

Optimal Unemployment Insurance in a Search Model with Variable Human Capital

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Abstract

The framework of a general equilibrium heterogeneous agent model is used to study the optimal design of an unemployment insurance scheme and the voting behaviour on unemployment policy reforms. Agents, who have a limited lifetime and participate in the labour market until they reach the retirement age, can either be employed or unemployed in each period of their working life. Unemployed agents receive job offers of different (match) qualities. Moreover, unemployed agents suffer a decline of their individual productivity during unemployment, whereas the productivity of employed agents increases over time.

An optimal unemployment insurance scheme is one that maximizes the expected lifetime utility of a newly born agent. Two types of unemployment insurance are considered, one with defined benefits and one with defined replacement ratios. A numerical version of the model is calibrated to the German economy.

The welfare maximising unemployment insurance system is determined in simulations. Under this optimal scheme, no payments are made to short-term unemployed agents. Long-term unemployed receive rather low (social assistance level) benefits, the optimal level of which depends on the assumed degree of risk aversion. Defined benefit systems provide a higher welfare than defined replacement ratios.

Furthermore, the question is addressed whether the majority of population would support the optimal system given the status quo. It turns out that if voters can choose between keeping their current unemployment system and jumping to the equilibrium associated with the optimal policy, there is a slight majority of just above 50% for the optimal policy.

Finally, a more realistic case is considered, in which voters do not choose between the long-run equilibria associated with policy changes, but take into account the transition process to the new equilibrium. As some of the relevant variables adjust very slowly to their new long-run equilibrium values, the effect of the transition process on voting behaviour cannot be neglected.

JEL Classification: C61, D58, D78, E24, E61, J64, J65

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1. Introduction

This paper explores the optimal design of an unemployment insurance scheme in the framework of a general equilibrium heterogeneous agents model. Agents, who have a limited lifetime and participate in the labour market until they reach the retirement age, can either be employed or unemployed in each period of their working life. Unemployed agents receive job offers of different (match) quality. Moreover, unemployed agents suffer a decline of their individual productivity during unemployment, whereas the productivity of employed agents increases over time.

An optimal unemployment insurance scheme is one that maximizes the expected lifetime utility of an agent. Two types of unemployment insurance are considered, one with defined benefits and one with defined replacement ratios. The (constrained) optimal scheme is determined by simulating a version of the model calibrated to the German economy under different assumptions regarding risk aversion and the openness of the economy.

Having obtained the optimal unemployment insurance systems, it is asked whether the switch from a the current more generous system to the optimal unemployment insurance would be supported by a majority of the voters. It turns out that if agents can choose between the steady state equilibria associated with the current and the optimal system, a close majority of the voters supports the optimal policy.

Finally, the model is extended to allow for transitional dynamics after a change of the unemployment insurance system. Simulations show that the adjustment process to the new steady state equilibrium after a regime switch can take a long time. Taking into account these transitional dynamics when voting on unemployment insurance leads to results that can be very different from those obtained when agents simply choose between log-run equilibria.

There is large and fast growing literature on the optimal design of unemployment insurance (UI) systems. One approach, pioneered by Shavell and Weiss (1979) and more recently adopted by Hopenhayn and Nicolini (1997), interprets UI design as a principal-agent problem. Optimal insurance designs are derived as second-best solutions in these models.

Another strand of literature more closely related to this paper evaluates UI systems in the framework of more or less complicated general equilibrium models. As such models are often too complicated to allow for an exact analytical solution, usually different institutional arrangements are compared using simulation techniques.

Acemoglu and Shimer (1999) construct a general equilibrium model with search in which highly productive jobs are also riskier. They show analytically that under these circumstances, if agents are risk averse, maximal output is attained only with an unemployment insurance.

Modelling an economy populated by liquidity constrained agents, Hansen and İmrohoroğlu (1992) study the consequences of moral hazard for optimal replacement ratios. They find that an optimal insurance has a relatively high benefit level to protect agents from large fluctuations of consumption. However, if moral hazard is introduced, replacement ratios at levels observed in reality may actually make the economy worse off than without any insurance at all.

Frederiksson and Holmlund (1999) construct a model of job search abstracting from capital. They consider an unemployment insurance with two benefit levels and show that optimality requires the benefits to decline. In a calibrated version of their model they find significant welfare gains from switching from an optimal one-level to a two level benefit unemployment insurance.

Costain (1999) incorporates stochastic job matches into a heterogeneous agents model. He only finds minor improvements of consumption smoothing with unemployment insurance. Also, the importance of moral hazard is relatively small.

Using version of the Costain (1999) model calibrated to the German economy, Heer (2003) numerically obtains optimal replacement ratios. He finds that the optimal UI scheme involves full insurance during the first year of unemployment. This result is driven by the somewhat counterintuitive effect that increasing the replacement ratio during the first year of unemployment results in aggregate savings.

Pallage and Zimmermann (2004) discuss the voting behaviour on UI systems. The authors model the Canadian unemployment insurance, which is more generous towards individuals living in regions with higher unemployment risk. Comparing this unconventional system to a “normal” insurance that treats all individuals in a country equally, the authors conclude that the existence of the current UI system can only be explained by the Canadian political process, as a majority of the voters would oppose it, whereas it is supported by a close majority in the parliament when the electoral process is taken into account.

The rest of this paper is laid out as follows. In section 2, the basic model is presented and the numerical solution strategy is explained. Section 3 described the calibration of the model and presents the optimal unemployment systems under various assumptions. Then the results of a vote on these optimal systems are discussed. Finally, the effects of considering the transitional

dynamics after the switch to an alternative UI scheme are presented. The last sections concludes.

2. The Model

2.1. *The Basic Set-up*

Consider an economy that is populated by a large constant number of finitely lived heterogeneous agents. Agents work for a certain number of periods and are retired thereafter. During their working life, they can either be unemployed or hold a job. When the first come into existence, all agents are identical. During their lives, however, they are exposed to idiosyncratic shocks, leading to increasing heterogeneity in asset holding, individual productivity, wages, and claims to social insurance.

Jobs are offered by firms that produce for a competitive product market. The labour market is modelled using a version of Pissarides's (2000) two-sided search approach.¹ Thus, the number of jobs offered and the probability of creating a match between an unemployed worker and a vacant job are determined by a matching function and a zero-profits constraint for firms.

There is also a government that can provide unemployment insurance or other benefits that are paid for by taxes.

2.2. *The Household Sector*

There large number of agents – or households – who live for L periods. The population is stationary, meaning that each period as many agents newly enter the economy as there are deaths, i.e. a share of $1/L$ of the total population is replaced each period.

Agents can be either employed or unemployed during the first L_r-1 periods of their lives, and are retired thereafter. In each period j_0 of their lives, they maximise their expected lifetime utility, a time-separable function of consumption

$$\max E_{j_0} \left[\sum_{j=j_0}^L \beta^{j-j_0} u(c_j) \right], \quad (1)$$

¹ *ibid.*, chapter 1.

subject to a budget constraint that must hold with probability one. Here, u is the period utility function with the usual properties, c_j the consumption in period j , and β is the discount factor.² E_{j_0} is the mathematical expectation operator conditional on all information available in period j_0 . u is assumed to exhibit a constant relative risk aversion γ :

$$u(c) = \begin{cases} \ln c & \text{if } \gamma = 1 \\ \frac{c^{1-\gamma} - 1}{1-\gamma} & \text{otherwise} \end{cases} \quad (2)$$

Let a_j be the assets owned at the beginning of period j and y_j the net income (including all kinds of benefits, but excluding interest income) received in the same period, then the period j_0 budget constraint for the remaining life span becomes

$$a_{j_0} + \sum_{j=j_0}^L R^{j_0-j} (y_j - c_j) = 0, \quad (3)$$

if the household leaves no bequest. Here, r is the risk-free interest rate, $R=1+r$. The budget constraint is required to hold for each individual.

Agents are characterised by a variable individual skill component. These skills q_j depend on the employment history of the agent: during unemployment, they deteriorate by a factor $1-\delta_u$ per period, and while employed, a worker enjoys a skill increase by the factor $1+\delta_e$. Immediately after being fired, q_j declines by $1-\delta_f$.

The “labour productivity” of an agent on a job is

$$p_j = \chi m_j (q_j + q), \quad (4)$$

which is a function of the job-specific “match quality” m_j and the current individual skill level, consisting of a variable component q_j and a fixed personal component q . χ is a scaling parameter.

² For better readability, I leave out the index i for the agent, i.e. I write c_j instead of $c_{i,j}$ for the consumption of individual i in period j .

Agents start their lives as without a job. Unemployed workers receive job offers at a Poisson rate ω , which is determined by the tightness of the labour market. So each period, after making their consumption decision, these agents may be presented with zero, one, or several job offers, each characterised by a match quality m , sampled form a log-normal distribution $G(m)$.

$$\ln m \sim N(-\frac{1}{2}\sigma_m^2, \sigma_m^2) \quad (5)$$

Agents consider the best offer and decide whether to accept or to reject this job.

The wage to be earned on a job is negotiated between the jobseeker and the firm. I assume Nash bargaining over the wage w upon job creation.

$$w_j = \arg \max_w (w - w_j^r)^\varepsilon (\tilde{p}_j - w)^{1-\varepsilon} \quad (6)$$

w_j^r is the household's reservation wage, and \tilde{p}_j is the agent's marginal productivity on the job. The parameter ε stands for the bargaining strength of the household.

As mentioned, the productivity of an agent improves while he is employed. I assume here that while an agent holds a job, the wage increases at the same rate. This means that in the wage negotiations the ex-ante surplus of a job is split between employer and employee according to their relative bargaining strengths.

Employees never quit, but face the risk of being laid off with probability λ per period.

A household's period net income y_j is simply the sum of net wage income and benefits. In general, it may be a function of an agent's entire current situation and history. A central assumption underlying the model is that any risk stemming from employment and income uncertainty must be borne by the individual, i.e. there is no way of insuring against fluctuations of y_j except saving. After retirement, income ceases to be random in any respect, thus for agents aged L_r or above, the individual consumption choice problem is deterministic.

A recursive formulation of the agents' decision problem making use of Bellman's approach can be written as

$$v_j(a_j, s_j, q_j, m_j, \Omega_j) = \max_{a_{j+1}, m_j} \{u_t(c_j) + \beta E_j v_{j+1}(a_{j+1}, s_{j+1}, q_{j+1}, m_{j+1}, \Omega_{j+1})\} \quad (7)$$

with $v_{L+1} = 0$, where the maximisation is subject to the budget constraint

$$a_{j+1} = R(a_j + y_j - c_j), \quad a_{L+1} = 0. \quad (\text{BC})$$

The value function $v_j(\cdot)$ represents the highest discounted lifetime value an agent of age j can achieve in the situation characterised by the function's parameters. $s_j \in \{e, u, r\}$ stands for the current employment status (employed, unemployed, retired) and Ω_j is meant to capture all relevant information about the individual employment history required to compute the net income y_j . \underline{m} is the minimum match quality required to make a job acceptable to an unemployed agent.

Under standard regularity conditions the agent's decision problem is well-defined and has the usual properties: The optimal savings choice requires smoothing the marginal utility of consumption over time according to the Euler equation

$$u'_j(c_j) = R\beta E_j[u'_{j+1}(c_{j+1})]. \quad (8)$$

Depending on the parameters R and β , some agents may behave like buffer-stock savers (see Deaton 1991 and Carroll 1997). In a world of perfect foresight they would choose a consumption level above their current resources, borrow money and repay it later in their lives. But with income uncertainty and the requirement to repay any debt with probability one, the agents' ability to borrow is practically limited. Consequently, they choose a certain target-amount of savings (the buffer-stock), that is an optimal compromise between the unachievable long-run consumption profile and the necessity to self-insure against unforeseen income fluctuations. However, agents approaching the retirement age L_r are likely to accumulate assets in order to provide for the final periods of their lifetime, when they cannot earn labour income any more.

The optimal job-acceptance rule implies a match threshold \underline{m} that depends on the current individual situation. If the best job offered to an unemployed agent in a certain period exhibits a match quality above \underline{m} , it is accepted, otherwise the agent chooses to remain unemployed. Ceteris paribus, a faster skill decline δ_u during unemployment or a higher value of a given job (higher δ_e or lower λ) imply a lower value of \underline{m} .

2.3. Firms

Firms produce goods according to a Cobb-Douglas production function

$$f(N, K) = L^{1-\alpha} K^\alpha \quad (9)$$

using the inputs capital K and labour N . Firms are price-takers at the capital market, where they rent assets at the interest rate r . Capital depreciates at the rate δ . It follows from the firm's profit maximisation that the capital used on a job of productivity $N=p_j$ is

$$K = \left(\frac{\alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}} p_j. \quad (10)$$

Firms negotiate wages with workers as discussed in the previous subsection. the resulting wage must lie between the reservation wage of the worker and the marginal productivity of the worker on the job

$$\tilde{p}_j = (1 - \alpha) \left(\frac{\alpha}{r + \delta} \right)^{\frac{\alpha}{1-\alpha}} p_j. \quad (11)$$

Thus, firms earn a rent on each job, the size of which depends on the productivity of the job and the worker's outside option at the time when the job was created. As jobs are destroyed at the constant rate λ per period, the expected discounted rent from a job is

$$\Pi_j = \frac{\left(\frac{1 + \delta_e}{R} (1 - \lambda) \right)^J - 1}{\frac{1 + \delta_e}{R} (1 - \lambda) - 1} \tilde{p}_j, \quad (12)$$

where J is the number of periods left until the agent retires.

To find employees in the first place, firms must engage in costly search. This is done by creating a vacancy at the cost k per period. Following Pissarides (2000), I assume that free market entry drives profits down to zero, meaning that in equilibrium the number of vacancies is such that total vacancy costs are equal to total expected rents.

2.4. Markets and Government

There is an imperfect labour market in the economy. Both households searching for a job and firms that want to hire must wait until they are matched. The total number of worker-firm matches per period is determined by a matching function M

$$M(U, V) = \tilde{\omega} U^{1-\eta} V^\eta \quad (13)$$

that is increasing in its arguments U (the number of unemployed workers) and V (the number of job offers). The probability of being matched is the same for all unemployed agents and all vacancies. The job arrival rate for a single unemployed individual is thus

$$\omega = \frac{M(U, V)}{U} = \tilde{\omega} \left(\frac{V}{U} \right)^\eta. \quad (14)$$

$\theta = V/U$ is called the market tightness (see Pissarides 2000). If an agent receives more than one offer in one period, he is assumed to consider only the best one. A match need not lead to the creation of a job, however, as the match specific productivity component m of an agent on this job may be so low that the resulting wage is below the agent's outside option.

Both firms and households have access to a competitive capital market. In the baseline case of a closed economy, the interest rate r must balance domestic capital supply, i.e. net household asset supply, and the capital demand of the production sector. As discussed in Aiyagari (1994) and Huggett (1993), the equilibrium interest rate is likely to lie below the agents' discount rate $1-\beta$, because households are exposed to uninsurable income risk. Households thus wish to hold more wealth to improve consumption smoothing in the case of undesirable income realisations.

In the case of a small open economy, the interest rate is equal to a "world" interest rate r^* . Any difference between the net asset supply of the household sector and the capital demand of the firms is invested at or borrowed from the world capital market.

There is also a government that raises taxes to provide social insurance. I consider two different concepts of unemployment insurance, one with fixed benefits and the other with fixed replacement rates. There may be up to three different benefit levels b_1 to b_3 or replacement ratios ρ_1 to ρ_3 , one for the first six months of unemployment, one for the next six

months, and one for the time thereafter. This should be flexible enough to capture the main features of existing benefit systems and also allow for a meaningful interpretation of the (constrained) optimal benefit scheme in terms of a benefit profile over time.

The government finances its expenditures by a proportional tax t on wages and benefits.

2.5. Long-run Equilibrium

Definition 1: steady-state equilibrium

The economy just described is defined to be in a steady-state rational expectations equilibrium if

1. households maximise their utility
2. firms maximise their profits
3. the capital market clears at the interest rate r
4. firms make zero profits on average given the tightness θ of the labour market
5. the government budget is balanced
6. r , θ , and the government policy are expected to remain unchanged
7. the cross-sectional distribution of the population with respect to the state variables is constant.

2.6. The Welfare Measure

In the next section, various policy regimes are compared with respect to the welfare they provide in equilibrium. The welfare measure used to this end is the expected lifetime utility of a newly born agent. As all agents are identical at the beginning of their lifecycle, this measure is representative. The resulting welfare criterion provides a ranking of policies that is independent of any reference population.

Whenever two levels of welfare W_1 and W_2 corresponding to two different policies are compared, a kind of “compensating variation” measure R is employed: $R(W_1, W_2)$ is the proportional consumption increase under policy 1 that makes the agent indifferent between the two policies. The functional form of R depends on the period utility function (2).

$$R(W_1, W_2) = \begin{cases} \exp\left(\frac{W_2 - W_1}{\xi}\right) - 1 & \text{if } \gamma = 1 \\ \left(\frac{W_2 + \xi}{W_1 + \xi}\right)^{\frac{1}{1-\gamma}} - 1 & \text{otherwise} \end{cases}, \quad (15)$$

where $\xi = \sum_{j=1}^L \beta^{j-1} = \frac{\beta^L - 1}{\beta - 1}$.

2.7. The Solution Algorithm

The algorithm used to numerically solve for the three parameters that characterise equilibrium, namely r , θ , and the tax rate t (given certain benefit levels), is similar to that used by Costain (1999) and Heer (2003) among others. It can be stated as follows:

Algorithm 1

- 1 guess initial parameters r_0 , θ_0 , and t_0
- 2 repeat for $n=0,1,\dots$
 - 2.1 derive optimal household behaviour, taking r_n , θ_n , and t_n as given
 - 2.2 simulate a large number I of individual histories and compute aggregate asset supply and demand, aggregate profits, the government budget balance, and welfare W_n
 - 2.3 update parameters to get r_{n+1} , θ_{n+1} , and t_{n+1}
- 3 until convergence

In step 2.3, the new parameters are chosen as follows. First, a “ceteris paribus guess” of parameter value is computed that would be compatible with equilibrium if household behaviour did not change. The next round parameter is then a convex combination of the current parameter and the ceteris paribus equilibrium guess.

Convergence is reached if all of the following conditions are met:

1. the newly guessed parameters are close to the current parameters:

$$|r_{n+1} - r_n| / (r_{n+1} + r_n) < \bar{\varepsilon}_1, \quad |\theta_{n+1} - \theta_n| / (\theta_{n+1} + \theta_n) < \bar{\varepsilon}_1, \quad |t_{n+1} - t_n| / (t_{n+1} + t_n) < \bar{\varepsilon}_1$$

2. the difference between aggregate asset supply A^s and asset demand A^d , firm profit, and the government budget imbalance are small:

$$|A_n^s - A_n^d| / (A_n^s + A_n^d) < \bar{\varepsilon}_2, \quad |\Pi_n - K_n| / (\Pi_n + K_n) < \bar{\varepsilon}_2, \quad |B_n - T_n| / (B_n + T_n) < \bar{\varepsilon}_2,$$

where Π is the aggregate quasi-rent earned by firms on existing jobs, K is aggregate vacancy costs, B are total benefit payments and T is the government’s tax revenue.

3. the welfare change between W_{n-1} and W_n is small: $|R(W_{n+1}, W_n)| < \bar{\varepsilon}_3$

It should be mentioned that the algorithm just described does not necessarily converge for all policies. If, for example, benefit are so high that the unemployment rate becomes close to one, there is no way to avoid a budget deficit. For this reason, the algorithm is stopped if it has not converged within 30 loops, which lies sufficiently above the 8 to 12 loops that are usually required.

3. Calibration and Experiments

3.1. Calibration of the Model

The model is calibrated to resemble some properties of the West German economy in the 1990ies. First of all, the period length is set at $\frac{1}{2}$ month. This results in a lifetime of 1,440 periods for all agents. While this number is rather large, the short period duration is likely to contribute to a more realistic simulated search behaviour than would be possible in a model at quarterly or annual frequency.

Table 1: Calibrated parameters.

parameter		value		
		$\gamma=1$	$\gamma=2$	$\gamma=3$
δ_e	growth rate of skills during employment ^a		0.016	
δ_u	skill loss per period during unemployment ^a		0.12	
δ_f	skill decline at job loss		0.2	
λ	job destruction rate ^a		0.045	
σ_m	standard deviation of log match quality		0.22	
α	capital share parameter		0.3	
δ	capital depreciation rate ^a		0.076	
η	match function parameter		0.5	
ε	household bargaining strength		0.95	
χ	scale parameter ^b		400	
β	discount factor ^a	0.961	0.950	0.934
k	vacancy cost ^b	321	298	249

^a per annum

^b per period

The choice of the labour market parameters δ_e , δ_u , δ_f and λ is motivated by the micro estimates obtained in Pollak (2005). The productivity improvement on the job δ_e is 1.6% per year. Job loss results in an immediate decrease of productivity of $\delta_f=20\%$, and during unemployment skills keep on declining at the rate $\delta_u=12\%$ per annum. The job destruction rate of $\lambda=4.5\%$ per year implies an average duration of 22 years between two unemployment spells. The standard deviation of the log match quality of job offers is chosen to be $\sigma_m=0.22$. The production technology is characterised by a capital share parameter $\alpha=0.3$ and a depreciation rate of $\delta=7.6\%$ per annum. The matching function parameter η is chosen to be 0.5. The relative bargaining strength of the household ε is set at 95%. While this value is much higher than the 50% often found in the literature, it seems to yield plausible results: for a market tightness of one and an unemployment rate of 4.3%, the cost of a vacancy is approximately 10% of the average gross wage. The scale χ is 400, which yields realistic DM wage levels. The parameter q is set to zero.

To highlight the importance of the prevailing risk aversion for the optimal choice of an unemployment insurance system, the experiments conducted in the following section are performed for three different values of the parameter of relative risk aversion, $\gamma \in \{1, 2, 3\}$, that will be referred to as the low, medium and high risk aversion case hereafter. For each of these values, a base calibration is chosen that makes the endogenous variables of the model close to values observed in reality. The unemployment insurance system used for all these calibrations is one with a constant replacement ratio of 67% during the first year of the unemployment spell and 57% thereafter. These were the legal replacement ratios for unemployed persons with at least one child in Germany in the years 1994 to 2004.

There is an issue concerning the treatment of young agents who have never held a job in their life in the unemployment insurance system. In a system with defined replacement ratios their benefit level is not well-defined. But even when policies with defined benefit levels are considered, one might want to exclude agents who have not worked before. One reason for doing so is the fact that – as all agents are created without a job in this model – the welfare measure would be dominated by the effects of benefits paid to very young unemployed agents, if they were allowed to participate in the insurance system. To avoid this effect, in all simulations only those agents who have held a job before are eligible for unemployment compensation.

Without an income in the first periods of their working life, agents need to be equipped with a positive initial wealth a_1 to be able to maintain a positive level of consumption throughout

their lives. This initial wealth should not be set too low, because otherwise welfare measure would be influenced very strongly by the chances of finding a first job very fast. In all simulations, agents are thus provided with initial assets of 14,583, corresponding to about three times a typical initial monthly wage. This initial wealth is given to new agents as a transfer paid out of the government budget.

For all tree base calibrations, the discount factor β is chosen such that the real interest rate matches the 1990 to 1996 average of 3.3% per annum. The vacancy cost k is calibrated to result in a job offer rate ω of 9 per period. Without loss of generality, the market tightness θ is normalised to one in these base cases. The calibrated parameters are summarised in Table 1.

In all numerical experiments, the number of simulated individual histories is 20,000, resulting in a total number of 28.8 million individual-age observations. The lifecycles are created using a pre-selected set of random numbers, which is necessary for the algorithm to converge. However, while the variance of the endogenous variables conditional on the chosen set of random numbers is only driven by the convergence criteria of the algorithm and can be made small, the variables unavoidably exhibit an unconditional variance that is due to the limited number of simulated cases. Table 2 lists the unconditional standard deviations of the interest rate, the market tightness, the tax rate, and the welfare measure. The figures were obtained applying the solution algorithm several times to the same problem, but basing the simulations on different sets of random numbers.³ The significance of differences between these parameters under different policies should be judged by these variances.

The parameters that indicate when the iteration of the solution algorithm is to be stopped are $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = 10^{-3}$, $\bar{\varepsilon}_3 = 5 \cdot 10^{-5}$.

Some equilibrium statistics for the base calibrations are reported in Table 3. As explained above, the real interest rate and the market tightness are the same in the tree scenarios. The resulting unemployment rates lie between 3.8% and 5.2%, increasing with risk-aversion. The tax rates reflect the varying costs for unemployment benefits in the scenarios. Aggregate (per capita) output can be split in a labour share ($1-\alpha=70\%$), which consists of aggregate gross labour income and aggregate vacancy costs, and the capital share consisting of aggregate

³ For these simulations, tighter convergence criteria than for the other simulations were used. See notes to Table 2.

capital income and the depreciation of the stock (not reported in the table). The share of the per-capita government budget that is used to pay for the initial asset holding of newly created agents amounts to 243 per annum in all simulations. For convenience, the average gross monthly wage is also reported in the table. As both wages and unemployment benefits are taxed at the same rate, this figure can be compared to the benefit levels of the alternative unemployment insurance policies considered in the next subsection.

Table 2: Measured unconditional standard deviations of the parameters.

variable	r	θ	t	W^a
unconditional standard deviation	$5.54 \cdot 10^{-7}$	0.024	$6.48 \cdot 10^{-6}$	$3.47 \cdot 10^{-4}$

^a s.d. of compensating variation

convergence criterion: $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = 10^{-4}$, $\bar{\varepsilon}_3 = 5 \cdot 10^{-6}$

Table 3: Equilibrium outcomes: base calibrations.

variable		values		
		$\gamma=1$	$\gamma=2$	$\gamma=3$
r	real interest rate ^a	0.033	0.033	0.033
θ	labour market tightness	1	1	1
t	wage tax rate	0.025	0.028	0.033
u	unemployment rate	0.038	0.043	0.052
	aggregate output ^b	77,039	76,272	75,222
	aggregate net labour income ^b	52,362	51,672	50,692
	aggregate capital income ^b	6,723	6,656	6,564
	government budget ^b	1,343	1,489	1,730
	aggregate vacancy costs ^b	222	229	233
	average monthly gross wage	6,206	6,170	6,145

^a per annum

^b per annum, per capita of total population

3.2. Optimal Benefit Levels

In this subsection, the welfare maximising defined replacement ratio and defined benefit unemployment insurance policies are presented. They are computed using a simple two-stage grid search strategy. The values considered for the monthly unemployment benefits b_1 , b_2 , and

b_3 cover the range between zero and 6,000 with a step length of 400. For the replacement ratios ρ_1 to ρ_3 , 13 values from 0 to 1.2 were taken into account.⁴

The optimal benefit levels and replacement ratios are presented in Table 4 and Table 5, together with some equilibrium statistics on these policies. It may be surprising that, regardless of the level of risk aversion assumed, no payments are made to agents whose unemployment spell lasts for less than one year. Even though the moral hazard effect of moderate benefits for short-term unemployed agents would not be too high, this form of “consumption insurance” would lead to welfare losses. Agents can efficiently provide for short periods of unemployment themselves by saving, thereby creating a positive external effect on the capital stock and thus output.

The optimal benefit for long-term unemployed workers, however, may be positive if the degree of risk aversion assumed is high. While b_1 is zero if the rate of relative risk aversion is one, it increases to 400 and even 2,000 per month if γ equals 2 or 3, respectively. The optimal long-run replacement ratios show a similar pattern: in the low risk aversion case, the replacement ratio should be zero. For the higher risk aversion cases, ρ_3 is 0.3 and 0.4.

Under the optimal policies, the real interest rates are below the base calibration level. This can be explained by the increased amount of precautionary saving that becomes necessary as the short-run unemployment benefits are abolished. The aggregate capital income even increases in the low risk aversion case, but it declines for $\gamma \geq 2$. Also, the unemployment rates are much lower than in the base case. In addition to the direct effect that the higher employment has on total output, there is an indirect effect. Less unemployment also means less skill depreciation and therefore a higher average productivity of employees on the job. This productivity increase amounts to about 0.8% for $\gamma=1$ and 0.1% in the $\gamma=3$ case. There are two further effects that tend to increase the average gross wages under the optimal regimes. First, the lower real interest rate leads to a higher capital usage per productive worker. Second, the

⁴ The grid search computation of the optimal benefit levels was performed as follows. In a first step, the best combination of benefits $(b_1, b_2, b_3) \in \{0, 1000, 2000, \dots, 6000\}^3$ was determined. Then, in the second step, the six or seven multiples of 400 closest to this best combination along each dimensions were searched.

The search for the optimal replacement ratio was done in a similar fashion starting out with the grid $\{0, 0.2, 0.4, \dots, 1\}^3$ in the first step and then considering the values $\{\rho_1^* - 0.2, \rho_1^* - 0.1, \dots, \rho_1^* + 0.2\} \times \{\rho_2^* - 0.2, \rho_2^* - 0.1, \dots, \rho_2^* + 0.2\} \times \{\rho_3^* - 0.2, \rho_3^* - 0.1, \dots, \rho_3^* + 0.2\}$ around the first step optimum $(\rho_1^*, \rho_2^*, \rho_3^*)$ in step two.

No negative benefits or replacement ratios were taken into account.

higher market tightness allows unemployed agents to choose from a greater number of job offers, which improves the average worker-firm match by about 0.04% ($\gamma=2$) to 2.7% ($\gamma=3$). Despite the fact that the lower benefits under the optimal system tend to worsen the workers' bargaining position, employees still strongly benefit from the improved average match quality.

Table 4: Equilibrium outcomes: optimal defined benefit.

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
b_1	monthly benefit:	months 1-6	0	0	0
b_2		months 7-12	0	0	0
b_3		months 13-	0	400	2,000
r	real interest rate ^a		0.031	0.029	0.028
θ	labour market tightness		2.93	3.94	4.93
t	wage tax rate		0.004	0.004	0.005
u	unemployment rate		0.014	0.011	0.011
	aggregate output ^b		81,519	82,264	82,859
	aggregate net labour income ^b		56,587	57,099	57,491
	aggregate capital income ^b		6,808	6,609	6,423
	government budget ^b		243	243	261
	aggregate vacancy costs ^b		233	243	250
	average monthly gross wage		6,402	6,445	6,490
	welfare improvement ^c		0.049	0.062	0.083

^a per annum

^b per annum, per capita of total population

^c compared to base calibration

Comparing the optimal benefit scenarios to the base case, the welfare differences are substantial. They range from an improvement worth a consumption increase of 4.9% in the low risk aversion case up to 8.3% in the high risk aversion scenario. With optimal benefits and replacement ratios of zero, the optimal defined benefit and defined replacement ratio policies are equal in the $\gamma=1$ case. For higher risk aversions, the defined benefit policies yield higher welfare levels than the replacement ratio systems.

One should note that the choice of the short-run benefits b_1 and b_2 has a much smaller impact on welfare than the long-term benefit b_3 . Figure 1 shows the welfare effects of varying each benefit parameter while the other two parameters remain at their optimal level. A badly chosen b_3 in the range between 0 and 2,500 may reduce welfare by more than 20%, whereas varying b_1 and b_2 in the same range affects welfare by less than 1%.

Table 5: Equilibrium outcomes: optimal replacement ratios.

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
ρ_1	replacement ratio:	months 1-6	0	0	0
ρ_2		months 7-12	0	0	0
ρ_3		months 13-	0	0.3	0.4
r	real interest rate ^a		0.031	0.030	0.026
θ	labour market tightness		2.93	3.88	5.70
t	wage tax rate		0.004	0.004	0.004
u	unemployment rate		0.014	0.012	0.010
	aggregate output ^b		81,519	82,181	83,646
	aggregate net labour income ^b		56,587	57,039	58,052
	aggregate capital income ^b		6,808	6,628	6,226
	government budget ^b		243	244	248
	aggregate vacancy costs ^b		233	243	252
	average monthly gross wage		6,402	6,440	6,543
	welfare improvement ^c		0.049	0.061	0.074

^a per annum

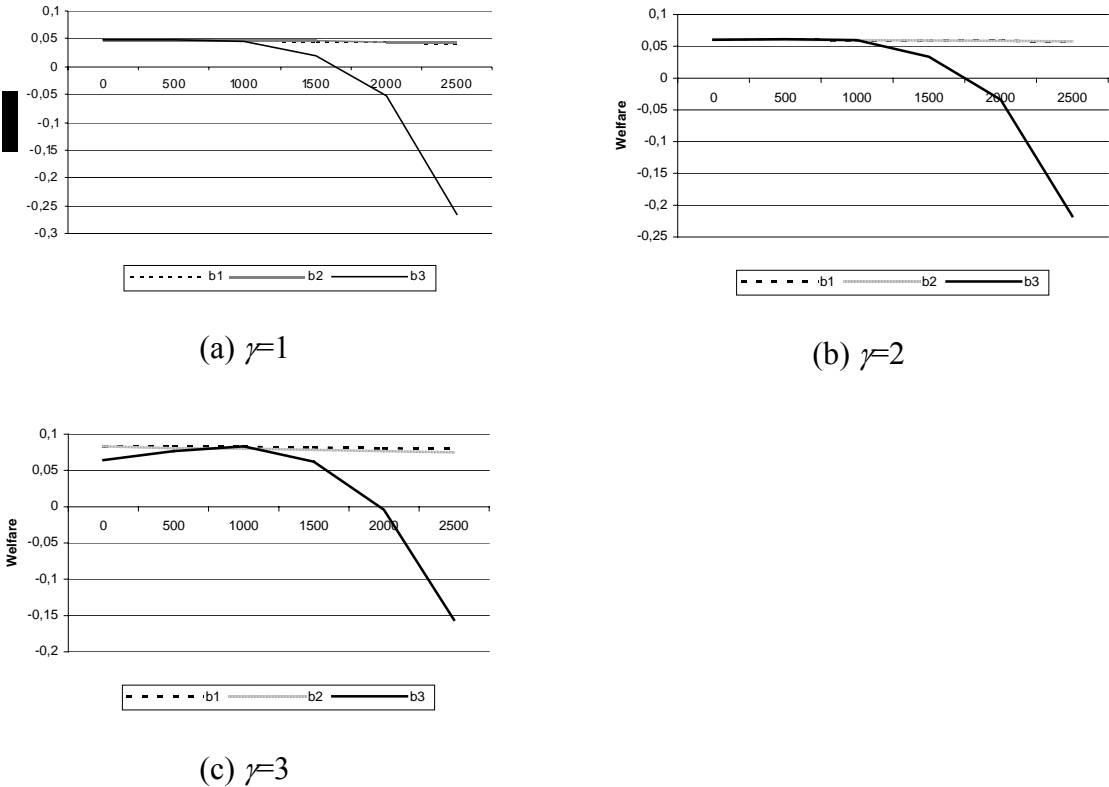
^b per annum, per capita of total population

^c compared to base calibration

In the light of these results one may conclude that the efficiency of the German unemployment insurance system could be significantly improved by replacing the high replacement ratio for long-term unemployed workers by a defined benefit of an appropriate amount. Indeed, that is exactly the kind of reform the government has recently adopted. Starting in January 2005, the former unemployment assistance available to workers who run

out of the usual unemployment insurance after six to twelve months⁵ is replaced by a benefit at social assistance level. The resulting benefit for long-term unemployed persons lies somewhere close to the optimal benefits computed for the $\gamma=2$ and the $\gamma=3$ case, depending on the household size and situation. However, an important difference between the unemployment systems considered in the simulations and existing benefits is that the latter are often means-tested.

Figure 1: Welfare effects of the variation of the benefit parameters.



To require – as has been done above – that every cent of additional domestic saving is invested in the home country is a strong assumption. In what follows, the opposite extreme case of a small, fully open economy is considered. Starting from the same base cases as before, the welfare maximising unemployment benefit profiles are obtained under the assumption that the interest rate remains unchanged. There is a world market supplying capital elastically at the prevailing interest rate.

⁵ Older persons may still receive the high unemployment insurance benefit for a longer period, up to 32 months in 2004 and up to 18 months if they become unemployed after February 2005.

Table 6: Equilibrium outcomes: optimal defined benefit (open economy).

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
b_1	monthly benefit:	months 1-6	0	400	6,000
b_2		months 7-12	0	0	400
b_3		months 13-	800	1,600	2,000
r	real interest rate ^a		0.033	0.033	0.033
θ	labour market tightness		2.84	3.45	3.35
t	wage tax rate		0.004	0.005	0.015
u	unemployment rate		0.014	0.013	0.016
	aggregate output ^b		80,857	80,870	80,462
	aggregate net labour income ^b		56,125	56,289	55,246
	aggregate capital income ^b		7,297	7,057	6,916
	government budget ^b		244	282	837
	aggregate vacancy costs ^b		231	238	240
	average monthly gross wage		6,353	6,367	6,333
	welfare improvement ^c		0.046	0.057	0.078

^a per annum

^b per annum, per capita of total population

^c compared to base calibration

Table 6 and Table 7 summarise the equilibrium outcomes under the optimal policies for the open economy. Without the positive effect of additional savings on domestic capital and thus productivity, the optimal benefit levels and replacement ratios are higher than in the closed economy scenario. While the optimal benefit for long term unemployed is now higher in the $\gamma=1$ and $\gamma=2$ cases, it still does not exceed the value of $b_3=2,000$ and $\rho_3=0.4$. The optimal benefit paid during the first six months of a spell is now as high as 6,000 in the high risk aversion case, which is approximately the level of the average wage.⁶ Interestingly, this stark increase of generosity compared to the optimal closed economy benefits comes at only a rather small cost in terms of unemployment. The unemployment rate in the low risk aversion case is the same as above, for higher risk aversion it changes by no more than $\frac{1}{2}$ percentage point.

⁶ as 6,000 is the highest benefit level considered in the simulations, even higher values for b_1 might lead to further welfare improvements.

Aggregate output and wages are somewhat lower than in the closed economy simulations. This is mainly explained by the lower capital intensity caused by the unchanged interest rate. The average individual productivity and match quality are almost the same as for the optimal policies in the closed economy. Capital income is now higher than in both the base calibration and the closed economy cases. The high risk aversion optimal defined benefit scenario with its very high benefit for the first six months of the unemployment spell is the only case where domestic capital supply falls short of demand. The net foreign position amounts to 1.5% of the total capital stock in this case. In the other cases, between 3.7% (defined benefit, $\gamma=1$) and 8.3% (defined replacement ratio, $\gamma=2$) of the domestic savings are invested abroad. Like in the closed economy case, the optimal fixed benefit regimes generate a higher welfare than the systems based on defined replacement ratios. Moreover, the simulations show that the switch to an optimal system yields higher welfare improvements in a closed economy.

Table 7: Equilibrium outcomes: optimal replacement ratios (open economy).

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
ρ_1	replacement ratio:	months 1-6	0	0	0.4
ρ_2		months 7-12	0	0	0.1
ρ_3		months 13-	0.3	0.4	0.4
r	real interest rate ^a		0.033	0.033	0.033
θ	labour market tightness		2.83	3.71	4.78
t	wage tax rate		0.004	0.004	0.008
u	unemployment rate		0.014	0.012	0.011
	aggregate output ^b		80,849	81,053	81,323
	aggregate net labour income ^b		56,119	56,254	56,243
	aggregate capital income ^b		7,331	7,642	7,588
	government budget ^b		245	245	439
	aggregate vacancy costs ^b		231	239	244
	average monthly gross wage		6,352	6,354	6,371
	welfare improvement ^c		0.046	0.054	0.062

^a per annum

^b per annum, per capita of total population

^c compared to base calibration

3.3. No Depreciation of Skills

It is a central assumption of the analysis performed so far that agents who lose their jobs suffer a permanent decrease of their productivity. This implies that unemployment does not only cause a transitory income reduction, but a permanent negative income shock. The risk of permanent income reductions of course is a much bigger threat to risk-averse agents than a temporary income drop.

This subsection addresses the question how important such permanent income shocks are for the design of an unemployment insurance system. The evolution of the agents' individual productivity is changed to no more depend on the employment history, but rather follow a deterministic path. Regardless of an agent's current employment status, his productivity increases at a rate of $\delta_e = \delta_u = 0.67\%$ per annum. There is no skill decline at job loss. The productivity increase is chosen to approximately match the expected productivity improvements used before. Without a permanent shock to productivity, only two transitory income shocks remain: Firstly, during unemployment, wage income is replaced by benefits, and secondly the acceptance of a new job is associated with a new random worker-firm match quality. While this latter shock is transitory in the sense that it does not affect wage income prospects beyond the duration of the current job, average employment durations are so long that this match quality shock may have an important impact on lifetime earnings.

Table 8 lists the variables that are calibrated differently than for the above simulations, and Table 9 presents the base calibrations for the case without skill depreciation.

Table 8: Calibrated parameters (no depreciation case).

parameter		value		
		$\gamma=1$	$\gamma=2$	$\gamma=3$
δ_e	growth rate of skills during employment ^a		0.0067	
δ_u	skill loss per period during unemployment ^a		-0.0067	
δ_f	skill decline at job loss		0	
β	discount factor ^a	0.965	0.962	0.958
k	vacancy cost ^b	235	262	277

^a per annum

^b per period

Table 9: Equilibrium outcomes: base calibrations (no depreciation case).

variable		values		
		$\gamma=1$	$\gamma=2$	$\gamma=3$
r	real interest rate ^a	0.033	0.033	0.033
θ	labour market tightness	1	1	1
t	wage tax rate	0.032	0.031	0.031
u	unemployment rate	0.052	0.048	0.046
	aggregate output ^b	85,645	84,453	83,766
	aggregate net labour income ^b	57,820	57,036	56,592
	aggregate capital income ^b	7,474	7,370	7,310
	government budget ^b	1,912	1,854	1,813
	aggregate vacancy costs ^b	220	227	231
	average monthly gross wage	7,000	6,874	6,804

^a per annum

^b per annum, per capita of total population

The equilibrium outcomes for the optimal defined benefit and replacement ratio cases are reproduced in Table 10 and Table 11, respectively. The optimal benefit levels are zero both for the first six months of an unemployment spell and for unemployment durations beyond one year. Only during months seven to twelve of a spell, low benefits are paid. The optimal replacement ratios are zero throughout with the exception of a 10% replacement rate for long-term unemployed in the high risk aversion ($\gamma=3$) case.

The potential for output and wage increases by implementing a better unemployment insurance system is smaller than in the cases with skill depreciation discussed above. This can simply be explained by the fact that a decrease of the equilibrium unemployment rate does not lead to an improvement of the average productivity here. However, there is still scope for considerable welfare improvements between 3% and 4.4%.

The equilibrium unemployment rates under the optimal policies are higher than in the case with productivity declines, which is simply the effect of longer job search, as the cost of unemployment to the agent is now lower.

Table 10: Equilibrium outcomes: optimal defined benefit (no depreciation case).

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
b_1	monthly benefit:	months 1-6	0	0	0
b_2		months 7-12	0	400	800
b_3		months 13-	0	0	0
r	real interest rate ^a		0.028	0.024	0.020
θ	labour market tightness		2.10	2.41	2.58
t	wage tax rate		0.004	0.004	0.004
u	unemployment rate		0.026	0.022	0.020
	aggregate output ^b		88,571	88,823	89,443
	aggregate net labour income ^b		61,524	61,569	62,093
	aggregate capital income ^b		6,819	6,051	5,323
	government budget ^b		243	254	263
	aggregate vacancy costs ^b		232	245	254
	average monthly gross wage		7,047	7,033	7,067
	welfare improvement ^c		0.030	0.038	0.044

^a per annum

^b per annum, per capita of total population

^c compared to base calibration

Compared to the cases with permanent productivity reductions discussed above, in this economy with reduced income risk, the potential welfare increases due to the introduction of an unemployment insurance system are very small. In the high risk aversion case ($\gamma=3$), where the benefits from any form of consumption insurance are most important, the welfare improvement from implementing the optimal benefit levels rather than no unemployment insurance at all amounts to 0.03%. In the scenario with skill depreciation, the optimal benefit equilibrium yields a welfare 1.8% above the no-insurance level.

Table 11: Equilibrium outcomes: optimal replacement ratios (no depreciation case).

variable			values		
			$\gamma=1$	$\gamma=2$	$\gamma=3$
ρ_1	replacement ratio:	months 1-6	0	0	0
ρ_2		months 7-12	0	0	0
ρ_3		months 13-	0	0	0.1
r	real interest rate ^a		0.028	0.023	0.023
θ	labour market tightness		2.10	2.44	2.37
t	wage tax rate		0.004	0.004	0.004
u	unemployment rate		0.026	0.021	0.021
	aggregate output ^b		88,571	88,823	88,428
	aggregate net labour income ^b		61,524	61,688	61,400
	aggregate capital income ^b		6,819	6,033	5,948
	government budget ^b		243	243	250
	aggregate vacancy costs ^b		232	245	250
	average monthly gross wage		7,047	7,031	6,997
	welfare improvement ^c		0.030	0.038	0.044

^a per annum^b per annum, per capita of total population^c compared to base calibration

3.4. Voting

So far, optimal unemployment insurance policies were derived under different assumptions concerning the risk aversion of workers and the openness of the economy. The welfare criterion used was the expected life-cycle utility of a newly born agent. While this welfare criterion is appealing from a theoretical point of view, it is not informative about the chances of a policy to be put in place in an economy with democratic voting mechanisms.

This sub-section addresses the question whether a switch from the status quo unemployment insurance system to the optimal policy would be approved by a majority of the voters. The status quo is the base calibration with the high replacement ratios of 67% during the first year of the unemployment spell and 57% thereafter. The voting mechanism is the same as the “helicopter drop” voting equilibrium defined in Pallage and Zimmermann (2004). The equilibrium population under the status quo policy is asked whether they enjoy a higher expected lifetime value under the current system or the long-run equilibrium corresponding

alternative policy, given their current state (i.e. age, employment situation, assets, etc.). Indifferent agents are assumed to vote for the alternative policy. Pallage and Zimmermann relate this voting mechanism to the following policy experiment: All agents living under the status quo regime are dropped at the border between two (big) countries, one of which implements the status-quo policy, the other the alternative system, and are allowed to choose in which country to live. An alternative interpretation would be that all members of the status-quo population are to vote on the immediate switch to the new policy, assuming that all macro variables would jump to their new long-run equilibrium values right after the possible policy change.

Table 12 presents the shares of votes in favour of the optimal policy for the scenarios discussed in section 3.2. In the closed economy setting, where the interest rate adjusts to balance the domestic capital market, just over fifty percent of the agents vote for the optimal policy, regardless of the level of risk aversion assumed. The best defined replacement ratio system, which yields slightly lower welfare than the defined benefit system for $\gamma=2,3$, gets a similar share of the votes as the optimal system for $\gamma=2$, but is not accepted in the high risk aversion case. In the open economy case, all optimal policies are accepted by a large majority of above 95% of all voters.

Table 12: Approval for policy switch to optimal unemployment system.

case	approval for optimal policy		
	$\gamma=1$	$\gamma=2$	$\gamma=3$
opt. def. benefit	51.1%	53.5%	50.8%
opt. def. repl. ratio	51.1%	54.6%	41.8%
opt. def. benefit (open econ.)	97.8%	97.7%	97.4%
opt. def. repl. ratio (open econ.)	97.7%	97.1%	96.5%

How can this voting behaviour be related to different demographic groups? First of all, retired agents must rely on their savings as the only source of consumption. This group is unambiguously worse off under a policy that leads to a lower interest rate than in the base case.⁷ These agents account for 25 percentage points of the votes for the status quo.

⁷ The only exception are agent who are already in last period of their lives and consume all their assets in the current period. This group is indifferent.

Figure 2 depicts the approval for the optimal defined benefit policy for employed and unemployed agents by age for the closed economy case. As one would expect, the alternative, which is less generous than the status quo system, is more popular among employed agents. Neither is it surprising that approval for the alternative system drops sharply around age 60. Agents getting close to their retirement age have accumulated a substantial amount of assets. Under the alternative policy with the lower interest rate, they would forego considerable capital income.

Among the age group between about 25 and 35 years, the support for the alternative policy is relatively low. Young agents who are unemployed or hold a relatively low paid job borrow against future income, which, under the status quo system, can never be below 57% of their current or last labour income, to improve their current consumption level. Such individuals would have problems repaying their debt in some states of the world if the alternative system with significantly lower benefits was introduced. Therefore, this group of agents opposes a regime switch.

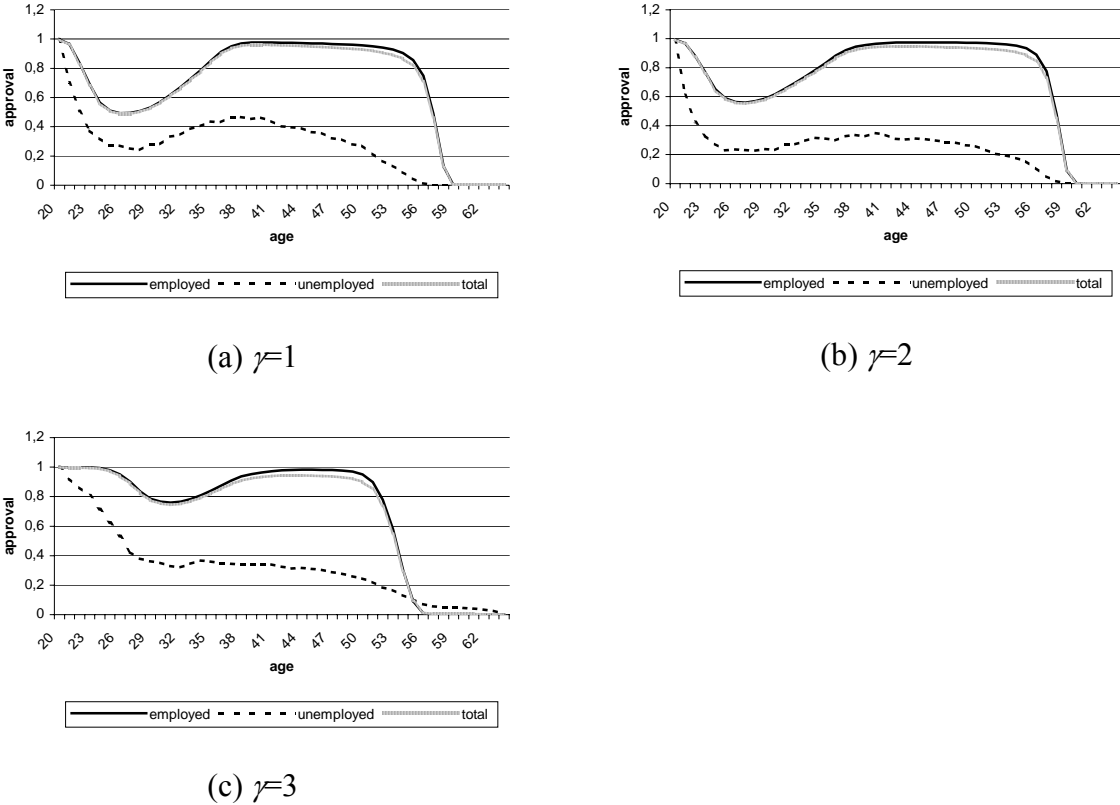
The high approval rates for the optimal policy in the case of the open economy are now easily explained. As the interest rate remains constant, retired agents are now indifferent between the status quo and the alternative system. As mentioned above, they are counted as votes for the alternative. Also, wealthy agents do not suffer capital income losses under the alternative and will therefore vote for the system with the higher labour market performance and lower taxes. Finally the group of young agents whose wealth would be too low under the optimal policy is very small in the open economy case, because here the optimal unemployment insurance system involves more generous benefits.

Having looked at the numerical examples, some general statements about the chances of getting a majority for a welfare enhancing unemployment insurance reform of the type presented can be made. First of all, the chances for such a reform are higher in a small open economy, where the effect on the equilibrium interest rate is small. Second, if there is a negative effect on the interest rate, approval for the reform is higher if the share of retired people in the total population is lower⁸ and if retirement income is less sensitive to the interest rate, e.g. in the presence of a pay-as-you-go pension system. Finally, the approval for the reform among young individuals with low or even negative wealth could be improved by

⁸ The computations were of course made for a stationary population with a given retirement age. Assuming however that the basic results would be similar for a slowly changing population and different retirement ages, a higher retirement age and faster population growth would seem to make the acceptance of the reform more likely.

either introducing the reform with a delay, thereby giving everyone the chance to prepare for the new situation, or by paying higher benefits to short-term unemployed agents, at least as transitional measure.

Figure 2: Approval for policy switch to optimal unemployment system, by age and employment status.



3.5. Transition to the new Equilibrium

In the previous subsection, the approval for the optimal unemployment insurance system was analysed assuming that voters can choose between the steady states associated with the different policies. Clearly, this is not a very realistic assumption. In what follows, the transition to the new steady state equilibrium after the introduction of the optimal policy is studied. Starting from the base cases described in section 3.1, the transition to the new steady state under the optimal defined benefit system is examined for the three levels of risk aversion $\gamma=1,2,3$. To allow for time varying parameters and a changing composition of the population, a different equilibrium concept than the one used above is required.

Definition 2: transitional equilibrium

The economy is defined to be in a transitional rational expectations equilibrium with initial population P if

1. households maximise their utility in all periods $\tau=0,1,\dots$
2. firms maximise their profits
3. the capital market clears every period at the interest rate r_τ
4. firms make zero profits on average every period given the tightness θ_τ of the labour market
5. the government budget is balanced for all τ
6. r_τ , θ_τ , and the government policy are anticipated correctly by the households
7. the initial cross-sectional distribution of the population with respect to the state variables is P .

Finding the steady-state equilibrium simply amounts to finding the equilibrium cross-sectional distribution of the population and three parameters, r , θ , and the tax rate t . A transitional equilibrium may involve a different population distribution, interest rate, market tightness and tax rate every period. Finding such an equilibrium is not a trivial task: it is impossible to solve the problem forward in time because the future values of r , θ , and t are not yet known. Solving backwards starting from a cut-off period far enough in the future is neither possible, as the composition of the population in any period is history-dependent.

The solution strategy I use circumvents this problem by attempting to solve for all periods' parameters simultaneously in an iterative process similar to the algorithm described in section 2.7. First, the problem is truncated at some period T . For all $\tau > T$, the parameters r_τ , θ_τ , and t_τ are assumed to remain at their steady-state equilibrium levels \bar{r} , $\bar{\theta}$, and \bar{t} . The algorithm then proceeds as follows:

Algorithm 2

- 1 guess initial parameters $r_\tau^0 = \bar{r}$, $\theta_\tau^0 = \bar{\theta}$, and $t_\tau^0 = \bar{t}$ for $\tau = 0 \dots T$
- 2 repeat for $n=0,1,\dots$
 - 2.1 derive optimal household behaviour for each cohort alive between periods 0 and T , taking r_τ^n , θ_τ^n , and t_τ^n as given

- 2.2 simulate a large number I of individual histories for the periods $\tau = 0 \dots T$ starting with an initial population corresponding to P , and compute aggregate asset supply and demand, aggregate profits, and the government budget balance for each period
- 2.3 update parameters for every period using the same mechanism as in Algorithm 1 to get r_τ^{n+1} , θ_τ^{n+1} , and t_τ^{n+1}
- 3 until $n=N$

I further made the assumption that the relevant parameters r_τ , θ_τ , and t_τ only change once a year instead of every period (i.e. 24 times a year). While not reducing the computational complexity of the problem, this simplification should improve the numerical properties of the algorithm for two reasons. First, with a smaller number of parameters to solve for, convergence is likely to be faster. Second, as the number of observed households per parameter is much larger, the variance in the market variables that govern the choice of the parameters (such as capital supply or the unemployment rate) is smaller.

The cut-off period chosen is $T=1,440$, meaning that a transition period corresponding to an agent's lifetime is allowed for. While it is not certain that all adjustments are completed (up to an interesting level of accuracy) within this time, a horizon of 60 years seems long enough to evaluate a labour market reform. For each of the 1,440 cohorts of the initial population, 20,000 agents are tracked over the time of 60 years, starting with the initial population corresponding to the base calibration⁹. Where necessary, cohorts are replaced by a new generation at the end of their lifetime. This results in a total of $20,000 \times 1,440 \times 60 = 4.1 \cdot 10^{10}$ agent-period observation or 691 million observations per year. The algorithm is iterated ten times. The resulting parameters meet the first convergence criterion defined in section 2.7 for all but a few periods, where they are only slightly beyond the value of $\bar{\varepsilon}_1$ used in the simulations above. Convergence is usually better for later periods.

Figure 3 illustrates the evolution of the parameters r_τ , θ_τ , and t_τ as well as the unemployment rate over the simulated 60 year transition period for the three degrees of risk aversion $\gamma=1,2,3$.

⁹ Using the base calibration population that exists under a much more generous benefit system than the regime actually used in the computation of the transition period creates the problem that some agents may be in a situation that yields an expected lifetime utility of $-\infty$. This can happen if their current wealth is so low that they cannot be certain to have a positive consumption under all circumstances. In such cases, the agents' consumption and reservation wage are set to zero until they return to a finite expected lifetime utility.

In the case of low risk aversion, where the benefits are set to zero, the tax rate reaches its long run equilibrium level almost immediately. What is more surprising is that the labour market tightness and the unemployment rate also jump to their new equilibrium level after only one year. Only the interest rate takes several years to adjust. In the other two cases $\gamma=2,3$, the adjustment of the variables is much slower. After an immediate change in the first two years, the labour market tightness and the unemployment rate slowly approach their steady state values, which are reached after about 35 to 45 years. The tax rate follows the decreasing unemployment rate. As in the low risk aversion case, the interest rate takes the longest time to come down to its new level.

Figure 3: Transition to the new steady state.



Table 13 shows the outcome of a vote on the optimal defined benefit system when agents correctly anticipate the transition process to the new long-rung equilibrium. These results are quite different from the fifty-something percent vote in favour of the optimal policies derived above under the assumption that the economy immediately jumps to the new steady-state. Now, the outcome is not only quantitatively different between the three cases, it changes qualitatively. In the low risk aversion case, the approval of 65% for the alternative policy is

very clear cut. At the other extreme, in the high risk aversion case, a 52% majority of the voters now rejects the optimal system. The $\gamma=2$ scenario with a 57.1% vote for the alternative policy lies somewhere in the middle.

Table 13: Approval for policy switch to optimal unemployment system, accounting for transition period.

case	approval for optimal policy		
	$\gamma=1$	$\gamma=2$	$\gamma=3$
opt. def. benefit	65.1%	57.1%	47.6%

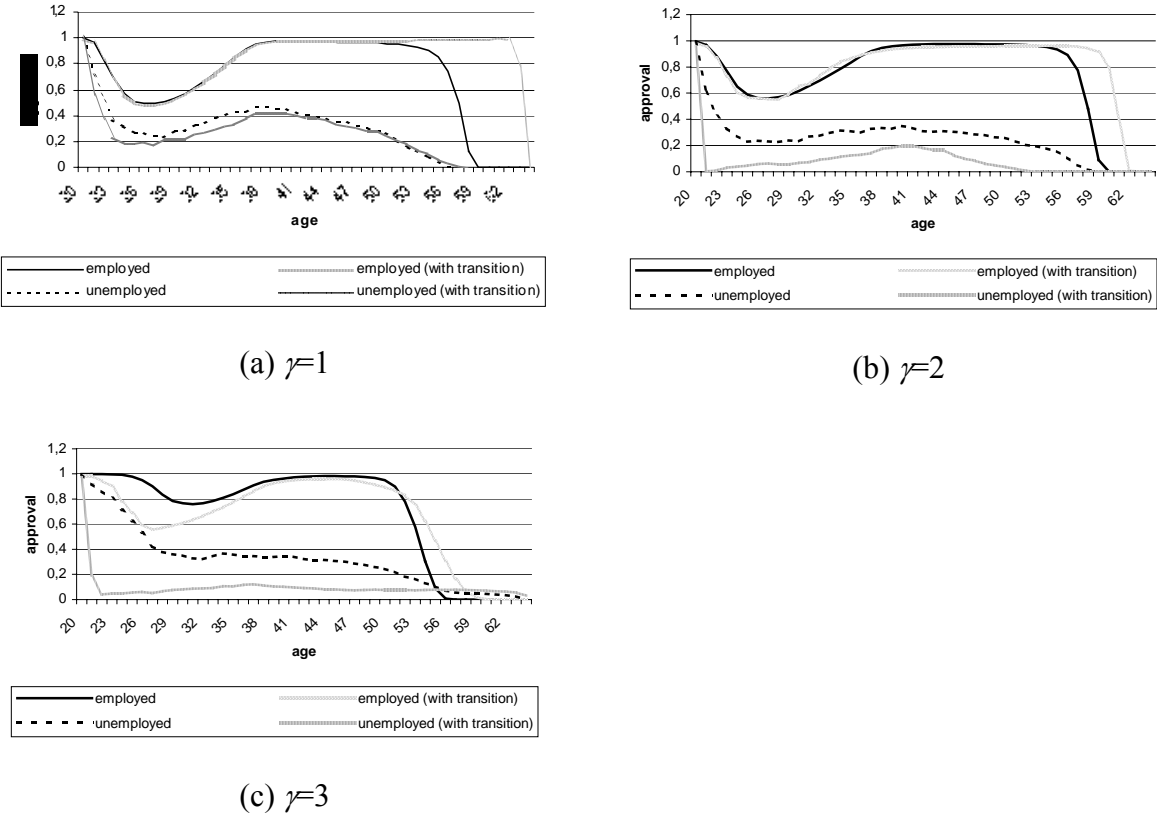
This change of the voting outcome compared to the case when the transition period is ignored is mainly caused by two effects working in opposite directions. Firstly, as the interest rate only decreases very slowly, the expected income loss for older (i.e. wealthier) workers compared to the status quo is small. This makes the policy switch more acceptable for agents who are close to their retirement age. Secondly, as the employment prospects only improve gradually and the tax rate also needs some time to reach its low steady state value, but the benefits immediately drop to their new level, some working age agents may find the optimal policy less appealing when they account for the transitional dynamics.¹⁰

Figure 4 shows the different voting outcomes with and without taking the transition into account by age and employment status. In case of low risk aversion, both the market tightness and the tax rate jump to their new equilibrium values immediately after the policy change. Only the interest rate needs some time to adjust. As a consequence, the voting behaviour is almost the same whether or not the transitional dynamics are considered, except for workers older than 60 years, who approve the optimal policy if the interest rate decreases only slowly. In the $\gamma=2$ case this effect is present, too, though less pronounced. Also, a much smaller share of the unemployed agents votes for the regime switch if accounting for the transition effects. Finally, if the adjustment process is considered in the high risk aversion case, most unemployed agents reject the unemployment insurance reform. Moreover, even a significant share of young employees vote against the optimal system. As the gradual adjustment of the

¹⁰ Retirees are still worse off if the regime switch is made. There is only one exception: In the first year after the policy change in the low risk aversion case, the interest rate is slightly above its previous level. Therefore, a group of agents older than 77 years, representing 3.6% of the population, now vote for the optimal policy in this case.

interest rate only has a small influence on the voting behaviour of older workers, the optimal unemployment insurance system is now rejected by the majority of the voters.

Figure 4: Approval for optimal unemployment system, with and without accounting for transition.



4. Summary and Conclusions

This paper presented a general equilibrium heterogeneous agent model with search unemployment and individual productivities that depend on the employment history. The model was calibrated to the German economy, using a simplified version unemployment insurance system in place until 2004 as the base case. Two basic types of alternative UI systems were considered, one with defined benefits and one with defined replacement ratios. Both systems allow for three different benefit levels, one for the first six months of an unemployment spell, one for months seven to twelve, and one for durations beyond one year. The welfare maximising UI systems were computed for different levels of risk aversion and for a closed and an open economy. Looking at the results, several patterns emerge. Firstly, the defined benefit systems was superior to the defined replacement ratio systems in terms of welfare in all cases considered. Secondly, while optimal benefits for long-term unemployed

individuals increase with the level of risk aversion, they are never very high. For a relative risk aversion of 3, the long-run benefit is about 30% of the average income, and the replacement ratio is 40%. Thirdly, the benefits paid to unemployed during the first year of the spell very low in the closed economy, where they have a negative external effect on aggregate saving and thus the capital stock, but can be quite high in the open economy. Finally, there is a considerable potential for welfare improvements by switching to the optimal UI system, even abolishing the whole status quo insurance would result in welfare gains.

As the next step, it was asked whether a majority of the population would support switching from the current UI system with relatively high replacement ratios to the optimal policy. When voters can choose between the long-run equilibria associated with the current and the alternative policy, the majority votes for the regime change. In the closed economy case, the approval for the optimal policy is just above 50%, regardless of the level of risk-aversion assumed. There are basically two important demographic groups who oppose the policy change, namely old agents (retired individuals and older workers), who would suffer interest income losses under the optimal policy, and young agents with low resources, who might have difficulties maintaining an adequate consumption level in some states if the optimal system was imposed. In the open economy cases, almost the whole population supports the optimal UI system.

Finally, the transitional dynamics after the introduction of an optimal unemployment insurance scheme were analysed. The speed of adjustment to the new steady-state equilibrium varies strongly between the low and high risk aversion scenarios considered. In the latter case, the change of the labour market variables takes very long. In all simulations, the adjustment of the interest rate was very slow. When agents take this transition process into account, the voting outcome looks different: In the low risk aversion case, the approval for the reform is increased from about 50% to almost two thirds, whereas in the high risk aversion scenario, the majority of the voters now opposes the optimal system.

This paper contributes to the literature by incorporating a history dependent individual productivity parameter in a heterogeneous agents model with search unemployment in the spirit of Costain (1999). The model thus not only captures the (negative) moral hazard effects of unemployment insurance, it also values the consumption insurance effect to a higher degree than models that do not allow for permanent income shocks.

A central message of this paper is that transitional effects after a policy change may be quite important. It was shown that taking these effects into account can qualitatively change the

outcome of a vote on an alternative policy. The same might be true of welfare comparisons of alternative policies, when a utilitarian welfare measure is employed.

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