturity-transformation model must be modified by enlarging the state space - to represent information about arrival times, and by making corresponding changes in the definitions of agents' types and of feasible allocations.

To enlarge the state space, replace the definition (7) by

$$
\begin{aligned}
&- = \{0,1\}^{\mathbf{I}} \times [0,1]^{\mathbf{I}}, \text{ and } p \in (0,1);\\
&\text{For all } i \leq I \quad \Pr(\omega_i = 1) = P;\\
&\text{For all } i \leq I \quad \omega_{I+i} \text{ is uniformly distributed;}\\
&\text{The projections of } \omega \text{ on its coordinates are independent r.v.'s.} \qquad (30)
\end{aligned}
$$

Replace the definition in (8) of agent i's type by

$$\mathcal{E}_i = \{\{\omega|\omega_i = 0 \text{ and } \omega_{I+i} \in A\} \cup \{\omega|\omega_i = 1 \text{ and } \omega_{I+i} \in B\}|A \in \mathcal{F} \text{ and } B \in \mathcal{F}\}, \qquad (31)$$

where $\mathcal{F}$ is the σ-algebra of Borel sets on $[0,1]$. That is, in each state a trader knows his own utility function and his own arrival time at the bank, but he knows nothing about the other traders.

Also replace the specification in (8) that the algebra $\mathcal{E}_0$ is trivial by the following definition, which intuitively specifies that information about all traders' arrival times may be used directly (that is, without having to be revealed by the traders' communication) as a basis for allocation.

$$\mathcal{E}_0 = \{\omega | \forall i \leq I \;\; \omega_{I+i} \in A_i | A_1 \in \mathcal{F}, \ldots, A_I \in \mathcal{F}\} \tag{32}$$

In order to formulate the sequential service constraint, define the arrival-order statistics by $\tau\colon \{0, \ldots, I\} \times$ - $\to \mathbf{I}$. That is, $\tau(1, \omega), \tau(2, \omega), \ldots \tau(I, \omega)$ are the first, second, $\ldots I$th traders in order of arrival determined by the coordinates $\omega_{I+1}, \omega_{I+2}, \ldots \omega_{2I}$ of ω. Ties can be assumed to be broken arbitrarily in the zero-probability event that several traders arrive simultaneously at the bank. Define the rank statistics $\rho\colon \mathbf{I} \times \{0, \ldots, I\} \times$ - $\to \{0, \ldots, I\}$, which are inverse to the order statistics in each state of nature, by $\rho(\tau(i, \omega), \omega) = i$.

Suppose that $\vec{a} = ((X_0^1, X_1^1), \ldots, (X_0^I, X_1^I)) \in \mathbf{A}^{-}$ is an allocation.[7] The intuitive content of the sequential service constraint is that the mechanism represents a financial intermediary (call it a bank) operating at a specific location that the trader visits at some time during date 0. When trader i visits, he communicates a message $m \in M$ determined by a communication strategy μ_i that is measurable with respect to $\mathcal{E}_i$, and he then receives $X_0^i(\omega)$ immediately. This quantity thus must not depend on information from traders who arrive later in state ω than i does, since those traders have not yet communicated their information to the bank. Since all traders are envisioned to arrive at the bank at some time before date 1, when the consumption amounts $X_1^j(\omega)$ are distributed, those date-1 quantities are not analogously constrained.

That is, the amount $X_0^{\tau(1,\omega)}(\omega)$ of consumption given to trader $\tau(1, \omega)$ at date 0 must depend only on the identity of $\tau(1, \omega)$ and the time $\omega_{I+\tau(1,\omega)}$, both of which the bank observes, and on that trader's utility parameter $\omega_{\tau(1,\omega)}$, which he has the opportunity to communicate to the bank. (Whether or not he actually does communicate his utility parameter in equilibrium is irrelevant to the formulation of this constraint, which expresses the limitation imposed by the exogenous sequential nature of the *opportunities* for the bank to acquire information.) Next, the information that the bank can use to determine the date-0 consumption of the second trader to arrives consists of both this information about the first trader, which the bank remembers, and also the corresponding information about the second trader himself. And so forth. Formally, $((X_0^1, X_1^1), \ldots, (X_0^I, X_1^I))$

[7]In this section, since - is a continuum, $\mathbf{A}^{-}$ denotes the set of Borel-measurable functions from - to $\mathbf{A}$.

satisfies the *sequential service constraint* if

$$\forall i \quad X_0^{\tau(i,\omega)} = \mathrm{E}\left[X_0^{\tau(i,\omega)} | \tau(1,\omega), \ldots, \tau(i,\omega), \omega_{\tau(1,\omega)}, \ldots, \omega_{\tau(i,\omega)}, \omega_{I+\tau(1,\omega)}, \ldots, \omega_{I+\tau(i,\omega)}\right]. \tag{33}$$

In view of this constraint, **F** should be defined by

$$\mathbf{F} = \left\{\vec{a} | \vec{a} \in \mathbf{A}^{\text{-}} \text{ and } \vec{a} \text{ satisfies (33)}\right\}. \tag{34}$$

B. The efficient, symmetric, state-contingent allocation

In this section, we consider the solution of the optimization problem posed in equation (5), that is,

$$\text{Maximize } \sum_{i \in \mathbf{I}} \mathrm{E}[u_i(\vec{c}(\omega), \omega)] \text{ subject to } \vec{c} \in \mathbf{F},$$

with **F** defined by (34). Subsequently we will consider the problem of implementing this allocation, which is efficient by lemma 1.

The key to solving problem (36) is the observation, formalized below in lemma 4, that the arrival-order statistics $\tau(i,\omega)$ provide all of the relevant information about traders' arrival times. More precise arrival-time information is relevant neither to traders' enjoyment of utility nor to the technical feasibility of allocations in the sequential service environment.[8] In view of this observation, define mappings $\sigma^i \colon - \to \{0,1\}^i$ for $1 \leq i \leq I$ by

$$\forall j \leq i \quad \sigma_j^i(\omega) = \omega_{\tau(j,\omega)}. \tag{35}$$

Define the set of 0–1 sequences of length at most I, including the null sequence, as $\mathcal{S}$. For $s \in \mathcal{S}$, let $\ell(s)$ denote the length of s. Define $\langle 0 \rangle$ to be the sequence consisting of I consecutive zeros. Define $\theta^*(s) = \sum_{i<\ell(s)} s_i$ and $\pi(s) = P^{\theta^*(s)}(1-P)^{\ell(s)-\theta^*(s)}$. Define the weak and strict extension-ordering relations on $\mathcal{S}$ by

$$\begin{aligned} r \leq s \quad &\iff \quad \ell(r) \leq \ell(s) \text{ and } \forall i \leq \ell(r)\ [r_i = s_i]; \\ r < s \quad &\iff \quad \ell(r) < \ell(s) \text{ and } \forall i \leq \ell(r)\ [r_i = s_i]. \end{aligned} \tag{36}$$

Lemma 4 *Suppose that* $\vec{a} = ((X_0^1, X_1^1), \ldots, (X_0^I, X_1^I))$ *solves problem (5) in the sequential-service*

[8] Each trader will use his information about his precise arrival time to make inference about his probable rank in the arrival queue (which he does not observe directly) though, so this information is relevant to implementation.

environment. Then there exists a vector $x \in \mathbb{R}_+^{\mathcal{S}}$ *such that*

$$\sum_{r \le \langle 0 \rangle} x_r = I \quad \textit{and} \quad \forall s \in \mathcal{S} \quad \left[\left[s_{\ell(s)} = 1 \Longrightarrow x_s = 0\right] \ \textit{and} \ \sum_{r \le s} x_r \le I\right] \tag{37}$$

and, almost surely for all i*,*

$$X_j^{\tau(i,\omega)} = \begin{cases} x_{\sigma^i(\omega)} & \textit{if } j = 0 \textit{ and } \sigma_i^i(\omega) = \omega_{\tau(i,\omega)} = 0; \\ \frac{R}{\theta(\omega)}\left(I - \sum_{r \le \sigma^I(\omega)} x_r\right) & \textit{if } j = 1 \textit{ and } \sigma_i^i(\omega) = \omega_{\tau(i,\omega)} = 1; \\ 0 & \textit{otherwise.} \end{cases} \tag{38}$$

If $\vec{a}$ *and* x *are related according to (38), then*

$$\begin{aligned} \sum_{i \in \mathbf{I}} E[u_i(\vec{a}(\omega), \omega)] &= \sum_{\substack{\ell(s)=I \\ \theta^*(s)>0}} \pi(s) \left[\left(\sum_{r \le s} v(x_r)\right)\right. \\ &\left. + \theta^*(s) v\left(\frac{R}{\theta^*(s)}\left(I - \sum_{q \le s} x_q\right)\right)\right] + \pi(\langle 0 \rangle) \sum_{r \le \langle 0 \rangle} v(x_r). \end{aligned} \tag{39}$$

Proof: One can alternatively characterize $\vec{a}$ in terms of a vector of random variables $((Y_0^1, Y_1^1), \ldots, (Y_0^I, Y_1^I))$, where $Y_j^i(\omega) = X_j^{\tau(i,\omega)}(\omega)$ a.s. for each i and j. Consider the state-contingent allocation $\vec{c} = ((Z_0^1, Z_1^1), \ldots, (Z_0^I, Z_1^I))$, defined by $Z_j^{\tau(i,\omega)}(\omega) = \mathrm{E}[Y_j^i(\omega)|\sigma^i]$ a.s. It is easily verified that $\vec{c} \in \mathbf{F}$, and for every i, $\mathrm{E}[u_i(\vec{a}(\omega), \omega)] \le \mathrm{E}[u_i(\vec{c}(\omega), \omega)]$, with strict inequality for at least one i if $\vec{a} \ne \vec{c}$. (This inequality must hold because v is strictly concave and $\vec{c}$ is obtained by taking conditional expectation with respect to $\vec{a}$.) That is, $\vec{a} \ne \vec{c}$ would contradict the hypothesis of the lemma. By construction, $\vec{c}$—that is to say, $\vec{a}$—can be characterized in terms of a vector $x \in \mathbb{R}_+^{\mathcal{S}}$. This vector must actually satisfy (38), by the same considerations that prove the efficiency assertion in lemma 3. (Note that, in the context of (38), condition (37) states that traders of type 1 consume exclusively at date 2.) Condition (39) is verified by straightforward computation. ∎

By lemma 4, a solution to optimization problem (5) can be found by optimizing over a set of vectors in $\mathbb{R}_+^{\mathcal{S}}$. Specifically, given the strict concavity of the right side of (39), a solution is characterized by a vector that satisfies the first-order conditions for optimization of (39) subject to the constraint (37). That is, the following lemma holds.

Lemma 5 *A necessary and sufficient condition for a state-contingent allocation*

$\vec{a} = ((X_0^1, X_1^1), \ldots, (X_0^I, X_1^I))$ *to solve problem (5) in the sequential-service environment is that there should exist a vector* $x \in \mathbb{R}_+^{\mathcal{S}}$ *that satisfies (37), (38), and for all* $r \in \mathcal{S}$ *such that* $r_{\ell(r)} = 0$,

$$\pi(r)v'(x_r) - R\left[\sum_{\substack{\ell(s)=I \\ \theta^*(s)>0 \\ r\leq s}} \pi(s)v'\left(\frac{R}{\theta^*(s)}\left(I - \sum_{q\leq s} x_q\right)\right)\right] + 0^{\theta^*(r)}\pi(\langle 0\rangle)v'(x_{\langle 0\rangle}) = 0. \tag{40}$$

C. Iterated elimination of strictly dominated strategies

Like theorem 1, the corresponding result for the sequential-service environment will guarantee that the mechanism implementing the efficient allocation has a unique Bayesian Nash equilibrium. However, due to the sequential-service constraint, uniqueness cannot be established by the dominant-strategy argument used to prove theorem 1. Rather a concept of iterated elimination of strictly dominated strategies must be used. To state this concept, generalize the concept of strict dominance for allocation mechanism (M, α) in the following way. First, for functions $f\colon - \to \mathbb{R}$ and $g\colon - \to \mathbb{R}$, define $f < g$ if $\Pr(f(\omega) \leq g(\omega)) = 1$ and $\Pr(f(\omega) < g(\omega)) > 0$. Next, for an arbitrary subset $K \subseteq M^I$, define the relation $<_i^K$ on the strategy set of trader i (that is, the subspace of M^- that is measurable with respect to $\mathcal{E}_i$) by specifying that $\phi <_i^K \phi'$ whenever $\phi = \mu_i$ for some $\mu \in K$, $\phi' = \mu'_i$ for some $\mu' \in K$, and

$$\forall \mu \in K \;\; \forall \mu' \in K \;\; [[\phi = \mu_i \text{ and } \phi' = \mu'_i] \Longrightarrow U_i(\alpha \circ \mu(\omega), \omega) < U_i(\alpha \circ \mu'(\omega), \omega)] \tag{41}$$

(with $U_i(\alpha \circ \mu(\omega), \omega)$ and $U_i(\alpha \circ \mu'(\omega), \omega)$ being treated as functions of ω). Finally, define $M_i^0 = M$ for each i, and for each n, let $K^n = \prod_{j\leq I}(M_j^n)^-$ and define

$$M_i^{n+1} = \{\phi \in M_i^n | \text{not} \;\; \exists \phi' \in (M_i^n)^- \;\; \phi <_i^{K^n} \phi'\}. \tag{42}$$

(Note that, for every j including i, $(M_j^n)^-$ refers to the space of functions that are measurable with respect to $\mathcal{E}_j$.)

The following lemma can be proved by an argument analogous to that in Myerson (1991) regarding iterated elimination of strictly dominated strategies.

Lemma 6 *For each i, let $M_i^* = \bigcap_{n \in \mathbf{N}} M_i^n$. If each M_i^n contains a single element, then the unique element of $\prod_{i \leq I} M_i^*$ is the unique Bayesian Nash equilibrium of allocation mechanism (M, α).*

D. A monotonicity lemma

The first-order condition (40) just derived for the sequential-service environment has analogous structure to the first-order condition (13) in the simultaneous-communication environment studied in section 4. The monoticity assertion of lemma 3, which provides the key to establishing theorem 1 regarding the dominant-strategy implementability of the symmetric, efficient allocation in that environment, is proved by examining condition (13). A monotonicity result for a sequential-service environment is provable on the basis of condition (40), and it plays an analogous role to lemma 3 in establishing implementability.

To understand intuitively the way that this monotonocity lemma will be formulated, it helps to know the order in which strictly dominated strategies will be eliminated. Essentially, that order is according to backward induction on the arrival time of a trader at the bank. we will establish that, if a trader arrives at the bank sufficiently late at date 0, then he can be sure that everyone who arrives subsequently will give a truthful report, and that therefore the optimal report for the trader in question is also truthful. Here "sufficiently late" means "not before some time t," and we will show by working backward in time that actually t can be taken to be zero. That is, truthful reporting is optimal for a trader regardless of what time he arrives at the bank.

The foregoing discussion should make it clear that, when we are characterizing implementation, we will have to consider the situation of a trader i who arrives at the bank at some time during date 0 after $I - J - 1$ traders have already arrived, and before the last J traders will arrive, and who knows that those J traders who follow him will give truthful reports. Some nonnegative amount of the endowment good will have already been given to the earlier traders who have reported themselves to be of type 0, and an amount y remains to be allocated.

Suppose that the $I - J - 1$ traders who arrive prior to trader i have given a vector of reports $p \in \mathcal{S}$, with $\ell(p) = I - J - 1$. In terms of a representation like the one developed in lemma 4, the bank must allocate consumption in a way specified by a vector $\gamma \in \mathbb{R}_+^{\mathcal{S}'}$, where $\mathcal{S}' = \{s \in \mathcal{S} | \ell(s) \leq J\}$. The amount $\gamma_\emptyset$ represents the amount of date-0 consumption to be given to trader i, and for $\ell(s) = n > 0$, γ_s represents the amount of date-0 consumption to be given to the nth trader to arrive after i if s is the vector of reports of the first n traders to arrive following i. If i reports being of type 0, then the optimization problem of the bank is to maximize

$$v(\gamma_\emptyset) + \sum_{\ell(s)=J} \pi(s) \left[\left(\sum_{\emptyset < r \leq s} v(\gamma_r) \right) + (\theta^*(p) + \theta^*(s)) v \left(\frac{R}{\theta^*(p) + \theta^*(s)} \left(y - \sum_{q \leq s} \gamma_q \right) \right) \right].$$

(The term

$$(\theta^*(p) + \theta^*(s)) v \left(\frac{R}{\theta^*(p) + \theta^*(s)} \left(y - \sum_{q \leq s} \gamma_q \right) \right)$$

is taken to be zero if $\theta^*(p) = \theta^*(s) = 0$, since in that case there is no trader who wishes to consume at date 1.)

If i reports being of type 1, then the optimization problem of the bank is to optimize

$$\sum_{\ell(s)=J} \pi(s) \left[\left(\sum_{\emptyset < r \leq s} v(\gamma_r) \right) + (\theta^*(p) + \theta^*(s) + 1) v \left(\frac{R}{\theta^*(p) + \theta^*(s) + 1} \left(y - \sum_{q \leq s} \gamma_q \right) \right) \right].$$

Analogously to what we have done in lemma 3, these formulae can be subsumed in a general formula. Specifically they correspond to $\eta = 0$ and $\eta = 1$ in

$$\begin{aligned} &(1-\eta) v \left(\frac{\Gamma_\emptyset(\eta)}{1-\eta} \right) + \sum_{\ell(s)=J} \pi(s) \left[\left(\sum_{\emptyset < r \leq s} v(\Gamma_r(\eta)) \right) \right. \\ &\left. + (\theta^*(p) + \theta^*(s) + \eta) v \left(\frac{R}{\theta^*(p) + \theta^*(s) + \eta} \left(y - \sum_{q \leq s} \Gamma_q(\eta) \right) \right) \right]. \end{aligned} \tag{43}$$

Our goal is to prove the following statement, which formalizes the idea that a trader of type 1 should reveal his type truthfully if all traders who will arrive at the bank after him are also going to reveal their types truthfully.

Lemma 7 *For each $\eta \in [0,1]$, let $\Gamma(\eta) \in \mathbb{R}_+^{\mathcal{S}'}$ maximize (43) subject to the constraint (37) (with $\mathcal{S}'$ replacing $\mathcal{S}$ in the statement of the constraint). Suppose that v satisfies condition (12) and also the condition that*

$$\forall \gamma \quad \frac{d}{d\gamma} \frac{v''(\gamma)}{v'(\gamma)} \geq 0 \qquad \textit{(Absolute risk aversion is non-increasing everywhere).} \tag{44}$$

Then

$$v\left(\Gamma_{\emptyset}(0)\right) < \sum_{\ell(s)=J} \pi(s) v\left(\frac{R}{\theta^*(p)+\theta^*(s)+1}\left(y - \sum_{q\leq s}\Gamma_q(1)\right)\right). \tag{45}$$

Proof: Throughout the following arguments, we will assume that the function Γ, the image of which is defined at each η by optimization of (43), is continuously differentiable in the interval $(0,1)$ and is continuous at $\eta = 0$ and $\eta = 1$. This assumption can be proved by using an implicit-function theorem to establish differentiability and arguing from the Inada conditions on v to establish continuity at the endpoints.

We will also assume that $\theta^*(p) > 0$ This assumption avoids a complication that occurs when all traders report being impatient, in which case it is optimal for the bank to use all remaining endowment to provide date-0 consumption when the last trader to arrives. The first-order condition will generally balance the marginal utility of trader i against the expected marginal utility of patient traders, but when $\theta^*(p) = 0$ the first-order condition will balance the marginal utility of trader i against an expected value that includes the marginal utility of the last trader to arrive at the bank in this special case. The logic of the proof is unaffected by this complication in the form of the first-order condition when $\theta^*(p) = 0$, but extra terms would appear in all of the derivations if it were to be considered explicitly.

The function Γ satisfies the following first-order condition at each value of $\eta \in (0,1)$.

$$0 = \begin{cases} v'\left(\frac{\Gamma_{\emptyset}(\eta)}{1-\eta}\right) \\ \quad - R\sum_{\ell(s)=J}\pi(s)v'\left(\frac{R}{\theta^*(p)+\theta^*(s)+\eta}\left(y - \sum_{q<s}\Gamma_q(\eta)\right)\right) & \text{FOC for } \Gamma_{\emptyset}; \\ \\ \pi(r)v'\left(\Gamma_r(\eta)\right) \\ \quad - R\sum_{\substack{\ell(s)=J \\ r\leq s}}\pi(s)v'\left(\frac{R}{\theta^*(p)+\theta^*(s)+\eta}\left(y - \sum_{q\leq s}\Gamma_q(\eta)\right)\right) & \text{FOC for } \Gamma_r,\ \emptyset < r. \end{cases} \tag{46}$$

This condition implies the following three martingale-marginal-utility equations:

$$v'\left(\frac{\Gamma_{\emptyset}(\eta)}{1-\eta}\right) = \sum_{\ell(s)=1}\pi(s)v'\left(\Gamma_s(\eta)\right)\ ; \tag{47}$$

for $0 < \ell(r) < J$ and $r_{\ell(r)} = 0$,

$$\pi(r)v'\left(\Gamma_r(\eta)\right) = \sum_{\ell(s)=\ell(r)+1} \pi(s)v'\left(\Gamma_s(\eta)\right) \ ; \tag{48}$$

and, for $\ell(r) = J$ and $r_J = 0$,

$$v'\left(\Gamma_r(\eta)\right) = Rv'\left(\frac{R}{\theta^*(p) + \theta^*(s) + \eta}\left(y - \sum_{q \le r} \Gamma_q(\eta)\right)\right) \tag{49}$$

Since $R > 1$, equation (46) implies that

$$v'\left(\Gamma_\emptyset(0)\right) > \sum_{\ell(s)=J} \pi(s)v'\left(\frac{R}{\theta^*(p) + \theta^*(s)}\left(y - \sum_{q<s} \Gamma_q(0)\right)\right) \ . \tag{50}$$

By an argument analogous to lemma 3, involving the martingale conditions (47)–(49),

$$\frac{d}{d\eta}\left[\sum_{\ell(s)=J} \pi(s)v'\left(\frac{R}{\theta^*(p) + \theta^*(s) + \eta}\left(y - \sum_{q \le s} \Gamma_q(\eta)\right)\right)\right] \le 0 \ . \tag{51}$$

By the fundamental theorem of calculus and continuity at the endpoints 0 and 1, (51) implies that

$$\begin{aligned} &\sum_{\ell(s)=J} \pi(s)v'\left(\frac{R}{\theta^*(p) + \theta^*(s)}\left(y - \sum_{q \le s} \Gamma_q(0)\right)\right) \\ &\ge \sum_{\ell(s)=J} \pi(s)v'\left(\frac{R}{\theta^*(p) + \theta^*(s) + 1}\left(y - \sum_{q \le s} \Gamma_q(1)\right)\right) \ . \end{aligned} \tag{52}$$

Equations (50) and (52) imply that

$$v'\left(\Gamma_\emptyset(0)\right) > \sum_{\ell(s)=J} \pi(s)v'\left(\frac{R}{\theta^*(p) + \theta^*(s) + 1}\left(y - \sum_{q \le s} \Gamma_q(1)\right)\right) \ . \tag{53}$$

Finally, by assumption (44) (non-increasing absolute risk aversion), there is a function $\phi\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that $\phi' < 0$, $\phi'' > 0$, and for all c, $v'(c) = \phi(v(c))$. Thus (53) can be rewritten as

$$\phi\left(v\left(\Gamma_\emptyset(0)\right)\right) > \sum_{\ell(s)=J} \pi(s)\phi\left(v\left(\frac{R}{\theta^*(p) + \theta^*(s) + 1}\left(y - \sum_{q \le s} \Gamma_q(1)\right)\right)\right) \ . \tag{54}$$

By convexity of ϕ and Jensen's inequality,

$$\phi\left(v\left(\Gamma_{\emptyset}(0)\right)\right) > \phi\left(\sum_{\ell(s)=J} \pi(s) v\left(\frac{R}{\theta^*(p)+\theta^*(s)+1}\left(y - \sum_{q \le s} \Gamma_q(1)\right)\right)\right) . \tag{55}$$

Since $\phi' < 0$ this equation implies that

$$v\left(\Gamma_{\emptyset}(0)\right) < \sum_{\ell(s)=J} \pi(s) v\left(\frac{R}{\theta^*(p)+\theta^*(s)+1}\left(y - \sum_{q \le s} \Gamma_q(1)\right)\right) , \tag{56}$$

which is the desired conclusion. ■

E. A mechanism with a unique, efficient equilibrium

As we have explained at the beginning of section D, lemma 7 implies the analogue of theorem 1 in the sequential-service environment.

Theorem 2 *Suppose that v satisfies conditions (12) and (44). Let $M = \{0, 1\}$ be the set of signals for each trader. Let $x: \mathcal{S} \to \mathbb{R}_+$ be the vector satisfying the optimality conditions (37) and (39). Define $\alpha: - \times M^I \to \mathbf{A}$ by*

$$\begin{aligned}
[\alpha(\omega, m)]_i \quad = \quad & \Bigg((1 - m_i)\, x(m_{\tau(1,\omega)}, \ldots, m_{\tau(\rho(i,\omega),\omega)}), \\
& m_i \frac{R}{\sum_{j \le I} m_j}\left(I - \sum_{j \le I} x(m_{\tau(1,\omega)}, \ldots, m_{\tau(j,\omega)})\right)\Bigg) .
\end{aligned} \tag{57}$$

Then the profile of truthful-communication strategies $\hat{\mu}_i(\omega) = \omega_i$ is the unique profile that survives iterated elimination of strictly dominated strategies. The mechanism thus implements the symmetric, ex-ante efficient allocation by a unique Bayesian Nash equilibrium.

Proof: Define $S^i \equiv \{\mu_i : \{0,\ 1\} \times [0,\ 1] \longrightarrow \{0,\ 1\}\}$ and $S^i_t \equiv \{\mu_i \in S^i : \mu_i(\omega) = \omega_i$ for $\omega_{I+i} \ge t\}$,where t is a number between 0 and 1. That is, S^i is the space of trader i's reporting strategies as functions of state ω, and S^i_t is the collection of such strategy functions that involve truthful reporting if trader i arrives at the bank after time t.

Our goal is to prove that there exists a unique Bayesian Nash equilibrium which is the element of $\prod_{i \in I} S^i_0$. Given the form of state contingent utility functions, truth-telling is obviously the optimal strategy for trader i if $\omega_i = 0$, no matter at which time it arrives at the bank and what strategies the other traders adopt. Next we prove that truth-telling is also the dominant strategy for trader i in states in which $\omega_i = 1$.

Consider the reporting decision of trader i if his arrives at the bank later than time $t_1 \equiv 1-\varepsilon_1$. Once he arrives at the bank, y units of date 0 good have been given out to the traders who have already approached the bank. In deciding its reporting strategy, trader i compares the expected utility he will receive from announcing his type truthfully with that from lying. Since trader i does not observe other agents' arrival times, the conditional probability of trader i being the last one to approach the bank is $1- \ \delta_1 \equiv (1-\varepsilon_1)^{I-1}$ (given the uniform distributions of arrival times of all traders).

By telling the truth, i.e., $\mu_i = 1 = \omega_i$,trader i receives a utility equal to

$$(1-\delta_1)\ v\left((R(I-y))/(\theta^*(\mu))\right) + \delta_1 E\left(v\left((R(I-y))/(\theta^*(\mu))\right)|\tau(i,\omega) < I\right),$$

where μ is the vector of the reports of all the traders. If he lies about his type, trader i's utility will be $v(\Gamma(0,\, y))$. By Lemma 7, $v\left((R(I-y))/(\theta^*(\mu))\right) > v(\Gamma(0,\, y))$. So, if ε_1 is small enough (i.e., if δ_1 is close enough to 0) then reporting truthfully yields higher utility. (Since there is a finite number of agent types, there exists a $\delta_1 > 0$ such that the above inequality holds uniformly for all y.) This establishes that any strategy that involves untruthful communication by a trader when he arrives at the bank later than time t_1 is strictly dominated by a strategy in $S^i_{t_1}$.

Next, suppose that trader i arrives at the bank before time t_1 but no earlier than time $t_2 \equiv t_1 - \varepsilon_2$. Let δ_2 denote the probability that some trader(s) will arrive between time ω_{I+i} and t_1. The utility of trader i from truth-telling is then

$$(1-\delta_2)E\left(v\left((R(I-y))/(\theta^*(\mu))\right)|\omega_{I+j} \notin (\omega_{I+i}, t_1)\text{forall}j\right)$$

$$+\ \delta_2 E\left(v\left((R(I-y))/(\theta^*(\mu))\right)|\omega_{I+j} \in (\omega_{I+i}, t_1)\text{forsome}j\right).$$

From the above arguments, traders who arrive later than t_1 will report truthfully. This, along with Lemma 7, implies that the first term of the above expression is greater than the utility trader i receives from reporting lying , namely $v(\Gamma(0,y))$, provided that δ_2 is small. In other words, any strategy in $S^i_{t_1}$ that involves untruthful report at some time in $(t_2,\, t_1)$ is strictly dominated by a strategy in $S^i_{t_2}$.

Repeat the above process backward towards time 0. Let t_* denote the limit of such process. There is $0 \leq t_* < ... < t_n < ... < t_1 < 1$. It must be that $t_* = 0$ because otherwise one could go through the above reasoning process one step further to time $t_* - \varepsilon$, contradicting to the definition of t_*. The resulting strategy space $S^i_{t_*}$ then consists of only one element, namely $\mu_i = \omega_i$ for all ω.

According to Lemma 6, the unique element in $\prod_{i \in I} S^i_0$ is the unique Bayesian Nash equilibrium of the game (M, α). ∎

6. Conclusion

In a finite-agent version of the Diamond-Dybvig (1983) model with sequential service, we have analyzed the question of maturity transformation. Like Diamond and Dybvig, the (direct revelation) mechanism in our model implements the symmetric, ex ante efficient allocation as a truth-telling equilibrium. Unlike in Diamond and Dybvig, however, our mechanism, which does not contain government intervention (e.g., deposit insurance) of any sort, has no "bank-run" equilibrium.

The work of Diamond and Dybvig has been widely regarded as having shown both the desirability and the fragility of banking industry. In the "good" equilibrium of their model, banking deposit contract supports the efficient allocation, while the "bad" equilibrium entails bank run. When there is aggregate uncertainty, government regulation in the form of deposit insurance is called for to eliminate the run-equilibrium. Wallace (1988), however, showed that the deposit insurance policy is not feasible in the presence of sequential service. One the one hand, the result of Wallace shows that the deposit insurance scheme advocated by Diamond and Dybvig is unfounded. On the other hand, it seems to reinforce the belief that bank-runs are an intrinsic feature of the banking industry. The implication of our result is that bank-runs may not be inherent to banking contracts, at least in the environment of Diamond and Dybvig.

The existence of run-equilibrium of Diamond and Dybvig has been interpreted as an explanation of the numerous observed bank-runs in the United States history. In light of our finding, one would argue that the rational agents in the DD model who are ex ante utility-maximizers would adopt the arrangement of our model which does not leads to runs whatsoever. What this implies then is that certain features of the reality are not captured by the DD environment in order to have a theory that matches the US history.

To begin, one obvious feature of an actual banking system that our model fails to capture is its ongoing nature. If the population of the economy has a overlapping generation structure, then no trader is the last one to arrive the bank as the backward induction methodology in our model would require. The same problem will arise if the size of the population is not observable to individual traders so that no trader is certain whether or not she is almost the last one in line. When these features are present, the validity of our ”no-run” result needs to be reconsidered.

Another feature of a real banking industry which is absence in our model is the incentive problem of banking executives whose objectives may be different from that of a social planner. Suppose that the mechanism in our model is run by a banker, instead of the planner. The traders report their realized types to the banker who then distributes the resources individually back to the traders, supposedly in the way specified by the mechanism. But, if the traders cannot observe each others' reports, then there is no guarantee that the banker allocates the resources based on the true reported state of nature. The bank may keep part of the endowment for his own consumption and then claim that a great deal of resources has been already withdrawn by a number of short-lived agents that is in fact larger than actually reported. Anticipating this possibility and its consequences that less resources will be available at time 1, the long-lived agents may be tempted to withdraw early. This would increase the likelihood of a bank-run. Diamond (1984) and Krasa and Villamil (1992) have looked the issues of monitoring and incentives of bankers which our model does not address. This might be another explanation of why our contract is not observed, and why runs have historically occurred.

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