The Equity Premium in

Brock's Asset Pricing Model[*]

by

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1   Introduction

The equity premium puzzle that was raised by Mehra and Prescott (1985) stems from the fact that for no reasonable parameterizations of the Lucas (1978) asset pricing model is the theoretical equity premium as large as the empirically observed equity premium. As they discovered, the Lucas model does not have sufficient flexibility to predict the extent of the equity premium. It does predict that there is an equity premium, but it is insufficiently large to be compatible with data. The Brock (1982) asset pricing model *is* rich enough to predict a far more substantial equity premium. It gives pause for thought that if Mehra and Prescott had used Brock's model in their seminal article, the subsequent literature on the equity premium puzzle might have been less prolific and less contentious.[1] Referring to the equity premium and the risk free rate puzzles, Kocherlakota (1996) points out that "...there is now a vast literature that seeks to resolve these two puzzles." As Kocherlakota describes in that review of the literature, a number of alternate models[2] have been developed to explain the equity premium puzzle. We do not criticize those models.[3] Many of the effects that are captured in those models are important in the modeling of human behavior. Our point is that it is not *necessary* to change the assumption on preferences to get a higher equity premium. It is *sufficient* to include production

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[1]They based their decision, in part, on Mehra (1984), and in part on their experience with production models. In Mehra and Prescott (1985) they write, "If we had been successful in finding an economy which passed our not very demanding test, as we expected, we planned to add capital accumulation and production to the model using a variant of Brock's (1979, 1982) ...general equilibrium stationary structures and to perform additional tests." Although they make no mention of the technical difficulties in solving Brock's model, it is likely that this was also a reason for their not having done so. Neither the algorithms nor the hardware used to produce the results in Akdeniz and Dechert (1997) were available then.

[2]Among others, are the models on habit formation, such as Constantinides (1990), Campbell and Cochrane (1999), Boldrin, Christiano, and Fisher (2001), to cite a few. See also Campbell (2001) and the references there. McGrattan and Prescott (June 2001) show how taxes can account for both the growth in equity values as well as for the equity premium. In an earlier dynamic model of regulation and taxation, Brock and Turnovsky (1981) derived the impact of government policy on asset values. In a paper that has applications well beyond the equity premium puzzle, Weitzman (September 2004) argues that the modeling should be forward looking in the sense that "...the correct interpretation requires not frequentist *objective* estimates of the *past* mean and variance, but rather Bayesian *subjective* estimates of the *future* mean and variance." (Emphasis in the original.)

[3]Geweke (June 1999) does raise the point that "The benefits of an analytically rigorous economic theory will be realized only when harnessed to the same high standards for measurement." He found that "The posterior distribution for the mean of the risk free rate and the equity premium supports values consistent with ...dynamic stochastic general equilibrium models designed to address this question."

processes in the model.[4,5]

In this paper we have two basic points to make. First, there are parameterizations of the Brock model that have equity premia that are (1) more consistent with empirical evidence than those that were found by Mehra and Prescott in their solution of the Lucas asset pricing model;[6] and that are (2) consistent with empirical values for utility and production parameters. Second, the values of *all* relevant financial variables in the model are *functions* of the output level.[7] The values of the equity premium that are reported in empirical research are averages over time which means that the values have been *averaged over the business cycle.* However, one of the strong results from the Brock asset pricing model is that one should find that the equity premium is much higher at the bottom of a business cycle than it is at the top of the business cycle.[8]

In section 5 we present some scaling results that show, in part, why Brock's model with production leads to a higher equity premium than a consumption based model, or even a single firm model. What we show is that, along with systematic shocks, there must be idiosyncratic shocks that alter the *marginal* products of the firms *differently* from each other. Otherwise, as we show in section 5 the model reduces to a one firm model, and the equity premium is comparable to those reported in Mehra and Prescott (1985) for their consumption based asset pricing model. In section 4 we show how we model systematic and idiosyncratic shocks so that the (random) marginal products of the firms differ in each state.

1.1   A Parameterization of the Brock APM

In Brock (1979) and Brock (1982) it is shown how the stochastic economic growth model can be used to solve for the asset prices in a production based economy. How-

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[4]In referring to Brock's asset pricing model, Black (1995) remarked, "If we add non-separable utility, adjustment costs for moving capital from one sector to another, human capital, and a few other features, we will have a model of the kind I favor." This kind of model would indeed include many useful components that would make the results more realistic. Nevertheless, as we show in this paper, including production alone is sufficient to get the size of the equity premium that Mehra and Prescott were looking for.

[5]Campbell (2001) makes the comment that "Models with production also help one to move away from the common assumption that stock market dividends equal consumption ..." and he goes on to conclude that "... it will ultimately be more satisfactory to derive both dividends and consumption within a general equilibrium model." This is, of course, what Brock does in his asset pricing model.

[6]In that study the consumer's discount factor and risk aversion parameter were varied and the highest equity premium they found was 0.35 percent. In our study we focus most of our attention on keeping the values of the discount factor and risk aversion parameter consistent with empirical results.

[7]This property was emphasized in Akdeniz and Dechert (1997) and Akdeniz (2000) where it was shown that the asset prices are a function of the output level.

[8]In the introduction to their paper, Campbell and Cochrane (1999) state, "... standard business cycle models utterly fail to reproduce the level, variation, and cyclical co-movement of equity premia." As we show in this paper, Brock's APM does in fact reproduce the cyclical co-movement of equity premia with the level of output.

ever, there is only one parameterization for which a closed form solution exists for this model,[9] so we have to rely on numerical methods for solutions for the other cases. The advantage of numerical methods, as Judd (1995) points out, is that we can often solve for a much broader class of cases than is possible with analytical methods.

In this study we use that solution and explore the parameter space for solutions to the model that, to a certain extent, fit some of the stylized facts of asset markets. Many of the stylized facts (such as those that Cooley and Prescott (1995) report) relate to economic growth. In our model there is no labor and no population to grow, so we examine those results that do not relate directly to issues of economic growth. In particular, the equity premium puzzle is not related to the growth of the economy.

In this paper we explore the parameter space with a primary focus of presenting the results for values of the annual discount factor of β ranging from 0.95 to 0.99, and the relative risk aversion coefficient of γ ranging from -1 to -4.[10] These are typical values based on US data.[11]

2   The Growth Model

In this section we borrow heavily from Brock (1979). The essential elements of the growth model are:

$$V(y_0) = \max_{\{c_t, x_{it}\}} \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \tag{2.1}$$

$$\text{subject to:} \quad x_t = \sum_{i=1}^{N} x_{it} \tag{2.2}$$

$$y_{t+1} = \sum_{i=1}^{N} f_i(x_{it}, \xi_t) \tag{2.3}$$

$$c_t + x_t = y_t \tag{2.4}$$

$$c_t, x_{it} \geq 0 \tag{2.5}$$

$$y_0 \ \text{given} \tag{2.6}$$

Where:

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[9]The special case is for a logarithmic utility function, and Cobb–Douglas firms, where the value of the output elasticity (with respect to the input) is common to all firms.

[10]In our work we use $u(c) = (c^\gamma - 1)/\gamma$ for the utility function. In some of the studies cited, it is specified as $u(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$, and so our value of $\gamma = -1$ would correspond to $\gamma = 2$ in those models.

[11]Campbell and Cochrane (1999) use a value of $\gamma = 2.00$ (which would correspond to a value of -1.00 in our formulation. Since there is no universal value for the relative risk aversion parameter, we present results for several values of γ.

β discount factor on future utility

u per period utility function of consumption

c_t consumption at date t

y_t output at date t

x_t capital stock at date t

f_i production function of process i

x_{it} capital allocated to process i at date t

ξ_t random shock

For a description and interpretation of the model see Brock (1982). The main assumptions for this model are:

(A1) the utility function, u, is concave, increasing and twice continuously differentiable;

(A2) the production functions, f_i, are concave, increasing, twice continuously differentiable, and satisfy the Inada conditions;

(A3) the stochastic process $\{\xi_t\}$ is independent and identically distributed;

The first order conditions[12] for the intertemporal maximization are:

$$u'(c_{t-1}) = \beta \mathsf{E}_{t-1}\left[u'(c_t)f_i'(x_{it}, \xi_t)\right] \tag{2.7}$$

and the transversality condition is:

$$\lim_{t\to\infty} \beta^t \mathsf{E}_0\left[u'(c_t)x_t\right] = 0. \tag{2.8}$$

Equation (2.7) is the one that is used to derive a numerical solution to the growth model. Since the problem given by equations (2.1) to (2.6) is time stationary the optimal levels of c_t, x_t, and x_{it} are functions of the output level y_t, and can be written as:

$$c_t = c(y_t) \qquad x_t = h(y_t) \qquad x_{it} = h_i(y_t) \tag{2.9}$$

The first two functions in equation (2.9) can be expressed in terms of the individual firm investment functions:

$$h(y) = \sum_{i=1}^{N} h_i(y) \tag{2.10}$$

$$c(y) = y - h(y) \tag{2.11}$$

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[12]We assume that $x_{it} > 0$ wp 1. If not then the Kuhn–Tucker type conditions in Brock (1979) must be used. In our numerical studies we used functional forms that include a Cobb–Douglas term, so our assumption is satisfied.

Given these functions, one can then define next period's output, given the current level of output, y, and the shock, ξ:

$$Y(y,\xi) = \sum_{i=1}^{N} f_i(h_i(y),\xi) \ . \tag{2.12}$$

A numerical solution to this problem are policy functions (2.9) that satisfy

$$u'(c(y)) = \beta \mathsf{E}\left[u'(c(Y(y,\xi)))f_i'(h_i(Y(y,\xi)),\xi)\right] \quad i = 1,\cdots,n \tag{2.13}$$

for all y and for which the transversality condition (2.8) holds. Fortunately, the transversality condition does not have to be directly verified. A bounded solution to equation (2.13) satisfies the transversality condition.[13] Further details are in Akdeniz and Dechert (1997).

3   An Asset Pricing Model

The asset pricing model in Brock (1982) has its roots in the models of Brock and Mirman (1972) and Lucas (1978). In particular, Brock also solves for the rational expectations equilibrium. The difference between the Brock (1982) and Lucas (1978) models is that former includes production. By incorporating the shocks in with the production processes, Brock's model has the sources of uncertainty in the asset prices directly tied to economic fluctuations in output levels (and hence in profits).

There is one representative consumer whose preferences are given in equation (2.1). There are N different firms. Firms rent capital from the consumers at the rate r_{it} to maximize their profits:

$$\pi_{i,t+1} = f_i(x_{it},\xi_t) - r_{it}x_{it} \tag{3.1}$$

Each firm rents capital given ξ_t. Here r_{it} denotes the interest rate on capital in industry i at date t and is determined within the model. Asset shares are normalized so that there is one perfectly divisible equity share for each firm. Ownership of the share in firm i at date t entitles the consumer to the firms profits at date t. It is also assumed (as in Lucas (1978)) that the optimum levels of asset prices, capital, consumption and output form a rational expectations equilibrium.

The Model:

The representative consumer takes asset prices, profits and rents as given and solves the following problem:

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[13]See Judd (1992) and Judd (1998) for the solution to the one firm growth model and the argument about the transversality condition.

$$\max \qquad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \qquad (3.2)$$

$$\text{subject to:} \quad c_t + x_t + P_t \cdot Z_t \leq \pi_t \cdot Z_{t-1} + P_t \cdot Z_{t-1} + \sum_{i=1}^{N} r_{i,t-1} x_{i,t-1} \qquad (3.3)$$

$$c_t, Z_t, x_{it} \geq 0$$
$$r_{it} = f_i'(x_{it}, \xi_t) \qquad (3.4)$$
$$\pi_{it} = f_i(x_{i,t-1}, \xi_{t-1}) - f_i'(x_{i,t-1}, \xi_{t-1})x_{i,t-1} \qquad (3.5)$$

where:

P_{it} price of one share of firm i at date t
Z_{it} number of shares of firm i owned by the consumer at date t
π_{it} profits of firm i at date t

The details of the model are in Brock (1982). The first order conditions are:

$$P_{it}u'(c_t) = \beta\mathsf{E}_t\left[u'(c_{t+1})\left(\pi_{i,t+1} + P_{i,t+1}\right)\right] \qquad (3.6)$$

and

$$u'(c_t) = \beta\mathsf{E}_t\left[u'(c_{t+1})f_i'(x_{i,t+1}, \xi_{t+1})\right] \qquad (3.7)$$

from which we get the prices for the assets. In addition the transversality conditions

$$\lim_{t\to\infty} \beta^t\mathsf{E}_0\left[u'(c_t)\sum_{i=1}^{N} P_{it}Z_{it}\right] = 0 \qquad (3.8)$$

$$\lim_{t\to\infty} \beta^t\mathsf{E}_0\left[u'(c_t)x_t\right] = 0. \qquad (3.9)$$

are needed to fully characterize the optimum. Brock (1979) shows that there is a duality between the growth model (2.1) – (2.6) and the asset pricing model (3.2) – (3.5), and that the solution to the growth model is also the solution to the asset pricing model. Once the growth model is solved, the asset pricing functions can be solved for by equation (3.6). As for the transversality condition, Judd (1992) points out that it implies that we are looking for the bounded solution[14] to the growth model.

Other assets can be included in this model. Assets that are in zero net supply in equilibrium do not affect the real side of the model, and so the solution does not change. For example, we can introduce one period discount bonds into the model. If

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[14]The optimal solution remains in a bounded interval: $0 < a < y_t < b < \infty$ for all t.

the bond pays \$1 (with probability one) at date $t + 1$, then the equilibrium price of the bond, $B_{t,t+1}$, at date t must satisfy

$$u'(c_t)B_{t,t+1} = \beta \mathsf{E}_t \left[u'(c_{t+1}) \right]$$

or

$$B_{t,t+1} = \frac{\beta \mathsf{E}_t \left[u'(c_{t+1}) \right]}{u'(c_t)} \tag{3.10}$$

This, then, defines the one period risk free rate of return in period t as $1/B_{t,t+1}$. In similar fashion we can introduce a two period discount bond that pays \$1 (with probability one) at date $t + 2$. The equilibrium price of this bond, $B_{t,t+2}$, at date t must satisfy

$$
\begin{aligned}
u'(c_t)B_{t,t+2} &= \beta^2 \mathsf{E}_t \left[u'(c_{t+2}) \right] \\[2mm]
&= \beta \mathsf{E}_t \left[u'(c_{t+1}) \beta \frac{\mathsf{E}_{t+1} \left[u'(c_{t+2}) \right]}{u'(c_{t+1})} \right] \\[2mm]
&= \beta \mathsf{E}_t \left[u'(c_{t+1}) B_{t+1,t+2} \right]
\end{aligned}
$$

or

$$B_{t,t+2} = \frac{\beta \mathsf{E}_t \left[u'(c_{t+1}) B_{t+1,t+2} \right]}{u'(c_t)} \tag{3.11}$$

An n period discount bond that pays \$1 (with probability one) at date $t + n$ can be similarly defined.[15]

Since (in equilibrium) there is 1 share of each asset, the equally weighted market portfolio is:

$$M_t = \sum_{i=1}^{N} P_{it}$$

and the dividends (profits) are:

$$\pi_t = \sum_{i=1}^{N} \pi_{it}.$$

Define the return on each asset by:

$$R_{it} = \frac{p_{i,t+1} + \pi_{i,t+1}}{p_{it}}$$

and the (equally weighted) return on the market portfolio by:

$$R_{Mt} = \frac{M_{t+1} + \pi_{t+1}}{M_t}.$$

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[15]In an analogous way one can also derive the price of an Arrow–Debreu security, i.e., an asset that pays \$1 if state i occurs at date t and \$0 otherwise.

From the first order condition (3.6), the return on each asset satisfies:

$$u'(c_t) = \beta \mathsf{E}\left[u'(c_{t+1})R_{it}\right]$$

which is the efficiency condition from the growth model. By summing equation (3.6), we get that the return on the market portfolio satisfies:

$$u'(c_t) = \beta \mathsf{E}\left[u'(c_{t+1})R_{Mt}\right]$$

and so it too is efficient. (This is one of the hypotheses of the CAPM, which in this model is a consequence of the optimizing behavior of the consumer.[16]) Now define the profit and output functions by:

$$\pi_i(y,\xi) = f_i(h_i(y),\xi) - h_i(y)f_i'(h_i(y),\xi) \tag{3.12}$$

$$Y(y,\xi) = \sum_{i=1}^{n} f_i(h_i(y),\xi) \tag{3.13}$$

and the asset pricing functions by:

$$p_i(y)u'\left(c(y)\right) = \beta \mathsf{E}\left[u'\left(c\left(Y(y,\xi)\right)\right)\left(p_i\left(Y(y,\xi)\right) + \pi_i(y,\xi)\right)\right] \quad i = 1,\ldots,n. \tag{3.14}$$

Once we have the pricing functions[17]we next define the return functions

$$R_i(y,\xi) = \frac{p_i\left(Y(y,\xi)\right) + \pi_i(y,\xi)}{p_i(y)} \tag{3.15}$$

$$R_M(y,\xi) = \frac{\displaystyle\sum_{i=1}^{n}\left[p_i\left(Y(y,\xi)\right) + \pi_i(y,\xi)\right]}{\displaystyle\sum_{i=1}^{n}p_i(y)} \tag{3.16}$$

as well as the one period risk free bond price

$$B(y) = \frac{\beta \mathsf{E}\left[u'(c(Y(y,\xi)))\right]}{u'(c(y))} \tag{3.17}$$

As these formulæ make clear, the expected return on assets, $\mathsf{E}\left[R_i(y,\xi)\right]$, the expected return on the (equally weighted) market portfolio, $\mathsf{E}\left[R_M(y,\xi)\right]$, and the bond price, $B(y)$, are all functions of the output level, y. The equity premium in this model is given by

$$e(y) = \mathsf{E}\left[R_M(y,\xi)\right] - \frac{1}{B(y)} \quad . \tag{3.18}$$

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[16]See Akdeniz and Dechert (1997) and Akdeniz (2000) for more details on this issue.

[17]Solving equation (3.14) is easier than it might appear: these equations are *linear* in the functions p_i. The details are in Akdeniz and Dechert (1997).

In order to study the term structure of interest rates in this model define the following recursion:

$$
\begin{aligned}
G_0(y) &= u'(c(y)) \\
G_{n+1}(y) &= \mathsf{E}\left[u'(c(Y(y,\xi)))G_n(Y(y,\xi))\right] \quad n = 0, 1, \ldots
\end{aligned}
$$

Then the n period risk free bond price when the output level is y is

$$
\frac{G_n(y)}{u'(c(y))} \tag{3.19}
$$

In the next section we will explore the nature of the relationship of these functions in terms of the output level, y.

4 A Numerical Solution

Except for a very special case of the utility and production functions, there is no closed form solution for the optimal investment functions.[18] In order to analyze the properties of the solutions to the asset pricing model we must use numerical techniques instead. Akdeniz and Dechert (1997) report the technical details of the numerical solution which we will not repeat here.[19] In this study we use that solution and explore the parameter space for solutions to the model that, to a certain extent, fit some of the stylized facts of asset markets. Our primary focus will be on the equity premium and the term structure of interest rates that come out of the Brock asset pricing model.

Akdeniz (2000) and Akdeniz and Dechert (1997) showed that CAPM results *do* hold in the model, but with the proviso that they *depend* on the level of output.[20] I.e., an asset's beta is not constant over time, but fluctuates with the level of output. Furthermore, at each level of output the market portfolio (which is efficient in this model) *is* on the security market line (at that level of output).

In section 4.3 we report the results of simulations using the numerical solution for the model. In keeping with the results found by Akdeniz and Dechert, one of the important features to note is that the equity premium sharply depends on the level of output. In the tables, for each value of γ and β there are five values of the equity premium which, reading down the table, occur for low, below average, average, above average and high output levels.[21]

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[18]The special case is for a logarithmic utility function, $u(c) = \log(c)$, and for Cobb–Douglas firms, $f_i(x, \xi_i) = \xi_i x^\alpha$, where the value of α is common to all firms. In this case the optimal investment policy functions are linear in output, $\hat{x}_i(y) = \gamma_i y$.

[19]Basically, equation (2.13) is solved for the policy functions, $h_i(y)$, and then equation (3.14) is solved for the pricing functions, $p_i(y)$.

[20]As pointed out by Campbell and Cochrane (1999), "...the slope of the conditional mean-variance frontier ...changes through time with a business cycle pattern ..." This is *precisely* what Akdeniz (2000) shows in the context of Brock's APM.

[21]The tables are based on the long run distribution of output. The levels used in the tables are the 10, 30, 50, 70 and 90th percentiles.

4.1   Computational Details

We use the projection method[22] to solve equation (2.13) for the coefficients of the policy functions[23]

$$h_i(y) = \sum_{j=0}^{m} a_{ij} T_j(y) \tag{4.1}$$

where $T_j(y)$ is the $i\underline{{}^{th}}$ Chebyshev polynomial shifted to the interval which is the support of the invariant measure, $[y_{\min}, y_{\max}]$. One of the steps is to use Newton's method to solve for the coefficients, $\{a_{ij}\}$. Since Newton's method is only locally convergent, we need a good initial set of values for these coefficients. In Judd (1992) good results were obtained by choosing a linear approximation that passes through the origin and the steady state.[24] In the model in this paper we are dealing with the multi firm stochastic case, and need an alternative procedure. We chose to first create a deterministic model by taking the expected value of the firm's production functions. For this deterministic model, we can readily solve for the steady state levels of capital for each firm. It turns out that the linear approximations of the policy functions which passes through the origin and the steady states are not good enough in general to get Newton's method to converge. So, by using the value function for this deterministic case, we also computed the derivative of the policy functions at the steady state and used linear approximations *at the steady state* as an initial point for the Newton routine. See Figure 1 for a pictorial representation of this method. As long as the values $y_{\min}$ and $y_{\max}$ are reasonably close to the deterministic steady state, the Newton method converges for this initialization. The values of $y_{\min}$ and $y_{\max}$ then have to be determined by trial and error.[25]

4.2   Systematic and Idiosyncratic Shocks

For firm production functions we use Cobb–Douglas functional forms with depreciation. Total output in state s and period t and is:

$$y_{s,t+1} = \sum_{i=1}^{n} \left[\theta_{is} x_{it}^{\alpha_{is}} + (1 - \delta_{is}) x_{it} \right] \tag{4.2}$$

where n is the number of firms. For the numerical work below, we took the depreci-

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[22]See Akdeniz and Dechert (1997) for complete details. Also see Judd (1992) and Judd (1998) for a further description of the technique as applied to problems in economic analysis.

[23]Actually, we are solving for polynomial approximations to the policy functions.

[24]In that article, a deterministic growth model with a single firm was analyzed.

[25]In order to determine the support of the invariant distribution of output we need to solve $y = \min_{\xi} Y(y, \xi)$ and $y = \max_{\xi} Y(y, \xi)$ for the crossing points on the 45 degree line. See equation (2.12) for the definition of $Y(y, \xi)$.
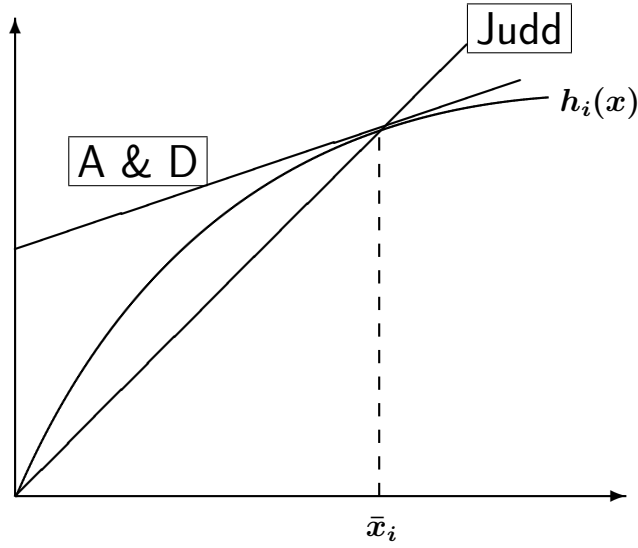
Figure 1: Initialization of $\displaystyle\sum_{j=0}^{m} a_{ij}T_j(y)$

ation terms to be deterministic, and so

$$\delta_{is} = \delta_i \quad i = 1.\ldots,n$$

In order to model both systematic shocks as well as idiosyncratic shocks we constructed the $(\alpha_{is}, \theta_{is})$ parameters as follows. First, construct a partition of the state space, $\{\Omega_j\}$ for $j = 1,\ldots,m$, and select capital levels, $k_1 < k_2 < \cdots < k_m$, one for each partition. Next, select values of α_{is}. These can be chosen at random, or they can be chosen to fit a pattern based on stylized facts. Then the parameters θ_{is} are chosen so that

$$\theta_{is} = \delta_i k_j^{1-\alpha_{is}} \quad s \in \Omega_j$$

This implies that for all the firms, i,

$$\theta_{is} k_j^{\alpha_{is}} + (1 - \delta_i)k_j = k_j \quad \forall s \in \Omega_j$$

holds for all $j = 1.\ldots,m$. A visual example of the effect of this parameterization is in Figure 2.

4.3   Numerical Results

In this section we will present some results from simulation studies of the solution to Brock's asset pricing model with various parameterizations. The examples will show that there are values of the parameters of the model that give values of excess returns (at the median level of output) in excess of 5%.
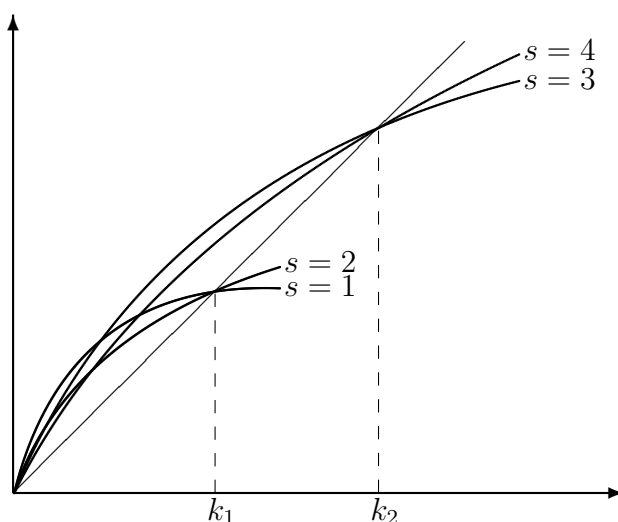
Figure 2: $\Omega_1 = \{1, 2\}$ $\Omega_2 = \{3, 4\}$

First, we present some results that show that the equity premium can be quite large, even for some fairly reasonable values of the parameters. In the examples, the parameter values are for a model with three firms and eight states of uncertainty. For these examples, the range of utility parameters are $\beta = 0.95, 0.97, 0.99,$ and $\gamma = -1.0, -2.0, -3.0, -4.0$.

The resulting values of the equity premium are listed in Tables 2 – 14.[26] The first column in the tables are levels of the invariant distribution of output. The second column is the excess return on the market portfolio, and the remaining columns are the excess returns on the individual firm shares. Each table caption shows the particular values of γ and β, and also the value of the standard deviation of consumption, σ_c.

Typical results for the term structure of interest rates are given in Tables 15 and 16. Notice that the interest rates depend on the output level, with high interest rates associated with low output levels, and low rates (negative) with high levels. The one period interest rates are the risk free rates that are used to measure the equity premium.

Figures 3 – 14 show the consumption levels for a simulation of the model for 120 periods. Notice in both the tables and the figures how the standard deviation of consumption, σ_c, falls as γ decreases, while the equity premium increases.[27]

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[26]See Tables 15 and 16 for some corresponding risk free rates in the columns labeled $t = 1$.

[27]This is of course a standard observation: as the consumer becomes increasingly risk averse, he smooths intertemporal consumption more and demands higher rates of return on risky assets.

4.4   Comparative Dynamics

The impact of depreciation on the results can be substantial. For the three firm model above with depreciation levels increased by 1% for each firm in each state we get in increase in the equity premium at all levels of output. The results are in Table 10 and should be compared with the results in Table 9. As can be seen from these results, a change in depreciation has a substantial impact on the equity premium. Arguably, in the US economy depreciation rates have increased in the past 20 years. There is more rapid replacement of high technology capital which has become an ever increasing proportion of the US capital stock. This is certainly one of the testable hypotheses of this model.

5   Scaling and Uncertainty

In this section we present some scaling results that hold for this model. We will show an invariance principle for two types of scalings: one where the productivity parameters, $\theta_{i,s}$, are scaled, and one where the economy is expanded by replication. We then show the impact of uncertainty in multiplicative models, and why our parameterization of the Brock model produces the results that we have presented in this paper.

5.1   Scaling by Replication

Let the solution to the one firm problem:

$$\max \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

$$c_t + x_{t+1} = y_t \qquad t = 0, 1, \ldots$$

$$y_t = f(x_t, \xi_t) \qquad t = 1, 2, \ldots$$

be $x_t = h(y_t)$. Now consider the following two firm problem:

$$\max \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

$$c_t + x_{1,t+1} + x_{2,t+1} = y_t \qquad t = 0, 1, \ldots$$

$$y_t = f(x_{1,t}, \xi_t) + f(x_{2,t}, \xi_t) \qquad t = 1, 2, \ldots$$

where the two production functions both have the same functional form as in the one firm case. Since the utility function is a concave function of consumption, and the

production function is a concave function of input, from the first order conditions it is the case that the optimal investments in the two firms satisfy

$$x_{1,t} = x_{2,t}$$

Let us call this common value z_t. Thus the solution to the two firm problem is equivalent to

$$\max \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

$$c_t + 2z_{t+1} = y_t \qquad t = 0, 1, \ldots$$

$$y_t = 2f(z_t, \xi_t) \qquad t = 1, 2, \ldots$$

Now define

$$\tilde{c}_t = \frac{1}{2}c_t \qquad \tilde{y}_t = \frac{1}{2}y_t$$

and this equivalent problem becomes

$$\max \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(2\tilde{c}_t)\right]$$

$$\tilde{c}_t + z_{t+1} = \tilde{y}_t \qquad t = 0, 1, \ldots$$

$$\tilde{y}_t = f(z_t, \xi_t) \qquad t = 1, 2, \ldots$$

Note the similarity with the one firm problem at the beginning of this section. Now this is as far as we can push the scaling result, and we see that indeed enlarging the economy by replication will have, *in general*, an impact on the result. However, for the parameterization in this paper the utility function is of the form $u(c) = c^\gamma$ and so

$$\mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(2\tilde{c}_t)\right] = \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t 2^\gamma \tilde{c}_t^\gamma\right] = 2^\gamma \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t)\right]$$

and so for this utility function, the two firm problem has the same solution as

$$\max \quad \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t)\right]$$

$$\tilde{c}_t + z_{t+1} = \tilde{y}_t \qquad t = 0, 1, \ldots$$

$$\tilde{y}_t = f(z_t, \xi_t) \qquad t = 1, 2, \ldots$$

Thus, the optimal policy functions are the same as for the one firm case which means that the equity premium (among other things) is invariant to scaling by replication.

5.2   Scaling the $\theta_{i,s}$ Parameters

Consider scaling the productivity parameters, $\theta_{i,s}$, as follows:

$$\tilde{\theta}_{i,s} = A^{\alpha_{i,s}-1}\theta_{i,s}$$

where $A > 0$. The total output is

$$
\begin{aligned}
y &= \sum_{i}^{n} \left[\tilde{\theta}_{i,s} x_{i,s}^{\alpha_{i,s}} + (1 - \delta_{i,s})x_{i,s} \right] \\
&= \sum_{i}^{n} \left[A^{\alpha_{i,s}-1}\theta_{i,s} x_{i,s}^{\alpha_{i,s}} + (1 - \delta_{i,s})x_{i,s} \right] \\
&= \sum_{i}^{n} \left[A^{-1}\theta_{i,s}(Ax_{i,s})^{\alpha_{i,s}} + (1 - \delta_{i,s})x_{i,s} \right]
\end{aligned}
$$

By multiplying both side of this last equality by A we get:

$$Ay = \sum_{i}^{n} \left[\theta_{i,s}(Ax_{i,s})^{\alpha_{i,s}} + (1 - \delta_{i,s})Ax_{i,s} \right]$$

Therefore, if we scale the dynamic optimization equation,

$$Ac_t + Ax_{t+1} = Ay_t$$

and follow the reasoning above for the case that the utility function is of the form $u(c) = c^\gamma$, we readily see that the solution will be invariant to this rescaling as well.

5.3   The Real Impact of Production Uncertainty

Consider the Bellman equation for the growth model (2.1)

$$V(y) = \max_{\{x_i\}} \left\{ u\left(y - \sum_{i=1}^{n} x_i \right) + \beta \mathsf{E}\left[V\left(\sum_{i=1}^{n} f_i(x_i, \Xi) \right) \right] \right\}$$

for which the first order conditions are:

$$u'\left(y - \sum_{i=1}^{n} x_i\right) = \beta\mathsf{E}\left[V'\left(\sum_{i=1}^{n} f_i(x_i, \Xi)\right) f_j'(x_j, \Xi)\right] \qquad j = 1, \ldots, n$$

which in turn implies that for all j and k,

$$\mathsf{E}\left[V'\left(\sum_{i=1}^{n} f_i(x_i, \Xi)\right) f_j'(x_j, \Xi)\right] = \mathsf{E}\left[V'\left(\sum_{i=1}^{n} f_i(x_i, \Xi)\right) f_k'(x_k, \Xi)\right] \qquad (5.1)$$

From this, it is clear that if the uncertainty is multiplicative, so that $f_i(x_i, \xi) = \xi f_i(x_i)$, then for all firms

$$f_i'(x_i) = f_j'(x_j) \ . \qquad (5.2)$$

Thus, the marginal products of all firms are equal, *regardless* of the shock. So uncertainty only affects the overall level of output, and not the disaggregated investment decisions. Put another way, under these conditions we can construct a technology set that depends only on the level of aggregate savings, and not on the risk preferences of the consumer. For a given level of investment, x, solve equation (5.2) subject to $\sum x_i = x$, and define

$$f(x) = \sum_{i=1}^{n} f_i(x_i)$$

Then we can reduce the problem to a one firm problem with the (random) production function, $\Xi f(x)$. In this case, there is very little equity premium unless other factors are adjusted or taken into account, e.g., a very high degree of relative risk aversion, habit persistence in consumption, etc.

Finally let us look at a special case of the parameterization that we have chosen: the total output from equation (4.2) is of the special form

$$y_{s,t+1} = \sum_{i=1}^{n} f_i(x_i, \xi) = \sum_{i=1}^{n} \left[\theta_s x_{it}^{\alpha_i} + (1 - \delta_i)x_{it}\right] \ .$$

Notice that the α_i and δ_i parameters are not random, and the θ_s shock is *common* to all firms. In this case equation (5.1) can be rewritten as

$$\alpha_j x_j^{\alpha_j - 1} - \alpha_i x_i^{\alpha_i - 1} = (\delta_i - \delta_j)\frac{\mathsf{E}\left[V'\left(\sum\left[\Theta x_i^{\alpha_i} + (1 - \delta_i)x_i\right]\right)\right]}{\mathsf{E}\left[V'\left(\sum\left[\Theta x_i^{\alpha_i} + (1 - \delta_i)x_i\right]\right)\Theta\right]} \qquad (5.3)$$

Note that the ratio on the right hand side of this equation is a function of $x = \sum x_i$ (as well as the particular form of the utility function). Call this term

$$\zeta_u(x) = \frac{\mathsf{E}\left[V'\left(\sum\left[\Theta x_i^{\alpha_i} + (1-\delta_i)x_i\right]\right)\right]}{\mathsf{E}\left[V'\left(\sum\left[\Theta x_i^{\alpha_i} + (1-\delta_i)x_i\right]\right)\Theta\right]}$$

where the subscript u emphasizes the dependence on the particular utility function. Then equation (5.3) is:

$$\alpha_j x_j^{\alpha_j-1} - \alpha_i x_i^{\alpha_i-1} = (\delta_i - \delta_j)\zeta_u(x) \qquad (5.4)$$

Now if $\delta_i = \delta_j$, then we have the same situation as above. There is a single technology that we can construct which is independent of the utility function. The shocks do not impact the marginal product of the technology in a significant way. Even without the equality of depreciation rates, equation (5.4) shows that what matters is the level of investment, x, and not the distributional aspects of the shock.

What these results show is that for the case that there is only systematic shocks and no idiosyncratic shocks, the model behaves as a one firm model. What our numerical results show is that the presence of idiosyncratic shocks along with systematic shocks matter in the sense that the equity premium is higher when there are more sources of idiosyncratic shocks.

6   Conclusion

In this paper, numerical results point to the fact that by including production directly into the model we can obtain equity premia that are more consistent with US data than can be had from models that are based on consumption alone.

It is also clear from both the data and simulations, that the typical application of Brock's asset pricing model has a number of shortcomings that need to be addressed before it might become a widely accepted basis for macro economic research.[28] From a macro economic point of view, one shortcoming is that the model does not include growth. While one can view it as a "detrended" model, most macro economists build growth into their models, and would expect to see it in this one as well. Another problem with the model can be seen from Figures 15 and 16.[29] Market prices and profits do not follow the patterns in these figures. One reason for this is the iid nature of the shocks over time. To the extent that random shocks is a reasonable model for

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[28]Although the model is widely cited, the only numerical solution is the one in Akdeniz and Dechert (1997) which was also used in Akdeniz (2000). Judd (1992) has a solution for the Brock and Mirman (1972) model.

[29]Figures 15 and 16 show the price and profit series for the market portfolio, while Figures 17 and 18 show the percentage changes in the two series.

the fluctuations in output, surely there is substantial serial correlation in them. This means that Brock's model should, at the very least, use a Markov model for the systematic shocks.[30] These shortcomings are not inherent in Brock's model, but are rather complications which add to the computational complexity of implementing a numerical solution of the model. This explains, in part, why the model has not been extensively used.

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[30]Notice that Brock's model with iid shocks *does* exhibit serial correlation in the output and consumption processes. This is due to the "spillover" effects of savings. I.e., when output is high, a greater fraction of it is used as investment, which somewhat insulates the economy from a bad shock in the next period. A Markov series for the shocks would emphasize this effect and produce longer intervals between peaks and troughs in output. Also, the effect of crashes being steeper than recoveries can be incorporated with Markov shocks, but not with iid shocks. See Danthine and Donaldson (1986) for a model of asset prices where the shocks are modeled as a Markov process in the output series, Y_t.

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Tables and Figures

State	α_1	α_2	α_3	θ_1	θ_2	θ_3
1	0.200	0.204	0.204	0.268	0.178	0.351
2	0.420	0.414	0.414	0.651	0.439	0.868
3	0.392	0.378	0.378	0.235	0.158	0.312
4	0.352	0.495	0.495	0.770	0.360	0.710
5	0.448	0.457	0.357	0.226	0.149	0.316
6	0.348	0.357	0.557	0.779	0.506	0.609
7	0.548	0.457	0.457	0.211	0.149	0.295
8	0.448	0.557	0.557	0.608	0.308	0.609

Table 1: Parameter Values of the 3 Firms

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	5.37%	5.56%	5.62%	5.04%
30%	3.83%	3.95%	4.04%	3.59%
50%	3.17%	3.26%	3.36%	2.98%
70%	2.75%	2.82%	2.93%	2.58%
90%	2.21%	2.26%	2.37%	2.08%

Table 2: Equity Premia at $\gamma = -1.0$ $\beta = 0.95$ $\sigma_c = 0.0927$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	4.73%	4.82%	4.9%	4.55%
30%	4.17%	4.24%	4.32%	4.03%
50%	3.46%	3.5%	3.58%	3.34%
70%	2.55%	2.59%	2.66%	2.46%
90%	1.4%	1.42%	1.48%	1.34%

Table 3: Equity Premia at $\gamma = -1.0$ $\beta = 0.97$ $\sigma_c = 0.0961$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	3.41%	3.43%	3.47%	3.35%
30%	2.68%	2.69%	2.73%	2.64%
50%	2.21%	2.21%	2.25%	2.17%
70%	1.77%	1.78%	1.81%	1.74%
90%	1.29%	1.29%	1.32%	1.27%

Table 4: Equity Premia at $\gamma = -1.0$ $\beta = 0.99$ $\sigma_c = 0.0755$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	7.85%	8.13%	8.28%	7.35%
30%	6%	6.17%	6.33%	5.66%
50%	4.95%	5.07%	5.23%	4.68%
70%	3.89%	3.97%	4.12%	3.68%
90%	2.65%	2.7%	2.83%	2.5%

Table 5: Equity Premia at $\gamma = -2.0$ $\beta = 0.95$ $\sigma_c = 0.0815$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	6.37%	6.49%	6.64%	6.09%
30%	4.87%	4.94%	5.08%	4.68%
50%	3.91%	3.96%	4.09%	3.77%
70%	2.99%	3.02%	3.13%	2.87%
90%	2.06%	2.07%	2.17%	1.98%

Table 6: Equity Premia at $\gamma = -2.0$ $\beta = 0.97$ $\sigma_c = 0.0708$

$\mathsf{P}\left\{Y < y\right\}$	Market	Firm 1	Firm 2	Firm 3
10%	5.11%	5.14%	5.22%	5.02%
30%	4.09%	4.11%	4.17%	4.03%
50%	3.37%	3.38%	3.44%	3.32%
70%	2.6%	2.6%	2.65%	2.56%
90%	1.65%	1.66%	1.69%	1.63%

Table 7: Equity Premia at $\gamma = -2.0$ $\beta = 0.99$ $\sigma_c = 0.0632$

$\mathsf{P}\left\{Y < y\right\}$	Market	Firm 1	Firm 2	Firm 3
10%	10.3%	10.7%	10.9%	9.62%
30%	7.81%	8.02%	8.25%	7.38%
50%	6.29%	6.42%	6.63%	5.97%
70%	4.69%	4.78%	4.96%	4.46%
90%	2.8%	2.84%	3%	2.64%

Table 8: Equity Premia at $\gamma = -3.0$ $\beta = 0.95$ $\sigma_c = 0.0717$

$\mathsf{P}\left\{Y < y\right\}$	Market	Firm 1	Firm 2	Firm 3
10%	8.77%	8.95%	9.18%	8.36%
30%	6.57%	6.66%	6.86%	6.32%
50%	5.15%	5.21%	5.38%	4.97%
70%	3.67%	3.71%	3.84%	3.54%
90%	2.27%	2.29%	2.39%	2.19%

Table 9: Equity Premia at $\gamma = -3.0$ $\beta = 0.97$ $\sigma_c = 0.0626$

$\mathsf{P}\left\{Y < y\right\}$	Market	Firm 1	Firm 2	Firm 3
10%	10.4%	10.7%	10.9%	10%
30%	8.33%	8.44%	8.62%	8.06%
50%	6.08%	6.15%	6.29%	5.91%
70%	3.69%	3.73%	3.84%	3.57%
90%	1.87%	1.89%	1.97%	1.8%

Table 10: Equity Premia at $\gamma = -3.0$ $\beta = 0.97$ $\sigma_c = 0.0677$ $\delta_i + 0.01$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	7.34%	7.38%	7.5%	7.2%
30%	5.76%	5.78%	5.87%	5.68%
50%	4.55%	4.56%	4.63%	4.49%
70%	3.16%	3.17%	3.22%	3.12%
90%	1.68%	1.68%	1.72%	1.66%

Table 11: Equity Premia at $\gamma = -3.0$ $\beta = 0.99$ $\sigma_c = 0.0557$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	12.2%	12.7%	13%	11.4%
30%	9.85%	10.1%	10.4%	9.32%
50%	7.95%	8.11%	8.35%	7.58%
70%	5.51%	5.59%	5.78%	5.27%
90%	2.7%	2.73%	2.88%	2.57%

Table 12: Equity Premia at $\gamma = -4.0$ $\beta = 0.95$ $\sigma_c = 0.0664$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	11.5%	11.5%	11.9%	11.4%
30%	7.25%	7.23%	7.46%	7.14%
50%	4.53%	4.79%	4.97%	4.03%
70%	4.71%	4.59%	4.75%	4.81%
90%	3.28%	3.16%	3.28%	3.39%

Table 13: Equity Premia at $\gamma = -4.0$ $\beta = 0.97$ $\sigma_c = 0.0517$

$\mathsf{P}\{Y < y\}$	Market	Firm 1	Firm 2	Firm 3
10%	9.14%	9.19%	9.35%	8.97%
30%	7.53%	7.55%	7.67%	7.43%
50%	5.95%	5.96%	6.05%	5.87%
70%	3.95%	3.95%	4.01%	3.9%
90%	1.7%	1.7%	1.74%	1.67%

Table 14: Equity Premia at $\gamma = -4.0$ $\beta = 0.99$ $\sigma_c = 0.0514$

P$\{Y < y\}$	$t = 1$	$t = 2$	$t = 3$
10%	5.28%	10.9%	16.9%
30%	1.85%	4.05%	6.56%
50%	-0.297%	-0.11%	0.474%
70%	-1.85%	-3.2%	-4.12%
90%	-2.83%	-5.41%	-7.69%

Table 15: Term Structure at $\gamma = -2.0$ $\beta = 0.97$

P$\{Y < y\}$	$t = 1$	$t = 2$	$t = 3$
10%	4.93%	10.2%	15.8%
30%	1.15%	2.65%	4.48%
50%	-1.32%	-2.05%	-2.33%
70%	-2.89%	-5.21%	-7.04%
90%	-3.22%	-6.34%	-9.24%

Table 16: Term Structure at $\gamma = -3.0$ $\beta = 0.97$
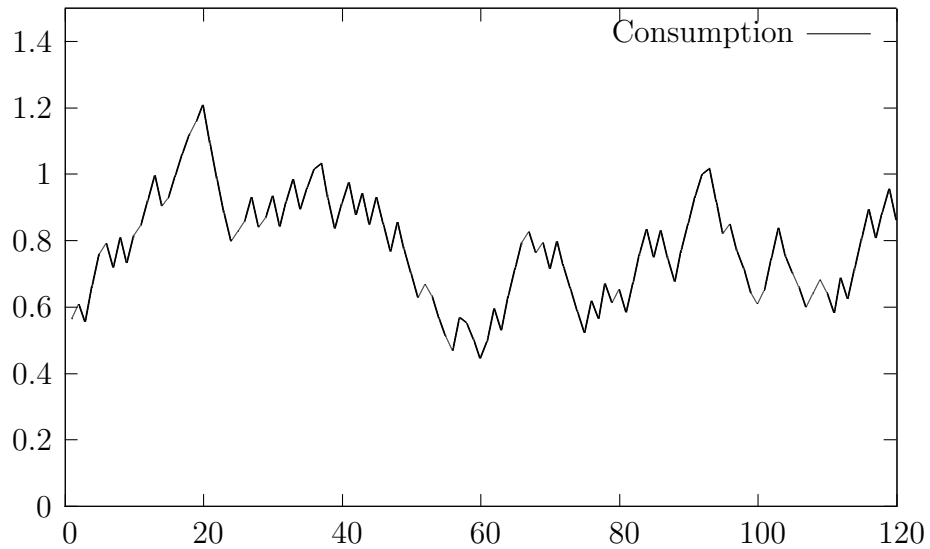
Figure 3: Consumption at $\gamma = -1.0$ $\beta = 0.95$
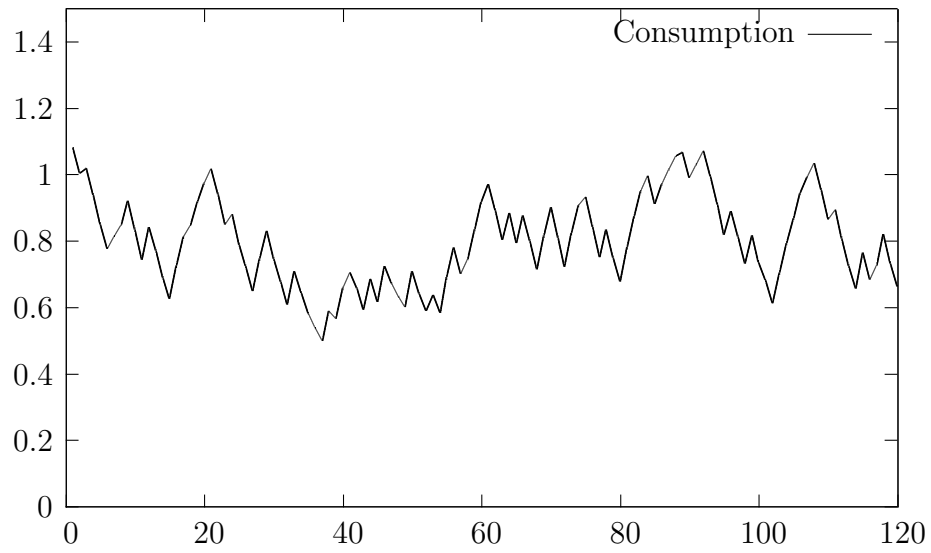
Figure 4: Consumption at $\gamma = -1.0$ $\beta = 0.97$
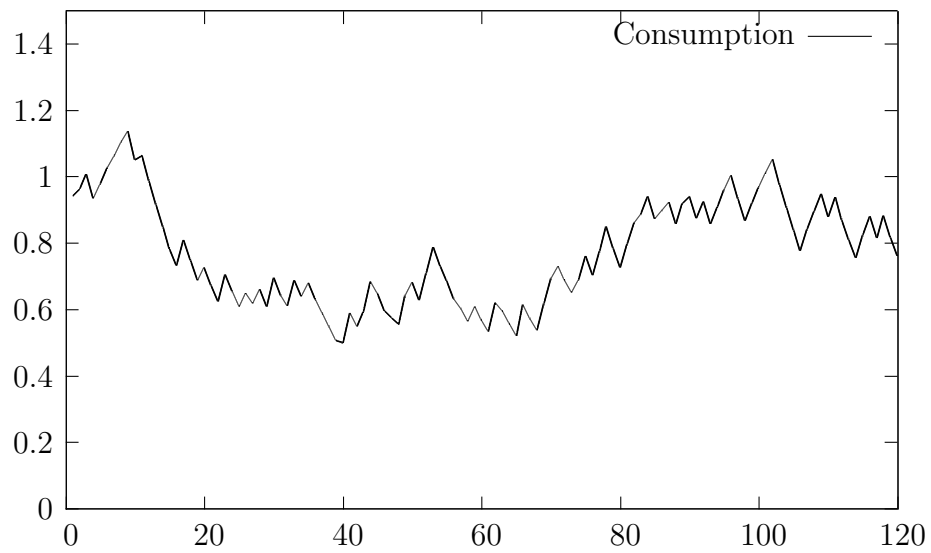
Figure 5: Consumption at $\gamma = -1.0$ $\beta = 0.99$
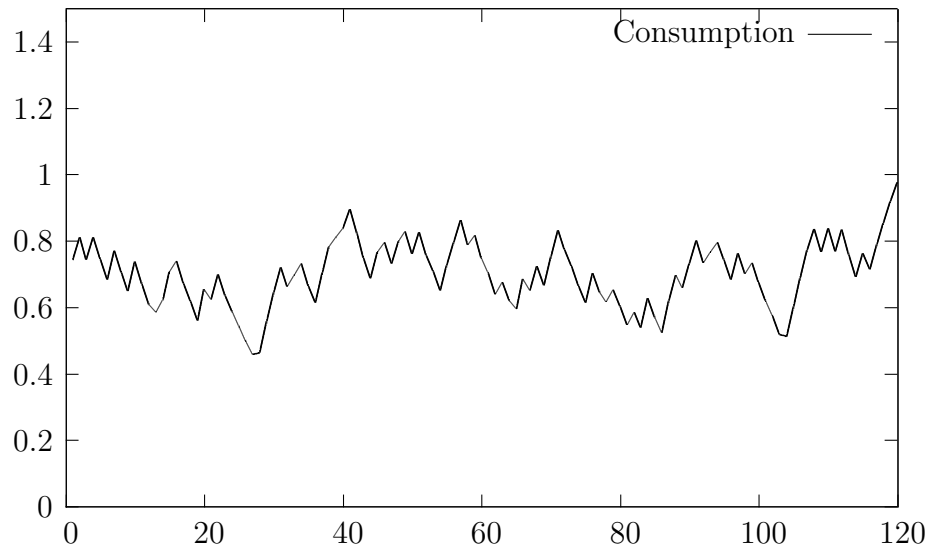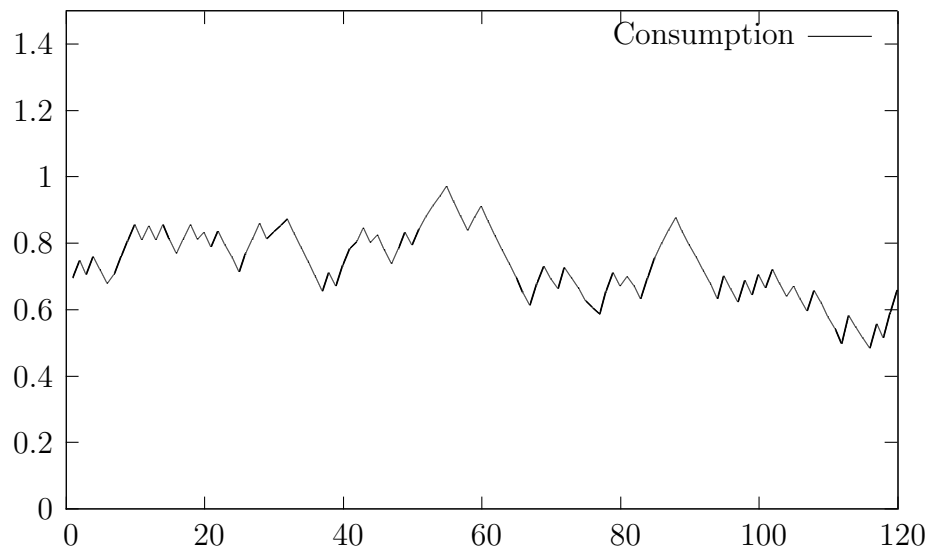
Figure 6: Consumption at $\gamma = -2.0$ $\beta = 0.95$
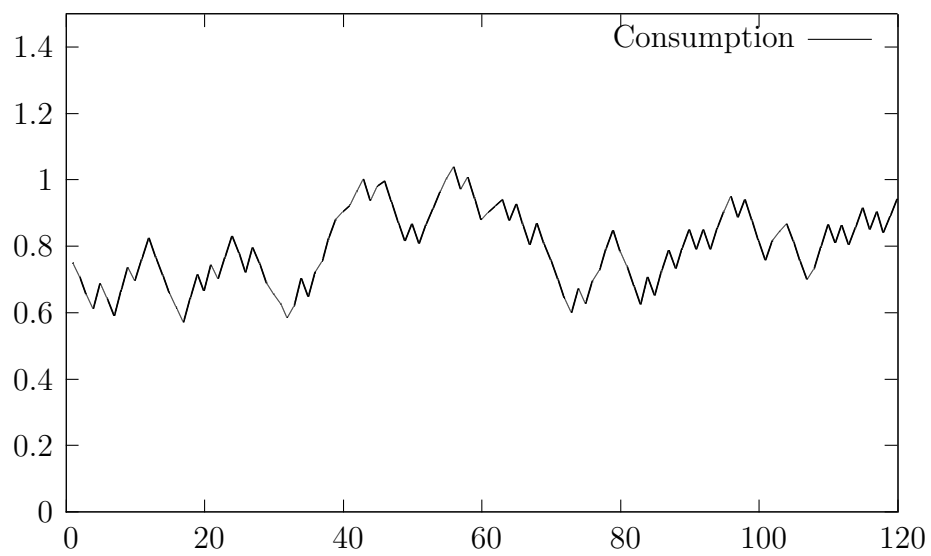
Figure 7: Consumption at $\gamma = -2.0$ $\beta = 0.97$
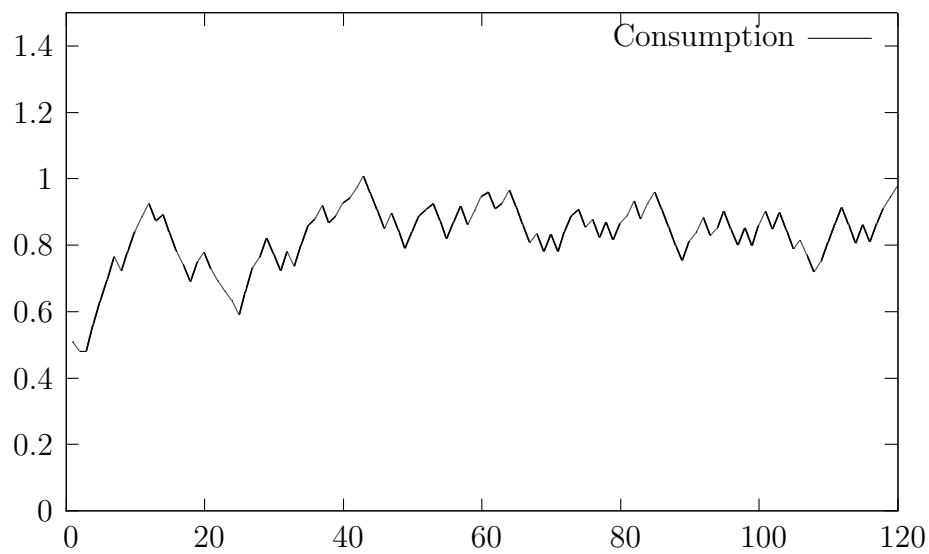
Figure 8: Consumption at $\gamma = -2.0$ $\beta = 0.99$
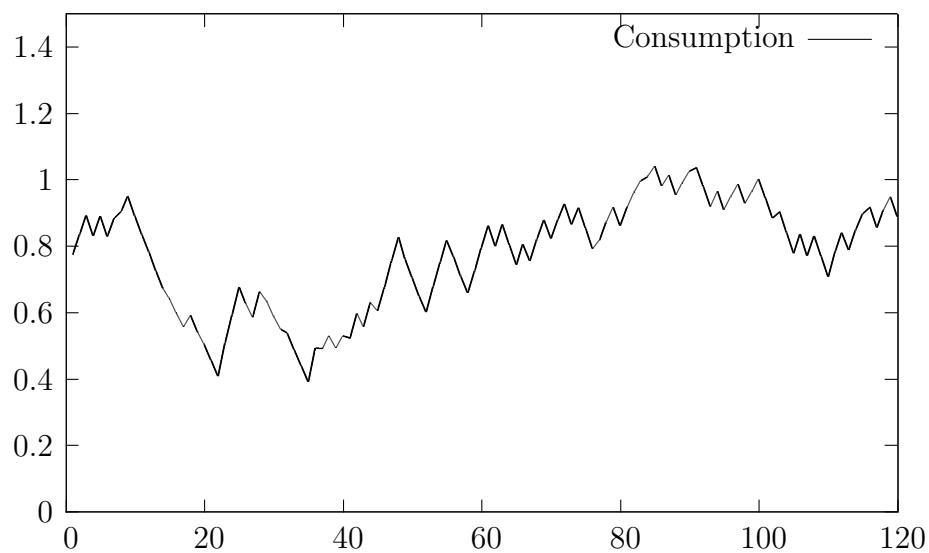
Figure 9: Consumption at $\gamma = -3.0$ $\beta = 0.95$
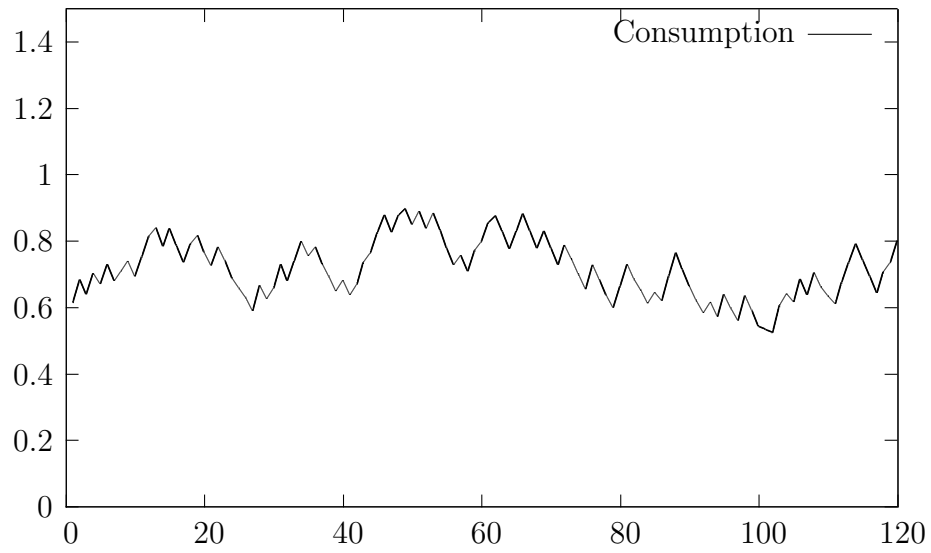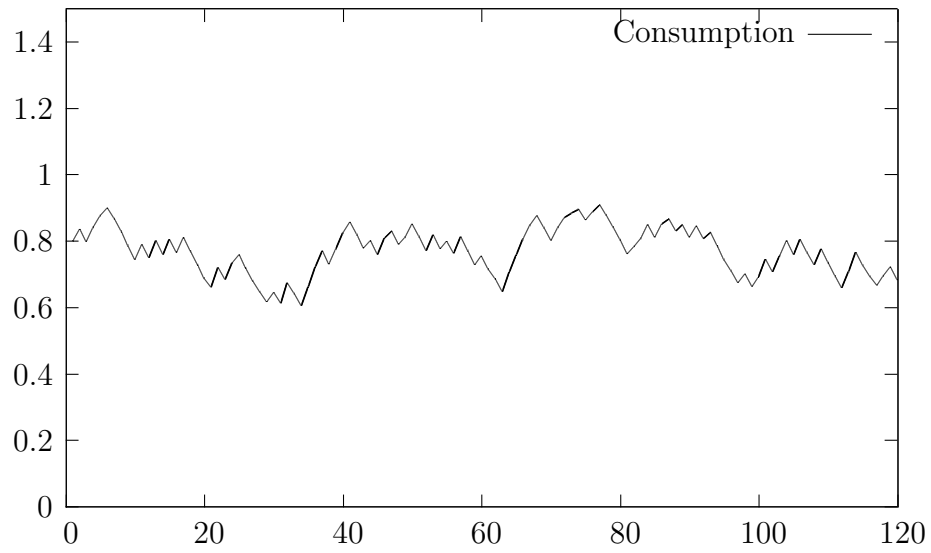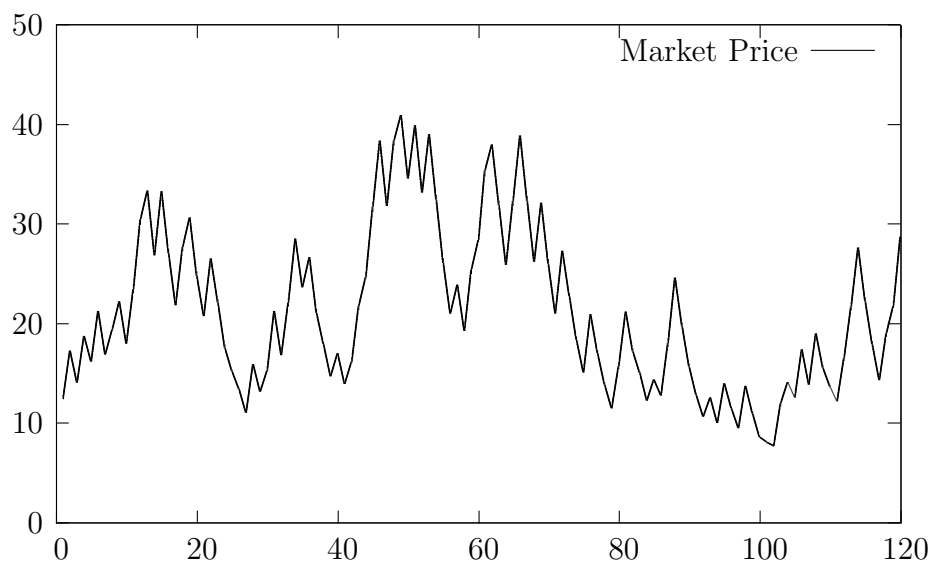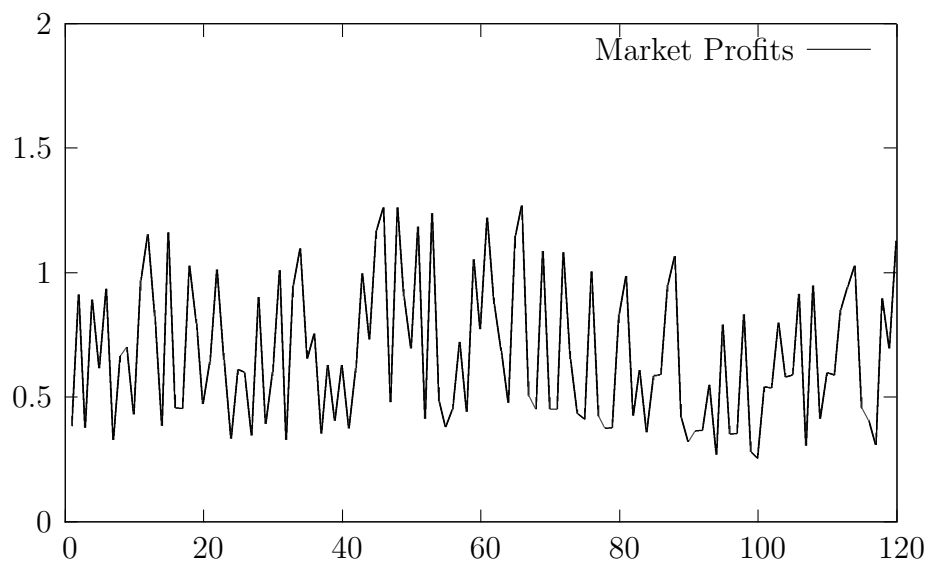
Figure 10: Consumption at $\gamma = -3.0$ $\beta = 0.97$
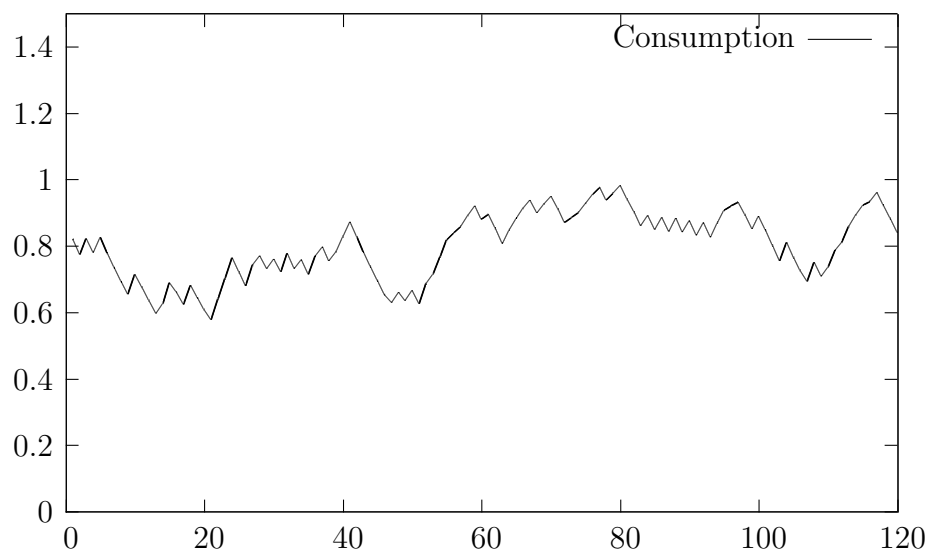
Figure 11: Consumption at $\gamma = -3.0$ $\beta = 0.99$
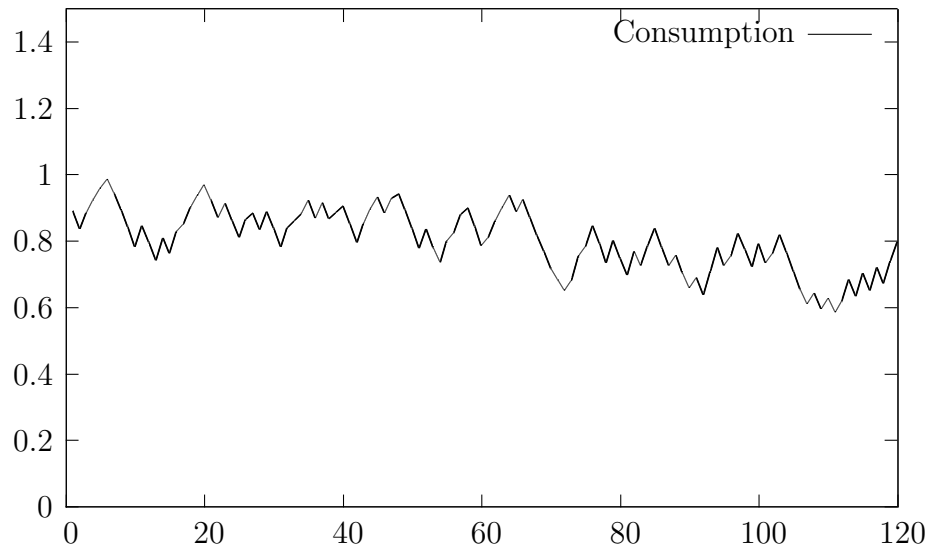
Figure 12: Consumption at $\gamma = -4.0$ $\beta = 0.95$

Figure 13: Consumption at $\gamma = -4.0$ $\beta = 0.97$

Figure 14: Consumption at $\gamma = -4.0$ $\beta = 0.99$

Figure 15: Market Price at $\gamma = -3.0$ $\beta = 0.97$

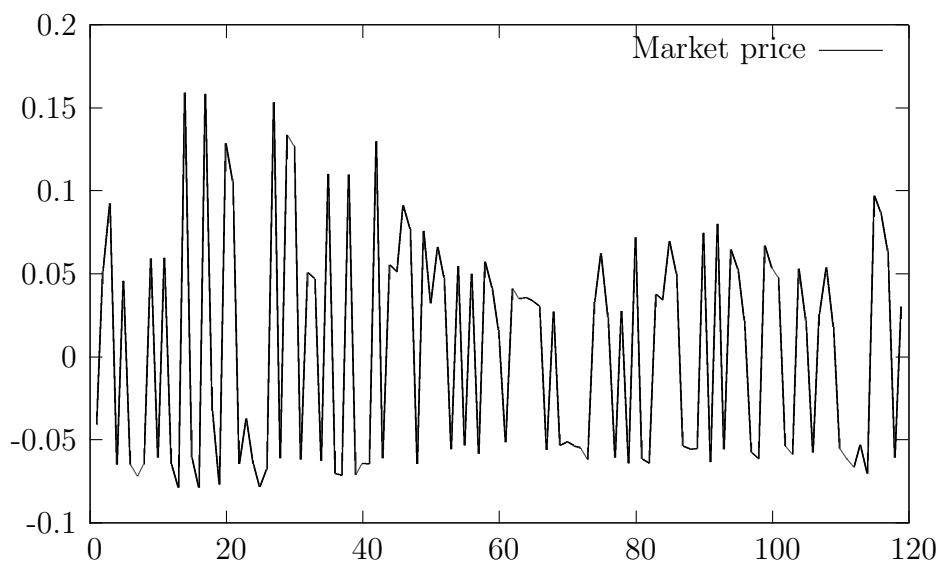Figure 16: Market Profits at $\gamma = -3.0$ $\beta = 0.97$

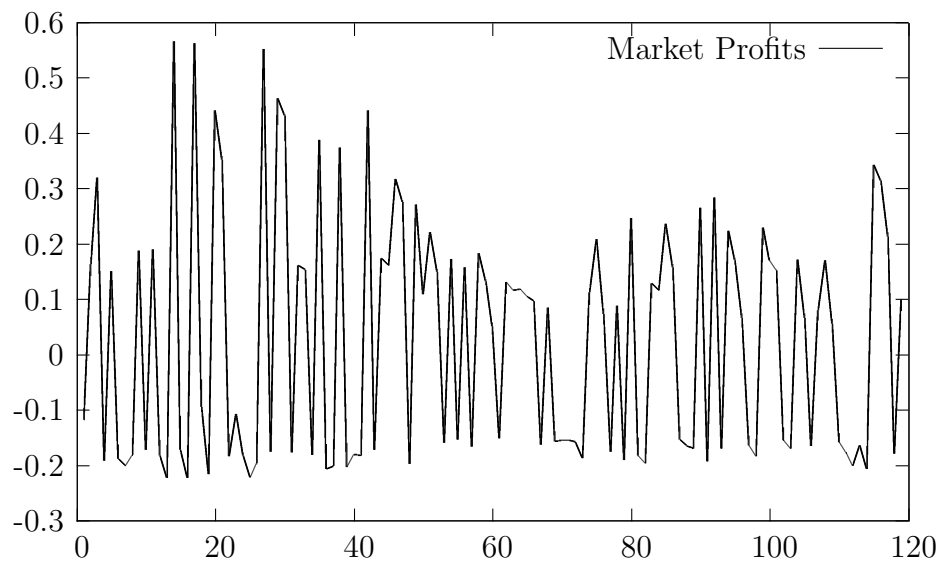Figure 17: Changes in Market Price at $\gamma = -3.0$ $\beta = 0.97$

Figure 18: Changes in Market Profits at $\gamma = -3.0$ $\beta = 0.97$