Keynesian Dynamics and the Wage-Price Spiral. Estimating and Analyzing a Baseline Disequilibrium Approach

Pu Chen
Faculty of Economics
Bielefeld University, Bielefeld
Germany

Carl Chiarella
School of Finance and Economics
University of Technology, Sydney
Australia

Peter Flaschel
Faculty of Economics
Bielefeld University
Germany

Willi Semmler
Faculty of Economics
Bielefeld University, Bielefeld
Germany

November 10, 2004

Abstract:

In this paper, we reformulate the theoretical baseline DAS-AD model of Asada, Chen, Chiarella and Flaschel (2004) to allow for its somewhat simplified empirical estimation. The model now exhibits a Taylor interest rate rule in the place of an LM curve and a dynamic IS curve and dynamic employment adjustment. It is based on sticky wages and prices, perfect foresight of current inflation rates and adaptive expectations concerning the inflation climate in which the economy is operating. The implied nonlinear 6D model of real markets disequilibrium dynamics avoids striking anomalies of the old Neoclassical synthesis and can be usefully compared with the model of the new Neoclassical Synthesis when the latter is based on both staggered prices and wages. It exhibits typical Keynesian feedback structures with asymptotic stability of its steady state for low adjustment speeds and with cyclical loss of stability – by way of Hopf bifurcations – when certain adjustment speeds are made sufficiently large.

In the second part we provide system estimates of the equations of the model in order to study its stability features based on empirical parameter estimates with respect to its various feedback channels. Based on these estimates we find that the dynamics is strongly convergent around the steady state, but will loose this feature if the inflationary climate variable adjusts sufficiently fast. We also study to which extent more active interest rate feedback rules or downward wage rigidity can stabilize the dynamics in the large when the steady state is made locally repelling by a faster adjustment of inflationary expectations. We find support for the orthodox view that (somewhat restricted) money wage flexibility is the most important stabilizer in this framework, while monetary policy should allow for sufficient steady state inflation in order to avoid stability problems in areas of the phase space where wages are still not very flexible in a downward direction.

Keywords: DAS-DAD growth, wage and price Phillips curves, nonlinear estimation, stability, economic breakdown, persistent cycles, monetary policy.

JEL CLASSIFICATION SYSTEM: **E24**, **E31**, **E32**.

1 Introduction

In this paper, we reformulate, simplify and also extend from the empirical perspective a mature, but traditionally oriented theoretical model of disequilibrium AS-AD dynamics with both traditional, but also quite recent microfoundations, as for example provided in Blanchard and Katz' (1999) analysis of the dynamics of money wages. Our model of now in fact D(isequilibrium)AS-D(isequilibrium)AD growth is in its qualitative features based on our earlier theoretical presentation and analysis of a model of DAS-AD growth dynamics, and represents a significant reformulation of the conventional neoclassical AS-AD framework, see Asada, Chen, Chiarella, Flaschel (2004) for details. It replaces the LM curve of the earlier paper by a Taylor interest rate policy rule, as in the New Keynesian approaches. The model, as well as its theoretical analog, exhibits sticky wages as well as sticky prices, underutilized labor as well as capital stock, myopic perfect foresight of current wage and price inflation rates and adaptively formed medium-run calculations concerning the inflation climate in which the economy is operating. Moreover we now employ a dynamic IS-equation in the place of the originally static one of the Asada, Chen, Chiarella, Flaschel (2004) paper and will also make use of a dynamic form of Okun's law in addition.

The resulting nonlinear 5D model of labor and goods market disequilibrium dynamics (with a Taylor rule based treatment of the financial part of the economy) avoids the striking anomalies of the conventional AS-AD model of the old Neoclassical synthesis when analyzed under the assumption of myopic perfect foresight.[1] Instead it exhibits Keynesian feedback dynamics proper with in particular asymptotic stability of its unique interior steady state solution for low adjustment speeds of wages, prices, and expectations among others. The loss of stability occurs cyclically, by way of Hopf bifurcations, when some of these adjustment speeds are made sufficiently large, even leading eventually to purely explosive dynamics sooner or later. This latter fact – if it occurs – implies the need to look for appropriate extrinsic (behavioral) nonlinearities that can bound the dynamics in an economically meaningful domain, such as (some) downward rigidity of wages and prices and the like, if the economy departs too much from its steady state position. This procedure of making an explosive dynamics bounded and thus viable stands in stark contrast to the New Keynesian approach to macrodynamics where on a similar level of formalization total instability is desirable and achieved by the choice of an appropriate Taylor policy rule and where the economy is then made a bounded one simply by assumption (and thus always sitting in the steady state if exogenously given stochastic processes are removed from the dynamics).

Our approach is indeed – formally seen – closely related to the New Keynesian one. We use the same formal structure for the variables that drive wage and price inflation rates (utilizations rates and real wages), but with a microfoundations that are for example based on Blanchard and Katz's (1999) reconciliation of Wage Phillips Curves and current labor market theories. The basic difference in the wage-price module is on this basis that we augment this structure by hybrid expectations formation where the forward-looking part is based on a neoclassical type of dating and where expectations are of cross-over

[1]These anomalies include in particular saddle point dynamics that imply instability unless some poorly motivated – and indeed inconsistent – jumps are imposed on certain variables, here in fact on both the price and the wage level, despite the existence of a money wage Phillips Curve (WPC), see Asada, Chen, Chiarella and Flaschel (2004) for details.

type, i.e., we have price inflation expectations in the Wage Phillips Curve (WPC) and wage inflation expectations in the Price Phillips Curve (PPC). Our formulation of the wage-price module allows therefore an interesting comparison to situations where New Keynesian authors allow for both staggered prices and wages. Concerning the IS-curve we make use of a law of motion for the rate of capacity utilization of firms that depends on the level of capacity utilization (the dynamic multiplier), the real rate of interest and finally on the real wage and thus on income distribution. New Keynesian authors generally use a purely forward-looking IS-curve (with only the real rate of interest effect), a procedure that – like the purely forward-looking New Keynesian Phillips curve – has been criticized from the empirical point of view, see in particular the recent paper by Fuhrer and Rudebusch (2004) with regard to the relevance of forward-looking behavior in the IS-representation of goods-market dynamics. Since we distinguish between the rate of employment of the labor force and that of the capital stock, the rate of capacity utilization, we finally employ some form of Okun's law to relate these to variables with each other.

The model of this paper can therefore, on the one hand, usefully compared to the New Keynesian one with staggered wage and price dynamics, but is on the other hand radically different from this approach with respect to implications, since we are not forced into a framework with four forward-looking variables where we have to look for four unstable roots in order to get the conclusion that the model is always sitting in its steady state (assuming boundedness as solution procedure) as long as only isolated shocks occur and is thus driven as far as business cycle implications are concerned solely by the stochastic processes that are added to its deterministic core. We have forward-looking behavior (with neoclassical dating) and will find asymptotic stability in the traditional sense of the word over certain ranges in the parameter space, while – in the case of local instability – we look for behavioral nonlinearities that allow the dynamics to remain bounded in an economically meaningful range in the place of an imposition of such boundedness on admissible solution curves and looking for determinacy.

We, by contrast, therefore obtain and can prove – still from the purely theoretical perspective – based on empirically still unrestricted sizes of the considered adjustment speeds of wages, prices, and quantities, the existence of damped, persistent or explosive fluctuations in the real and the nominal part of the dynamics, in the rates of capacity utilization of both labor and capital, and of wage and price inflation rates which here induce interest rate adjustments by the monetary authority through the attempt to stabilize the observed output and price level fluctuations, providing us with a Keynesian theory of an income distribution driven cycle, including a modern approach to monetary policy in such a context. This even holds in the case of myopic perfect foresight, where the structure of the od Neoclassical synthesis radically dichotomizes into independent classical supply-side and real dynamics – that cannot be influenced by monetary policy – and a subsequently determined inflation dynamics, that are purely explosive if the price level is taken as a predetermined variable, a situation that forces conventional approaches to these dichotomizing dynamics to assume convergence by an inconsistent application of the jump-variable technique,[2] see again Asada, Chen, Chiarella and Flaschel (2004) for details. In our new matured type of Keynesian labor and goods market dynamics we however can treat myopic perfect foresight of both firms and wage earners without

[2]since the nominal wage is transformed into a non-predetermined variable there, despite the initial assumption of only gradually adjusting money wages.

the need for the methodology of this 'rational expectations' solution in the context of an unstable saddlepoint dynamics, be it of old or new Keynesian type.

From the global perspective, if our theoretical model loses asymptotic stability for higher adjustment speeds, in the present framework specifically of prices and our inflationary climate expression, purely explosive behavior is the generally observed outcome, as it can be demonstrated by means of numerical simulations. The considered, so far only intrinsically nonlinear, model type therefore cannot be considered as being completely specified under such circumstances, since some mechanism is then required to bound the fluctuations to economically viable regions. Downward money wage rigidity was the mechanism we have often used for this purpose and which we will try here again (with limited success however as we shall see), in contrast to its successful application in the numerical investigations in Asada, Chen, Chiarella and Flaschel (2004).

The here estimated somewhat simplified feedback structure of their theoretical model, now indeed no longer (in general) supports the view (of Keynes and others) that downward money wage rigidity will stabilize the economy (as was shown in the structurally more elaborated earlier paper). Instead, this downward rigidity may now even cause economic breakdown when applied to situations that were strongly stable (convergent to the steady state) without it. This is due to our estimation of the dynamics of capacity utilization rates where we find, on the one hand, besides the usual negative dependence on the real rate of interest, a strong negative dependence on the real wage. On the other hand, we find in the wage-price block of the model the sign restrictions of New Keynesian wage and price inflation equations (but do not have their sign reversals in their reduced-form expressions later on, due to our different handling of forward-looking expectations and the inclusion of backward-looking ones). As far as the money wage Phillips curve is concerned we also confirm the form specified in Blanchard and Katz (1999) and find a similar general form to hold for the price inflation Phillips curve. These estimated curves then by and large suggest that real wage changes depend positively on economic activity, and this the more the stronger nominal wages react to the employment gap on the market for labor. In sum we therefore get that growth rate of real wages depends negatively on its level, and this stabilizing feedback chain to being the stronger the more flexible nominal wages react to labor market imbalances in the upward as well as in the downward direction. Complete downward wage flexibility may therefore become a problem, and this already in situations where the economy is producing fairly damped cycles (if the monetary authority is using too low an inflation target).

In the numerical simulations of the estimated model we will indeed then find that the reestablishment of money wage flexibility in severely depressed regions of the phase space (however coupled with some midrange downward wage rigidity) will avoid this breakdown, however at the costs of persistent economic fluctuations to some extent below the normal operating level of the economy. In the present framework (of a profit-led goods demand regime, where real wage increases decrease economic activity) finally established downward money wage flexibility is therefore good for economic stability and this the more the closer this stability feature is established to the steady state from below. The opposite conclusion however holds with respect to price flexibility. Excluding by our parameter estimates significant price level deflation – but not wage level deflation – from occurring, we thereby obtain and study a baseline model of the DAS-DAD variety with a rich set of stability implications and with various types of business cycle fluctuations that it can generate endogenously as well as exogenously (by adding stochastic processes to the considered deterministic dynamics). The dynamic outcomes of this baseline disequilibrium AS-AD or DAS-DAD model can be usefully contrasted with those of the currently fashionable New Keynesian alternative (the new Neoclassical Synthesis) that in our view is more limited in scope, at least as far as the treatment of interacting Keynesian feedback mechanisms and the thereby implied dynamic possibilities are concerned. A detailed comparison with this New Keynesian approach is provided in Chiarella, Flaschel and Franke (2004, Ch.1). This comparison reveals in particular that one does not really need the typical (in our view strange) dynamics of rational expectation models, based on the specification of certain forward looking variables, if such forward-looking behavior is coupled with backward-looking behavior for the medium-run evolution of the economy (and neoclassical dating in the forward-looking part) and if certain non-linearities in economic behavioral make the obtained dynamics bounded far off the steady state. In our approach standard Keynesian feedback mechanisms are coupled with a wage-price spiral having – besides partial forward-looking behavior – a considerable degree of inertia, with the result that these feedback mechanisms by and large work as expected (as known from partial analysis), in their interaction with the added wage and price level dynamics.

The present paper therefore intends to provide a empirically supported baseline model of the Keynesian DAS-DAD variety, not plagued by the theoretical anomalies of the old Neoclassical Synthesis and the empirical anomalies of the new Neoclassical Synthesis of the New Keynesian approach. It does so on the basis of the fully specified DAS-AD growth dynamics of Asada, Chen, Chiarella and Flaschel (2004), by transforming these dynamics into a reduced DAS-DAD format that can be estimated empirically. It discusses the feedback structure of this reduced form and its stability implications, first on a general level and then on the level of the sign and size restrictions obtained from empirical estimates of the six laws of motion of the dynamics. These estimates also allow us to show asymptotic stability for the estimated parameter sizes and to determine stability boundaries (with respect to the speed of adjustment of the inflationary climate) where the need for further (behavioral) nonlinearities therefore becomes a matter of fact, here only studied to a certain degree, but further investigated and discussed in a companion paper to the present one (see Asada, Chiarella, Flaschel and Hung, 2004).

Section 2 considers for later comparison the New Keynesian macrodynamic model with staggered wage and price setting in a deterministic and continuous time framework. Section 3 then presents our reformulation of the baseline Keynesian DAS-AD growth dynamics of Asada, Chen, Chiarella and Flaschel (2004) as a DAS-DAD growth dynamics in order to make this model applicable to empirical estimation. Section 4 considers the feedback chains of the reformulated model and derives cases of local asymptotic stability and of loss of stability by way of Hopf-bifurcations. In section 5 we then estimate the model to find out sign and size restrictions for its behavioral equations and which type of feedback mechanisms may apply to the US-economy after World War II. Section 6 investigates in detail the stability properties of the estimated dynamics. Section 7 analyzes on the one hand the stability problems that occur when there is a floor to money wage deflation and the role of monetary policy in such a context. On the other hand, it studies the role of such a floor and its removal in more or less severe depressions from the global point of view, when the steady state becomes a local repeller due to a faster , adjustment of the inflationary climate expression considered to surround current perfectly foreseen inflation. We here find that the combination of some downward rigidity in wage deflation coupled with its returning flexibility when depressions become sufficiently severe will on the one hand avoid the explosive fluctuations of the completely unrestricted case, while also avoiding complete the economic collapse that would come about when the floor to wage deflation would be a global one. Section 8 concludes. Details of the estimation results are presented in an appendix to the paper.

2 New Keynesian macrodynamics

In this section we consider briefly the modern analog to the old neoclassical synthesis (in its Keynesian format), the New Keynesian approach to macrodynamics, and this already in its advanced form, where both staggered price setting and staggered wage setting are assumed. We here follow Woodford (2003, p.225) and Ereg et al. (2000) in their formulation of staggered wages and prices, where their joint evolution is coupled with the usual forward-looking output dynamics, coupled with a derived law of motion for real wages now in addition.[3] We shall only briefly look at this extended approach here in order to allow us to consider the similarities and differences between our later and these New Keynesian dynamics later on.

Woodford (2003, p.225) makes basically use of the following two loglinear equations for describing the joint evolution of wages and prices:[4]

$$\begin{aligned}\Delta \ln w_t &\overset{WPC}{=} \beta E_t(\Delta \ln w_{t+1}) + \beta_{wy} \ln Y_t - \beta_{w\omega} \ln \omega_t \\ \Delta \ln p_t &\overset{PPC}{=} \beta E_t(\Delta \ln p_{t+1}) + \beta_{py} \ln Y_t + \beta_{p\omega} \ln \omega_t\end{aligned}$$

where all parameters are assumed as positive. Our first aim here is to derive the continuous time analog to these two equations (and the other equations of the full model) and to show on this basis how this extended model is solved in the spirit of the rational expectations school.

In a deterministic setting we obtain from the above

$$\begin{aligned}\Delta \ln w_{t+1} &\overset{WPC}{=} \frac{1}{\beta}[\Delta \ln w_t - \beta_{wy} \ln Y_t + \beta_{w\omega} \ln \omega_t] \\ \Delta \ln p_{t+1} &\overset{PPC}{=} \frac{1}{\beta}[\Delta \ln p_t - \beta_{py} \ln Y_t - \beta_{p\omega} \ln \omega_t]\end{aligned}$$

and thus get, if we assume (in all of the following and without much loss in generality) that the parameter β is not only close to one, but in fact set equal to one:

$$\begin{aligned}\Delta \ln w_{t+1} - \Delta \ln w_t &\overset{WPC}{=} -\beta_{wy} \ln Y_t + \beta_{w\omega} \ln \omega_t \\ \Delta \ln p_{t+1} - \Delta \ln p_t &\overset{PPC}{=} -\beta_{py} \ln Y_t - \beta_{p\omega} \ln \omega_t\end{aligned}$$

Denoting by π^w the rate of wage inflation and by π^p the rate of price inflation (both indexed by the end of the corresponding period) then gives rise to (when transferred to continuous time, with $\ln Y = y$ and $\theta = \ln \omega$):

[3]Further approaches that incorporate both wage and price staggering are provided in Christiano, Eichenbaum and Evans (2001) and Sbordone (2001, 2002).

[4]Δ the backward oriented difference operator.

$$\boxed{\begin{array}{lcl} \dot{\pi}^w & \overset{WPC}{=} & -\beta_{wy}y + \beta_{w\omega}\theta \\ \dot{\pi}^p & \overset{PPC}{=} & -\beta_{py}y - \beta_{p\omega}\theta \end{array}}$$

With respect to the output dynamics of the New Keynesian approach:

$$y_t = y_{t+1} - \alpha_{yi}(i_t - \pi^p_{t+1} - i_0), \quad i.e., \quad y_{t+1} - y_t = \alpha_{yi}(i_t - \pi^p_{t+1} - i_0)$$

we get on this basis the reduced form law of motion

$$\boxed{\dot{y} \overset{IS}{=} \alpha_{yi}[(\beta_{i\pi} - 1)\pi^p + (\beta_{iy} + \beta_{py})y + \beta_{p\omega}\theta)]}$$

where we have already inserted an interest rate policy rule in order to (hopefully) get determinacy as in the New Keynesian baseline model, which is known to be indetermined for the case of an interest rate peg. As Taylor interest rate policy rule we here have chosen the simple rule:

$$i = i_t = i_o + \beta_{i\pi}\pi + \beta_{iy}y$$

see Walsh (2003, p.247), which is of a classical Taylor rule type.

There remains finally the law of motion for real wages to be determined, which due to $\theta = \ln\omega$ simply reads

$$\boxed{\dot{\theta} = \pi^w - \pi^p}$$

We thus get from this extended New Keynesian model an autonomous linear dynamics, in the variables π^w, π^p, y and θ. The in general uniquely determined steady state of the dynamics is given by $(0.0, 0, i_o)$. From the definition of θ we obtain, in direct generalization of the baseline New Keynesian model with only staggered price setting, that the model exhibits four forward-looking variables. Searching for a zone of determinacy of the dynamics (appropriate parameter values that make the steady state the only bounded solution of the dynamics to which the economy then immediately returns after isolated shocks of any type) thus demands to establish conditions such that all roots of the Jacobian have positive real parts.

The Jacobian of the 4D dynamical system under consideration reads:

$$J = \begin{pmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \alpha_{yi}(\beta_{i\pi} - 1) & \alpha_{yi}(\beta_{iy} + \beta_{py}) & \alpha_{yi}\beta_{p\omega} \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

For the determinant of this Jacobian we therefrom get

$$-|J| = (\beta_{wy}\beta_{p\omega} + \beta_{py}\beta_{w\omega})\alpha_{yi}(\beta_{i\pi} - 1) \gtreqless 0 \quad \text{iff} \quad \beta_{i\pi} \gtreqless 1$$

We thus get that an active monetary policy ($\beta_{i\pi} > 1$) is no longer appropriate to ensure determinacy (for which a positive determinant of J is a necessary condition). There arises the necessity to specify an extended interest rate policy rule from which one can obtain determinacy (the steady state as the only stable solution and the only realized situation in this deterministic setup) as in the New Keynesian baseline model, which is known to be indetermined in the case of an interest rate peg, but which is always determined for $\beta_{i\pi} > 1$.

There are a variety of critical arguments raised in the literature against the New Phillips Curve (NPC) of the baseline model of Keynesian macrodynamics, see in particular Mankiw (2001) and recently Eller and Gordon (2003) for particular strong statements.[5] These and other criticisms in our view will also apply to the above extended wage and price dynamics. In view of these and other critiques we here propose the following modifications to the above presentation of the wage-price dynamics which will remove from it completely the questionable feature of a sign reversal for the role of output and wage gaps, caused by the fact that future values of the considered state values are used on the right hand side of their determining equations, which implies that the time rates of change of these variables depend on output and wage gaps with a reversed sign in front on them. These sign reversals are at the root of the problem when the empirical relevance of such NPC's is investigated. We instead will make use of the following expectations augmented wage and price Phillips curves:

$$\begin{aligned}
\Delta \ln w_{t+1} &\overset{WPC}{=} \kappa_w \Delta \ln p_{t+1} + (1-\kappa_w)\pi_t^m + \beta_{wy} \ln Y_t - \beta_{w\omega} \ln \omega_t] \\
\Delta \ln p_{t+1} &\overset{PPC}{=} \kappa_p \Delta \ln w_{t+1} + (1-\kappa_p)\pi_t^m + \beta_{py} \ln Y_t + \beta_{p\omega} \ln \omega_t]
\end{aligned}$$

We have modified the New Keynesian approach to wage and price dynamics here only with respect to the terms that concern expectations, in order to obtain the situation of in fact a wage-price spiral from it. We first assume that expectation formation is of a crossover type, with perfectly foreseen price inflation in the WPC of workers and perfectly foreseen wage inflation in the PPC of firms. Furthermore, we make use in this regard of a neoclassical dating in the considered PC's, which means that – as in the reduced form PC then often considered – we have the same dating for expectations and actual wage and price formation on both sides of the PC's. Finally, following Chiarella and Flaschel (1996), we assume expectations formation to be of a hybrid type, where a certain weight is given to current (perfectly foreseen) inflation rates and the counterweight attached to a concept which we have dubbed the inflationary climate π^m that is surrounding the currently evolving wage-price spiral. We thus assume that workers as well as firms to a certain degree pay attention of whether the current situation is embedded in a high inflation regime or in a low inflation one.

[5] With respect to the New Phillips curve it is stated in Mankiw (2001): "Although the new Keynesian Phillips curves has many virtues, it also has one striking vice: It is completely at odd with the facts."

These relatively straightforward modifications of the New Keynesian approach to expectations formation will imply for the dynamics of what we call a matured traditional Keynesian approach – completed in the next section – radically different solutions and stability features, with in particular no need to single out the steady state as the only relevant situation for economic analysis in the deterministic setup here considered. Concerning microfoundations for the assumed wage-price spiral we here only note that the WPC can be microfounded as in Blanchard and Katz (2000), using standard labor market theories, giving rise to nearly exactly the form shown above (with the unemployment gap in the place of the logarithm of the output gap) if hybrid expectations formation is embedded into their approach in addition. Concerning the PPC a similar procedure may be applied based on desired markups of firms. Along these lines one in particular gets an economic motivation for the inclusion of – indeed the logarithm – of the real wage (or wage share) with negative sign into the WPC and with positive sign into the PPC, without any need for loglinear approximations. We furthermore use the (un-)employment gap and the capacity utilization gap in these two PC's, respectively, in the place of a single measure (the log of the output gap). We conclude that the above wage-price spiral is an interesting alternative to the – theoretically rarely discussed and empirically questionable – New Keynesian form of wage-price dynamics. This wage-price spiral will be embedded into a complete Keynesian approach in the next section, exhibiting a dynamic IS-equation as in Rudebusch and Svensson (1999), now also including real wage effects and thus a role for income distribution, exhibiting furthermore Okun's law as the link from goods to labor markets and exhibiting of course the classical type of a Taylor interest rate policy rule in the place of an LM-curve.

3 Keynesian disequilibrium dynamics: Empirically oriented reformulation of a baseline model

In this section we reformulate the theoretical disequilibrium model of AS-AD growth of Asada, Chen, Chiarella and Flaschel (2004) in order to make it applicable for empirical estimation in a somewhat simplified way and for the study of the role of contemporary interest rate policy rules in such a framework. We dismiss now the LM curve of the original approach and replace it here by a Taylor type policy rule and use in addition dynamic IS as well as employment equations in the place of the originally static ones, where with respect to the former the dependence of consumption and investment on income distribution now only appears in an aggregated format. We use Blanchard and Katz (2000) error correction terms both in the wage and the price Phillips curve and thus give income distribution a role to play in wage as well as in price dynamics. Finally, we will have again inflationary inertia in a world of myopic perfect foresight through the inclusion of a medium-run variable, the inflationary climate in which the economy is operating, and its role for the wage - price dynamics of the considered economy.

We start from the observation that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found for this in Keynes' General Theory) allow for under- (or over-)utilized labor *as well as* capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes, IS-LM is (or should be) based on imperfectly flexible wages and prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that also medium-run aspects count both in wage and price adjustments, here formulated in simple terms by the introduction of the concept of an inflation climate. We have moreover model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS-AD model that we shall consider thus treats – as already described in the preceding section – expectations in a hybrid way, with crossover myopic perfect foresight of the currently evolving rates of wage and price inflation on the one hand and an adaptive updating of an inflation climate expression with exponential or any other weighting schemes on the other hand.

We consequently assume two Phillips Curves in the place of only one. In this way, we can discuss wage and price dynamics separately from each other, in their structural forms, now indeed both based on their own measure of demand pressure, namely $V^l - \bar{V}^l, V^c - \bar{V}^c$, in the market for labor and for goods, respectively. We here denote by V^l the rate of employment on the labor market and by $\bar{V}^l$ the NAIRU-level of this rate, and similarly by V^c the rate of capacity utilization of the capital stock and $\bar{V}^c$ the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation rates, $\hat{w}, \hat{p}$, are both augmented by a weighted average of corresponding cost-pressure terms, based on forward looking myopic perfect foresight $\hat{p}, \hat{w}$, respectively, and a backward looking measure of the prevailing inflationary climate, symbolized by π^m.

We thereby arrive at the following two Phillips Curves for wage and price inflation, which in this core version of Keynesian AS-AD dynamics are – qualitatively seen – formulated in a fairly symmetric way.[6] We stress that we include forward-looking behavior here, without the need for an application of the jump variable technique of the rational expectations school in general and the New Keynesian approach in particular as will be shown in the next section.[7]

The structural form of the wage-price dynamics:

$$\hat{w} = \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o) + \kappa_w\hat{p} + (1-\kappa_w)\pi^m, \qquad (1)$$

$$\hat{p} = \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o) + \kappa_p\hat{w} + (1-\kappa_p)\pi^m. \qquad (2)$$

Somewhat simplified versions of these two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2004), Flaschel, Kauermann and Semmler (2004) and Chen and Flaschel (2004) and were found there to represent a significant improvement over the conventional single reduced-form Phillips curve, there with wage flexibility being greater than price flexibility with respect to their demand pressure measure in the market for goods and for labor,[8] respectively, and with workers being

[6]With respect to empirical estimation one could also add the role of labor productivity growth, yet will here not do so, but concentrate on the cycle component of the model, caused by changing income distribution in a world of stable goods market and interest rate dynamics. With respect to the distinction between real wages, unit wage costs we shall therefore detrend the corresponding time series such that the following types of PC's can still be applied.

[7]For a detailed comparison with the New Keynesian alternative to our model type see Chiarella, Flaschel and Franke (2004).

[8]for lack of better phrases we associate the degree of wage and price flexibility with the size of the parameters β_{w_1}, β_{p_1}, though of course the extent of these flexibilities will also depend on the size of the fluctuations of the excess demands in the market for labor and for goods, respectively.

more short-sighted than firms with respect to their cost pressure terms. Note that such a finding is not possible in the conventional framework of a single reduced-form Phillips curve. Inflationary expectations over the medium run, π^m, i.e., the inflationary climate in which current inflation is operating, may be adaptively following the actual rate of inflation (by use of some linear or exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other possibilities for updating expectations. For simplicity of the exposition we shall make use of the conventional adaptive expectations mechanism in the theoretical part of this paper:

$$\boxed{\dot{\pi}^m = \beta_{\pi^m}(\hat{p} - \pi^m) \qquad (3)}$$

Note that for our current version of the wage-price spiral, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by (with $\kappa = 1/(1 - \kappa_w \kappa_p)$):

$$\hat{\omega} \;=\; \kappa[(1-\kappa_p)(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o)) - (1-\kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))]$$

This follows easily from the following obviously equivalent representation of the above two PC's:

$$\begin{aligned} \hat{w} - \pi^m &= \beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o) + \kappa_w(\hat{p} - \pi^m), \\ \hat{p} - \pi^m &= \beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o)) + \kappa_p(\hat{w} - \pi^m), \end{aligned}$$

by solving for the variables $\hat{w} - \pi^m$ and $\hat{p} - \pi^m$. It also implies the following two across-markets or reduced form PC's:

$$\begin{aligned} \hat{p} = &\quad \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o) + \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o))] + \pi^m, \\ \hat{w} = &\quad \kappa[\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o)) + \kappa_w(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))] + \pi^m, \end{aligned}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market.

The remaining laws of motion of the private sector of the model are as follows:

$$\boxed{\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) && (4) \\ \hat{V}^l &= \alpha_{V^l_1}(V^c - \bar{V}^c) + \alpha_{V^l_2}\hat{V}^c && (5) \end{aligned}}$$

The first law of motion is of the type of a dynamic IS-equation, see also Rudebusch and Svensson (1999) in this regard, here represented by the growth rate of the capacity utilization rate of firms. It reflects the dependence of output changes on aggregate income and thus on the rate of capacity utilization by assuming a negative, i.e., stable dynamic multiplier relationship in this respect, it shows the joint dependence of consumption and investment on the real wage (which in the aggregate may in principle allows for positive or negative signs before the parameter α_ω, depending on whether consumption or investment is more responsive to real wage changes), and shows finally the negative influence of the real rate of interest on the evolution of economic activity. Note here that we have generalized this law of motion in comparison to the one in the original baseline model of Asada, Chen, Chiarella and Flaschel (2004), since we now allow for the possibility that also consumption, not only investment, depends on income distribution as measured by the real wage. We note that we also use $\ln \omega$ in the dynamic multiplier equation, since this variable will be used later on to estimate this equation.

In the second law of motion, for the rate of employment, we assume that the employment policy of firms follows – in the form of a generalized Okun Law – the rate of capacity utilization (and the thereby implied rate of over- or underemployment of the employed workforce) partly with a lag (measured by $1/\beta_{V_1^l}$), and partly without a lag (through a positive parameter $\alpha_{V_2^l}$). Employment is thus assumed to adjust to the level of current activity in somewhat delayed form which is a reasonable assumption from the empirical point of view. The second term, $\alpha_{V_2^l}\hat{V}^c$, is added to take account of the possibility that Okun's Law may hold in level form rather than in the form of a law of motion, since this latter dependence can be shown to be equivalent to the use of a term $(V^c/\bar{V}^c)^{\alpha_{V_2^l}}$ when integrated, i.e., the form of Okun's law in which this law was originally specified by Okun (1967) himself.

The above two laws of motion therefore reformulate the static IS-curve and the employment this curve implies, as employed in Asada, Chen, Chiarella and Flaschel (2004), in a dynamic form. They only reflect implicitly the there assumed dependence of the rate of capacity utilization on the real wage, due to on smooth factor substitution in production (and the measurement of the potential output this implies in Asada, Chen, Chiarella and Flaschel (2004))), which constitutes another positive influence of the real wage on the rate of capacity utilization and its rate of change. This simplification helps to avoid the estimation of separate equations for consumption and investment C, I and for potential output Y^p as they were discussed and used in detail in this earlier paper.

Finally, we no longer to employ here a law of motion for real balances as was still the case in Asada, Chen, Chiarella and Flaschel (2004). Money supply is now accommodating to the interest rate policy pursued by the central bank and thus does not feedback into the core laws of motion of the model. As interest rate policy we assume the following classical type of Taylor rule:

$$\dot{r} = -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) \tag{6}$$

Note that we allow for interest rate smoothing in this rule. Furthermore, the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target $\bar{\pi}$ of the central bank. There is in addition the assumption of a positive influence of an output gap in the law of motion for the nominal rate of interest, here measured by the rate of excess capacity of firms. Note finally that we could have included (but have not done this here yet) a new kind of gap into the above Taylor rule, the real wage gap, since we have in our model a dependence of aggregate demand on income distribution and the real wage. The state of income distribution matters for the dynamics of our model and thus should also play a role in the decisions of the central bank. All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with it.

We note that the steady state of the considered dynamics is basically the same as the one considered in Asada, Chen, Chiarella and Flaschel (2004), with $\epsilon_o = 0, V^c_o = \bar{V}^c, V^l_o = \bar{V}^l, \pi^m_o = \bar{\pi}$. The values of ω_o, r_o are in principle determined as in Asada, Chen, Chiarella and Flaschel (2004), but are here just assumed as given, underlying the linear approximation of the IS curve of the present model around the steady state of the original framework (when adjusted to the considered modifications of the baseline model). As the model is formulated now it exhibits five gaps, to be closed in the steady state and has five laws of motion, which when set equal to zero, exactly imply this result, since the determinant of the Jacobian of the dynamics is shown to be always non-zero in the next section of the paper.

The steady state of the dynamics is locally asymptotically stable under certain sluggishness conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability by way of cycles (by way of so-called Hopf-bifurcations) if the system becomes too flexible, and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high, as we shall show below. If the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior – like downward money wage rigidity – to come into being at least far off the steady state in order to bound the dynamics to an economically meaningful domain in the considered 5D state space. Asada, Chiarella, Flaschel and Hung (2004) provide a variety of numerical studies for such an approach with extrinsically motivated nonlinearities and thus undertake its detailed numerical investigation. In sum, therefore, our dynamic AS-AD growth model here and there will exhibit a variety of features that are much more in line with a Keynesian understanding of the characteristics of the trade cycle than is the case for the conventional modelling of AS-AD growth dynamics or its radical reformulation by the New Keynesians (where – if non-determinacy can be avoided by the choice of an appropriate Taylor rule – only the steady state position is a meaningful solution in the related setup we considered in the preceding section).

Taken together the model of this section consists of the following five laws of motion (with the derived reduced form expressions as far as the wage-price spiral is concerned and with reduced form expressions by assumption concerning the goods and the labor market dynamics):[9]

[9] As the model is formulated we have no real anchor for the steady state rate of interest (via investment behavior and the rate of profit it implies in the steady state) and thus have to assume here that it is the monetary authority that enforces a certain steady state values for the nominal rate of interest.

$$
\begin{aligned}
\hat{V}^c &\stackrel{Dyn.IS}{=} -\alpha_{V^c}(V^c - \bar{V}^c) \pm \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r-\hat{p}) - (r_o - \bar{\pi})) && (7)\\
\hat{V}^l &\stackrel{O.Law}{=} \beta_{V_1^l}(V^c - \bar{V}^c) + \beta_{V_2^l}\hat{V}^c && (8)\\
\dot{r} &\stackrel{T.Rule}{=} -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) && (9)\\
\hat{\omega} &\stackrel{RWPC}{=} \kappa[(1-\kappa_p)(\underline{e}ta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o))\\
&\quad - (1-\kappa_w)(\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o))] && (10)\\
\dot{\pi}^m &\stackrel{I.Climate}{=} \beta_{\pi^m}(\hat{p} - \pi^m) \quad \text{or} && (11)\\
\pi^m(t) &= \pi^m(t_o)e^{-\beta_{\pi^m}(t-t_o)} + \beta_{\pi^m}\int_{t_o}^{t} e^{\beta_{\pi^m}(t-s)}\hat{p}(s)ds
\end{aligned}
$$

representing in correspondence to the baseline model of New Keynesian macroeconomics the IS-dynamics, Okun's Law and the Taylor Rule, but including now also the dynamics of the real wage, and the updating of the inflationary climate expression. We have to make use in addition of the following reduced form expression for the price inflation rate or the PPC:

$$
\begin{aligned}
\hat{p} &= \kappa[\beta_{p_1}(V^c - \bar{V}^c) + \beta_{p_2}(\ln\omega - \ln\omega_o)\\
&\quad + \kappa_p(\beta_{w_1}(V^l - \bar{V}^l) - \beta_{w_2}(\ln\omega - \ln\omega_o))] + \pi^m && (12)
\end{aligned}
$$

which has to be inserted into the above laws of motion in various places in order to get an autonomous nonlinear system of differential equations in the state variables: capacity utilization V^c, the rate of employment V^l, the nominal rate of interest r, the real wage rate ω, and the inflationary climate expression π^m. We stress that one can consider the eq. (12) as a sixth law of motion of the considered dynamics which however – when added — leads a system determinant which is zero and which therefore allows for zero-root hysteresis for certain variables of the model (in fact in the price level if the target rate of inflation of the CB is zero and if interest rate smoothing is present in the TR). We have written the laws of motion in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then our formulation of the dynamics of income distribution and of the inflationary climate in which the economy is operating.

With respect to the empirically motivated restructuring of the original theoretical framework, the model is as pragmatic as the approach employed by Rudebusch and Svensson (1999). By and large we believe that it represents a working alternative to the New Keynesian approach, in particular when the current critique of the latter approach is taken into account. It overcomes the weaknesses and the logical inconsistencies of the old Neoclassical synthesis, see Asada, Chen, Chiarella and Flaschel (2004), and it does so in a minimal way from a mature, but still traditionally oriented Keynesian perspective (and is thus not really 'New'). It preserves the problematic stability features of the real rate of interest channel, where the stabilizing Keynes effect or the interest rate policy of the central bank is interacting with the destabilizing, expectations driven Mundell effect. It preserves the real wage effect of the old Neoclassical synthesis, where – due to an unambiguously negative dependence of aggregate demand on the real wage – we had that price flexibility was destabilizing, while wage flexibility was not. This real wage channel is not really a topic in the New Keynesian approach, due to the specific form of wage-price dynamics there considered, see the preceding section, and it is summarized in the figure 1 for the situation where investment dominates consumption with respect to real wage changes. In the opposite case, the situations considered in this figure will be reversed with respect to their stability implications.

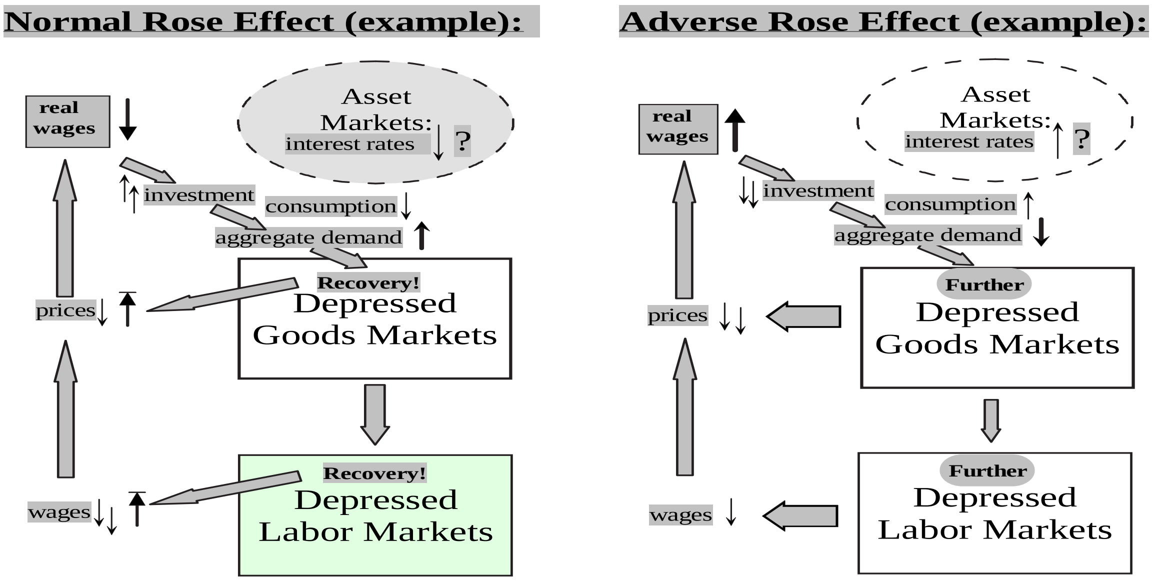

Figure 1: Rose effects: The real wage channel of Keynesian macrodynamics .

The feedback channels just discussed will be the focus of interest in the now following stability analysis of our D(isquilibrium)AS-D(isquilibrium)AD dynamics. We have employed reduced-form expressions in the above system of differential equations whenever possible. We have thereby obtained a dynamical system in five state variables that is in a natural or intrinsic way nonlinear (to its reliance on growth rate formulations). We note that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms with respect to some stability assertions it gives rise to. A rigorous proof of local asymptotic stability and its loss by way of Hopf bifurcations can be found in Asada, Chen, Chiarella and Flaschel (2004), there for the original baseline model. For the present model variant we supply a more detailed stability proofs in Asada, Chiarella, Flaschel and Hung (2004), where also more detailed numerical simulations of the model will be provided.

4 5D Feedback-guided stability analysis

In this section we illustrate an important method to prove local asymptotic stability of the interior steady state of the dynamical system (7) – (11) (with eq. (12) wherever needed) through partial motivations from the feedback chains that characterize this empirically oriented baseline model of Keynesian dynamics. Since the model is an extension of the standard AS-AD growth model, we know from the literature that there is a real rate of interest effect typically involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). Instead of the stabilizing Keynes-effect, based on activity-reducing nominal interest rate increases following price level increases, we have here however a direct steering of economic activity by the interest rate policy of the central bank. Secondly, if the correctly anticipated short-run real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is the activity stimulating (partial) effect of increases in the rate of inflation (as part of the real rate of interest channel) that may lead to accelerating inflation under appropriate conditions. This is the so-called Mundell-effect that normally works opposite to the Keynes-effect, but through the same real rate of interest channel as this latter effect. Due to our use of a Taylor rule in the place of the conventional LM curve, the Keynes-effect is here implemented in a more direct way towards a stabilization of the economy (coupling nominal interest rates directly with the rate of price inflation) and it is supposed to work the stronger the larger the parameters γ_p, γ_{V^c} are chosen. The Mundell-effect by contrast is the stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have a positive influence of this climate variable both on price as well as on wage inflation and from there on rates of employment of both capital and labor.

There is a further important potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital and thus negatively on the real wage ω. We thus get – since consumption may also depend (positively) on the real wage – that real wage increases can depress or stimulate economic activity depending on whether investment or consumption is dominating the outcome of real wage increases (we here neglect the stabilizing role of the additional Blanchard and Katz type error correction mechanisms). In the first case, we get from the reduced-form real wage dynamics:

$$\hat{\omega} \quad = \quad \kappa[(1-\kappa_p)\beta_w(V^l-\bar{V}^l)-(1-\kappa_w)\beta_p(V^c-\bar{V}^c)].$$

that price flexibility should be bad for economic stability, due to the minus sign in front of the parameter β_p, while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation as it was already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal and price flexibility to adverse Rose effects as far as real wage adjustments are concerned (if it is assumed – as in our theoretical baseline model – that only investment depends on the real wage). Besides real rate of interest effects, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella, Flaschel, Groh and Semmler (2000) for further discussion of these as well as of other feedback mechanisms of such Keynesian growth dynamics. We observe again that our theoretical DAS-AD growth dynamics in Asada, Chen, Chiarella and Flaschel (2004) – due to their origin in the baseline model of the Neoclassical Synthesis, stage I – allows for negative influence of real wage changes on aggregate demand solely, and thus only for cases of destabilizing wage level flexibility, but not price level flexibility. In the empirical estimation of the model (7) – (11) we will indeed find that this case seems to be the one that characterizes our empirically and broader oriented dynamics (7) – (11).

This adds to the description of the dynamical system (7) – (11) whose stability properties are now to be investigated by means of varying adjustment speed parameters appropriately. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, $\pi_o^m = \bar{\pi}$, if $\beta_{\pi^m} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\alpha_\omega = 0, \beta_{w_2} = 0, \beta_{p_2} = 0$ is assumed, since this decouples the ω-dynamics from the remaining system dynamics V^c, V^l, r. We will consider the stability of these 3D subdynamics – and its subsequent extensions – in informal terms here only, reserving rigorous calculations to the alternative scenarios provided in Asada, Chiarella, Flaschel and Hung (2004). We nevertheless hope to be able to show to the reader how one can indeed proceed systematically from low to high dimensional analysis in such stability investigations from the perspective of the partial feedback channels implicitly contained in the considered 5D dynamics. This method has been already applied successfully to various other, often more complicated, dynamical systems, see Asada, Chiarella, Flaschel and Franke (2003) for a variety of typical examples.

Before we start with our stability investigations we establish that loss of stability can in general only occur in the considered dynamics by way of Hopf-bifurcations, since the following proposition can be shown to hold true under mild – empirically plausible – parameter restrictions.

Proposition 1:

> *Assume that the parameter γ_r is chosen sufficiently small and that the parameters $\beta_{w_2}, \beta_{p_2}, \kappa_p$ fulfill $\beta_{p_2} > \beta_{w_2}\kappa_p$. Then: The 5D determinant of the Jacobian of the dynamics at the interior steady state is always negative in sign.*

Sketch of proof: We have for the sign structure in this Jacobian under the given assumptions the following situation to start with (we here assume as limiting situation $\gamma_r = 0$ and have already simplified the law of motion for V^l by means of the one for V^c through row operations that are irrelevant for the size of the determinant to be calculated):

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ + & + & 0 & + & + \\ - & + & 0 & - & 0 \\ + & + & 0 & + & 0 \end{pmatrix}.$$

We note that the ambiguous sigh in the entry J_{11} in the above matrix is due to the fact that the real rate of interest is a decreasing function of the inflation rate which in turn depends positively on current rates of capacity utilization.

Using the second row and the last row in their dependence on the partial derivatives of $\hat{p}$ we can reduces this Jacobian to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & + \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

without change in the sign of its determinant. In the same way we can now use the third row to get another matrix without any change in the sign of the corresponding determinants

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & - & 0 \\ 0 & + & 0 & + & 0 \end{pmatrix}$$

The last two columns can under the observed circumstances be further reduced to

$$J = \begin{pmatrix} 0 & 0 & - & \pm & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}$$

which finally gives

$$J = \begin{pmatrix} 0 & 0 & - & 0 & 0 \\ + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 \end{pmatrix}.$$

This matrix is easily shown to exhibit a negative determinant which proves the proposition, also for all values of γ_r which are chosen sufficiently small. ∎

Proposition 2:

Assume that the parameters $\beta_{w_2}, \beta_{p_2}, \alpha_\omega$ and β_{π^m} are all set equal to zero. This decouples the dynamics of V^c, V^l, r from the rest of the system. Assume furthermore that the partial derivative of the first law of motion depends negatively on V^c, i.e., the dynamic multiplier process, characterized by α_{V^c}, dominates this law of motion with respect to the overall impact of the rate of capacity utilization V^c.[10] Then: The interior steady state of the implied 3D dynamical system

$$\begin{aligned} \hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) && (13) \\ \hat{V}^l &= \beta_{V_1^l}(V^c - \bar{V}^c) && (14) \\ \dot{r} &= -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) && (15) \end{aligned}$$

is locally asymptotically stable if the interest rate smoothing parameter γ_r and the employment adjustment parameter β_{V^l} are chosen sufficiently small in addition.

Sketch of proof: In the considered situation we have for the Jacobian of these reduced dynamics at the steady state:

$$J = \begin{pmatrix} - & + & - \\ + & 0 & 0 \\ + & + & - \end{pmatrix}$$

[10] i.e., $\alpha_{V^c} > \alpha_p \kappa \kappa_p \beta_w$.

The determinant of this Jacobian is obviously negative if the parameter γ_r is chosen sufficiently small. Similarly, the sum of the minors of order 2: a_2, will be positive if β_{V^l} is chosen sufficiently small. The validity of the full set of Routh-Hurwitz conditions then easily follows, since trace J $= -a_1$ is obviously negative and since det J is part of the expressions that characterize the product $a_1 a_2$. ■

Proposition 3:

Assume now that the parameter α_ω is negative, but chosen sufficiently small, while the error correction parameters β_{w_2}, β_{p_2} are still kept at zero. Then: The interior steady state of the resulting 4D dynamical system (where the state variable ω is now included)

$$\begin{aligned}
\hat{V}^c &= -\alpha_{V^c}(V^c - \bar{V}^c) - \alpha_\omega(\ln\omega - \ln\omega_o) - \alpha_r((r - \hat{p}) - (r_o - \bar{\pi})) && (16)\\
\hat{V}^l &= \beta_{V_1^l}(V^c - \bar{V}^c) && (17)\\
\dot{r} &= -\gamma_r(r - r_o) + \gamma_p(\hat{p} - \bar{\pi}) + \gamma_{V^c}(V^c - \bar{V}^c) && (18)\\
\hat{\omega} &= \kappa[(1-\kappa_p)\beta_{w_1}(V^l - \bar{V}^l) - (1-\kappa_w)\beta_{p_1}(V^c - \bar{V}^c) && (19)
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof: It suffices to show in the considered situation that the determinant of the resulting Jacobian at the steady state is positive, since small variations of the parameter α_ω must then move the zero eigenvalue of the case $\alpha_\omega = 0$ into the negative domain, while leaving the real parts of the other eigenvalues – shown to be negative in the preceding proposition – negative. The determinant of the Jacobian to be considered here – already slightly simplified – is characterized by

$$J = \begin{pmatrix} 0 & + & - & - \\ + & 0 & 0 & 0 \\ 0 & + & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

This can be further simplified to

$$J = \begin{pmatrix} 0 & 0 & 0 & - \\ + & 0 & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & + & 0 & 0 \end{pmatrix}$$

without change in the sign of the corresponding determinant which proves the proposition. ■

We note that this proposition also holds where $\beta_{p_2} > \beta_{w_2}\kappa_p$ holds true as long as the thereby resulting real wage effect is weaker than the one originating from α_ω. Finally – and in sum – we can also state that the full 5D dynamics must also exhibit a locally stable steady state if β_{π^m} is made positive, but chosen sufficiently small, since we have already shown that the full 5D dynamics exhibits a negative determinant of its Jacobian at the steady state under the stated conditions. Increasing β_{π^m} from zero to a small positive value therefore must mover the corresponding zero eigenvalue into the negative domain of the plane of complex numbers.

Summing up, we can state that a weak Mundell effect, the neglect of Blanchard-Katz error correction terms, a negative dependence of aggregate demand on real wages, coupled with nominal wage and also to some extent price level inertia (in order to allow for dynamic multiplier stability), a sluggish adjustment of the rate of employment towards actual capacity utilization and a Taylor rule that stresses inflation targeting therefore are here (for example) the basic ingredients that allow for the proof of local asymptotic stability of the interior steady state of the dynamics (7) – (11). We expect however that indeed a variety of other and also more general situations of convergent dynamics can be found, but have to leave this here for future research and numerical simulations of the model. Instead we now attempt to estimate the signs and also the sizes of the parameters of the model in order to gain insight into the question to what extent for example the US economy after World War II supports one of the real wage effects considered in figure 1 and also the possibility of overall asymptotic stability for such an economy, despite a destabilizing Mundell effect in the real interest rate channel. Due to proposition 1 we know that the dynamics will generally only loose asymptotic stability in a cyclical fashion (by way of a Hopf-bifurcation) and will indeed do so if the parameter β_{π^m} is chosen sufficiently large. We thus arrive at a radically different outcome for the dynamics implied by our mature traditional Keynesian approach as compared to the New Keynesian dynamics. the next topic naturally here is if the economy can be assumed to be in the convergent regime of its alternative dynamical possibilities. This of course can only be decided by an empirical estimation of its various parameters which is the subject of the now following section.

5 Estimating the model

We now provide some estimates for the signs and sizes of the parameters of the model of this paper and will do so – with respect to the wage-price spiral – on the level of its structural form (where it has not yet been reduced to the dynamics of real wages, see eq. (12)). The further aim of these estimates is of course to determine whether the implied autonomous reduced form 5D dynamics we considered in the preceding section – obtained when equation (12) is inserted into (7) – (11) – exhibits asymptotic stability of (convergence to) its interior steady state position. in its theoretical form these dynamics exhibit the following sign structure in their Jacobian, calculated at its interior steady state:

$$J = \begin{pmatrix} \pm & + & - & \pm & + \\ \pm & + & - & \pm & + \\ + & + & - & \pm & + \\ - & + & 0 & - & 0 \\ + & + & 0 & \pm & 0 \end{pmatrix}.$$

There are therefore a variety of ambiguous effects embedded in the general form of its dynamics, due to the Mundell-effect and the Rose-effect in the dynamics of the goods-market, and the opposing Blanchard-Katz error correction terms in the reduced form price Phillips curve. In section 4 we have then considered certain special cases of the general model which allowed for the derivation of asymptotic stability of the steady state and its loss of stability by way of Hopf bifurcations if certain speed parameters become sufficiently large. In the present section we now provide empirical estimates for the laws of motion (7) – (11) of our disequilibrium AS-AD model, by means of the structural form of the wage and price Phillips curve, coupled with the dynamic multiplier equation, Okun's law and the interest rate policy rule. These estimates, on the one hand, serve the purpose of confirming the parameter signs we have specified in the initial theory-guided formulation of the model and to determine the sizes of these parameters in addition. On the other hand, we have three different situations where we cannot specify the parameter signs on purely theoretical grounds and where we therefore aim at obtaining these signs from the empirical estimates of the equations where this happens.

There is first of all, see eq. (7), the ambiguous influence of real wages on (the dynamics of) the rate of capacity utilization, which should be a negative one if investment is more responsive than consumption to real wage changes and a positive one in the opposite case. There is secondly, with an immediate impact effect if the rates of capacity utilization for capital and labor are perfectly synchronized, the fact that real wages rise with economic activity through money wage changes on the labor market, while they fall with it through price level changes on the goods market, see eq. (9). Finally, we have in the reduced form equation for price inflation a further ambiguous effect of real wage increases, which there lower $\hat{p}$ through their effect on wage inflation, while speeding up $\hat{p}$ through their effect on price inflation, effects which work into opposite directions in the reduced form price PC (12). Mundell-type, Rose-type and Blanchard-Katz error-correction feedback channels therefore make the dynamics indeterminate on the general level.

In all of these three cases empirical analysis will now indeed provide us with definite answers which ones of these opposing forces will be the dominant ones. Furthermore, we shall also see that the Blanchard and Katz (2000) error correction terms do play a role in the US-economy, in contrast to what has been found out by these authors for the money wage PC in the U.S. However, we will not attempt to estimate the parameter β_{π^m} that characterizes the evolution of the inflationary climate in our economy. Instead, we will use moving averages with linearly declining weights for its representation, which allows to bypass the estimation of the law of motion (11). We consider this as the simplest approach to the treatment of our climate expression (comparable with recent New Keynesian treatments of hybrid expectation formation), which should later on be replaced by more sophisticated ones, for example one that makes use of the Livingston index for inflationary expectations as in Laxton et al. (2000) which in our view mirrors some adaptive mechanism in the adjustment of inflationary expectations.

We take an encompassing approach to conduct our estimates. The structural laws of motion of our economy, see section 3, have been formulated in an intrinsically nonlinear way (due to certain growth rate formulations). We note that single equations estimates have suggested to only use $\alpha_{V_2^l}$ in the equation that describes the dynamics of the employment rate. We use moreover both the logs of real wages and unit-wage costs (in detrended form) in the equation describing goods market dynamics, in an attempt to estimate separately the influence of income distribution on aggregate demand as induced via consumption (a positive one) and via investment (a negative one).[11] In the wage-price spiral we use however – in line with Blanchard and Katz (2000) – the log of unit wage costs throughout, again removing their significant downward trend in the employed data appropriately.

[11]Using a single measure for the influence of income distribution on aggregate demand basically aggregates the separated outcome as we shall see our further estimates below.

We do this in conjunction with time-invariant estimates of all the parameters of our model. This in particular implies that Keynes' (1936) explanation of the trade cycle, which employed systematic changes in the propensity to consume, the marginal efficiency of investment and liquidity preference over the course of the cycle, find no application here and that – due the use of detrended measures for income distribution changes and unit-wage costs – also the role of technical change is downplayed to a significant degree, in line with its neglect in the theoretical equations of the model presented in section 3. As a result we expect to obtain from our estimates long-phased economic fluctuations, but not long -waves yet, since important fluctuations in aggregate demand (based on time-varying parameters) are still ignored and since the dynamics is then driven primarily by slowly changing income distribution, indeed a slow process in the overall evolution of the U.S. economy after world war II.

To show that such an understanding of the model is a suitable description of (some of) the dynamics of the observed data, we first fit a corresponding 6D VAR model to the data to find out the dynamics in the six independent variables there employed. We then identify a linear structural model that parsimoniously encompasses the employed VAR. Finally, we contrast our nonlinear structural model, i.e., the laws of motion (1) to (5) in structural form, with the linear structural VAR model and show through a J Test that the nonlinear model is indeed preferred by the data. In this way we show that our nonlinear structural model represents a proper description of the data.

The relevant variables for the following investigation are the wage inflation rate, the price inflation rate, the rates of utilization of labor and of capital, the nominal interest rate, the log of the real wage and / or of average unit wage cost, to be denoted in the following by: $d\ln w_t, d\ln p_t, V_t^l, V_t^c, r_t, rw_t,$ and uc_t, where $uc_t(rw_t)$ is the cycle component of the log of the time series for the unit real wage cost (the real wage), both filtered by the bandpass filter.[12]

5.1 Data Description

The empirical data of the corresponding time series are taken from the Federal Reserve Bank of St. Louis data set (see http:/www.stls.frb.org/fred). The data are quarterly , seasonally adjusted and are all available from 1948:1 to 2001:2. Except for the unemployment rates of the factors labor, U^l, and capital, U^c, the log of the series are used (see table 1).

[12]For details of bp filter see Baxter and King (1991).

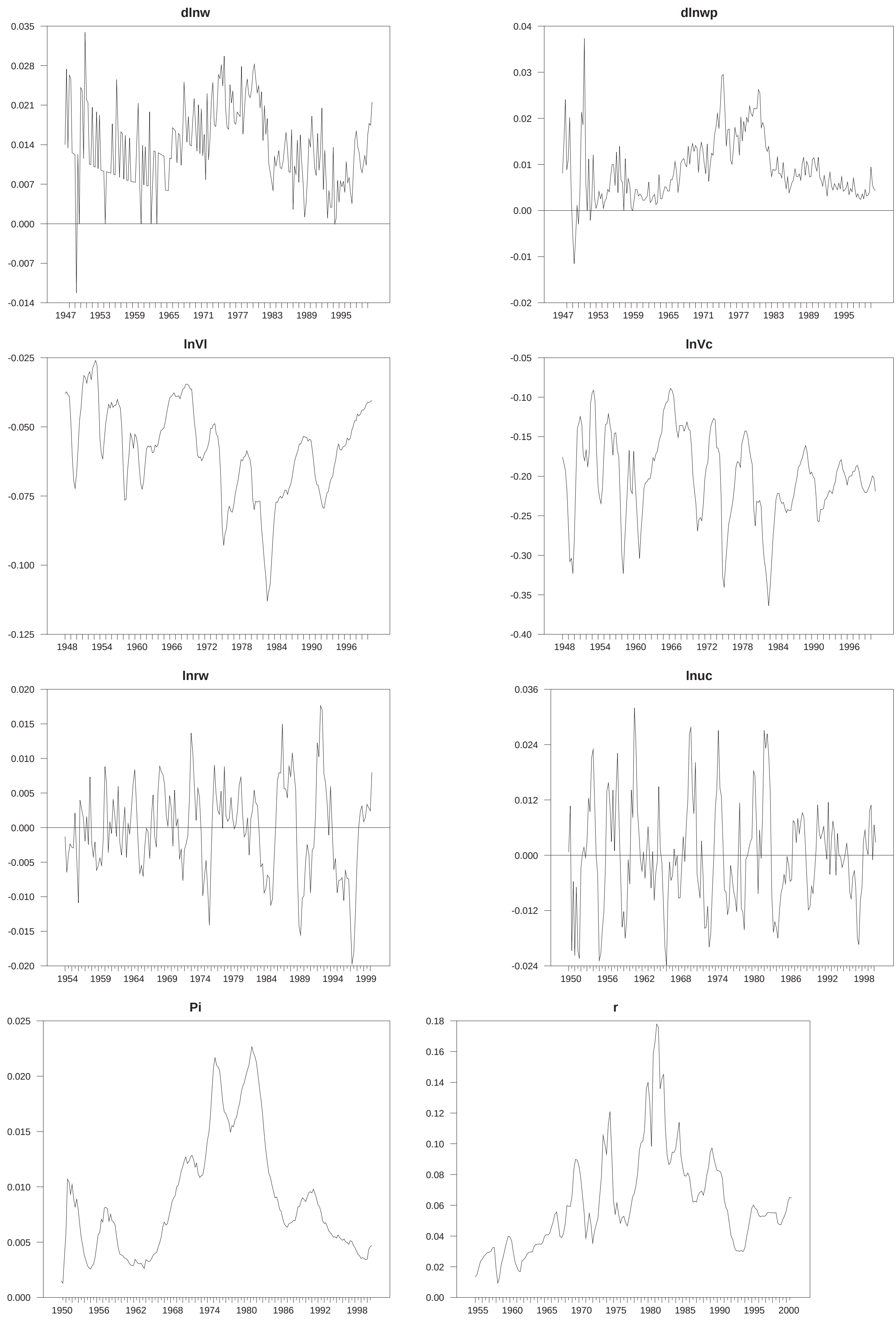

Figure 2: The fundamental data of the model.

Variable	Transformation	Mnemonic	Description of the untransformed series
$U^l = 1 - V^l$	UNRATE/100	UNRATE	Unemployment Rate
$U^c = 1 - V^c$	1-CUMFG/100	CUMFG	Capacity Utilization: Manufacturing, Percent of Capacity
$\ln w$	ln(COMPNFB)	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
$\ln p$	ln(GNPDEF)	GNPDEF	Gross National Product: Implicit Price Deflator, 1992=100
$\ln yn = \ln y - \ln l^d$	ln(OPHNFB)	OPHNFB	Nonfarm Business Sector; Output Per Hour of All Persons, 1992=100
$UC = \ln w - \ln p - \ln yn$	$\ln\left(\frac{COMPRNFB}{OPHNFB}\right)$	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Output Unit, 1992=100
$RW = \ln w - \ln p$	———	———	log of the real wage
r	———	———	Federal Funds Rate

Table 1: Data used for the empirical investigation

Note that we now use $\ln w_t$ and $\ln p_t$, i.e., logarithms, in the place of the original level magnitudes. Their first differences $d \ln w_t, d \ln p_t$ thus give the current rate of wage and price inflation. We use π_t in this section to denote here specifically a moving average of price inflation with linearly decreasing weights over the past 12 quarters, interpreted as a particularly simple measure for the inflationary climate expression of our model, and we denote by $V^l, V^c(U^l, U^c)$ the rates of (under-)utilization of labor and the capital stock. The graphs of the time series of these variables are shown in the figure 2.

There is a pronounced downward trend in part of the employment rate series (over the 1970's and part of the 1980's) and in the wage share (normalized to 0 in 1996). The latter is not the topic of this paper and will only briefly be considered in the concluding section. Wage inflation shows three to four trend reversals, while the inflation climate representation clearly show two periods of low inflation regimes and in between a high inflation regime.

We expect that the 6 independent time series for wages, prices, capacity utilization rates, labor productivity and the interest rate (federal funds rate) are stationary. The graphs of the series for wage and price inflation, capacity utilization rates and labor productivity growth, $d \ln w_t, d \ln p_t, V_t^l, V_t^c, d \ln yn_t$ confirm our expectation. In addition we carry out the DF unit root test for each series. The test results are shown in table 2.

Variable	Sample	Critical Value	Test Statistic
$d \ln w$	1947:02 TO 2000:04	-3.41000	-3.74323
$d \ln p$	1947:02 TO 2000:04	-3.41000	-3.52360
$\ln V^l$	1948:02 TO 2000:04	-1.95000	-0.75474
$\ln V^c$	1948:02 TO 2000:04	-3.41000	-4.15536
r	1955:01 TO 2000:04	-1.95000	-0.94144
lnuc	1950:01 TO 2000:04	-3.41000	-7.09932

Table 2: Summary of DF-Test Results

The applied unit root test confirms our expectations with the exception of V^l and r. Although the test cannot reject the null of unit root, there is no reason to expect the rate of unemployment and the federal funds rate as being unit root processes. Indeed we expect that they are constrained in certain limited ranges, say from zero to 0.3. Due to the lower power of the DF test, this test result should only provide hints that the rate of unemployment and the federal funds rate exhibit strong autocorrelations, respectively.

5.2 Estimation of the unrestricted VAR

Given the assumption of stationarity, we can construct a VAR model for these 6 variables to mimic the DGP of these 6 variables by linearizing our given structural model in an obvious way.

$$\begin{pmatrix} d\ln w_t \\ d\ln p_t \\ \ln V_t^l \\ \ln V_t^c \\ r_t \\ uc_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix} d74 + \sum_{k=1}^{P} \begin{pmatrix} a_{11k} & a_{12k} & a_{13k} & a_{14k} & a_{15k} & a_{16k} \\ a_{21k} & a_{22k} & a_{23k} & a_{24k} & a_{25k} & a_{26k} \\ a_{31k} & a_{32k} & a_{33k} & a_{34k} & a_{35k} & a_{36k} \\ a_{41k} & a_{42k} & a_{43k} & a_{44k} & a_{45k} & a_{46k} \\ a_{51k} & a_{52k} & a_{53k} & a_{54k} & a_{55k} & a_{56k} \\ a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{pmatrix} \tag{20}$$

To determine the lag length of the VAR we apply sequential likelihood tests. We start with a lag length of 24, at which the residuals can be taken as a WN process. The sequence likelihood ratio test procedure gives a lag length of 11. The test results are listed below.

- $H_0 : P = 20$ v.s. $H_1 : P = 24$
  Chi-Squared(144)= 147.13 with Significance Level 0.91
- $H_0 : P = 16$ v.s. $H_1 : P = 20$
  Chi-Squared(144)= 148.92 with Significance Level 0.41
- $H_0 : P = 12$ v.s. $H_1 : P = 16$
  Chi-Squared(36)= 118.13 with Significance Level 0.94
- $H_0 : P = 11$ v.s. $H_1 : P = 12$
  Chi-Squared(36)= 42.94 with Significance Level 0.19
- $H_0 : P = 10$ v.s. $H_1 : P = 11$
  Chi-Squared(36)= 51.30518 with Significance Level 0.04

According to these test results we use a VAR(12) model to represent a general model that should be a good approximation of the DGP. Because the variable uc_t is treated as exogenous in the structural form (1) – (6) of the dynamic system , we factorize the VAR(12) process into a conditional process of $d\ln w_t, d\ln p_t, \ln V_t^l, \ln V_t^c, r_t$ given uc_t and the lagged variables, and the marginal process of uc_t given the lagged variables:

$$
\begin{pmatrix} d\ln w_t \\ d\ln p_t \\ \ln V_t^l \\ \ln V_t^c \\ r_t \end{pmatrix} = \begin{pmatrix} c_1^* \\ c_2^* \\ c_3^* \\ c_4^* \\ c_5^* \end{pmatrix} + \begin{pmatrix} b_1^* \\ b_2^* \\ b_3^* \\ b_4^* \\ b_5^* \end{pmatrix} d74 + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} uc_t \tag{21}
$$

$$
+ \sum_{k=1}^{P} \begin{pmatrix} a_{11k}^* & a_{12k}^* & a_{13k}^* & a_{14k}^* & a_{15k}^* & a_{16k}^* \\ a_{21k}^* & a_{22k}^* & a_{23k}^* & a_{24k}^* & a_{25k}^* & a_{26k}^* \\ a_{31k}^* & a_{32k}^* & a_{33k}^* & a_{34k}^* & a_{35k}^* & a_{36k}^* \\ a_{41k}^* & a_{42k}^* & a_{43k}^* & a_{44k}^* & a_{45k}^* & a_{46k}^* \\ a_{51k}^* & a_{52k}^* & a_{53k}^* & a_{54k}^* & a_{55k}^* & a_{56k}^* \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + \begin{pmatrix} e_{1t}^* \\ e_{2t}^* \\ e_{3t}^* \\ e_{4t}^* \\ e_{5t}^* \end{pmatrix}
$$

$$
uc_t = c_6 + \sum_{k=1}^{P} \begin{pmatrix} a_{61k} & a_{62k} & a_{63k} & a_{64k} & a_{65k} & a_{66k} \end{pmatrix} \begin{pmatrix} d\ln w_{t-k} \\ d\ln p_{t-k} \\ \ln V_{t-k}^l \\ \ln V_{t-k}^c \\ r_{t-k} \\ uc_{t-k} \end{pmatrix} + e_{6t} \tag{22}
$$

We now examine whether uc_t can be taken as "exogenous" variable. The partial system (21) is exactly identified. Hence the variables uc_t are weakly exogenous for the parameters in the partial system.[13] For the strong exogeneity of uc_t, we test whether $d\ln w_t, d\ln p_t, \ln V_t^l, \ln V_t^c, r_t$ Granger cause uc_t. The test is carried out by testing the hypothesis: $H_0 : a_{ijk} = 0, (i = 6; j = 1, 2, 3, 4, 5; k = 1, 2, ..., 12)$ in (22) based on the likelihood ratio

- Chi-Squared(60)=57.714092 with Significance Level 0.55972955

The result of the test is uc_t is strongly exogenous with respect to the parameters in (21). Hence we can investigate the partial system (21) taking uc_t as exogenous.

5.3 Estimation of the Structural Model

As discussed in section 3, the law of motion for the real wage rate, eq. (10), represents a reduced form expression for the two structural equations for $d\ln w_t$ and $d\ln p_t$. Noting again that the inflation climate variable is defined here as a linearly declining function of past price inflation rates, the dynamics of the system (1) – (6) is equivalently presented

[13]For a detailed discussion of this procedure, see Chen (2003).

by the following equations:[14]

$$
\begin{aligned}
d\ln w_t &= \beta_{w_1}\ln V_{t-1}^l - \beta_{w_2}uc_{t-1} + \kappa_w d\ln p_t + (1-\kappa_w)\pi_t + c_1 + e_{1t} && (23)\\
d\ln p_t &= \beta_{p_1}\ln V_{t-1}^c + \beta_{p_2}uc_{t-1} + \kappa_p d\ln w_t + (1-\kappa_p)\pi_t + c_2 + e_{2t} && (24)\\
d\ln V_t^l &= \alpha_{V_2^l} d\ln V_t^l + e_{3t} && (25)\\
d\ln V_t^c &= -\alpha_{V^c}\ln V_{t-1}^c - \alpha_r(r_{t-1} - d\ln p_t) - \alpha_\omega uc_{t-1} + c_4 + e_{4t} && (26)\\
dr_t &= -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c}\ln V_{t-1}^c + c_5 + e_{5t} && (27)
\end{aligned}
$$

Obviously, the model (23) – (27) is nested in the VAR(12) of (21). Therefore we can use (21) to evaluate the empirical relevance of the model (23) – (27). First we test whether the parameter restrictions on (21) implied by (23) – (27) are valid.

The linearized structural model (23) – (27) puts 349 restrictions on the unconstrained VAR(12) of the system (21). Applying likelihood ratio methods we can test the validity of these restrictions. For the period from 1965:1 to 2000:4 we cannot reject the null of these restrictions. The test result is the following:

- Chi-Squared(349)= 361.716689 with Significance Level 0.34902017

Obviously, the specification (23) – (27) is a valid one for the data set from 1965:1 to 2000:4. This result shows strong empirical relevance for the laws of motions as described in (1) – (6) as a model for the U.S. economy from 1965:1 to 2000:4. It is worthwhile to note that altogether 349 restrictions are implied through the structural form of the system (1) – (6) on the VAR(12) model. A p-value of 0.39 thus means that (1) – (6) is a much more parsimonious presentation of the DGP than VAR(12), and henceforth a much more efficient model to describe the economic dynamics for this period.

To get a result that is easier to interpret from the economic perspective, we transform the model (23) – (27) back to its originally nonlinear form (1) – (6), now using in addition at first two distributional variables to measure the influence of consumption and investment on aggregate demand:[15]

$$
\begin{aligned}
d\ln w_t &= \beta_{w_1}V_{t-1}^l - \beta_{w_2}uc_{t-1} + \kappa_w d\ln p_t + (1-\kappa_w)\pi_t + c_1 + e_{1t} && (28)\\
d\ln p_t &= \beta_{p_1}V_{t-1}^c + \beta_{p_2}uc_{t-1} + \kappa_p d\ln w_t + (1-\kappa_p)\pi_t + c_2 + e_{2t} && (29)\\
d\ln V_t^l &= \alpha_{V_2^l} d\ln V_t^c + e_{3t} && (30)\\
d\ln V_t^c &= -\alpha_{V^c}V_{t-1}^c + \alpha_\omega rw_{t-1} - \alpha_u uc_{t-1} - \alpha_r(r_{t-1} - d\ln p_t) + c_4 + e_{4t} && (31)\\
dr_t &= -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c}V_{t-1}^c + c_5 + e_{5t} && (32)
\end{aligned}
$$

The estimation results for this particular case are listed in the appendix to this paper. They do not differ in their qualitative implications from the estimates to be considered below where only the measure uc is used to consider the role of income distribution

[14]Note here that the difference operator d is to be interpreted as backward in orientation and that the nominal rate of interest is dated at the beginning of the relevant period. The linearly declining moving average π_t in turn concerns the past twelve price inflation rates.

[15]Note that $dr_t = -\gamma_r r_{t-1} + \gamma_p d\ln p_t + \gamma_{V^c}V_{t-1}^c + c_5 + e_{5t}$ can also be represented by $r_t = (1-\gamma_r)r_{t-1} + ...$ in the equations to be estimated below.

in the dynamic multiplier equation and where the Taylor rule is represented by $r_t = (1-\gamma_r)r_{t-1} + ...$

This model therefore differs from the model (23) – (27) by referring now again to the explaining variables V^c and V^l instead of $\ln V^c$ and $\ln V^l$ which were necessary to construct a linear VAR(12) system. In addition we have considered the log of the real wage rw as an explaining variable to account for the influence of consumption demand on the rate of capacity utilization and the log of average unit wage costs uc as the explaining variable for the profitability effect on investment behavior. We compare on this basis the model (28) – (32) with the model (23) – (27) in a nonnested testing framework. Applying the J test to such a nonlinear estimation procedure, we get significant evidence that the model (28) – (32) is to be preferred to the model (23) – (27).

Model	J test
H_1 : Model of (23) – (27) is true	$t_\alpha = 4.611$
H_2 : Model of (28) – (32) is true	$t_\phi = -0.928$

We have already omitted in the following summary of our model estimate the insignificant parameters in the shown quantitative representation of the structural model and also the stochastic terms. By putting furthermore the NAIRU expressions and all other expressions that are here still assumed as constant into overall constant terms, we therefore finally get following estimation result:[16]

$$\begin{aligned}
d\ln w_t &= 0.11V^l_{t-1} - 0.09uc_{t-1} + 0.62d\ln p_t + 0.38\pi_t - 0.10 && (33)\\
d\ln p_t &= 0.03V^c_{t-1} + 0.07uc_{t-1} + 0.34d\ln w_t + 0.66\pi_t - 0.03 && (34)\\
d\ln V^l_t &= 0.20d\ln V^c_t && (35)\\
d\ln V^c_t &= -0.11V^c_{t-1} - 0.17(r_{t-1} - d\ln p_t) + 0.48rw_{t-1} - 0.80uc_{t-1} + 0.10 && (36)\\
dr_t &= -0.10r_{t-1} + 0.43d\ln p_t + 0.05V^c_{t-1} - 0.03 && (37)
\end{aligned}$$

If one disregards the possibly questionable application of both the real wage and the average unit wage costs as explaining variables in the equation driving goods market dynamics, employing the single measure uc in their place, one gets as approximate parameter values (with basically the same statistical characteristics):

$$\begin{aligned}
d\ln w_t &= 0.12V^l_{t-1} - 0.09uc_{t-1} + 0.57d\ln p_t + 0.43\pi_t - 0.11 && (38)\\
d\ln p_t &= 0.03V^c_{t-1} + 0.06uc_{t-1} + 0.32d\ln w_t + 0.68\pi_t - 0.03 && (39)\\
d\ln V^l_t &= 0.21d\ln V^c_t && (40)\\
d\ln V^c_t &= -0.09V^c_{t-1} - 0.17(r_{t-1} - d\ln p_t) - 0.74uc_{t-1} + 0.08 && (41)\\
r_t &= 0.90r_{t-1} + 0.41d\ln p_t + 0.05V^c_{t-1} - 0.04 && (42)
\end{aligned}$$

Next we compare the preceding situation with the case where the climate expression π is based on 24 quarter horizon in the place of the 12 quarter horizon we have employed so far.

[16] We have here reduced the weights in the cost pressure expressions slightly in order to meet the unity restrictions on these weights that are postulated by theory and thus here only represent the degree of forward-looking behavior in a somewhat stylized from, see the appendix to this paper for the exact results and their evaluation by means of t-statistics, R^2 terms and Durbin-Watson coefficients.

$$\begin{aligned}
d\ln w_t &= 0.13V^l_{t-1} - 0.08uc_{t-1} + 0.69d\ln p_t + 0.31\pi 24_t - 0.12 && (43)\\
d\ln p_t &= 0.03V^c_{t-1} + 0.09uc_{t-1} + 0.50d\ln w_t + 0.50\pi 24_t - 0.03 && (44)\\
d\ln V^l_t &= 0.21d\ln V^c_t && (45)\\
d\ln V^c_t &= -0.09V^c_{t-1} - 0.17(r_{t-1} - d\ln p_t) - 0.74uc_{t-1} + 0.09 && (46)\\
r_t &= 0.90r_{t-1} + 0.43d\ln p_t + 0.05V^c_{t-1} - 0.03 && (47)
\end{aligned}$$

We see that the application of a time horizon of 24 quarters for the formation of the inflationary climate variable does not alter the qualitative properties of the dynamics significantly as compared to the case of a moving average with linearly declining weights over 12 quarters only (which approximately corresponds to a value of $\beta_{\pi^m} = 0.15$ in an adaptive expectations mechanism as used for the theoretical version of the model in section 3). Even choosing only a six quarter horizon for our linearly declining weights preserves the qualitative features of our estimated model and also by and large the stability properties of the dynamics as we shall see later on, though inflationary expectations over the medium run are then updated with a speed comparable to the ones used in hybrid New Keynesian approaches to their price PC:

$$\begin{aligned}
d\ln w_t &= 0.11V^l_{t-1} - 0.10uc_{t-1} + 0.40d\ln p_t + 0.60\pi 6_t - 0.10 && (48)\\
d\ln p_t &= 0.03V^c_{t-1} + 0.03uc_{t-1} + 0.18d\ln w_t + 0.82\pi 6_t - 0.02 && (49)\\
d\ln V^l_t &= 0.21d\ln V^c_t && (50)\\
d\ln V^c_t &= -0.09V^c_{t-1} - 0.17(r_{t-1} - d\ln p_t) - 0.74uc_{t-1} + 0.08 && (51)\\
r_t &= 0.90r_{t-1} + 0.44d\ln p_t + 0.05V^c_{t-1} - 0.04 && (52)
\end{aligned}$$

We thereby arrive at the general qualitative result that wages are more flexible than prices with respect to their corresponding measures of demand pressure and that wage earners are more short-sighted than firms with respect to the weight they give their current (perfectly foreseen) measure of cost pressure as compared to the inflationary climate that surrounds this situation. Blanchard and Katz (2000) type error correction mechanisms play a role both in the WPC and also in the PPC for the U.S. economy and have the sign that is predicted by theory, in contrast to what is found out by these two authors. We have the validity of Okun's law with an elasticity coefficient of around 20 percent and have the correct signs for the dynamic multiplier process as well as with respect to the influence of changing real rate of interests on economic activity. Finally, the impact of income distribution on the change in capacity utilization is (in sum) a negative one and thus of an orthodox type, meaning that rising average unit wage costs will decrease economic activity, and will therefore imply at least from a partial perspective that increasing wage flexibility is stabilizing, while increasing price flexibility (again with respect to its measure of demand pressure) is not. We note, finally, that the different definitions of the inflationary climate used in these system estimates primarily and not unexpectedly changes the weights in the employed cost pressure terms, while it leaves intact the qualitative nature of all other parameter estimates.

We conclude from the above that it should be legitimate to use (38) – (42) for the further evaluation of the dynamic properties of our theoretical model of section 3, in order to see what more can be obtained as compared to the theoretical results of section 4 when empirically supported parameter sizes are approximately taken into account. To make this approximation a sensible procedure we finally also report single equations estimates for our 5D system in order to get a feeling for the intervals in which the parameter values may sensibly assumed to be in.

$$\begin{aligned}
d\ln w_t &= 0.19V^l_{t-1} - 0.07uc_{t-1} + 0.16d\ln p_t + 0.84\pi_t - 0.17 && (53)\\
d\ln p_t &= 0.05V^c_{t-1} + 0.05uc_{t-1} + 0.09d\ln w_t + 0.91\pi_t - 0.04 && (54)\\
d\ln V^l_t &= 0.17d\ln V^c_t && (55)\\
d\ln V^c_t &= -0.13V^c_{t-1} - 0.18(r_{t-1} - d\ln p_t) - 0.79uc_{t-1} + 0.12 && (56)\\
r_t &= 0.90r_{t-1} + 0.44d\ln p_t + 0.05V^c_{t-1} - 0.03 && (57)
\end{aligned}$$

Again the weights concerning the cost pressure items are changed significantly and also certain speeds of adjustment. We do not expect however that this changes the stability properties of the dynamics in a qualitative sense and have to check this in the following sections from the theoretical as well as numerical perspective.

The above by and large similar representation of the signs and the sizes of the parameter values of our dynamics thus reveal various interesting assertions on the relative importance of demand pressure influences as well as cost pressure effects in the wage-price spiral of the U.S. economy. The Blanchard and Katz error correction terms have the correct signs and are of relevance in general. Okun's law holds as a level relationship between the capacity utilization rate and the rate of employment, basically of the form $V^l/\bar{V}^l = (V^c/\bar{V}^c)^b$ with an elasticity parameter b of about 20 percent. The dynamic IS equation shows the from the partial perspective the stabilizing role of the multiplier process and a significant dependence of the rate of change of capacity utilization on the current real rate of interest. There is a significant and positive effect of real wages – we conjecture: via consumption – on the growth in activity levels and an even stronger negative effect of real unit wage costs – we conjecture: via investment – on this growth rate of capacity utilization, which in aggregated form however gives that the economy is profit-led as far as aggregate goods demand is concerned, i.e., real wage cost increases significantly decrease economy activity.

Finally, for the Taylor interest rate policy rule, we get the result that interest rate smoothing takes place around the ten percent level, that monetary policy is to be considered as passive ($\gamma_p = 0.43$) in such an environment as far as the inflation gap is concerned, and that there is only a weak direct influence of the output gap on the rate of change of the nominal rate of interest. We stress here that this result will change considerably if interest rate smoothing is removed from the model which however implies in our modelling framework that the steady state rate of interest is no longer uniquely determined. Finally, turning back to the case of interest rate smoothing, it is not possible to recover the steady state rate of interest from the constant in the above estimated Taylor rules in a statistically significant way, since the expression implied for this rate by our formulation of the Taylor rule would be:

$$r_o = (const + \gamma_p\bar{\pi} + \gamma_{V^c})/\gamma_r$$

which does not determine this rate with any reliable statistical confidence. This also holds for the other constants that we have assumed as given in our formulation of Keynesian DAS-DAD dynamics.

In sum we therefore get that the system estimates of this section provides us with a result that confirms theoretical sign restrictions. It moreover provides by and large definite answers with respect to the role of income distribution in the considered disequilibrium AS-AD or DAS-DAD dynamics, confirming in particular the orthodox point of view that economic activity is – in sum – likely to depend negatively on real wage or unit wage costs. We have also a negative real wage effect in the dynamics of income distribution, in the sense that the growth rate of real wages, see our reduced form real wage dynamics (**??**), depends – through Blanchard and Katz error correction terms – negatively on the real wage. Its dependence on economic activity levels however is somewhat ambiguous, but in any case small. Real wages – possibly – therefore only weakly increase with increases in the rate of capacity utilization which in turn however depends in an unambiguous way negatively on the real wage, implying in sum that the Rose (1967) real wage channel is there, but not dominating the dynamic outcomes.

Finally, the estimated adjustment speed of the price level is so small that the dynamic multiplier effect dominates the overall outcome of changes in capacity utilization on the growth rate of this utilization rate, which therefore establishes a further stabilizing mechanism in the reduced form of our multiplier equation. The model and its estimates thus by and large confirm the conventional Keynesian view on the working of the economy and thus provide in sum a result very much in line with the traditional ways of reasonings from a Keynesian perspective, with one important qualification however, as we will show in the next sections, namely that downward money wage flexibility is good for economic stability, in line with Rose's (1967) model of the employment cycle, but in opposition to what Keynes (1936) stated on the role of rigid money wages. Yet, the role of income distribution in aggregate demand and wage vs. price flexibility was not really a topic in the General Theory, which therefore did not comment on the possibility that wage declines may lead the economy out of a depression via a channel different from the conventional now so-called Keynes-effect.

6 Stability analysis of the estimated model

In the preceding section we have provided definite answers with respect to the type of real wage effect present in the data of the U.S. economy after World War II, concerning the dependence of aggregate demand on the real wage, the degrees of wage and price flexibilities and the degree of forward-looking behavior in the wage and price PC. The resulting combination of effects and the estimated sizes of the parameters (in particular the relative degree of wage vs. price flexibility) suggest that their particular type of interaction is favorable for stability. We stress however that the inflation climate is so far only measured by a moving average of past inflation rates with linearly declining weights. The role of a varying parameter β_{π^m} – which when increased will definitely destabilize the economy – is thus not yet considered at present.

We start the stability analysis of the model with estimated parameters from the following reference situation (the system estimate where the inflationary climate is measured as by the twelve quarter moving average):

$$\begin{aligned}
d\ln w_t &= 0.12V^l_{t-1} - 0.09uc_{t-1} + 0.57d\ln p_t + 0.43\pi_t - 0.11\\
d\ln p_t &= 0.03V^c_{t-1} + 0.06uc_{t-1} + 0.32d\ln w_t + 0.68\pi_t - 0.03\\
d\ln V^l_t &= 0.21d\ln V^c_t\\
d\ln V^c_t &= -0.09V^c_{t-1} - 0.17(r_{t-1} - d\ln p_t) - 0.74uc_{t-1} + 0.08\\
r_t &= 0.90r_{t-1} + 0.41d\ln p_t + 0.05V^c_{t-1} - 0.04
\end{aligned}$$

and consider first the 3D core situation obtained by totally ignoring adjustments in the inflationary climate term, by setting $\pi^m = \bar{\pi}$ in the theoretical model, and by interpreting the estimated law of motion for V^l in level terms, by moving from the equation $\hat{V}^l = b\hat{V}^c$ to the equation $V^l = \bar{V}^l(V^c/\bar{V}^c)^b$, with $b = 0.21$ (and $\bar{V}^l = \bar{V}^c = 1$ for reasons of simplicity and without much loss of generality). On the basis of our estimated parameter values we furthermore have that the expression $\beta_{p_1} - \kappa_p\beta_{w_1}$ is approximately zero, i.e., the weak influence of the state variable ω in the reduced form PPC will not be of relevance in the following reduced form of the dynamics. Finally, the expression

$$(1-\kappa_p)\beta_{w_1}(V^c)^b - (1-\kappa_w)\beta_{p_1}V^c$$

is also – due to the measured size of the parameter b – close to zero, though probably exhibiting a positive derivative at the steady state.

Under these assumptions, the laws of motion (7) – (11) – with the reduced form PPC inserted again – can be reduced to the following qualitative form (where the here still indetermined signs of a_1, b_1, c_1 do not matter for the following stability analysis):

$$\hat{V}^c = a_1 - a_2\, V^c - a_3\, r - a_4\, \ln\omega \tag{58}$$

$$\dot{r} = b_1 + b_2\, V^c - b_3\, r \pm b_4 \ln\omega \tag{59}$$

$$\hat{\omega} = c_1 \pm c_2\, V^c - c_4\, \ln\omega \tag{60}$$

since the dependence of $\hat{p}$ on V^c is a weak one, to be multiplied with 0.17 in the comparison with the direct impact of V^c on its rate of growth, and thus does not modify the sign measured for the direct influence of this variable on the growth rate of the capacity utilization rate significantly. Note with respect to this qualitative characterization of the remaining 3D dynamics, that the various influences of the same variable in the same equation have been aggregated here into a single expression, the sign of which has been obtained from the quantitative estimates shown above. We thus have to take note here in particular of the fact that the reduced form expression for the price inflation rate has been inserted into the first two laws of motion for the activity dynamics and the interest rate dynamics, which have been rearranged on this basis so that the influence of the variables V^c and ω appears at most only once, though both terms appear via two different channels in these laws of motion, one direct channel and one via the price inflation rate. The result of our estimates of this equation is that the latter channel is not changing the signs of the direct effects of capacity utilization (via the dynamic multiplier) and the real wage (via the aggregate effect of consumption and investment behavior). Furthermore, the parameter a_2 may be uncertain in sign, but will in any case be close to zero.

A similar treatment applies to the law of motion for the nominal rate of interest, where price inflation is again dissolved into its constituent part (in its reduced form expression) and where the influence of V^l in this expression is again replaced by V^c through Okun's Law. Since the direct real wage effect in the reduced form price PC is small and approximately equal to the indirect real wage effect in this reduced form that comes with weight κ_p through the cost pressure channel of the original structural price PC, we can here too assume that the parameter b_4 is small in size and will therefore not influence the following stability investigation significantly. Finally, the law of motion for real wages themselves is obtained from the two estimated structural laws of motion for wage and price inflation in the way shown in section 3. We have the stated weak, possibly positive influence of capacity utilization on the growth rate of real wages, since the wage Phillips curve slightly dominates the outcome here and an unambiguously negative influence of real wages on their rate of growth due to the signs of the Blanchard and Katz error correction terms in the wage and the price dynamics.

On this basis, we arrive – if we set the considered small magnitudes equal to zero – at the following sign structure for the Jacobian at the interior steady state of the above reduced model for the state variables V^c, r, ω:

$$J = \begin{pmatrix} - & - & - \\ + & - & 0 \\ 0 & 0 & - \end{pmatrix}.$$

We therefrom immediately get that the steady state of these dynamics must be asymptotically stable, since the trace is negative, the sum a_2 of principal minors of order two is always positive, and since the determinant of the whole matrix is negative. The coefficients $k_i, i = 1, 2, 3$ of the Routh Hurwitz polynomial of this matrix are therefore all positive as demanded by the Routh Hurwitz stability conditions. The remaining stability condition is

$$k_1 k_2 - k_3 = (-traceJ)k_2 + detJ > 0.$$

With respect to this condition we immediately see that the determinant of J, given by:

$$J_{33}(J_{11}J_{22} - J_{12}J_{21})$$

is dominated by the terms that appear in k_1k_2, i.e., this final Routh-Hurwitz condition is also of correct sign as far as the implication of local asymptotic stability is concerned. The weak and maybe ambiguous real wage effect or Rose effect that is included in the working of the dynamics of the private sector thus creates no harm for the stability of the steady state of the considered dynamics. Ignoring the Mundell effect by assuming $\beta_{\pi^m} = 0$ therefore allows for an unambiguous stability result, basically due to the stable interaction of the dynamic multiplier with the Taylor interest rate policy rule, augmented by a real wage dynamics that in itself is stable due to the estimated signs (and sizes) of the Blanchard error correction terms, where the estimated negative dependence of the change in economic activity on the real wage is welcome from an orthodox point of view, but does not really matter for the stability features of the model. The neglectance of the Mundell effect therefore leaves us with a situation that is close in spirit to the standard textbook considerations of Keynesian macrodynamics.

The figure 3 shows simulations of the estimated dynamics where the parameter β_{π^m} is now no longer zero, but set equal to $0.075, 0.15, 0.30$ in correspondence to the measures $\pi 24, \pi 12, \pi 6$ of the inflationary climate used in our estimates (these values approximately arise when we estimate β_{π^m} by means of these moving averages). We use a real wage shock (increase by ten percent) to investigate the response of the dynamics to such a shock. The obtained impulse-responses are very similar to each other, also in the case where we use $\pi 12$ in combination with the single equation estimates of our parameter values. In the considered range for the parameter β_{π^m} the responses of the dynamics are by and large of the shown type, i.e., the system has strong, though cyclical stability properties over this whole range, independently of the particular combination of the speed of adjustment of the inflationary climate and the set of other parameter values we have estimated in the preceding section.

We note that the system is subject to zero root hysteresis, since the laws of motion for V^l, V^c are here linearly dependent (since $\alpha_{V_1^l}$), has been estimated as being zero), i.e., it need not converge back to the initially given steady state value of the rate of capacity utilization which was assumed to be 1. Note also that the parameter estimates are based on quarterly data, i.e., the plots in figure 3 correspond to 25 years and thus show a long period of adjustment, due to the fact that all parameters have been assumed as time invariant so that only the slow process of changing income distribution and its implications for Keynesian aggregate demand is driving the economy.

Next we test the stability properties of the model if one of its parameters is varied in size. We find (also for parameter variations that are not shown) that all partial feedback chains (including the working of the Blanchard / Katz error correction terms) translate themselves into corresponding 'normal' eigenvalue reaction patterns for the full 5D dynamics, with the exception of the speed parameter β_{w_1} where the eigenvalue diagram shows that increasing wage flexibility may become destabilizing if made sufficiently large. This shows that are partial insight may be misleading due to the fact that the corresponding feedback chain is only a small component of the many minors that have to be investigated in the application of the Routh-Hurwitz conditions to the full 4D dynamics (where Okun's law is applied in level form).

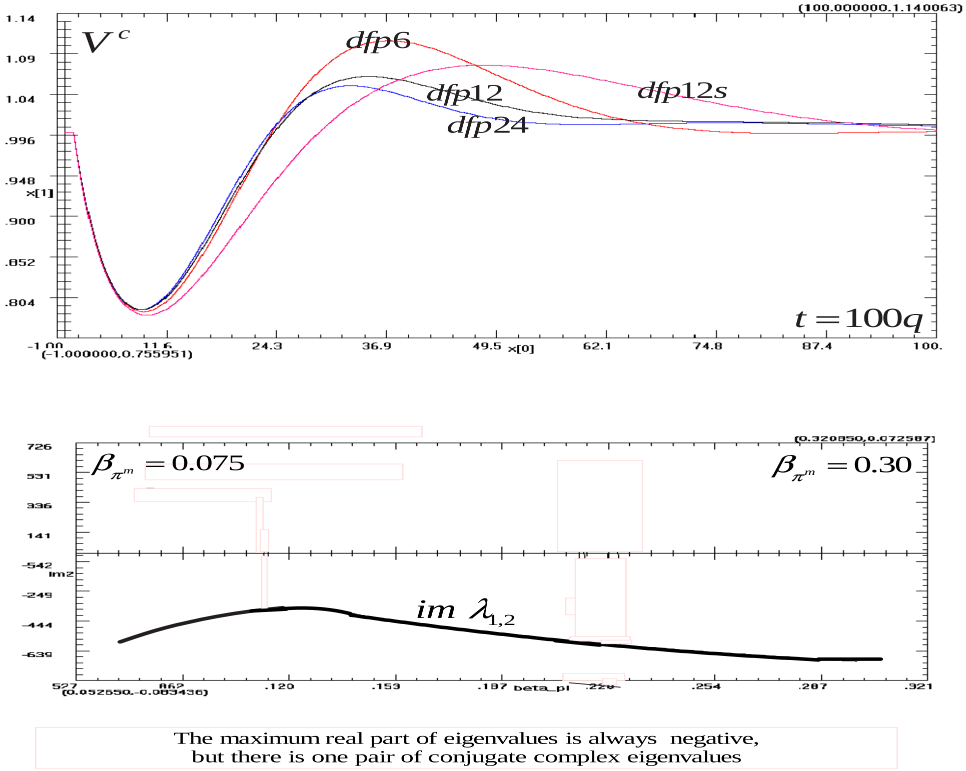

Figure 2: Responses to real wage shocks in the range of estimated parameter values.

Increasing price flexibility is destabilizing via the Mundell-effect, since the growth rate $\hat{V}^c$ of economic activity can thereby be made to depend positively on its level (via this real rate of interest channel, see eq. (12)), leading to an unstable augmented dynamic multiplier process in the trace of J under such circumstances. Furthermore, such increasing price flexibility will give rise to a negative dependence of the growth rate of the real wage on economic activity (whose rate of change in turn depends negatively on the real wage) and thus lead to further sign changes in the Jacobian J. Increasing price flexibility is therefore bad for the stability of the considered dynamics from at least two perspectives.

The destabilizing role of price flexibility is enhanced if we add to the above stability analysis for the 3D Jacobian the law of motion for the inflationary climate surrounding the current evolution of price inflation. Under this extension we go back to a 4D dynamical system, the Jacobian J of which is obtained by augmenting the previous one in its sign structure in the following way (see again eq. (12)):

$$J = \begin{pmatrix} - & - & - & + \\ + & - & 0 & + \\ 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix}.$$

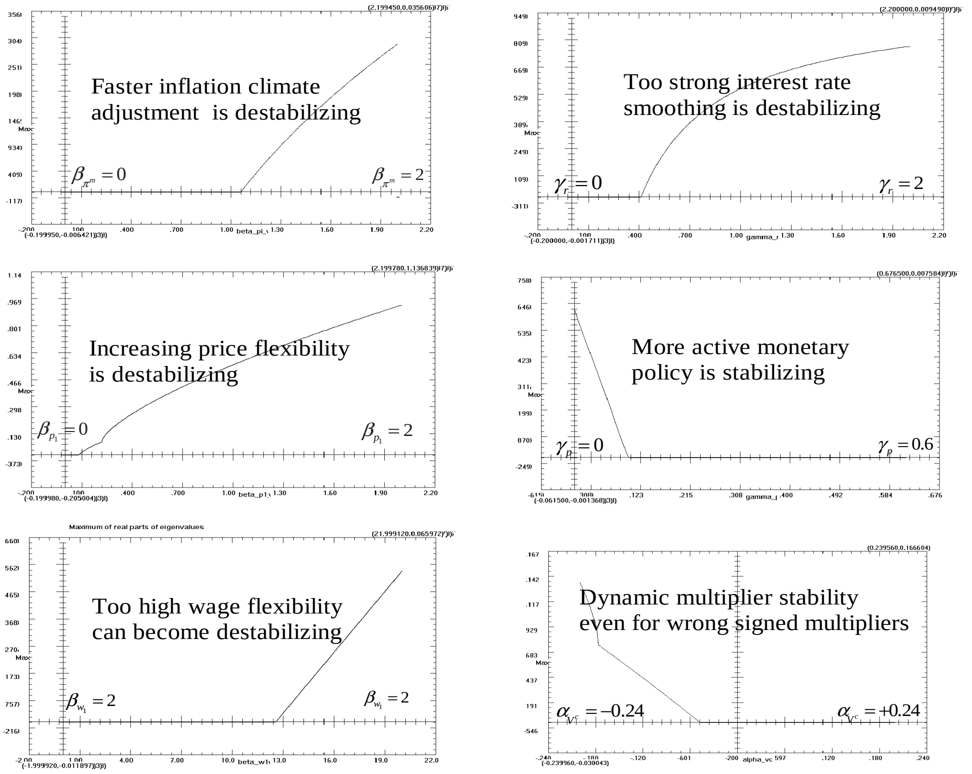

Figure 3: Eigenvalue diagrams for varying parameter sizes.

As the positive entries J_{14}, J_{41} show, there is now a new destabilizing feedback chain included, leading from increases in economic activity to increases in inflation and climate inflation and from there back to increases in economic activity, again through the real rate of interest channel (where the inflationary climate is involved due to the expression that characterizes our reduced form PPC). This destabilizing, augmented Mundell effect must become dominant sooner or later as the adjustment speed of the climate expression β_{π^m} is increased. This is obvious from the fact that the only term in the Routh-Hurwitz coefficient a_2 that depends on the parameter β_{π^m} exhibits a negative sign, which implies that a sufficiently high β_{π^m} will make the coefficient a_2 negative eventually. The Blanchard / Katz error correction terms in the fourth row of J, obtained from the reduced form price Phillips curve, that are (as only further terms) associated with the speed parameter β_{π^m}, are of no help here, since they do not appear in combination with the parameter β_{π^m} in the sum of principal minors of order 2. In this sum the parameter β_{π^m} thus only enters once and with a negative sign implying that this sum can be made negative (leading to instability) if this parameter is chosen sufficiently large.

Assuming – as a mild additional assumption – that interest rate smoothing is sufficiently weak furthermore allows for the conclusion that the 4D determinant of the above Jacobian exhibits a positive sign throughout. We thus in sum get that the local asymptotic stability of the steady state of the 3D case extends to the 4D case for sufficiently small parameters $\beta_{\pi^m} > 0$, since the eigenvalue that was zero in the case $\beta_{\pi^m} = 0$ must become negative due to the positive sign of the 4D determinant (since the other three eigenvalues must have negative real parts for small β_{π^m}). Loss of stability can only occur through a change in the sign of the Routh-Hurwitz coefficient a_2, which can occur only once by way of a Hopf-bifurcation where the system looses its local stability through the local death of an unstable limit cycle or the local birth of a stable limit cycle. This result is due to the destabilizing Mundell-effect of a faster adjustment of the inflationary climate the economy is embedded into, which in the present dynamical system works through the elements J_{14}, J_{41} in the Jacobian J of the dynamics and thus through the positive dependence of economic activity on the inflationary climate expression and the positive dependence of this climate expression on the level of economic activity.

We therefrom in sum get that the 4D dynamics will be convergent for sufficiently small speeds of adjustments β_{π^m}, while it will be divergent for parameters β_{π^m} chosen sufficiently large. The Mundell effect thus works as expected from a partial perspective. There will be a unique Hopf bifurcation point $\beta^H_{\pi^m}$ in between where the system loses asymptotic stability in a cyclical fashion. Yet sooner or later purely explosive behavior will be indeed be established (as can be checked by numerical simulations), where there is no room any more for persistent economic fluctuations in the real and the nominal magnitudes of the economy. In such a situation global behavioral nonlinearities must be taken into account in order to limit the dynamics to domains in the mathematical phase space that are of economic relevance. Compared to the New Keynesian approach briefly considered in section 2 of this paper we thus have that – despite many similarities in the wage-price block of our dynamics – we have completely different implications for the resulting dynamics which is convergent (and thus determined from the historical perspective) when estimated empirically (with structural Phillips curves that are not all at odds with the facts) and which – should loss of stability occur via a faster adjustments of the inflationary climate expression – must be bounded by appropriate changes in economic behavior far off the steady state and not just by mathematical assumption as in the New Keynesian case. Furthermore, we have employed in our model type a dynamic IS-relationship in the spirit of Rudebusch and Svensson's (1999) approach, also confirmed in its backward orientation by a recent article of Fuhrere and Rudebusch (2004). One may therefore state that the results achieved in this and the preceding section can provide an alternative of mature, but traditional Keynesian type that does not lead to the radical – and not very Keynesian – New Keynesian conclusion that the deterministic part of the model is completely trivial and the dynamics but a consequence of the addition of appropriate stochastic processes.

7 Instability, global boundedness and monetary policy rules

Based on the estimated parameter values the preceding section has shown that the model then exhibits strong convergence properties with only mild fluctuations around the steady state in the case of small shocks, but with a long downturn and a long-lasting adjustment in the case of strong shocks (as in the case of figure 2, where a 10 percent increase in real wages is shocking the economy. Nevertheless, the economy is reacting fairly stable to such a large shock and thus seems to be of the type of a strong shock absorber. Figure 2 however is based on estimated linear Phillips curves, i.e. in particular, on wage adjustment that is as flexible in an upward as well as in a downward direction. It is however much more plausible that wages behave differently in a high and in a low inflation regime, see Chen and Flaschel (2004) for a study of the WPC along these lines which confirms this common sense statement. Following Filardo (1998) we here go even one step further and indeed assume a three regime scenario shown in figure 4 where we make use of his figure 4 and for illustrative purposes of the parameter sizes there shown for yearly data (though they there refer to output gaps on the horizontal axis, inflation surprises on the vertical axis and a standard reduced form Phillips curve relating these two magnitudes):

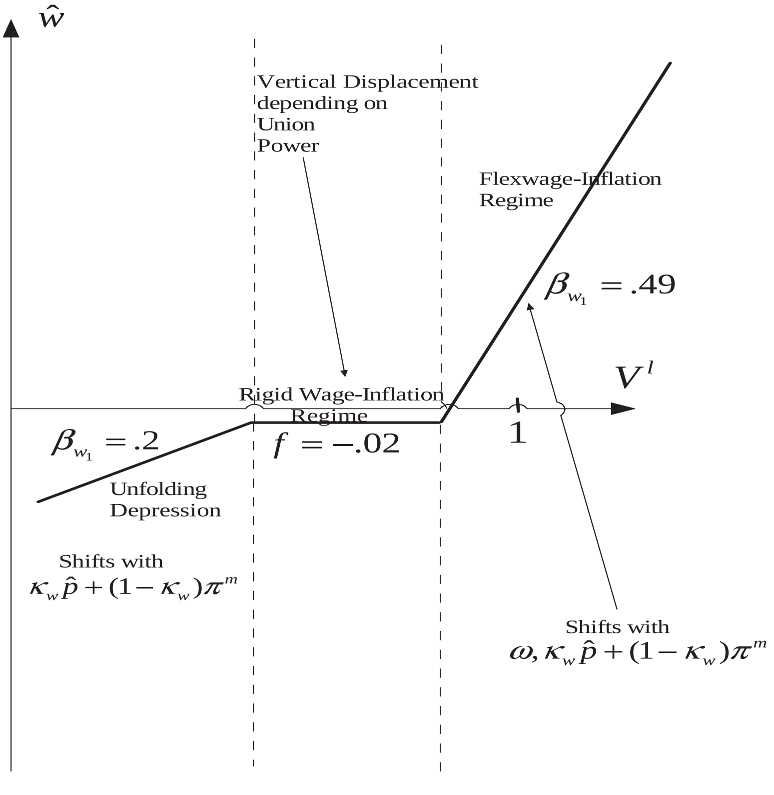

Figure 4: Three possible regimes for wage inflation.

The figure 4 suggests that the WPC of the present model is only in effect if there holds simultaneously that wage inflation is above a certain floor f – here (following Filardo) shown to be negative, while this floor is claimed and measured to be positive for six European countries in Hoogenveen and Kuipers (2000) – and that the employment rate is still above a certain floor $\underline{V}^l$, where wage inflation starts to become (downwardly) flexible again. In this latter area (where wage inflation according to the original linear curve is below f and the employment rate below $\underline{V}^l$,) we assume as form for the resulting flex wage-inflation curve the following simplification and modification of the original one:

$$\hat{w} \;=\; \beta_{w_1}(V^l - \underline{V}^l) + \kappa_w \hat{p} + (1-\kappa_w)\pi^m, \quad i.e.,$$

we do not consider the Blanchard and Katz error correction term to be in operation then any more. In sum, we therefore assume a normal operation of the economy if both lower floors are not yet reached, constant wage inflation if only the floor f has been reached and further falling wage inflation or deflation rates (from the side of demand pressure) if both floors have been passed. Downward wage inflation or wage deflation rigidity thus does not exist for all states of a depressed economy, but can give way to its further downward adjustment.[17]

In figure 5 we consider the situation depicted in figure 2 for a twelve quarter moving average inflation climate and again contractive real wage shocks. Compared to the reaction of the rate of capacity utilization V^c in figure 2 we have for the accompanying interest rate reaction no overshooting, but after the initial decrease in the interest rate reacting to the resulting depression and disinflation a monotonic increase in this rate back to its steady state level. We use the interest rate now in the place of V^c in order to check that it does not become negative during a viable operation of the economy. The shown time series is the same for no floor in the money wage rate and a strict floor $f = 0$ that excludes any decline in the money wage rate from the dynamics.

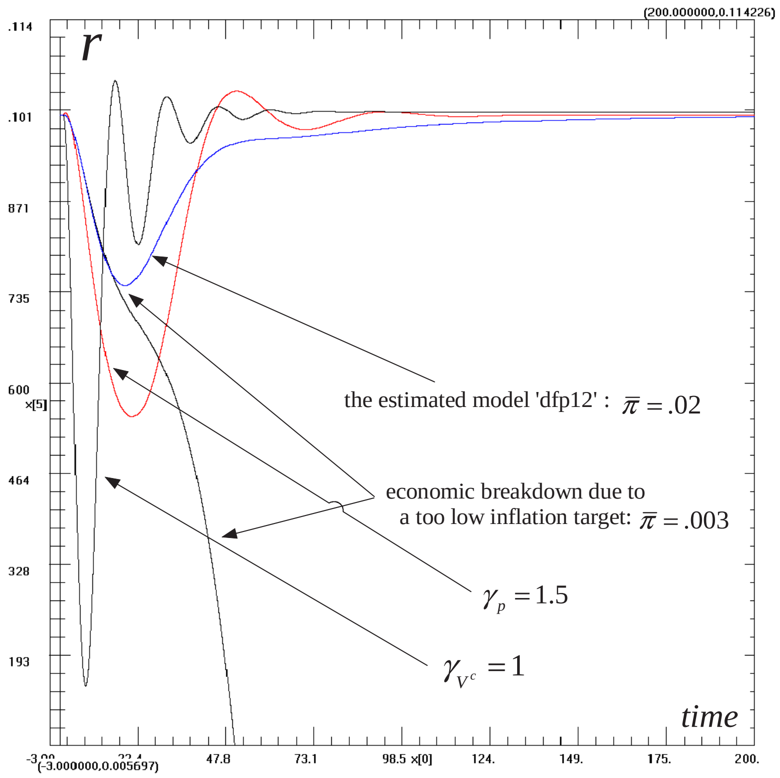

Figure 5: Downward money wage rigidity and too restrictive inflation targets.

[17] An example for this situation is given by the German economy, at least since 2003.

If, in the estimated situation, the central bank now reduces the target rate of inflation from 2 percent to 0.3 percent, we get a dramatic change for the case of complete downward money wage rigidity. The turning point in economic activity – here shown through the rate r solely – gets lost and the economy is subject to economic breakdown over a horizon of about 6 years (since the time scale on the horizontal axis is referring to quarters). The explanation of such an occurrence is a simple one: Declining price levels in combination with downwardly rigid money wages lead to systematic real wage increases which in turn reduce economic activity and thus lead to further reductions in the price level and so on. An inflation target that is too tight can therefore be disastrous in the case of downward money wage rigidity.

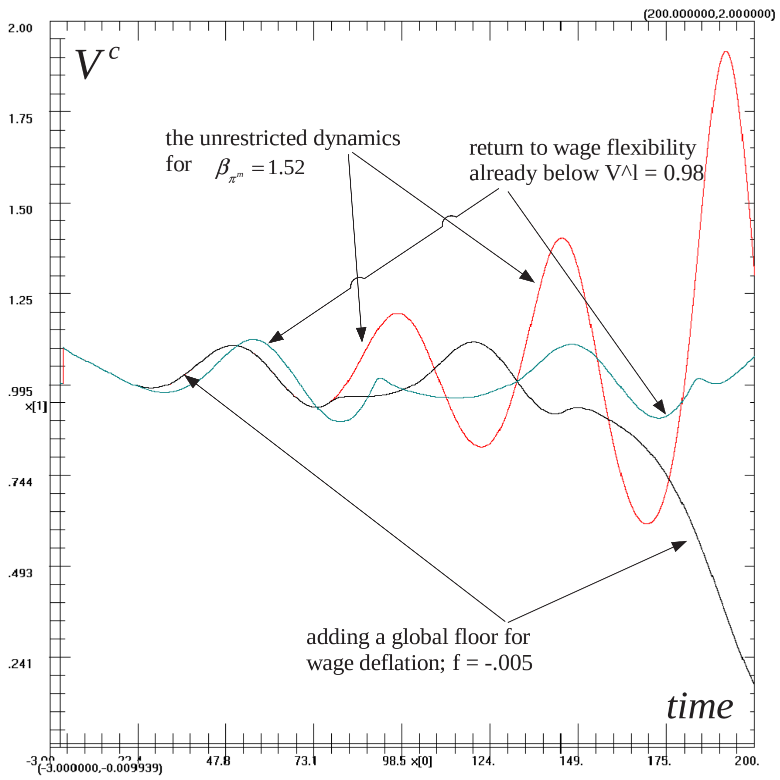

Figure 6: Explosive unrestricted fluctuations and single vs. two times kinked WPC's.

Of course, the monetary authority may try to stick to this target and to make its interest rate policy more active, either by increasing its reaction to the inflation gap or to the capacity utilization gap. The result is also depicted in figure 5 and it is shown there that both of these policies can prevent economic breakdown, the stronger reaction to the inflation gap with a deep and long depression and the stronger reaction to the activity gap with a less deep, but more volatile result in the fluctuations of the capacity utilization of firms (here only indicated through movements in the nominal rate of interest). The lesson from this figure thus is that even in a very stable environment too tight an inflation target can be very problematic and only be overcome in its consequences by a strong response of interest rates to inflation and output gaps (if there is a strong nonlinearity in the money wage PC, but less strong in fact than what was estimated to be the case in Hoogenveen and Kuipers (2000)).

This result holds for an economy that exhibits strong convergence properties if not restricted by behavioral nonlinearities of the discussed type. Let us next investigate a situation where the economy is destabilized by a change in the speed of adjustment of the inflationary climate that is surrounding it (see figure 3 top-left). We assume now in the place of the value 0.15 the value 1.52 for the parameter β_{π^m} and leave all other parameters as they were estimated in the case $dp12$. The result, now again in terms of capacity utilization, is shown in figure 6 by the business cycle time series with symmetrically increasing amplitudes.

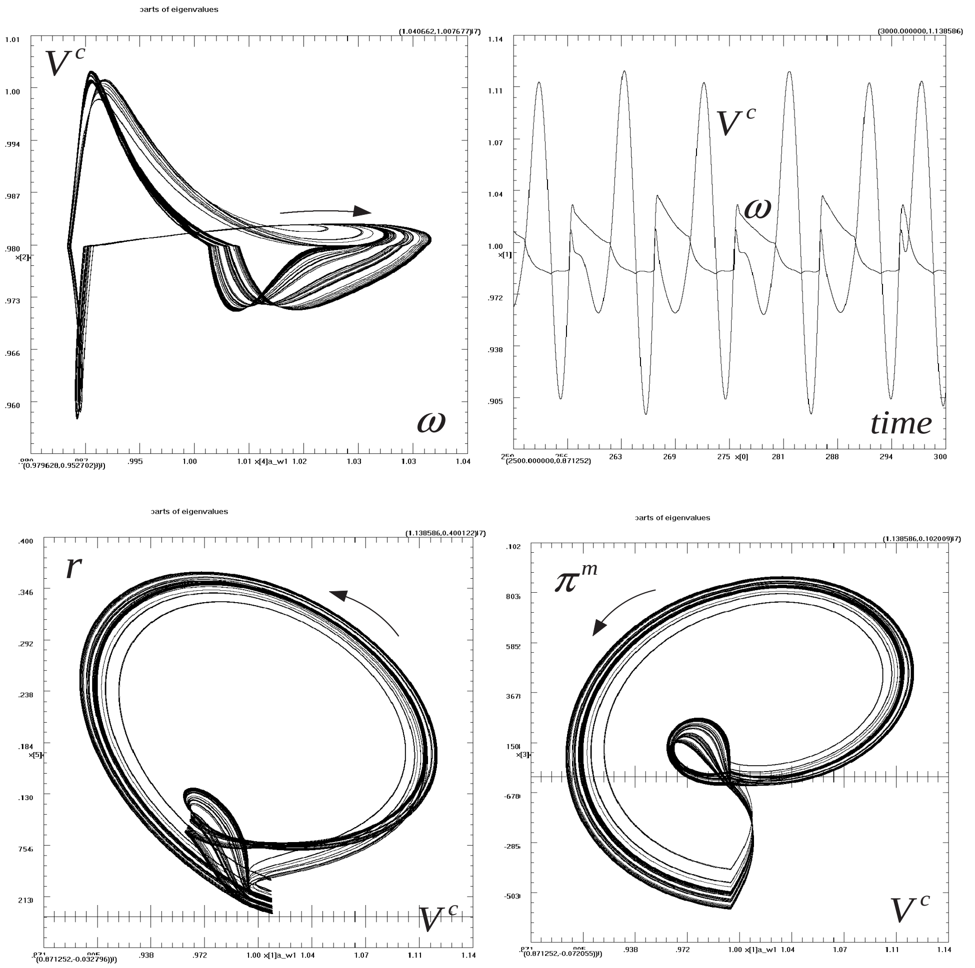

Figure 7: Economic viability and irregular dynamics through kinked WPC's.

With the change in the adjustment speed of the inflationary climate expression the economy is no longer viable in the long run and it becomes even less viable if a global floor $f = -.005$ is introduced into the estimated WPC as shown in figure 6. Yet assuming a WPC as discussed in connection with figure 4 overcomes not only this latter monotonic downturn, but also the explosive fluctuations of the unrestricted case. Some downward flexibility of money wages in the middle regime, augmented – in the simulation show – by downward flexibility of money wages (as in the unrestricted situation) provides viability to the evolution of the trajectories of the dynamics as is only indicated in figure over a 50 year horizon. In figure 7 we therefore show the attractor of the resulting dynamics in the form of various planar projections of the 4D dynamics and also again a time series of the rate of capacity utilization over a time span of 250 years.[18]

We see in figure 7 projections of – from the mathematical point of view – complex attractor, which – from the economic point of view – is however only somewhat irregular. We can detect phases where economic activity rises without much change in real wages (and also the opposite) and phases where inflation without much change or even a decline in economic activity. The figure 7, bottom-right, shows the typical overshooting mechanism of here the interaction of capacity utilization with the inflationary climate, with stagflation occurring top-right in this phase plot. The interactions of interest rates and economic activity is a cyclical one, with increased tensions and sudden turns at low rates of interest. The interaction of the employment rate is partly clockwise, but also partly counterclockwise. In sum, we therefore for the moment find that twice kinked Phillips curves as considered in Filardo (1998), here used for our WPC, can tame explosive behavior or even avoid economic breakdown, and will in fact give rise to even complex dynamics if the adjustment of inflationary expectations introduces local instability in an otherwise stable system with estimated parameter sizes. We finally note that, on an average, the employment rate stays significantly below the normal level 1, the interest rate above its target level and the inflation climate below the target of the central bank, while capacity utilization fluctuates fairly symmetrically around its normal level.

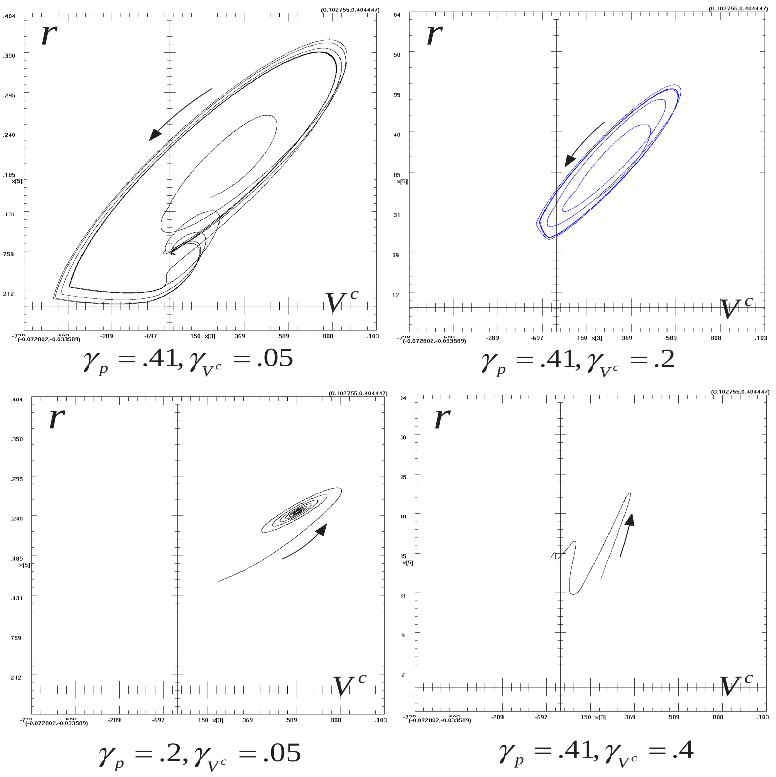

Figure 8: Monetary policy in regimes of irregular persistent business fluctuations .

In figure 8 finally we consider the issue of what monetary policy can achieve in such an

[18]Note that the phase length of the considered fluctuations must be a long one, since all parameters of the model are kept constant along these fluctuations, instead of varying systematically with as for example discussed in the 'Notes on trade Cycle' in Keynes General Theory.

environment. Top-left we show again the obtained complex dynamics for the estimated parameters of the interest rate policy rule, there projected into the interest rate – inflationary climate phase plane. Increasing the reaction of the monetary authority to the utilization gap (from the estimated value 0.05 to 0.2) removes the complexity of the dynamics and give rise to a limit cycle with considerably less amplitude and less deflation than was the case before. We even get convergence and thus complete disappearance of the business cycle for the further increased parameter value $\gamma_{V^c} = 0.4$, though not towards the interior steady state of the unrestricted model, but towards a depressed situation with high interest rates and the stable deflation our kinked WPC does allow for. By contrast, however, increasing the reaction of the central bank to the size of the observed inflation gap will make the economy not more, but less stable, while indeed a reduction in the already passive reaction strength (shown in figure 8, bottom-left) will again remove the initial persistent cycle and lead to damped fluctuations, again not around the interior steady state of the unrestricted dynamics, but to a steady state with capacity utilization and nominal interest rate above and the other state variables below these steady state values.

We conclude from these few simulation examples, that our estimated Keynesian disequilibrium dynamics gives rise to a variety of interesting situations when certain kinks in money wage behavior and changes in the adjustment speed of our inflationary climate expression is taken account of. These changes furthermore show that more further investigation of such behavioral nonlinearities are needed, see Chen and Flaschel (2004) and Flaschel, Kauermann and Semmler (2004) for some attempts into this direction in the case of the U.S. economy, and that the role of climate expressions must be further analyzed in future extensions of the analysis here presented.

8 Conclusions and outlook

We have considered in this paper a significant extension and modification of the traditional approach to AS-AD growth dynamics, primarily by way of an appropriate reformulation of the wage-price block of the model, that allows us to avoid the dynamical inconsistencies of the traditional Neoclassical Synthesis. It also allowed us to overcome the empirical weaknesses and theoretical indeterminacy problems of the New Neoclassical Synthesis, the New Keynesian approach, that arise from the existence of only purely forward looking behavior in baseline models of staggered price and wage setting. Conventional wisdom, based on the rational expectations approach, however is here used to avoid the latter indeterminacy problems by appriate extensions of the baseline model that enforce its total instability (the existence of only unstable roots), implying that the steady state represents the only bounded trajectory in the deterministic core of the model (to which the economy then immediately returns when hit by a demand, supply or policy shock).

By contrast, our alternative approach – which allows for sluggish wage as well as price adjustment and also for certain economic climate variables, representing the medium-run evolution of inflation (and in Asada, Chen, Chiarella and Flaschel (2004) also of excess profitability) – completely bypasses the purely formal imposition of such boundedness assumptions. Instead it allows to demonstrate in a detailed way, guided by the intuition behind important macroeconomic feedback channels, local asymptotic stability under certain plausible assumptions (indeed very plausible from the perspective of Keynesian feedback channels), cyclical loss of stability when these assumptions are violated (if speeds of adjustment become sufficiently high), and even explosive fluctuations in the case of further increases of the crucial speeds of adjustment of the model. In the latter case extrinsic behavioral nonlinearities have to be introduced in order to tame the explosive dynamics, for example as in Chiarella and Flaschel (2000, Ch.6,7) where a kinked Phillips curve (downward wage rigidity) is employed to achieve global boundedness.

The stability features of this – in our view properly reformulated – Keynesian dynamics are based on specific interactions of traditional Keynes- and Mundell-effects or real rate of interest effects with so-called Rose or real-wage effects, see Chiarella and Flaschel (2000) for their introduction, which in the present framework – for the estimated model – simply means that increasing wage flexibility is stabilizing and increasing price flexibility destabilizing, based on the estimated fact that aggregate demand here depends negatively on the real wage and due to the extended types of Phillips curves we have employed in our new approach to traditional Keynesian growth dynamics. The interaction of these three effects is what explains the obtained stability results under the in this case not very important assumption of myopic perfect foresight, on wage as well as price inflation, and thus gives rise to a traditional type of Keynesian business cycle theory, not at all plagued by the anomalies of the textbook AS-AD dynamics, see Chiarella, Flaschel and Franke (2004) for a detailed treatment and critique of this textbook approach.

Our model therefore provides an array of stability results, which however are narrowed down to damped oscilations when the model is estimated with data for the U.S. economy after World War II. Yet, also in this strongly conbvergent case, there can arise stability problems if the estimated linear WPC is modified to allow for some downward money wage rigidities. In such a case, prices may fall faster than wages in a depression, leading to real wage increases and thus to further reductions in economic activity, setting in motion a downward deflations spiral with further and further increasing real wages until in fact economic breakdown occurs. We have shown in this regard how the reestablishment of downward wage flexibility in situations that have become very severe may avoid this economic breakdown, leading then to persistent business fluctuations of more or less irregular type and thus back to a further array of interesting stability scenarios. Monetary policy can try to avoid such situations and preserve damped oscillatory behavior, primarily through the adoption of a target rate of inflation that is chosen sufficiently large, and in case of the establishment of persistent fluctuations of the above type, reduce such fluctuations by an activation of its reaction to output gaps or a reduction of its reaction to inflation gaps, as was shown by way of numerical examples. It is therefore not obvious into which direction monetary policy should be changed in order to make the economy less and not indeed more volatile.

The model of this paper will be numerically explored in a companion paper, Asada, Chiarella, Flaschel and Hung (2004), in order to analyze in greater depth, with and without the empirical background here generated, the interaction of the various feedback channels present in the considered dynamics. At that point we will make use of LM curves as well as Taylor interest rate policy rules, kinked Phillips curves and Blanchard and Katz error correction mechanisms in order to investigate in detail the various ways by which locally unstable dynamics can be made bounded and thus viable. The question then is which assumption on private behavior and fiscal and monetary policy – once viability is achieved – can reduce the volatility of the resulting persistent fluctuations.

Our work on related models suggests that interest rate policy rule may not be sufficient to tame the explosive dynamics in all conceivable cases, or even make it convergent. But when viability is achieved – for example by downward wage rigidity – we can then investigate parameter corridors where monetary policy can indeed reduce the endogenously generated fluctuations of this approach to Keynesian business fluctuations.

Taking all this together, our general conclusion here is that this framework not only overcomes the anomalies of the Neoclassical Synthesis, Stage I, but also provides a coherent alternative to its second stage, the New Keynesian theory of the business cycle, as for example sketched in Gali (2000). This alternative is based on disequilibrium in the market for goods and labor, on sluggish adjustment of prices as well as wages and on myopic perfect foresight interacting with certain economic climate expression with a rich array of dynamic outcomes that provide great potential for further generalizations. Some of these generalizations have already been considered in Chiarella, Flaschel, Groh and Semmler (2000) and Chiarella, Flaschel and Franke (2004). Our overall approach, which may be called a disequilibrium approach to business cycle modelling of mature Keynesian type, thus provides a theoretical framework within which to consider the contributions of authors such as Zarnowitz (1999) who also stresses the dynamic interaction of many traditional macroeconomic building blocks.

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9 Appendix: Estimation Results

```
System-Estimate: \pi24


Linear Systems - Estimation by System Instrumental Variables

Iterations Taken 2
Quarterly Data From 1965:01 To 2002:04
Usable Observations    141
 Total Observations    152      Skipped/Missing       11

Dependent Variable DW
Centered R**2     0.575957      R Bar **2   0.483774
Uncentered R**2   0.924109      T x R**2     130.299
Mean of Dependent Variable      0.0144658956
Std Error of Dependent Variable 0.0067780095
Standard Error of Estimate      0.0048699237
Sum of Squared Residuals        0.0027273581
Durbin-Watson Statistic             1.729474

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DP                        0.742146755  0.174278024
4.25841  0.00002059 2.  \PI24                        0.370928080
0.258063828      1.43735  0.15061853 3.  VL{1}
0.130083208  0.050842531      2.55855  0.01051094 4.  UCBP{1}
-0.083710765  0.035544840     -2.35508  0.01851897 5.  Constant
-0.119681460  0.048707479     -2.45715  0.01400452


Dependent Variable DP
Centered R**2     0.741782      R Bar **2   0.682891
Uncentered R**2   0.937025      T x R**2     132.120
Mean of Dependent Variable      0.0107048312
Std Error of Dependent Variable 0.0061013171
Standard Error of Estimate      0.0034358019
Sum of Squared Residuals        0.0013457398
Durbin-Watson Statistic             1.756548

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
6.  DW                        0.542721944  0.058720985
9.24238  0.00000000 7.  \PI24                        0.545524216
0.075159486      7.25822  0.00000000 8.  VC{1}
```

```
0.029690971  0.006570948      4.51852  0.00000623 9.  UCBP{1}
0.091781957  0.022814773      4.02292  0.00005748 10. D74
0.006062371  0.001735776      3.49260  0.00047834 11. Constant
-0.027228957  0.005453445     -4.99298  0.00000059


Dependent Variable DLVL
Centered R**2     0.635807      R Bar **2   0.567907
Uncentered R**2   0.635925      T x R**2      89.665
Mean of Dependent Variable      0.0000643400
Std Error of Dependent Variable 0.0035883459
Standard Error of Estimate      0.0023587552
Sum of Squared Residuals        0.0006565197
Durbin-Watson Statistic             1.431341

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
12. DLVC                     0.2091319356 0.0144254835     14.49740  0.00000000
13. Constant                 0.0001859823 0.0001763756      1.05447  0.29166905


Dependent Variable DLVC
Centered R**2     0.403475      R Bar **2   0.280056
Uncentered R**2   0.404114      T x R**2      56.980
Mean of Dependent Variable      -0.000581654
Std Error of Dependent Variable  0.017816411
Standard Error of Estimate       0.015117141
Sum of Squared Residuals        0.0265092413
Durbin-Watson Statistic             1.334401

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
14. VC{1}                    -0.093805470  0.026035675     -3.60296  0.00031462
15. RRATE{1}                 -0.173880204  0.044069044     -3.94563  0.00007959
16. UCBP                     -0.744305360  0.127462642     -5.83940  0.00000001
17. Constant                  0.086219485  0.021848091      3.94632  0.00007936


Dependent Variable RATE
Centered R**2     0.889129      R Bar **2   0.866190
Uncentered R**2   0.982857      T x R**2     138.583
Mean of Dependent Variable      0.0712450355
Std Error of Dependent Variable 0.0305780815
Standard Error of Estimate      0.0111854768
Sum of Squared Residuals        0.0145133273
Durbin-Watson Statistic             1.717594

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
18. DP                        0.426569670  0.169847604      2.51148  0.01202245
19. VC{1}                     0.045860894  0.020205006      2.26978  0.02322101
20. RATE{1}                   0.900706175  0.035391750     25.44961  0.00000000
```

```
21. Constant                    -0.034790497  0.016931833     -2.05474  0.03990422


Covariance\Correlation Matrix of Residuals
            DW             DP             DLVL           DLVC           RATE
DW    0.00001934297  -0.8808113407  -0.0769496687  -0.0757498777  -0.1304136539
DP   -0.00001196783   0.00000954425 -0.0508887611   0.1207371612   0.1294293357
DLVL -0.00000073027  -0.00000033924  0.00000465617 -0.3872194428  -0.0668326528
DLVC -0.00000456807   0.00000511447 -0.00001145673  0.00018800880  0.5109124741
RATE -0.00000581914   0.00000405674 -0.00000146311  0.00007107380  0.00010293140


System-Estimate: \pi12

Linear Systems - Estimation by System Instrumental Variables

Iterations Taken 2
Quarterly Data From 1965:01 To 2002:04
Usable Observations    141
 Total Observations    152      Skipped/Missing       11

Dependent Variable DW
Centered R**2     0.605060      R Bar **2   0.519204
Uncentered R**2   0.929317      T x R**2     131.034
Mean of Dependent Variable      0.0144658956
Std Error of Dependent Variable 0.0067780095
Standard Error of Estimate      0.0046998345
Sum of Squared Residuals        0.0025401711
Durbin-Watson Statistic             1.745489

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DP                        0.602645750  0.176417048
3.41603  0.00063541 2.  \PI12                        0.475264131
0.210906153       2.25344  0.02423149 3.  VL{1}
0.123497405  0.036017983      3.42877  0.00060632 4.  UCBP{1}
-0.088926183  0.033803346     -2.63069  0.00852113 5.  Constant
-0.113137436  0.034283953     -3.30001  0.00096681


Dependent Variable DP
Centered R**2     0.806935      R Bar **2   0.762903
Uncentered R**2   0.952915      T x R**2     134.361
Mean of Dependent Variable      0.0107048312
Std Error of Dependent Variable 0.0061013171
Standard Error of Estimate      0.0029708892
Sum of Squared Residuals        0.0010061848
Durbin-Watson Statistic             1.702034

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
```

```
6.  DW                       0.336057879  0.073119122
4.59603  0.00000431 7.  \PI12                    0.681704965
0.081901222      8.32350  0.00000000 8.  VC{1}
0.034327378  0.006749461      5.08594  0.00000037 9.  UCBP{1}
0.064237346  0.021147879      3.03753  0.00238525 10. D74
0.007869569  0.001745077      4.50958  0.00000650 11. Constant
-0.029547353  0.005457849     -5.41374  0.00000006


Dependent Variable DLVL
Centered R**2     0.628144      R Bar **2   0.558815
Uncentered R**2   0.628265      T x R**2      88.585
Mean of Dependent Variable      0.0000643400
Std Error of Dependent Variable 0.0035883459
Standard Error of Estimate      0.0023834420
Sum of Squared Residuals        0.0006703339
Durbin-Watson Statistic             1.440226

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
12. DLVC                     0.2126134352 0.0148315058     14.33526  0.00000000
13. Constant                 0.0001880074 0.0001775830      1.05870  0.28973571


Dependent Variable DLVC
Centered R**2     0.401444      R Bar **2   0.277605
Uncentered R**2   0.402086      T x R**2      56.694
Mean of Dependent Variable      -0.000581654
Std Error of Dependent Variable  0.017816411
Standard Error of Estimate       0.015142848
Sum of Squared Residuals        0.0265994794
Durbin-Watson Statistic             1.334156

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
14. VC{1}                    -0.089399344  0.025944109     -3.44584  0.00056928
15. RRATE{1}                 -0.172584186  0.043438797     -3.97304  0.00007096
16. UCBP                     -0.740257970  0.126969851     -5.83019  0.00000001
17. Constant                  0.082545222  0.021763561      3.79282  0.00014895


Dependent Variable RATE
Centered R**2     0.889087      R Bar **2   0.866140
Uncentered R**2   0.982850      T x R**2     138.582
Mean of Dependent Variable      0.0712450355
Std Error of Dependent Variable 0.0305780815
Standard Error of Estimate      0.0111875877
Sum of Squared Residuals        0.0145188059
Durbin-Watson Statistic             1.720381

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
```

```
18. DP                      0.412678154  0.170978227      2.41363  0.01579450
19. VC{1}                   0.049245320  0.020250914      2.43176  0.01502575
20. RATE{1}                 0.902140081  0.035309913     25.54920  0.00000000
21. Constant               -0.037507088  0.016965122     -2.21084  0.02704726
```

```
Covariance\Correlation Matrix of Residuals
            DW              DP             DLVL            DLVC            RATE
DW    0.00001801540  -0.6927029730  -0.0446864666  -0.0453206036  -0.1118783388
DP   -0.00000785413  0.00000713606  -0.0375454185   0.1912816213   0.1742782419
DLVL -0.00000041356 -0.00000021869  0.00000475414  -0.4108331850  -0.0870687804
DLVC -0.00000264207  0.00000701826 -0.00001230350  0.00018864879   0.5126235990
RATE -0.00000481863  0.00000472420 -0.00000192644  0.00007144659  0.00010297025
```

```
System-Estimate: \pi6

Linear Systems - Estimation by System Instrumental Variables

Iterations Taken 2
Quarterly Data From 1965:01 To 2002:04
Usable Observations    141
 Total Observations    152      Skipped/Missing       11

Dependent Variable DW
Centered R**2     0.609609      R Bar **2   0.524741
Uncentered R**2   0.930132      T x R**2     131.149
Mean of Dependent Variable      0.0144658956
Std Error of Dependent Variable 0.0067780095
Standard Error of Estimate      0.0046726921
Sum of Squared Residuals        0.0025109159
Durbin-Watson Statistic             1.664760

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DP                      0.420721233  0.218072640
1.92927  0.05369722 2.  \PI6                      0.617009742
0.240374437      2.56687  0.01026213 3.  VL{1}
0.110618073  0.029588639      3.73853  0.00018510 4.  UCBP{1}
-0.097200730  0.034370975     -2.82799  0.00468415 5.  Constant
-0.100307296  0.028014846     -3.58051  0.00034293


Dependent Variable DP
Centered R**2     0.832232      R Bar **2   0.793969
Uncentered R**2   0.959084      T x R**2     135.231
```

```
Mean of Dependent Variable      0.0107048312
Std Error of Dependent Variable 0.0061013171
Standard Error of Estimate      0.0027694288
Sum of Squared Residuals        0.0008743499
Durbin-Watson Statistic             1.747540

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
6.  DW                        0.177078861  0.083857839
2.11166  0.03471601 7.  \PI6                    0.824803784
0.089209229      9.24572  0.00000000 8.  VC{1}
0.028436521  0.006116308      4.64930  0.00000333 9.  UCBP{1}
0.030970638  0.021627119      1.43203  0.15213584 10. D74
0.007316561  0.001775362      4.12116  0.00003770 11. Constant
-0.023573580  0.004863736     -4.84680  0.00000125


Dependent Variable DLVL
Centered R**2     0.624941      R Bar **2   0.555015
Uncentered R**2   0.625062      T x R**2      88.134
Mean of Dependent Variable      0.0000643400
Std Error of Dependent Variable 0.0035883459
Standard Error of Estimate      0.0023936855
Sum of Squared Residuals        0.0006761082
Durbin-Watson Statistic             1.443444

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
12. DLVC                     0.2139935327 0.0148279167     14.43180  0.00000000
13. Constant                 0.0001888101 0.0001779486      1.06104  0.28867294


Dependent Variable DLVC
Centered R**2     0.400707      R Bar **2   0.276715
Uncentered R**2   0.401350      T x R**2      56.590
Mean of Dependent Variable      -0.000581654
Std Error of Dependent Variable  0.017816411
Standard Error of Estimate       0.015152169
Sum of Squared Residuals        0.0266322345
Durbin-Watson Statistic             1.333938

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
14. VC{1}                    -0.087901027  0.025899056     -3.39399  0.00068883
15. RRATE{1}                 -0.172313629  0.042725620     -4.03303  0.00005506
16. UCBP                     -0.738904028  0.126750047     -5.82962  0.00000001
17. Constant                  0.081306043  0.021707330      3.74556  0.00017999


Dependent Variable RATE
Centered R**2     0.889093      R Bar **2   0.866147
Uncentered R**2   0.982851      T x R**2     138.582
```

Mean of Dependent Variable	0.0712450355
Std Error of Dependent Variable	0.0305780815
Standard Error of Estimate	0.0111872870
Sum of Squared Residuals	0.0145180254
Durbin-Watson Statistic	1.719811

Variable	Coeff	Std Error	T-Stat	Signif
18. DP	0.436354636	0.172619081	2.52785	0.01147643
19. VC{1}	0.049608273	0.020259936	2.44859	0.01434167
20. RATE{1}	0.900497767	0.035458792	25.39561	0.00000000
21. Constant	-0.037940151	0.016971735	-2.23549	0.02538517

Covariance\Correlation Matrix of Residuals

	DW	DP	DLVL	DLVC	RATE
DW	0.00001780791	-0.3668130112	-0.0735320861	0.0071825216	-0.1002736397
DP	-0.00000385464	0.00000620106	-0.0987427905	0.2880260143	0.1636919701
DLVL	-0.00000067949	-0.00000053844	0.00000479509	-0.4197416135	-0.0904398589
DLVC	0.00000041656	0.00000985732	-0.00001263208	0.00018888110	0.5124650125
RATE	-0.00000429376	0.00000413623	-0.00000200957	0.00007146653	0.00010296472