

Financial Market Structure and the Ergodicity of Prices

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September 1997

Abstract

The properties of prices, especially with respect to initial conditions related to market startup and unusual shocks to the market environment, are of concern to regulators assessing alternative financial market structures. A natural way to investigate the importance of initial conditions is to evaluate the ergodicity of the price process. A consistent nonparametric test for ergodic failure is introduced for this purpose. We compare the ergodic properties of prices across (i) a computerized market, characterized by an electronic limit order book and a separate batch opening protocol; and (ii) a traditional open-outcry floor market. The work is enabled in part by unusual matched high-frequency trading data on identical financial instruments traded in both markets over the same 24-hour period. We find that differences in market structure matter, in the sense that prices in the automated market exhibit ergodic failure, while prices generated by floor trading are ergodic. Variations in information environments over the course of the day also are considered, but cannot account for the results.

KEY WORDS: auctions, financial market structure, ergodicity, consistent nonparametric test procedures, high frequency data

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1. Background

A primary contribution of work on auction mechanisms and financial market microstructure is that the form of the trading institution affects behavior and the stochastic properties of prices (e.g., Friedman, 1993, on auctions and Madhavan, 1992, on trading market structure). In this paper, we focus on the persistence properties of prices, and more narrowly, on the ergodicity of price processes across trading market structures. Our vehicle for the analysis is a newly developed test for the ergodicity of a time series. The experiment considered is a comparison of pricing across computerized and traditional floor trading markets in precisely the same financial instruments. The work is motivated by several considerations.

Computerized trading markets constitute the fastest growing institution in financial market structure, and are sharply differentiated from open-outcry floor trading venues. While both generally consist of a form of continuous double auction, the former implements this paradigm through mathematical algorithms that process allowable trader messages into prices and quantity allocations via the operation of an electronic limit order book. There is no similar consolidated order book on the typical floor, where trading is conducted orally. An automated system accepts and processes orders which do not represent improvement of the existing best market quotes, while the rules of floor trading mandate matching or improving quotes in order to have standing in the market. These fundamental differences already have been shown to have effects on pricing equilibria (Glosten, 1994) and on the dynamics of second moment properties of prices (Bollerslev and Domowitz, 1991).

Ergodic failure is important in the assessment of market designs. The properties of prices, including those with respect to initial conditions related to market startup and unusual shocks to the market environment, are of continuing concern to regulators (see, for example, Domowitz, 1990). There are two notable points relevant to the markets under consideration and to the general analysis in this paper.

First, computerized markets begin with an automated batch auction procedure to set an initial price, while many floor markets, including that under study here, simply start continuous trading without any opening protocol. In regulatory communication, Donovan (1988), for example, articulates the concern that continuous trading over the remainder of the day can be persistently influenced by the design and operation of a separate opening auction. This point has been taken seriously by some exchange policy makers. For example, it is clearly stated in the description of the automated APT market, a part of the London International Financial Futures and Options Exchange (LIFFE), that traders may not enter orders prior to continuous trading. The rationale is quotable: "...to ensure that the market trades its way to an opening price without having to impose an *artificial* algorithm that attempts to match trades on the market opening" (LIFFE, 1991, section 5.2.4; italics added). The opening price represents an initial condition for the day's trading. If the opening price is not representative, and system design encourages ergodic failure, pricing over the remainder of the day cannot fully reflect market fundamentals. Such a result violates the public interest requirements embodied in existing market regulation pertaining to reliable price discovery (e.g., U.S. Commodity Exchange Act of 1974, section 3).

Second, the acceptance and processing of orders on an electronic book away from the best prices in the market can decrease the speed of information revelation through prices, also hindering efficient price discovery. This may result in increased dependence on an initial condition following a shock to the market, as well as that induced by natural trading halts, including, but not limited to, the close of the market for the day.

A natural way to investigate the importance of initial conditions is to evaluate the ergodicity of a process. One loose characterization of ergodicity is the lack of dependence of the process on the initial condition in the long run, a conceptualization made rigorous in section 2 below. We introduce a consistent nonparametric test for ergodicity in this paper, and apply it to an examination of the issues above.

Failure of the ergodic assumption has largely been studied for linear processes in the parametric unit root framework (see Stock, 1995, for an overview and references). Design and analysis of unit root tests have been feasible through clear definitions and applications of the concept of an $I(0)$ random process applied to Markovian environments, signifying stationary ergodic behavior of the process under consideration. Recently, however, Granger (1995) demonstrates that the $I(0)$ concept is not well defined for certain linear environments and nonlinear models more generally.

The test proposed is differentiated from the unit root paradigm in several respects. The nonparametric nature of the procedure does away with the need for specifying precise models, beyond a characterization of transition densities, for both the null and alternative hypothesis. Nonlinear processes, in particular, are accommodated. The concept of $I(0)$ is replaced by its underlying motivation, namely the definition of ergodicity itself, obviating the criticisms noted above. The test does not require that failure of the ergodic property be linked to nonstationarity of the process. In particular, it is capable of capturing ergodic failure that is not characterized by explosiveness, as in trend or unit root models. The potential importance of this generalization is illustrated by the analysis of the properties of interactive Markov chains, as in Conlisk (1976), and by nonergodic solutions to dynamic programs (e.g., Majumdar, et al., 1989, and Eckstein, et al., 1991).

We present the methodology underlying the test in the next section, with additional detail in the Appendix. The data, including some information about the institutional differences between market structures, are described in section 3. We turn to the substantive application and results in section 4. The basic finding is simply stated: price processes are found to be ergodic in the case of floor trading, while they are nonergodic as produced by the automated market. We discuss systems theory and differences in information environments that could represent explanations of such results. Some concluding remarks are offered in the last section.

2. A Consistent Test of Ergodicity

2.1 Concepts and Definitions

We consider a univariate Markov process on \mathfrak{R} , defined by a one-step transition function $p_{\tau}(\xi, A)$, for $\xi \in \mathfrak{R}$ and $A \in B(\mathfrak{R})$, where $B(\mathfrak{R})$ is the Borel σ -algebra.

Although $p_T(\xi, A)$ must be estimated, accounting for the use of the time subscript, it is convenient for the moment to treat it as though it were known. We denote the corresponding transition density by $f_T(\cdot, \cdot)$. Starting from an initial density $g_0(x)$, the probability of the process falling in any Borel set A at period s is defined by

$$\Pr_{g_0}(x_s \in A) = \int_A g_0(\xi) p_T^s(\xi, d\eta) \equiv \int_A g_s(\eta) d\eta.$$

This expression implicitly defines the Markov operator $P_T: D(\mathfrak{R}) \rightarrow D(\mathfrak{R})$, via $g_s = P^s g_0$, where $D(\cdot)$ is the space of densities and $p^s(\cdot, \cdot)$ is the s -step transition probability, defined recursively in the usual fashion.

If a stationary density g^* exists for P_T , we say the stochastic process is *stationary-ergodic* iff

$$\lim_{s \uparrow \infty} \frac{1}{s} \sum_{i=0}^{s-1} P_T^i g(x) = g^*(x)$$

for all $x \in X$ and for all $g \in D(\mathfrak{R})$. More generally, the process is said to be *ergodic* iff

$$\lim_{s \uparrow \infty} \left| \frac{1}{s} \sum_{i=0}^{s-1} P_T^i g_1(x) - P_T^i g_2(x) \right| = 0$$

for all $x \in X$ and for all $g_1, g_2 \in D^2(\mathfrak{R})$. These definitions are standard; see Loève (1978) and Isaacson and Madsen (1976), for example.

The intuition behind the testing strategy to follow may now be clarified. A nonparametric estimate of f_T permits recovery of p_T , hence P_T , via integration. Given two initial densities, random samples from the associated Cesàro average densities are selected, and a test of equality of two distributions is used to evaluate the limits in the definition of ergodicity above. Of course, the condition “for all $g_1, g_2 \in D^2(\mathfrak{R})$ ” must be taken seriously; this is treated in depth in Domowitz and El-Gamal (1993), and embodied in the algorithms described in the Appendix, via randomization. This description ignores the complications induced by the necessity of estimating the law of motion, and we now turn to the relevant issues.

2.2 The Maintained Hypothesis

We assume that any given time series contains two components: (i) a systematic transition density $p(x_{t+1} | S_{t+1})$ defining the density of x_{t+1} conditional on its previous value x_t and a “state” variable S_{t+1} ; and (ii) an idiosyncratic shock process. The law of motion for the observed series is given by

$$\tilde{p}(x_{t+1} | S_{t+1}) = \begin{cases} p(x_{t+1} | S_{t+1}) & \text{if shock}_t = 0 \\ (1 - \alpha)p(x_{t+1} | S_{t+1}) + \alpha v & \text{if shock}_t = 1 \end{cases}$$

where v is a measure with full support, α is some scalar in $(0, 1)$, and shock_t is a stochastic process taking values in $\{0, 1\}$, independent of $\{x_t, S_t\}$.

The state variable S_t permits us to encompass regime switching and other forms of nonstationarity, which may or may not be accompanied by ergodic failure. We assume that the process governing S_t is independent of $\{x_t\}$, and has a unique invariant distribution $\sigma = (\sigma_1, \dots, \sigma_N)$, defining the asymptotic frequency $\sigma_i > 0$ of $S_t = i$.

Hamilton (1989), for example, achieves this through an aperiodic irreducible Markov transition matrix for S_t . The test also will fail to reject in the presence of other forms of nonstationary, but ergodic behavior. These include the asymptotically stationary models of Kampe de Fariet and Frenkiel (1962), simple heteroskedasticity, as illustrated by Makkadem (1987), stable time-varying parameter models as in Rao (1978), and Markov models exhibiting general forms of nonstationarity of the transition kernel that admit so-called weak ergodicity (e.g., Isaacson and Madsen, 1976). Formalization for such cases requires an extension of the state space of $\{s_t\}$ from finite to infinite, a complication we avoid to eliminate the need for measure-theoretic niceties that are extraneous to the analysis.

Since the state is typically not observable, we are interested in testing the ergodicity of the marginal process on x . This is facilitated by defining

$$p^*(x_{t,\cdot}) = \sum_{i=1}^N \sigma_i p(x_{t,\cdot} | S_{t+1} = i)$$

and

$$\tilde{p}^*(x_{t,\cdot}) = \sum_{i=1}^N \sigma_i \tilde{p}(x_{t,\cdot} | S_{t+1} = i)$$

where the first equation defines the average systematic component for the observable marginal process $\{x_t\}$, and the second defines the average law for the observed process itself. In Domowitz and El-Gamal (1997) we show that the transition density $p(x_{t,\cdot} | S_t)$ is ergodic if and only if the averaged transition $p^*(x_{t,\cdot})$ is ergodic, providing a foundation for the empirical application of the test to potentially nonstationary laws of motion for x .

The process on shock $_t$ is motivated purely by technical considerations, with the following intuition. In order to guarantee the consistency of a nonparametric estimate of $\tilde{p}^*(x_{t,\cdot})$ under both the null and alternative, the shock process must be sufficiently persistent. Nevertheless, such shocks must be sufficiently infrequent in the limit so that the αv component does not dominate $\tilde{p}^*(x_{t,\cdot})$ allowing a consistent estimate of $p^*(x_{t,\cdot})$. This is achieved in Domowitz and El-Gamal (1997) by assuming that the partial sum of shock $_t$ tends to infinity, as $T \uparrow \infty$, while the average of the shock process tends to zero, almost surely, which can be satisfied by a reasonably large number of possible scenarios.

2.3 Estimation and Testing

The test is constructed by nonparametrically estimating a Markovian transition density p_T on X , and using this estimate in the algorithm described below. The assumptions on the state and shock process, together with standard conditions for kernel density estimation, are sufficient to show that this estimator p_T is consistent for \tilde{p}^* . Further, a consistent test of the ergodicity of p_T is asymptotically consistent for the systematic component p^* , as $T \uparrow \infty$, given the assumptions concerning the shock process. These propositions are proved in Domowitz and El-Gamal (1997). Finally, the result cited in section 2.2 yields the consistency of the test for p .

We estimate the transition density over a compact set X using the kernel density estimator

$$f_T(x, x') = j_T(x, x') / m_T(x)$$

where

$$j_T(x, x') = \frac{1}{Th_T^2} \sum_{t=1}^{T-1} K\left(\frac{x-x_t}{h_T}\right) K\left(\frac{x'-x_{t+1}}{h_T}\right)$$

and

$$m_T(x) = \frac{1}{Th_T} \sum_{t=1}^T K\left(\frac{x-x_t}{h_T}\right).$$

Standard assumptions with respect to continuity of $m(\cdot)$ and $j(\cdot, \cdot)$, as well as with respect to the limiting properties of $K(\cdot)$ and h_T are imposed; see, for example, Roussas (1969).

Let

$$p_T(x, A) = \int_A f_T(x, y) dy$$

where f_T is estimated on a compact set $X \in B(\mathcal{R})$, according to the formulas above. We can now state the basic result underlying the test used in the application to follow, expressed formally in Domowitz and El-Gamal (1997, Theorem 2). Let the conditions discussed above on the nature of the law of motion, the laws governing the state and shock processes, and assumptions permitting consistent kernel estimation hold. Consider the following algorithm:

1. Choose a compact set in the support of f_T .
2. Randomly draw two initial densities g and g' , using *Algorithm A* described in the Appendix.

3. Construct two i.i.d. samples of size n from the Cesàro averages $\frac{1}{s} \sum_{i=0}^{s-1} P_T^i g$ and

$$\frac{1}{s} \sum_{i=0}^{s-1} P_T^i g'$$

using *Algorithm B* described in the Appendix.

4. Conduct a test of equality of two distributions, e.g., Kolmogorov-Smirnov, obtaining the p-value for this test.
5. Repeat steps 2-4 to obtain a number of p-values. Under the null of ergodicity, the p-values so obtained should be uniformly distributed on $[0, 1]$.

Then the test in steps 1-4, applied to the estimated p_T , obtains the correct asymptotic size under the null of ergodicity of p , and asymptotic power unity against the alternative of nonergodicity of p , as first $T \uparrow \infty$, then $X \uparrow \mathcal{R}$, $s = O(T) \uparrow \infty$, and then $n \uparrow \infty$.

We close this section with a few practical remarks related to the methodology. The asymptotics require taking limits as $T \uparrow \infty$, followed by limits with respect to s and n . This is not very restrictive, because s and n are control parameters in the simulations; the important assumption remains the familiar one of $T \uparrow \infty$. Monte Carlo analyses in Domowitz and El-Gamal (1993, 1996, 1997) suggest that values of $s=50$ and $n=200$ produce good size and power performance. Details of the algorithm in the Appendix also include the choice of k , the dimensionality of the polynomial representation of initial densities, taken as $k=10$ here. While k should be arbitrarily large to ensure that the set of such representations is dense in the space of densities, moderate k gives very good approximations to many densities. These values for s , n , and k are used in the application to follow.

The method also requires the estimation of the transition density over a compact interval X , which is assumed to grow. Two points are relevant in this regard. First, although the theory treats this interval as though fixed *ex ante*, as a practical matter we choose the interval by taking the middle 90 percent of any given sample. This is similar to standard procedures involved in the estimation and/or testing of structural breaks in regression frameworks. Second, the order of the limits with respect to X is crucial, and theoretically rules out the application of the procedure to random walk problems, for example. More formally, the test obtains the correct asymptotic size under both stationary and nonstationary ergodic cases under the null, and unit power against stationary nonergodic alternatives. The power of the test is indeterminate in the case of nonstationary, nonergodic alternatives. On the other hand, the issue appears to be a technical one of proof methodology, in that Monte Carlo experiments and applications to data generally thought to be characterized by a unit root demonstrate that the test has good power performance against a variety of unit root and random walk alternatives (Domowitz and El-Gamal, 1997).

Finally, the replication recommended in step 5 is not required for the consistency of the test. Given the randomized draw of initial densities, such replication rules out the possibility that two researchers, each with the same data and method, might reach two different conclusions, through the law of large numbers. Formalization of this idea is contained in Delicado and Placencia (1997), and references therein.

3. Institutions and Data

3.1. Market Mechanisms

The two markets under consideration are the Globex automated trading system and the open-outcry floor market of the Chicago Mercantile Exchange (CME). A complete description of the rules governing the Globex algorithm is provided in Domowitz (1990), but the basic setup is easily summarized. Anonymous limit bids and offers are entered via computer terminals, without a central trading location (other than the computer processing the data). New orders are filled at the best available price. In case of ties, orders are filled on a first-in, first-out basis, with some allowance for undisplayed order flow to be transacted at a lower priority. The maximum possible quantity will be traded by completely filling an order, subject to liquidity available in the system. Unfilled quantities remain on the electronic order book until cancelled or filled. The limit order book is continuously displayed to participants, and information on transactions is instantaneous. Trading is opened at the beginning of a session by an automated batch auction, producing multiple trades at a single opening price, before the commencement of the continuous double auction. Unfilled orders at the open are routed automatically to the continuous limit order book.

In contrast, trading on the CME occurs on a central floor, allowing the identification of counterparties. There is no centralized limit order book. Recorded data on quotations is sporadic, at best. Once a trader calls out a bid (offer), any subsequent bid (offer) must be higher (lower) than the standing order in the market, until the next transaction occurs. Contracts are traded when an outstanding bid or

offer is accepted by another trader. If more than one trader attempts to accept the bid, say, there are rules of thumb on the trading floor that determine how such orders are split. Although time priority is in force, strong physical presence on the floor can be a dominating factor in determining whose order is filled first. There is no separate opening protocol; continuous trading simply starts at the opening bell of the session.

3.2. *Data*

Our comparisons of these two market structures are based on trading data for the September futures contracts on the S&P 500 (SP) index, the Deutschemark (DM), the Yen, and the Swiss Franc (SF), over the period 7/1/94 through 9/1/94. Trading hours will be relevant in the discussion of results. On Central Standard Time, the trading week opens with Globex trading on Sunday at 6:30 p.m., and closes on the floor on Friday. If Monday is a holiday and Tuesday is not, trading starts on Monday at 6:30 p.m. The floor stops trading in the SP contract at 3:15, and Globex opens at 3:45, with continuous trading up to a half hour before the floor again opens at 8:30 a.m. Currency floor trading ends at 2:00 p.m., with a Globex opening at 2:30. It stays open until 6:45 a.m., and floor trading resumes at 7:20.

Data are time-stamped to the minute, and any multiple observations within a minute are ordered in terms of time of arrival. Globex prices are computed as the log of the midpoint of the best bid and ask quotes at any point in time, a common transformation and practice for proxying transactions prices (see, for example, Engle, 1996). Although transactions prices are available, there are far fewer of them, and we prefer the quote midpoints for sample size considerations. We have, however, verified that the results reported below are maintained if transactions prices are used, replacing the automatic cross-validation procedure in the Appendix with Silverman's original rules of thumb; the former tends to oversmooth, based on the relative paucity of observations. Quotes are only sporadically available from the CME, and log transactions prices are used. The average number of observations per series for the floor is 73,465, with a range from 67,059 to 81,150. For Globex, the average is 38,996, ranging from 25,992 to 55,800.

4. **Price Dynamics Across Trading Mechanisms**

The results of our analysis are presented in graphical form. For each series, a plot of the time series is given together with a kernel-smoothed density of p-values over 100 replications of the test.

4.1. *Currency Futures*

Sharp results are obtained for the DM and SF futures prices, presented in figures 1-4 and 5-8, respectively. Ergodicity is rejected for trading in the automated market, but we fail to reject the ergodic null for the floor. Rejections in the former case are clearly exemplified by the strong peak in the density of p-values in the range of 0 to 0.1, approximately, with the frequency of remaining p-values dropping sharply thereafter. In the case of the floor, the distribution of values is nearly uniform, taking into account the scale of the vertical axis. For example, were the density of p-values

for the floor-traded DM graphed on the same scale as for Globex, the graph would be virtually a horizontal line.

Beyond arguments with respect to trading mechanisms, discussed below, the reason for the rejections can be clarified based on the time series plots. Consider figures 5 and 7 for the SF contract, for example. The Globex price pattern is marked by sharp increases and decreases in the series. Following each such movement, the price often remains for some time within a relatively narrow range before another jump. This suggests the existence of multiple ergodic classes, confirmed by the testing procedure. Although market fundamentals necessarily result in similar patterns on the floor, the movements are significantly smoother. Prices tend to oscillate around the short-lived trends, as well as around peaks and troughs, and so multiple ergodic classes cannot be identified.

The pattern of trading in the Yen fails to reveal the same sort of disparity between market systems. Abrupt shifts in Globex are much smaller than observed in the DM or SF, and the market tends not to linger around particular price points. We, therefore, fail to reject the ergodic null in both cases. We conjecture that the result is due in part to increased intervention activity aimed at stabilizing the Yen, with Japanese central bank announcements during Globex hours, as suggested by Bonser-Neal (1996). This would obviate jumps between ergodic subclasses, but we cannot confirm the hypothesis, since official data on interventions are not available for the period in question.

4.2. *S&P 500 Futures*

We present two sets of results for the SP series. Figures 13-16 contain results for the log levels of the series. Although the Globex series is marked by the same sort of jump behavior as observed for the currencies, there is also a weak rejection of the ergodic null for the floor-traded series. Both series, however, are characterized by a secular trend that reflects the general bull market in stocks of the period under study. It is, therefore, no surprise that the test is biased towards rejection, regardless of market mechanism.

The obvious response is simply to detrend both series. Results are reported in figures 17-20. The detrended series again exhibit behavior as described for the currencies on Globex, and the mitigation of such jumps for the floor. The p-value densities now clearly show a rejection of the ergodic null for Globex, and failure to reject for the floor-traded series.

4.3. *Information Environments*

The use of matched data within the same 24-hour period for both markets controls for differences in underlying market fundamentals to a large extent. One might conjecture, however, that variations in outside information flow over the 24-hour period explain the results.

Floor trading in the SP contract, for example, occurs during the same hours as trading in the stocks underlying the index in New York. There is very little trading in the vast majority of stocks comprising the S&P 500 index during the hours of Globex operations. Occasional shocks overnight, followed by little information flow, might

account for the patterns and results discussed above, compared with a steady flow of new information during the day, and the associated smoothing of price response.

This line of reasoning cannot account for the results obtained for the currencies, however. The analogue of the New York stock market is the interbank currency spot market, which is the main source of information revelation through prices. Interbank currency trading in the DM and SF, for example, is particularly heavy from 2:00 a.m. to 10:00 a.m. CST, largely covering Globex hours. Asian markets, with an emphasis on Yen trading, open even earlier, by about 8 hours. Similar strong activity is observed for New York banks, from roughly 7:00 a.m. through 3:00 p.m., now encompassing CME hours for the most part. Although outside information flow is thin for a short period during the Globex session, it is difficult to ascribe the overall pattern of results to information differences.

4.4 Automated Mechanism Design and Ergodic Failure

Domowitz and Wang (1994) identify two general conditions for the existence of stationary invariant distributions of prices in an order book environment such as Globex. First, arrival processes of bids, offers, cancellations of orders, and instructions to hit the bid or lift the offer, must be stationary. The information environments discussed above may contribute to a failure of this requirement.

Second, stationarity of price distributions depends on net inflows and outflows to the system, summed over all priority classes in the queue generated by the order book. The intuition is the same as for single-server queues, that the mean number of arrivals be strictly less than the mean number of departures for stationarity of queue length. In more practical terms, the requirement is that the probability that an order is immediately executed upon arrival is strictly positive. Relative illiquidity in the overnight market (compared with the floor) could conceivably generate violation of such a condition. On the other hand, Bollerslev and Domowitz (1991) show that the limit order book generally increases persistence in pricing dynamics, consistent with the results presented here.

We also have noted some differences across markets in terms of display and dissemination of market data, with the automated market reflecting greater transparency of price information. Greater price transparency does not necessarily translate into more efficient pricing, however, as demonstrated by Madhavan (1995).

5. Conclusion

We find that, in three of four cases, trading on an automated market results in nonergodic price processes, while floor trading produces prices that retain the ergodic property. The implication is that the automated mechanism does not encourage information revelation through prices in such a way as to escape the effects of initial conditions induced by an opening auction, trading halts or other large shocks to the market environment. Such failure is inconsistent with requirements pertaining to reliable price discovery, in the regulatory assessment of alternative market structures. Given that both market mechanisms are in the form of standard double auctions, the source of this result appears to lie in the operation of a consolidated limit order book in the automated venue. This feature makes the automated market particularly vulnerable to nonstationarity in order and information flow, as well as to any illiquidity in the market environment.

The findings are particularly relevant when combined with the existence of an opening protocol in the automated market. A separate opening algorithm may produce misleading information about fundamentals, which are only slowly reversed over the course of the regular trading day. Such an effect is exacerbated by ergodic failure. Interestingly, the London International Financial Futures Exchange has refused to institute an opening protocol even for its automated APT system, citing such openings as being "artificial," and reinforcing this intuition in practice.

The vehicle for our analysis is a newly developed test of ergodicity for time series. Beyond the particular empirical issue addressed here, the test should be useful in other situations of practical interest. Policy conclusions other than those considered in this paper also depend on an evaluation of ergodicity. A leading example is the analysis of income distributions across countries (e.g., Quah, 1997). The test may be used to assess the proper "balance" of the right and left hand sides of a dynamical relationship, expressed in terms of persistence properties; see, for example, Granger (1995). Granger also introduces a concept to replace that of $I(0)$, which he calls "short memory in mean." The idea is reminiscent of mixing conditions, and it is shown in Domowitz and El-Gamal (1997) how the test presented here can be modified to test the null of mixing with respect to a stochastic process. Dynamic behavioral models may easily result in nonergodic processes (e.g., Arthur, 1989, Majumdar et al., 1989, and Eckstein et al., 1991), but explicit verification is usually difficult, and must be ascertained from the data. Finally, statistical inference problems exist with respect to evaluation of estimates based on nonergodic processes (e.g., Basawa and Scott, 1983). An operational test for the ergodicity of the relevant time series will enable a researcher to identify appropriate inference methods in empirical applications.

Appendix

This appendix is devoted to a statement of the algorithms used in the test. The implementation of these algorithms used in the data analysis was conducted in GAUSS™ version 3.2.39 for Solaris 5.x, and using the GAUSS™ Maximum Likelihood Version 4.0.22/1 module. All computations were performed on a Sun Ultra2 with two 168 Mhz procesors. For a given series, the code reads in the data and conducts the test by implementing the following steps.

Algorithm 1

1. Calculate cross-validation bandwidth for kernel estimation, using the GAUSS maximum likelihood module, and initializing the maximum likelihood search by a Silverman rule of thumb $h_T = \eta \times T^{-1/5}$, where η was selected to be the difference between the 75th and 25th percentiles, a choice less sensitive to outliers than Silverman's choice of $\eta = \sigma$, the standard deviation of the series. For a survey of methods of bandwidth selection for kernel density estimation, see Jones et al. (1996). For discussion of the applicability of essentially the same selection methods in time series contexts, see Bosq (1996, pp. 88-91).
2. Obtain kernel density estimates m_T , j_T , and f_T as discussed in section 1.3, using standard Normal kernels. The estimated transition density f_T is estimated on a 100×100 grid, and implies a finite approximation to the Frobenius-Perron operator on density space: $(P_T g)(x) = \int g(y) f_T(y, x) dy$. Justification for the use of finite approximations of the Frobenius-Perron operator on the density space is given by Li (1976). Alternative analysis justifying finite approximations to continuous Markov processes is contained in Guihenneuc-Jouhaux and Robert (1996).
3. Perform n tests (equal to 100 in the applications here), choosing values for s , n , and k (50, 200, and 10, respectively in our applications), as follows:
 - a) Use *Algorithm A* below to generate random coefficients (c_1, \dots, c_k) and (c'_1, \dots, c'_k) ; draw n data points, each i.i.d. from $g(x) = \sum_{i=1}^k c_i x^i$ and $g'(x) = \sum_{i=1}^k c'_i x^i$.
 - b) Use *Algorithm B* below to randomly draw n i.i.d. samples each from $(1/s) \sum_{i=1}^s P_T^{i-1} g$ and $(1/s) \sum_{i=1}^s P_T^{i-1} g'$.
 - c) Use *Algorithm C* below to perform a Kolmogorov-Smirnov test of the equality of the distributions of the two samples, and obtain a p-value for the test.
1. Plot the time series, the estimated marginal and transition densities, and a kernel-smoothed density of the p-values so obtained. Under the null hypothesis of ergodicity, this density should be uniform over $[0, 1]$, and under the alternative, it should put a significant weight on low values of p .

Algorithm A

1. Choose a compact interval $[a,b]$, the support of initial densities.
2. Generate a random vector p_0, \dots, p_k on the $k+1$ -dimensional simplex as follows:
 - a) Generate U_1, \dots, U_k i.i.d. $U[0,1]$.
 - b) Generate the order statistics $U_{(1)}, \dots, U_{(k)}$ and $U_{(0)}=1, U_{(k+1)}=1$.
 - c) Set $p_i = U_{(i+1)} - U_{(i)}$.
1. Compute c_0, \dots, c_k as follows:
 - a) For $i=1, \dots, k$, generate V_i i.i.d. $U[0,1]$.
 - b) Set $c_i = (i+1)p_i$ if $V_i > 0.5$; otherwise set $c_i = -((i+1)/i) p_i$.
 - c) Set $c_0 = p_0 - \sum_{i: c_i < 0} c_i$.
1. Generate i.i.d. variables from the density $g(x) = \sum_{i=0}^k c_i x^i$ using the algorithm of Ahrens and Dieter (1974).
2. Transform the i.i.d. draws on $[0,1]$ by multiplying them by b and adding a , to produce i.i.d. draws on the compact set $[a,b]$.

Algorithm A provides samples of i.i.d. draws from a polynomial density of order k , whose coefficients are drawn randomly. The product of each application of the algorithm is a sample of n i.i.d. draws from the constructed density $g(x)$, $x_i^g, i=1, \dots, n$. To obtain a sample of n i.i.d. draws from the Cesàro averaged density $\sum_{j=0}^{s-1} P_T^j g$, the following algorithm is used:

Algorithm B

1. For each of the n observations x_i^g , draw a value $t_i \in \{0, \dots, s\}$, with probability $1/(s+1)$ for each value.
2. For the i^{th} observation, iterate for $j=1, \dots, t_i$:
 - a) Given x_i^g , normalize $f_T(\tilde{x}_i^g, \cdot)$ to sum to one, where \tilde{x}_i^g is the closest point on the 100×100 grid to x_i^g .
 - b) Replace x_i^g with a random draw from the multinomial random variables with probabilities defined by the normalized $f_T(\tilde{x}_i^g, \cdot)$.
 - c) Repeat steps (a) and (b) t_i times.

The joint product of *Algorithms A* and *B*, applied twice, is two i.i.d. samples from the Cesàro averages based on two randomly drawn initial densities. *Algorithm C* is simply the implementation of the two-sample Kolmogorov-Smirnov test of equality of two distributions to those two samples. The algorithm was implemented by coding the procedures *kstwo* and *probks* from Press et al. (1988, pp. 493-494) in GAUSS™.

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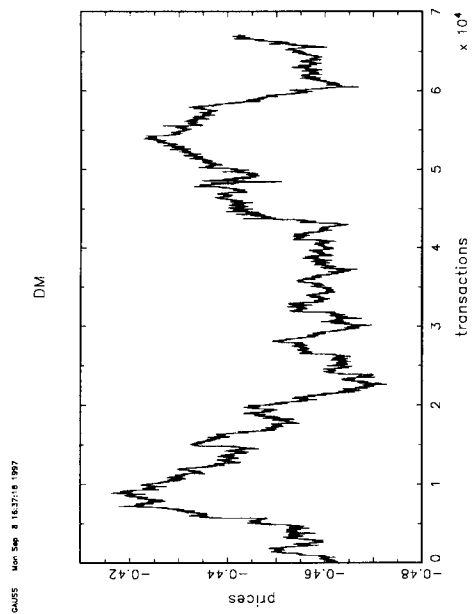


Figure 1: Floor prices of DM

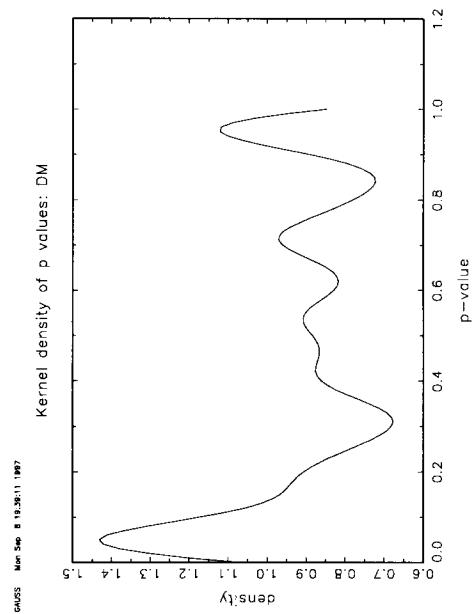


Figure 2: Density of p-values of test for DM

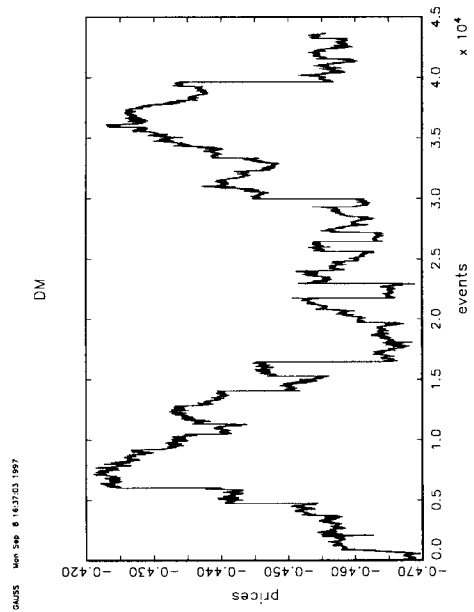


Figure 3: Globex prices of DM

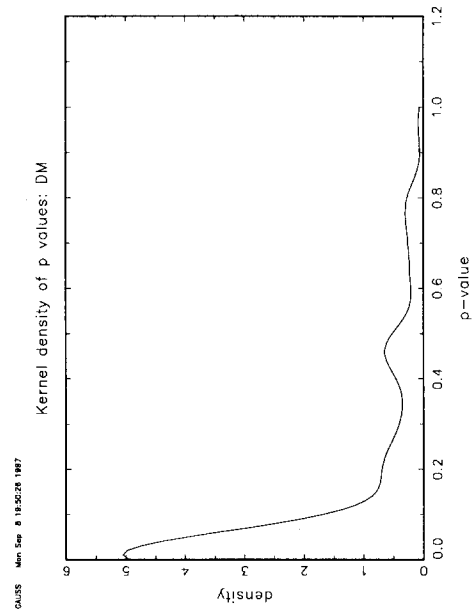


Figure 4: Density of p-values of test for DM

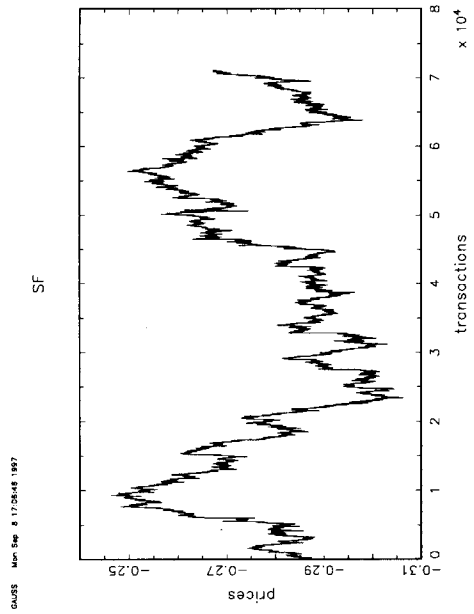


Figure 5: Floor prices of SF

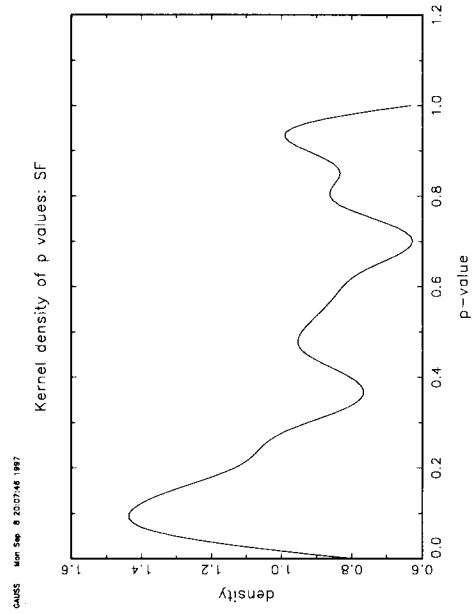


Figure 6: Density of p-values of test for SF

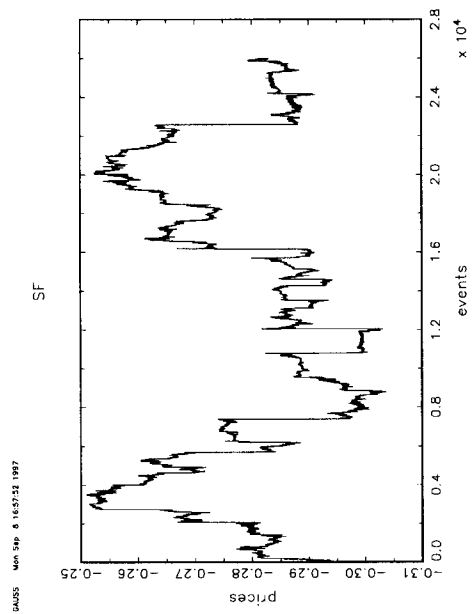


Figure 7: Globex prices of SF

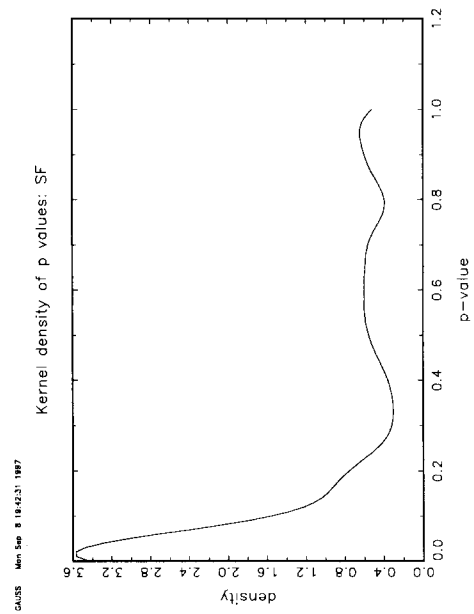


Figure 8: Density of p-values of test for SF

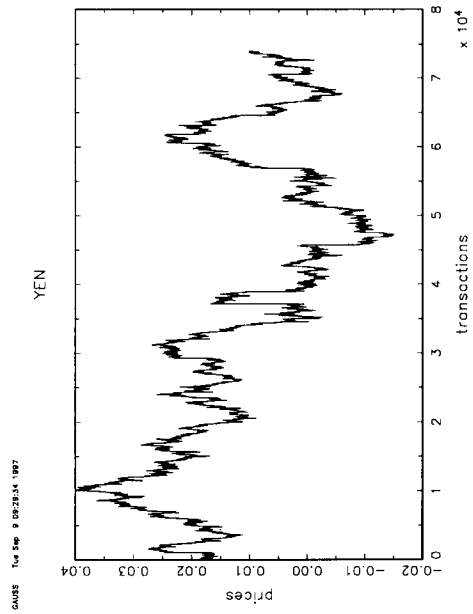


Figure 9: Floor prices of Yen

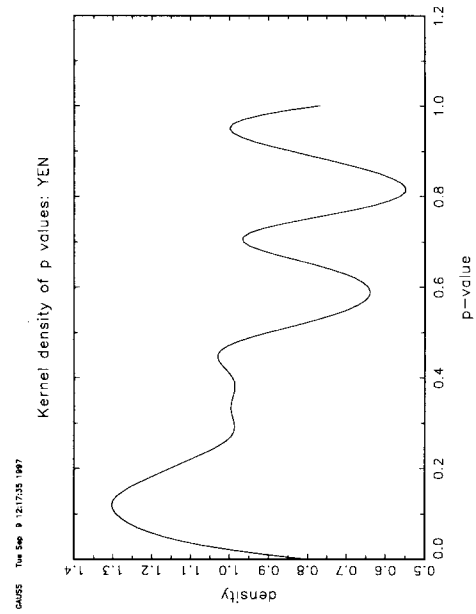


Figure 10: Density of p-values of test for Yen

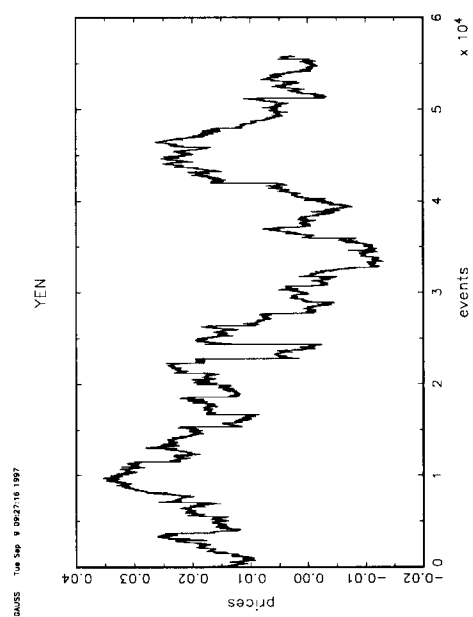


Figure 11: Globex prices of Yen

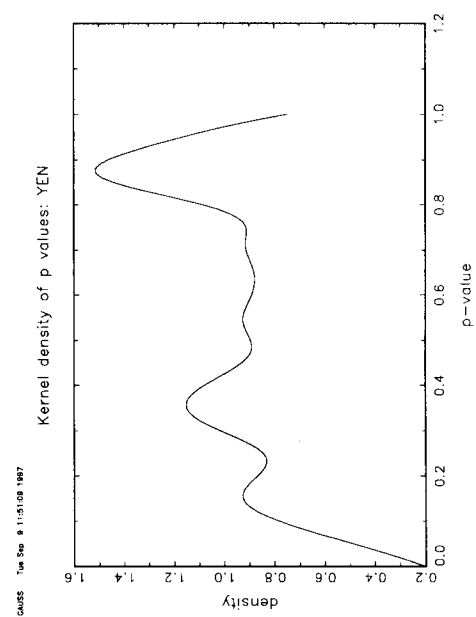


Figure 12: Density of p-values of test for Yen

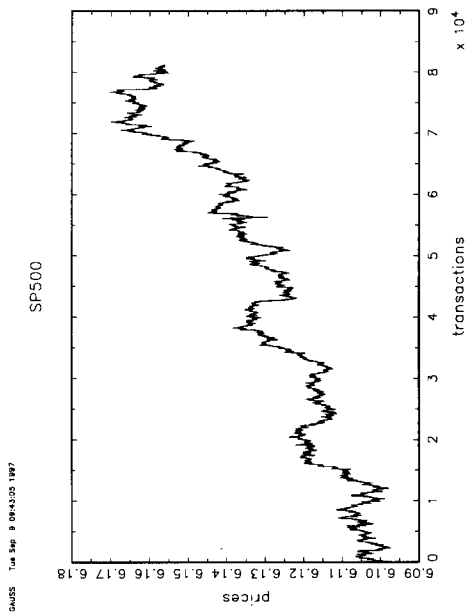


Figure 13: Floor prices of SP500

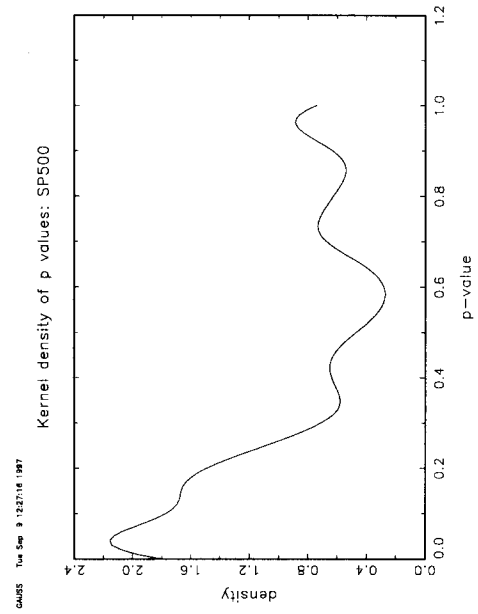


Figure 14: Density of p-values of test for SP500

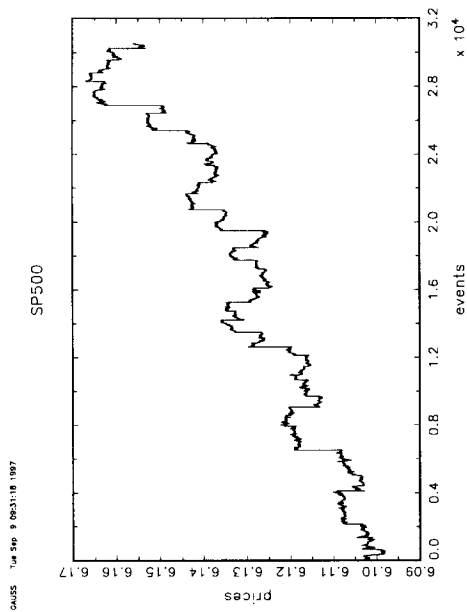


Figure 15: Globex prices of SP500

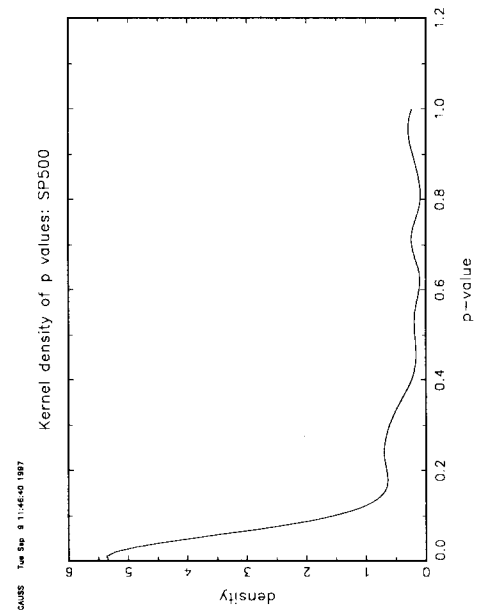


Figure 16: Density of p-values of test for SP500

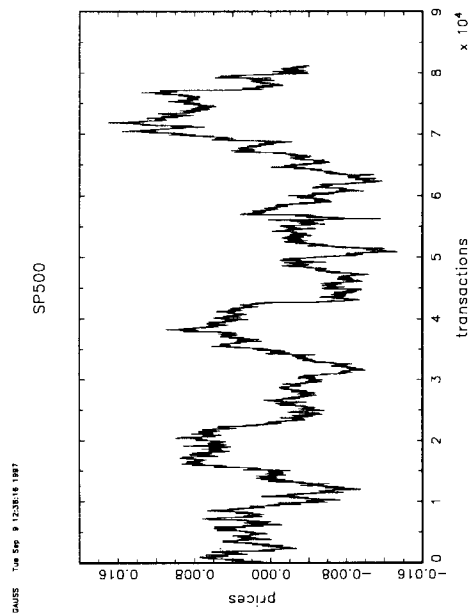


Figure 17: Detrended floor prices of SP500

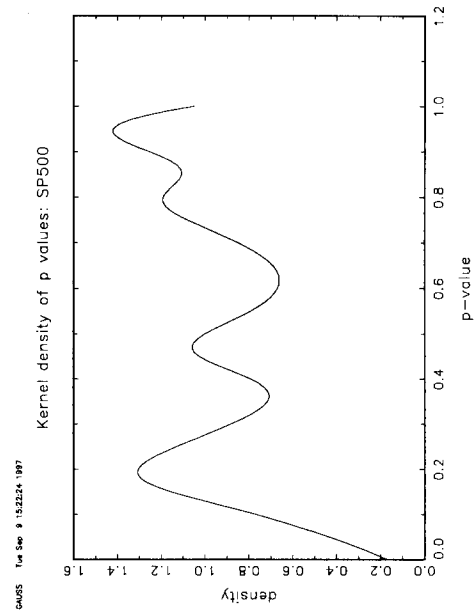


Figure 18: Density of p-values of test for SP500

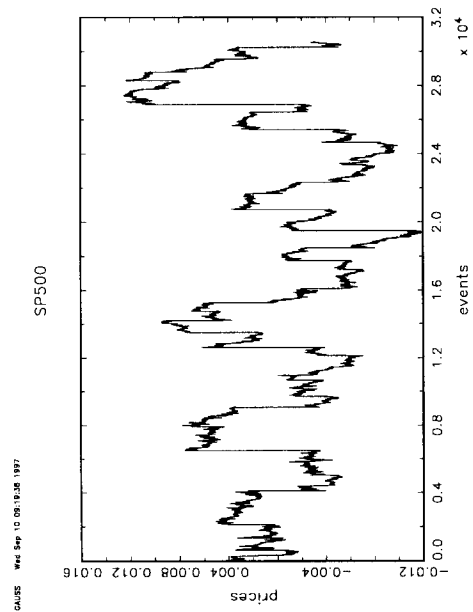


Figure 19: Detrended Globex prices of SP500

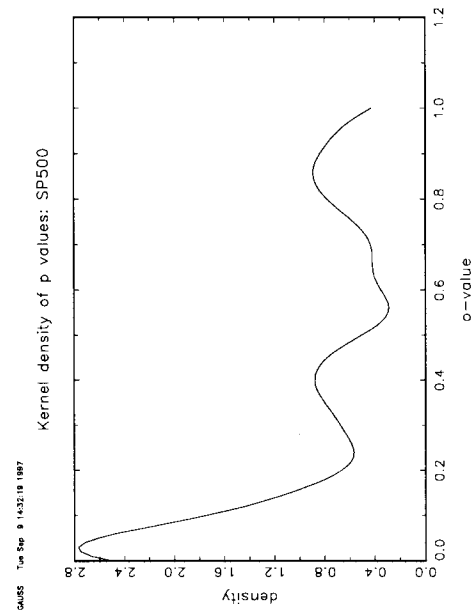


Figure 20: Density of p-values of test for SP500