

**Demand Uncertainty, Endogenous Timing and Costly Waiting:  
Jumping the Gun in Competitive Markets<sup>1</sup>**

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## **Abstract**

We demonstrate a new coordination failure in a general equilibrium model where the set of active consumers is random and consumers choose when to access the market. Consumers face a cost of transacting early, yet we show the existence of competitive equilibria in which inefficient early transactions occur. This coordination failure is impossible when the number of active consumers is known.

## 1. Introduction

There are several strands of literature addressing the question of whether markets tend to unravel in time, with some or all transactions occurring too early from a socially optimal perspective. Diamond and Dybvig [1983] and subsequent papers show that a coordination failure can develop, making bank runs a self-fulfilling prophecy. Admati and Pfleiderer [1989] consider an asset market operated by a price-setting market maker, where informed traders choose when to trade and noise traders absorb losses. Roth and Xing [1994] document a number of matching markets in which transactions tend to become earlier and earlier. None of these models is in the general equilibrium tradition, where agents are price takers and markets clear. Although there is now a substantial literature on general equilibrium with restricted participation,<sup>4</sup> the restrictions are usually exogenously specified.

In a simple general equilibrium model with asymmetric information, we show that transactions can take place too early, based on the idea that the market cannot observe the absence of one trader. This unraveling of markets is purely informational. Market power and risk aversion play no role whatsoever. In our setting with asymmetric information, several modeling difficulties must be overcome before finding a general equilibrium framework that endogenizes the choice of when to transact. Li and Rosen [1998] demonstrate an unraveling possibility in a competitive matching model with symmetric information. In their model, insurance markets are missing, so agents insure themselves against productivity risk by transacting early, accepting the possibility of ex post inefficiency. Welfare results are ambiguous in Li and Rosen [1998], in

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<sup>4</sup> See, for example, Balasko, Cass, and Shell [1995], Balasko, Cass, and Siconolfi [1990], and Lisboa [1995].

contrast to our setting in which unraveling unambiguously lowers welfare. Our paper considers a very special model, but we are able to endogenize the choice by consumers of when to arrive at markets. Specifically, the set of active consumers is random, and firms must produce output ex ante. Next, active consumers commit to visit the market in one of  $T$  rounds or subperiods. Based on the current round's price and agents' beliefs, firms choose how much of their inventory to sell, consumers choose how much to buy, and the market clears.

Since firms do not know the actual set of active consumers, and an active consumer knows that she is active, there is asymmetric information. Thus, firms will draw inferences from the arrival of consumers. It turns out that, without restricting beliefs off the equilibrium path, there would be a plethora of equilibria, just as in the signalling literature (for example, see Cho and Kreps [1987]). Gale [1992] embeds adverse selection into a general equilibrium model and faces the same issue. Since all active consumers are assumed to have the same utility function, we are able to impose a natural restriction on beliefs, along the lines of trembling-hand perfection, in order to pin down reasonably the inferences of firms.

To address the issue of unraveling of markets or "jumping the gun," we assume that consumers face a reduction in utility when they transact early, before the last round of trading. For example, consider the market for air travel between a pair of cities on a given date. The number of seats is more or less fixed in advance, consumers may purchase tickets and prices may adjust for many days before the date of travel, and the cost of purchasing early can be the inflexibility, which declines as the gap between purchase date and travel date narrows. We assume that utility is independent of the round in which consumption is purchased, except for a separable function of the round that reflects early arrival cost. Our specification has similarities to

that of Brusco and Jackson [1997]. In their model, traders exogenously assigned to round 1 can carry commodities to round 2 if they choose to pay a fixed cost. They show that market clearing can be inefficient, because a trader who chooses to intermediate trade between round 1 and round 2 does not internalize all of the benefits. In contrast to Brusco and Jackson [1997], we are interested in unraveling of markets, and not financial intermediation. There is no need for financial intermediation in our setting, because firms costlessly maintain a market presence throughout the market period.

In section 2, we set up the model and present some results. Proposition (2.3) states that an equilibrium exists in which all active consumers choose the last round. Proposition (2.5) states that this equilibrium is ex ante Pareto optimal, although we discuss why the result is not robust. In section 3, we consider the possibility of competitive equilibria in which early transactions occur. Proposition (3.1) shows that, when a regularity condition is satisfied and the cost of early arrival is sufficiently small, there will exist equilibria in which all active consumers choose the first round. Proposition (3.2) demonstrates the importance of demand uncertainty, showing that inefficient early arrival is impossible when the total number of active consumers is known. An example is presented with two potential consumers, who randomize over which round to attend when they are active. Since all possible arrival patterns occur with positive probability, there are no zero-probability events to worry about. Section 4 contains some concluding remarks.

## **2. The Model**

There are two types of agents in the economy, consumers and firm-owning entrepreneurs. There are two physical commodities, the commodity produced by firms,  $x$ , and the numeraire

commodity,  $y$ . The total number of potential consumers is denoted by  $N$ , but the number of active consumers is random and generally less than  $N$ . The market period is divided into a finite number of subperiods or rounds, with the number of rounds denoted by  $T$ . Each active consumer chooses a single round to attend, followed by the traditional choice of consumption to maximize utility, given the prevailing prices at the chosen round.

We consider two sources of uncertainty. First, there is nature's choice of the set of active consumers. Letting  $S$  denote the set of subsets of  $\{1, 2, \dots, N\}$ , an "aggregate demand state"  $s \in S$  specifies the number of active consumers as well as their identities. The probability of aggregate demand state  $s$  is denoted by  $\mu(s)$ . The second source of uncertainty is the breakdown of consumers across trading rounds, a form of "market uncertainty" reflecting uncertainty agents have about what round others have chosen.<sup>5</sup> Let  $\eta_i^t$  denote the probability with which consumer  $i$  chooses round  $t$ , conditional on being active. The complete description of the state of the world specifies the set of consumers choosing each round  $t$ . Let  $\sigma^t \in S$  denote the set of consumers active and choosing round  $t$ , let  $\sigma = (\sigma^1, \dots, \sigma^T)$  denote the state of the world, and let  $\Omega$  denote the set of possible states. Then given  $\mu$  and  $\eta = (\dots, \eta_i^t, \dots)$ , a probability measure over  $\sigma$  is well defined and denoted by  $\pi(\sigma)$ . Let  $n^t(\sigma)$  be the number of active consumers choosing round  $t$  when the state of the world is  $\sigma$ , and let  $n(\sigma) \equiv \sum_t n^t(\sigma)$  be the total number of active consumers when the state of the world is  $\sigma$ . We assume that all active consumers face the same distribution

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<sup>5</sup> Our modelling of the states of the world as partly endogenous is similar to the approach taken by Aumann [1987]. Of course, we assume equilibrium behavior rather than deriving it from first principles as Aumann does.

of market conditions:  $\pi(n(\sigma) = n^* \mid i \text{ is active})$  is independent of  $i$  for all  $n^*$ .<sup>6</sup>

We have in mind the following time line and market structure. Before anything is observed about demand, firms produce their output of commodity  $x$ , using the numeraire as an input. Commodities are perfectly storable across rounds but perishable after round  $T$ . Next, nature chooses the set of active consumers. Each active consumer chooses which round to attend, not knowing which other consumers are active. The market in round  $t$  consists of a spot market, with a single price representing the price of commodity  $x$  in terms of the numeraire. Consumers must purchase during their chosen round, and simply equate their marginal rate of substitution to the price ratio. Firms are assumed to maintain a market presence throughout, and must choose how much output to sell in the current round and how much output to retain for future rounds. The price in round  $t$  when the state is  $\sigma$ ,  $p^t(\sigma)$ , must be measurable with respect to the history of arrivals,  $(n^1(\sigma), n^2(\sigma), \dots, n^t(\sigma))$ . The history is observed by firms at the market but not by consumers who have not yet arrived.<sup>7</sup>

Consumers are assumed to have identical preferences over certain consumption, as represented by the quasilinear utility function,  $u(x) + y$ . We assume that  $u' > 0$ ,  $u'(0) < \infty$ , and  $u'' < 0$  hold. All active consumers are endowed with  $\omega$  units of commodity  $y$  and zero units of commodity  $x$ . We assume that  $\omega$  is large enough so that we can ignore corner solutions in which numeraire consumption is zero. Consumers are assumed to be von Neumann-Morgenstern

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<sup>6</sup> This assumption is satisfied, for example, if nature first selects the total number of active consumers,  $n(\sigma)$ , and then selects each potential consumer to be active with probability  $n(\sigma)/N$ .

<sup>7</sup> Allowing consumers to observe the history of arrivals, excluding the current number of arrivals, before deciding whether to attend the market would not affect any of our results. A consumer should not be able to observe  $n^t(\sigma)$  before choosing whether to arrive in round  $t$ , because  $n^t(\sigma)$  will be affected by that choice.

expected utility maximizers. To capture the idea that early arrival represents inefficient “jumping the gun,” we assume that there is a disutility of arriving in round  $t$ ,  $c(t)$ , where  $c'(t) \leq 0$  and  $c(T)$  is normalized to zero. Letting  $(x_i^t(\sigma), y_i^t(\sigma))$  be the consumption of consumer  $i$  choosing round  $t$  when the state of the world is  $\sigma$ ,<sup>8</sup> the expected utility of consumer  $i$  choosing round  $t$  is given by:

$$\sum_{\sigma \in \Omega} \pi(\sigma | i \in \sigma^t) [u(x_i^t(\sigma)) + y_i^t(\sigma)] - c(t) \quad (1)$$

Firms are risk neutral, seeking to maximize expected profits denominated in terms of the numeraire. Each firm is owned by an entrepreneur who cares only about numeraire consumption. Let  $y_f$  denote the numeraire input used by firm  $f$ , and let  $x_f$  denote firm  $f$ 's output. These quantities are related according to:  $x_f = g_f(y_f)$ . The production function,  $g_f(y_f)$ , is assumed to be strictly monotonic, strictly concave, and continuously differentiable for  $f = 1, \dots, F$ . We also impose the Inada condition,  $g_f'(y_f) \rightarrow \infty$  as  $y_f \rightarrow 0$ . Let  $q_f^t(\sigma)$  denote the quantity supplied by firm  $f$  in round  $t$  when the state is  $\sigma$ .

### *Zero Probability Events*

Because the choice of which round to attend is endogenous, events that have zero probability in equilibrium play an important role in the analysis, similar to the role played in the game theoretic concept, sequential equilibrium. For example, suppose that all active consumers choose round  $T$  with probability 1. Then any state  $\sigma$  in which some consumer chooses an earlier round occurs with probability 0,  $\pi(\sigma) = 0$ . However, the behavior of firms when a consumer chooses an early round affects consumers' round choices. We require firms to maximize profits,

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<sup>8</sup> For consumers not in  $\sigma^t$  when the state is  $\sigma$ , we have  $(x_i^t(\sigma), y_i^t(\sigma)) = (0, 0)$ .



even conditional on zero-probability events. Which quantities firms should supply to the market in turn depend upon how firms update their beliefs about the state of the world. The definition of competitive equilibrium, definition (2.2) below, pins down the inferences of firms by assuming that consumers make mistakes with vanishingly small probabilities.

**Definition 2.1:** For each active consumer,  $i$ , define an  $\epsilon$ -tremble as follows. With probability  $(1-T\epsilon)$ , consumer  $i$  chooses the round according to  $\eta_i$ . With probability  $\epsilon$  (independent of  $i$  and  $t$ ), the consumer will make a “mistake” and choose round  $t$ . Denote the associated probability measure over states as  $\pi^\epsilon(\sigma)$ .

Since definition (2.2) below assumes that firms believe that consumers make  $\epsilon$ -trembles with vanishingly small probabilities, our competitive equilibrium captures the notion of trembling hand perfection. In a competitive equilibrium, consumers and firms exactly optimize. The role of taking limits as  $\epsilon$  approaches zero is to guarantee that firms optimize off the equilibrium path as well. Because of the symmetry of the model, we impose the further restriction that all consumers make the same trembles.<sup>9</sup> Beliefs satisfying definition (2.1) have the desirable property that, if  $\pi(n^1(\sigma) = n^2(\sigma) = \dots = n^t(\sigma) = 0) = 1$  holds, then we have

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \pi^\epsilon(n(\sigma) = n^* \mid n^1(\sigma) = n^2(\sigma) = \dots = n^{t-1}(\sigma) = 0, n^t(\sigma) = 1) &= [n(\sigma) \pi(\sigma) / \sum_{\sigma' \in \Omega} n(\sigma') \pi(\sigma')] \\ &= \pi(n(\sigma) = n^* \mid i \text{ is active}). \end{aligned}$$

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<sup>9</sup> The probability of trembling, and choosing round  $t$ , is assumed to be independent of  $t$  for expositional convenience only. None of our results are affected by allowing  $\epsilon$  to depend on  $t$ .

In other words, if a consumer deviates and chooses a round too early to be consistent with equilibrium, then the consumer and all firms share the same beliefs about the total number of active consumers.

As far as consumers' beliefs about zero-probability events are concerned, there is only one situation to worry about. This is the situation in which some consumer  $i$  chooses a round,  $t$ , for which  $\eta_i^t = 0$ . When consumer  $i$  is considering a deviation to arrive in round  $t$ , for which  $\eta_i^t = 0$ , he believes that all other consumers' round choices are unaffected. Although slightly abusing our notation,  $\pi(\sigma \mid i \in \sigma^t)$  is defined as follows when  $\eta_i^t = 0$ . The state of the world  $\sigma$  specifies an aggregate demand state,  $s$ , the round choices of active consumers other than  $i$ , and consumer  $i$ 's choice of round  $t$ .<sup>10</sup> Then  $\pi(\sigma \mid i \in \sigma^t)$  is the probability of  $s$  and the round choices of the consumers other than  $i$  who are active in  $s$ , as determined by  $\mu$  and  $\eta$ .

**Definition 2.2:** A *competitive equilibrium* is a collection of prices,  $\dot{p}^t(\sigma)$ , round choices,  $\eta^*$ , and an allocation,  $\{ \dot{x}_i^t(\sigma), \dot{y}_i^t(\sigma), \dot{q}_f^t(\sigma), \dot{x}_f, \dot{y}_f \}$ ,  $i = 1 \dots N$ ,  $f = 1 \dots F$ ,  $t = 1 \dots T$ ,  $\sigma \in \Omega$ , such that

(i) for all  $\sigma$  and (active) consumers  $i$  such that  $i \in \sigma^t$ ,  $\dot{x}_i^t(\sigma)$  and  $\dot{y}_i^t(\sigma)$  are the solutions to

$$u'(\dot{x}_i^t(\sigma)) = \dot{p}^t(\sigma) \quad \text{and} \quad \dot{p}^t(\sigma) \dot{x}_i^t(\sigma) + \dot{y}_i^t(\sigma) = \omega,$$

(ii)  $i \notin \sigma^t$  implies  $\dot{x}_i^t(\sigma) = \dot{y}_i^t(\sigma) = 0$ ,

(iii) for all  $i$  and  $t$  such that  $\eta_i^{*t} > 0$ ,  $t$  solves

$$\max_t \sum_{\sigma \in \Omega} \pi(\sigma \mid i \in \sigma^t) [u(\dot{x}_i^t(\sigma)) + \dot{y}_i^t(\sigma)] - c(t),$$

(iv) for  $f = 1 \dots F$ ,  $\dot{q}_f^t(\sigma)$ ,  $\dot{x}_f$ , and  $\dot{y}_f$  are the limiting solutions, as  $\epsilon \rightarrow 0$ , of

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<sup>10</sup> If consumer  $i$  is not active and choosing round  $t$  when the state of the world is  $\sigma$ , then let  $\pi(\sigma \mid i \in \sigma^t) = 0$ .

$$\begin{aligned} & \max \sum_{\sigma \in \Omega} \sum_{t=1}^T \pi^{\varepsilon}(\sigma) p^t(\sigma) q_f^t(\sigma) - y_f \\ & \text{subject to} \\ & \quad x_f = g_f(y_f) \quad \text{for all } \sigma, \\ & \quad \sum_{t=1}^T q_f^t(\sigma) = x_f \quad \text{for all } \sigma, \\ & \quad y_f \geq 0, \quad q_f^t(\sigma) \geq 0 \quad \text{for all } \sigma, \\ & \quad q_f^t(\sigma) \text{ is measurable w.r.t. the history } (n^1(\sigma), \dots, n^t(\sigma)). \end{aligned}$$

(v)  $\tilde{p}^t(\sigma)$  is measurable w.r.t. the history  $(n^1(\sigma), \dots, n^t(\sigma))$ ,

(vi)  $\sum_{i=1}^W x_i^*(\sigma) = \sum_{j=1}^F q_j^*(\sigma) \quad \text{for all } \sigma, t.$

In definition (2.2), equalities in (i), (iv), and (vi) are specified because utility is strictly monotonic. Also, to keep notation as uncluttered as possible, we have not been explicit about entrepreneurs' numeraire consumption and market clearing for commodity  $y$ . This can be done in the obvious way for any competitive equilibrium.

Our results require that a competitive equilibrium exists for the corresponding static economy with one round of trading. Let  $W_t(y_t)$  denote the total social surplus if there is only one firm, producing  $y_t$ , and where all consumers arrive in the last round and share the output equally.

$$W_f(y_f) \equiv \sum_{\sigma \in \Omega} \pi(\sigma) n(\sigma) u\left(\frac{g_f(y_f)}{n(\sigma)}\right) - y_f.$$

We assume that there is an interior optimum to the welfare maximization problem with one firm.<sup>11</sup>

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<sup>11</sup> The expression for social welfare, and assumption 1 below, can be expressed in terms of the exogenously given probability distribution,  $\mu$ , but the current formulation simplifies the proof.

*Assumption 1:*  $\lim_{y_f \rightarrow \infty} W_f'(y_f) < 0$  for  $f = 1, \dots, F$ .<sup>12</sup>

In Proposition (2.3) below, we consider replications of the original economy. In an  $r$ -fold replication, there are  $r$  identical consumers for each consumer in the original economy. These  $r$  consumers are active in exactly the same aggregate demand states, although they separately choose which rounds to attend.

**Proposition 2.3:** *Consider the  $r$ -fold replication of the original economy. Then for sufficiently large  $r$ , and given Assumption 1 and our other maintained assumptions, a competitive equilibrium exists in which all active consumers choose round  $T$ .*

**Proof:** We will construct a competitive equilibrium, in which all consumers choose round  $T$ , as follows. Let  $\eta_i^{*T} = 1$  for all  $i$ .

The Inada conditions imply  $W_f'(0) > 0$ , which along with continuity and Assumption 1 implies that there is a solution to  $W_f'(y_f) = 0$ . Strict concavity of  $u$  and  $g_f$  implies  $W_f''(y_f) < 0$ , so there is a unique solution to  $W_f'(y_f) = 0$ , which we denote by  $\bar{y}_f$ . Without loss of generality, we can restrict attention to  $y_f$  lying in the compact set  $(0, \bar{y}_f)$ . By compactness and concavity, there must exist a unique  $(y_1, y_2, \dots, y_F)$  which solves the surplus maximization problem:

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<sup>12</sup> The Inada condition and the strict concavity of  $g_f$  are strong assumptions used to simplify the proofs. All of our results would hold if we instead assume that  $g_f$  is weakly concave (allowing constant returns) and that  $\lim_{y_f \rightarrow 0} W_f'(y_f) > 0$  for some  $f$ . Multiple equilibria would then be possible, and some firms might strictly prefer to produce zero output. Since these issues are not related to our main point, we opt for simplicity.

$$\max \sum_{\sigma \in \Omega} \pi(\sigma) n(\sigma) u \left( \frac{\sum_{f=1}^F g_f(y_f)}{n(\sigma)} \right) - \sum_{f=1}^F y_f.$$

*Step 1: on the equilibrium path*

For all  $\sigma$  such that  $n^1(\sigma) = n^2(\sigma) = \dots = n^{T-1}(\sigma) = 0$ , let  $\hat{p}^T(\sigma)$  and  $\hat{y}_f$  for  $f = 1, \dots, F$  be the unique solution to<sup>13</sup>

$$u' \left( \frac{\sum_{f=1}^F g_f(y_f)}{n(\sigma)} \right) = p^T(\sigma) \tag{2}$$

$$g'_f(y_f) = \frac{1}{\sum_{\sigma \in \Omega} \pi(\sigma) p^T(\sigma)}$$

That a unique solution exists follows from the fact that (2) is equivalent to

$$g'_f(y_f) \sum_{\sigma \in \Omega} \pi(\sigma) u' \left( \frac{\sum_{f=1}^F g_f(y_f)}{n(\sigma)} \right) = 1 \quad \text{for } f = 1, \dots, F,$$

which is the unique stationary point of the surplus maximization problem above. Notice that  $\hat{p}^T(\sigma)$  is the market-clearing price for the one-round economy with  $n(\sigma)$  consumers. Also, for all  $t \neq T$ , let  $\hat{p}^t(\sigma)$  be given by

$$\hat{p}^t(\sigma) = \sum_{\sigma' \in \Omega} \pi(\sigma') p^T(\sigma'). \tag{3}$$

Since the right side of (3) is an unconditional expectation of the round  $T$  price,  $\hat{p}^t(\sigma)$  does not depend on  $\sigma$ , so the measurability requirement is trivially satisfied. Consumption is uniquely

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<sup>13</sup> If we have  $n(\sigma) = 0$ , then let  $\hat{p}^T(\sigma) = 0$ .

determined and chosen to satisfy (i) and (ii). From (3), firms are indifferent as to when to sell their output, so it is optimal for them to wait until the last round. Condition (2) guarantees that markets clear, and that  $\dot{y}_f$  maximizes firm  $f$ 's expected profits.

*Step 2: single deviations to an earlier round*

For  $\sigma$  for which there is a single deviation in round choice, where we have  $n^t(\sigma) = 1$  for some  $t < T$ , and  $n^\tau(\sigma) = 0$  for all  $\tau \neq t, T$ , then prices are given as follows. For periods  $\tau$  where  $\tau < t$ , prices are given by (3), which is consistent with measurability. For  $T > \tau \geq t$ , we have

$$\dot{p}^\tau(\sigma) = \sum_{\sigma' \in \Omega} \left[ \frac{n(\sigma) \pi(\sigma)}{\sum_{\sigma' \in \Omega} n(\sigma') \pi(\sigma')} \right] p^T(\sigma) \quad (4)$$

Again,  $\dot{p}^\tau(\sigma)$  depends only on the history of arrivals (and, in particular, the deviation by consumer  $i$  to arrive early), since the formula given in (4) is independent of  $\sigma$ . Equation (4) guarantees that firms are indifferent as to when to sell their output. Given  $\dot{p}^\tau(\sigma)$ , consumer  $i$ 's demand is uniquely determined by

$$u'(\dot{x}_i^t(\sigma)) = \dot{p}^t(\sigma), \quad (5)$$

and to satisfy market clearing, let the total quantity supplied in round  $t$  equal  $\dot{x}_i^t$ , where the distribution across firms can be chosen arbitrarily. Then the price in round  $T$ ,  $\dot{p}^T(\sigma)$ , is chosen to satisfy

$$u' \left( \frac{\sum_{f=1}^F r g_f(y_f^*) - x_i^t}{r n(\sigma) - 1} \right) = p^T(\sigma) . \quad (6)$$

From the concavity of  $u$ , it follows that (4)-(6) uniquely determine  $\hat{x}_i^t(\sigma)$  and all prices in response to the early arrival by consumer  $i$ . Conditions (i) and (ii) determine the consumption of all consumers. Except for the amount supplied to consumer  $i$ , firms sell all of their output in round  $T$ , all parties are maximizing, and markets clear.

*Step 3: optimality of waiting until round T*

If consumer  $i$  chooses round  $T$ , the price is determined by the first equation in (2). If, instead, consumer  $i$  chooses round  $t$ , the (nonrandom) price equals

$$\sum_{\sigma \in \Omega} \left[ \frac{n(\sigma) \pi(\sigma) u' \left( \frac{r \sum_f g_f(y_f^*) - x_i^t}{r n(\sigma) - 1} \right)}{\sum_{\sigma' \in \Omega} n(\sigma') \pi(\sigma')} \right]. \quad (7)$$

Therefore, for sufficiently large  $r$ , the price faced by consumer  $i$  when he deviates is arbitrarily close to<sup>14</sup>

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<sup>14</sup> If  $\hat{x}_i^t$  in expression (7) does not remain bounded as  $r$  converges to infinity, a deviation to round  $t$  is even more unattractive, since  $u'' < 0$  implies the price would be even higher than in (8).

$$\sum_{\sigma \in \Omega} \left[ \frac{n(\sigma) \pi(\sigma) u' \left( \frac{\sum_f g_f(y^*_f)}{n(\sigma)} \right)}{\sum_{\sigma' \in \Omega} n(\sigma') \pi(\sigma')} \right]. \quad (8)$$

If there is nontrivial demand uncertainty,  $n(\sigma) \neq n(\sigma')$  for  $\sigma, \sigma'$  occurring with positive probability, then the strict quasi-convexity of the indirect utility function implies that the random prices given in (2) provide strictly higher utility than the nonrandom price given in (7), since the expected prices in the two cases are arbitrarily close, as implied by (2), (4), and (8). If there is trivial demand uncertainty, then it is easy to see that prices will be the same in all states and again there is no incentive to deviate.

*Step 4: multiple deviations to an earlier round*

Obviously, the prices and allocation that results when two or more consumers arrive before round T has no bearing on the equilibrium decision to choose round T. The remainder of the equilibrium construction proceeds as follows. Given any history, firms continue to believe that all the other active consumers will choose round T, except for  $\epsilon$  trembles (see definition (2.1)). A system of equations similar to (4)-(6) will determine the price in round t, where (4)-(6) must be modified to take into account firms' beliefs, the number of consumers present in round t, and the amount of output that has been sold thus far. We omit the details. ■

**Remark 2.4:** It seems strange that an assumption about replicating the economy would play any role in the existence proof. A careful reading indicates that the number of replications,  $r$ , only affects the incentive to arrive early. When consumer  $i$  arrives early, he faces an "average" price,



rather than waiting until the last round and facing a price that clears the market based on the actual number of consumers. The complication is that consumer  $i$ 's demand in round  $t$ ,  $\bar{x}_i^t$ , is greater than what he would receive in high demand states and less than what he would receive in low demand states. Thus, without replicating the economy, consumer  $i$ 's early arrival affects pricing in round  $T$  as well. We cannot rule out the possibility that the price consumer  $i$  would face in round  $t$ , which equals the expected price in round  $T$  following the early arrival, becomes less than the expected price in round  $T$  where everyone chooses round  $T$ . A consumer takes the *function*, mapping states of the world into prices, as given. However, with small numbers, a consumer can affect the realized price by choosing a different round, which changes the state of the world. This kind of manipulation runs counter to the spirit of the model, so we simply assume that the number of replications is large enough so that consumer  $i$ 's early arrival does not affect the realized price in round  $T$ .

**Proposition 2.5:**     *The competitive equilibrium constructed in Proposition (2.3) is ex ante Pareto optimal.*

**Proof:**           We will show that the appropriate planner's problem is solved at the C.E. allocation. Since any transaction made in round  $t < T$  can also be made in round  $T$ , the planner can restrict attention to allocations in which all transactions occur in round  $T$  and which depend only on the aggregate demand state. Since an aggregate demand state,  $s$ , specifies a set of consumers, the event that consumer  $i$  is active in aggregate demand state  $s$  is denoted by  $i \in s$ . Let  $Y_i(s)$  represent the numeraire consumption, net of numeraire endowment, received by the

entrepreneur owning firm  $f$  in aggregate demand state  $s$ . Therefore, the solution to the following planning problem is Pareto optimal. Choose  $\{Y_f(s), y_f, x_i(s), y_i(s)\}$  for  $f = 1, \dots, F, i = 1, \dots, N, s \in S$  to solve

$$\begin{aligned} \max \quad & \sum_{s \in S} \mu(s) [\sum_f Y_f(s) + \sum_{i \in s} u(x_i(s)) + \sum_{i \in s} y_i(s)] & (9) \\ \text{subject to} \quad & \sum_f Y_f(s) + \sum_{i \in s} y_i(s) = \sum_{i \in s} \omega - \sum_f y_f & \text{for all } s \\ & \sum_{i \in s} x_i(s) = \sum_f g_f(y_f) & \text{for all } s. \end{aligned}$$

The constraints in (9) are equalities due to monotonicity, and potential nonnegativity constraints are not binding because numeraire endowments are sufficiently large and the production function satisfies the Inada condition. Plugging the first constraint in (9) into the objective function, we have an equivalent problem, to choose  $y_f$  and  $x_i(s), s \in S$ , to solve

$$\begin{aligned} \max \quad & \sum_{s \in S} \mu(s) [\sum_{i \in s} \omega - \sum_f y_f + \sum_{i \in s} u(x_i(s))] & (10) \\ \text{subject to} \quad & \sum_{i \in s} x_i(s) = \sum_f g_f(y_f) & \text{for all } s. \end{aligned}$$

Given a solution to (10), any values of  $Y_f(s)$  and  $y_i(s)$  satisfying the first constraint in (9) will together constitute a Pareto optimal allocation. Letting  $n_s$  denote the number of active consumers in aggregate demand state  $s$ , necessary and sufficient conditions for a solution to (10) are

$$\begin{aligned}
x_i(s) &= \frac{\sum_f g_f(y_f)}{n_s} \quad \text{for all } i \in s & (11) \\
g_f'(y_f) &= \frac{1}{\sum_{s \in \mathcal{S}} \mu(s) u'(\frac{\sum_f g_f(y_f)}{n_s})}
\end{aligned}$$

Equation system (2) characterizes the competitive equilibrium allocation for all states of the world that occur with positive probability in equilibrium, where all active consumers choose round T. Since  $\sigma$  is of the form  $(0, 0, \dots, 0, s)$ , we can rewrite the competitive equilibrium allocation, which depends on  $\sigma$ , with an allocation that depends on the corresponding  $s$ . Therefore, we have the following relationship between  $\sigma$  and the corresponding  $s$ :  $\pi(\sigma) = \mu(s)$  and  $n(\sigma) = n_s$ . Definition (2.2), part i, and equation (2) imply

$$\begin{aligned}
x_i^{*T}(s) &= \frac{\sum_f g_f(y_f^*)}{n_s} \quad \text{for all } i \in s & (12) \\
g_f'(y_f^*) &= \frac{1}{\sum_{s \in \mathcal{S}} \mu(s) u'(\frac{\sum_f g_f(y_f^*)}{n_s})}
\end{aligned}$$

From (12), it follows that the competitive equilibrium allocation satisfies (11), and is therefore ex ante Pareto optimal. ■

**Remark 2.6:** The Pareto optimality of the competitive equilibrium constructed in Proposition (2.3) depends on the quasi-linear specification of the utility function. Quasi-linear utility implies that consumers are risk neutral with respect to numeraire consumption. If instead consumers were risk averse, then they would like to insure themselves against the risk associated with

random numeraire consumption. Our results concerning inefficient “jumping the gun” equilibria in section 3 are therefore all the more interesting, since they do not rely on missing insurance opportunities. Furthermore, the quasi-linear assumption is appropriate when commodity  $x$  represents a tiny portion of a consumer’s expenditure, so that income effects are negligible. The existence theorem does not rely on quasi-linearity in an essential way, and we conjecture that a similar proof can be applied when the utility function  $u(x,y)$  is smooth, strictly increasing and strictly concave, as long as the economy is “regular” in an appropriate sense.

### 3. Jumping the Gun

Proposition (3.1) below shows that, under a regularity condition ensuring that a consumer’s absence might not be detected by the market, and assuming that the cost of early arrival is sufficiently small, there exist competitive equilibria in which all active consumers choose round 1. To illustrate the importance that demand uncertainty plays, proposition (3.2) shows that, when the total number of active consumers is known with certainty, all competitive equilibria have all active consumers choosing round T.

**Proposition 3.1:** *Assume that for  $k = 1, \dots, N$ , we have  $\pi(n(\sigma) = k) > 0$ , where  $N$  is the number of potential consumers. Under Assumption 1 and our maintained assumptions, if the cost of arriving in round 1,  $c(1)$ , is sufficiently small, then there exists a competitive equilibrium in which all active consumers choose round 1.*

**Proof:** For  $f = 1, \dots, F$  and for  $k = 1, \dots, N$ , let  $\hat{y}_f$  and  $\hat{p}(k)$  be the unique solution to

$$u' \left( \frac{\sum_f g_f(\hat{y}_f)}{k} \right) = \hat{p}(k) \quad (13)$$

$$g'_f(\hat{y}_f) = \frac{1}{\sum_{k=1}^N \pi(n(\sigma) = k) \hat{p}(k)}$$

As shown for (2) in the proof of Proposition (2.3), we know a unique solution to (13) exists.

Construct a competitive equilibrium as follows:  $\eta_i^* = 1$  for all  $i$  and  $\dot{y}_f = \hat{y}_f$  for  $f = 1, \dots, F$ . For a given  $\sigma$ , let  $\tau$  be the first round in which a consumer arrives. That is,  $n^t(\sigma) = 0$  for  $t < \tau$  and  $n^\tau(\sigma) > 0$  holds. Equilibrium prices and supplies are as follows.

$$\begin{aligned} \text{For } t < \tau: \quad & \dot{p}^t(\sigma) = 0 \text{ and } \dot{q}_k^t(\sigma) = 0 \text{ for } f = 1, \dots, F. \\ \text{For } t = \tau: \quad & \dot{p}^t(\sigma) = \hat{p}(n^t(\sigma)) \text{ and } \dot{q}_k^t(\sigma) = \dot{y}_f \text{ for } f = 1, \dots, F. \\ \text{For } t > \tau: \quad & \dot{p}^t(\sigma) = 0 \quad \text{if } n^t(\sigma) = 0 \\ & \dot{p}^t(\sigma) = u'(0) \quad \text{if } n^t(\sigma) > 0, \text{ and} \\ & \dot{q}_k^t(\sigma) = 0 \text{ for } f = 1, \dots, F. \end{aligned} \quad (14)$$

The prices defined in (14) and conditions (i) and (ii) in definition (2.2) determine consumption bundles. We now check to see that the constructed prices and allocation form a competitive equilibrium. On the equilibrium path, consumers choose round 1, firms sell all their output in round 1, which is optimal given consumer behavior, and  $\dot{y}_f$  maximizes expected profit. Firms are also optimizing off the equilibrium path. Obviously, there is no incentive to sell before round  $\tau$ , since the price is zero. There is no incentive to save output for a later round, since with probability one, no more consumers will arrive and the price will be zero. Thus, condition (iv) of

definition (2.2) is satisfied. The measurability condition, (v), is clearly satisfied. Market clearing is satisfied, because firms supply zero when demand is zero before round  $\tau$ , supply equals demand in round  $\tau$ , and any consumer who arrives after round  $\tau$  faces the choke price, where demand is zero. We will be done if we can show that consumers are optimally choosing round 1.

To simplify the expressions, let  $\pi_k$  denote active consumer  $i$ 's probability that the number of active consumers is  $k$ :  $\pi_k = \pi(n(\sigma) = k | \text{consumer } i \text{ is active})$ . Then, by choosing round 1, consumer  $i$ 's utility *net of initial endowment*, conditional on  $n(\sigma) = 1$ , is given by

$$u(\sum_f g_f(y_f^*)) - [\sum_f g_f(y_f^*)]u'(\sum_f g_f(y_f^*)) - c(1) \quad (15)$$

and conditional on  $n(\sigma) > 1$ , is given by

$$(1/(1 - \pi_1)) \sum_{k=2}^N \pi_k \left( u(\sum_f g_f(y_f^*)/k) - [\sum_f g_f(y_f^*)/k]u'(\sum_f g_f(y_f^*)/k) - c(1) \right) \quad (16)$$

By choosing round T, the best deviation, consumer  $i$ 's utility, conditional on  $n(\sigma) = 1$ , is given by

$$u(\sum_f g_f(y_f^*)) - [\sum_f g_f(y_f^*)]u'(\sum_f g_f(y_f^*)) \quad (17)$$

and conditional on  $n(\sigma) > 1$ , is given by  $u(0)$ .

Concavity of  $u$  implies  $u(x) - u'(x)x > u(0)$  for  $x > 0$ . Setting  $x = \sum_f g_f(y_f^*)/k$ , we see that expression (16) is bounded above  $u(0)$  for  $c(1)$  sufficiently small. Expression (15) is arbitrarily close to expression (17) for  $c(1)$  sufficiently small. It follows that expected utility of choosing

round 1, a weighted average of (15) and (16), is greater than the expected utility of choosing round T, a weighted average of (17) and  $u(0)$ . ■

The intuition behind the proof is that all output will be sold in round 1 unless firms know that consumer  $i$  has deviated to round T, which only occurs when consumer  $i$  is the only active consumer. Deviating to round T saves  $c(1)$  when consumer  $i$  is the only active consumer, but sacrifices the gains from trade when consumer  $i$  is not the only active consumer. For sufficiently small  $c(1)$ , it is better to choose round 1. The assumption that  $\pi(n(\sigma)=k) > 0$  for  $k = 1, \dots, N$  simplifies the proof but is far too strong. A similar proof goes through as long as there is a positive probability that a consumer's absence in round 1 will not be missed, which occurs if  $\pi(n(\sigma)=k) > 0$  and  $\pi(n(\sigma)=k+1) > 0$  for some  $k$ . Then a consumer who deviates to round T will lose all gains from trade when  $n(\sigma)=k+1$ . When the consumer's absence is noticed,  $c(1)$  is saved but the consumption is unchanged. Notice also that proposition (3.1) holds without requiring the economy to be large.

**Proposition 3.2:** *If the cost of early arrival is positive and the total number of consumers is not random [i.e.,  $c(T-1) > 0$  and there is an  $\bar{n}$  such that  $n(\sigma) = \bar{n}$  for all  $\sigma$ ], then all active consumers choose round T in any competitive equilibrium.*

**Proof:** We will prove this proposition with a series of claims. First, define  $Q^t(\sigma)$  as the quantity of consumption unsold as of the beginning of period  $t$ :

$$Q^t(\sigma) \equiv \sum_f x_f - \sum_f \sum_{\tau=1}^{t-1} q_f^\tau(\sigma).$$

*Claim 1: Let  $\sigma$  occur with positive probability in a competitive equilibrium. Suppose that we have  $Q^t(\sigma) > 0$ ,  $n^t(\sigma) > 0$ ,  $n^T(\sigma) > 0$ , and that consumers are known not to arrive between round  $t$  and round  $T$  (that is,  $\eta_i^\tau = 0$  for all  $i$  and all  $\tau$  such that  $t < \tau < T$ ). Then the competitive equilibrium prices satisfy  $\dot{p}^t(\sigma) = \dot{p}^T(\sigma)$ .*

*Proof of Claim 1:* In round  $t$ , both  $n^t(\sigma)$  and  $n^T(\sigma)$  are known, and it is known that all consumption not sold in round  $t$  will be sold in round  $T$ , so  $\dot{p}^T(\sigma)$  is known.<sup>15</sup> If  $\dot{p}^t(\sigma) > \dot{p}^T(\sigma)$ , then for firms to be maximizing profits according to definition (2.2), condition (iv), we must have  $q_f^t(\sigma) = 0$  for all  $f$ . But then, to clear the market in round  $T$ , we must have  $\dot{p}^T(\sigma) = u'(0)$ , which implies  $\dot{p}^t(\sigma) > u'(0)$ . This, however, is inconsistent with market clearing in round  $t$ . If  $\dot{p}^t(\sigma) < \dot{p}^T(\sigma)$ , then we must have  $q_f^t(\sigma) = 0$  for all  $f$ . But market clearing in round  $t$  implies  $\dot{p}^t(\sigma) = u'(0)$ , which implies  $\dot{p}^T(\sigma) > u'(0)$ , a contradiction.

*Claim 2: If we have  $\eta_i^\tau = 0$  for all  $i$  and all  $\tau$  such that  $t < \tau < T$ , then in any competitive equilibrium,  $\eta_i^t = 0$  for all  $i$ .*

*Proof of Claim 2:* In round  $t$ , both  $n^t(\sigma)$  and  $n^T(\sigma)$  are known, and it is known that all consumption not sold in round  $t$  will be sold in round  $T$ , so  $\dot{p}^T(\sigma)$  is known. Suppose we have a

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<sup>15</sup> Even though the number of active consumers is known, consumers are free to choose any round, so there are many possible states of the world. We cannot yet drop the dependence of the price on  $\sigma$ .



state  $\sigma$ , occurring with positive probability at the competitive equilibrium, in which some consumer  $i$  chooses round  $t$ . For the trivial case in which  $Q^i(\sigma) = 0$ , consumer  $i$  is clearly better off choosing round  $T$  than round  $t$ . We now consider the various cases in which  $Q^i(\sigma) > 0$ . In case 1, where  $n^i(\sigma) > 1$  and  $n^T(\sigma) > 0$ , market clearing implies

$$u' \left( \frac{Q^i(\sigma) - Q^T(\sigma)}{n^i(\sigma)} \right) = \dot{p}^i(\sigma) \quad \text{and}$$

$$u' \left( \frac{Q^T(\sigma)}{n^T(\sigma)} \right) = \dot{p}^T(\sigma).$$

From Claim 1, it follows that

$$u' \left( \frac{Q^i(\sigma)}{n^i(\sigma) + n^T(\sigma)} \right) = \dot{p}^i(\sigma) = \dot{p}^T(\sigma) \quad (18)$$

holds for  $\sigma$  in case 1, and since we have  $n^i(\sigma) > 1$ , equation (18) also holds when consumer  $i$  chooses round  $T$ . Since prices in the two states are unaffected by consumer  $i$ 's round choice, and early arrival is costly, consumer  $i$  is better off choosing round  $T$  than round  $t$ , given  $\sigma$  in case 1.

In case 2, where  $n^i(\sigma) > 1$  and  $n^T(\sigma) = 0$ , firms cannot be holding consumption for round  $T$ , since the price in round  $t$  must be positive. Therefore, we have  $Q^T(\sigma) = 0$ . Market clearing in round  $t$  implies

$$u' \left( \frac{Q^i(\sigma)}{n^i(\sigma)} \right) = \dot{p}^i(\sigma). \quad (19)$$

If consumer  $i$  deviates to round  $T$ , and the resulting state is  $\sigma'$ , we have

$$u' \left( \frac{Q^t(\sigma') - Q^T(\sigma')}{n^t(\sigma) - 1} \right) = \dot{p}^t(\sigma') \quad \text{and} \quad (20)$$

$$u'(Q^T(\sigma')) = \dot{p}^T(\sigma').$$

Claim 1 applied to state  $\sigma'$  implies that  $\dot{p}^t(\sigma') = \dot{p}^T(\sigma')$ , so from (20) we derive

$$u' \left( \frac{Q^t(\sigma)}{n^t(\sigma)} \right) = \dot{p}^t(\sigma) = \dot{p}^T(\sigma). \quad (21)$$

From (19) and (21), we see that consumer  $i$  faces the same terms of trade, whether arriving in round  $t$  or round  $T$ ,  $\dot{p}^t(\sigma) = \dot{p}^T(\sigma)$ . Once again, consumer  $i$  is better off choosing round  $T$ .

In case 3, where  $n^t(\sigma) = 1$  and  $n^T(\sigma) = 0$ , then consumer  $i$  is the only remaining consumer. Consumption is  $Q^t(\sigma)$ , whichever round is chosen, so consumer  $i$  is better off choosing round  $T$ .

Finally, in case 4, where  $n^t(\sigma) = 1$  and  $n^T(\sigma) > 0$ , claim 1 implies

$$u' \left( \frac{Q^t(\sigma)}{1 + n^T(\sigma)} \right) = \dot{p}^t(\sigma) = \dot{p}^T(\sigma). \quad (22)$$

If consumer  $i$  deviates to round  $T$  and the resulting state is  $\sigma'$ , then we have

$$u' \left( \frac{Q^t(\sigma)}{1 + n^T(\sigma)} \right) = \dot{p}^T(\sigma'). \quad (23)$$

Again, the terms of trade faced by consumer  $i$  is the same in round  $T$  as in round  $t$ , so consumer  $i$  is better off choosing round  $T$  to save on the early arrival cost, establishing claim 2.

To complete the proof of proposition (3.2), iteratively apply claim 2. First, it follows that  $\eta_i^{T-1} = 0$  for all  $i$ . Given  $\eta_i^{T-1} = 0$  for all  $i$ , it follows that  $\eta_i^{T-2} = 0$  for all  $i$ , and so on. Therefore, in any competitive equilibrium, all consumers must be choosing round  $T$  with probability 1. ■

Proposition (3.1) does not rely on small numbers or market power. Essentially, the distribution of aggregate demand states must, with high probability, hide the absence of any particular consumer. Notice that, if a small number of consumers are exogenously required to choose round 2, inefficient jumping the gun remains a possibility. However, if a large number of consumers exogenously must choose round 2, then jumping the gun is inconsistent with equilibrium. To avoid the coordination failure documented here, we must do more than impose open markets; we must impose thick markets. Since agents are price takers and prices are well defined even in rounds when no trades occur in equilibrium, this coordination failure is conceptually distinct from those in the market games literature.<sup>16</sup>

**Remark 3.3:** Our maintained assumptions rule out the case of constant returns to scale, but all of our propositions can be shown to hold when all production functions exhibit constant returns and the Inada condition is replaced with a condition guaranteeing that some production will take place. Of course, firms with higher marginal costs must produce zero output in equilibrium, so the system of equations, (2), must be adjusted accordingly. Also, the aggregate output is determined uniquely, while the breakdown of output across firms could be indeterminate. To show that the coordination failure we document is consistent with constant returns, we offer the

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<sup>16</sup> For example, see Peck, Shell, and Spear [1992].

following example.

*Example:*

We will now construct an example which relies on a different construction from that of Proposition (3.1). In particular, the inefficiency is not only that trading occurs in the wrong round, but that trading can occur at the wrong prices as well. That is, marginal rates of substitution are not equated across consumers. Also, since all possible states of the world occur with positive probability at the competitive equilibrium, it is clear that the example does not depend on beliefs off the equilibrium path.

There are two rounds of trade and two potential consumers, each with the utility function defined as follows. For  $x \in [0,1]$ , let  $u(x) = x - x^2/2$ . This function is strictly increasing and strictly concave on the relevant domain.<sup>17</sup> (In partial equilibrium language, each active consumer has the demand curve  $D(p) = 1 - p$ .) The cost of arriving early is  $c$ ,  $c(1) = c$  and  $c(2) = 0$ . With probability  $1/2$ , nature selects one of the consumers (at random) to be active; with probability  $1/2$ , nature selects both consumers to be active. The number of firms is arbitrary, with each firm possessing the same constant-returns-to-scale production function,  $x_f = (1/a) y_f$ . The parameter,  $a$ , represents the marginal cost of production.

We look for a symmetric competitive equilibrium, in which each consumer, conditional on being active, chooses round 1 with probability  $\eta$  and round 2 with probability  $(1-\eta)$ . Thus, there are five possible arrival patterns, which at some abuse of notation we denote as follows:

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<sup>17</sup> In equilibrium, the possibility of consumption greater than 1 never comes into play, and it would be easy to complicate the function to satisfy the maintained assumptions for all  $x \geq 0$ .

arrival pattern	description: $(n^1, n^2)$	prior probability
$\sigma_1$	(1,0)	$\eta/2$
$\sigma_2$	(0,1)	$(1-\eta)/2$
$\sigma_3$	(2,0)	$\eta^2/2$
$\sigma_4$	(1,1)	$\eta(1-\eta)$
$\sigma_5$	(0,2)	$(1-\eta)^2/2$

Consumers, conditional on being active, believe that they are the only active consumer with probability  $1/3$  and that both consumers are active with probability  $2/3$ . Let  $x$  denote the aggregate quantity of output produced by firms.

Let us now consider the possible histories. If one consumer arrives in round 1,  $n^1 = 1$ , then only arrival patterns 1 and 4 are possible. Firms' posteriors are given by:

$$\pi(\sigma_1 | n^1 = 1) = 1/(3-2\eta) \quad \text{and} \quad \pi(\sigma_4 | n^1 = 1) = 2(1-\eta)/(3-2\eta).$$

For simplicity, we will look for an equilibrium in which the entire inventory is sold in round 1 when  $n^1 = 1$ . Then the following prices are market clearing:

$$p^2(\sigma_1) = 0 \quad p^2(\sigma_4) = 1 \quad \text{and} \quad p^1(\sigma_1) = p^1(\sigma_4) = 1 - x$$

For firms to be willing to sell all their output in round 1, the price must be at least as high as the expected price in round 2, so a necessary condition on  $x$  is:

$$1 - x \geq 2(1-\eta)/(3-2\eta).$$

If two consumers arrive in round 1, then all output is sold, so we have

$$p^1(\sigma_3) = 1 - x/2 \quad \text{and} \quad p^2(\sigma_3) = 0.$$

If no consumers arrive in round 1, then all output is carried over. Market clearing prices are

$$p^2(\sigma_2) = 1 - x, \quad p^2(\sigma_5) = 1 - x/2, \quad \text{and} \quad p^1(\sigma_2) = p^1(\sigma_5) = 0.$$

Constant returns to scale and profit maximization imply the zero profit condition,

$$x = 4(1-a)/[3 + 2\eta(1-\eta)].$$

To finish the construction of an equilibrium, we must find values for marginal production cost,  $a$ , the choice parameter,  $\eta$ , and the early arrival cost parameter,  $c$ , such that consumers are indifferent between choosing round 1 and round 2 (and therefore, willing to mix). When selecting round 1, a consumer's expected utility is:

$$W^1 = 1/3 [ x^2/2 ] + 2/3 [ \eta x^2/8 + (1-\eta) x^2/2 ] + \omega - c, \quad (24)$$

and when selecting round 2, her expected utility is:

$$W^2 = 1/3 [ x^2/2 ] + 2/3 [ (1-\eta) x^2/8 ] + \omega \quad . \quad (25)$$

The following parameters give rise to an equilibrium in which  $x = 1/2$ :  $a = 9/16$ ,  $\eta = 1/2$ , and  $c = 1/24$ .<sup>18</sup>

#### 4. Concluding Remarks

Proposition (3.1) and our example rely on the round T market being thin. When early arrival is costly we conjecture that, as the number of replications approaches infinity, in all symmetric competitive equilibria in which  $\eta_i = \eta_j$  for all consumers  $i$  and  $j$ , then  $\eta_i^t \rightarrow 1$  for some  $t$ . In other words, there can only be one round whose market is thick. Our intuition is that prices for the first round  $t$  with  $\eta_i^t > 0$  fully reveal the aggregate demand state, which implies that prices must be constant (on the equilibrium path) from that point onward. But if there is another round with a thick market, a consumer choosing round  $t$  could instead choose the later round, without affecting prices and reducing the early arrival cost,  $c(t)$ .

The model should be extended to allow for heterogeneous utility functions, which would require several modifications. First, the definition of equilibrium should reflect the fact that relevant histories include the number of consumers as well as their trades. Our restriction on the beliefs of firms, based on  $\epsilon$ -trembles (see definition (2.1)), is no longer compelling with heterogeneity. Then perhaps one should use a refinement based on who is most likely to deviate. If no refinement on beliefs is specified, it is clear that existence of efficient and inefficient

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<sup>18</sup> A slightly more complicated equilibrium can be constructed in which some output is carried over when one consumer shows up in round 1. For this example, we have:  $a = 1/2$ ,  $\eta = 1/2$ ,  $c = 11/338$ ,  $x = 8/13$ , and inventory carried over when  $n^1 = 1$  is  $1/13$ .

equilibria will be easy. Our goal here is to present a new sort of coordination problem that may lead to “jumping the gun” in competitive markets, but which is not based upon a questionable refinement nor on a framework where “almost anything” can happen.



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