Regulating Nonlinear Environmental Systems under Knightian Uncertainty

William Brock[1] and Anastasios Xepapadeas[2]

1. Introduction

Joe Stiglitz has recently been very active in questioning standard approaches to policy making such as the "Washington Consensus" and thinking about alternatives to it (Stiglitz, April 1998; October 1998). We think it is fair to say that his critiques have stimulated valuable controversy and debate. Prestigious economists have weighed in on both sides of the debate. The reader will find much discussion of this issue in a few minutes of search on the Internet. Joe's paper (Stiglitz, April 1998) examines financial instabilities and the role played by incomplete and imperfect capital markets during financial liberalizations. Economists argue vigorously about the relative roles played by market imperfections and government imperfections in causing financial instabilities. Joe's Prebisch Lecture (Stiglitz, October 1998) goes further and not only challenges much of conventional development policy but also proposes rather major modifications. One analytical and potentially econometrically tractable way of thinking about such disputes is to use concepts of scientific model uncertainty.

An intelligent policy maker might operate in the face of scientific model uncertainty by using concepts from econometrics like Bayesian Model Averaging coupled with recent advances in decision theory such as modelling "Knightian Uncertainty". This approach was taken by Brock and Durlauf (2001) in an attempt to constructively critique policy applications of empirical growth analysis and to suggest a modified approach that is still empirically disciplined. The idea is to first objectively represent the amount of scientific uncertainty in what we can learn from empirical exercises when there are levels of uncertainty present such as theory uncertainty and model uncertainty above and beyond the usual sampling uncertainty in parameter estimates for a given model. This "true" amount of uncertainty is typically larger than representations of uncertainty in conventional econometric studies. Second, given levels of uncertainty that must be faced by the policy maker, the policy maker should indulge in "robust" policy making that appropriately makes some attempt to hedge against worst cases as well as maximize the usual estimated net benefit.

In this paper we illustrate how the conceptualization of Knightian Uncertainty can be applied to the classic problem of regulating human impacted ecosystems and how it can lead to a type of precautionary principle. Joe has written extensively in the environmental area. For example, we believe that a version of Joe's pair of classical papers on growth and exhaustible resources (Stiglitz (1974a,b)) could be extended to include stochastic shocks and model uncertainty about the impact of human activities upon the regenerative power of the ecosystem as well as uncertainty about the elasticity of substitution between ecosystem inputs and human produced inputs into the economic process. In such an extension of Joe's work, one could develop a policy analysis framework under Knightian Uncertainty which could lead to potentially useful conceptions of macrogrowth precautionary principles as well as useful insights into the interaction among uncertainties in different parts of the system.

An example of what this approach might look like is Pizer (1996), except that we would add uncertainty about the elasticity of substitution between inputs and, especially, nonlinear regeneration dynamics for the ecosystem which allow multiple stable states for appropriate parameter values. In this way Pizer's Bayesian analysis would allow data to speak to these uncertainties as well as the uncertainties that he models. We believe this kind of analysis would help explain which uncertainties matter the most and how scientific resources should be allocated across attempts to reduce uncertainties. Our current paper makes a very modest start on this challenging project by considering optimal management of a human impacted ecosystem under deterministic nonlinear ecosystem dynamics under Knightian Uncertainty.

The analysis of dynamic environmental systems where the accumulation of pollutants cause environmental damages, such as phosphorus in a lake, greenhouse gasses in the atmosphere, or acid deposit in soils, has received extensive attention in the literature of environmental and resource economics.[3] In these environmental problems many agents (e.g. countries, firms, farmers) contribute through their individual actions (e.g. emissions) to the accumulation of the pollutant stock, and the damages caused by the pollutant have global characteristics; that is, they affect all agents involved in the problem. In the analysis of environmental systems with the above characteristics, the natural system is described by linear, in most cases, or nonlinear transition equations describing pollutant accumulation. Uncertainty has also been introduced into this framework by modeling the natural system through Ito stochastic differential equations and application of the expected utility hypothesis.[4]

In this paper we consider an environmental system where many agents contribute to the accumulation of a pollutant with global characteristics. We analyze cooperative and noncooperative solutions under uncertainty which is associated with the process of pollution accumulation. We allow for the presence of nonlinear feedbacks in the natural system which could result in multiple steady state equilibria. The novelty in our approach lies in that: (i) we seek to explore situations where there is a potential heterogeneity in risk aversion between a regulator, acting as a Stackelberg leader that seeks to implement a cooperative solution, and individual agents that behave in a noncooperative way; and (ii) we analyze the implications of this heterogeneity for regulation when nonlinear dynamics could steer the dynamic system towards alternative basins of attraction.

Heterogeneity in risk aversion is a possibility that appears once we start considering as a possible way to model uncertainty the ideas of the ”least favorable prior” decision theory (Gilboa and Schmeilder, 1989) which results in the use of maximin expected utility theory. Sims (2001), for example, hints at this heterogeneity by indicating that the same maximin criterion should not be imposed on private agents and optimizing policy makers.

In this paper heterogeneity in risk aversion is introduced in the following way. In the cooperative solution, the regulator faces Knightian uncertainty which is of the *e*-contamination type. That is, the regulator is uncertainty averse (first-order risk averse) while at the noncooperative solution the agents are risk averse (second-order risk averse).

This heterogeneity could be defended along different lines. A regulator could face a dynamical system with at least two different time scales with unobservables at a slow time scale that can cause bifurcations. These unobservable slow moving dynamics may or may not be influenced by responses of the regulatees to controls chosen by the regulator. In any event, these unobservable dynamics can cause flips to undesirable steady states,[5] which hurt the regulator's objective. It would be interesting but complex to formalize this interaction between time scales, unobservable slow moving bifurcational dynamics, multiple stable states in the fast moving dynamics and the regulator's information set being coarser than the regulatees' information set which influences the dynamics that the regulator is attempting to control. In this paper we abstract away this complexity by positing that the regulator, a Stackelberg leader, views his/her problem as facing a regulatory objective that is *e*-contaminated Knightian.

In a problem of global pollution a regulator that seeks to implement a cooperative solution under uncertainty could face divergent beliefs and not consensus, on the part of the agents involved in the global problem, regarding the natural structure and the dynamics of the system as well as its behavior under alternative policy shocks. This however implies that if the regulator can not impose his/her own beliefs, then he/she faces a problem of choice under Knightian uncertainty and can be considered as uncertainty averse.[6] On the other hand the individual agents acting noncooperatively do not need to face divergent beliefs; they choose by maximizing subjective expected utility and thus can be regarded as risk averse.

Another line of approach could be to consider the case where the regulator's employment contract and its incentive schedule are “as if” the regulator gets punished more severely if something unusual happens in response to his/her instrument choice than if something opposite in sign that is positive happens. To protect against this possibility the regulator could operate under the “least favorable prior”, implying uncertainty aversion.

Using an open loop - most rapid approach path (MRAP) concept[7] as the equilibrium concept for the noncooperative solution, we show that the deviation between the cooperative optimal steady state (OSS) and the noncooperative OSS can be broken down into two components: one which is due to the public bad externality of the global pollutant, while the other is due to the heterogeneity in risk aversion between the regulator and the agents. The second effect can be identified as a precautionary effect. Thus the regulatory instrument should account both for the public bad externality and the uncertainty aversion effect, which implies that under heterogeneity in the type of risk aversion regulation is more stringent. We also show that in the presence of multiple equilibria, market-based instruments such as taxes or tradable permits might have some difficulties in attaining the steady state chosen by the regulator because of hysteresis effects.

Finally we examine regulation when the regulator faces random shocks to the initial values of the regulated system that can move the system to an undesired basin of attraction. We derive the optimal regulation in a framework where the parameters of the *e*-contamination in Knightian uncertainty are *endogenized*. Thus, at a second level our paper contributes to the literature by having the *e*-contamination parameter and the implied worst case outcome *derived* from the underlying structure of the problem rather than imposed in a somewhat ad hoc matter which has been the most common way of handling Knightian uncertainty of the *e*-contamination type.

2. The Cooperative Solution

There are $i = 1, \ldots, n$ players (e.g. countries) that emit pollutant a_i per unit time with global effects. Gross benefits from a_i are pa_i where p is some fixed price (small countries, small players). The environmental cost of the accumulated pollutant for player i is $c_i(x)$, $c_i' > 0, c_i'' > 0$. The pollutant accumulates according to

$$\dot{x} = \sum_{i=1}^{n} a_i - bx + f(x),\ x(0) = x^0,\ x \in X \subset \Re_+ \qquad (1)$$

where $f(x)$ is a convex-concave function reflecting the nonlinearity associated with feedbacks of the natural system.

The cooperative problem assuming symmetric players is:

$$\max_a \int_0^\infty e^{-\rho t}[pa - nc(x)]dt,\ \sum_{i=1}^{n} a_i = a \text{ , s.t. (1) and } 0 \le a \le a^{\max} \qquad (2)$$

Equation (1) can be written as: $a = \dot{x} + bx - f(x)$. Substituting into (2), the problem can be rewritten in the MRAP formulation as:

$$\max \int_0^\infty e^{-\rho t}[p(bx - f(x)) - nc(x) + \rho px]dt \text{ s.t. } 0 \le x \le x^{\max} \qquad (3)$$

The cooperative OSS is determined by:

$$\max_x \; W(x, b) = \max_x \; [p(bx - f(x)) - nc(x) + \rho px] \tag{4}$$

The optimality condition is:

$$p(b + \rho - f'(x)) = nc'(x) \tag{5}$$

Suppose a solution x^c to (5) exists. Then the approach to the cooperative optimal steady state is according to

$$\begin{array}{ll} a = 0 & \text{if } x > x^c \\ a = bx^c - f(x^c) & \text{if } x = x^c \\ a = a^{\max} & \text{if } x < x^c \end{array} \tag{6}$$

Given the nonlinearity in transition equation (1), the first-order condition (5) might have more than one solution. Brock and Starrett (1999) determine conditions under which there are an odd number of solutions for (5). Therefore,

$$\text{if } -[pf''(x^c) + nc''(x^c)] \begin{array}{l} < 0 \\ > 0 \end{array} \text{ then } \begin{array}{l} x^c \text{ local maximum} \\ x^c \text{ local minimum} \end{array} \tag{7}$$

This is shown in figure 1. The OSSs are defined by the intersection of the *R* curve and the *C* curve. Local maxima are (x^c_{r1}, x^c_{r3}) and local minimum is x^c_{r2}.

[Figure 1]

If we assume an adjustment mechanism in the neighborhood of the OSS of the form $\dot{x} = \phi[p(b + \rho - f'(x)) - nc'(x)]$, $\phi > 0$ it is clear that since the slope at any equilibrium point x^c is given by (7), local maxima are locally stable equilibria, while local minima are locally unstable. The sign of $f''(x)$ depends on the curvature of the feedback function $f(x)$ in the neighborhood of an equilibrium point, while $c''(x) > 0 \; \forall \; x \in X$. It follows then, that some $\bar{n}$ exists such that for $n > \bar{n}$ (7) is negative for all *x*. In this case only one globally stable OSS exists. Therefore cooperation of many players acts as a stabilizer and could eliminate multiple equilibria. This is shown in figure 1 where the C_m curve drawn for $n_m > \bar{n}$ intersects *R* only once at *F* to define a unique OSS.

Suppose that the planner managing the cooperative solution faces Knightian uncertainty with regard to the parameters of the natural system. Assume that the uncertainty for *b* - that is, uncertainty about the self-cleaning process in a shallow lake where phosphorus accumulates, or about the CO_2 absorption capability of oceans - is of the *e*-contamination type

$$P(e) = (1 - e)b + em \ , \ m \in M(B)$$

where *b* represents a point mass of unity at *b* and *M* represents the entire set of probability measures with support [*b*-*B*,*b*+*B*]. Following Epstein and Wang (1994, p. 288, equations (2.3.1) and (2.3.2)) we shall assume the planner wishes to choose total emissions a to maximize $W(x, b)$. Then the *e*-contamination MRAP problem under Knightian uncertainty is to maximize

$$\int WdP(e) = (1 - e)W(b, x) + e\left[\inf \int W(x, b)dm\right] \tag{8}$$

It can be shown by using the envelope theorem in (4) that $W(x, b)$ is increasing in *b.* Since *M* contains all probability measures over *b* values with support [*b*-*B*,*b*+*B*], then (8) can be written as

$$\max_x \int WdP(e) = \max_x \ (1 - e)W(x, b) + eW(x, b - B) \tag{9}$$

The equivalent to optimality condition (5) under Knightian uncertainty is

$$p(b - eB + \rho - f'(x)) = nc'(x) \tag{10}$$

By comparing (5) to (10) the following result can be stated.

Proposition 1 *Under uncertainty aversion the regulator is first-order risk averse for all the local maxima* $x^c(e)$ *of the welfare maximization problem (8), and it holds that* $\frac{dx^c(e)}{de} < 0$. *Under risk aversion the regulator is second-order risk averse and* $\frac{dx^c(e)}{de} = 0$. *For proof see Appendix.*

In terms of figure 1 the solution for the first-order risk averse regulator is given by the intersections of the *U* curve with the *C* curve. Local maxima are (x_1^c, x_3^c) and local minimum is x_2^c. On the other hand, the solution for a second order risk averse regulator is given by the intersections of the *R* curve and the *C* curve. The deviations $x_{1r}^c - x_1^c$ and $x_{3r}^c - x_3^c$ can be characterized as the reductions in the socially-optimal steady state for the accumulation of the pollutant due to precautionary effect.

It can also be noted in the same figure that if *e* or *B* are sufficiently large such that $p(b - eB + \rho - f'(x))$ shifts further down like curve U_1 then there is only one globally stable OSS for the uncertainty averse regulator, at a. In this case uncertainty aversion eliminates multiple equilibria, and directs the system towards the smallest concentration of the pollutant. This effect is not, however, present when the regulator is second-order risk averse.

3. The Noncooperative Solution

In the noncooperative case each player (country) *i* maximizes its own payoff given the best response $\bar{a}_j$ of the rest of the $j \neq i$ players. Thus in the deterministic case each player solves the open loop problem

$$\max_{a_i} \int_0^\infty e^{-\rho t}\left[pa_i - c(x)\right]dt$$
$$s.t.\ \dot{x} = a_i + \sum_{j \neq i}^{n} \overline{a}_j - bx + f(x)\ ,\ 0 \le a_i \le a_i^{\max} \tag{11}$$

Because players are symmetric, a_i is the same for all *i.* Using the MRAP formulation the noncooperative OSS is determined under symmetry by

$$\max_x W^i(x, b) = \max_x \left[p\left(bx - \sum_{j \neq i}^{n} \overline{a}_j - f(x)\right) - c(x) + \rho px\right]\ \forall i$$

and the optimality condition is

$$p(b + \rho - f'(x)) = c'(x) \tag{12}$$

If a solution x^n to (12) exists, then the approach to the noncooperative locally stable OSS follows a MRAP. Furthermore, as in the cooperative case, local maxima are locally stable and local minima are locally unstable.

By comparing (5) to (12) the public bad-type of externality characterizing the global pollutant can be easily identified through the $nc'(x)$ term. Then the well known result that the pollutant accumulation at the noncooperative solution exceeds the pollutant accumulation at the cooperative solution immediately follows. As shown in figure 1, where the two solutions are compared, the *C* curve shifts down to the *N* curve for the noncooperative case, and the solution is determined by the intersection of the *N* curve with the *R* curve.

For the noncooperative game, under uncertainty, the solution of the *e*-contaminated problem is characterized by the optimality condition $p(b - e_1 B + \rho - f'(x)) = c'(x)$. It is clear that if $e \neq e_1$, that is the *e*-contamination parameters are different between the players and the regulator, then a new discrepancy is introduced because the $p(b - eB + \rho)$ part of the optimality condition will not be the same in the two solutions. Thus the *U* curve is different for the two problems in figure 1, since the *e*-contamination parameters are different between the cooperative and the noncooperative solutions.

If the players are risk averse, with $\tilde{b} = b + e\omega$ where ω is a random variable with zero mean and finite variance, then each player solves the problem $\max_x EW^i(x, b + e\omega)$ with FONC

$p(b + \rho - f'(x)) = c'(x)$.

The deviation between the cooperative and the noncooperative solutions, as shown in figure 1 for the two locally stable OSSs, is $x_3^n - x_3^c$, $x_1^n - x_1^c$. This deviation can be broken into two parts which can be attributed to two different sources:

1. The public bad externality *PB* = $(x_1^n - x_{r1}^c)$ or $(x_3^n - x_{r3}^c)$ due to the shift of the *C* curve to the *N* curve.
2. The uncertainty aversion effect *U* = $(x_{r3}^c - x_3^c)$ or $(x_{r1}^c - x_1^c)$ due to the shift of the *R* curve to the *U* curve. This effect can be identified as a precautionary effect stemming from the fact that the regulator is uncertainty averse with respect to the values of the natural system.

Under these conditions regulation that seeks to attain the socially-optimal outcome, as this outcome is determined under uncertainty aversion, should correct not only for the public bad externality which is the standard approach in a global pollution problem, *but also* for the uncertainty aversion effect. This effect is induced by the fact that while the regulator managing the cooperative solution exhibits first-order risk aversion, the individual players determining the noncooperative solution exhibit second-order risk aversion.

4. Regulation

Given the discrepancy between the cooperative and the noncooperative OSS, the regulator seeks to implement the cooperative OSS by introducing a regulatory instrument. Assume that the regulator uses a linear tax τ on emissions to implement x^c. Then, using the MRAP formulation, the noncooperative OSS under regulation is determined as

$$\max_x W^i(x, b, \tau) = \max_x \left[(p - t)\left(bx - \sum_{j \neq i}^{n} \overline{a}_j - f(x)\right) - c(x) + \rho(p - t)x\right]$$

The FONC imply that the optimal τ should be chosen so that

$$(p - \tau)(b + \rho - f'(x)) - c'(x) = 0 \text{ implies } x = x_i^c, i = 1,3 \qquad (13)$$

The tax impact is determined in the following proposition.

Proposition 2 *Let x_i^n be an unregulated noncooperative steady state (τ= 0). Then $\frac{dx_i^n}{d\tau} < 0$ if x_i^n is locally stable (local maximum), and $\frac{dx_i^n}{d\tau} > 0$ if x_i^n is locally unstable (local minimum). For proof see Appendix.*

[Figure 2]

Assume that the regulator wants to implement x_1^c. It is clear from (13) that an increase in the tax rate τ will shift the *R* curve defined by $(p - \tau)(b + \rho - f'(x))$ downward. The purpose is to shift the *R* curve downward to R_τ (Figure 2) so that it intersects the *N* curve at the point *E* corresponding to x_1^c. If the initial noncooperative steady state was x_1^n, then the regulator's steady state is attained in a straightforward way. The reduction $x_1^n u$ is attributed to the correction for the uncertainty aversion effect while the reduction ux_1^c is attributed to the correction for the public bad externality. If the initial noncooperative steady state was x_3^n, then there could be complications. The tax should shift the *R* curve sufficiently to eliminate the basin *JK* that attracts the system to x_3^n. When this basin shrinks to zero, the system is attracted to x_1^c, which is the only stable equilibrium. However, if the *JK* basin is large enough so that when it has been eliminated the lower branch of the R_τ curve intersects the *N* curve to the left of *E* as does the curve $R_{\tau 1}$ in figure 2, then the system is attracted to a steady state which is *below* the desired one. To bring the system back to *E* , the tax needs to be reduced so that the curve $R_{\tau 1}$ shifts upward to R_τ. The fact that we need to raise the tax beyond the desired point and then reduce it is a hysteresis effect resulting from the nonlinearity of the transition equation of the natural system.[8] This discussion suggests that in the presence of hysteresis effects a command-and-control regulation that sets a nontransferable limit might be more effective in implementing the desired steady state.

4.1 Regulation under Large Rare Shocks

The above results imply that in the presence of multiple locally stable steady states, regulation design depends on the specific basin of attraction where the system is slaved. Once the regulator knows the basin of attraction of the system, then the regulation discussed in the previous sections applies. The regulator might however be uncertain of the system's basin of attraction. This is because the specific basin of attraction depends on initial conditions which in cases of a natural system could very well be subjected to large rare random shocks which can move them from one basin of attraction to the other. It is clear that optimal regulation should take into account such an event.

Let *q* denote the probability that a large shock moves the initial value $x(0) = x^0$ of our system to the high pollutant accumulation basin of attraction. In order to expose the effects of large rare shocks in a clearer way, assume that *B*=0 in (9) so that there is no uncertainty regarding the natural parameter *b* of the system. Then the optimal regulation problem for the regulator is to determine an optimal tax τ^* such that

$$\tau^* = \arg\max \left[(1 - q)W\left(x_1^{\,n}(\tau)\right) + qW\left(x_3^{\,n}(\tau)\right)\right] \tag{14}$$

$x_i^{\,n}(\tau), i = 1,3$ is a solution of (13) and *W* is defined by (4).

Proposition 3 For an optimal tax that solves (14), $\dfrac{d\tau^*}{dq} > 0$.

For proof see Appendix.

Thus the regulator will react to an increase in the probability that a random shock might move the system to a “bad” basin of attraction by increasing the optimal tax.

Problem (14) can be interpreted in a way that is very close to the *e*-contamination formulation of Knightian uncertainty (9) with $q = e$. In the Knightian formulation (9) the second term has been transformed to reflect the worst case scenario which is the worst possible value that Nature can choose for *b.* In (14) the worst possible choice of Nature would be to shock the initial condition in such a way that the system moves to the high pollutant accumulation basin of attraction and converges eventually to the high pollutant accumulation steady state $x_3^{\,n}$.

Thus (14) can be regarded as an *e*-contamination formulation of Knightian uncertainty regarding the basin of attraction of the system, with the *e*-contamination parameter being the probability that Nature would choose the worst possible case. This way we provide a straightforward interpretation of the *e*-contamination parameter. This parameter can even be endogenized if we take into account that the probability of the system ending in the high pollutant accumulation basin of attraction can be affected by the choice of the optimal tax τ.

Assume that initially the system is in the basin of attraction of the low pollutant accumulation steady state. Let the timing be such that the regulator chooses τ and Nature adds a random shock. The system will jump to the basin of attraction of the high pollutant accumulation steady state if the shock is such that the initial value passes to the right of point *J* in figure 2, which is the locally unstable equilibrium that separates the two locally stable basins of attraction. However, by setting a tax, the regulator affects the position of this basin-separation point, since increasing that tax reduces the high pollutant accumulation basin of attraction *JK.* In a situation like this the regulator could have two alternative courses of action. One is to choose a tax so that the probability of a shock moving the system to a high pollutant accumulation basin of attraction is zero. The second is to choose the tax by optimally taking into account the effect of the tax on the basin separation point.

To make the probability of the system being shocked to a high pollutant accumulation basin of attraction zero, the high pollutant accumulation steady state should be eliminated. This means that the tax should be chosen so that curve $R_{\tau 1}$ in figure 2 shifts downward until the point where the intersections at *J* and *K* are eliminated. The following proposition defines this tax.

Proposition 4 *Let* $\bar{\tau}$ *be a tax rate such that*

$(p - \bar{\tau})(b + \rho - f'(x(\bar{\tau}))) - c'(x(\bar{\tau})) = 0$ *has two solutions:*

$$x_1(\bar{\tau}) \;:\; pf''(x_1(\bar{\tau})) + c''(x_1(\bar{\tau})) > 0$$
$$x_2(\bar{\tau}) \;:\; pf''(x_2(\bar{\tau})) + c''(x_2(\bar{\tau})) = 0$$

Then for any $\tau > \bar{\tau}$ *the high pollutant accumulation basin of attraction is eliminated and the regulated system has only one steady state with low pollution accumulation.*

For Proof see Appendix.

The tax rate $\bar{\tau}$ can be obtained iteratively by gradual increases until the point of tangency between the *N* curve and the $R_{\tau 1}$ is reached.

To determine the optimal tax rate by taking into account the effect of the tax on the basin separation point let $F_S(z) = \Pr[S > z]$ be the cumulative distribution function of a random shock *S* . If $x^0 + S < x_2^n(\tau)$ where $x_2^n(\tau)$ is the locally unstable steady state of the regulated system, corresponding to point *J* in figure 2, then the system converges to the locally stable low pollutant accumulation steady state $x_1^n(\tau)$ corresponding to point *H* in figure 2. If $x^0 + S > x_2^n(\tau)$ then the system converges to the locally stable high pollutant accumulation steady state $x_3^n(\tau)$ corresponding to point *K* in figure 2. Then the optimal taxation problem is defined as

$$\max_{\tau} \left[F_S\left(x_2^n(\tau)\right)W\left(x_1^n(\tau)\right) + \left(1 - F_S\left(x_2^n(\tau)\right)\right)W\left(x_3^n(\tau)\right)\right] \qquad (15)$$

with FONC

$$F_S'\left(x_2^{\,n}(\tau)\right)\frac{dx_2^{\,n}(\tau)}{d\tau}\left[W\left(x_1^{\,n}(\tau)\right)-W\left(x_3^{\,n}(\tau)\right)\right]+F_S\left(x_2^{\,n}(\tau)\right)\frac{\partial W\left(x_1^{\,n}(\tau)\right)}{\partial x_1^{\,n}}\frac{dx_1^{\,n}(\tau)}{d\tau}$$
$$+\left(1-F_S\left(x_2^{\,n}(\tau)\right)\right)\frac{\partial W\left(x_3^{\,n}(\tau)\right)}{\partial x_3^{\,n}}\frac{dx_3^{\,n}(\tau)}{d\tau}=0 \qquad (16)$$

Comparing (15) to (14) and (9) it is clear that the *e*-contamination parameter is now determined by $1-F_S\left(x_2^{\,n}(\tau)\right)$ and it has been endogenized since the probability that Nature will choose the high accumulation basin of attraction depends on the optimal tax choice. In (16) it can be seen that in the FONC the first term reflects the marginal effect of a tax change on the probability that the shock will take the system to the high pollutant accumulation basin of attraction which is $F_S'\left(x_2^{\,n}(\tau)\right)\frac{dx_2^{\,n}(\tau)}{d\tau}$ weighted by the difference in welfare between the low and the high pollutant accumulation which is $W\left(x_1^{\,n}(\tau),b\right)-W\left(x_3^{\,n}(\tau),b-B\right)$.

5. Concluding Remarks

This paper introduces a new framework of analysis of environmental regulation issues, using a non-linear representation of the natural system, where there is heterogeneity in risk aversion between regulator and regulatees, the regulator being first-order risk averse (uncertainty averse) facing Knightian uncertainty of the *e*-contamination type, while the regulatees are second-order risk averse.

We are able to identify a precautionary effect in addition to the public bad externality effect contributing to the deviation between cooperative and noncooperative solutions. The precautionary effect is induced by risk aversion heterogeneity. The first-order risk averse regulator should choose policy instruments in a way that allows for both the precautionary effect and the public bad effect.

Nonlinearities and multiplicity of basins of attraction reduce the effectiveness of market-based instruments such as taxes or tradable emission permits. Because of an hysteresis effect, the achievement of the cooperative solution could require setting taxes initially below the optimal level and then changing them to move towards their optimal level.

Finally we consider regulation under large rare shocks that could move the system to an undesirable basin of attraction. In this case the regulator can choose taxes optimally by taking into account the effects of the tax choice on the probability that the system will move to an undesirable basin of attraction. In this way we obtain an endogenization of the *e*-contamination parameter of Knightian uncertainty, which is an advance relative to the ad hoc way in which this parameter has been chosen up to now.

In this paper we examined only one possible combination of risk aversion heterogeneity and game form between the regulator and the regulatees in a nonlinear system, namely the one in which the regulator is first-order risk averse and leads, while the regulatees are second-order risk averse and follow. This seems to be the most appropriate choice for the specific environmental problem. Different types of regulation problems could fit different combinations of risk aversion and game forms. This implies that our methodological approach, by allowing for heterogeneity in risk aversion and nonlinear dynamics, could lead to a more realistic analysis of general classes of regulation under uncertainty. It should be noticed that the endogenization of the *e*-contamination parameter of Knightian uncertainty can also be used as a general approach for analyzing regulation under rare shocks in nonlinear systems.

Appendix

Proof of Proposition 1: Taking the total derivative of (10) with respect to *e* we obtain

$$\left[pf''\left(x^c(e)\right) + nc''\left(x^c(e)\right)\right] \frac{dx^c(e)}{de} = -pbB < 0$$

Since $pf''\left(x^c(e)\right) + nc''\left(x^c(e)\right) > 0$ at a local maximum it follows that $\frac{dx^c(e)}{de} < 0$.

Consider now a risk averse regulator and a mean preserving spread around *b*, to be $e\omega$ where ω is a random variable with zero mean and finite variance. The OSS for the regulator is determined as the solution of the expected welfare maximization problem $\max_x EW(x, b + e\omega)$ with FONC

$$E\left[p(b + e\omega + \rho - f'(x)) - nc'(x)\right] = p(b + \rho - f'(x)) - nc'(x) = 0.$$

The FONC does not change due to the linearity of $p(b + ew)$ and $Ew = 0$. Thus $\frac{dx^c(e)}{de} = 0$ trivially. QED

Proof of Proposition 2: At a steady state $b + \rho - f'(x) > 0$ because $c'(x) > 0$ in (13). Totally differentiating the FONC we obtain $\frac{dx}{d\tau} = -\frac{b + \rho - f'(x)}{\left[(p - \tau)f''(x) + c''(x)\right]}$. Evaluating this in the neighborhood of $\tau = 0$ we obtain that $\frac{dx_i^n}{d\tau} < 0 \ (> 0)$ if x_i^n is locally stable (unstable). QED

Proof of Proposition 3: The optimal regulation is defined as $\max_\tau G(\tau) = \max_\tau \ (1 - q)W\left(x_1^n(\tau)\right) + qW\left(x_3^n(\tau)\right)$ with FONC

$$(1-q)\{p[(b+\rho)-f'(x_1^n(\tau))]-nc'(x_1^n(\tau))\}\frac{dx_1^n(\tau)}{d\tau}+$$
$$q\{p[(b+\rho)-f'(x_3^n(\tau))]-nc'(x_3^n(\tau))\}\frac{dx_3^n(\tau)}{d\tau}=0 \tag{17}$$

Assume that second-order conditions are satisfied so that $\frac{\partial^2 G}{\partial \tau^2}=G_{\tau\tau}<0$. At $q=0$ we have, since $\left(\frac{dx_1^n(\tau)}{d\tau},\frac{dx_3^n(\tau)}{d\tau}\right)<0$ from Proposition 2, that

$$\frac{\partial W(x_1^n(\tau),b)}{\partial x_1^n}=p[(b+\rho)-f'(x_1^n(\tau))]-nc'(x_1^n(\tau))=0 \tag{18}$$

The existence of a solution for this equation requires, from the implicit function theorem, that $-p[f''(x_1^n(\tau))+nc''(x_1^n(\tau))]\neq 0$ which is true for a local maximum. In this case $x_1^n(\tau^*)=x_1^c$. In the same way we can prove the existence of a solution for the optimal regulation problem for $q=1$ with $x_3^n(\tau^*)=x_3^c$. Using (17) we obtain:

$$\frac{d\tau^*}{dq}=-\frac{1}{G_{\tau\tau}}\left[(1-q)\frac{\partial W(x_1^n(\tau),b)}{\partial x_1^n}\frac{dx_1^n(\tau)}{d\tau}+q\frac{\partial W(x_3^n(\tau),b)}{\partial x_3^n}\frac{dx_3^n(\tau)}{d\tau}\right]$$

Evaluating at $q=0$ we obtain $\frac{d\tau^*}{dq}<0$ because: (i) $G_{\tau\tau}<0$ by second-order conditions, (ii) $\frac{\partial W(x_1^n(\tau),b)}{\partial x_1^n}=0$ from (18), (iii) $\frac{dx_3^n(\tau)}{d\tau}<0$ from proposition 2 and (iv) $\frac{\partial W(x_3^n(\tau),b)}{\partial x_1^n}>0$ because $x_3^n(\tau)>x_1^n(\tau^*)=x_1^c$. QED

Proof of Proposition 4: For $\tau=\bar{\tau}$ the system has one hyperbolic equilibrium point at $x_1(\bar{\tau})$ which is locally stable and a nonhyperbolic equilibrium point at $x_2(\bar{\tau})$. The nonhyperbolic point is a point of tangency of the *N* curve with the $R_{\tau 1}$ curve. For any $\tau>\bar{\tau}$ the nonhyperbolic point is eliminated, the $R_{\tau 1}$ curve shifts further to the right and there is only one globally stable point of low pollutant accumulation. QED.

References

Brock, W. "A Polluted Golden Age," in V.L. Smith (Ed.) *Economics of Natural and Environmental Resources*, New York: Gordon & Breach, 1977.

Brock, W. and S. Durlauf. "Growth Empirics and Reality." *The World Bank Economic Review*, 15(2)(2001): 229-272.

Brock, W. and D. Starrett. "Nonconvexities in Ecological Management Problems." *SSRI Working Paper* #2026, 1999.

Carpenter, S., W. Brock, and P. Hanson. "Ecological and Social Dynamics in Simple Models of Ecosystem Management." *Conservation Ecology*, 1999, online at: http://www.consecol.org/journal/vol3/

Dasgupta, P.S. *Control of Resources*, Oxford: Basil Blackwell, 1982.

Epstein, L. and T. Wang. "Intertemporal Asset Pricing under Knightian Uncertainty." *Econometrica* 63 (1994):283-322.

Forster, B.A. "Optimal Capital Accumulation in a Polluted Environment." *Southern Economic Journal* 39 (1973): 544-7.

Gilboa, I. and D. Schmeilder. "Maxmin Expected Utility with Non-Unique Prior." *Journal of Mathematical Economics* 18 (1989): 141-53.

Ludwig, D., R. Hilborn and C. Walters. "Uncertainty, Resource Exploitation and Conservation: Lessons from History." *Science* 260 (1993): 17-36.

Maler, K.-G. *Environmental Economics: A Theoretical Inquiry*, Baltimore: Johns Hopkins University Press, 1974.

Nordhaus, W. "Expert Opinion to Climatic Change." *American Scientist* 82(1994): 45-52.

Plourde, G. and D. Yeung. "A Model of Industrial Pollution in a Stochastic Environment." *Journal of Environmental Economics and Management* 16 (1989): 97-105.

Pizer, W. *Modelling Long-Term Policy under Uncertainty*, Ph.D. Thesis, Department of Economics, Harvard University, 1996.

Sims, C. "Pitfalls to Minimax Approach to Model Uncertainty." Website, Princeton University Dept. of Economics, 2001.

Stiglitz, J. "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth." *Review of Economic Studies* 41(1974a): 123-137.

Stiglitz, J. "Growth with Exhaustible Natural Resources: The Competitive Economy," *Review of Economic Studies* 41(1974b): 139-152.

Stiglitz, J. "Towards a New Paradigm for Development: Strategies, Policies, and Processes," 1998 Prebisch Lecture at UNCTAD, Geneva, October 19, 1998.

Stiglitz, J. "Knowledge for Development: Economic Science, Economic Policy, and Economic Advice." Paper prepared for the Annual World Bank Conference on Development Economics, Washington, D.C., April 20-21, 1998.

Tahvonen, O. and J. Kuuluvainen. "Economic Growth, Pollution, and Renewable Resources." *Journal of Environmental Economics and Management* 24(1993): 101-18.

Woodward, R. and R. Bishop. "How to Decide when Experts Disagree: Uncertainty-Based Choice Rules in Environmental Policy."

Land Economics 73(1997): 492-507.

Xepapadeas, A. "Environmental Policy Design and Dynamic Nonpoint-Source Pollution." *Journal of Environmental Economics and Management* 23(1992a): 22-39.

Xepapadeas, A. "Optimal Taxes for Pollution Regulation: Dynamic, Spatial and Stochastic Characteristics." *Natural Resource Modelling* 6(1992b): 139-170.

---

[1] Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison Wisconsin, wbrock@ssc.wisc.edu. William Brock thanks NSF Grant SES-9911251 and the Vilas Trust.

[2] Department of Economics, University of Crete, University Campus, Rethymno 74100 Greece, xepapad@econ.soc.uoc.gr. Anastasios Xepapadeas thanks the Secretariat for Research, University of Crete, Research Program 1266.

[3] See for example Forster (1973), Maler (1974), Brock (1977), Dasgupta (1982), Tahvonen and Kuuluvainen (1993).

[4] For example Plourde and Yeung (1989), Xepapadeas (1992a, 1992b).

[5] See for example Carpenter, Brock, and Hanson (1999).

[6] Divergence of experts' beliefs regarding the implications of global warming has been established in Nordhaus (1994). Woodward and Bishop (1997) model choices under expert disagreement as a problem of choice under pure uncertainty. Ludwig, Hilborn and Walters (1993) argue that consensus is not possible among experts in ecosystem management which also implies that the regulator might act as a Knightian.

[7] We doubt that the use of an alternative equilibrium concept such

as the closed loop (feedback-subgame perfect) changes the substantive conclusions or the methodological advances that we are developing in this paper.

[8] A similar result can be shown to hold with tradable permits regulation under competitive markets.

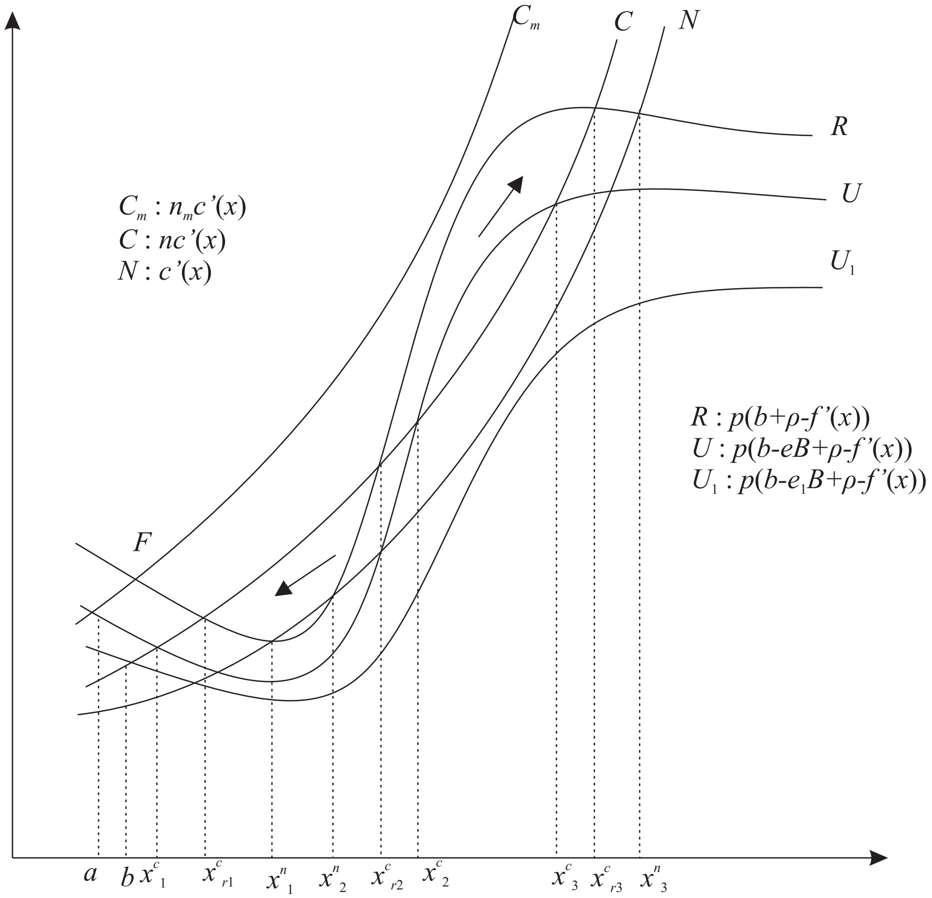

Figure 1: Cooperative and noncooperative equilibrium

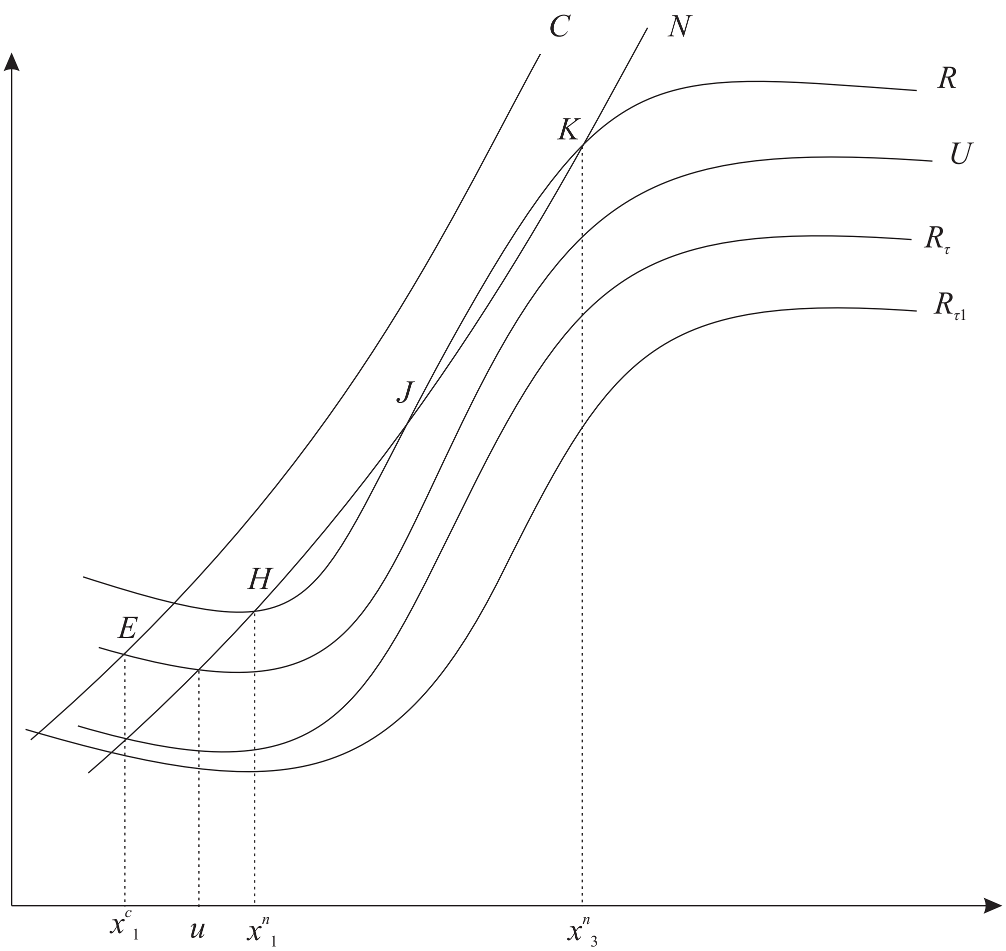

Figure 2: Regulation