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On the optimality of transparent monetary policy*

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Working Paper 613

April 2001

ABSTRACT

We analyze the optimal design of monetary rules. We suppose there is an agreed upon social welfare function that depends on the randomly fluctuating state of the economy and that the monetary authority has private information about that state. We suppose the government can constrain the policies of the monetary authority by legislating a rule. In general, well-designed rules trade-off the need to constrain policymakers from the standard time consistency problem arising from the temptation for unexpected inflation with the desire to give them flexibility to react to their private information. Surprisingly, we show that for a wide variety of circumstances the optimal rule gives the monetary authority no flexibility. This rule can be interpreted as a strict inflation targeting rule where the target is a prespecified function of publicly observed data. In this sense, optimal monetary policy is transparent.

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Beginning with Kydland and Prescott (1977), there has been a long debate over the question of how tightly should rules constrain the discretion of the monetary authority in setting monetary policy. In practice, in the United States, the Federal Open Market Committee (FOMC) does not articulate a plan for monetary policy as a simple function of publicly observed data, nor does it explain its past actions as a simple function of past data. One motivation for this ambiguity is that the committee wants to preserve the flexibility to react to private information it has about the state of the economy at its discretion. Cukierman and Meltzer (1986) provide a formal rationale for such ambiguity. A number of economists argue that better outcomes would be achieved if the FOMC set monetary policy as a simple function of observed data. They argue that by so doing the committee can improve the transparency its actions and that this transparency can increase welfare even when the cost of the rule in terms of diminished flexibility for the monetary authority is taken into account. (See, among others, Goodfriend 1986, Bernanke and Mishkin 1997, Blinder 1997 and Svensson and Faust 2000.)

We address the question of how tightly rules for monetary policy should constrain the decisions of the monetary authority by building on an insightful paper by Canzoneri (1985). In it he asks the question: How should the government legislate rules to constrain the policies of the monetary authority when the monetary authority has private information about the economy? One option is for the government to set rules that only loosely constrain the decisions of the monetary authority, leaving it the flexibility to change policy in response to its private information. Another option is for the government to set rules for monetary policy that tightly constrain these decisions, leaving the monetary authority little room to change policy in response to its private information. Canzoneri discusses the pros and cons of several simple types of rules. These rules trade-off the need to constrain policymakers from the standard time consistency problem arising from the temptation to stimulate the economy with a surprise inflation with the desire to give them flexibility to react to their private information. He argues that while it is most interesting to solve for the optimal rule for monetary policy, it is extremely difficult and an open problem.

The point of this paper is to answer the question posed by Canzoneri, namely to find the rule for monetary policy that optimally trades off these needs and desires. We consider a simple model of the monetary policy similar to that of Kydland and Prescott (1977) and Barro and Gordon (1983). There is an agreed-upon social welfare function that depends on the random state of the economy. At one extreme, if the state is observed equally well by both private agents and the monetary authority, then there is no tension between time consistency and flexibility: the government can legislate an optimal rule for monetary policy as a function of publicly observed data and leave the monetary authority no flexibility in implementing that rule.

We analyze the opposite extreme, one in which the monetary authority observes the state exactly and private agents have no direct information about the state. Here there is a tension. A rule with less flexibility mitigates the time consistency problem in which the monetary authority is tempted to claim repeatedly that its information about the current state of the economy justifies a monetary stimulus to output, but such a rule has the cost of leaving little room for the monetary authority to fine tune its policy to its private information. A rule with more flexibility lets the monetary authority fine tune its policy but allows more room for the monetary authority to stimulate the economy with a surprise inflation.

We consider rules for monetary policy that set the range of inflation rates that can be chosen by the monetary authority as a function of publicly observed data. We interpret these rules as inflation targets. As discussed by Bernanke and Mishkin (1997), in practice, inflation targets often take the form of ranges of acceptable inflation rates. Rules with wide ranges of acceptable inflation allow more flexibility while those with narrow ranges allow less flexibility. These rules for monetary policy, once set, determine the rules of the game played by the monetary authority in setting monetary policy and private agents in the economy.

We solve for the optimal rule for setting monetary policy namely, the one that, in equilibrium, leads to the highest level of social welfare. Surprisingly, we find that, under a large set of circumstances, the optimal rule for monetary policy leaves no discretion at all for the monetary authority to vary policy in response to its private information about the state of the economy. That is, the optimal rule for monetary policy lays down precise guidelines for monetary policy as a simple function of publicly observed data and leaves no room for flexibility whatsoever. We argue that this rule can be interpreted as a strict inflation targeting rule where the target is a prespecified function of publicly observed data.

More formally our model can be described as follows. Each period, the monetary authority observes one of N possible privately observed states of the economy denoted θ_i. These states are i.i.d. over time. With full commitment, the monetary authority would prefer to choose higher inflation when higher values of this state are realized and lower inflation when lower values of this state are realized. The incentive problem arises because the state of the economy is not publicly observed. A rule for monetary policy is an interval of inflation rates that constrains the range of inflation rates that the monetary authority is allowed to choose. In this sense, this rule sets the feasible actions available to the monetary authority in the repeated game that is then played between the monetary authority and the public.

We use standard recursive techniques to analyze this repeated game. In it payoffs are the sum of the payoff from current actions and a continuation payoff reflecting the discounted value of payoffs from next period on. To maximize current period payoffs the monetary authority would like to stimulate the economy with a surprise inflation. Thus, for a flexible monetary policy to be incentive compatible it must be that if the monetary authority chooses a higher rate of inflation in the current period it must receive a correspondingly lower continuation value from next period on. The lower continuation payoff occurs because the higher inflation in the current period leads agents to expect inflation in future periods and hence set their wages correspondingly high. We think of this lower continuation value as reflecting a reduction in the credibility of the monetary authority. In contrast, if the monetary authority is allowed no flexibility in setting monetary policy there is no incentive compatibility problem and continuation payoffs can all be the same and as high as possible. Hence, in designing the optimal rule for monetary policy there is a tradeoff between flexibility and credibility.

We find that under a wide variety of circumstances the gains from flexibility are outweighed by the costs of lost credibility. The key to this result lies in the incentive compatibility constraints. These constraints require that the monetary policy be structured so that when given state θ_i is realized, the monetary authority be indifferent between reporting the true state θ_i and claiming instead that the state is the next higher one θ_{i+1}. Thus, if the policy is to allow the monetary authority to choose higher inflation when state θ_{i+1} is realized than it does when state θ_i is realized, then at the higher state θ_{i+1} this policy must trade off current inflation against future payoffs at the marginal rate of substitution of the authority at the lower state θ_i, not at its true, higher, marginal rate of substitution. In this way, incentive compatibility requires that any policy with flexibility trades off this flexibility and credibility in an inefficient manner. We use this logic to show that the optimal policy must specify that monetary policy be independent of the private information of the monetary authority.

In terms of the literature on monetary policy, our paper is most closely related to that of Canzoneri (1985). It is also related to the work of Cukierman and Meltzer (1986) and Faust and Svensson (2000). At a technical level it is draws on the literature on recursive approaches to dynamic games. We use the techniques of Abreu, Pearce, and Stachetti (1991) and use insights related to those in the work of Athey, Bagwell, and Sanchiricho (2000).

1. The economy

Time is discrete. There is one government, one monetary authority, and a continuum of agents. In period 0, the government chooses an inflation target zone that is described by bounds on inflation rates that the monetary authority can choose. (See Bernanke and Mishkin 1997 for a discussion of how, in practice, inflation targets are typically specified as ranges). It then delegates the job of setting monetary policy within these bounds to the monetary authority. We interpret the government as allowing more or less flexibility for the monetary authority depending on the gap between the upper and lower bounds on inflation imposed in the inflation targets. The monetary authority and the agents then play an infinitely repeated game in which monetary policy is constrained by the bounds chosen by the government.

Consider the following game. At the beginning of each period, agents choose individual action z_t from some compact set. We interpret z as (the growth rate of) nominal wages. We let x_t denote the average nominal wage. Next, the monetary authority observes the current realization of its private information θ_t. This private information θ_t is an i.i.d. mean 0 random variable with support $\theta \in \{\theta_0, , \ldots, \theta_N\}$ with $p(\theta)$ denoting the probability of θ. Then the monetary authority chooses money growth $\mu_t \in \left[\underline{\mu}, \bar{\mu}\right]$ where $\underline{\mu}$ and $\bar{\mu}$ are the bounds imposed by the inflation target zone chosen by the government once-and-for-all in period 0.

The monetary authority maximizes a social welfare function that depends on unemployment, inflation, and its private information θ. Each period, inflation π_t is equal to the money growth rate μ_t chosen by the monetary authority. Unemployment is determined by a Phillips curve. The unemployment rate is given by

$$(1) \qquad u_t = U + x_t - \mu_t$$

where x is the average of z across agents and U is a positive constant, which we interpret as the natural rate of unemployment. Social welfare in period t is a function of u_t and π_t and the unobserved state θ_t. Our leading example will be the Kydland and Prescott objective function which has the form

$$(2) \qquad -u_t^2/2 - (\pi_t - \theta_t)^2/2.$$

Using (1) and $\pi_t = \mu_t$ we can write this objective function in terms of nominal wage growth x_t, money growth μ_t and the unobserved shock so that the objective in (2) can be written

$$(3) \qquad R(x_t, \mu_t, \theta_t) = -\frac{1}{2}\left[(U + x_t - \mu_t)^2 + (\mu_t - \theta_t)^2\right].$$

In our example the private information is about the inflation target.

We develop our model for general specifications of the social welfare function $R(x_t, \mu_t, \theta_t)$ which imply (3) as a special case. In the general setup, we interpret θ_t to be private information of the monetary authority regarding the impact of a monetary stimulus on social welfare in the current period.

A. Two Ramsey benchmarks

In what follows we will be interested in a game in which the monetary authority cannot commit to its policy beyond the constraint that $\mu \in \left[\underline{\mu}, \bar{\mu}\right]$ imposed by the inflation target zone set by the government in period 0. Before analyzing this game it is useful to consider two alternative games with commitment that we think of as benchmarks.

Our first benchmark, *the Ramsey policy*, denoted $\mu^R(\theta)$, yields the highest payoff that can be achieved even with commitment. The gap between the associated Ramsey payoff and the payoff in the game without commitment measures the welfare loss of the lack of commitment.

Our second benchmark, *the expected Ramsey policy*, denoted μ^{ER}, is the optimal policy of the monetary authority when it can commit once-and-for-all to a monetary policy that is independent of its private information. This policy is a useful benchmark because in the game without commitment the government can ensure that it implements the expected Ramsey policy as an equilibrium by choosing an inflation target zone that imposes the expected Ramsey money growth rate with no flexibility: $\underline{\mu} = \bar{\mu} = \mu^{ER}$. We use this benchmark in proving our main result, namely it is optimal for the government to choose an inflation target zone that imposes the expected Ramsey policy with no flexibility.

For the Ramsey benchmark consider a game with commitment with the following timing scheme. Before the realization of its type, the monetary authority commits to a schedule for money growth rates $\mu(\theta)$ indicating what money growth rate will be implemented once its type is realized. Next, private agents choose their nominal wages z with associated average nominal wages x. Then the government's type θ is privately realized and money growth rate $\mu(\theta)$ is implemented. The equilibrium allocations and policies in this game solve the *Ramsey problem*

$$\max_{x,\mu(\theta)} \sum_{\theta} R(x, \mu(\theta), \theta) p(\theta)$$

subject to

$$x = \sum_{\theta} \mu(\theta) p(\theta) d\theta$$

For our example (3), the Ramsey policy is $\mu^R(\theta) = \theta$. Note that the Ramsey policy has the monetary authority choosing a money growth rate that is increasing in its type, and thus, with full commitment, it is optimal to allow the monetary authority flexibility in choosing monetary policy to reflect its private information. This feature of the environment leads to tension in the repeated game between flexibility and credibility.

For the second benchmark, consider a variant of this game in which the monetary authority is restricted to choosing a money growth μ that does not vary with its type. This equilibrium allocations and policies in this game solve the *expected Ramsey problem*

$$v^{ER} = \max_{x,\mu} \sum_{\theta} R(x, \mu, \theta) p(\theta) \tag{4}$$

subject to

$$x = \mu.$$

Let μ^{ER} denote the expected Ramsey policy. For our example (3), the expected Ramsey policy is $\mu^{ER} = 0$.

Clearly, for our example (3), the Ramsey policy yields strictly higher welfare than the expected Ramsey policy. More generally, as long as

$$\frac{\partial^2 R(x, \mu, \theta)}{\partial \mu \partial \theta} > 0, \tag{5}$$

the Ramsey policy $\mu^R(\theta)$ is strictly increasing in θ and the Ramsey policy yields strictly higher welfare than the expected Ramsey policy.

B. The repeated game

A monetary policy in this environment is a sequence of functions $\{\mu_t(h_t, \theta_t)\}_{t=0}^{\infty}$ where $\mu_t(h_t, \theta_t) \in [\underline{\mu}, \bar{\mu}]$ specifies the money growth rate chosen by the monetary authority in period t following history $h_t = (\mu_0, \mu_1, \ldots, \mu_t)$, and the current realization of the private information θ_t.

Each period, each agent chooses the action z_t as a function of the history of money growth rates h_t. We assume that each agent's objective is to choose nominal wage growth equal to expected inflation. Taking monetary policy $\mu_t(h_t, \theta_t)$ as given, consumers set $z_t(h_t)$ equal to expected inflation

(6) $$z_t(h_t) = \sum_{\theta} \mu_t(h_t, \theta)p(\theta).$$

In the repeated game, the monetary authority maximizes the discounted sum of social welfare

(7) $$(1-\beta)\sum_{t=0}^{\infty}\sum_{\theta_t} \beta^t R(x_t(h_t), \mu_t(h_t, \theta_t), \theta_t)p(\theta_t)$$

where the future histories h_t are recursively generated from the choice of monetary policy $\mu_t(h_t, \theta_t)$ in the natural way, starting from the null history. The term $(1-\beta)$ normalizes the discounted payoffs to be in the same units as the per-period payoffs.

A *perfect Bayesian equilibrium* of this repeated game, given an inflation target zone $[\underline{\mu}, \bar{\mu}]$, is a monetary policy $\{\mu_t(h_t, \theta_t)\}_{t=0}^{\infty}$, a strategy for wage setting by agents $\{z_t(h_t)\}_{t=0}^{\infty}$, and average wages $\{x_t(h_t)\}_{t=0}^{\infty}$ such that (6) is satisfied in every period following every history

h_t, average wages equal individual wages in that $x_t(h_t) = z_t(h_t)$, and the monetary policy is incentive compatible in the standard sense that, in every period, following every history h_t and realization of the private information θ_t, the monetary authority prefers to choose money growth rate $\mu_t(h_t, \theta_t)$ rather than any other choice of money growth $\mu \in \left[\underline{\mu}, \bar{\mu}\right]$. Note that since average wages $x_t(h_t)$ always equal wages of individual agents $z_t(h_t)$, we need only record average wages from now on.

C. A recursive formulation

Here we formulate the problem of characterizing the set of (perfect Bayesian) equilibrium payoffs of our repeated game recursively along the lines of Abreu, Pearce and Stachetti (1991).

The basic idea is the following. Since there are no physical state variables the set of equilibrium payoffs that can be obtained from any period t on is the same that can be obtained from period 0. Thus, the payoff to any equilibrium strategies for the repeated game can be broken down into payoffs from current actions for the players and continuation payoffs that are themselves drawn from the set of equilibrium payoffs. Following this logic, Abreu, Pearce and Stachetti (1991) show that the set of equilibrium payoffs can be found using a recursive method.

In our environment, this recursive method is as follows. Consider an operator on sets of the following form. Let W be some compact subset of the real line and $\underline{w}$ and $\bar{w}$ be the smallest and largest element of W respectively. The set W may be interpreted as a candidate set of equilibrium payoffs levels of social welfare. In our recursive formulation the current actions are average wages x and a choice of money growth $\mu(\theta)$ for every realized value of the state θ. The continuation payoffs represent the discounted utility for the monetary authority from next period on and are denoted by $w(\mu)$. These payoffs depend on the publicly observable action μ of the monetary authority. Clearly, these payoffs cannot vary directly with the privately observed state θ. Moreover, it is easy to show, along the lines of Chari and Kehoe (1992), that it is superflous to let these payoffs depend on the actions x of the private agents.

We say that actions $x, \mu(\theta) \in \left[\underline{\mu}, \bar{\mu}\right]$, and function $w(\mu)$ are *enforceable by* W if,

(8) $\quad w(\mu) \in W$

(9) $\quad x = \sum_{\theta} \mu(\theta) p(\theta)$

and the incentive constraints

(10) $\quad (1-\beta)R(x, \mu(\theta), \theta) + \beta w(\mu(\theta)) \geq (1-\beta)R(x, \hat{\mu}, \theta) + \beta w(\hat{\mu})$

for all θ and for all $\hat{\mu} \in \left[\underline{\mu}, \bar{\mu}\right]$. Constraint (8) requires that each continuation payoff $w(\mu)$ be drawn from the set of candidate equilibrium payoffs W while constraint (9) requires that average wages equal expected inflation. Constraint (10) requires that for each privately observed state θ, the monetary authority prefer its money growth rate $\mu(\theta)$ and continuation value $w(\mu(\theta))$ rather than a money growth rate $\hat{\mu}$ and corresponding continuation value $w(\hat{\mu})$.

We find it useful to simplify the incentive constraint (10) as follows. In the current period, given a specified set of current actions $\mu(\theta_0), \ldots, \mu(\theta_N)$ and a current realized state θ, there are two types of potential deviations $\hat{\mu}$ by the monetary authority: undetectable deviations and detectable deviations. In an *undetectable deviation*, the monetary authority chooses a money growth rate specified for some other privately observed state. That is, in state θ it chooses

(11) $\quad \hat{\mu} \in \{\mu(\theta_0), \ldots, \mu(\theta_N)\}.$

but $\hat{\mu} \neq \mu(\theta)$. In this type of deviation the monetary authority is effectively misrepresenting the true state of the economy and the incentive constraint can be written

$$(12) \quad (1-\beta)R(x,\mu(\theta),\theta)+\beta w(\mu(\theta)) \geq (1-\beta)R(x,\mu(\hat{\theta}),\theta)+\beta w(\mu(\hat{\theta}))$$

for all $\hat{\theta}$.

In a *detectable deviation* the monetary authority chooses some money growth rate not equal to any of those specified for any of the states. That is, it chooses

$$(13) \quad \hat{\mu} \notin \{\mu(\theta_0),\ldots,\mu(\theta_N)\}.$$

For any such growth rate, private agents know for sure that the monetary authority has deviated. It should be clear that a detectable deviation can be deterred by some continuation payoffs $w(\hat{\mu}) \in W$, if and only if it it can be deterred by the lowest possible continuation payoff $w(\hat{\mu}) = \underline{w}$. Using this logic we write the incentive constraint for undetectable deviations as

$$(14) \quad (1-\beta)R(x,\mu(\theta),\theta)+\beta w(\theta) \geq (1-\beta)R(x,\hat{\mu},\theta)+\beta\underline{w}$$

for all $\hat{\mu} \in \left[\underline{\mu},\bar{\mu}\right]$.

The payoff corresponding to $x, \mu(\theta)$, and $w(\theta)$ is

$$(15) \quad \Pi(x,\mu(\theta),w(\theta)) = \sum_{\theta}\left[(1-\beta)R(x,\mu(\theta),\theta)+\beta w(\theta)\right]p(\theta)$$

Define the operator T that maps sets so payoffs W into new sets of payoffs $T(W)$ according to

$$(16) \quad T(W) = \{\Pi(x,\mu(\theta),w(\theta))|x,\mu(\theta),w(\theta) \text{ are enforceable by } W\}.$$

As demonstrated by Abreu, Pearce, and Stacchetti, the set of equilibrium payoffs is the largest set W that is a fixed point of this operator, namely

$$(17) \quad W^* = T(W^*).$$

For any given set of candidate equilibrium payoffs W, we are interested in finding the largest payoff that is enforceable by W, namely, the largest element $\bar{v} \in T(W)$. We find this payoff by solving the following problem, termed the *best payoff problem,*

$$\text{(18)} \quad \bar{v} = \max_{x,\mu(\theta)\in[\underline{\mu},\bar{\mu}],w(\mu)} \sum_{\theta} \left[(1-\beta)R(x,\mu(\theta),\theta) + \beta w(\mu(\theta))\right] p(\theta)$$

subject to constraints x, $\mu(\theta)$, and $w(\mu)$ are enforceable by W, in that they satisfy (8), (9), (12), and (14).

When we solve this problem with $W = W^*$, (17) implies that the resulting payoff is the highest equilibrium payoff. We refer to this equilibrium as the *best equilibrium* and denote its payoff as $\bar{w}^*$.

2. The optimality of transparent policy

In this section we show the optimal inflation target zone requires that the monetary authority follows the expected Ramsey policy with no flexiblity. This result means that optimal monetary policy is transparent in the sense that it depends only on publicly observed information.

We proceed as follows. We begin with some technical assumptions on the probabilities of various unobserved states and conditions on the payoff function. We then consider a version of the best payoff problem in which we relax a number of constraints. We show that for this relaxed problem the optimal policy for any given W does not vary with the unobserved state. We argue that the optimal policy for the relaxed problem is feasible for the original problem and hence solves the original problem for any given W. Since the equilibrium set of payoffs solves this best problem for some particular set W^*, the optimal equilibrium policy does not vary with the unobserved state either. This establishes that optimal policy depends only on publicly observed information.

We make the following assumptions. We assume that the values of θ_i, $i = 0, 1, \ldots, N$ in the support of the monetary authority's private information are equally spaced. To simplify the notation, we let $\mu_i = \mu(\theta_i)$ and $w_i = w(\mu(\theta_i))$ and refer to i as the *type* of the monetary authority. Recall that this type changes every period and is i.i.d. across periods. Let the probabilities of type i being realized in any period be p_i and the c.d.f.'s be $P_i = \sum_{j=0}^{i} p_i$. We assume that these probabilities satisfy

$$\text{(A1)} \quad \frac{1 - P_i}{p_{i+1}} > \frac{1 - P_{i+1}}{p_{i+2}}$$

for all i. Assumption (A1) is a monotone hazard condition.

We make three assumptions on the payoff function. First, we assume

$$\text{(A2)} \quad \frac{\partial}{\partial \mu} R(x, \mu, \theta_{i+1}) > \frac{\partial}{\partial \mu} R(x, \mu, \theta_i) \text{ for all } i, x \text{ and } \mu.$$

which is a single crossing condition. Second, we assume that

$$\text{(A3)} \quad \frac{\partial}{\partial \mu} R(x, \mu, \theta_i) - \frac{\partial}{\partial \mu} R(x, \mu, \theta_{i-1}) \geq \frac{\partial}{\partial \mu} R(x, \mu', \theta_{i+1}) - \frac{\partial}{\partial \mu} R(x, \mu', \theta_i)$$

for all i whenever $\mu' \geq \mu$, which essentially requires that third derivatives are negative. Finally, we assume that

$$\text{(A4)} \quad \frac{\partial}{\partial \mu} R(x, \mu, \theta_i) > 0$$

for all x and μ in $[\underline{\mu}, \bar{\mu}]$ and for all i. This assumption guarantees the current period payoffs are always increasing in unanticipated inflation. This will imply that in the game the monetary authority is always tempted to stimulate the economy with a surprise inflation. It is immediate to check that the Kydland-Prescott example (3) satisfies assumptions (A2) and (A3). It satisfies assumption (A4) if $U \geq 2\bar{\mu} - \underline{\mu} - \theta_{-1}$.

In the relaxed problem we will replace the incentive constraints with the weaker condition that the policy μ_i be increasing in the type. We do so based on the following lemma.

Lemma 1. (Discrete single crossing implies monotonicity) Under (A2), any incentive compatible allocation has $\mu_{i+1} \geq \mu_i$

Proof. Incentive compatibility requires

$$(1-\beta)R(x, \mu_i, \theta_{i+1}) + \beta w_i \leq (1-\beta)R(x, \mu_{i+1}, \theta_{i+1}) + \beta w_{i+1} \tag{19}$$

$$(1-\beta)R(x, \mu_i, \theta_i) + \beta w_i \geq (1-\beta)R(x, \mu_{i+1}, \theta_i) + \beta w_{i+1} \tag{20}$$

From (19)

$$(1-\beta)[R(x, \mu_{i+1}, \theta_{i+1}) - R(x, \mu_i, \theta_{i+1})] \geq \beta(w_i - w_{i+1}) \tag{21}$$

$$(1-\beta)[R(x, \mu_{i+1}, \theta_i) - R(x, \mu_i, \theta_i)] \leq \beta(w_i - w_{i+1}) \tag{22}$$

These inequalities together imply

$$R(x, \mu_{i+1}, \theta_{i+1}) - R(x, \mu_i, \theta_{i+1}) \geq R(x, \mu_{i+1}, \theta_i) - R(x, \mu_i, \theta_i) \tag{23}$$

Notice that (A2) implies

$$R(x, \mu', \theta_{i+1}) - R(x, \mu, \theta_{i+1}) > R(x, \mu', \theta_i) - R(x, \mu, \theta_i) \text{ for each } i \text{ and } \mu' > \mu$$

$$R(x, \mu', \theta_{i+1}) - R(x, \mu, \theta_{i+1}) < R(x, \mu', \theta_i) - R(x, \mu, \theta_i) \text{ for each } i \text{ and } \mu' < \mu.$$

Hence, under (A2), (23) implies $\mu_{i+1} \geq \mu_i$. Q.E.D.

Note that under (A4) any incentive compatible allocation also has $w_{i+1} \leq w_i$.

Consider a version of the best payoff problem in which we impose that the monetary policy μ_i be non-decreasing in type i,we impose only a subset of the incentive constraints for undetectable deviations (12), we omit the incentive constraints for detectable deviations (14), and we replace the constraints $w_i \in W$ for with simply $w_i \leq \bar{w}$ $i = 0, \ldots, N$.. We refer to this problem as our *relaxed problem,* and it is given by

$$(24) \quad \max_{x,\mu_i,w_i} \sum_{i=0}^{N} p_i \left[(1-\beta) R(x, \mu_i, \theta_i) + \beta w_i \right]$$

subject to constraints

$$(25) \quad w_i \leq \bar{w}$$

$$(26) \quad x = \sum_i p_i \mu_i$$

$$(27) \quad \mu_i \leq \mu_{i+1}$$

$$(28) \quad (1-\beta)R(x, \mu_i, \theta_i) + \beta w_i \geq (1-\beta)R(x, \mu_{i+1}, \theta_i) + \beta w_{i+1}$$

for all $i = 0, 1, \ldots, N$.

We first show that the optimal policy for the relaxed problem does not vary with the unobserved state θ_i.

Proposition 1. Under (A1)-(A4), the optimal monetary policy in the relaxed problem is independent of the unobserved state, so $\mu_0 = \ldots = \mu_N$.

Proof. We prove the proposition with a variational argument. Assume, by way of contradiction, that for some $i = 0, \ldots, N$, $\mu_i < \mu_{i+1}$. Consider the following variation: increase μ_i by $\Delta\mu_i$ and decrease μ_{i+1} by $\Delta\mu_{i+1} = -p_i\Delta\mu_i/p_{i+1}$, hold all other money growth rates at their original levels, and let the continuation values w_i be adjusted so at to change the left and right hand sides of the incentive constraints (28) equally, thus ensuring the variations satisfies these constraints. Under this variation expected inflation is unchanged since $\Delta x = p_i \Delta\mu_i + p_{i+1}\Delta\mu_{i+1} = 0$. By design, the incentive constraints will continue to be satisfied. By showing that this variation is feasible and improves welfare we show the contradiction that establishes our result.

It is helpful to write the impact of this variation on expected utility in two parts. The first part is that due to raising μ_i by $\Delta\mu_i$, for a constant x. We choose Δw_i to hold fixed the right-side of type $i-1's$ incentive constraint (28). Thus,

$$(29) \quad \beta\Delta w_i = -(1-\beta)\frac{\partial R(x, \mu_i, \theta_{i-1})}{\partial \mu}\Delta\mu_i.$$

Note (A4) implies that Δw_i is negative and hence (25) is satisfied. This combination of $\Delta\mu_i$ and Δw_i raises type $i's$ discounted utility by

$$(30) \quad \Delta U_i = (1-\beta)(\frac{\partial R(x, \mu_i, \theta_i)}{\partial \mu} - \frac{\partial R(x, \mu_i, \theta_{i-1})}{\partial \mu})\Delta\mu_i$$

which by assumption (A2) is positive. Intuitively, this variation trades off higher inflation for lower continuation utility for type i at the marginal rate of substitution of the lower type $i-1$. The single crossing assumption implies that higher types have higher marginal rates of substitution and thus type i gains from this trade.

To keep the incentive constraints unchanged for types $i+1$ and higher we raise their continuation utilities by ΔU_i. To see that this part of the variation satisfies (25), recall that incentive compatibility for type $i+1$ requires that

$$(31) \quad (1-\beta)[R(x, \mu_{i+1}, \theta_{i+1}) - R(x, \mu_i, \theta_{i+1})] \geq \beta(w_i - w_{i+1}).$$

Hence $\mu_{i+1} > \mu_i$ and (A4) implies that $w_i > w_{i+1}$. Since $\mu_j \geq \mu_{j-1}$ it follows that $w_i > w_{i+1} \geq w_j$ for all $j \geq i+2$. Thus, since $w_i \leq \bar{w}$ so is w_j for all $j \geq i$ and so (25) is satisfied.

The impact of the first part of the variation on expected utility is to raise it by

$$(32) \quad \sum_{j=i}^{N} p_j \Delta U_i.$$

The second part of the variation is that due to lowering μ_{i+1} by $\Delta\mu_{i+1}$, for a given x. We choose the increment to the continuation utility for type $i+1$ to keep the right-hand side of the incentive constraint for type i unchanged. Thus the incremental impact of this part of the variation on the utility of type $i+1$ is

$$(33) \quad \Delta U_{i+1} = (1-\beta)(\frac{\partial R(x, \mu_{i+1}, \theta_{i+1})}{\partial \mu} - \frac{\partial R(x, \mu_{i+1}, \theta_i)}{\partial \mu})\Delta\mu_{i+1}.$$

which by assumption (A2) is negative. This variation trades off lower inflation for higher continuation utility for type $i+1$ at the marginal rate of substitution of the lower type i. To keep the incentive constraints unchanged for types $i+2$ and higher we lower their continuation utilities by ΔU_{i+1}. This variation clearly satisfies (25).

The impact of the second part of the variation on expected utility is to lower it by

$$(34) \quad \sum_{j=i}^{N} p_j \Delta U_{i+1}.$$

Since $\Delta\mu_{i+1} = -p_i \Delta\mu_i / p_{i+1}$ we can write the total change in utility, the sum of (32) and (34), as $(1-\beta)p_i\Delta\mu_i$ multiplied by

$$(35) \quad \frac{1-P_{i-1}}{p_i}[\frac{\partial R(x, \mu_i, \theta_i)}{\partial \mu} - \frac{\partial R(x, \mu_i, \theta_{i-1})}{\partial \mu}] - \frac{1-P_i}{p_{i+1}}[\frac{\partial R(x, \mu_{i+1}, \theta_{i+1})}{\partial \mu} - \frac{\partial R(x, \mu_{i+1}, \theta_i)}{\partial \mu}].$$

Assumption (A1) is that $(1-P_{i-1})/p_i > (1-P_i)/p_{i+1}$. Assumption (A3), together with $\mu_{i+1} \geq \mu_i$, guarantees that

$$\frac{\partial R(x, \mu_i, \theta_i)}{\partial \mu} - \frac{\partial R(x, \mu_i, \theta_{i-1})}{\partial \mu} \geq \frac{\partial R(x, \mu_{i+1}, \theta_{i+1})}{\partial \mu} - \frac{\partial R(x, \mu_{i+1}, \theta_i)}{\partial \mu}.$$

Hence, (35) is positive.

Thus, if $\mu_i < \mu_{i+1}$ for some $i = 1, \ldots, N$ there is a feasible variation that improves the objective. This is a contradiction and hence, at the solution to the relaxed problem $\mu_0 = \ldots = \mu_N$. *Q.E.D.*

The following proposition is then immediate.

Proposition 2. Under (A1)-(A4), the expected Ramsey value v^{ER} is an upper bound on the best equilibrium value $\bar{w}^*$. The government can uniquely implement the best equilibrium value by requiring that the monetary authority choose the expected Ramsey policy in each period by setting $\underline{\mu} = \bar{\mu} = \mu^{ER}$.

Proof. We have shown that in the relaxed problem it is optimal to have the money growth rate independent of the type. By construction, the expected Ramsey payoff is the highest payoff that can be achieved with a monetary policy that is independent of type. Hence, the best equilibrium value $\bar{w}^*$ is necessarily less than or equal to the expected Ramsey payoff v^{ER}.

If the government sets the inflation target zone to be the single point μ^{ER} then there are no incentive problems arising from the choice of money growth and all of the dropped constraints in the dynamic game are trivially satisfied. The associated payoff is clearly the expected Ramsey payoffs

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t \sum_{\theta} R(\mu^{ER}, \mu^{ER}, \theta)p(\theta) = v^{ER}.$$

By choosing $\bar{\mu} = \underline{\mu} = \mu^{ER}$ the government then uniquely implements the expected Ramsey policies and payoffs. Q.E.D.

3. Conclusion

How should the rules governing the conduct of monetary policy be set? We have argued that optimal rules tightly constrain the discretion of the monetary authority. This rule specifies that monetary policy not react to the private information of the monetary authority. In our simple setting there is no publicly observed state and hence the optimal rule specifies a constant inflation rate. If we extended the model to have a publicly observed state then the optimal rule would respond to this state but not to the private information. To achieve this the government would specify a rule for setting monetary policy a function of public information, with no room for discretion. We interpret this rule as a strict inflation targeting rule specified as a function of observables.

References

[1] Abreu, Dilip. 1986. Extremal Equilibria of Oligopolistic Supergames, *Journal of Economic Theory*, 39(1), 191-225.

[2] Abreu, Dilip. 1988. On the Theory of Infinitely Repeated Games with Discounting. *Econometrica*, 56(2), 383-96.

[3] Abreu, Dilip, Pearce, David and Stacchetti, Ennio. 1986. Optimal Cartel Equilibria with Imperfect Monitoring *Journal of Economic Theory* 39(1), 251-69.

[4] Abreu, Dilip, Pearce, David and Stacchetti, Ennio. 1990. Toward a Theory of Discounted Repeated Games with Imperfect Monitoring. *Econometrica*, 58(5), 1041-63.

[5] Athey, Susan, Bagwell Kyle, and Sanchirico, Chris. 1998. Collusion and Price Rigidity. MIT Working Paper 98-23.

[6] Barro, Robert J. and Gordon, David. 1983. Rules, discretion and reputation in a model of monetary policy. Journal of Monetary Economics, 12, 101-122.

[7] Canzoneri, Matthew B. 1985. Monetary Policy Games and the Role of Private Information, *American Economic Review*, 75(5), 1056-70.

[8] Chang, Roberto. 1998. Credible Monetary Policy in an Infinite Horizon Model: Recursive Approaches. *Journal of Economic Theory*, 81(2), 431-61.

[9] Chari, V. V., Kehoe, Patrick J. 1990. Sustainable Plans. *Journal of Political Economy*, 98(4), 783-802.

[10] Chari, V. V., Kehoe, Patrick J., and Prescott, Edward C.1989. Time Consistency and Policy, in *Modern business cycle theory*, edited by Robert J. Barro,Cambridge, Mass.: Harvard University Press, 265-305.

[11] Cukierman, Alex, Meltzer, Allan H.A. 1986. A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information. *Econometrica;* 54(5), 1099-1128.

[12] Faust, Jon, Svensson, Lars E. O. 1998. Transparency and Credibility: Monetary Policy with Unobservable Goals, NBER working paper 6452.

[13] Faust, Jon, Svensson, Lars E. O. 1999. The Equilibrium Degree of Transparency and Control in Monetary Policy. NBER working paper 7152.

[14] Goodfriend, Marvin. 1986. Monetary Mystique: Secrecy and Central Banking. *Journal of Monetary Economics*; 17(1), 63-92.

[15] Green, Edward-J., and Porter, Robert H. 1984. Noncooperative Collusion under Imperfect Price Information. *Econometrica*, 52(1), 87-100.

[16] Kydland, Finn, and Prescott, Edward C. 1077. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85, 473-491.

[17] Persson, Torsten, and Tabellini, Guido. 1994. *Monetary and Fiscal Policy: Volume 1, Credibility*. MIT Press, Cambridge, MA.

[18] Phelan, Chris, and Stachetti, Ennio. 1999. Sequential Equilibria in a Ramsey Tax Model. Federal Reserve Bank of Minneapolis Staff Report. (Forthcoming *Econometrica*.)