Consumption Over the Life Cycle*

Pierre-Olivier Gourinchas
Stanford University

Jonathan A. Parker
University of Wisconsin

November 1997

(First Draft: April 1995)

* Comments welcome. We wish to thank Olivier Jean Blanchard, Ricardo Caballero, and Chris Carroll, for detailed comments and encouragement. We also thank Daron Acemoglu, Orazio Attanasio, Charles Fleischman, Franco Modigliani, Steve Pischke, Nicholas Souleles, and participants at seminars at the Board of Governors of the Federal Reserve System, Columbia, Harvard, Michigan, MIT, Princeton, Stanford, Wisconsin, and Yale. Parker gratefully acknowledges financial support from the Sloan Foundation.

Abstract

This paper employs a synthetic cohort technique and Consumer Expenditure Survey data to construct average age-profiles of consumption and income over the working lives of typical households across different education and occupation groups. Even after controlling for family and cohort effects, typical consumption profiles are not flat, and seem to track income at young ages. Using these profiles, we estimate a structural model of optimal life-cycle consumption expenditures in the presence of realistic income uncertainty. The model fits the profiles quite well. In addition to providing tight estimates of the discount rate and risk aversion, we find that consumer behavior changes strikingly over the life-cycle. Young consumers behave as "buffer-stock" agents. Around age 43, the typical household starts accumulating liquid assets for retirement and its behavior mimics more closely that of a certainty equivalent consumer. This change in behavior is mostly driven by the life-cycle profile of expected income. Our methodology provides a natural decomposition of saving into its precautionary and retirement components.

List of Tables

1. Introduction.

This paper analyzes consumption and savings behavior of households over their working lives, focusing on estimation of age-specific consumption functions and their implications. We are motivated by four observations.

First, household consumption and savings decisions are arguably among the most important determinants of economic growth and business cycles. Consumer expenditures account for two thirds of national output and a large percent of output fluctuations. The difference between consumption and income– savings– determines the stock of wealth, which in turn determines the interest rate, the level and perhaps the growth rate of output. To understand business cycles and growth, we must thus first understand household consumption behavior.

Second, better methodology, data, and creative use of natural experiments have lead to more frequent and more convincing rejections of the most widely-used model of consumption behavior, the certainty equivalent life-cycle hypothesis (henceforth, CEQ LCH). At the individual level, empirical studies typically test the central implication of the CEQ LCH -the Martingale hypothesis-, by testing whether consumption responds to *expected* changes in income. Despite generally poor-quality individual data on consumption, the CEQ LCH is often rejected.[1] On aggregate data, the CEQ LCH is even more convincingly rejected.[2]

Third, consumption smoothing does not seem to be a good characterization of low frequency consumption movements and savings behavior. According to the 1992 Survey of Consumer Finances (SCF), only 15% of the respondents reported retirement as their primary motive for saving while 42% cited liquidity needs.[3] Life-cycle savings seem to occur late in the working lives of consumers. Median holdings of very liquid assets for households under age 50 are $3,900 while median holdings of non-housing non-business wealth are just under $13,000.[4]

Lastly, recent theoretical work (Zeldes (1989), Deaton (1991), and Carroll (1992)) demonstrates that the Martingale hypothesis and consumption smoothing can be bad approximations to consumer behavior when agents face large amounts of individual uncertainty. Carroll (1993a) and Hubbard, Skinner and Zeldes (1994) among others have shown that income uncertainty can generate a positive covariance between expected income changes and consumption at low and high frequen-

[1]It should also be noted that many of these papers technically test the permanent income hypothesis rather than the LCH. The differences can be pronounced at the aggregate level, but both theories predict no response of consumption to expected changes in income at the household level.

[2]Some of the most recent papers are Lusardi (1993), Shea (1995), Souleles (1994), Levenson (1993), Carroll and Summers (1991) and Campbell and Mankiw (1989). See Deaton (1992) for an excellent survey of the field or Dynan (1993) for a recent approach to estimating Euler equation with a precautionary term.

[3]See Carroll (1993a) and Carroll (1992).

[4]As reported in Carroll and Samwick (1994). This does not include Social Security and pension wealth, which constitute a large fraction of wealth at retirement. Note that distribution of wealth and its constituents is always strongly skewed, with a mean far exceeding the median.

cies through precautionary savings.

In this paper, we examine individual consumption data, estimate a structural model with realistic levels of income uncertainty, and finally reinterpret life-cycle consumption and asset accumulation behavior within the context of the model. We measure, exploit and analyze the systematic age-pattern of consumption profiles. Age-heterogeneity in consumption behavior results from (a) the interaction between, and relative strengths of, retirement and precautionary motives for saving at different ages and (b) the changing slope of the income profile. We are successful along several dimensions.

First, we provide new evidence on the failure of the Certainty Equivalent (CEQ) LCH at the microeconomic level. We do this by demonstrating that consumption age-profiles averaged across time and households are not flat and are related to expected profiles in income. Importantly, this result still holds after controlling for family composition and cohort effects — two potential reasons for the observed hump-shape of consumption. Our approach involves using the best available data on consumption expenditures in the US, the Consumer Expenditure Survey (CEX) from 1980 to 1993, giving us data on around 40,000 households. Using weak identifying assumptions, we construct consumption and income profiles across the working lives of "typical" men of five different educational attainments and five different occupational groupings. Consumption and income profiles are both significantly hump-shaped, and consumption tracks income reasonably well early in life.

Second, embedding realistic income uncertainty into the canonical model of life-cycle consumer behavior substantially improves the fit of the predicted life-cycle consumption profile. To demonstrate this, we write down a consumer maximization problem with a retirement period and explicit income uncertainty. The solution to this model will be the standard CEQ LCH consumption rules when uncertainty is ignored. Consumption will depend on the interest rate, the intertemporal elasticity of substitution, the discount factor and the present discounted value of income. However, under uncertainty, consumption will also depend on the *path* of expected income. Thus consumer behavior will vary systematically over the life cycle. When expected income growth and the discount rate are low relative to the interest rate, consumers' behavior will remain similar to that of standard life-cycle consumers. If, on the other hand, expected income growth or the discount rate are large relative to the interest rate, consumers will behave as "buffer-stock" agents, consuming roughly their income and saving only small amounts to buffer against bad income draws. As households age, income growth declines. Consequently, the retirement savings motive will enter consumers' horizons. They will save more and behave more like certainty-equivalent consumers. The model can potentially deliver average consumption profiles which are more concave than income profiles. It is important to emphasize that we do not assume our results here. With a sufficiently low discount rate, the average consumption profile would be very similar to that of the certainty equivalent case.

Positing that the average income profile for a given group corresponds to the expected income profile and incorporating calibrated individual-specific income shocks, we estimate consumption functions for consumers in each occupation and education group. By simulating the lives of many consumers using these consumption functions we create predicted average consumption profiles. We then estimate the discount rate and the intertemporal elasticity of substitution by a Method of Simulated Moments procedure. The average household has an intertemporal elasticity of substitution of 2.04 and a discount rate of 3.9%.[5] The discount factor is tightly estimated, and the estimated discount rates decline weakly with educational attainment. It is worth stressing that the estimated coefficients are within a "reasonable" range. In particular, buffer-stock behavior arises early in life *due to the steepness of the income profiles at young ages.*

The model fits the data quite well and does an excellent job of capturing the main features of the consumption profiles. To the best of our knowledge, this represents the first structural estimation of consumption functions over the life cycle which incorporates precautionary savings.

Third, we find strikingly different consumption functions for households at different ages: consumers behave like "buffer-stock" consumers early in their working lives and more like CEQ LCH consumers as retirement nears. We show that *households make the transition from buffer-stock to LCH behavior just after age 42*. This confirms our initial intuition: relative movements of the consumption and income profiles reveal a great deal of information about the relative strength of the two savings motives. We conclude that a large fraction of consumers consists of *target savers*, for whom the Euler equation, as typically tested, should be expected to fail. This is, in part, a confirmation of Carroll (1993a) and (1993b) which argue, based on asset data, that buffer-stock models apply only to households before ages 45 to 50.

Fourth, this paper contributes to the debate on the determinants of wealth accumulation. In our model, saving and consumption at each age are determined by the interactions between precautionary and retirement savings motives. Defining all wealth accumulation at retirement as life-cycle savings, we can decompose saving into precautionary and life-cycle saving.[6] Early in life, households would like to borrow against expected future labor income. Consequently, life-cycle savings are negative. However, uncertainty causes households to build a buffer stock of savings, implying that precautionary saving is positive. Late in life, labor-income uncertainty is mostly resolved, and consumers run down their buffer stocks, while retirement saving becomes positive and large. Thus, the calibrated model matches the basic features of asset data. Both the fact that most households hold few, if any, assets and the fact that most households do not start saving for retirement until late in life have often been interpreted either as evidence against the LCH or evidence against forward-looking consumers. Our fitted model suggests that these facts are instead consistent with the LCH augmented to include income uncertainty

[5] The latter estimate is very sensitive to the assumption about the real after tax interest rate, 3% in our benchmark analysis. We discuss robustness issues in section 5.

[6] We do not enter the debate on the relative importance of retirement versus bequest savings. Implicitly, our retirement savings measure will include both.

and consistent with forward-looking optimizing behavior.

Our paper builds on many previous studies of life-cycle consumption behavior.

Several papers have used micro-consumption and income data to construct life-cycle profiles of consumption and income. Kotlikoff and Summers (1981) construct synthetic life-cycle profiles of consumption and income and present some evidence that consumption falls significantly below income only after age 50. Carroll and Summers (1991) report that consumption tracks income across countries, education and occupation groups, providing additional evidence that life-cycle savings do not seem to occur until late in life. Both studies find that consumption tracks income over the lives of households until around age 50. However, these studies are using *cross-sectional* data to infer *time-series* behavior. Thus, the close correlation between consumption and income may come from cohort-effects: on average, young families have larger lifetime resources and hence consume more.[7] Further, changes in family size and consumer-needs over the life cycle may impart a hump-shape to consumption which does not come from a failure of CEQ consumption smoothing. We address these issues by adjusting both consumption and income for life-cycle changes in family size and for cohort effects.

More recently, Attanazio and Browning (1995) and Attanasio, Bank, Meghir and Weber (1995), using data from the UK Family Expenditure Survey (FES) and the US Consumer Expenditure Survey (CEX) respectively, have examined life-cycle behavior adjusted for cohort and family size effects. Attanasio et al. (1995) shows that the residuals from a regression of consumption on family composition and labor supply variables are uncorrelated with age. However, if the true consumption profile is hump shaped over the life cycle, this regression suffers from an omitted variable bias, which will incorrectly assign the hump to changes in demographics. Both papers also draw life-cycle implications from certainty equivalent Euler equation estimation under flexible representations of preferences. They do not reject the certainty equivalent life-cycle model.[8] Here again, however, the instruments used in the Euler equation estimation are likely to be correlated with the omitted precautionary term. This overestimates the share of the consumption hump attributed to labor supply and family size variables.[9]

We are also building on previous studies which parameterize and simulate life-cycle consumption models with uncertainty. Hubbard et al. (1994) and Carroll (1993a) show that the optimal consumption choices of consumers lead to profiles

[7] See for example Attanasio and Weber (1995).

[8] Since adjusted profiles are not reported, it is not clear whether their finding is merely the result of adding covariates -labor supply and family size- so that the instrumental variable technique they employ no longer has the power to detect the relationship. We note that one of the best papers which uses occupation and education to predict expected income growth, Lusardi (1993), finds that consumption does track income.

[9] Both of these papers separately deflate components of consumption, which can eliminate a consumption-income parallel. If liquidity constraints are binding or consumers are buffer-stock then *nominal* consumption will track *nominal* income. Finally, both papers look only at nondurable consumption, which, as we demonstrate and discuss, is not simply a scaled down version of total consumption.

which are hump-shaped and track income over the early part of life for some parameterizations. Hubbard, Skinner and Zeldes (1995) go further and choose simulated profiles so as to try to reproduce constructed profiles of assets over the life-cycle.[10] They re-interpret low-asset holding by most households as driven by means-tested government programs.[11] Our approach goes beyond those studies by estimating a structural model of consumption.

Further, Palumbo (1994) uses individual consumption, income and asset data to estimate individual consumption functions for retirees. We choose to rely on average profiles precisely because we do not believe that the individual-level data are of sufficient quality to support the employed technique in general.[12]

Finally, nearly all previous Euler equation estimations of these parameters ignore the precautionary term in the Euler equation, a potentially serious flaw. Two recent papers immune to this problem are Carroll and Samwick (1994) and Barsky, Juster, Kimball and Shapiro (1995). The former, using asset data and a theoretical framework similar to ours, finds that the discount rate is poorly identified. The latter, using survey questions about preferences over lotteries and income paths, estimates an intertemporal elasticity of substitution and a discount rate both lower than what we estimate. We are exploiting lower frequency movements in the data then typical Euler-equation tests. High-frequency Euler equation tests might reject the CEQ PIH, while the CEQ PIH could still be a reasonably accurate model for low frequency analysis.

The structure of the paper is as follows. In section 2, we lay out a model of consumer maximization and its implications for the construction of consumption and income profiles. We describe the numerical dynamic programming techniques used to solve the model and present characterizations of optimal behavior. The third section describes the data, discusses empirical issues involved in constructing our life-cycle profiles, and presents graphs of the profiles. Section 4 introduces the method of Simulated Moments methodology for estimating the model. Finally, we present the results of the estimation and conclude. Appendices contain more detailed descriptions of the CEX data, the numerical optimization, and econometric technique.

---

[10] These authors do not correct for family-size or cohort effects.

[11] We do not address this alternate interpretation, but simply note that we believe that the heterogeneity in skills, abilities, and wealth across people starting their working lives makes the low-asset trap of their model very relevant for a small subset of the population and much less relevant for the typical household. See also Carroll and Samwick (1994) for a critique of the implications of their approach.

[12] For instance, Palumbo (1994) must use a scaled-up measure of food consumption as his measure of household consumption and must make various assumptions about each individual's expected health dynamics.

2. Consumption Behavior with Stochastic Income.

We begin by setting up a model of consumer behavior incorporating two saving motives: retirement and precautionary. The life-cycle saving motive results from the finite lifetime of individuals and from their retirement period. Income uncertainty at the individual level provides incentives for precautionary savings.

By nature, we are dealing with a non-stationary problem, as expected income follows a deterministic path and the permanence of the shocks depends upon the consumer's age. The consumer's program must be solved recursively, keeping track of consumption rules at each age. But this results precisely from our initial intuition: the systematic age-pattern of consumption functions will reflect the interaction of the two saving motives and will translate into some definite life-cycle profile. Conversely, the age-pattern of the profiles we construct will allow us to identify life-cycle consumption functions.

2.1. The Canonical Model with Labor Income Uncertainty.

Our starting point is the basic discrete-time, life-cycle model of consumption behavior. Consumers live for N periods and work for $T < N$, where both T and N are exogenous and fixed. In every period $t < T$, the consumer receives a stochastic income Y_t and consumes C_t. There is only one asset in the economy, totally liquid and yielding a constant gross, after-tax, real interest rate R. Our unit of analysis is the household. We assume that preferences take the standard additively separable expected utility form, with a discount factor β:

$$U = E\left[\sum_{t=1}^{N}\beta^t u\left(C_t, Z_t\right) + \qquad \beta^{N+1}V_{N+1}(W_{N+1}, Z_{N+1})\right], \tag{2.1}$$

where W_t represents total financial wealth and Z_t is a vector of household characteristics (e.g. family size). V_{N+1} represents the value to the consumer of any assets left at the time of death, capturing any bequest motive. The consumer maximizes (2.1) given an initial wealth level W_1, and the constraint that terminal wealth is non-negative $W_{N+1} \geq 0$. The dynamic budget constraint is:

$$W_{t+1} = R\left(W_t + Y_t - C_t\right). \tag{2.2}$$

We further assume that the felicity function $u\left(.,.\right)$ is of the Constant Relative Risk Aversion ($CRRA$) form, with intertemporal elasticity of substitution $1/\rho$, and multiplicatively separable in Z:[13]

$$u\left(C, Z\right) = v\left(Z\right)\frac{C^{1-\rho}}{1-\rho}.$$

[13] Equivalently, ρ is the coefficient of relative risk aversion. This is a well known feature of additively separable expected utilities. This paper will not explore alternatives such as non-expected utilities (Kreps and Porteus (1978)) or habit formation (Heaton (1990)).

If income were certain, the solution to this program would be standard: the consumer would choose a consumption path such that:

$$\frac{C_{t+1}}{C_t} = \left(\beta R \frac{v(Z_{t+1})}{v(Z_t)}\right)^{\frac{1}{\rho}}. \tag{2.3}$$

With constant individual characteristics, (2.3) implies a constant growth rate of consumption. Consumption increases (respectively decreases) over time when the interest rate is larger (respectively smaller) than the discount rate. The growth rate of consumption (as opposed to its level) is independent of the income profile. The consumption level is then determined by the lifetime budget constraint and the terminal value function. The desire to smooth consumption over the entire lifetime will induce households to save for retirement and for bequest during their working lives.

When individual characteristics vary over the life cycle, the growth rate of consumption may change accordingly. For instance, if the marginal utility of consumption increases with family size, consumption will grow as family size increases, and decrease as children leave the household. These variations in individual characteristics may induce a positive correlation between consumption and income over the life cycle.

The certainty (or certainty-equivalent) LCH provides extremely valuable insights on the determinants of consumption and savings. However, by deliberately assuming away individual uncertainty, it may be missing an important part of consumer behavior.

With individual income uncertainty and prudence, households will hold precautionary savings to insure themselves against future contingencies. The variation in this precautionary motive has far-reaching and striking implications. The main consequence of income uncertainty is to increase the slope of the consumption profile (provided that consumers are prudent). Hubbard et al. (1994) demonstrate that this uncertainty can lead to hump-shaped consumption profiles as households save for precautionary reasons early in life and run down these assets during retirement due to lower levels of uncertainty and an increased probability of death. Carroll (1992) and Deaton (1991) analyze the case in which consumers are also *impatient*: in the absence of uncertainty, households would like to borrow in order to finance a high level of current consumption. In addition, Deaton (1991) imposes liquidity constraints while Carroll (1992) sets up a model in which consumers choose never to borrow. In either rendition, assets play the role of a *buffer stock* against bad income shocks. Consumers have a target level of liquid assets, above which impatience dominates and assets are decumulated, and below which the precautionary motive dominates and assets are accumulated. Thus the theory predicts a correlation of expected income growth and consumption growth at both high and low frequency. Over the life-cycle, consumption will appear to track income.

In the rest of the paper, we explicitly incorporate uninsurable idiosyncratic income uncertainty in addition to our retirement saving motive. We adopt Carroll's

(1992) formulation, and decompose the labor income process into a permanent, P_{jt}, and a transitory, U_{jt}, components (where j indexes occupation and education groups):

$$\begin{aligned} Y_{jt} &= P_{jt}U_{jt} \\ P_{jt} &= G_{jt}\,P_{jt-1}N_{jt} \end{aligned} \tag{2.4}$$

The transitory shocks, U_{jt}, are independent and identically distributed, take the value 0 with probability $p \geq 0$, and are otherwise log-normally distributed so that $\ln U_{jt}$ has mean zero and variance σ^2_{uj}. The log of the permanent component of income, $\ln P_{jt}$, evolves as a random walk with drift. G_{jt} is a deterministic growth factor (specific to age t and group j) while $\ln N_{jt}$, the shock to the permanent component of income, is independently and identically normally distributed with mean zero and variance σ^2_{nj}.[14] Thus income evolves as a *nonstationary*, serially correlated process, with both permanent and transitory shocks, and a positive probability of zero income in every period.[15]

Two points are worth noting. First, permanent shocks are only as permanent as the length of the working life: all shocks are ultimately transitory, as consumers retire and die. In the last working period, transitory and permanent shocks are equivalent. As a consequence, the propensity to consume out of "permanent" shocks to income will decrease with age, a point emphasized by Clarida (1991). This property holds true for the CEQ LCH also. Second, in this setup consumers will choose never to borrow against future labor income. This follows from (a) there being a strictly positive probability that labor income will be arbitrarily close to zero for the rest of the working life and (b) the Inada condition $\lim_{c\rightarrow 0} u'(c) = \infty$.[16] It is important to note that this holds true, even when p, the probability of strictly zero income, is set to zero. Suppose the household were to borrow in the next to last working period. Then, with strictly positive probability it would be left without any wealth in the last working period. The household would then have an infinite expected marginal utility. Simple backward induction implies that it will never be optimal to borrow. Thus, in this setup, the precautionary motive acts as a *self-imposed* liquidity constraint.[17]

Going from the model to the data, we need to make three assumptions. First, note that in order to solve the consumer's problem as stated, we need to specify both the nature of uncertainty during retirement and a bequest function. While there have been good attempts at modelling consumer behavior after retirement,[18]

[14] According to this decomposition, the permanent level of income is that level that would obtain without transitory shocks, not the present discounted value of future income streams. This matches Friedman's (1957) interpretation.

[15] While Abowd and Card (1989) found that change in labor income was best characterized by an $MA(2)$ process, they also found little gain in moving from an $MA(1)$ to an $MA(2)$.

[16] A condition always satisfied with isoelastic utility and positive risk aversion.

[17] If instead we had assumed a strictly positive lower bound on income, consumer could borrow up to the present discounted value of certain future income.

[18] See Hubbard et al. (1994) and Palumbo (1994).

we feel that we know too little about the form that uncertainty takes after retirement to use our methodology and draw inferences from post-retirement behavior. Uncertainty arises from different sources— medical expenses, the timing of death and asset returns. *Inter-vivos* bequests are important. Although these sources of uncertainty are also present to some extent in the last working years, labor income uncertainty seems to be the most important form of uncertainty. Further, high quality information on household asset holdings together with consumption and income are not available. Given that investment income, social security, and pensions represent the main sources of income during retirement, it is currently difficult to establish consumption patterns as a function of total wealth.

Even with a proper treatment of retirement issues, one would have to make a guess about the bequest function. Therefore we decided instead to make use of Bellman's optimality principle, and truncate our problem at retirement.[19] Defining the value function at time t, V_t, our problem becomes:

$$\begin{aligned} V_\tau\left(X_\tau, P_\tau, Z_\tau\right) &= \max_{c_\tau,\ldots,c_T} E_\tau \left[\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T-\tau}\, V_T\left(X_T, P_T, Z_T\right) \right] \\ s.t.\ X_{t+1} &= R\ \left(X_t - C_t\right) + Y_{t+1}, \end{aligned} \tag{2.5}$$

where we define cash on hand X_{t+1} as total available financial resources at time $t+1:$

$$X_{t+1} = W_{t+1} + Y_{t+1} = R\left(X_t - C_t\right) + Y_{t+1}.$$

Second, the model imposes a single vehicle for precautionary and retirement savings, since there is only one asset. In practice, much of retirement savings is accumulated in the form of illiquid assets, only available after retirement.[20] This suggests that the relevant model of consumption behavior should incorporate an additional asset which is illiquid and accessible only after retirement. However, this would substantially complicate the problem by introducing another control variable (how much to save in liquid versus illiquid assets) and state variable (illiquid assets). In order to keep our estimation procedure feasible, we instead assume that consumers will receive at the age of retirement some accumulated illiquid wealth, proportional to their permanent income. This illiquid wealth accumulates exogenously and cannot be borrowed against. Effectively, this imposes a borrowing constraint $W_t \geq 0$. We denote accumulated illiquid wealth as H_t and total financial wealth after retirement as $A_t = H_t + W_t$.

[19] Our approach does not emphasize the relative importance of retirement versus bequest wealth. The salvage value function accomodates a bequest motive. See Modigliani and Brumberg (1956) and the ensuing debate with Kotlikoff and Summers (1981) about the relative importance of life cycle and bequest savings.

[20] Social security wealth is definitely illiquid and is only available as annuities after retirement. Early withdrawal of pension and savings vehicles targeted for retirement purposes, such as IRA's, 401k plans and Keogh, is often penalized, if allowed at all. One might also consider housing wealth as part of retirement wealth. Empirical evidence suggests that households run down their housing wealth only extremely late in life.

Lastly, we need to postulate a salvage value function which summarizes the consumer's problem at retirement time. We choose a functional form which maintains the tractability of the problem and is flexible enough to allow robustness checks:

$$V_T\left(X_T, H_T, Z_T\right) = k\, v\left(Z_T\right)\, \left(X_T + H_T\right)^{1-\rho} \tag{2.6}$$

This functional form is exactly correct if the only source of uncertainty after retirement is the time of death. When we move to estimation, we will calibrate the parameters of the associated consumption rule at retirement using information on consumer wealth, income, and consumption.

2.2. Solving for Optimal Consumer Behavior.

The setup of the problem combined with our particular choice of retirement value function makes the problem homogeneous of degree $1-\rho$ in the permanent component of income. Since a second state variable would render our estimation procedure unfeasible, we assume here that individual characteristics are constant throughout the life cycle: $Z_t = Z = 1$. Furthermore, since our data does not track households over time, we cannot calibrate the family-size process for each household over its life cycle. We will instead directly control for family effects when constructing our profiles. This allows us to write the optimal consumption rule as a function of a single state variable, x_t, the ratio of cash on hand to permanent income:

$$x_{t+1} \equiv X_{t+1}/P_{t+1} = \left(x_t - c_t\right) \frac{R}{G_{t+1} N_{t+1}} + U_{t+1}. \tag{2.7}$$

We can then derive the Euler equation in any period prior to retirement:

$$\begin{aligned} u'\left(c_t\left(x_t\right)\right) &= \beta R\, E\left[u'\left(c_{t+1}\left(x_{t+1}\right) G_{t+1} N_{t+1}\right)\right] \\ &= \beta R\, \left\{p\, E\left[u'\left(c_{t+1}\left(x_{t+1}\right) G_{t+1} N_{t+1}\right) \middle| U_{t+1} = 0\right] + \right. \\ &\quad \left. (1-p)\, E\left[u'\left(c_{t+1}\left(x_{t+1}\right) G_{t+1} N_{t+1}\right) \middle| U_{t+1} > 0\right]\right\}, \end{aligned} \tag{2.8}$$

where lowercase letters are normalized by the permanent component of income, and $c_t(x_t)$ represents the optimal consumption rule at time t as a function of normalized cash on hand x_t. Next period expected marginal utility can be decomposed according to Bayes formula into expected marginal utility conditional on zero future income, and expected marginal utility conditional on a strictly positive income.

The solution to the consumer problem consists of a set of consumption rules $\left\{c_t\left(x_t\right)\right\}_{1 \le t \le T}$. In the last working period, under our previous assumptions, consumption will be linear in cash on hand:

$$c_T\left(x_T\right) = \gamma_0 + \gamma_1 x_T, \tag{2.9}$$

where $\gamma_0 = \gamma_1 h_T$.[21] Consumption in the next to last period is then found as the solution to (2.8) for all values of cash on hand, where we replace c_T using (2.9).

[21] In the case of full certainty after retirement, it is straightforward to show that: $\gamma_1 = \frac{1-\kappa}{1-\kappa^{N-T+1}}$ where $\kappa = \beta^{1/\rho} R^{1/\rho - 1}$.

Solving recursively generates c_{T-1}, ..., c_1.[22] A complete description of the solution method is provided in Appendix A.

2.3. Characterization of Individual Consumption Behavior.

Figure 2.1.a shows the consumption rules at various ages, when the permanent income profile is flat ($G_t = G = 1$), there is no retirement period ($\gamma_0 = 0, \gamma_1 = 1$), working life starts at age 25 and ends at age 65, and consumers are impatient.[23] When permanent income growth is constant, the finite horizon problem converges in the limit to the infinite horizon one, as we move further away from retirement.[24] Consumption is always positive, increasing and concave in cash on hand. One can also show that cash on hand can only increase if the income draw is sufficiently large.[25] Early in life, households will exhibit the standard *buffer stock behavior*: for low level of assets, typically less than the permanent component of their income ($x \leq 1$), households will consume most, but never all, of their financial wealth, and move to the next period with a very low level of assets. At high levels of cash on hand, households will consume a smaller fraction of cash on hand, but always enough so that they expect to run down their assets.

As death nears, the consumer faces less and less uncertainty from labor income shocks. It is then rational for an impatient consumer to start *running down* their buffer of assets: the consumption rules converge progressively towards the 45^0 line. In the last period, obviously, the household consumes everything. Consumers save only in order to buffer income shocks. We note also that from age 25 to 55, our consumer has roughly the same consumption rule. Buffer stock savings are run down quite late in life.

Figure 2.1.b shows consumption rules, for a household facing the same expected income growth, now assuming that the retirement consumption rule, at age 65 is more realistically characterized by:[26]

$$\gamma_0 = 0.384, \ \gamma_1 = 0.049.$$

[22] The consumption rules have to be found numerically, as no closed form solution exists for this problem. We did this using a discretization method (see Judd (1993)). We solved the Euler equation (2.8) recursively on a grid of normalized cash-on-hand values. The consumption function was then interpolated between the points on the grid. In order to capture the curvature of the consumption rules at low values of cash on hand, the discretization grid was finer for $x \in [0, 2]$. See Caballero (1990) for a closed form solution under exponential utility.

[23] Other relevant parameters are $\beta = 0.931$, $\rho = 1.13$ and $R = 1.05$. These parameters generate buffer-stock behavior. Income uncertainty is the average amount, discussed later, and presented in Table 2.

[24] In other words, the solution to the supremum problem in the infinite horizon case is obtained as the fixed point of the associated functional equation. Theorem 9.12 in Stokey, Lucas and Prescott (1989) applies even though returns are unbounded, as long as the discount factor is strictly less than 1.

[25] This condition is analyzed in more details in Deaton (1991) and Ayagari (1993). In the infinite horizon case, this guarantees that cash on hand has an ergodic distribution.

[26] We discuss calibration of γ in section 4.

The two households have the same consumption rules early in life– governed by the common solution to the infinite horizon problem. In other words, both households will behave as standard buffer stock households in their youth. Now however, the agent will have to *accumulate* enough wealth for retirement purposes. As retirement nears, savings must *increase*. Note also that with impatient consumers, the retirement savings motive matters only late in life.

Figures 2.2.a and 2.2.b display randomly drawn profiles of consumption for households facing typical paths of income, retirement rules, and income uncertainty.[27] In both profiles, consumption tracks income early in life, and diverges later in life. Notice further that unexpected transitory shocks are better smoothed later in life, despite the fact that they contain greater information about total resources for the remainder of the life. Smoothing is easier later in life since retirement saving also acts as a large buffer.

What are the necessary conditions to generate buffer-stock behavior early in life? Previous characterizations have addressed the problem in a stationary environment. Specifically, Deaton (1991) defines buffer-stock consumers as consumers who would borrow against future income, were it not for uncertainty. In the CEQ LCH, the slope of normalized consumption is given by (see (2.3)):

$$\ln(\frac{c_{t+1}}{c_t}) = \frac{1}{\rho}\ln(R\beta) - \ln(G) \simeq \frac{1}{\rho}(r-\delta) - g, \tag{2.10}$$

where G is the gross rate of income growth. The ratio of consumption to income increases whenever the right hand side is positive. A higher preference for the present or a lower interest rate will make buffer stock behavior more likely. However, households with a low discount rate and facing a high interest rate may still decide to behave as buffer-stock consumers, saving only a small fraction of their resources *if their income profile is steep enough.*

With i.i.d. income growth G, uncertainty, and an infinite horizon, Deaton (1991) shows that, agents are buffer-stock if and only if:

$$R\beta E\left[G^{-\rho}\right] < 1. \tag{2.11}$$

As Carroll (1992) emphasizes, buffer-stock agents have a *desired level of cash on hand relative to permanent income.* The existence of such target is both necessary and sufficient for buffer stock behavior in the infinite horizon framework. At low levels of assets, agents on average save to build the buffer, and cash on hand is expected to increase. For large levels of normalized cash on hand, the precautionary motive vanishes and agents increase consumption. This target level of cash on hand is defined as the fixed point of the mapping from current to expected future cash on hand:[28]

[27] What is typical will be discussed in detail in Sections 4 and 5.

[28] In our finite horizon framework, this characterization will be necessary but not sufficient in the following sense. When the interest rate is lower than the discount rate, agents will have a target level of cash on hand throughout their working lives. For large levels of cash on hand, impatience

$$\bar{x}_t = E_t\left[x_{t+1}|x_t = \bar{x}_t\right]. \tag{2.12}$$

Figure 2.3 presents the mapping $\bar{x}_t \rightarrow E_t\left[x_{t+1}|x_t = \bar{x}_t\right]$ at various ages. It is convex, increasing and initially above the 45^0 line, so that in general it need not have a fixed point.

3. Data and Consumption and Income Profiles.

3.1. Profile Construction Methodology.

The construction of our profiles is motivated by the model presented in the previous section. We want the income profiles to be usable as inputs to the consumer optimization problem and the consumption profiles to be comparable to average consumption paths from the optimal program. That is, we are interested in constructing the income growth profile $\{G_t\}_{t=25}^{65}$ and the average consumption profile $\left\{\bar{C}_t\right\}_{t=25}^{65}$. Consumption at a certain age t will be the log-average across the distribution of cash-on-hand, permanent component of income, and consumer characteristics for that age:

$$\bar{C}_t \equiv \exp\left[\int\int \ln c_t(x_t)\, P_t\, dF(x_t)\, dF(P_t)\right]$$

In order to construct such a profile, we must address three issues. First, due to its excessive noisiness, we do not exploit the limited panel nature of the Consumer Expenditure Survey, but instead rely on data from repeated cross-sections. Consequently, in our sample, birthyear and age will be correlated. Households observed at age sixty, say, will have been born long before those we observe at young ages and will have on average lower lifetime resources, and lower levels of income and consumption at each age. Ignoring birthyear effects would lead to a negative bias in our estimate of the slope of income and consumption growth, especially, late in life.[29] Second, the model refers to household consumption, adjusted for family size ($Z = 1$). Since household size is hump-shaped over the life cycle, the correlation found in previous studies between consumption and income over the life cycle may disappear after correcting for family size.[30] Finally, we are interested in exploiting some variation in expected income profiles across households. Thus, while we do

will cause consumers to run down these assets. At low levels of cash on hand, consumers must save either for liquidity needs or retirement. However, near retirement, this target level will reflect life-cycle considerations and is likely to be extremely large. The behavior of such agents is unlike buffer stock behavior as previously described.

[29]This point has been emphasized recently by Attanasio and Weber (1995).

[30]Attanasio et al. (1995) make this argument, and find that, after correcting for family size, consumption is no longer significantly dependent on expected changes in income at high frequency. These authors do not demonstrate that the profiles are flat after making this correction. It is worth noting that both this paper and ours assume that family size is exogenous. If the buffer stock model is correct and having children is a form of consumption, then the decision to have children is affected by the expected path of income. By correcting for family size composition over

conduct our exercise for the average household, we also focus our analysis on income and corresponding consumption profiles for subgroups of the US population defined by occupation and education groups. We assign households to these groups on the basis of the male head. Male labor force participation over the life cycle is high and stable, giving us more data and robust profiles.

We posit the following effects model for the natural logarithm of consumption for individual i, in education/occupation group j, of age a, in year t:

$$\ln C_{ji}^{at} = f_i^a + \pi_j^a + b_i + y^t + \varepsilon_{ji}^{at} \tag{3.1}$$

That is, consumption of individual i is determined by a family size effect, f_i^a; an effect, π_j^a, specific to their age, a, and education/occupation group, j; a cohort effect, b_i; a year effect, y^t; and an idiosyncratic, individual effect ε_{ji}^{at}. We are interested in recovering π_j^a. By doing so, we create a profile which has a constant family size over the life cycle, and also correct for the fact that we do not actually follow the same individuals over their entire lives. As discussed in Deaton (1985), it is not possible to remove the linear component of the time and cohort effects without also removing the average (across education/occupation groups) age profile of consumption.[31] We make the identifying restriction that time effects are related to business cycles and thus are well captured by the partial correlation of consumption with the regional unemployment rate.

Our procedure can then be summarized as follows. First we put all the data into real terms using the Gross Domestic Product implicit price deflator for personal consumption expenditures.[32] Second we generate family size adjustments— f_i^a in equation (3.1)— and apply them to all the consumption and income data so that households have a constant effective size over the life cycle.[33] Then, to construct unsmoothed profiles, we estimate the following model, over households with male heads aged 25 to 65, by weighted least squares with weights based on the CEX population weights:

$$\ln X_i - \hat{f}_i = \pi a_i + \eta b_i + \nu \mathcal{U}_i + \tau Ret_i + \varepsilon_i, \tag{3.2}$$

where X is either consumption or income, a_i is a complete set of age dummies crossed with education or occupation group dummies, b is a complete set of cohort

the lifecycle, one is removing that portion of changes in consumption driven by expected income changes. As we will see, however, the profiles presented subsequently suggest that correcting for family size attenuates but does not eliminate the consumption income parallel.

[31] This follows from the annoying identity that interview year less age equals birthyear.

[32] Again, it is important not to use different deflators for different items within consumption or for income and consumption. This could break the relationship between cash on hand and consumption in nominal terms which is the relationship predicted by the buffer-stock theory.

[33] We construct f_a^i by running equation (3.2) without separate age effects by subgroup and with family size dummies on the right hand side. The effective family size is then 2.8 (the sample average). We also experimented with exogenous family size adjustments –assuming f_a^i is simply family size raised to the power -0.7. This led to profiles which were noisier and flatter early in life.

dummies (less one), $\mathcal{U}$ is the Census region unemployment rate in year t, and Ret is a dummy for each group which is equal to 1 when the respondent is retired. Profiles are constructed by predicting $\ln X_i$ for each age and grouping, setting the cohort and unemployment rates are at their average values and the retirement dummies to zero. Smooth profiles are estimated by replacing the age and cohort dummies by fifth order polynomials, and extending the highest age to 70 to avoid some of the endpoint problems commonly encountered with polynomial smoothing.

Income profiles are used to construct estimates of $\{G_t\}$ for the consumer problem. Recalling that, $\ln Y_t^i = \ln P_t^i + \ln U_t^i = \ln G_t + \ln P_{t-1}^i + \ln N_t^i + \ln U_t^i$, after removing the cohort, family, and time effects, our procedure is in effect taking a sample average over a large number of individuals, M, with the same characteristics:

$$\frac{1}{M}\sum_{i=1}^{M}\ln Y_t^i = g_t + \frac{1}{M}\sum_{i=1}^{M}\ln Y_{t-1}^i + \frac{1}{M}\sum_{i=1}^{M}\ln N_t^i + \frac{1}{M}\sum_{i=1}^{M}\ln U_t^i - \frac{1}{M}\sum_{i=1}^{M}\ln U_{t-1}^i$$

Applying the Law of Large Numbers, the probability limits of the last three term are all zero. Hence, we get

$$plim\left(\frac{1}{M}\sum_{i=1}^{M}\ln Y_t^i - \frac{1}{M}\sum_{i=1}^{M}\ln Y_{t-1}^i\right) = \ln G_t.$$

Thus simply first differencing our log-average income levels gives the income growth rates which are input into the simulated model. $\bar{Y}_t = \exp[\frac{1}{M}\sum_{i=1}^{M}\ln Y_t^i]$ provides an estimate of average income by age.

3.2. The Consumer Expenditure Survey and Our Use of it.

The main data source for the consumption and income profiles is the Consumer Expenditure Survey (CEX). The CEX contains information about consumption expenditures, demographics, income and assets, for a large sample of the US population. The Survey is conducted by the Bureau of Labor Statistics in order to construct baskets of goods for use in the bases for the Consumer Price Index, and has been run continuously since 1980. We use data from 1980 to 1993 from the family, member, and detailed expenditure files. The survey is known to have excellent coverage of consumption expenditures, to have reasonable data on liquid assets, and to have income information of moderate quality.[34] The survey interviews about 5500 households each quarter. In a household's first interview, the CEX procedures are explained to them and information is collected so that they can be assigned a population weight. They are then interviewed four more times (once every three months) about detailed consumption expenditures over the previous three months. In interviews two and five, income information is collected, and in the final interview asset information is collected. Families rotate through the

[34] See Lusardi (1993), Attanasio (1994), and Branch (1994).

process, so that about 25% of households leave and are replaced in each quarter. About half of all households make it through all the interviews.

Each household contributes one datapoint to our sample. For each household we construct a measure of household income and consumption, and assign it to an occupation group, an education group, a birth cohort, an interview year, and a Census region. In order to obtain a high quality sample which tracks men and has the required information, we drop a significant portion of the data and make a series of adjustments. A detailed description of the data preparation is contained in Appendix B, however, we will make note of three major points here, and then turn to our definitions of consumption and income.

First, we dropped households which are classified as incomplete income reporters, which had any of the crucial variables missing, or which reported changes in age over the course of the survey greater than one year or negative. Households dropped when constructing profiles by occupation remain in the education profiles. We did not use the occupational classifications of Armed forces, Service workers, and Farming, forestry, or fishing due to small cell sizes. Similarly we do not analyze the group of households with male heads holding less than 9 years of schooling due to very few younger households. Second, we dropped all households with male heads younger then 25 or older than 70, since as discussed above we are choosing to focus on the working life. Cell sizes are reported in Table 3.1. Third, while topcoding is very infrequent in consumption information, the household annual income variable reflects summation over a topcoded item for roughly half a percent of our households. Since, in most years, topcoding occurs at $100,000 in income subcategories, reported individual annual labor income is the source of almost all income topcoding problems. However, households are also asked the gross amount of their paycheck and what length of time-period this paycheck covers. By multiplying these two variables together, we construct a second measure of annual labor income. Topcoding on this variable occurs only for a few cases. We correct our measure of after-tax family income by replacing the reported annual labor income in family income with our constructed measure whenever the family income variable is topcoded. We are able to correct almost all topcoding.

Finally, we consider how best to construct measures of income and consumption which match the concepts in the theoretical model. First, we choose to define consumption as total household expenditures less those on education, medical care, and mortgage interest. These categories of expenditure do not provide current utility but rather are either investments or negative income shocks.[35] They are also excluded from our income definition.

It should be noted that our model refers to nondurable consumption at annual frequencies. Since we are averaging expenditures across a large number of individuals and looking across one-year horizons, the distinction between durables and nondurables is less likely to matter. Since the buffer stock model gives strong

[35]We are arguing here that user cost of housing -repairs, maintenance, utilities, and housing services- captures the expenditures made for consumption on housing.

Table 3.1: Cell Sizes for Consumption Profiles

	Households	Percent
Total	43031	
Education Group		
Some Highschool	4409	10.2
Highschool Degree	12906	30.0
Some College	10027	23.3
College Degree	6570	15.3
Grad/Prof School	6087	14.1
Occupation Group		
Managerial and Professional Specialty	13215	34.3
Technical, Sales, and Admin. Support	6976	18.1
Precision Production, Craft and Repair	5061	13.1
Operators, Fabricators, and Laborers	7116	18.5
Self-Employed	3479	9.0

Note: this table shows counts used in constructing unsmoothed consumption profiles. More observations are used for smoothed profiles and fewer are available for income profiles.

predictions about consumption tracking income, it is important not to break the consumption-income link when studying consumption.[36]

Our measure of income is comprised of after-tax family income less Social Security tax payments, mortgage interest, expenditures on medical care, spending on education, pension contributions and after-tax asset and interest income. For the first five adjustments, the related expenditures do not provide current utility but are either non-liquid investments or, in the case of medical care, simply losses of income. Further these expenditures involve a large amount of commitment and are hard to substitute intertemporally. Were we to include pension contributions in income, for example, our measure of liquid assets which could be used to buffer bad income shocks would include pension wealth. We remove asset income since the input to our theoretical model is a profile of income net of liquid asset returns. Attanasio (1994) looks at savings behavior and finds large savings by the typical household over the entire life. If we consider a larger measure of income and take the relative levels of consumption and income seriously, we also find significant savings over the life cycle. Our preferred interpretation of pension and housing wealth however is that it is illiquid, and thus that the typical household has little in liquid assets with which to buffer income shocks early in life.

[36]When computing power has increased, this issue is likely to be surmountable by adding a state variable for the stock of durable goods and taking a stand on the size and relevance of adjustment costs.

3.3. Life Cycle Profiles.

Figure 3.1 presents the estimated consumption and income profiles for our entire sample.[37] Even after correcting for cohort, time, and family effects, both profiles are still hump shaped and still track each other early in life. Consumption lies above income from age 25 to 28. Reflecting on our own experiences, we may interpret this as underreporting the assistance which is provided by intergenerational transfers early in life. After these first few years, consumption rises with income from age 30 to age 45, when consumption drops significantly below income. This tracking is however a lot less than is observed in profiles constructed by simply averaging cross-sections. As stated above, the two main reasons for this are the changes in family size over the life cycle and the different wealth and incomes of different cohorts. Figure 3.2 displays the profile of average family size over the life cycle. Figures 3.3a and 3.3b present the consumption and income profiles without the cohort adjustment and without the family size adjustment, respectively. In each case the unadjusted profiles are more hump shaped, and seem to track each other more closely. These profiles are more directly comparable to those shown in Kotlikoff and Summers (1981) and Carroll and Summers (1991) which do not correct for family size over the life cycle or for cohort effects.

Despite the fact that the asset data in the CEX is of lesser quality than data from sources like the Survey of Consumer Finances (SCF), we can see an interesting age pattern in the profile of asset accumulation, which mimics that found in more accurate surveys. Figure 3.4 shows a life-cycle profile of the ratio of total liquid assets to income. Liquid assets in the CEX are the value of holdings of stocks and bonds, and the cash held in savings and checking accounts. We observe that the typical household accelerates its building of a stock of liquid assets around age 45. This age pattern also shows up in the profiles of income and consumption and is the crucial feature that will help us to pin down the time-heterogeneity in consumer behavior: until around age 45, people consume roughly their income, saving very little in the form of liquid assets.

Figures 3.5 and 3.6 give some evidence that consumption and income track each other across subgroups of the population defined by education and occupation levels. These graphs are unfortunately noisy. However, despite the noise in the data, one can see that the occupation and education groups with the most pronounced humps in income present the most pronounced humps in consumption. Further, we can formally reject the null hypothesis that the consumption profiles are flat. Table 3.2 presents F-tests of the equality of 40 age dummies in a profile regression which also includes age (when appropriate, interacted with subgroup). We get very strong rejection of a constant slope in all our profiles.[38] We could also proceed to

[37]We get reasonable relative levels of consumption and income as does Attanasio (1994), who uses relative levels in his analysis of saving in the US. However, the levels may not tightly identified by the data. In our results section, we will present evidence on estimation which instead uses only information from the changes in income and consumption.

[38]Of course it is possible that we have omitted a key preference shifter which varies over the lifecycle. As Attanasio et al. (1995) note, labor supply is an obvious candidate. Thus, we also

Table 3.2: F-Tests for Flatness of Consumption Profiles

	F-Stat.		F-Stat.
Whole Sample	1121		
Education Group		Occupation Group	
Some Highschool	637	Managerial and Professional Specialty	934
Highschool degree	1015	Technical, Sales, and Admin. Support	803
Some College	1023	Precision Production, Craft and Repair	676
College Degree	804	Operators, Fabricators, and Laborers	790
Grad/Prof School	533	Self-Employed	604

Note: the F-statistic is distributed as a $F(39, 43000)$. The critical value is 1.40.

test whether the income profiles are significant in predicting consumption profiles across occupation and education groups. But this is essentially a Hall and Mishkin (1982)-style test of the CEQ consumption Euler equation. Lusardi (1993) performs such a test, merging Panel Study of Income Dynamics (PSID) data with the CEX consumption to get the best possible combination of data, and she rejects the null hypothesis that the consumption profiles are unrelated to the income profiles.

Finally, Figure 3.6 displays the profile for total, nondurable, and food consumption, all rescaled to the same mean. This figure demonstrates that total consumption over the life cycle is not simply a scaled up version of nondurable consumption. Expenditures on durable goods involve spending income when the good is purchased, and receiving a utility flow over time. To the extent that consumers would like to borrow against future income, purchases of durable goods tighten this constraint by moving expenditures forward in time relative to utility flow. If consumers are buffer stock when young and if nondurable and durable consumption are substitutes, then one would expect to see total consumption rising slower early in life and peaking later than nondurable consumption. The relative profiles of these consumption categories are at least consistent with what we shall conclude subsequently: that households are behaving as buffer stock consumers early in their lives.

test for flatness conditioning on labor income variables. We include in the regressions, annual hours worked by the male head and annual hours worked by the female head. We are still able to reject flatness in the total and in all subgroups. We do not use profiles constructed after removing this correlation in our later analysis, since our theoretical model implicitly assumes that utility is additively separable in leisure and consumption expenditures.

4. Estimation Strategy.

4.1. Method of Simulated Moments (MSM) Estimation.

According to section 2, consumption at age t for individual i depends on cash on hand x_t^i, the realization of permanent component of income P_t^i, the entire path of expected permanent income growth $\mathfrak{G}_T = \{G_t\}_{t=1}^T$, and the parameters of the consumption problem $\tilde{\theta}' = (\beta, \rho, R, \gamma_0, \gamma_1, p, \sigma_u^2, \sigma_n^2)$, an 8x1 vector.[39] In practice, it will not be possible to estimate directly all the elements of $\tilde{\theta}$. Instead, we will calibrate most of the parameters using existing micro data and will focus on the estimation of the structural parameters of the utility function β and ρ. In other words, we rewrite $\tilde{\theta} = (\theta', \chi')'$ where $\theta = (\beta, \rho)'$ and $\chi = (R, \gamma_0, \gamma_1, p, \sigma_u^2, \sigma_n^2)'$. The elements of χ will be calibrated from micro data and we will estimate only the elements of θ. Defining the vector of state variable $z_t^i = (x_t^i, P_t^i)$, for individual i, we postulate the following data-generating process:

$$\ln C_t^i = \ln C_t\left(z_t^i, \theta; \chi, \mathfrak{G}_T\right) + \epsilon_t^i = \ln\left(c_t\left(x_t^i, \theta; \chi, \mathfrak{G}_T\right) P_t^i\right) + \epsilon_t^i, \tag{4.1}$$

where ϵ_t^i is an idiosyncratic shock that represents measurement error in consumption levels and satisfies $E\left[\epsilon_t^i | z_t^i\right] = 0$.[40] We are interested in estimating θ. Direct estimation of (4.1) is not possible since we have only poor information on individual assets x_t^i.[41] We do however observe χ and average consumption at each age, $\ln \bar{C}_t$, as defined in the previous section. This suggests that we can look directly at *the unconditional expectation of log-consumption at each age*:

$$\ln \hat{C}_t(\theta; \chi, \mathfrak{G}_T) \equiv E\left[\ln C_t\left(z, \theta; \chi, \mathfrak{G}_T\right)\right] = \int \ln C_t(z, \theta; \chi, \mathfrak{G}_T)\, dF_t(z). \tag{4.2}$$

This says that average consumption of households equals the average level of consumption over both values of cash-on-hand and levels of permanent income. Our approach consists in matching the T moment conditions:

$$E\left[\ln C_t - \ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right)\right] = 0.$$

Defining the sample moment at age t, $g_t\left(\theta; \chi, \mathfrak{G}_T\right) = \ln \bar{C}_t - \ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right)$, and $g\left(\theta; \chi, \mathfrak{G}_T\right)$the vector of moments, the estimation procedure minimizes

$$g\left(\theta; \chi, \mathfrak{G}_T\right)' W_T\ g\left(\theta; \chi, \mathfrak{G}_T\right)$$

[39] For ease of notation and consistency with our theoretical model, we assume in this subsection that age runs from 1 to T.

[40] ϵ_t^i may also encompass missing variables such as class. Our approach remains correct as long as ϵ_t^i and the state variables are independent.

[41] This is the approach taken by Palumbo (1994) who uses Maximum Likelihood and PSID data to estimate structural parameters during the retirement period. One could also estimate consumption functions non-parametrically, much like Gross (1994), using Kernel methods, estimates investment functions of firms facing liquidity constraints.

where W_T is a weighting matrix. With a weighting matrix equal to the identity, this is equivalent to minimizing the distance between the average and the estimated profile. That is, considering the life-cycle profile, we minimize:

$$S(\theta; \chi, \mathfrak{G}_T) = \sum_{t=1}^{T} \left(\ln \bar{C}_t - \ln \hat{C}_t(\theta; \chi, \mathfrak{G}_T) \right)^2. \tag{4.3}$$

Unfortunately we do not directly observe $\ln \hat{C}_t(\theta; \chi, \mathfrak{G}_T)$, since we do not observe the distributions of permanent income or of cash-on-hand. Further, we do not have analytic solutions for the consumption functions or how they change with alteration of our key parameters. Thus, instead of computing the actual expectation with respect to the true distribution of z, we use a Monte-Carlo integration method and perform a *Method of Simulated Moments* (MSM) estimation. We draw a random sample of shocks to income $\{U_t^i, N_t^i\}_{i=1}^{L}$, as defined in (2.4), calculate the associated paths of consumption and cash on hand, and compute the simulated sample average:

$$\ln \tilde{C}_t(\theta; \chi, \mathfrak{G}_T) = \frac{1}{L} \sum_{j=1}^{L} \ln C_t\left(z_t^i, \theta; \chi, \mathfrak{G}_T\right).$$

We then choose parameters to minimize:

$$\hat{S}(\theta; \chi, \mathfrak{G}_T) = \sum_{t=1}^{T} \left(\ln \bar{C}_t - \ln \tilde{C}_t(\theta; \chi, \mathfrak{G}_T) \right)^2.$$

In practice, we simulate $\ln \tilde{C}_t(\theta; \chi, \mathfrak{G}_T)$ by running $20,000$ independent income processes (temporary and permanent) for 40 years, and computing in each year the associated consumptions. That is, we average over $20,000$ profiles as in Figures 2.2.a and 2.2.b.

Under the regularity conditions stated in Pakes and Pollard (1989) and Duffie and Singleton (1993), the MSM estimator $\hat{\theta}$ is both consistent and asymptotically normally distributed. Denoting the number of observations at age t as $I(t)$, $I = \frac{1}{T}\sum_{t=1}^{T} I(t)$, $\tilde{g}_t = \ln \bar{C}_t - \ln \hat{C}_t(\theta; \chi, \mathfrak{G}_T)$, and $\tilde{g} = (\tilde{g}_1, ..., \tilde{g}_T)'$:

$$\sqrt{I}\left(\hat{\theta} - \theta_0\right) \rightsquigarrow \mathfrak{N}(0, V),$$

where V is estimated by:

$$\hat{V} = \left(\hat{D}'\hat{D}\right)^{-1} \hat{D}'\hat{\Omega}\hat{D} \left(\hat{D}'\hat{D}\right)^{-1} \tag{4.4}$$

$$\hat{D} = \left.\frac{\partial \tilde{g}}{\partial \theta'}\right|_{\theta=\hat{\theta}}. \tag{4.5}$$

$$\hat{\Omega} = avar(\tilde{g}) \tag{4.6}$$

Efficient estimation is obtained when $W_T = \Omega^{-1}$. In practice, we use $\hat{W}_T = \hat{\Omega}^{-1}$. This methodology also provides a useful overidentifying restriction test. If the model is correctly specified, the statistic

$$\chi_{T-2} = I\, \tilde{g}\,(\theta; \chi, \mathfrak{G}_T)'\, \hat{\Omega}^{-1} \tilde{g}\,(\theta; \chi, \mathfrak{G}_T)$$

is distributed asymptotically as Chi-squared with $T-2$ degrees of freedom.

As discussed previously, the levels of our profiles might be misestimated. To test the robustness of our results, we also estimate without using information on the level of consumption. Assuming that the bias is constant through time, the moment conditions become:

$$E\left[\ln C_t - \ln \hat{C}_t\,(\theta; \chi, \mathfrak{G}_T) - \alpha\right] = 0 \quad ; \quad \forall\, 1 \leq t \leq T,$$

for some unknown constant α. Since we are not interested in estimating this parameter, we instead rewrite the moment condition in first difference:

$$E\left[\Delta \ln C_t - \Delta \ln \hat{C}_t\,(\theta; \chi, \mathfrak{G}_T)\right] = 0 \quad ; \forall\, 2 \leq t \leq T,$$

where Δ is the time-difference operator. This amounts to performing the estimation in first differences, that is, minimizing:[42]

$$\tilde{S}\,(\theta; \chi, \mathfrak{G}_T) = \sum_{t=2}^{T} \left(\Delta \ln \bar{C}_t - \Delta \ln \tilde{C}_t(\theta; \chi, \mathfrak{G}_T)\right)^2.$$

The rest of the procedure is identical, with the exception that we now have only $T-1$ moments.

4.2. Remaining Calibration and Data.

In order to find the consumption rules for the consumers in our dynamic program, we must still specify the elements of χ. Computing power currently limits us to searching over only two parameters, and to checking robustness and relative explanatory power as we change other parameters. We now present our calibration of the remaining parameters in our model.

1. The after-tax real interest rate is set as $r = R - 1 = 3\%$ per annum. This is roughly the average real interest rate on high grade municipal bonds over the sample period of our data.

2. The variance of the permanent and transitory components of shocks to income, σ_u^2 and σ_n^2, are taken from Carroll and Samwick (1994). Carroll and Samwick (1994) estimate these parameters from the Panel Study of Income Dynamics (PSID), which provides repeated high-quality measures of household income. The estimation procedure is based on income differences of

[42] with an identity weighting matrix.

Table 4.1: Variance of Income Shocks

Group	Variance of Permanent Shock	Variance of Transitory Shock
Total	0.0217	0.0440
Occupation		
Managerial and Prof. Speciality	0.0180	0.0357
Tech., Sales, and Admin. Support	0.0235	0.0361
Precision Prod., Craft, and Repair	0.0175	0.0432
Operators and Laborers	0.0299	0.0458
Self Employed	0.0165	0.0926
Education		
Some Highschool	0.0214	0.0658
Highschool Degree	0.0277	0.0431
Some College	0.0238	0.0342
College Graduate	0.0146	0.0385
Graduate School	0.0115	0.05

Source: Carroll and Samwick (1994).

different lengths and correctly assigns the relative importance of transitory and permanent income shocks even in the presence of significant moving average correlation of transitory shocks, up to an $MA(2)$. The procedure and data employed are designed to estimate the parameters for the income process in Carroll (1992)– that is, exactly the income process we have specified.[43] Table 4.1 displays the variances of the permanent and transitory shocks across education and education groups.

3. We follow Carroll (1992) and set the probability of zero income to $p = 0.005$. This calibration comes from the PSID and again is estimated with the goal of calibrating exactly the income process which we are considering.

4. Given our assumption on the retirement value function, the optimal consumption function in the last working period is linear in the permanent component of income and the level of cash on hand, or in normalized terms: $c_T = \gamma_0 + \gamma_1 x_T$. This results, as discussed earlier, from the assumption that post-retirement income– pension and Social Security income– is illiquid, cannot be borrowed against before retirement, and that the only source of uncertainty after retirement is the time of death. In order to calibrate the parameters of this consumption rule, we first construct an income profile

[43] The definitions of occupation in the PSID and CEX do not exactly overlap, so that we are required to make rather crude adjustments to one cell.

Table 4.2: DATA FOR RETIREMENT CONSUMPTION RULE

	CONS.	INCOME	LIQUID ASSETS	TOTAL RESOURCES	γ_0	γ_1
TOTAL AVERAGE	18349	23839	164616	352490	0.384	0.049
EDUCATION GROUP						
Some Highschool	13631	16112	48296	156788	0.531	0.079
Highschool Graduate	14556	17791	146852	311519	0.409	0.044
Some College	18768	24865	215380	427021	0.354	0.042
College Graduate	19628	27375	344287	607399	0.297	0.031
Graduate School	24410	32690	345442	665461	0.342	0.035
OCCUPATION GROUP						
Managerial and Prof.	22461	37732	406053	706256	0.240	0.030
Tech., Sales, Admin.	21394	30809	288376	520212	0.292	0.039
Precision Prod., Craft	17010	26726	252064	436494	0.253	0.037
Operators, Laborers	15557	23438	229031	370511	0.238	0.039
Self Employed	19654	25350	9097	201430	0.658	0.087

Source: CEX 1980-1993.

which adds Social Security and pension contributions to the income measure we consider. We do not use housing wealth to calibrate this parameter since most elderly do not run down the asset value of their housing. The difference between this profile and the consumption profile is accumulated at the assumed interest rate and gives an estimate of total resources at retirement, $W_T + H_T$. A similar calculation using our main income measure gives a measure of liquid assets at retirement, W_T. Finally we calculate the required parameters using date from smoothed profiles at retirement as:

$$\gamma_1 = \frac{C_T}{W_T + H_T + Y_T} \tag{4.7}$$
$$\gamma_0 = \gamma_1 \frac{H_T}{Y_T}.$$

The results are summarized in the Table 4.2.[44]

5. Since households generally begin life with some assets, we capture this by assuming that initial cash on hand, x_0, is equal to 0.3 times permanent income at age 25, a number consistent with the CEX.[45]

[44] We find significantly larger assets on both counts than appears in wealth data (Venti and Wise (1993)).

[45] As it turns out, the results are mostly insensitive to this assumption. See section 5.3.

5. Estimation Results.

We first estimate both the discount factor and the intertemporal elasticity of substitution for the average household. We then turn to disaggregated results, by education and occupation groups. Lastly, we discuss the robustness of our results to the calibrated parameters.

5.1. Results for the Entire Sample.

We start by asking what the standard Life-Cycle theory would predict, assuming away all uncertainty. Although this constitutes a crude attempt at matching the data, it serves as a useful benchmark against which to evaluate the rest of our results. To give the best chance to the CEQ LCH, we perform first difference estimation of the CEQ LCH, not asking it to fit the mean of the consumption profile, as discussed at the end of section 4.1. Under certainty, equation (2.3) holds, implying, after controlling for individual characteristics, a constant growth rate of consumption over the working period:

$$\Delta \ln \bar{C}_t = \frac{1}{\rho} \ln (\beta R) \equiv \xi. \tag{5.1}$$

We estimate ξ from the coefficient on age in a least-squares regression on individual data. This procedure seems trivial only because of our earlier efforts to remove changing family-size and cohort effects. It is precisely this simplicity which gives the CEQ LCH its power. From our estimate of ξ, we use the delta method to recover the discount factor and its standard error, postulating a real interest rate of three percent and a coefficient of relative risk aversion of 0.49.[46] The latter choice matters little since consumption is estimated to be nearly flat. The former matters a lot, and changes our estimates one for one.[47] We estimate a discount rate of 2.56% with a standard error of 0.05.[48] However, and not surprisingly given Figure 3.1, the certainty model performs poorly when it comes to explaining the dynamics of consumption across the life cycle. The estimated profile does not capture the hump shape in consumption, as Figure 5.1 demonstrates. Were we to present figures which showed both our fitted values and the data unadjusted for family size and cohort effects, the CEQ LCH would look better and our procedure less naive.

Next we use our structural model to estimate both the discount rate and the coefficient of relative risk aversion. The resulting estimates are reported in Table 5.1.[49]

[46] This is the estimate from our MSM procedure with uncertainty. While in our stochastic model we are able to estimate the coefficient of relative risk aversion and the discount factor,

Table 5.1: Structural Estimation

Method	Level	Level	Differences
Optimal Weighting	No	Yes	No
β	0.9615	0.9625	0.9603
S.E.	(0.0043)	(0.0042)	(0.0048)
δ (%)	4.0017	3.8891	4.1313
S.E.	(0.472)	(0.4527)	(0.537)
ρ	0.5437	0.4897	0.558
S.E.	(0.2092)	(0.2068)	(0.2404)
χ^2	288.22	288.05	514.66

Note: MSM estimation for entire group in levels and first differences. Cell size is 43031. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 38 and 37 degrees of freedom respectively. The critical values at 5% are 53.10 and 51.91. Efficient estimates are reported in the second column with a weighting matrix $\hat{\Omega}$ computed from the first step.

Estimation in levels yields tight estimates of both parameters of interest.[50] The structure which our model imposes on the data gives us strong predictions about preferences. Although first-difference estimates are slightly less precise, they do not appear to contradict the level estimates. Moreover, the overidentifying restrictions are more strongly rejected for the difference method. Therefore, in what follows, we will refer only to level estimates. Efficient estimation has a minor impact on the point estimates or their standard deviations. In both level and first difference estimation, we can reject strongly the overidentifying restrictions. The 5% critical value for a χ^2 (38) is 53.10, and 51.91 for a χ^2 (37). This is not entirely surprising, given the number of moments we use. Moreover, the initial and last moments (average consumption at young ages and just before retirement) are each possibly misspecified if we have misspecified the initial distribution of cash on hand or the

here they are not separately identified.

[47]This estimation procedure only captures the substitution effect, as is clear from (5.1). Income and wealth effects change the *level* of the consumption path, not its *slope*.

[48]The standard error is not robust to serial correlation and probably is underestimated by an order of magnitude.

[49]MSM estimation is performed in 3 steps. First, we do a broad grid search over the parameter space, then a finer grid search and lastly use an optimization routine. This ensures that the program identifies the global minimum. The programs are CPU-intensive: one evaluation of the objective function requires approximately 45 minutes on an experimental P6 (or "686") chip running at 133 Mhz (courtesy of Intel Corp.). A 25x25 grid search therefore requires 12 days. For details, see Appendix *A*.

[50]An earlier version of this paper presented pictures of our objective function which demonstrated that the optima lay in a valley in (β, ρ) space. Many interpreted this as implying that the separate identification of β and ρ was tenous– that we could really only accurately estimate a linear relation between parameters. We want to emphasize that the valley picture is correct, but that the valley has a clear minima which gives us estimates of both parameters.

retirement rule.

Our estimates of the discount rate are close to the interest rate. In the efficient estimation case, we cannot reject the hypothesis that the two are equal at standard levels of confidence. It is worth noting that the discount factor which we estimate is within a reasonable range. Using information on the elasticity of assets with respect to uncertainty, Carroll and Samwick (1994) estimate that discount rate is in the vicinity of 10-15% and argue that even higher discount rates are needed to rationalize the findings of Hubbard et al. (1994). Our lower discount rate, however, does not imply that households are not impatient enough to generate buffer stock behavior. Lower levels of impatience generate buffer stock behavior when combined with steep income profiles.

With our estimates in hand, we can now address how well the stochastic model fits the life-cycle consumption profile. Figure 5.4 plots the simulated and actual consumption data along with the income profile, and a 95% pointwise confidence interval for the simulated profile. The stochastic life-cycle model does a much better job at fitting the consumption profile than the certainty line.[51] Consumption tracks income until around age $40 - 42$ and then falls sharply, as the household starts building up its retirement savings. Simulated consumption never exceeds income, except in the first periods of life.[52] The tight structure imposed by the model is able to deliver good predictions in terms of consumption dynamics, despite having only two free parameters to work with.

Why are we able, within the context of our model to obtain such tight estimates of the discount rate? Figure 5.5 plots various simulated profiles for different β between 0.91 and 0.95, corresponding to a discount rate between 4.71 and 9.98 percent. It is immediately obvious that the profiles are quite sensitive to this parameter. With a higher discount factor, the agent is willing to save more and earlier for retirement purposes. The consumption path exhibits less of a hump shape, and may even be increasing over the entire working life. On the other hand, for more impatient consumers, consumption parallels income until much later in life and then falls more precipitously to build assets for retirement. This implies a stronger concavity of the consumption profile. Thus, our method will yield tight estimates of the discount factor precisely because the discount factor drives the hump shape in consumption.

We now turn to the question of how household behavior changes over the life cycle. Figure 5.6 displays the average household saving rate when there are no initial assets ($x_0 = 0$). Two distinct phases in the consumer's life are visible. Until about age 40, households built their buffer and then consume roughly their income. Around age 40, retirement considerations induce an increase in saving. From then until the age of retirement, households consume much less than their

[51]Unfortunately, the two cases we analyzed are not nested, so that hypotheses testing is not possible.

[52]This result is simply an artifact due to our assumptions regarding the initial level of cash on hand.

current income.

We can put some additional structure on these two phases by looking at the target level of cash on hand at each age, as defined in section 2. Figure 2.3, already reported, plots next period expected cash on hand as a function of current cash on hand for various ages. Figure 5.7 directly computes the target level of cash on hand for consumers aged $25-48$. One can see from the graph that the target level of cash on hand remains small early in life, around 1.3 times permanent income. Shortly after 40, the target increases substantially, as consumers try to build their retirement nest-egg. This figure shows a dramatic change in behavior. When the target level of liquid wealth is small, agents are "buffer stock". Their consumption closely follows their income. Around age 41, agents desire to accumulate assets for retirement. With a large stock of wealth relative to permanent income, consumers can smooth high frequency movements in income. Their behavior more closely mimics that of certainty-equivalent consumers.

None of these results are assumed in our model. If we had found either consumers to be more patient or flatter adjusted income profiles, households could have behaved as life-cycle consumers for their entire lives. Similarly, very impatient consumers could have been buffer stock consumers all of their working lives, relying on illiquid wealth to finance consumption during retirement.

We can also decompose total saving at each age into life-cycle and buffer-stock saving. Our previous discussion might lead the reader to think that agents have no concern for retirement when they are young and no concern for labor income uncertainty later in life. This is incorrect since consumers are rational and perfectly foresee their retirement needs. To proceed, we define total saving as the discounted variation in financial wealth from one period to the next, using our simulated profile:[53]

$$S_t = \left(W_{t+1} - W_t\right)/R = (R-1)/R\ W_t + Y_t - C_t.$$

Saving is equal to investment plus labor income minus consumption, i.e. to disposable income minus consumption. Next, for the estimated parameters, we compute the consumption path $\left\{C_t^{LC}\right\}$, that would occur under certainty.[54] We then define life-cycle saving as the difference between total income and life-cycle consumption

$$S_t^{LC} = \left(W_{t+1}^{LC} - W_t^{LC}\right)/R = (R-1)/R\ W_t^{LC} + Y_t - C_t^{LC}$$

and buffer stock saving is defined as the complement. Figure 5.8 plots the precautionary saving, liquid and total life-cycle saving. The latter is defined by adding back to income pension and social security contributions. Given the estimated discount rate, CEQ life-cycle consumers would like to borrow early in life. However, precautionary saving motives cause them to hold a positive buffer stock of wealth.

[53] The discount comes from our assumption that income is received and consumption occurs at the beginning of the period. See (2.2).

[54] In order to do this, we calibrate the certainty case, so as to yield the same consumption rule at retirement. Consumers effectively have strong bequest motives under these assumptions.

Table 5.2: BENCHMARK CERTAINTY CASE, $\rho = 0.4897$

GROUP	δ (percent)	S.E.
TOTAL	2.56	0.045
EDUCATION		
Some Highschool	3.12	0.125
Highschool Graduate	3.12	0.074
Some College	3.02	0.092
College Graduate	2.94	0.111
Graduate School	2.78	0.128
OCCUPATION		
Managerial and Prof.	2.71	0.083
Tech., Sales, Admin.	2.89	0.109
Precision Prod., Craft	2.95	0.119
Operators, Laborers	2.95	0.102
Self-Employed	2.99	0.169

Note: estimation based on first differences of profiles from Figures 3.5 and 3.6

Around age 40, in accordance with our previous characterization, life-cycle savings becomes larger than precautionary savings. The need to build retirement savings sets in. As asset levels increase, the expected variance of consumption declines, decreasing the precautionary saving motive. This latter effect, which our previous decomposition masked, induces the agent to *run down* the buffer. As a result, the total saving rate later in life is smaller than under the certainty equivalent framework.

5.2. Disaggregated evidence.

We next fit the model to each occupation and education cell separately. To begin with, we present the results for the CEQ LCH case across education and occupation groups in Table 5.2. Note that here and subsequently, we do not incorporate possible cross-cells correlation.[55]

Again, the results indicate that the discount rate is accurately estimated and is roughly equal to the interest rate. Higher education levels tend to have a higher discount factor. Figures 5.2 and 5.3 present these fitted profiles. One can see the increase in the slope for higher educational groups.

To estimate across cells using our stochastic model, we simply follow the procedure described above. However, due to computing power constraints, we are unable to search over the entire (β, ρ) space for all cells. Since the results are more

[55] Note also that the same individuals are allocated both to an education and an occupation cell. Therefore, results across education/occupation are not independent.

Table 5.3: ESTIMATES FROM THE STOCHASTIC MODEL, $\rho = 0.4897$

GROUP	β	S.E.	δ	S.E.	χ^2
EDUCATION					
Some Highschool	0.9607	($1.33\ 10^{-5}$)	4.09	(0.001)	256.73
Highschool Graduate	0.9592	($2.84\ 10^{-5}$)	4.25	(0.003)	107.41
Some College	0.9629	($2.37\ 10^{-4}$)	3.84	(0.025)	85.25
College Graduate	0.9679	($1.62\ 10^{-5}$)	3.31	(0.002)	115.36
Graduate School	0.9656	($2.00\ 10^{-5}$)	3.56	(0.002)	165.96
OCCUPATION					
Managerial and Prof.	0.9621	($1.75\ 10^{-4}$)	3.94	(0.018)	118.11
Tech., Sales, Admin.	0.9669	($2.49\ 10^{-5}$)	3.42	(0.003)	74.92
Precision Prod., Craft	0.9641	($4.60\ 10^{-5}$)	3.72	(0.005)	123.43
Operators, Laborers	0.9655	($4.29\ 10^{-5}$)	3.57	(0.005)	36.53
Self Employed	0.9555	($6.57\ 10^{-3}$)	4.66	(0.719)	67.09

Note: *MSM* estimation in levels over β. Cell size given in Table 3.1. The last colum reports a test of the overidentifying restrictions distributed as a Chi-squared with 38 degrees of freedom. The critical value at 5% is 53.10.

sensitive to the discount factor, we fix the intertemporal elasticity of substitution at its aggregate value, $1/0.49 = 2.04$, and search across values of the discount rate. Each cell's optimization is run with a different income profile (Figures 3.8 and 3.9), income uncertainty (Table 4.1), and retirement consumption rules (Table 4.2), while we impose a constant probability of zero income ($p = 0.5\%$)and real interest rate ($R = 1.03$). The results are summarized in Table 5.3. Except for the last occupation cell (Self-Employed), the parameters are remarkably close to those estimated using the aggregate profile. As we observed with the benchmark case, the discount rate decreases weakly with education. and ranges from 3.31% to 4.25%. The associated fitted profiles are displayed in Figures 5.9 and 5.10. The fit is quite good for most cells, except Self-Employed and College Graduates.[56] For all groups except the first educational group, the consumption profile is humped. As in the aggregate case, we reject the overidentifying restrictions in all cases. We conclude, given the similarity in the estimates that our results are quite robust to heterogenous shocks and processes.

Further, as in the aggregate case, the estimated discount factors are consistently lower in our estimation of the stochastic model, than in the CEQ LCH baseline case. We can reject that the interest rate and the discount rate are equal in all cells, except Self-Employed.

[56]Note that for all cells the standard errors are smaller than for the aggregate estimation. This simply reflects the fact that ρ is fixed. The correlation between ρ and β in the aggregate estimation increases the standard errors. We hope in the future to be able to estimate both parameters for each cells.

Table 5.4: Robustness Checks, $\rho = 0.4897$

		β	S.E.	δ	S.E.	χ^2
1	$R = 1.05,\ \gamma_0 = 0.493;\ \gamma_1 = 0.035$	0.9441	($3.28\ 10^{-4}$)	5.92	(0.036)	266.25
2	$p = 0.05;$	0.9631	($8.56\ 10^{-5}$)	3.83	(0.009)	275.07
3	$\sigma_u^2 = 0;\ \sigma_n^2 = 4.79\,10^{-4}$	0.9632	($6.26\ 10^{-5}$)	3.81	(0.007)	435.67
4	$\gamma_0 = 0.535;\ \gamma_1 = 0.074$	0.9612	($5.4\ 10^{-5}$)	4.03	(0.006)	471.92
5	$x_0 = 0.0$	0.9623	($1.9\ 10^{-4}$)	3.91	(0.020)	489.96
6	Large Income; $\gamma_0 = 0.241;\ \gamma_1 = 0.035$	0.9899	(0.1676)	1.01	(17.10)	30596

Notes: *MSM* estimation in levels over β. Cell size is 43031. The last column reports a test of the overidentifying restrictions distributed as a Chi-squared with 38 degrees of freedom. The critical value at 5% is 53.10. (1) assumes an interest rate of 5% and recomputes accordingly the last working period consumption rule; (4) computes the last working period consumption rule using asset data from Venti and Wise (1993). Large income in (6) includes mortage payments and pension contributions.

5.3. Robustness Checks and Extensions.

Our estimation procedure depends on the calibrated parameters, χ. In this section we investigate the robustness of our results to these parameters. Due to computing constraints, we have only been able to check the robustness with respect to the discount factor. In what follows, we again maintain a constant intertemporal elasticity of substitution equal to 2.04. The results are reported in Table 5.4.

The estimate of the discount rate is, as mentioned, most sensitive to our choice of interest rate. This is not surprising, since, early in life the difference between the interest rate and the discount rate is a key determinant of whether consumers exhibit buffer stock behavior. Late in life, consumers will behave in a manner more consistent with the CEQ LCH, in which the change in consumption is driven by the product βR.

So far we assumed a real interest rate of 3% a year. Although this is close to the long run real rate on liquid assets, some savings are held in longer term assets, yielding on average higher returns. Therefore, we re-estimate the discount rate assuming a real interest rate of 5%.[57] The estimate of the discount factor is now $\beta = 0.9441$, with a standard deviation of $3.28\ 10^{-4}$. This implies a discount factor of 5.92%, almost exactly two hundred basis points higher than our previous estimate. Thus it is important to note that our results do not provide a tight estimate of the discount factor per se. However, *our model gives precise estimates of the difference between the interest rate and the discount rate.* This indicates that we capture mostly the substitution effect. The simulated profile reported in Figure

[57]With a 5% interest rate, we recompute the last period consumption rule as $\gamma_0 = 0.493;\ \gamma_1 = 0.035$. Thus illiquid assets are more important and consumption is less sensitive to current cash on hand.

5.11.1 is roughly similar to our benchmark case, although consumption is smaller later in life to reflect the smaller γ.

We next check the robustness of our results to the probability of zero income, p. A higher p, by increasing uncertainty, should lead to a larger buffer stock, implying less life-cycle savings late in life. Thus, an increase in p is likely to yield a higher discount rate, to counteract the increase in precautionary savings. Our results indicate that this effect is quite weak. When $p = 5\%$, a tenfold increase, the discount rate actually decrease marginally to 3.83%. On the other hand, given unemployment benefits, government assistance programs, one might argue that households can never experience zero income. Decreasing p has even smaller effects on the estimated discount rate. Looking at the simulated profile, we see that the change in p affects mostly consumption early in life. After age 40, asset level can buffer transitory fluctuations in income.

Next, we investigate the sensitivity of our results to the variance of permanent and transitory components, σ_u^2, and σ_n^2. We decreased the uncertainty faced by the agent from both sources of shocks. Intuitively, this should lead to a smaller buffer and should give a lower estimate of the discount rate. Our test eliminates altogether transitory shocks and reduces the variance of the permanent shocks to $\sigma_n^2 = 4.8\ 10^{-3}$. Our estimated discount rate, 3.81%, confirms our intuition and reemphasizes that our results are reliant upon the underlying individual uncertainty. Looking at the simulated profile, we see that consumption is higher early in life and lower later, as the lower buffer translates into smaller accumulated assets.

An important assumption of our model concerns the retirement rule. As described in the previous section, the consumption rule at retirement is calibrated from CEX data, using both income and asset reports. To the extent that asset data are not accurate, our consumption rule is likely to be mismeasured. This, in turn, will affect the life-cycle profile as we near retirement. In order to test robustness to our hypothesis, we calibrate the retirement consumption rule using asset data reported in Venti and Wise (1993). The resulting values are $\gamma_0 = 0.535$; $\gamma_1 = 0.074$. Looking at (4.7), this indicates both a substantially smaller asset accumulation (as γ_1 increases) and a larger share of illiquid assets (as γ_0 increases). With a larger γ_0, the agent can rely on illiquid saving at retirement time. This should lead to a smaller liquid asset accumulation and to a lower discount rate. A larger γ_1 has ambiguous effects. It increases the level of consumption out of cash on hand in the last working period. This is compatible both with a higher discount rate (as the household will consume more of a given wealth) and a lower discount rate (if the households accumulates more wealth). Thus, the actual consumption profile may indicate more or less preference for the present. We estimate a discount rate of 4.03%, slightly higher than the benchmark estimate. This suggests that the presence of illiquid assets play an important role in the precision of our estimates and that the increase in γ_1 dominates. Looking at Figure 5.11.4, we see that assumptions about the last working period consumption rule affect the simulated profile substantially.

We then check the validity of the assumption $x_0 = 0.3$. As for the retirement rule, this is likely to affect our estimates by shifting the consumption profile at young ages. We reestimate our aggregate problem assuming that $x_0 = 0$. The estimated β, 0.9623, is extremely close to our original one. With a lower initial level of cash on hand, households build theirs buffers early in life. This results in slightly lower consumption for the first few years. However, from then on, the consumption profile is similar. Thus, this assumption does not have a large impact on our estimates.

Our estimation procedure is also extremely sensitive to our assumed income profile. Permanent income growth is a key variable and determines to a large extent buffer-stock behavior, as we have demonstrated. Our definition of income subtracts pension contributions and mortgage payments, as they are likely to reflect illiquid saving, for which our theory is ill-equipped. However, it is possible in some circumstances to draw on voluntary pension contribution. Similarly, housing wealth is not entirely illiquid. Thus we reestimate our model assuming that these components of income contribute to liquid savings instead of illiquid savings. The associated values of the consumption rule at retirement are $\gamma_0 = 0.241$; $\gamma_1 = 0.035$. Since pension contribution are now part of liquid savings, the accumulated illiquid savings and γ_0 are lower. In effect, this amounts to shifting upwards the income profile and increases substantially measured savings. Not surprisingly, this can only be matched by a lower discount rate. The estimated discount rate is 1.01%, well below the interest rate. The parameters are poorly estimated and the overidentifying restriction is enormous. This increase in savings can only be matched by assuming that consumers are extremely patient. However, this fails to capture the life-cycle pattern of consumption: the simulated profile grows exponentially.

We conclude that, except with respect to our definition of liquid income, our estimates and inference are reasonably robust to the calibration of our model.

6. Conclusion.

Macroeconomic models generally represent the consumer as an infinitely-lived, rational, representative agent, who behaves in accordance with the Permanent Income Hypothesis. Some analyses provide explicit microeconomic justification for this assumption, by deriving an insurance system which protects individuals from any idiosyncratic consumption risk (e.g. Rogerson (1988)) or by assuming that individual budget constraints never bind, so that aggregate behavior mimics individual behavior (e.g. Barro (1974)). Other models incorporate certain forms of individual heterogeneity when aggregating (e.g. Blanchard (1985)). The resulting representative agent will not, in general, have the same characteristics as individual consumers.

However, nearly all currently employed macroeconomic models include some form of a representative agent facing representative shocks. Such a representative agent framework constitutes an extremely powerful tool, mainly because the restrictions that optimizing behavior place on individuals will carry over to the aggregate economy. In this paper, while we work in an explicitly partial equilibrium environment, we find important and substantial deviations from the canonical representative consumer model.

Using individual-level data to construct average profiles of income and consumption over the working lives of households, we demonstrated that consumption remains hump-shaped, even after controlling for family and cohort effects. We then developed a model of consumption behavior embedding realistic levels of income uncertainty and estimated *individual* consumption functions using the Method of Simulated Moments. The model fits well and yields tight estimates of the discount rate and intertemporal elasticity of substitution. To the best of our knowledge, this is the first paper which uses explicit individual uncertainty and the life-cycle profile of consumption to identify structural parameters of the utility function. The results indicate that consumers hold only a buffer stock of liquid assets in order to offset labor income fluctuations, until around age 42. Then, they start saving actively for retirement purposes. These two phases in consumer behavior are quite distinct and are at the heart of our identification procedure.

This age-heterogeneity of consumer behavior has important implications for aggregate consumption. In particular, it may rationalize Campbell and Mankiw (1989) finding that roughly 40% of all agents are "hand to mouth". In our interpretation, this may simply reflect the consumption of young households. In later work, we plan to investigate in more detail the aggregate implications of the age-heterogeneity of consumers.

References

Abowd, John and David Card, "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 1989, *57*, 411–45.

Attanasio, Orazio, "Personal Saving in the US," in James M. Poterba, ed., *International Comparisons of Household Saving*, University of Chicago Press, 1994.

__ **and Guglielmo Weber**, "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey," *Journal of Political Economy*, Dec 1995, *103* (6), 1121–57.

__ **, James Bank, Costas Meghir, and Guglielmo Weber**, "Humps and Bumps in Lifetime Consumption," Nov 1995. mimeo.

Attanazio, Orazio and Martin Browning, "Consumption over the Life Cycle and over the Business Cycle," *American Economic Review*, Dec 1995, *85*, 1118–1137.

Ayagari, Rao, "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics*, 1993, *109*, 659–684.

Barro, Robert, "Are Government Bonds Net Wealth?," *Journal of Political Economy*, 1974, *82*, 1095–1117.

Barsky, Robert, Thomas Juster, Miles Kimball, and Matthew Shapiro, "Preference Parameters and Behavioral Heterogeneity: an Experimental Approach in the Health and Retirement Survey," 1995. manuscript, University of Michigan.

Blanchard, Olivier J., "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 1985, *93*, 223–47.

Branch, E. Raphael, "The Consumer Expenditure Survey: a comparative analysis," *Monthly Labor Review*, 1994, pp. 47–55.

Bureau Of Labor Statistics, *Consumer Expenditure Survey, 1993: Interview Survey*, Washington, DC: US Department of Commerce, Bureau of the Census, 1993, and years 1980-1992. Ann Arbor, MI: Inter-university Consortium of Political and Social Research.

Caballero, Ricardo, "Consumption Puzzles and Precautionary Saving," *Journal of Monetary Economics*, 1990, *25*, 113–36.

Campbell, John Y. and N. Gregory Mankiw, "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence," in Oliver Jean Blanchard and Stanley Fischer, eds., *N.B.E.R. Macroeconomics Annual*, Cambridge: MIT Press 1989, pp. 185–215.

Carroll, Christopher D., "The Buffer Stock Theory of Saving: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, 1992, *2*, 61–135.

__ , "Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis," 1993. mimeo, Federal Reserve Board.

__ , "How Does Future Income Affect Current Consumption?," *Quarterly Journal of Economics*, 1993, *109*, 111–147.

__ **and Andrew A. Samwick**, "The Nature of Precautionary Wealth," 1994. manuscript.

__ **and Lawrence H. Summers**, "Consumption Growth Parallels Income Growth: Some New Evidence," in B.D. Bernheim and John B. Shoven, eds., *National Savings and Economic Performance*, Chicago University Press 1991.

Clarida, Richard, "Aggregate Stochastic Implications of the Life-Cycle Hypothesis," *Quarterly Journal of Economics*, 1991, *106*, 851–867.

Council Of Economic Advisors, *The Economic Report of the President*, Washington, DC: Government Printing Office, 1995.

Deaton, Angus, "Panel Data from Time Series and Cross Sections," *Journal of Econometrics*, 1985, *30*, 109–26.

__ , "Saving and Liquidity Constraints," *Econometrica*, 1991, *59*, 1121–1142.

__ , *Understanding Consumption*, Oxford:Clarendon Press, 1992.

Duffie, Darrell and Kenneth Singleton, "Simulated Moments Estimation of Markov Models of Asset Prices," *Econometrica*, July 1993, *61*, 929–952.

Dynan, Karen, "How Prudent Are Consumers?," *Journal of Political Economy*, Dec 1993, *101(6)*, 1104–1113.

Friedman, Milton, *A Theory of the Consumption Function*, Princeton, Princeton University Press, 1957.

Gross, David B., "The Investment and Financing Decisions of Liquidity Constrained Firms," 1994. MIT mimeo.

Hall, Robert E. and Frederic S. Mishkin, "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, 1982, *50*, 461–481.

Hansen, Lars, "Large Sample Properties of Method of Moments Estimators," *Econometrica*, 1982, *50*, 1029–1054.

Heaton, John, "The Interaction Between Time-nonseparable Preferences and Time Aggregation," Jun 1990. Sloan School MIT, Working Paper 3181-90-EFA.

Hubbard, Glenn, Jonathan S. Skinner, and Stephen P. Zeldes, "The Importance of Precautionary Motives for Explaining Individual and Aggregate Saving," in Allan H. Meltzer and Charles I. Plosser, eds., *Carnegie-Rochester Conference Series on Public Policy*, Vol. 40 Jun 1994, pp. 59–125.

__ **, Jonathan Skinner, and Stephen Zeldes**, "Precautionary Saving and Social Insurance," *Journal of Political Economy*, Apr 1995, *103*, 360–397.

Judd, Kenneth, "Projection Methods for Solving Aggregate Growth Models," *Journal of Economic Theory*, 1992, *58*, 410–452.

__ , "Numerical Methods in Economics," 1993. manuscript, Hoover.

Kotlikoff, Laurence J. and Lawrence H. Summers, "The Contribution of Intergenerational Transfers in Aggregate Capital Accumulation," *Journal of Political Economy*, 1981, *86*, 706–32.

Kreps, David and Evan Porteus, "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 1978, *46*, 185–200.

Levenson, Alec R., "Do Consumers Respond to Future Income Shocks? Evidence from Social Security Reform in Taiwan," 1993. mimeo, Milken Institute for Job and Capital Formation.

Lusardi, Annamaria, "Euler Equations in Micro Data: Merging Data from Two Samples," *Center for Economic Research*, 1993. Tillburg University, Progress Report No.5.

Modigliani, Franco and Richard Brumberg, "Utility Analysis and the Consumption Function: an Interpretation of Cross-Section Data," in Kenneth K. Kurihara, ed., *Post-Keynesian Economics*, New Brunswick, N.J. Rutgers University Press 1956, pp. 388–436.

Pakes, Ariel and David Pollard, "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 1989, *57*, 1027–1057.

Palumbo, Michael G., "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle," 1994. University of Houston Mimeo.

Rogerson, Richard, "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 1988, *21*, 3–16.

Shea, John, "Union Contracts and the Life Cycle-Permanent Income Hypothesis," *American Economic Review*, 1995, *85*, 186–200.

Souleles, Nicholas, "The Response of Household Consumption to Income Tax Refunds," 1994. M.I.T. Mimeo.

Stokey, Nancy L., Robert E Lucas, and Edward Prescott, *Recursive Methods in Economics Dynamics*, Harvard University Press, Cambridge, MA., 1989.

Venti, Steven and David Wise, "The Wealth of Cohorts: Retirement Saving and the Changing Assets of Older Americans," 1993. NBER Working Paper 4600.

Zeldes, Stephen P., "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics*, 1989, *104*, 275–98.

Appendices

A. Solving the consumer's problem.

In this appendix we describe our approach to solving numerically the consumer problem.

A.1. Euler equation.

The algorithm exploits the recursive structure of the consumer problem by solving the Euler equation. Given a consumption rule at age $t+1$, $c_{t+1}(.)$, the algorithm solves for the consumption rule $c_t(x_t)$ that satisfies for any x_t:

$$\begin{aligned} u'(c_t(x_t)) &= \beta R\, E\left[u'(c_{t+1}(x_{t+1}))\right] \qquad \text{(A.1)} \\ &= \beta R\, \left(p\, E\left[u'(c_{t+1}(x_{t+1}))\middle|\, U_{t+1}=0\right] + (1-p)\, E\left[u'(c_{t+1}(x_{t+1}))\middle|\, U_{t+1}>0\right]\right). \end{aligned}$$

A.2. Gauss-Hermite quadrature.

Assume for the time being that we know how to compute $c_{t+1}(.)$ for all values of cash on hand. Our first problem consist in evaluating the expectation in (A.1). One can rewrite the Euler equation using the Intertemporal budget constraint,

$$x_{t+1} = X_{t+1}/P_{t+1} = (x_t - c_t)\frac{R}{G_{t+1}N_{t+1}} + U_{t+1}, \qquad \text{(A.2)}$$

as:

$$\begin{aligned} u'(c_t(x)) &= \beta R \left(p\, E\left[u'\left(c_{t+1}\left((x-c_t)\frac{R}{G_{t+1}N}\right)G_{t+1}N\right)\right] + \right. \\ &\qquad \left.(1-p)\, E\left[u'\left(c_{t+1}\left((x-c_t)\frac{R}{G_{t+1}N}+U\right)G_{t+1}N\right)\middle|\, U>0\right]\right). \end{aligned}$$

Since N and U are log normally distributed, the natural way to evaluate these integrals is to perform a two dimensional Gauss-Hermite quadrature:

$$\begin{aligned} E\left[u'(c_{t+1}(x_{t+1})\, G_{t+1}N)\right] &= \int u'\left(c_{t+1}\left((x-c_t)\frac{R}{G_{t+1}N}+U\right)G_{t+1}N\right) dF(N)\; dF(U) \\ &= \int_{-\infty}^{\infty} f_t(n,u)\; e^{-n^2} e^{-u^2} du\, dn \\ &= \sum_{i,j} f_t(n_i,u_j)\,\omega_{ij}, \end{aligned}$$

where $f_t(n,u) = \frac{1}{\pi}\, u'\left(c_{t+1}\left((x-c_t)\,\frac{R}{G_{t+1}}\, e^{-\sqrt{2}\sigma_n n} + e^{-\sqrt{2}\sigma_u u}\right)G_{t+1}e^{-\sqrt{2}\sigma_n n}\right)$. The weights ω_{ij} and nodes n_i, u_j are tabulated in Judd (1993). In practice, we performed a quadrature of order 12.

One can then find the root of the Euler equation at any point x using a standard Newton method. In practice, we constrain the root to be positive and less than x, the current cash on hand. As discussed in the text, this restriction is always satisfied when there are no illiquid assets. Since illiquid assets cannot be borrowed against, it is also satisfied in their presence.

A.3. Consumption rules.

We initialize the algorithm with the consumption rule at retirement: $c_T(x_T) = \gamma_0 + \gamma_1 x_T$. One can show that the consumption rules for this problem are continuously differentiable as long as there are no liquidity constraints. However, in the presence of liquidity constraints, the consumption rules may exhibit a kink. See Deaton (1991)and Ayagari (1993). We effectively impose a liquidity constraint by not allowing the household to borrow against illiquid assets. This indicates that smooth approximation methods, as advocated by Judd (1992) are inappropriate in that case. Instead, we will use a standard discretization method: we specify an exogenous grid for cash-on-hand: $\{x^j\}_{j=1}^{J} \subset [0, x^{\max}]$. In order to capture the curvature of the consumption rule at low values of cash on hand, the grid will be finer for $x \in [0, 2]$. In practice, for each value of cash on hand on the grid, x^j, we find the associated consumption c^j that satisfies (A.1). In choosing the size and coarseness of the grid, we face the usual trade-off between precision and computing time. Adding points on the grid gives a finer approximation of the consumption rules. Since the consumption rule at age $t+1$ is the input necessary to get the consumption rule at age t, imprecisions could compound over time. On the other hand, the Euler equation is the innermost loop of the entire algorithm. With 100 points on the grid and 40 time period, we must solve 4000 solutions to (A.1). This takes approximately 45 minutes on a P6 chip running at $133Mhz$, courtesy of Intel Corporation. We also face a difficult decision regarding the range of cash on hand, $x^{\max}$. For small values, cash on hand in sample is likely to move out of the grid. Consumption will then be evaluated using extrapolation methods, always much less precise than interpolation. On the other hand, increasing the range with a fixed number of points implies less precise estimation of the curvature. One solution consists in endogenizing the grid so that, for instance, cash on hand remains within the grid with probability 0.95. We adopted the simpler approach consisting in checking that cash on hand, in the simulations, remains *strictly* inside the grid. In practice, we took $x^{\max} = 40$ and $J = 100$, with 50 points between 0 and 2. We checked the quality of the approximation by solving the stationary infinite horizon problem and checking the rate of convergence to the fixed point of the functional Bellman equation.

B. Data.

We use the CEX family, member and detailed expenditure files for years 1980 to 1993, as kindly provided by the NBER. Most of our information about the CEX is obtained from Bureau of Labor Statistics (1993, and years 1980-1992) and conversations with BLS statisticians. Households are discarded if they are missing any of the information necessary for the regressions, if they report changes in age from the second to fifth interview of more than a year or less than zero years, if they are classified as incomplete income reporters, or if their reporting implies less than $1000 in annual income or consumption.

We use information about the reference person to assign the household to cells, unless the reference person is female. In this case we use the spouses information. If there is no spouse, or his information is missing, the household is discarded. When this cut was made it eliminated 20% of the sample. All information besides individual labor income and consumption is taken from the family files. Values are assigned to a household based on information gathered in the fifth interview, otherwise information is used from the second interview, or, if it is not available, the household is discarded. Households should not be matched across 1985 to 1986, and are not. Care is taken to assure consistency in our data despite variable classification changes through time, and across reference person and spouse. Information was kindly provided by the Division of the CEX in the Bureau of Labor Statistics about various issues including the matching of occupation codes from 1980-81 to later years.

Pension contributions, income, Social Security contributions, and all asset income all refer to the past twelve months. Our definition of pension contributions is the sum over the CEX subcategories and thus includes private pensions, public pensions, Railroad Retirement pensions, and self-employed, IRA, and Keogh plans. If the after-tax family income variables is topcoded, reference person and spouse labor incomes are subtracted and we add, for each, the variable created by multiplying the earnings in last paycheck by the appropriate pay period. These labor income variables are the sole variables from the member files used. Assets and asset income refers to the sum over savings accounts, checking accounts, bonds, and stocks, as of the time of interview. Each household is assigned to a year based on the midpoint between the first and fifth interview if both data are available; otherwise simply the single interview date is used. Age is the average of both interviews if both are available, otherwise it is the single one available. Due to some extreme reports, we reset reported tax rates above 50% back to 50%, and below zero percent to zero. We perform a similar exercise for Social Security contribution rates and pension contribution rates, using 25% as the upper bound.

Consumption data is compiled from the detailed expenditure files as all expenditures by a household except for those for health care, mortgage interest, and education. The consumption level is then the average monthly expenditure times twelve. Five percent of households have consumption data for 4, 7, 10, 13, or 14 months and these households' consumption are treated as if they were over 3, 6, 9, and 12 months. That is the recall interview period extended beyond the basic three months and some expenditures are recorded in a later month. BLS statisticians recommend treating these expenditures as if they occurred in the preceding month. Those covering 1 or 2 months (one percent of the sample) were dropped.

The unemployment rates merged to the CEX are the regional unemployment rates for civilian population from the Household survey conducted and published by the Bureau of Labor Statistics in "Employment and Earnings." The GNP IPD PCE is from Council Of Economic Advisors (1995).

C. Method of Simulated Moments (MSM).

According to section 2, consumption at age t for individual i depends on cash on hand x_t^i, the realization of permanent component of income P_t^i, the entire path of expected permanent income growth $\mathfrak{G}_T = \{G_t\}_{t=1}^T$, and the parameters of the consumption problem $\tilde{\theta}' = (\beta, \rho, R, \gamma_0, \gamma_1, p, \sigma_u^2, \sigma_n^2)$, an 8x1 vector. In practice, it will not be possible to estimate directly all the elements of $\tilde{\theta}$. Instead, we rewrite $\tilde{\theta} = (\theta', \chi')'$ where $\theta = (\beta, \rho)'$ and $\chi = (R, \gamma_0, \gamma_1, p, \sigma_u^2, \sigma_n^2)'$. We assume that θ belongs to some compact set $\Theta \subset \Re^2$. The elements of χ will be inputs into the estimation procedure. Defining the vector of state variable $z_t^i = (x_t^i, P_t^i)$, we postulate the following data-generating process:

$$\ln C_t^i = \ln C_t\left(z_t^i, \theta; \chi, \mathfrak{G}_T\right) + \epsilon_t^i = \ln(c_t\left(x_t^i, \theta; \chi, \mathfrak{G}_T\right) P_t^i) + \epsilon_t^i, \tag{C.1}$$

where ϵ_t^i is an idiosyncratic shock that represents measurement error in consumption levels. We are interested in estimating θ. If we were able to observe simultaneously the level of cash on hand of consumers and the level of their permanent component of income (as well as their consumption), θ could be estimated using Hansen's GMM (1982) on individual level data. More precisely, one would write the following moment conditions:

$$E\left[h\left(w^i, \theta_0; \chi, \mathfrak{G}_T\right)\right] = 0, \tag{C.2}$$

where θ_0 is the true parameter vector, and

$$\begin{aligned} w_t^i &= \left(\ln C_t^i, z_t^{i\prime}\right)' \\ h_t\left(w_t^i, \theta; \chi, \mathfrak{G}_T\right) &= \left(\ln C_t^i - \ln C_t\left(z_t^i, \theta; \chi, \mathfrak{G}_T\right)\right) \frac{\partial \ln C_t\left(z_t^i, \theta; \chi, \mathfrak{G}_T\right)}{\partial \theta'}. \end{aligned}$$

w is a Tx3 vector and $h\left(w, \theta, \mathfrak{G}_T\right)$ is a Tx2 vector.

The estimation procedure would then minimize:

$$g\left(\theta; \chi, \mathfrak{G}_T\right)' W g\left(\theta; \chi, \mathfrak{G}_T\right), \tag{C.3}$$

where $g\left(\theta; \chi, \mathfrak{G}_T\right) = vec\left(\frac{1}{I}\sum_{i=1}^{I} h\left(w^i, \theta; \chi, \mathfrak{G}_T\right)\right)$ is a $2T$x1 vector and W is a weighting matrix. In practice, the number of cross-section observations for each age varies in the sample, so that $I = I_t$. We do not write explicitly this extra time-dependence in order to keep the notations simpler. The first difficulty with (C.2) is that quality panel data on consumption, asset and income information for individual households are not available in any US dataset. Therefore direct estimation using (C.3) is not possible. We do however observe χ and average consumption at each age $\ln \bar{C}_t \equiv \frac{1}{I(t)}\sum_{i=1}^{I(t)} \ln C_t^i$. This suggests that we can circumvent the problem by looking directly at *the unconditional expectation of consumption at each age*:

$$\ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right) \equiv E\left[\ln C_t\left(z_t, \theta; \chi, \mathfrak{G}_T\right)\right] = \int \ln C_t\left(z, \theta; \chi, \mathfrak{G}_T\right) dF_t(z), \tag{C.4}$$

where the unconditional distributions of normalized cash on hand and permanent income depend on age t.We write the T moment conditions as

$$E\left[h\left(\ln C^i, \theta_0; \chi, \mathfrak{G}_T\right)\right] = 0, \tag{C.5}$$

where θ_0 is the true parameter vector, $\ln C^i = \{\ln C^i_t\}_{t=1}^T$ and $h\left(\ln C^i, \theta, \mathfrak{G}_T\right)$ is a Tx1 vector with t^{th} element:

$$h_t\left(\ln C^i, \theta; \chi, \mathfrak{G}_T\right) = \ln C^i_t - \ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right).$$

However, at this point we encounter a second difficulty. The unconditional distribution for the state variables at age t, $dF_t(z)$, is extremely cumbersome to evaluate, as well as the unconditional expectation (C.4).

The *Method of Simulated Moments*, as developed by Pakes and Pollard (1989) and Duffie and Singleton (1993) allows us to circumvent this difficulty. We can define a measurable transition function $\mathfrak{T}: \Re^2\text{x}\Re^2\text{x}\Re^T\text{x}\Theta \to \Re^2$ that describes the dynamics of the state variables $z_{t+1} = (x_{t+1}, P_{t+1}) = \mathfrak{T}\left(z_t, \nu_{t+1}, \mathfrak{G}_T, \theta\right)$, where $\nu_t = (U_t, N_t)$ according to:

$$\begin{aligned} x_{t+1} &= \frac{R}{G_{t+1}N_{t+1}}\left(x_t - c_t\left(x^i_t, \theta; \mathfrak{G}_T\right)\right) + U_{t+1} \\ P_{t+1} &= G_t\, P_t N_{t+1}. \end{aligned} \tag{C.6}$$

We omit the dependence in the calibrated parameters χ. $U_{t+1}$and N_{t+1} are respectively the permanent and transitory shocks to income. The first line of $(C.6)$ is the normalized budget equation while the second line follows from our assumptions about the income process. This transition function can then be used to rewrite the unconditional expectation (C.4):

$$\begin{aligned} \ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right) &= \int \ln C_t\left(z, \theta; \chi, \mathfrak{G}_T\right)\, dF_t(z) \\ &= \iint \ln C_t\left(\mathfrak{T}\left(z, \nu, \mathfrak{G}_T, \theta\right), \theta; \chi, \mathfrak{G}_T\right) dF_{t-1}(z)\, dF(\nu). \end{aligned} \tag{C.7}$$

Note that the transition function depends on θ, through the consumption rule. (C.7) provides a convenient way to calculate the unconditional expectation is to use a Monte-Carlo integration. Assume that we have an $\Re^2\text{x}\Re^T$-valued sequence of random variables $\{\hat{\nu}^i\}_{i=1}^{i=L}$ where $\hat{\nu}^i = (\hat{\nu}_1, ..., \hat{\nu}_T)'$, identically distributed and independent of $\{\nu^i\}_{i=1}^{i=I}$. From any initial distribution $F(z_0)$ and candidate θ, we can generate the path of state variables according to (C.6):

$$\hat{z}^i_{t+1} = \mathfrak{T}\left(\hat{z}^i_t, \hat{\nu}^i_{t+1}, \mathfrak{G}_T, \theta\right); \qquad \forall\, 1 \le t \le T \text{ and } 1 \le i \le L.$$

For large enough L, the unconditional expectation is then simulated by:

$$\ln \tilde{C}_t\left(\theta; \chi, \mathfrak{G}_T\right) \equiv \frac{1}{L}\sum_{i=1}^{L} \ln C_t\left(\hat{z}^i_t, \theta; \chi, \mathfrak{G}_T\right) \rightsquigarrow \ln \hat{C}_t\left(\theta; \chi, \mathfrak{G}_T\right).$$

For any parameter vector $\theta \in \Theta$ we can construct the T moments:

$$\begin{aligned}
\tilde{g}_t(\theta;\chi,\mathfrak{G}_T) &= \frac{1}{I(t)}\sum_{i=1}^{I(t)}\tilde{h}_t\left(\ln C_t^i,\theta;\chi,\mathfrak{G}_T\right)\\
&= \frac{1}{I(t)}\sum_{i=1}^{I(t)}\ln C_t^i - \ln\tilde{C}_t(\theta;\chi,\mathfrak{G}_T)\\
&= \ln\bar{C}_t - \ln\tilde{C}_t(\theta;\chi,\mathfrak{G}_T).
\end{aligned}$$

The estimation procedure minimizes:

$$\tilde{g}(\theta;\chi,\mathfrak{G}_T)' W \tilde{g}(\theta;\chi,\mathfrak{G}_T), \tag{C.8}$$

where $\tilde{g}(\theta;\chi,\mathfrak{G}_T) = (\tilde{g}_1,...,\tilde{g}_T)'$ is a Tx1 vector and W is a weighting matrix. Note that in the case where $W = I_T$, the identity matrix, the estimation procedure is equivalent to minimizing the sum of square residuals:

$$S(\theta;\chi,\mathfrak{G}_T) = \sum_{t=1}^{T}\left(\ln\bar{C}_t - \ln\tilde{C}_t(\theta;\chi,\mathfrak{G}_T)\right)^2.$$

However, even though we are minimizing the sum of squared residuals, asymptotic results still apply as long as $I(t)\to\infty$ where $I(t)$ is the number of observations at age t. Under the regularity conditions stated in Pakes and Pollard (1989) and Duffie and Singleton (1993), the MSM estimator $\hat{\theta}$ is both consistent and asymptotically normally distributed. Denoting $I = \frac{1}{T}\sum_{t=1}^{T} I(t)$,

$$\sqrt{I}\left(\hat{\theta}-\theta_0\right) \rightsquigarrow \mathfrak{N}(0,V),$$

with

$$\begin{aligned}
V &= (D'WD)^{-1} D'W\Omega WD (D'WD)^{-1}\\
D &= E\left[\partial\tilde{g}/\partial\theta'\right]\\
\Omega &= avar(\tilde{g})\\
W &= plim\ W_I.
\end{aligned}$$

In practice, the asymptotic variance-covariance matrix is estimated by:

$$\hat{V} = \frac{1}{I}\left(\hat{D}'W\hat{D}\right)^{-1}\hat{D}'W\hat{\Omega}W\hat{D}\left(\hat{D}'W\hat{D}\right)^{-1} \tag{C.9}$$

$$\hat{D} = \left.\frac{\partial\tilde{g}}{\partial\theta'}\right|_{\theta=\hat{\theta}}. \tag{C.10}$$

The t^{th} diagonal element of $\hat{\Omega}$ is given by

$$\hat{\Omega}_t = \frac{1}{I(t)} \sum_{i=1}^{I(t)} \left(\ln C_t^i - \ln \tilde{C}_t \left(\theta; \chi, \mathfrak{G}_T \right) \right)^2 .$$

Under the assumption of no serial correlation, the off-diagonal elements of $\hat{\Omega}$ are 0. However, a robust estimator can be constructed if $\hat{\Omega}_{t,t'}$ is defined as:

$$\hat{\Omega}_{t,t'} = \left(\ln \bar{C}_t - \ln \tilde{C}_t \left(\theta; \chi, \mathfrak{G}_T \right) \right) \left(\ln \bar{C}_{t'} - \ln \tilde{C}_{t'} \left(\theta; \chi, \mathfrak{G}_T \right) \right) .$$

This methodology also provides a useful overidentifying restriction test. If the model is correct, the statistic

$$\chi_{T-2} = I \, \tilde{g} \left(\theta; \chi, \mathfrak{G}_T \right)' \hat{\Omega}^{-1} \tilde{g} \left(\theta; \chi, \mathfrak{G}_T \right)$$

is distributed asymptotically as Chi-squared with $T-2$ degrees of freedom.

The optimal weighting matrix is $W = \Omega^{-1}$. The optimal weighting is implemented by first running the regression with an arbitrary weighting matrix, computing the associated $\hat{\Omega}^{-1}$, and then using this estimate in a second round of estimation.

In practice, we simulate $\ln \tilde{C}_t(\theta; \chi, \mathfrak{G}_T)$ by running $L = 20,000$ independents income processes for 40 years, and computing in each year the associated consumption and cash on hand. The first step used $W = I_T$. We also assume that the initial distribution of cash on hand is 0.3 times current income. This assumption captures the fact that most households do not start with no assets. Finally, for all households, we set the initial value of the permanent component of income to the estimated income level at age 25 from our profiles. We performed first a 25x25 grid search over the parameter space Θ. Then, we performed a second 25x25 grid search around the optimum. This guaranties that the procedure converges to the global minimum. Then, we used a standard minimization algorithm. Each grid search takes approximately 12 days of CPU time on a P6 or on a RSC6000. Once the optimum has been found, the gradient of the moment vector is evaluated numerically and the variance-covariance matrix estimated. For the disaggregated results and the robustness checks, a Brent algorithm was used.

Figure 2.1.a: Consumption Rules

β=0.963, ρ=0.490, γ_1=1, G=1

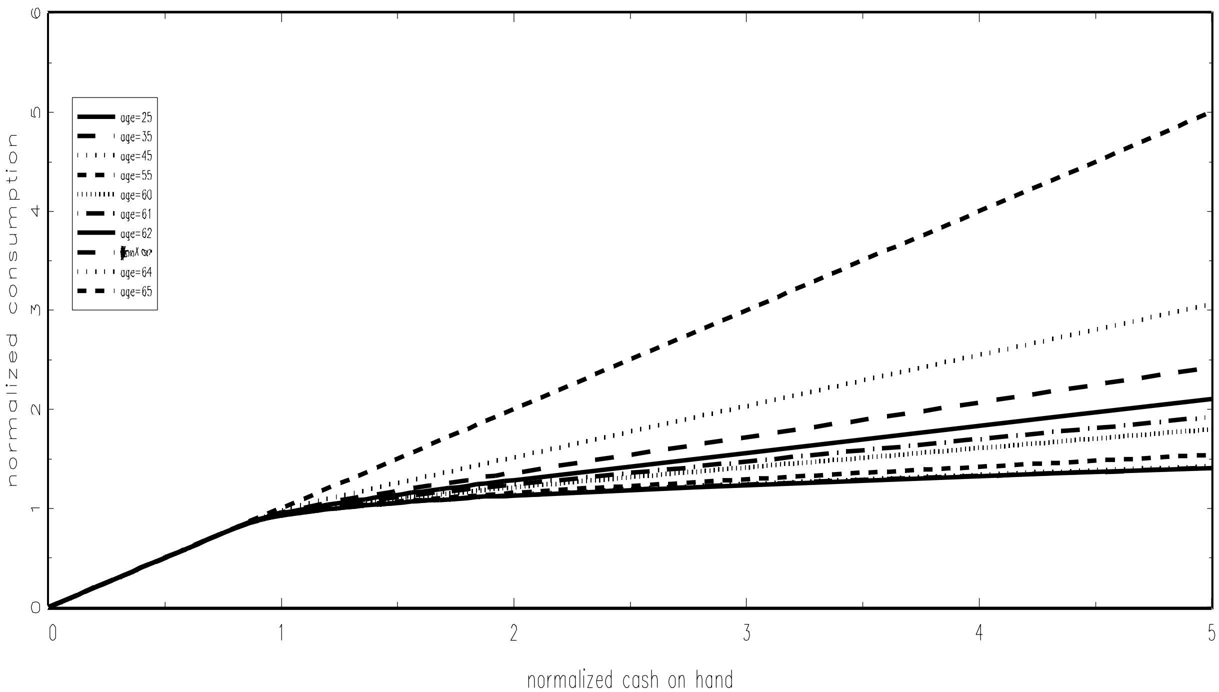


Figure 2.1.b: Consumption Rules

β=0.963, ρ=0.490,γ_0=0.384, γ_1=0.048, G=1

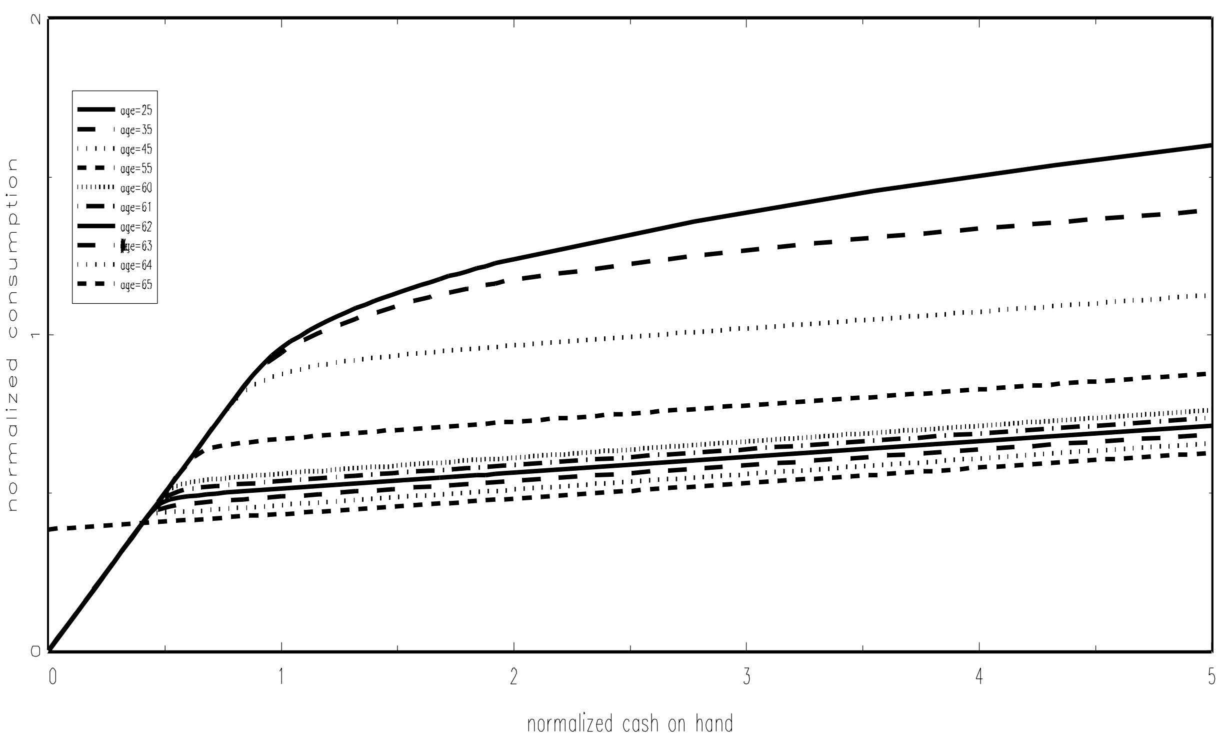


Figure 2.2.a: Individual Profile

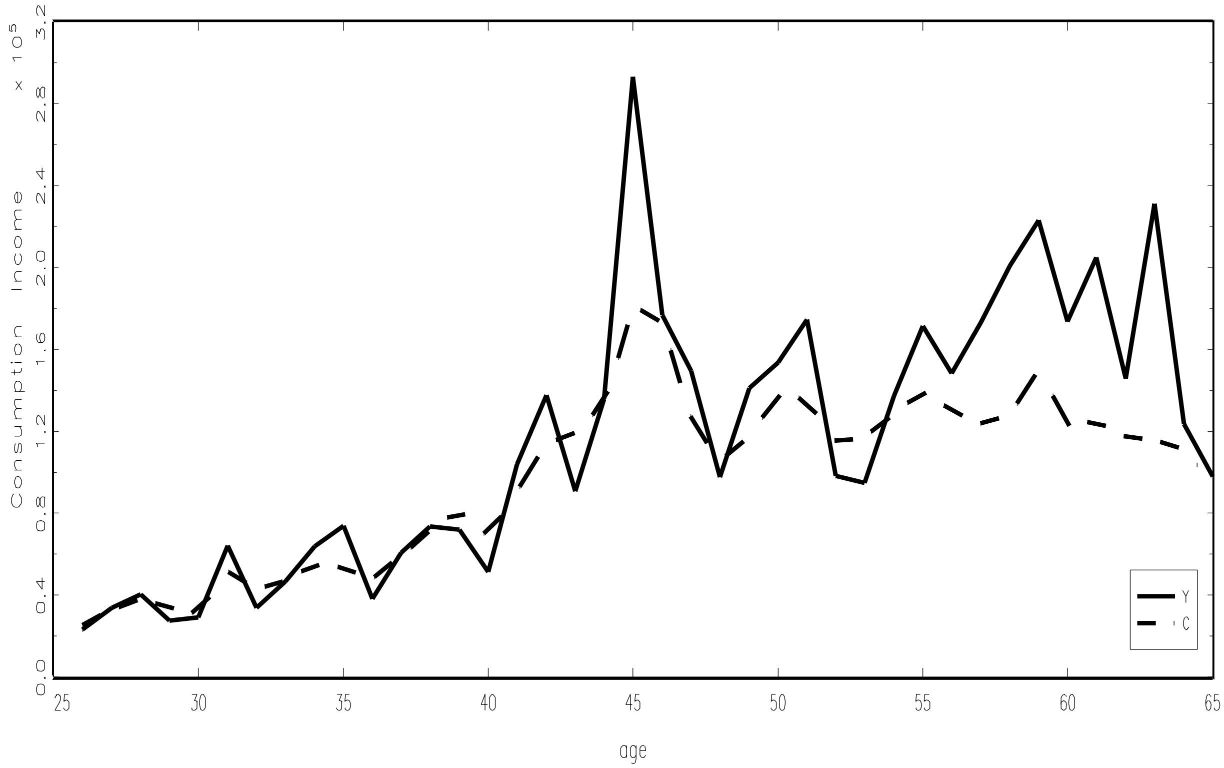


Figure 2.2.b: Individual Profile, 0-Income Shock

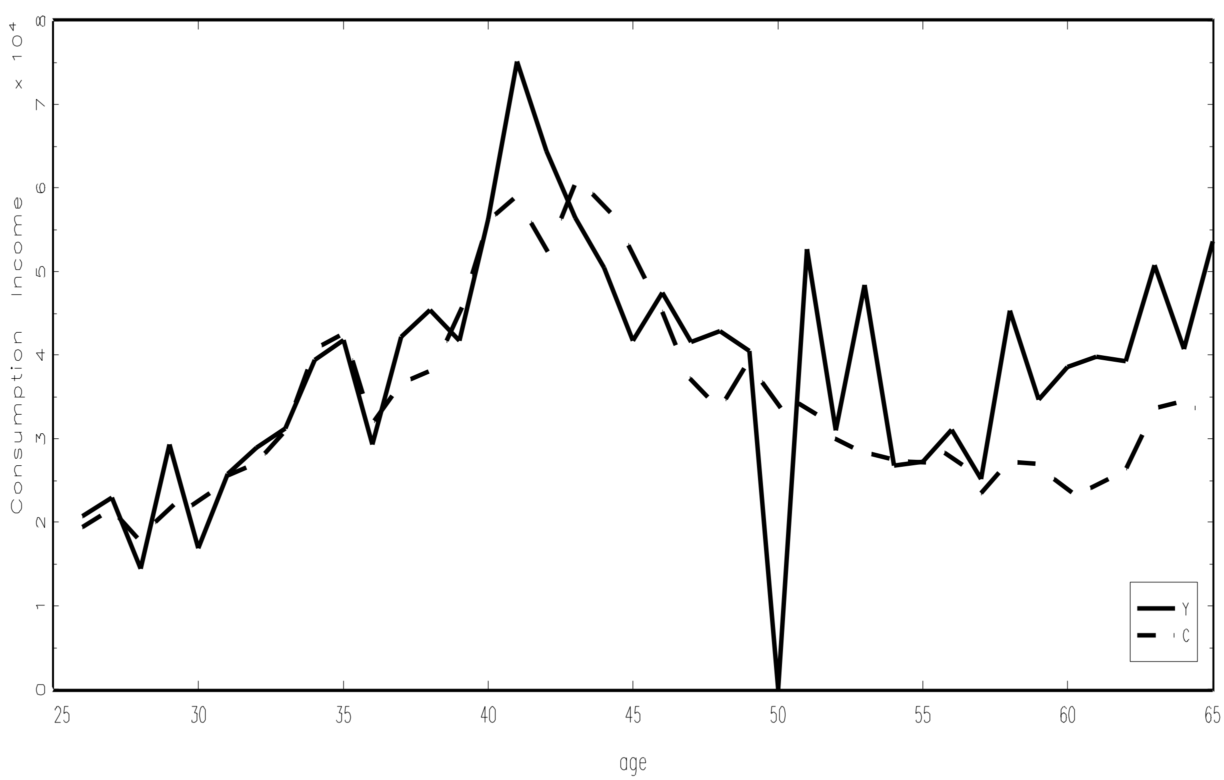


Figure 2.3: Expected Cash on Hand

β=0.963, ρ=0.490

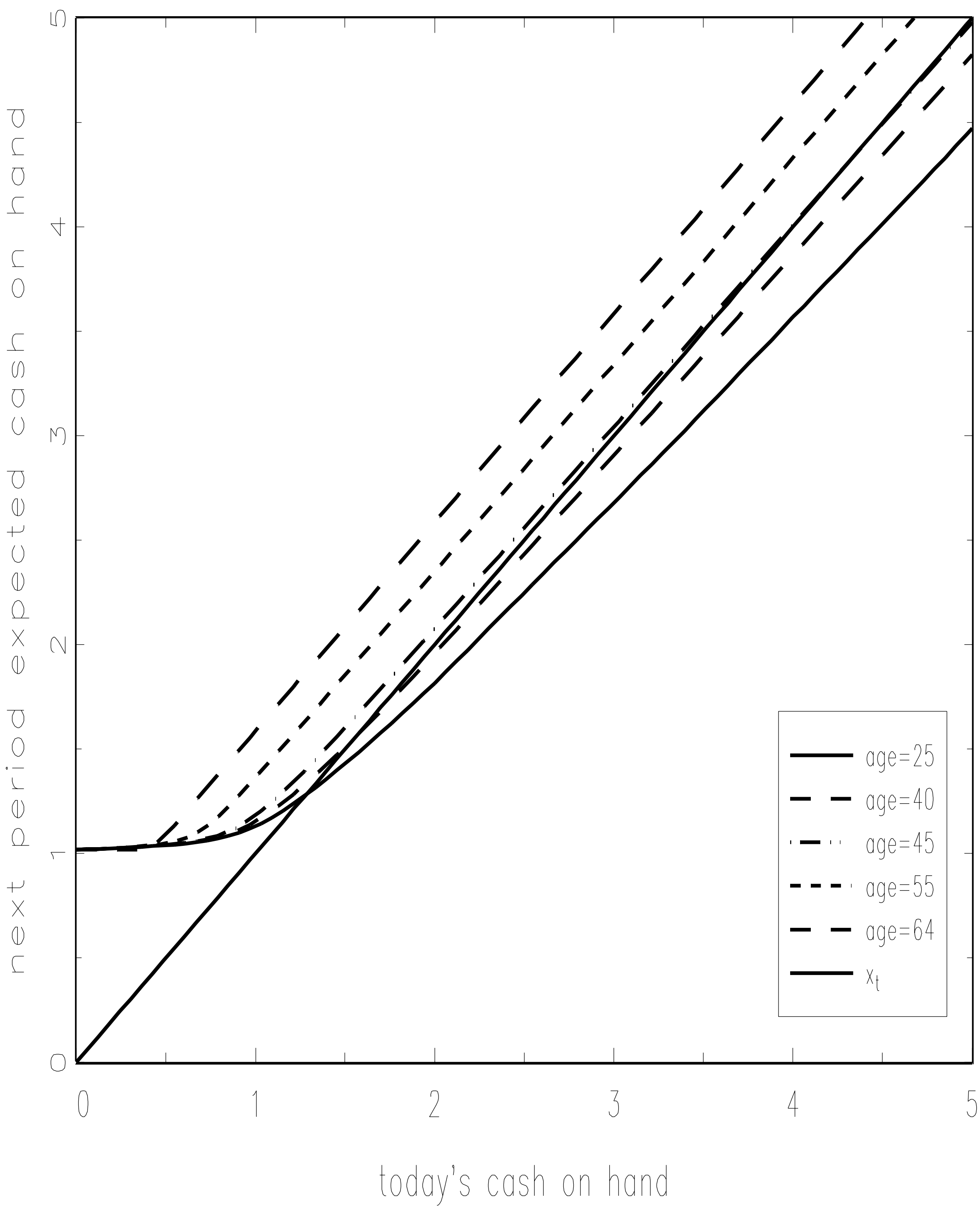


Figure 3.1
Household Consumption and Income over the Lifecycle

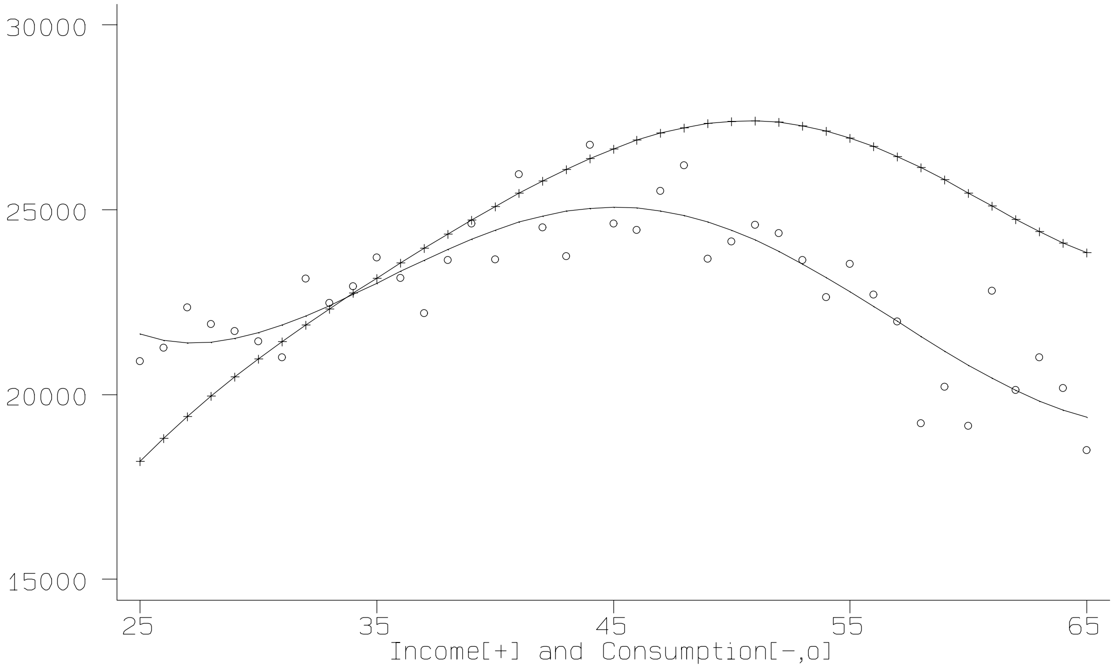


Figure 3.2
Family Size Over the Lifecycle

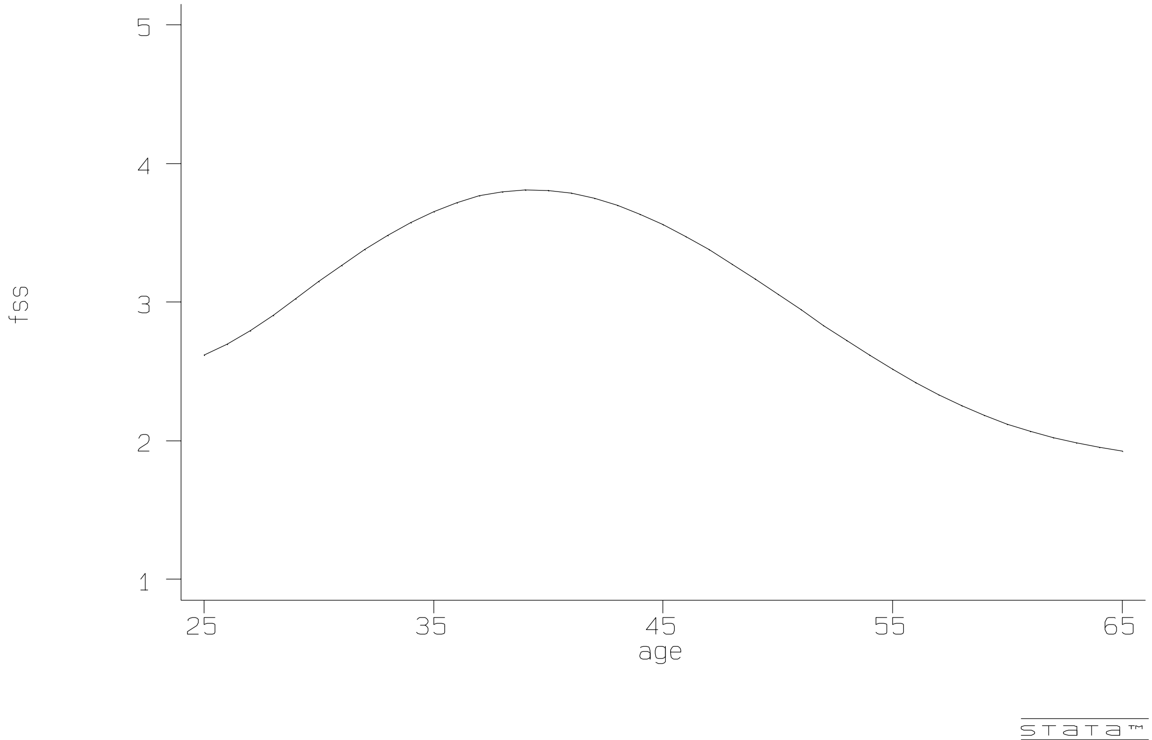


Figure 3.3a
Consumption and Income With and Without Cohort Adjustment

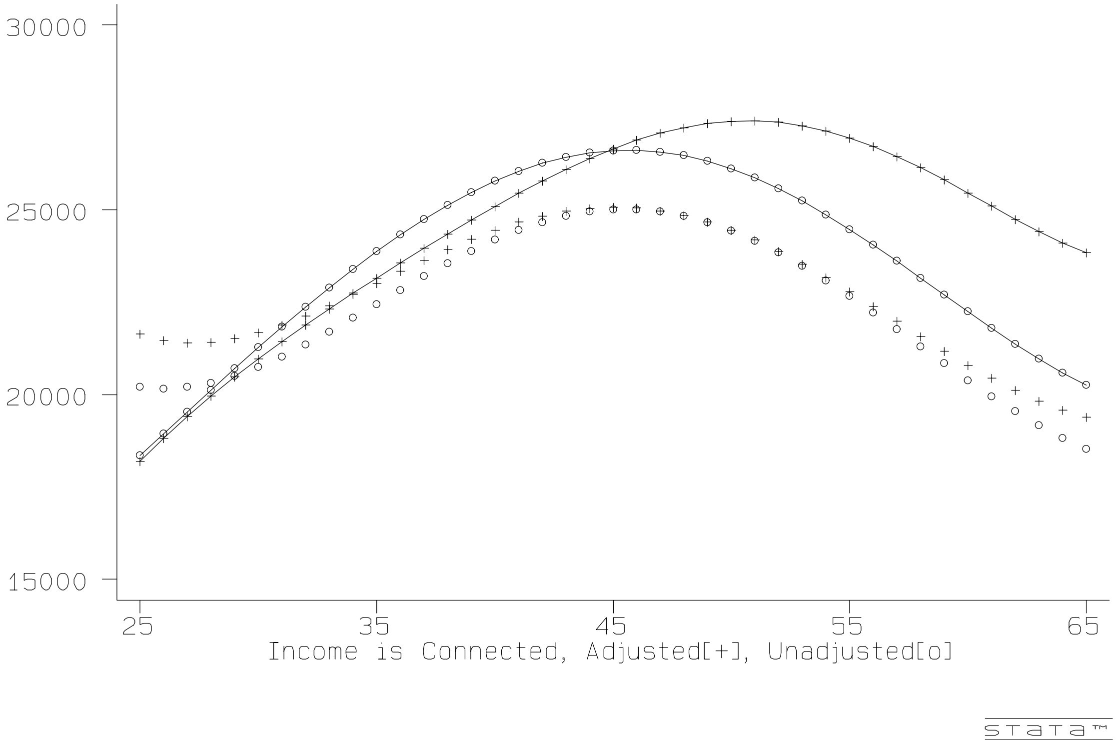


Figure 3.3b
Consumption and Income With and Without Family Adjustments

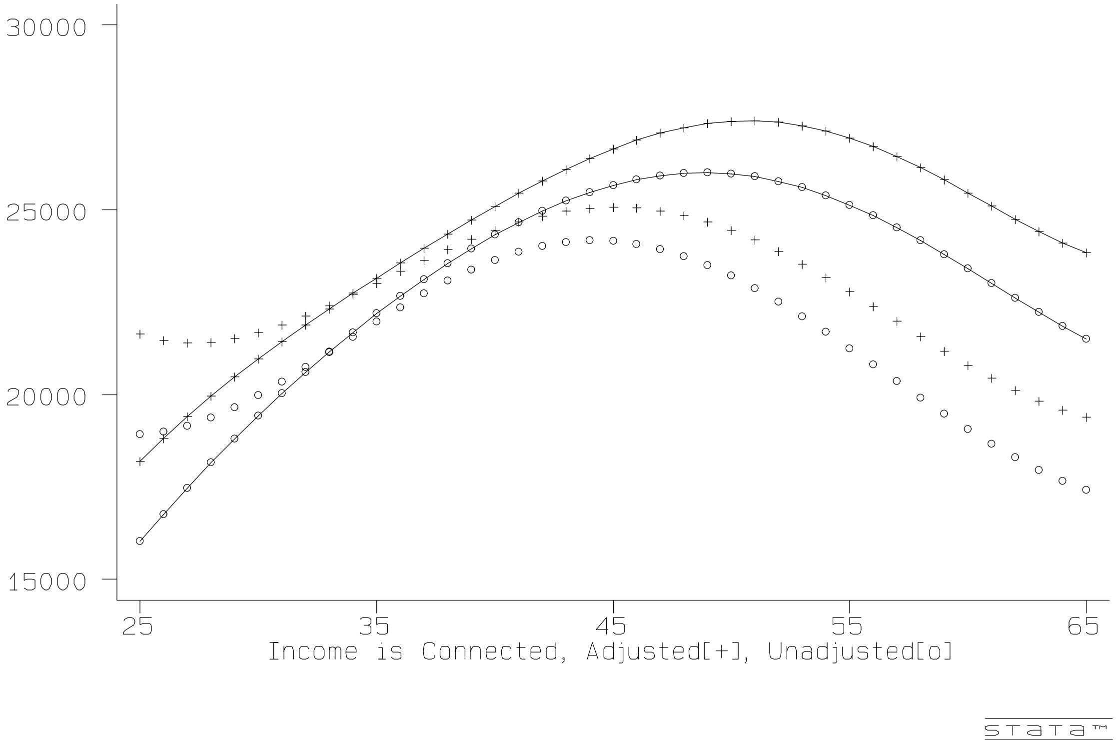


Figure 3.4
Total Liquid Assets to Income Ratio

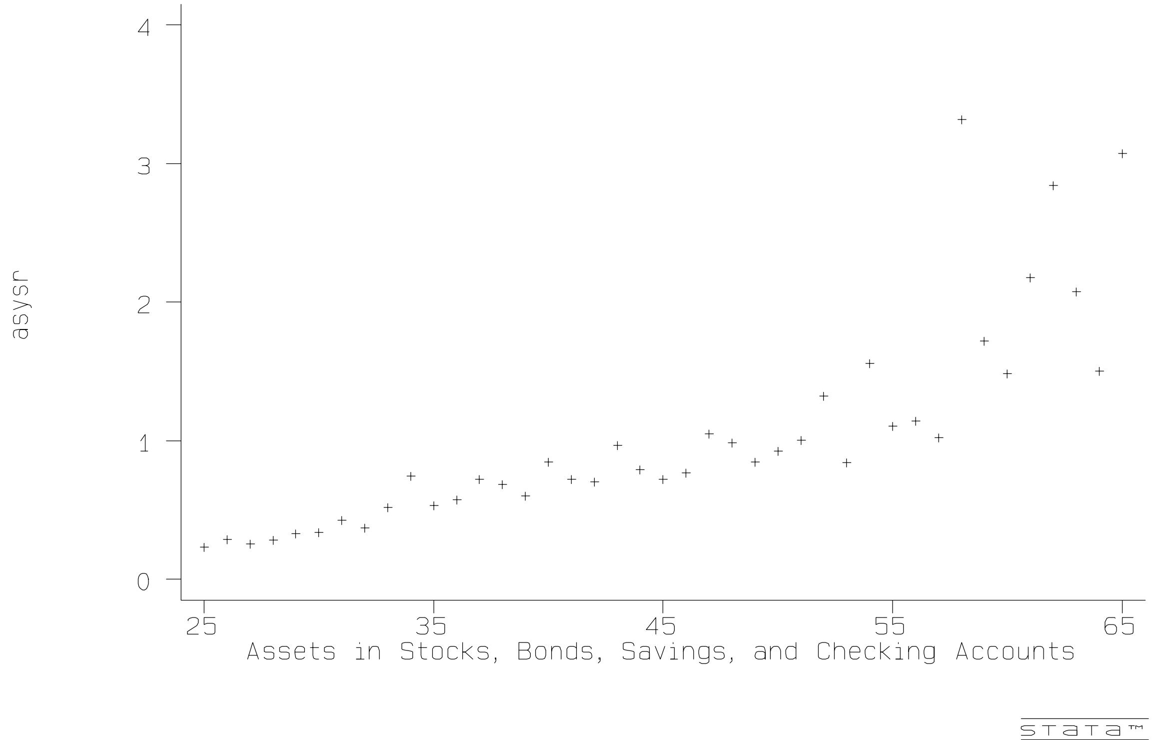


Figure 3.5
Consumption[o] and Income[+] over the Lifecycle

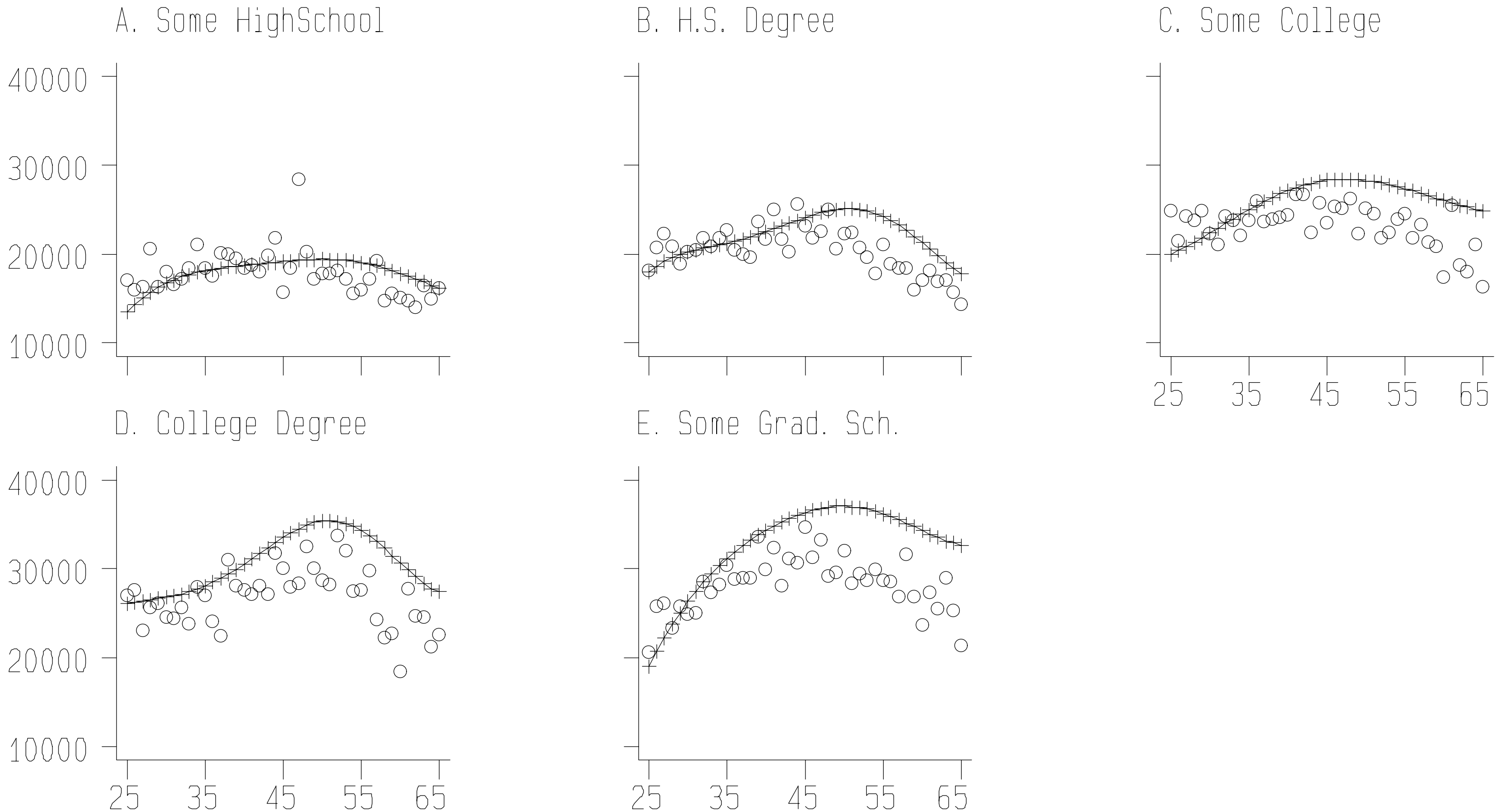


Figure 3.6
Consumption[o] and Income[+] over the Lifecycle

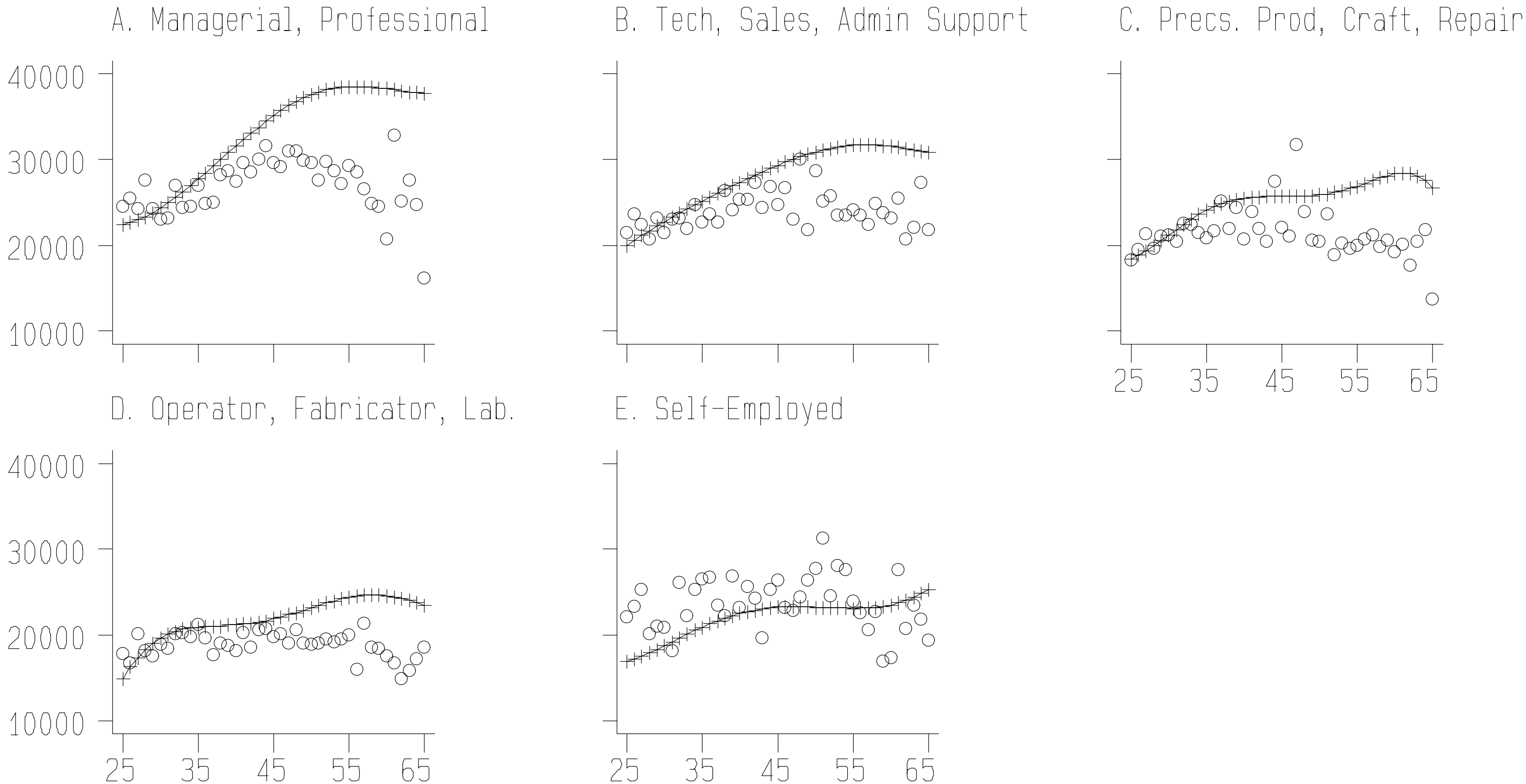
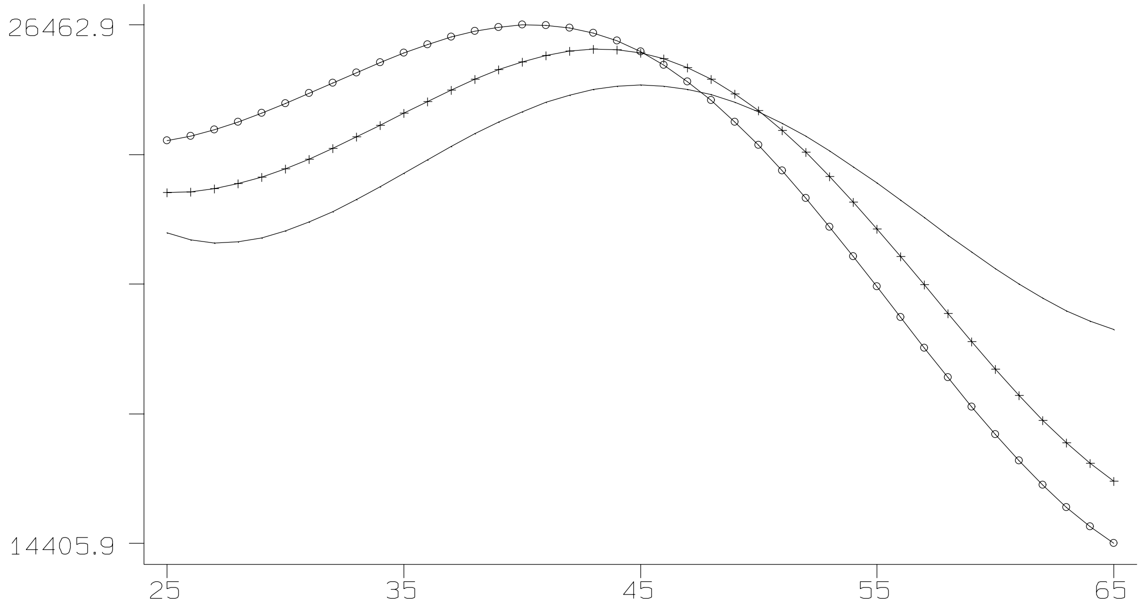


Figure 3.7
Rescaled Smoothed Consumption over the Lifecycle



Note: All series scaled to mean of uncmoothed total consumption

Total[-], Nondurable[o], and Food[+] Consumption

Figure 5.1
Income, Consumption and Consumption Predicted by CEQ LCH

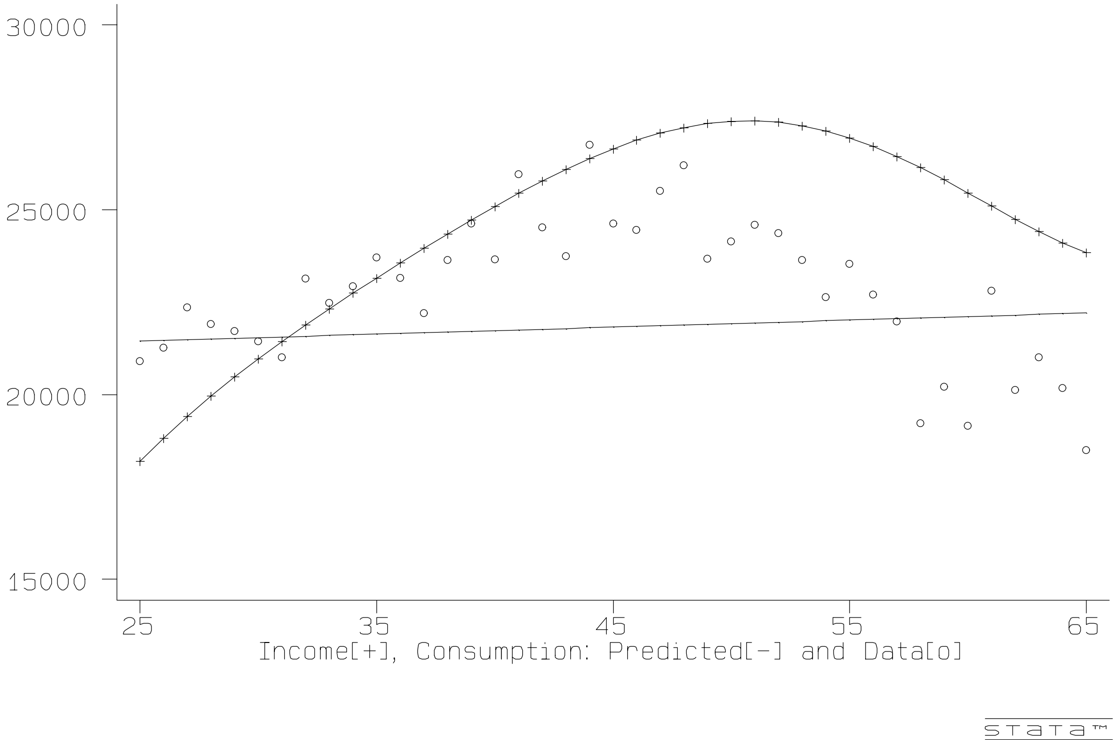


Figure 5.2
By Ed: Income, Consumption, and Consumption Predicted by CEQ LCH

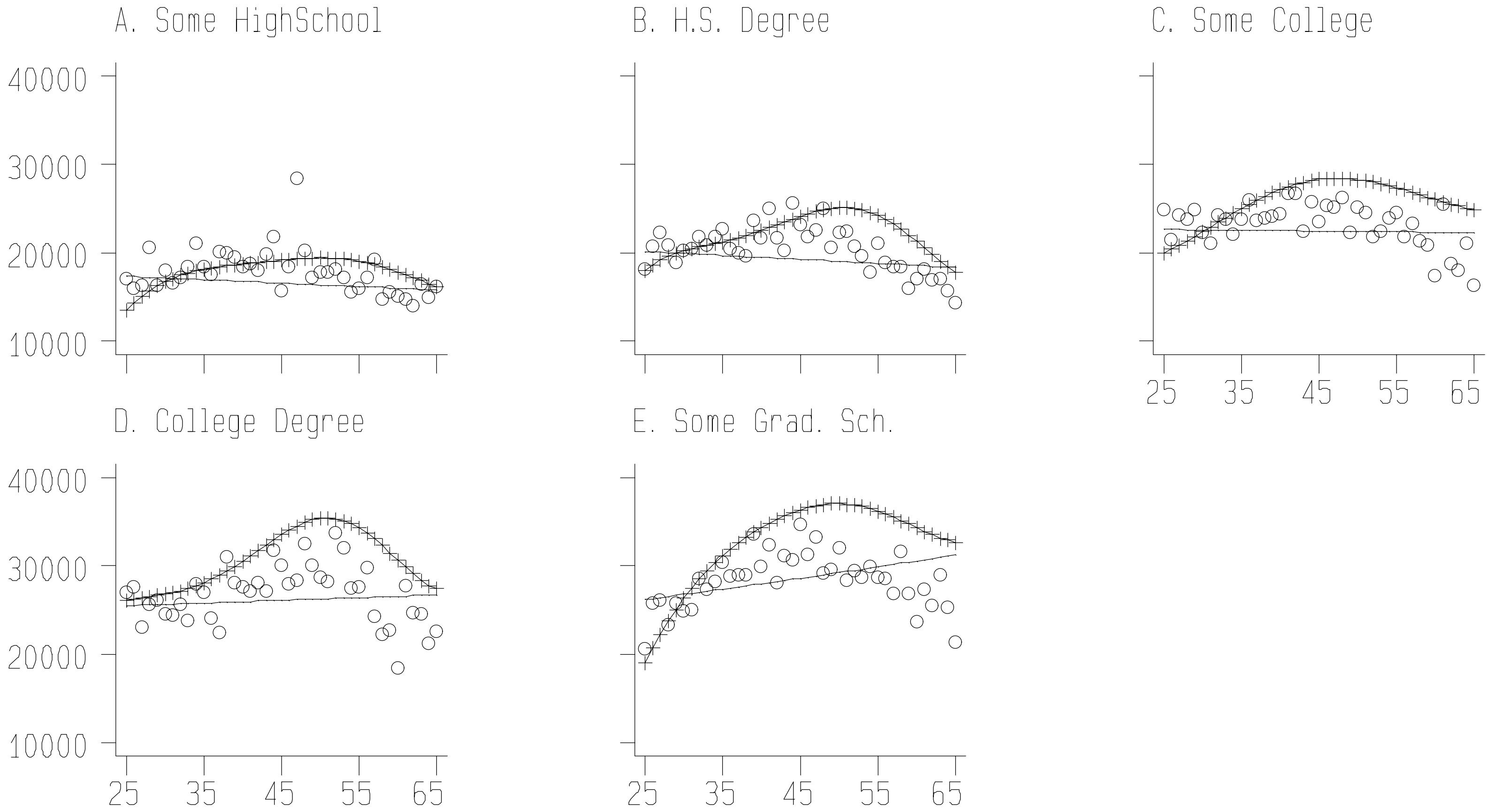


Figure 5.3
By Occ: Income, Consumption, and Consumption Predicted by CEQ LCH

Figure 5.4: Simulated and Actual Consumption Profiles (confidence bands), β=0.963, ρ=0.490

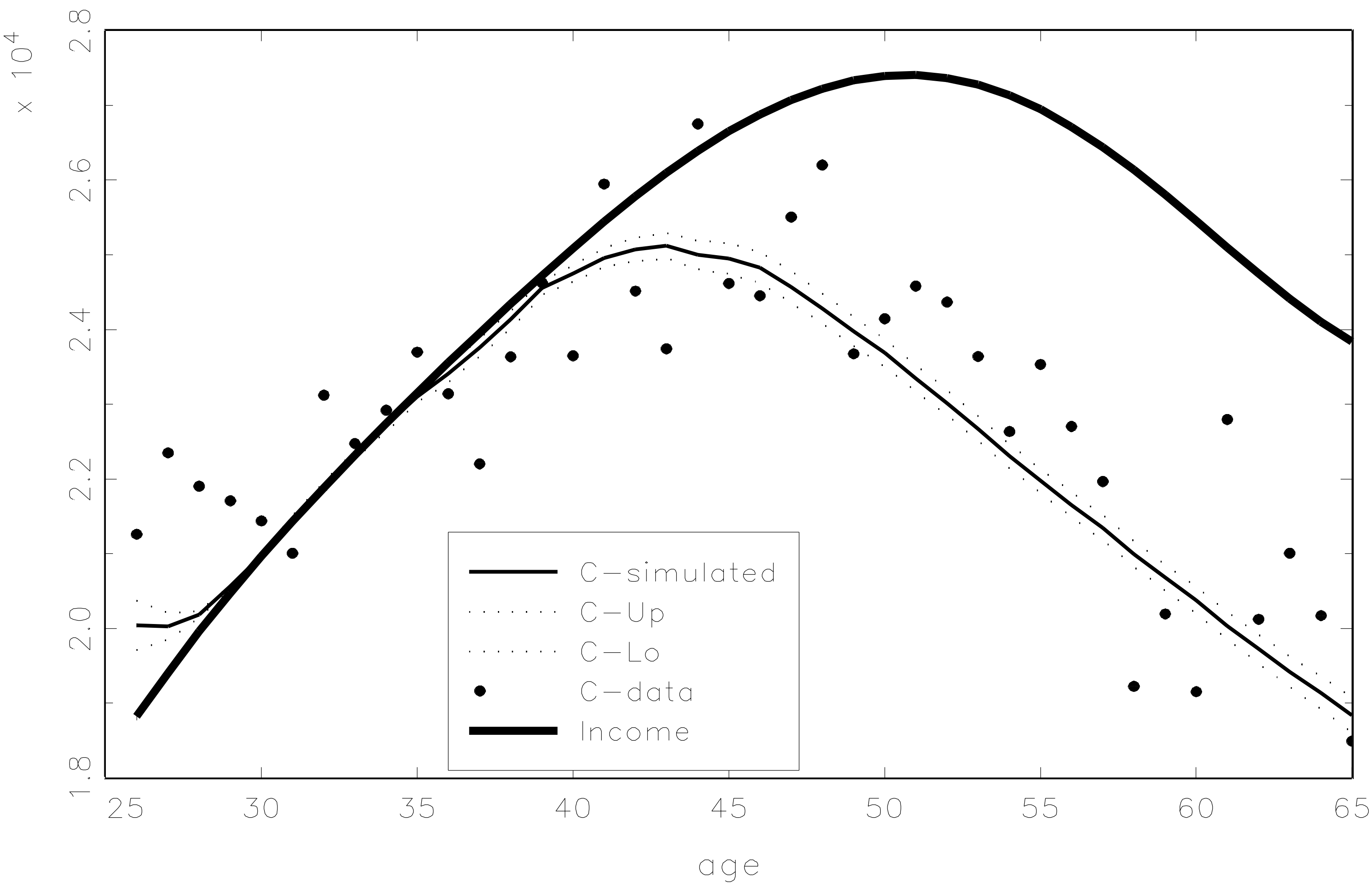


Figure 5.5: Simulated Consumption Profiles for different β

$R=3\%$, $\rho=0.490$

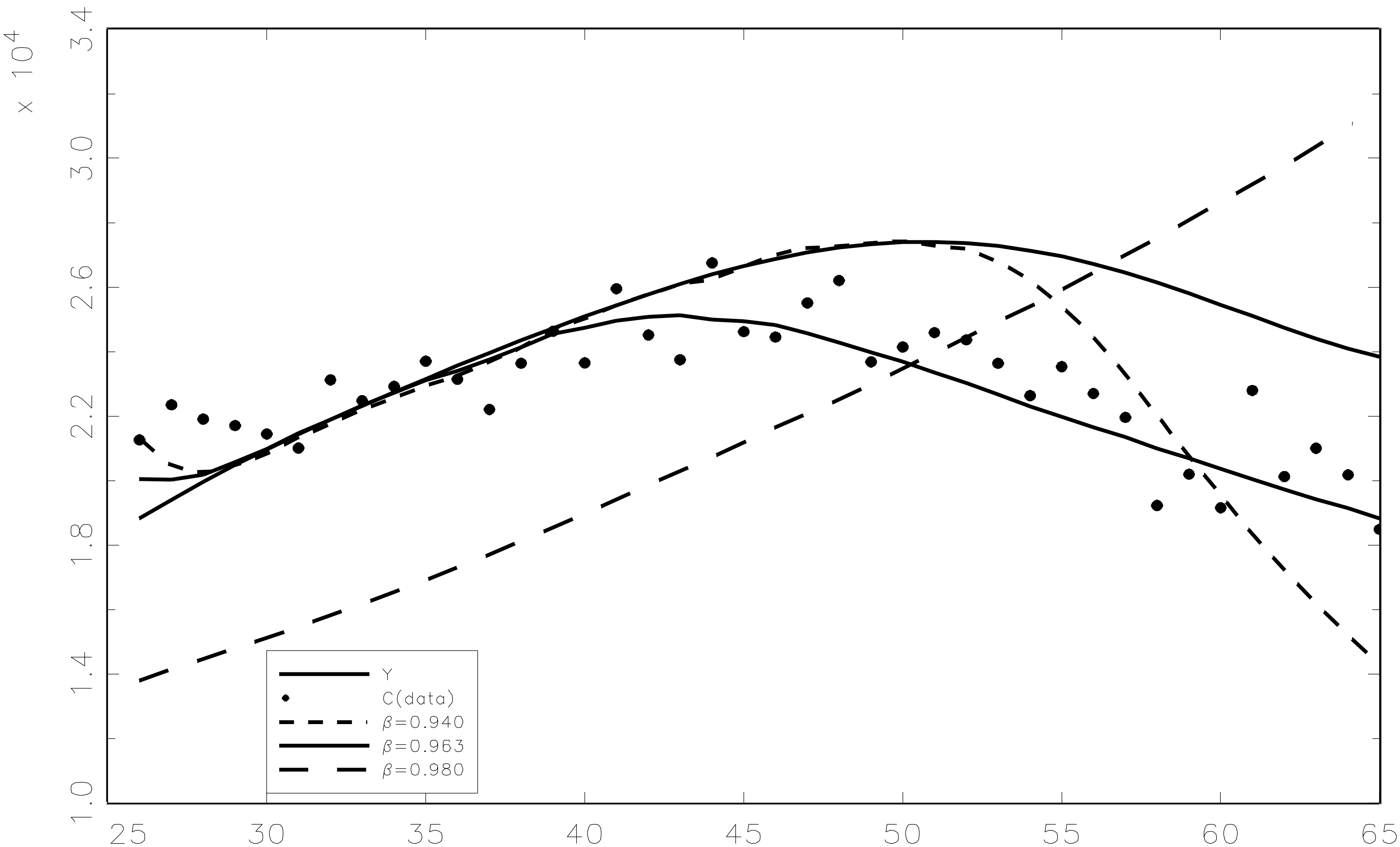


Figure 5.6: Savings Rate Profile
$(Y-C)/Y$, $x_0=0$

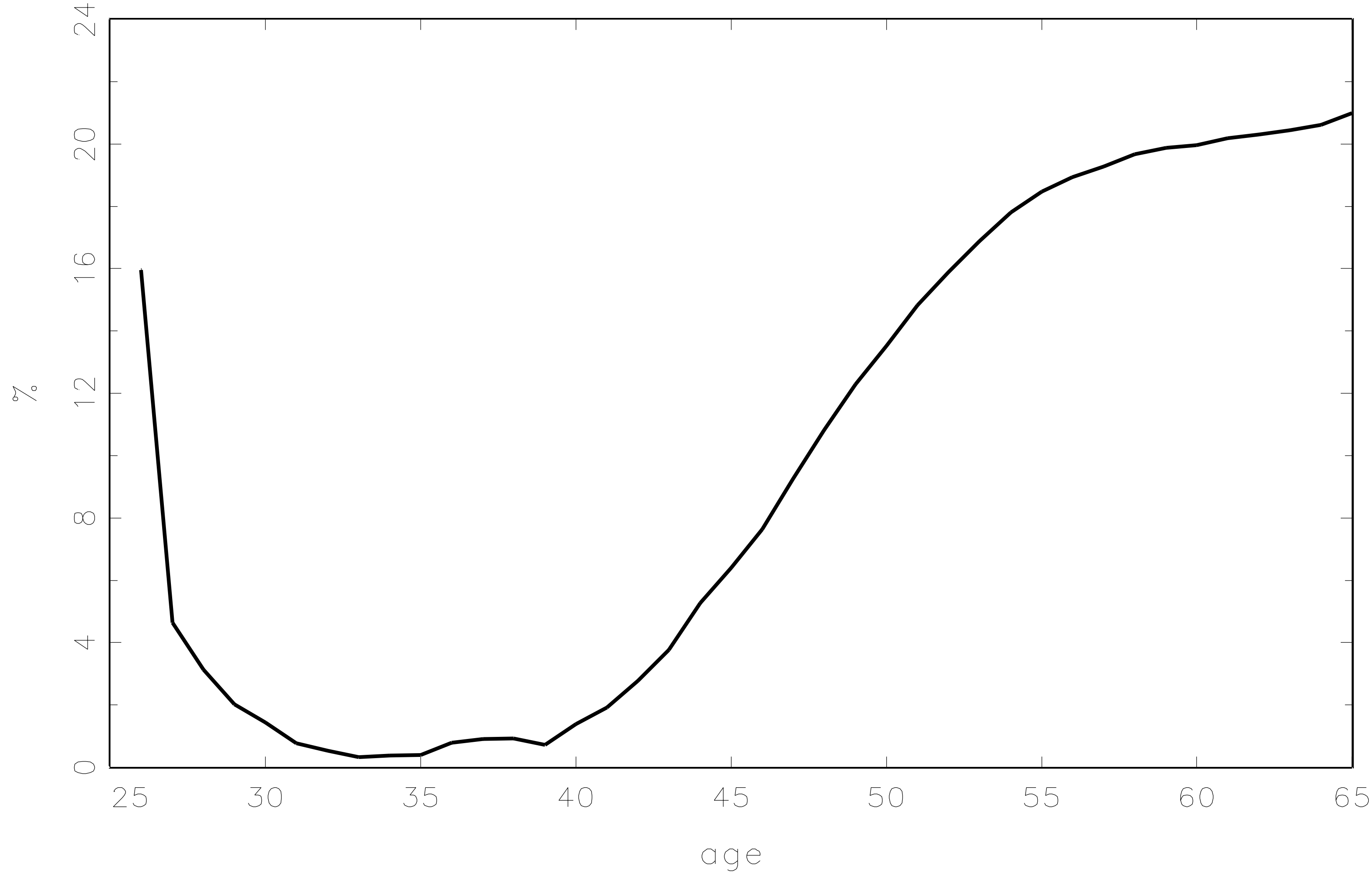


Figure 5.7: Target Cash on Hand

β=0.963, ρ=0.490, R=3%

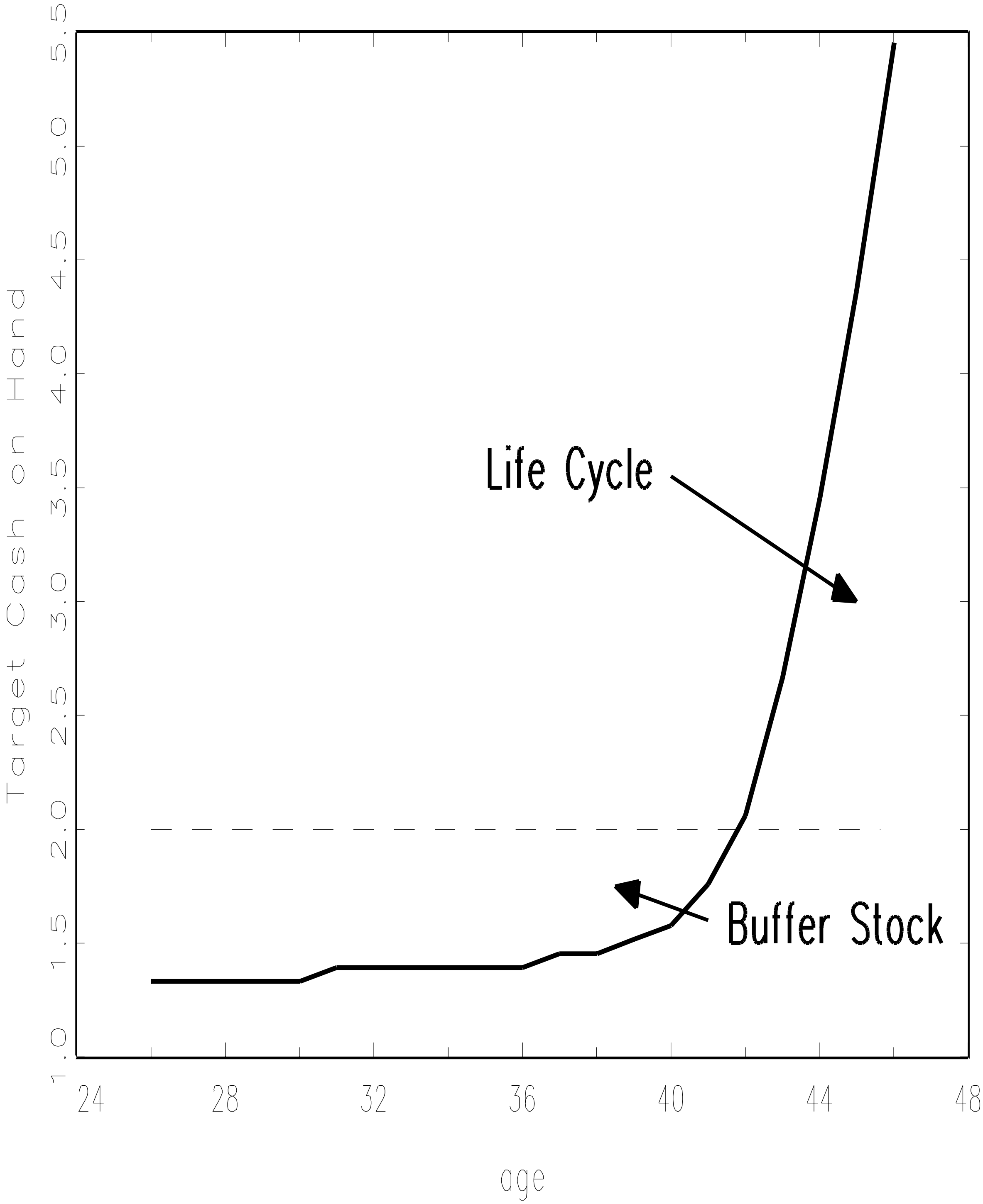


Figure 5.8: Life Cycle and Buffer Savings

$\beta=0.963$, $\rho=0.490$; $x_0=0$

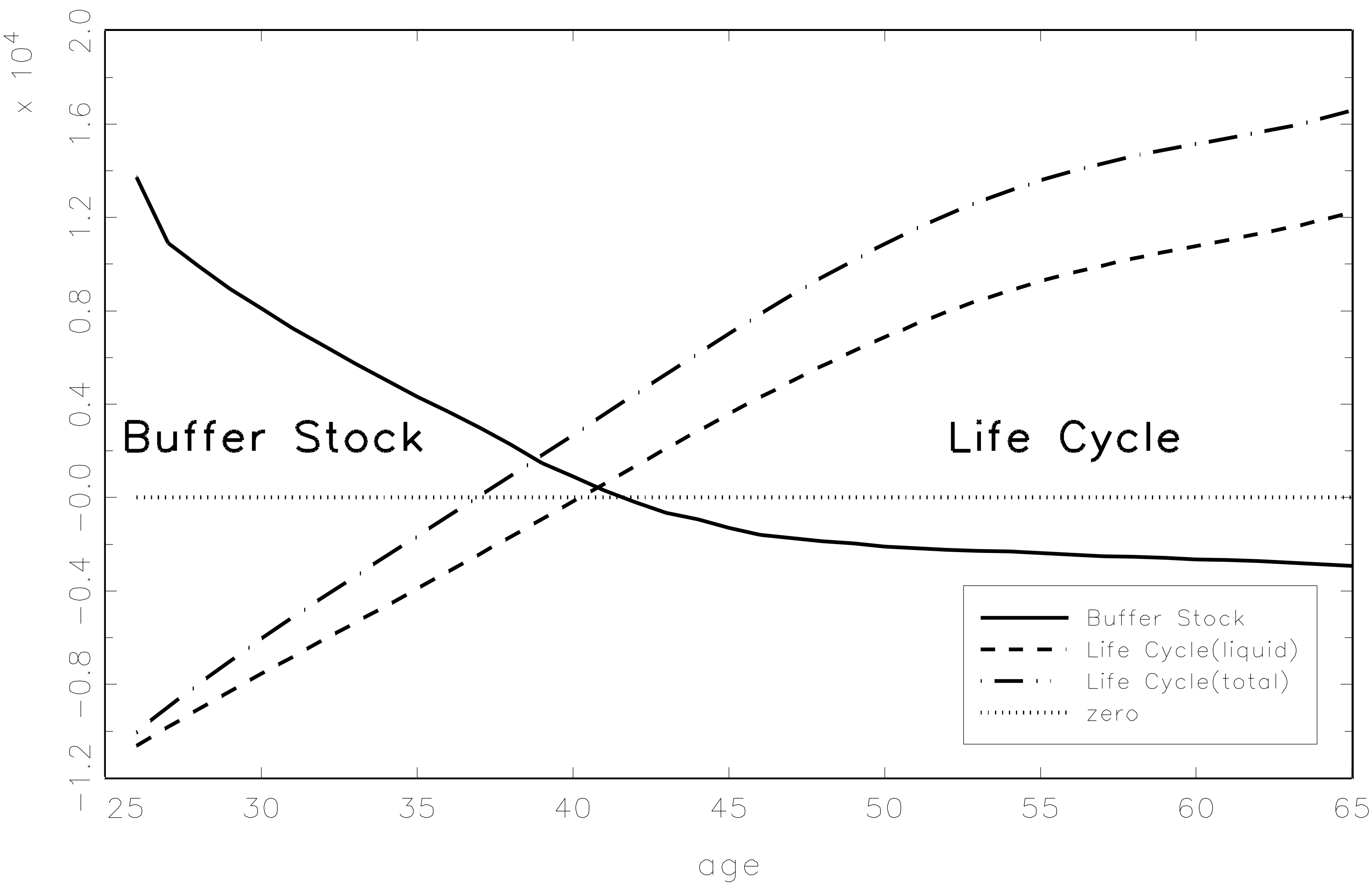


Figure 5.09.1: Simulated and Actual Consumption Profiles

β=0.961, ρ=0.490; education: Some HS

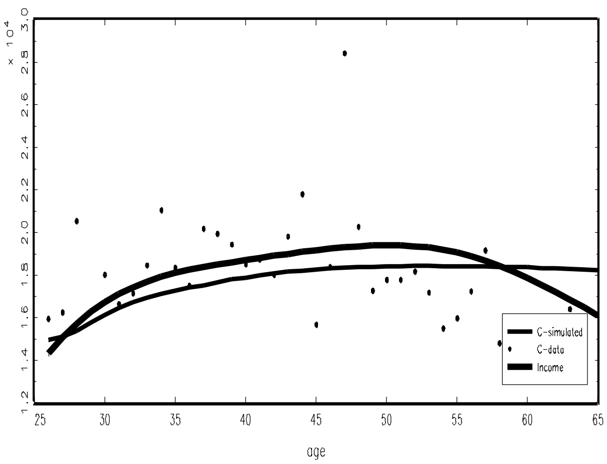


Figure 5.09.2: Simulated and Actual Consumption Profiles

β=0.959, ρ=0.490; education: HS Grad

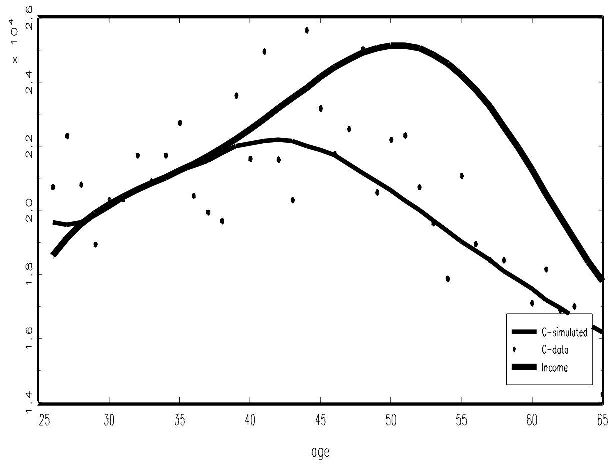


Figure 5.09.3: Simulated and Actual Consumption Profiles

β=0.963, ρ=0.490; education: Some Col

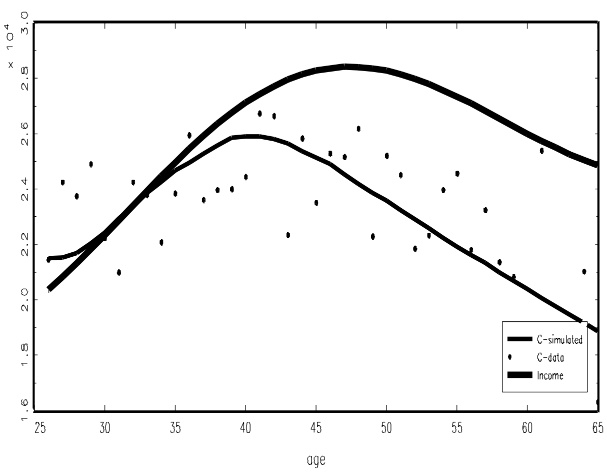


Figure 5.09.4: Simulated and Actual Consumption Profiles

β=0.968, ρ=0.490; education: Col Grad

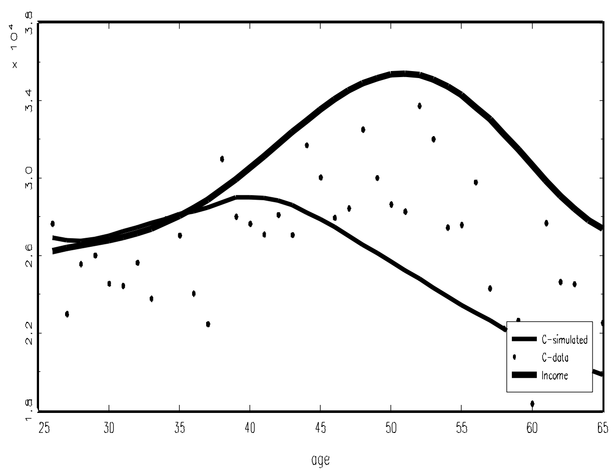


Figure 5.09.5: Simulated and Actual Consumption Profiles

β=0.966, ρ=0.490; education: Grad

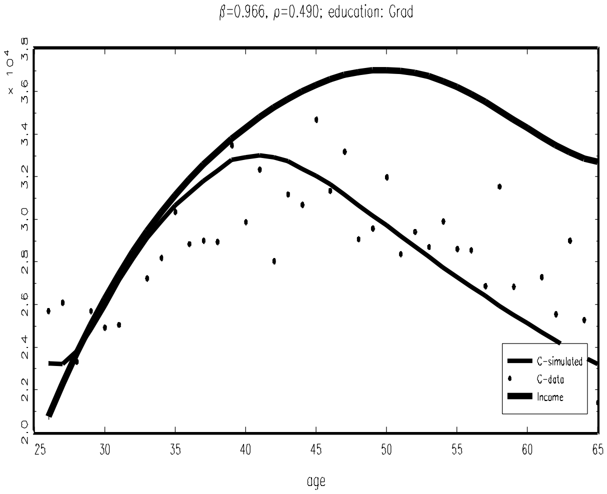


Figure 5.10.1: Simulated and Actual Consumption Profiles

β=0.962, ρ=0.490; occupation: Managers

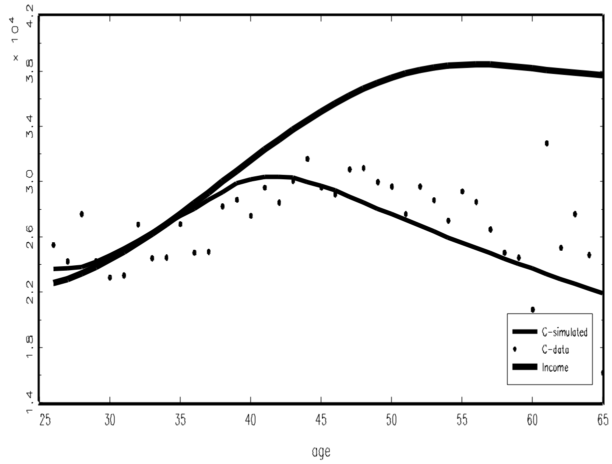


Figure 5.10.2: Simulated and Actual Consumption Profiles

β=0.967, ρ=0.490; occupation: Sales

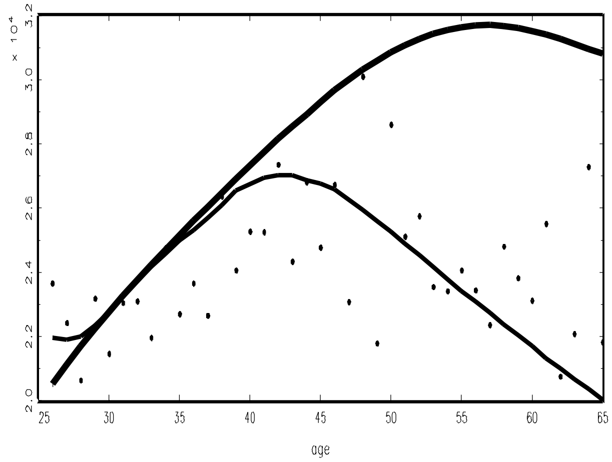


Figure 5.10.3: Simulated and Actual Consumption Profiles

β=0.964, ρ=0.490; occupation: Craft

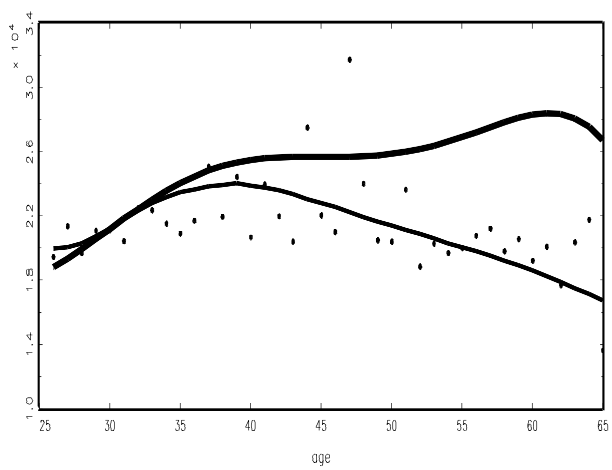


Figure 5.10.4: Simulated and Actual Consumption Profiles

β=0.966, ρ=0.490; occupation: Operator

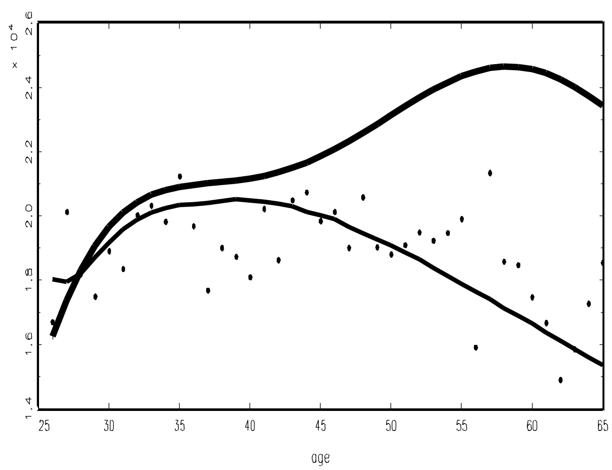


Figure 5.10.5: Simulated and Actual Consumption Profiles

β=0.955, ρ=0.490; occupation: Self

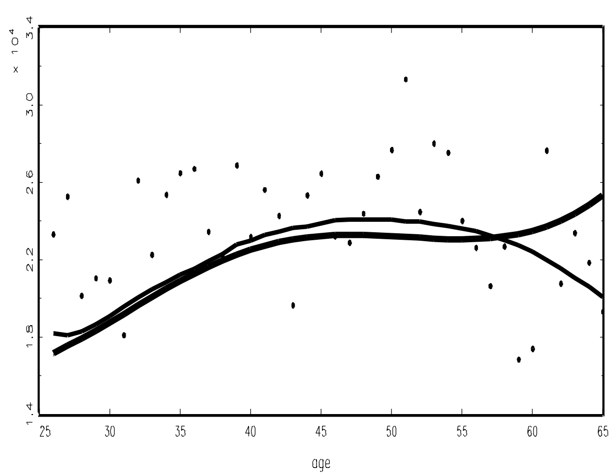


Figure 5.11.1: Simulated and Actual Consumption Profiles

β=0.944, ρ=0.490; R=1.05

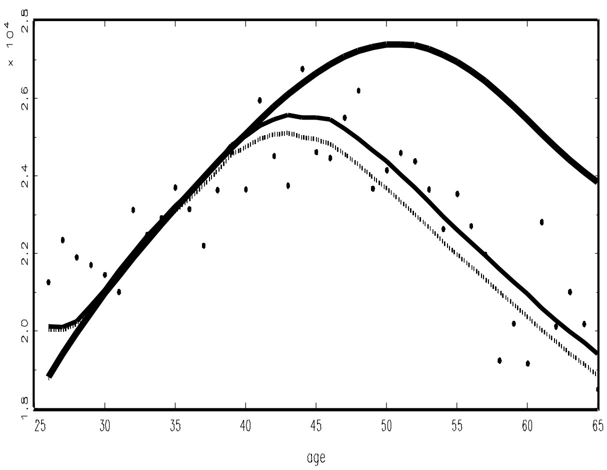


Figure 5.11.2: Simulated and Actual Consumption Profiles

β=0.963, ρ=0.490; p=0.5%

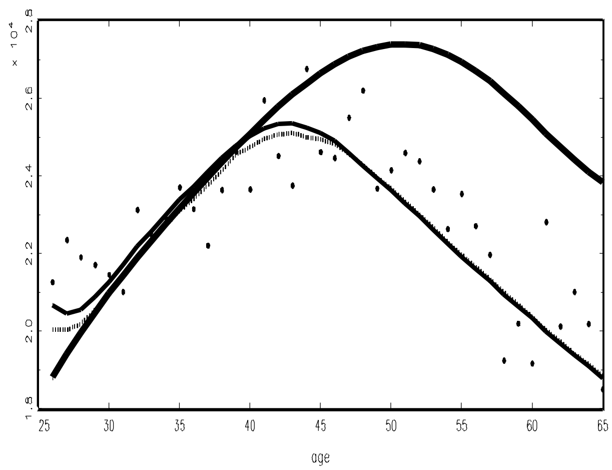


Figure 5.11.3: Simulated and Actual Consumption Profiles

β=0.963, ρ=0.490; variance

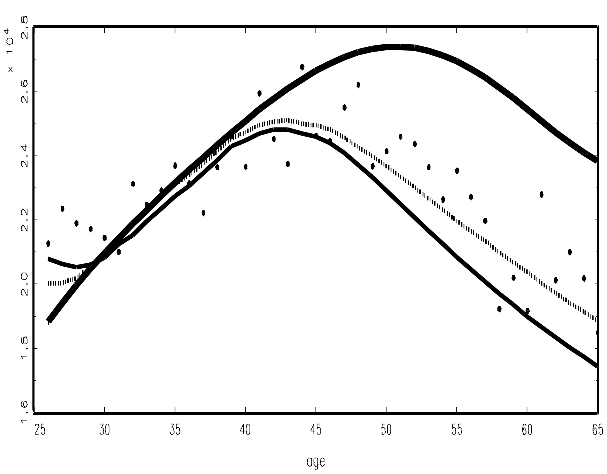


Figure 5.11.4: Simulated and Actual Consumption Profiles

β=0.961, ρ=0.490; Wise

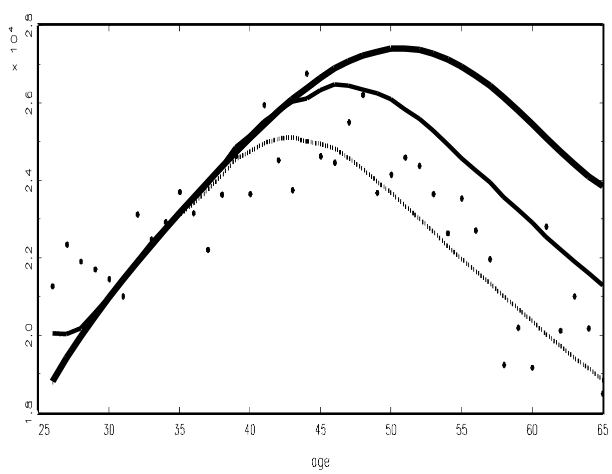


Figure 5.11.5: Simulated and Actual Consumption Profiles

β=0.962, ρ=0.490; x0=0