

# Uncertainty, Learning, and Optimal Technological Portfolios: A Dynamic General Equilibrium Approach to Climate Change\*

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## Abstract

How is the design of efficient climate policies affected by the potentials for induced technological change and for future learning about key parameter uncertainties? We address this question using a new integrated climate-economy model incorporating endogenous technological change to explore optimal technological portfolios against global warming in the presence of uncertainty and learning. We explicitly consider the interplays between induced innovation, the stringency of environmental policies, and possible environmental risks within the general equilibrium framework of probabilistic integrated assessment. We find that the value of resolving key scientific uncertainties would be non-trivial in the face of binding climate limits, but at the same time it can significantly decrease with induced innovation and knowledge spillovers that might otherwise be absent. The results also show that scientific uncertainties in climate change could justify immediate mitigation actions and accelerated investments in new energy technologies, reflecting risk-reducing considerations.

*Key words:* Uncertainty; Learning; Optimal technological portfolios; Endogenous technological change; Stochastic growth model; Probabilistic integrated assessment; Carbon-free technology; Expected value of information

*JEL classification:* Q28; D81; O33; C68

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# 1 Introduction

How will the design of efficient climate policies be affected by the potentials for induced innovation and for future learning about key parameter uncertainties? One of the most pressing problems facing the world economy nowadays is the design of climate-change policies in the presence of enormous uncertainty. And, this necessarily involves identifying efficient and diverse technological options for global emission reductions (e.g., required to prevent dangerous anthropogenic interference with the uncertain climate system by the UN Framework Convention on Climate Change, Article 2, 1992).<sup>1</sup> In particular, the optimal policy options may depend critically on the uncertainty and information about the relation between greenhouse gases and climate change. Moreover, it has often been suggested that induced innovation and environmental risk are closely related, although these factors are usually studied separately.

Several authors have made important contributions to the climate change policy under uncertainty, focusing on the questions of how optimal policies would change under uncertainty (Nordhaus, 1994; Nordhaus and Popp, 1997; Pizer, 1999; Webster et al., 2003) and of what happens if uncertainty is resolved over time (Manne and Richels, 1992; Nordhaus, 1994; Kolstad, 1996; Nordhaus and Popp, 1997; Ulph and Ulph, 1997; Kelly and Kolstad, 1999; Gollier et al., 2000). However, note that most of the existing studies on these issues are typically based on the modeling framework of a single policy option that is also simply subject to exogenously assumed technological change against global warming, which implies that there is no explicit close-loop interaction between induced technological progress and the stringency of environmental policies under uncertainty.

This paper explores the implications for optimal global change policy of the scientific uncertainties about the climate system within a dynamic integrated assessment framework of the world economy in which we allow for endogenous technological portfolios and the diffusion of innovative technologies in response to uncertainty and learning. First, we introduce the competing roles of alternative

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<sup>1</sup> Climate change is a long-term, global problem featuring complex interactions between environmental, socioeconomic, and technological processes over time, thereby involving uncertainties related to the coupled natural-human system and mitigation policies.

carbon mitigation options into in the framework of dynamic optimization problems.<sup>2</sup> The technological portfolios here refer to what kind of carbon mitigation efforts would occur in a carbon-constrained world, as compared to the business-as-usual world. We categorize these mitigation efforts into two broad clusters of options: conventional (fossil-fuel based) versus new carbon-free technologies.<sup>3</sup> Then, we integrate their related bottom-up cost information and technological improvement components into a simple Arrow-Romer type economic growth framework that is built on a recent Nordhaus' climate-economy model (Nordhaus and Boyer, 2000).

If innovation and environmental risk are endogenous, the direct impact of climate change on the economy would not be the only way in which global warming affects future economic welfare; the different rate and direction of technological progress (based on optimal intertemporal behavior) might also enhance capital accumulation, the society's propensity to save, and the rate of economic growth in a different way. Therefore, as compared to the literature, the analytical and numerical model developed here captures explicitly the possible links between endogenous technological change and the stringency of environmental policies in the presence of environmental risk. Despite its high-level abstractions, the modeling framework can thus help provide us with a better understanding for sources of technology-dependent domains of innovation and their interactions with the uncertain climate system.

Taking into account the possible competitions between the two stylized mitigation options, we first present a framework for evaluating systematically how the major scientific uncertainties, risks and alternative political preferences would affect the optimal technological portfolios strategies against global warming. We then examine the economic implications of scientific uncertainty and learning: How

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<sup>2</sup> Since the problems of choosing cost-efficient energy technologies are ones of scarcity and choice, appropriate response strategies that capture this behavior are intertemporal optimization techniques in the framework of dynamic general equilibrium.

<sup>3</sup> A somewhat stylized difference between conventional and new (carbon-free) energy technologies is that the latter are initially much costlier in mitigation than the former, but their costs are assumed to be decrease more rapidly with their diffusions, making the new technologies more competitive (Nakicenovic et al., 1998). In addition, the possibility of a carbon tax biases the technological portfolio more in favor of the new technologies. Note also that technological changes that govern the technological portfolios are inherently dynamic and uncertain in nature.

would the learning affect the society’s precautionary investment strategies (*e.g.*, investment in non-carbon energy technologies or in carbon abatement efforts)? What would be the value of new scientific knowledge about climate change in the presence of induced innovation and endogenous environmental risk?<sup>4</sup>

The rest of this paper is organized as follows. Section 2 sets up a theoretical background of the uncertainty analysis and describes the numerical model. Section 3 discusses optimal technological portfolios policies under uncertainty and risk, the effects of learning, and the value of information about uncertain climate change. Section 4 contains some concluding remarks.

## 2 The Model

### 2.1 *Theoretical background of the model*

In this section we present a simple stochastic growth model with externalities due to learning-by-doing, knowledge spillovers and climate-change control to explore analytically how the presence of environmental risk affects the economic performance and welfare in a carbon constrained world. In the model, environmental risk enters the economy as a multiplicative income disturbance while the economy-wide capital stock exerts a labor-augmenting technical progress (*via* learning-by-doing effect) in the spirit of Arrow (1962) and Romer (1986). In the Arrow-Romer economy, individual firms neglect their own contribution to the economy-wide stock of technical knowledge, but the “aggregate” capital stock in the economy is taken to represent the stock of knowledge in the economy so as to generate a positive externality on the production possibilities for individual firms. For this reason, aggregate production can exhibit increasing returns to scale as a whole, while a representative producer is subject to constant returns to scale with respect to capital and labor.

We then combine the Arrow-Romer technological change framework into a coupled natural-human system in the Nordhaus’ Dynamic Integrated Climate and

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<sup>4</sup> Note that the value of early knowledge can be extremely large to the extent that man-made investments and efforts are expensive and the stringency of policy goal is non-negligible.

Economy (DICE) model (Nordhaus, 1994; Nordhaus and Boyer, 2000).<sup>5</sup> In the DICE framework, the linkage between economic activity and climate change is summarized as an adjustment to labor productivity term that is purely exogenous to the standard neoclassical growth model without a climate sector. Production creates pollution and this pollution reduces properly measured output. As in the DICE model, global warming is assumed to reduce output through its impact on weather patterns and sea level rise. The net (environmentally-adjusted) labor productivity is then distinguished from the purely exogenous labor productivity term by describing the adjusted amount of output available for consumption and investment after gross output has been reduced by mitigation costs and climate damages. In addition, we assume there is some critical level of global warming that would result in an abrupt change in climate and its damage.

To be specific, the flow of instantaneous output of the individual firm  $i$ ,  $Y_i(t)$ , over the time period  $(t, t+dt)$  is produced according to the stochastic DICE technology (*i.e.*, Cobb-Douglas production with climate damage and controls):

$$Y_i(t) = \left[ \Omega(u(t), \theta) K_i(t)^\gamma (K(t)L_i(t))^{1-\gamma} u(t) \right] (dt + \sigma dz(t)) \quad (1)$$

where  $K_i(t)$  is the privately owned capital stock and  $u(t) \in [0,1]$  is an index of the “emission standard”. Labor input  $L_i(t)$  is supplied inelastically and normalized to unity. The labor-augmenting technical progress available to individual firms is assumed to be proportional to the “aggregate” capital stock  $K(t)$  as in Romer (1986).  $\Omega(u(t), \theta)$  denotes the environmental externality factor (in terms of productivity shock) common to all firms, subject to the environmental standard  $u(t)$  and environmental variable  $\theta$ . In addition, assume that  $dz(t)$  is the increment to a standard Wiener process with zero mean and unit variance, and  $\sigma$  measures the risk of the productivity shocks due to uncertain environmental parameter  $\theta$  embedded in

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<sup>5</sup> The DICE model is a Ramsey-style optimal growth model of the world economy including future CO<sub>2</sub> emissions, concentration and global mean temperature dynamics from economic activity. It has been widely accepted in the literature because of its simplicity, elegance and transparency in modeling framework.

the coupled environmental-technological feedback factor  $\Omega(u(t), \theta)$ .<sup>6</sup> For analytical simplicity, capital depreciation is neglected.

Note that compared to the DICE framework in (1),  $u(t)$  is defined as the rate of emission standard (*i.e.*, an index of the technology used), and  $1 - u(t)$  thus measures its resource cost in terms of output loss. Higher values for  $u(t)$  yields more goods (*via* less mitigation costs) but also more pollution. In the original DICE model,  $u(t)$  corresponds to  $1 - c_0 \mu(t)^{c_1}$  where  $\mu(t)$  is the rate of emissions abatement, and  $c_0$  and  $c_1$  are its cost coefficients. For example,  $u(t) = 1$  (*i.e.*,  $\mu(t) = 0$ ) implies the business-as-usual case (no policies) for the economy. The term  $\Omega(u(t), \theta)$  in (1) can be thought of as the climate feedback term,  $1 / [1 + d_1 T(t) + d_2 T(t)^{d_3}]$  specified in the DICE model, where  $T(t)$  is the average surface temperature relative to pre-industrialization in  $^{\circ}\text{C}$ , and  $d_0$ ,  $d_1$  and  $d_2$  are its related damage coefficients (which are related to the policy variables vector  $u(t)$  and environmental variables vector  $\theta$  here). Due to the trade-offs between environmental feedback and policy variables, we assume the augmented productivity impact function  $\Omega(u(t), \theta) \cdot u(t)$  in (1) to be hill-shaped with respect to  $u(t)$  and subject to  $\partial \Omega(u, \theta) / \partial u < 0$ .

Consider now a continuum of identical infinitely-lived individuals who maximize expected lifetime utility

$$V(0) = E_0 \int_0^{\infty} e^{-\rho t} \frac{1}{1-\alpha} [C(t)]^{1-\alpha} dt \quad (2)$$

where  $E_0$  is the expectation operator conditional on time 0 information,  $\rho$  is the rate of time preference,  $C(t)$  is the instantaneous rate of consumption, and  $\alpha$  is the degree of relative risk aversion. With the nonstochastic consumption,  $C(t)dt$ , the individual agent's capital accumulation evolves stochastically according to

$$dK_i(t) = [\Omega(u(t), \theta) K_i(t)^{\gamma} K(t)^{1-\gamma} u(t) - C(t)] dt + \Omega(u(t), \theta) K_i(t)^{\gamma} K(t)^{1-\gamma} u(t) \sigma dz(t) \quad (3)$$

while the stochastic aggregate resource constraint is given by

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<sup>6</sup> If the uncertainty about climate change resolves over time,  $\sigma$  in (1) will *fall* over time.  $\sigma = 0$  implies no uncertainty case.

$$dK(t) = [\Omega(u(t), \theta)K(t)u(t) - C(t)]dt + \Omega(u(t), \theta)K(t)u(t)\sigma dz(t) \quad (4)$$

The representative agent's stochastic optimization problem is then to choose his consumption-capital ratio and his rate of capital accumulation to maximize the expected intertemporal utility function (2), subject to the individual capital accumulation constraint (3), while taking the "aggregate" resource constraint (4), the initial values for capital  $K_0$  and the stochastic process  $z_0$ , and emissions standard  $u(t)$  as given.

Using the Ito's lemma and its related Bellman equation, solving this stochastic optimization is straightforward and leads to the following stochastic equilibrium relationship for the maximal expected life-time utility<sup>8</sup>

$$V = \frac{(C(t)/K(t))^{1-\alpha}}{(1-\alpha)[\rho - (1-\alpha)(g - (1/2)\alpha \Omega(u(t), \theta)^2 u(t)^2 \sigma^2)]} K(0)^{1-\alpha} \quad (5)$$

where the consumption-capital ratio is

$$C(t)/K(t) = \frac{\rho - (1-\alpha)\gamma \Omega(u(t), \theta)u(t) + \alpha(1-\gamma)\Omega(u(t), \theta)u(t)}{\alpha} + \Omega(u(t), \theta)^2 u(t)^2 \sigma^2 \left[ \gamma - \frac{1+\alpha}{2} \right] \quad (6)$$

and the growth rate of capital (and 'knowledge' stock) accumulation is

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<sup>7</sup> Note that  $K_i(t)$  is equal to the average level of economy-wide capital stock at the equilibrium and due to knowledge spillover aggregate production becomes linear in capital. For a discussion on this type of models with learning-by-doing and knowledge spillovers, see Barro and Sala-i-Martin (2004, Ch.4., pp.212-218). For analytical convenience, normalizing the number of firms to be unity, we can have  $K_i(t) = K(t)$  in equilibrium. In this case the aggregate and average values *coincide* in equilibrium, given all agents being identical and the aggregate and individual proportional shocks being identical and perfectly correlated.

<sup>8</sup> As is well known for the isoelastic preferences above, the maximized value function can be assumed to be of the time-separable form  $V(K_i(t), K(t), t) = e^{-\rho t} X(K_i(t), K(t))$  in a generalized power function and the optimal choice  $C(t)^{-\alpha} = X_{K_i}(K_i(t), K(t), t)$  and the value function must satisfy the usual Bellman equation of the problem. For details of solving this class of problem, see Malliaris and Brock (1982), Turnovsky (1995) or Corsetti (1997). The necessary conditions from this individual optimization can be then employed to determine the macroeconomic equilibrium and, using the resource constraint (4) and the equilibrium condition  $K_i(t) = K(t)$ , we can obtain (6) and (7).

$$(g \equiv) \frac{1}{K(t)} \left[ \frac{E_t dK(t)}{dt} \right] = \frac{\gamma \Omega(u(t), \theta) u(t) - \rho}{\alpha} + \Omega(u(t), \theta)^2 u(t)^2 \sigma^2 \left[ \frac{1+\alpha}{2} - \gamma \right]. \quad (7)$$

As shown in (5), we can identify various channels through which uncertainty about climate change and policy measures could affect welfare in the presence of endogenous technological change due to learning-by-doing and knowledge spillovers in our model. In particular, we can see in (5) that, given arbitrary environmental objective  $u(t)$ , the welfare impact of a change in the environmental risk  $\sigma^2$  depends critically on the relative size between the capital elasticity  $\gamma$ , the degree of relative risk aversion  $\alpha$ , and the environmental-technological performance factor  $\Omega(u, \theta)u$ .

More specifically, the results indicate that optimal savings rate and knowledge accumulation under uncertainty may differ from the results derived for the case of certainty if  $(1+\alpha)/2 \neq \gamma$ . For example, if we consider cases  $\alpha > 1$  (*i.e.*, highly risk averse as suggested in much of the recent empirical evidence including Obstfeld, 1994; Campbell, 1996) or  $\alpha = 1$  (*i.e.*, logarithmic preferences as in the DICE model), the presence of the environmental drift and risk factor would significantly enhance the (so-called) “precautionary savings effect” in aggregate knowledge accumulation in (7) and welfare change in (5). Note that in most cases,  $(1+\alpha)/2$  usually exceeds the capital elasticity  $\gamma$  ( $= 0.30$  in DICE). Then, we can see that the size of the stochastic premium terms in eqs. (5) - (7) *increase* significantly with the environmental-technological performance factor  $\Omega(u, \theta) \cdot u$  and its related risk  $\sigma^2$ .

Unlike the implications suggested by Nordhaus (1994, p.187-188), in this Arrow-Romer economy with endogenous technological change, (6) shows that greater uncertainty could lead to lower consumption rather than higher consumption for a reasonable range of parameter conditions, and that the extent of the change can be substantial, depending on the magnitude of environmental impact and policy objective embedded in  $\Omega(u, \theta) \cdot u$ .<sup>9</sup>

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<sup>9</sup> Nordhaus (1994) concludes that the surprising result in his DICE model is that greater uncertainty leads to higher consumption (although the extent of the change is modest) and perhaps the ultimate surprise is that great uncertainty about climate change has little effect on the optimal savings rate.



In fact, in light of current knowledge about the scope and consequences of climate change, uncertainty is likely to raise the stakes in future climate change. The coupling of endogenous technological progress and environmental risk in an optimal growth model may then change significantly the implications for the stringency of climate-change policies and the value of improved information about uncertain climate change. In the numerical model that follows, we implement these ideas and insights to calculate the effects and policy responses of uncertainty and risk under different assumptions.

## 2.2 *The extended DICE model: DICE-LBD*

Based on the theoretical background of the environmentally-extended, stochastic growth framework described above, this section specifies a numerical model for evaluating the effects of uncertain climate change on economic performance and welfare in a more realistic setting. The main building block of the numerical model developed here is the latest version of Nordhaus' DICE model (Nordhaus and Boyer, 2000).<sup>10</sup> It is an optimal growth model with a climate sector we call it DICE-LBD model, which is an extension of the DICE model. In essence, the DICE-LBD model adopts the Arrow-Romer type approach of the new growth theory (as described in Section 2.1) and adds to this the original DICE climate module to incorporate a closed-loop interaction between uncertain climate change and endogenous technological progress. It is further extended to add another dimension (namely, different state of the world) for different states of the world in uncertainty analysis.

The basic structure of the original DICE model will be familiar from the previous work (Nordhaus, 1994; Nordhaus and Boyer, 2000), so we will pause only to highlight the key extensions in our setup in the numerical model. We begin by modifying the macroeconomic growth block of the DICE model to incorporate technological competitions between the two broad groups of carbon mitigation

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<sup>10</sup> The DICE model is a Ramsey-style optimal growth model of the world economy including future CO<sub>2</sub> emissions, concentration and global mean temperature dynamics from economic activity. It has been widely accepted in the literature because of its simplicity, elegance and transparency in modeling framework.

options under uncertainty, taking into account the possible learning-by-doing and knowledge spillovers in energy technologies. In particular, we explicitly incorporate endogenous links between the stringency of climate policy and the direction and composition of future technological innovations to combat the global warming problem.<sup>11</sup>

In the model, the economy produces a single final good. Individual utility depends on consumption of the final good and on the quality of the environment (*i.e.*, global mean temperature). Unlike the single (smeared) mitigation option in the original DICE model, the environmental quality can be now augmented by reductions in carbon emissions both *via* conventional mitigation technology and *via* the supply of new carbon-free alternatives.<sup>12</sup> Due to the learning-by-doing and knowledge spillover effects in mitigation technologies, the numerical model is closer to the spirit of the Arrow-Romer endogenous growth framework as described in the previous section, whereas the original Nordhaus' DICE model is in the traditional Ramsey (1928)'s exogenous growth framework applied to climate change.

Economic activity is described by a production function in (8) and its uses side of output. Output at time  $t$ ,  $Y(t)$ , depends on the inputs of labor  $L(t)$  and general physical capital  $K_C(t)$ :

$$Y(t) = \Omega(D(t))K_C(t)^\gamma (A(t)L(t))^{1-\gamma} = C(t) + I(t) + I_\mu(t) + I_\zeta(t) \quad (8)$$

where  $A(t)$ , purely exogenous labor productivity, is assumed to increase with decreasing rate over time following  $A(t) = [1 + g_A(t)]A(t-1)$ . Labor is supplied inelastically, and is determined by exogenous population growth (with its rate of decline  $\delta_L$ ) and physical capital stock is accumulated in the usual fashion.  $D(t)$ , included in the environmental feedback term  $\Omega(D(t))$ , is climate damage which is an increasing function of global temperature change. Output (net of climate damage

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<sup>11</sup> In the context of dynamic economy, technological change is likely to be endogenous and it may be connected to the stringency of environmental policy. Technological change in mitigation technologies can be fundamentally related to uncertainty.

<sup>12</sup> Here, conventional technology implies policy-induced efficiency improvement and substitution into less carbon-emitting fossil fuel sources, while new carbon-free technology includes non-carbon activities to production such as backstop-like, hydrogen or renewables.

effect) is available for private consumption  $C(t)$ , private investment  $I(t)$ , and the two forms of carbon mitigation efforts including investments in conventional technology  $I_\mu(t)$  and in carbon-free technology  $I_\zeta(t)$ . Similarly to the usual capital, these energy-specific knowledge stocks are assumed to be generated by the accumulation of previous investment efforts.

In the economy, emissions from burning fossil fuels are identified as carbon, and they can be now reduced either by the direct carbon abatement effort  $\mu(t)$  or the indirect supply effort of carbon-free activities  $\zeta(t)$ . The carbon emissions are thus written as

$$E(t) = \sigma(t)[1 - \mu(t) - \zeta(t)]Y(t) \quad (9)$$

where  $\sigma(t)$ , the business-as-usual carbon intensity of production, is regarded as declining exogenously due to “autonomous energy-efficiency improvement” (AEEI) following  $\sigma(t) = \sigma(t-1)/[1 + g_\sigma(t)]$ .

On the other hand, the cost of each of the carbon mitigation options,  $\mu(t)$  and  $\zeta(t)$  in terms of output is assumed to be

$$I_i(t) = c_{0i} [K_i(t)]^{-\alpha_i} [i(t)]^{c_{1i}} Y(t), \text{ where } i(t) = \mu(t) \text{ and } \zeta(t) \quad (10)$$

respectively and where  $c_{0i}$  is a normalization parameter and  $\alpha_i$  is the learning elasticity index (*e.g.*, Messner, 1997; Anderson, 1999). We assume that the technological progress is represented as a decreasing function of cumulative installed capacity and pertains to investment costs for each of the technologies. Note that the accumulation of knowledge here occurs in part not as a result of direct deliberate efforts, but as a side effect of conventional economic activity. This type of knowledge accumulation is typically known as “learning-by-doing” (Arrow, 1962) and its simplest case is when the learning occurs as a side effect of the production of new capital. It is assumed that, as is typical in the relevant growth literature, the stock of knowledge can be formulated as a usual power function of the stock of capital, since the increase in knowledge is a function of the increase in capital, *i.e.*,

$K_i(t) = K_C(t)^{\phi_i}$  for each technology  $\mu$  and  $\zeta$ .<sup>13</sup> At each point in time  $t$ , depending on the rate of technological improvement and accumulated knowledge stocks, the economy determines the optimal portfolio for the mitigation options toward a specific environmental goal.

Having added distinct mitigation technologies and their accumulated knowledge stocks, productivity change in the economy is now modeled as a combination of mitigation efforts, induced technological progress, and climate change effects. Using the environmental feedback term  $\Omega(D(t)) \equiv 1/[1+D(t)] = 1/[1+d_1T(t)+d_2T(t)^{d_3}]$ , the potential output  $Y(t)$  in (8) can be now rewritten in terms of effective output  $Y^*(t)$  (*i.e.*, net of mitigation costs  $I_\mu(t)$  and  $I_\zeta(t)$ ):

$$Y^*(t) \equiv \frac{1}{1+d_1T(t)+d_2T(t)^{d_3}} K_C(t)^\gamma (A(t)L(t))^{1-\gamma} - I_\mu(t) - I_\zeta(t) \quad [= C(t) + I(t)] \quad (8')$$

Then, combining (10) with the relation  $K_i(t) = K_C(t)^{\phi_i}$  and defining the technological learning rates parameters  $\alpha_i^* = \alpha_i \phi_i$  (as defined in Appendix B) into (8'), we can get the following effective production function and labor-augmenting technical progress component in the spirit of Arrow-Romer growth framework:

$$Y^*(t) = K_C(t)^\gamma [A^*(t)L(t)]^{1-\gamma},$$

where  $A^*(t) \equiv A(t) \cdot \left( \frac{1 - c_{0\mu} K_C(t)^{-\alpha_\mu^*} \mu(t)^{c_{1\mu}} - c_{0\zeta} K_C(t)^{-\alpha_\zeta^*} \zeta(t)^{c_{1\zeta}}}{1 + d_1T(t) + d_2T(t)^{d_3}} \right)^{\frac{1}{1-\gamma}}. \quad (8'')$

In the Arrow-Romer economy, individual firms take the above adjusted labor productivity term as given, together with the “aggregate” knowledge stock, the two policy instruments, and environmentally-related variables. Here, the rate of emission standard function  $u(t)$  in (1) can be now thought of as having the two policy controls arguments  $u(\mu(t), \zeta(t))$  and the vector  $\theta$  in (1) summarizes the environmentally-

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<sup>13</sup> Following Romer (1996), pp.116-117,  $\phi_i$  represents the degree of the knowledge spillover (hence, its resulting technological learning rate for the economy) and this externality can be internalized by appropriate subsidies to production in the sense of Barro (1990).

related variables such as climate sensitivity, climate thresholds, climate damages, and temperature changes. As the technological learning rate increases, however, the economy-wide technology can start to escape from diminishing returns to scale to broad capital at the “aggregate” level.

The remaining portion of our DICE-LBD model explains simply the link between carbon emissions, global warming and climate damage which is directly adopted from the DICE’s climate sector in Nordhaus and Boyer (2000). The climate sector in the DICE model is summarized in Appendix A and a further detail of the equations can be found in Nordhaus and Boyer (2000).

In the model the dynamic equilibrium path of the coupled natural-human system is characterized as the solution to an intertemporal optimization problem, maximizing discounted utility of per capita consumption in (2) subject to economic and environmental resource constraint and policy instruments (for the complete equations of the model, see Appendix A). There are three control variables in the numerical model: the rate of physical investment  $I(t)$ , the rate of direct carbon-emissions mitigation options  $\mu(t)$  and the rate of supply for non-carbon activities  $\zeta(t)$ . However, the policy recommendation of the model is now highly dependent on the choice of uncertain parameter values and their probability distributions in the complex, non-linear relations of the model, which is also closely related to the what extent the rates and directions of technological change interact with environmental policies under uncertainty.

In effect, elements of uncertainties and the way of propagating uncertainties throughout the economy could affect significantly the optimal technological portfolios (and their policy implications), and it would be desirable to take into account this factor when reporting the model outcomes. Thus, desirable advice based on such a model outcome is not just in the form “if society sets its environmental goal in this way, then the outcome will be as follows,” but “if society sets its environmental goal in this way, then the outcomes will be within the ranges shown, together with how likely it is that it will happen.” Moreover, as knowledge about the uncertainties improves, our decisions and responses can be more focused and potentially wasteful decisions can be avoided significantly. So it would be useful to

take a close look at the effects of the resolution of uncertainty over time on optimal policies and economic welfare.

Since the DICE-LBD model uses the Nordhaus and Boyer (2000)'s global economy model as its basic building block, coefficients already present in the original model are left unchanged. For the new parameters, we need to identify all possible underlying technological options to represent their plausible technological changes over time that become important in the carbon-constrained cases. To this end, we follow the relevant literature to adopt plausible parameters values for the dynamics of the energy-economic system to curb future greenhouse warming. Specifically, the assumptions on the new technology parameters and their plausible ranges are made from some of previous studies including McDonald and Schrattenholzer (2001), Popp (2002), Gerlagh and Lise (2003) and Sims et al. (2003).<sup>14</sup> In particular, for a plausible range of the technological learning rate parameters, we adopt McDonald and Schrattenholzer (2001)'s study that presents a range of 8 - 30% learning rate for a large set of new energy technologies at large. However, note that in general, no good empirical estimates exist for this kind of technological parameters due to the lack of sufficient, empirical data so far. As Weyant (1997) emphasizes, there dose not exist single, established information on most of the uncertain technological parameters on this calibration issue. Obtaining good empirical estimates for these parameters would be one of the most difficult challenges of dealing with endogenous and induced technological change, and the analysis can be improved further as we get more technological information and experience later.

As far as uncertain climate change and its policy implications are concerned, one of the most important parameters is climate sensitivity. "Climate sensitivity" is defined as the equilibrium global-mean temperature change in response to a doubling of CO<sub>2</sub> concentrations. For this key uncertain climate parameter, we refer to several

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<sup>14</sup> The global general R&D by OECD countries is \$500 billion and, for US, 2% of R&D expenditure is for energy technology (= \$10 bil.) (Popp, 2002). According to Popp (2002) and Anderson (1997), the R&D investment in backstops is assumed to be about 1/10 of that of the conventional energy technologies in the base year. So, for the initial knowledge stocks of conventional and new technologies, we assume US \$10 billion and US \$1 billion, respectively. Following Gerlagh and Lise (2003) and Sims et al. (2003), the initial cost for new carbon-free technology is assumed to be 4 - 5 times (or \$400 - 500/tC avoided) higher than conventional technology.

previous studies surveyed in Dessai and Hulme (2003). Assumptions made on the plausible distributions for all other uncertain variables (except for climate sensitivity) are adopted from Norhaus (1994), Nordhaus and Popp (1996), and Pizer (1997).

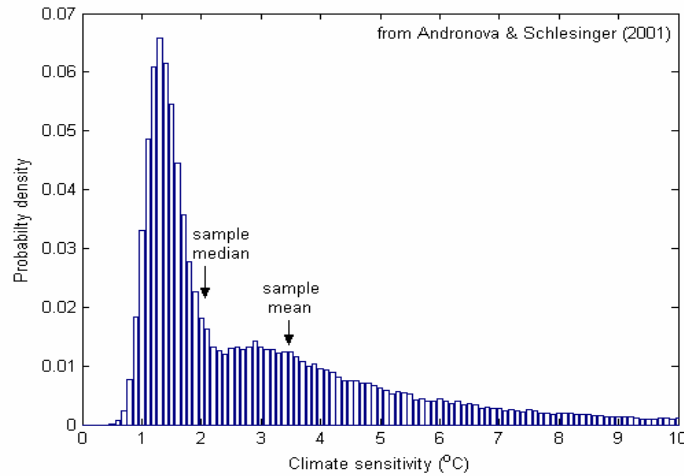
### 3 Results and Discussions

Current policy decisions must be made despite layers of uncertainties and the possibilities of the resolution of uncertainties. In light of this concern we explore the implications of uncertainty and learning for policies, focusing particularly on the optimal technological portfolios to cope with the uncertainties of climate change. Unlike many previous studies relying on a limited set of simple scenarios (or deterministic outcomes), this study allows explicitly for a “probabilistic” integrated assessment framework with uncertain climate change and induced technological progress, taking into account the possibility of policy-important climate threshold effects.<sup>15</sup> We first examine the range of possible outcomes and policy responses to see the consequences of uncertainties. In particular, we provide some metrics for assessing how to link the potential for dangerous climate change to desirable technological portfolio strategies when the future is uncertain. To see the implications for possible intertemporal conflicts between “wait-and-see” strategy and the precautionary principle in climate change policy, we then proceed to examine the effects of resolving uncertainties about climate change early rather than late, depending upon technological and environmental constraints.

First of all, to define our metric for the main determinant of uncertain climate change, we adopt a recent estimate from Andronova and Schlesinger (2001) on a probability distribution for uncertain climate sensitivity. Fig. 1 displays the density for climate sensitivity (with mean 3.4 and variance 8.6) adopted for our present study. As indicated in many other recent studies (*e.g.*, surveyed in Dessai and Hulme, 2003), it shows wider distributions than the Intergovernmental Panel on Climate Change (IPCC)’s range of 1.5 to 4.5°C (Cubashi et al., 2001).

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<sup>15</sup> In the simulation induced technological progress is measured as the increased role of new carbon-free technology (*e.g.*, share of “hydrogen” economy) that would respond to the impact of uncertainty and learning in the presence of environmental risks.



**Fig. 1. Distribution of uncertain climate sensitivity for our study**

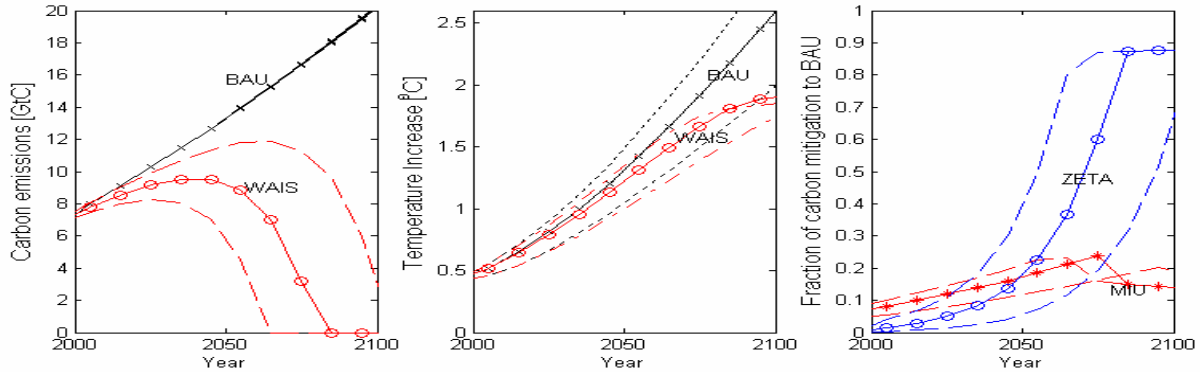
Under this probabilistic range of climate sensitivity, Fig. 2 shows band estimation for carbon emissions, global warming and technology choice over time for two scenarios: BAU vs. WAIS. In Fig. 2(a) and (b), BAU represents “no policy” and WAIS is “2.5 °C temperature stabilization policy.”<sup>16</sup> Shown are the estimated probabilistic ranges for (a) global carbon emissions, (b) global mean temperature increase, and (c) carbon mitigation technology portfolio over time. Lower and upper dashed lines in each panel refer to the first quartile and third quartile values in the distribution for each variable, respectively. In Fig. 2(c), MIU refers to “conventional technologies” and ZETA refers to “carbon-free technologies” (*e.g.*, non-carbon activities including solar/wind powers, carbon sequestration, hydrogen, biomass, and renewables) under the WAIS case. With the policy-relevant threshold chosen, we can see in Fig. 2(c) that carbon-free technologies would play an important role for carbon emission reductions over the this century (with wider variances in the middle of the century), which portrays the need for a substantial acceleration in the transition of

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<sup>16</sup> For a specific environmental threshold to avoid dangerous anthropogenic interference, in this paper we consider technology policies designed to limit the globally-averaged warming below 2.5°C that has been suggested as the temperature at which a collapse of the “West Antarctic ice sheet” (WAIS) might occur. For a detail on this issue, see Oppenheimer (1998) or Petit et al. (1997).

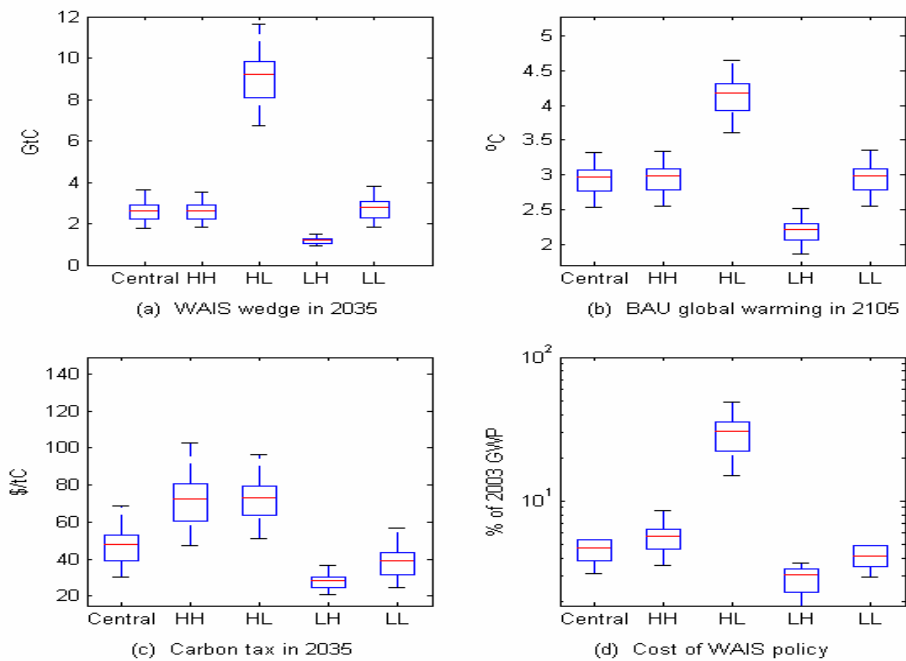


the energy system to non-fossil-fuel energy sources in comparison to the BAU reference scenario.



**Fig. 2. Band estimation for (a) carbon emissions, (b) global warming and (c) technology choice over time: BAU vs. WAIS.**

For the plausible distribution of the economic effects of the WAIS policy, we investigate the implication of alternative conjectural forces related to labor productivity growth and autonomous energy-efficiency improvement. Note that, in general, the cost and performance of carbon mitigation policies depend crucially on the evolution of purely exogenous labor productivity ( $A$ ), which is a critical determinant for the future economic growth, and on the evolution of autonomous energy-efficiency improvement ( $\sigma$ ). For this purpose, we compare four alternative cases about uncertain future economic environments surrounding the trends of two fundamental productivity-related parameters,  $g_A(t)$  and  $g_\sigma(t)$  that would dominate various economic and technological environments for the technological portfolios. In Fig. 3, the “WAIS wedge” is defined as the gap between global carbon emissions of BAU and WAIS in a specific year, which in turn implies the degree of policy stringency to minimize the negative impact of the BAU global warming. Not surprisingly, we can see in Figure 3 that the lower the labor productivity growth (or the higher the autonomous energy-efficiency improvements), the less costly for the economy to preserve the WAIS threshold constraint.

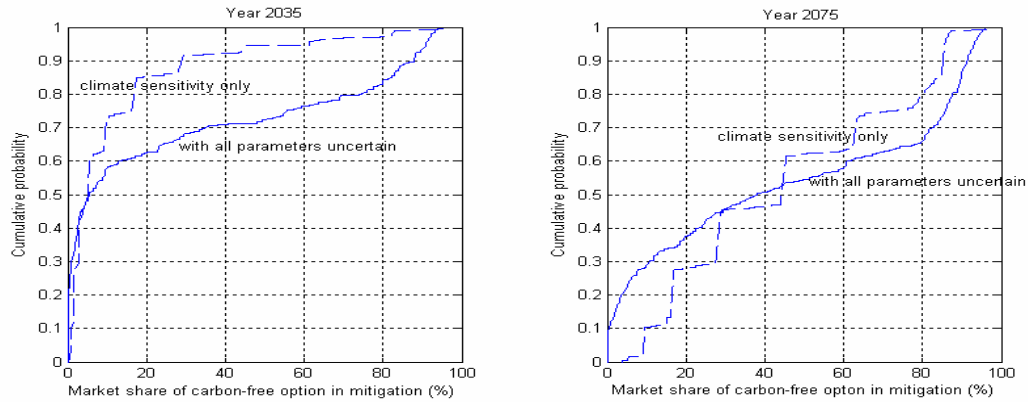


Note: “Central” assumes the reference cases for labor productivity growth (1.4%/yr) and AEEI growth (1.3%/yr), based on the historical trends. The extreme cases in the confidence region are then as follows: (i) “HH” case is with *higher* labor productivity growth and *higher* AEEI growth (2.5%/yr and 2.2%/yr), (ii) “HL” case is with *higher* labor productivity growth and *lower* AEEI growth (2.5%/yr and 0.5%/yr), (iii) “LH” case is with *lower* labor productivity growth and *higher* AEEI growth (0.8%/yr and 2.2%/yr), and (iv) “LL” case is with *lower* labor productivity growth and *lower* AEEI growth (0.8%/yr and 0.5%/y, respectively).

**Fig. 3. Distribution of the economic effects of WAIS policy**

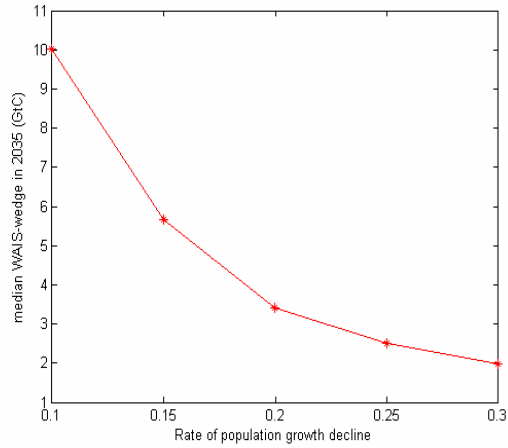
To see the effects of the scope of uncertainty on model outcomes, we simulate the distribution of the WAIS wedge in 2035 under alternative scope of parameter uncertainties in the model. According to the model simulation, the role of uncertain climate sensitivity (in determining the WAIS wedge in 2035) accounts for about 60-70% of total variation at the median WAIS wedge value. We also investigate the role played by each mitigation technology under alternative scope of parameter uncertainties. Fig. 4 compares the estimates for the distribution of the two technologies: conventional versus carbon-free technology. Shown in the left and

right panels are the distributions of efficient choice for the two options under the WAIS case in 2035 and in 2075, respectively. For each technology, the dashed line refers to the outcomes from “with only climate sensitivity uncertain” case, and the solid line from “with all parameters uncertain” case. Since the differences between the two cases are not large in both sample years, we can see that uncertainty about climate sensitivity would take a major part in determining the distribution of future technological portfolios.

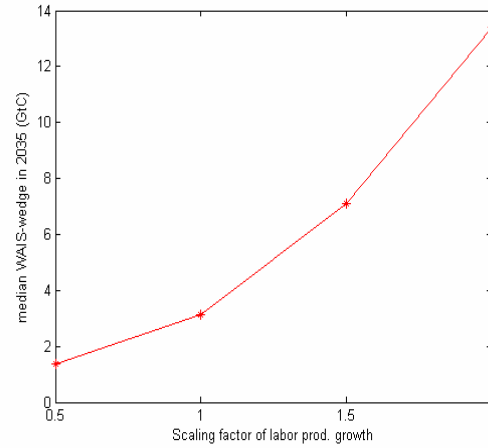


**Fig. 4. Distribution of mitigation options: Conventional vs. Carbon-free**

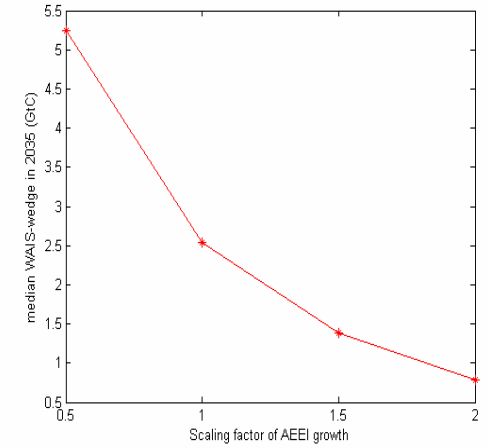
In the DICE-LBD model, the propagation of uncertainty in climate sensitivity displayed in Fig. 1 on the WAIS wedge can vary significantly with the assumptions on other important model parameters. So it would be interesting to see how the uncertain model outcome would respond to each of the other parameters. Fig. 5 depicts the sensitivity of the median “WAIS wedge” value in 2035 to some of the key model parameters: (a) decline rate of population growth, (b) scaling factor of labor productivity growth, (c) scaling factor of AEEI growth, (d) technological learning rate, and (e) pure rate of time preference. Similarly, Fig. 6 shows the sensitivity of probability of BAU global warming in 2105 exceeding the temperature threshold  $2.5^{\circ}\text{C}$  with respect to the same range of each of the other important parameters values.



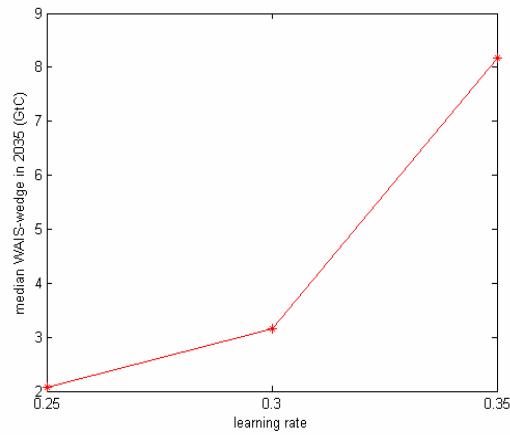
(a)



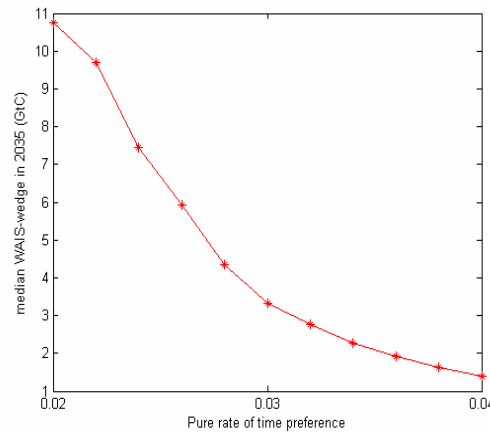
(b)



(c)

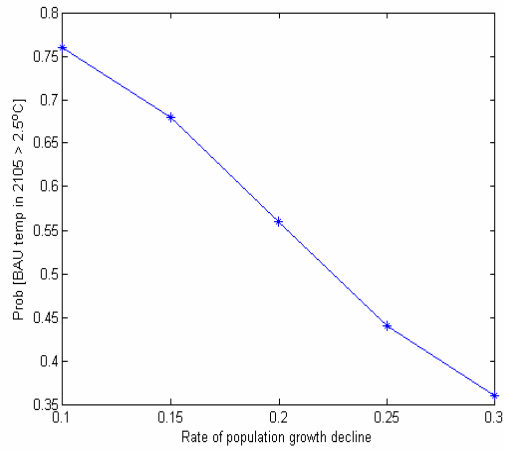


(d)

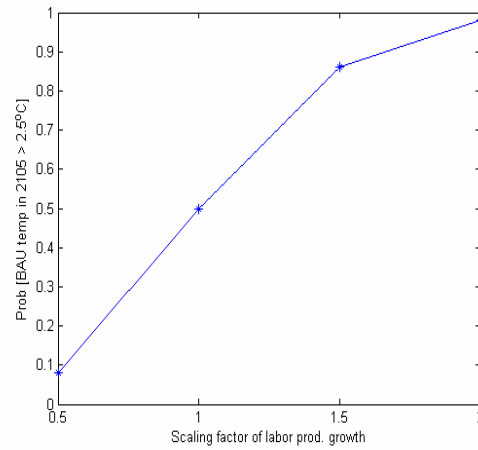


(e)

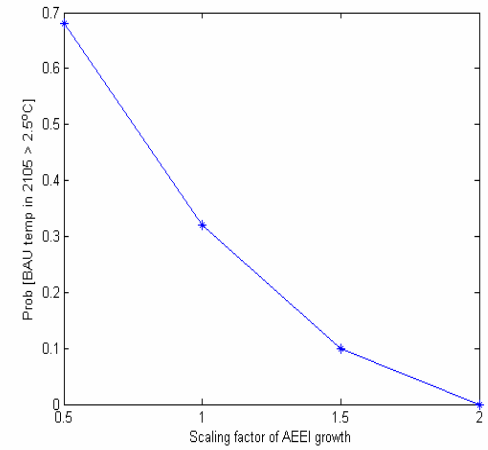
**Fig. 5. Sensitivity of the median WAIS “wedge” value in 2035 to various uncertain model parameters:**  
 (a) rate of population growth decline,  
 (b) scaling factor of labor prod. growth,  
 (c) scaling factor of AEEI growth,  
 (d) technological learning rate, and  
 (e) pure rate of time preference.



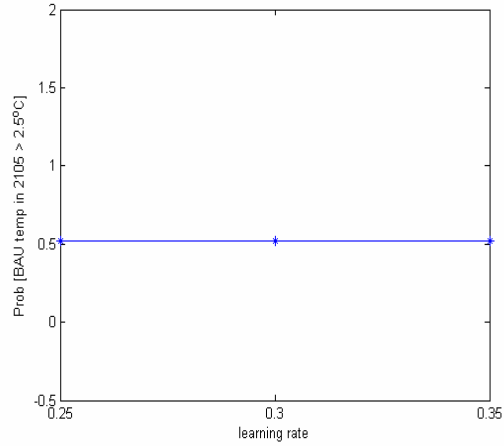
(a)



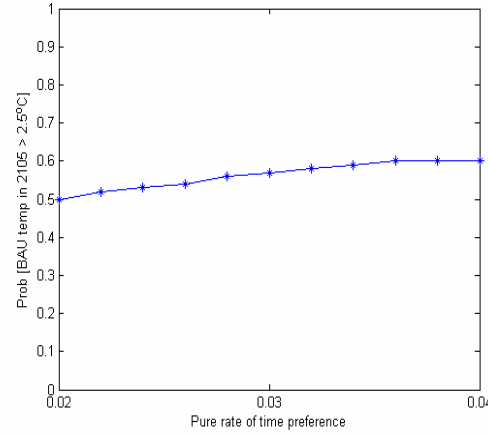
(b)



(c)



(d)



(e)

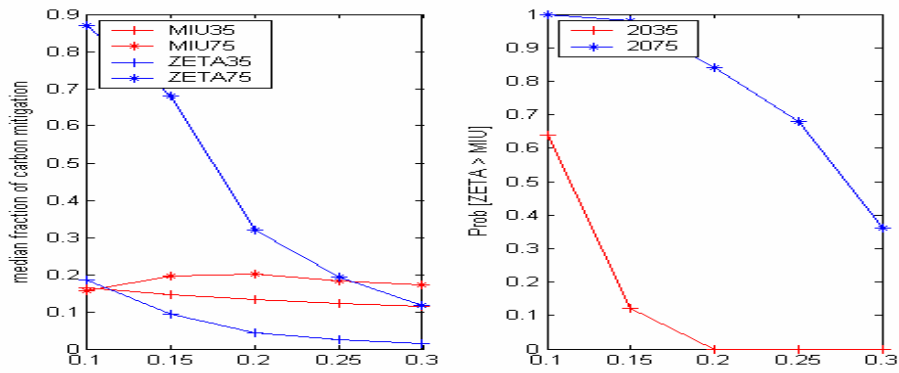
**Fig. 6. Sensitivity of Prob [BAU global warming in 2105 > 2.5°C] to various uncertain model parameters:**  
 (a) rate of population growth decline,  
 (b) scaling factor of labor prod. growth,  
 (c) scaling factor of AEEI growth,  
 (d) technological learning rate, and  
 (e) pure rate of time preference.

More specifically, to shed some light on the relative importance of future technological options under the uncertainties, we compare the estimates and possibilities distributions for each of the two technological options under various situations. First, the left panel of Fig.7a displays how the optimal technological portfolio values (in median) for MIU and ZETA respond to the uncertain rate of future population growth decline. Here, MIU35 and ZETA35 refer to “conventional technology” and “carbon-free technology” in 2035, respectively, and MIU75 and ZETA75 denote “conventional technology” and “carbon-free technology” in 2075, respectively. The right panel of Fig.7a shows that the probability of ZETA’s exceeding MIU (in the role of carbon mitigation) decreases with the rate of population growth decline, which implies a strong coupling between demography and technology substitution required to curb carbon emissions under the WAIS policy. The smaller population growth we have, the smaller chance of a new technology to dominate in the carbon constrained world.

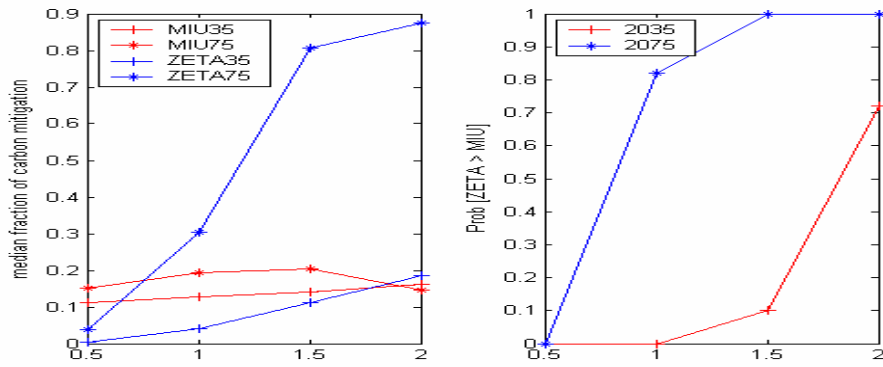
The results for the same sensitivities to the major uncertain economic parameters about a range of the trends of future labor productivity and AEEI growth are shown in Fig.7b and Fig.7c. The left panels in the figures depict how the optimal median technological portfolio values for MIU and ZETA respond to the degree of uncertain future labor productivity growth and to the degree of uncertain future AEEI growth, respectively. The results imply that the probability of ZETA’s exceeding MIU increases with the labor productivity growth (right panel of Fig.7b), whereas it decreases with AEEI growth (right panel of Fig.7c). While higher labor productivity growth requires more accelerated transition toward new carbon-free technologies required to preserve the temperature limit, the purely exogenous AEEI factor, as a substitute, weakens the potential of induced technological innovation.

The left panel of Fig.7d displays how the optimal median technological portfolio values for MIU and ZETA respond to the uncertain technological learning rate for ZETA. Note that, as expected, we can see that the median fraction of ZETA mitigation effort rises exponentially with the learning rate. The right panel of Fig.7d indicates that the probability of ZETA’s exceeding MIU (in

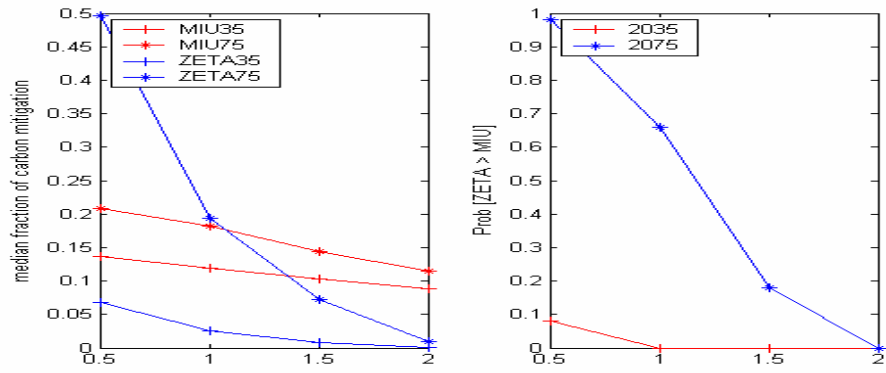
the role of carbon mitigation) increases with the technological learning rate for ZETA. The same analysis with respect to the uncertain pure rate of time preference is shown in Fig.7e. The left panel implies that the optimal median technological portfolio values for ZETA are highly responsive to the uncertain pure rate of time preference. Moreover, the right panel shows that the chance of ZETA's dominance in the role of carbon mitigation decreases greatly with the society's pure rate of time preference.



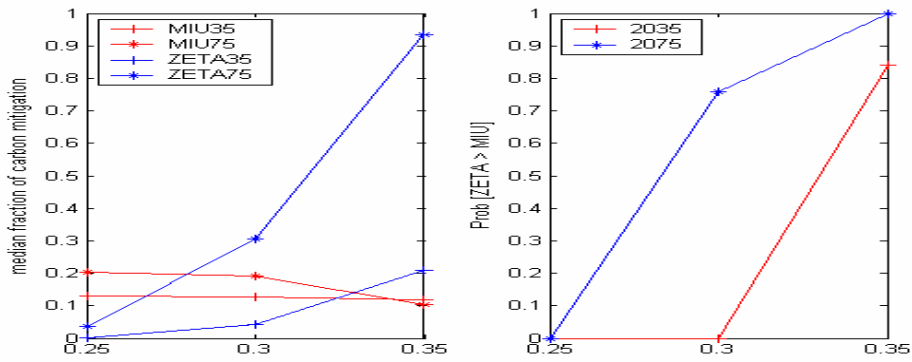
(a) Sensitivity to the rate of population growth decline



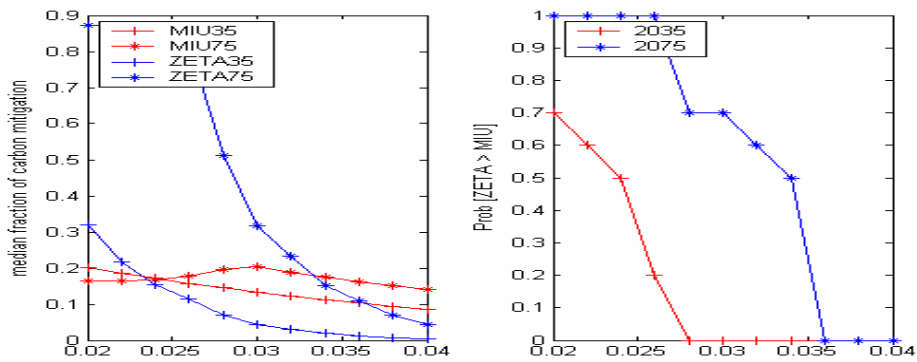
(b) Sensitivity to labor productivity growth



(c) Sensitivity to AEEI growth



(d) Sensitivity to technological learning rate



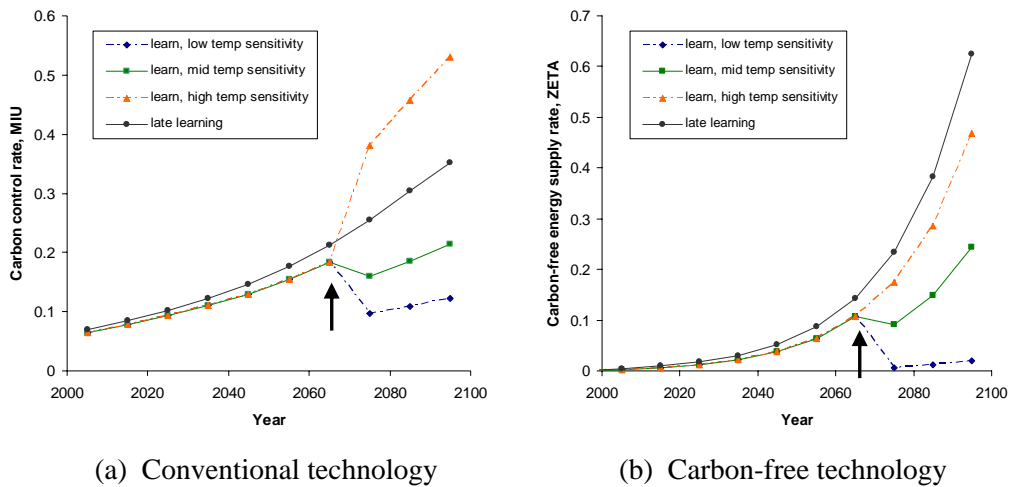
(e) Sensitivity to pure rate of time preference

Fig. 7. Effects of uncertainty on technological portfolios



Next, we examine the effects of learning about uncertainty and estimate the value of early information. The decisions are made in light of currently available information and they reflect the range of possible outcomes given the degree of uncertainty and risk aversion. As in the relevant literature (*e.g.*, Manne and Richels, 1991; Nordhaus, 1994; Nordhaus and Popp, 1997), we consider cases in which policymakers must undertake decisions before knowing which particular state of the world they are in. Before learning occurs, decisions must be made in ignorance of the true states of the world. However, they can become state dependent after the learning point since policymakers can then act with perfect information about the true states and the actual value of uncertain parameters.

Fig. 8(a) and (b) display the impact of learning about uncertain climate sensitivity in 2065 (compared to late learning in 2155) on optimal mitigation policies, MIU and ZETA, respectively. We assume equal probability of three true states of the world about climate sensitivity. Note that before learning in 2065, policies cannot be state contingent, and thus are constrained to be equal in all three states of the world. As seen in Fig 8, the presence of late learning requires more stringent mitigation policies, which implies that uncertainty can raise the costs of optimal policies significantly.



**Fig. 8. Learning, hedging and the value of early information**

The consequences of scientific uncertainty on policy stringency have been explored previously in models of no endogenous technological progress, including Manne and Richels (1992), Nordhaus (1994), Nordhaus and Popp (1997), and Pizer (1999). However, they found that learning has little or relatively modest effects on decisions. It is not surprising to see that that our experiments indicate relatively large consequences since the economy now responds more elastically to the stringency of environmental polices. More generally, in terms of overall macroeconomic policy, we compare the impact on consumption of uncertainties from the DICE-LBD and original DICE models. Greater uncertainty here is measured in the sense of later resolution of uncertainty. Table 1 shows the percentage difference between the expected value of a near-term consumption (2010-2020) for each of different years of resolution as compared to the value of consumption using late information. Contrary to previous result (Nordhaus, 1994, p.188, Table 8.4), our study shows that greater uncertainty lead to lower consumption rather than higher consumption. The result is broadly consistent with the precautionary savings principle implied in (6).

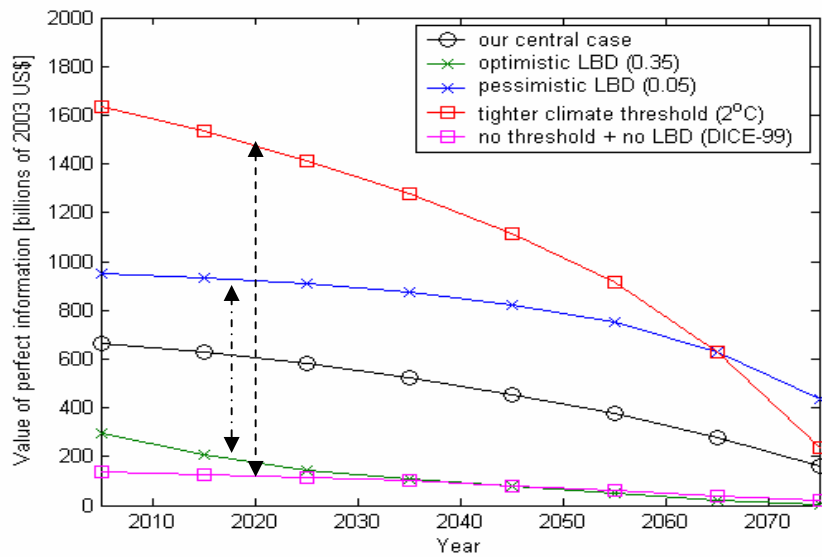
**Table 1. Comparison of optimal polices under learning with the DICE model (in impact of resolution on near-term consumption, 2015)**

Scenario	Impact of resolution on consumption, 2015 (%)	
	DICE-LBD	DICE
Perfect information	0.016	-0.011
2015 information	0.015	-0.010
2025 information	0.005	0.000
2035 information	0.000	0.000

Note: Shown is the percentage difference between the expected value of a near-term consumption (2010-2020) for each of different years of resolution as compared to the value of consumption using late information.

Fig. 9 quantifies the expected value of perfect information about the climate sensitivity depending upon the year in which uncertainties are revealed. We compare the present value of increased consumptions for each case relative to the

case where perfect information is attained in 2085 across alternative environments about the technological learning rate and the climate threshold. The results indicate that the value of early information is highly sensitive to how fast the uncertainty will narrow over time. Not surprisingly, the value of information can be extremely large with tight climate thresholds. However, it can significantly decrease with policy-induced technological progress that may occur in the presence of stringent climate goals.



**Fig. 9. Range of values of scientific knowledge about the climate sensitivity for different years of resolution**

The result implies that ignoring induced innovation can overstate significantly the welfare cost of reducing carbon emissions in a carbon constrained world, since endogenous technological progress in new carbon-free technologies lowers the cost of achieving a given environmental goal. Notice that unlike some previous studies with costly R&D efforts and their crowding-out effects (*e.g.*, Nordhaus, 2002; Popp, 2004), innovation in our model has a wider effect under a restrictive policy because a tighter climate threshold can trigger induced learning-by-doing and knowledge spillovers effects in the economy.

## 4 Conclusions

Using a simple climate-economy model of technology choice, this paper presents probabilistic integrated assessments and uncertainty analyses of optimal timing, costs and technology choice of carbon emission reductions in a carbon-constrained world. The key feature of the model developed here includes its extension of the existing integrated assessment (IA) modeling capabilities by incorporating environmental risk, endogenous technological choice and the diffusion of innovative technologies in the presence of uncertainty and learning, so that policymakers can gain clear insights into future energy technology strategies. This new approach enables us to explore the potential competitions (or trade-offs) between carbon mitigation technologies, while it captures a closed-loop interaction between environmental risk and induced innovation.

Uncertainty analysis in our study reveals that the endogenous portfolios between the traditional and new energy technologies are highly dependent on assumptions about the plausible range of uncertainties surrounding climate, technological and socioeconomic parameters and the stringency of the society's environmental goals. Moreover, the results imply that analyses ignoring this considerable uncertainty could lead to inefficient and biased technology-policy recommendations for the future. In particular, while ignoring the linkage between induced innovation and environmental risk can overstate significantly the welfare gains of wait-and-see strategies in combating global warming, innovation has a wider effect under a more stringent environmental goal. This study also find that the value of resolving key scientific uncertainties would be non-trivial under potentially catastrophic climate risks, but at the same time it can significantly decrease with induced innovation and knowledge spillovers that might otherwise be absent. In addition, the results indicate that scientific uncertainties could justify immediate mitigation actions and accelerated investments in new energy technologies, reflecting risk-reducing considerations.

We keep the underlying technology-choice model of the integrated assessment processes as simple as possible for its transparency and tractability. However, the probabilistic risk-management framework used for this study can be applied to all levels of model complexity and high dimensionality of the

problem. So the present work can be extended further to include: (i) endogenous technological portfolio component with emphasis on international technology spillovers in a disaggregated global economy, (ii) more diversification of technological portfolios (*e.g.*, “demand-side” efficiency technologies as well as “supply-side” technologies) to combat climate change, and (iii) state-of-the-art combination of the two traditions of endogenizing technological change via both R&D (learning-by-searching) and LBD (learning-by-doing) in the stochastic integrated assessment framework. Based on the availability for reliable data and parameter calibration in this direction, it would be useful to analyze the competition between carbon mitigation technologies in the presence of multi-regional, technological spillovers and catch-ups, and trade of carbon permits, which is left for future research.

## Appendix A: Equations of the model

This appendix presents the complete equations of the simple state-contingent model of endogenous technological change and choice for climate-change policy.

**Sets:**  $t$  = time periods (0 to 40)

i (subset) = two broad, stylized categories of mitigation options  
( $\mu$  = *conventional*,  $\zeta$  = *carbon-free*)

### 1). Equations of the model:

Utility

$$W = E_0 \sum_t \left[ \frac{1}{1 + \rho(t)} \right]^t L(t) \ln [C(t) / L(t)] \quad (\text{A1})$$

Economic and technological constraints

$$Y(t) = \Omega(D(t)) K_c(t)^\gamma (A(t)L(t))^{1-\gamma} = C(t) + I(t) + I_\mu(t) + I_\zeta(t) \quad (\text{A2})$$

$$\text{where } \Omega(D(t)) = 1 / [1 + D(t)] = 1 / [1 + d_1 T(t) + d_2 T(t)^{d_3}]$$

$$K_i(t) = I_i(t) + (1 - \delta_i)K_i(t-1), \quad i = c, \mu, \zeta \quad (\text{A3})$$

$$E(t) = \sigma(t)[1 - \mu(t) - \zeta(t)]Y(t) \quad (\text{A4})$$

$$I_i(t) = c_{0i} [K_i(t)]^{-\alpha_i} [i(t)]^{\alpha_i} Y(t), \quad i = \mu, \zeta \quad (\text{A5})$$

$$\zeta(t) = (1 + \varepsilon)\zeta(t-1) \quad (\text{A6})$$

#### Stochastic processes on generic productivities

$$A(t) = \varphi A(t-1) + (1 - \varphi)A^*(t-1) + \varepsilon_A(t) \quad (\text{A7})$$

$$\sigma(t) = \pi \sigma(t-1) + (1 - \pi)\sigma^*(t-1) + \varepsilon_\sigma(t) \quad (\text{A8})$$

and other uncertain parameters.

#### Environmental constraints

$$M_{AT}(t) = E(t-1) + a_{11}M_{AT}(t-1) + a_{21}M_{UP}(t-1) \quad (\text{A9})$$

$$M_{UP}(t) = a_{22}M_{UP}(t-1) + a_{12}M_{AT}(t-1) + a_{32}M_{LO}(t-1) \quad (\text{A10})$$

$$M_{LO}(t) = a_{33}M_{LO}(t-1) + a_{23}M_{UP}(t-1) \quad (\text{A11})$$

$$F(t) = 4.1 \ln \left\{ \left[ M_{AT}(t) / M_{AT}^{PI}(t) \right] / \ln 2 \right\} + O(t) \quad (\text{A12})$$

where  $O(t) = -0.1965 + 0.13465t$ ,  $t < 12$ ;  $= 1.15$ ,  $t \geq 12$

$$T(t) = T(t-1) + \kappa_1 \left\{ F(t) - (4.1/CS)T(t-1) - \kappa_2 [T(t-1) - T_{LO}(t-1)] \right\} \quad (\text{A13})$$

$$T_{LO}(t) = T_{LO}(t-1) + \kappa_3 \left\{ T(t-1) - T_{LO}(t-1) \right\} \quad (\text{A14})$$

**2). Parameters:** adopted mostly from Nordhaus and Boyer (2002), except for the assumptions on the technological and climate parameters and their plausible ranges made from Sims et al. (2003), McDonald and Schrattenholzer (2001), and Dessai and Hulme (2003). Assumptions on the probability distributions for all other uncertain variables are adopted from Nordhaus(1994), Nordhaus and Popp(1996), and Pizer (1997), etc.

$\alpha_\mu$	Technological learning index (or progress ratio) for MIU	/.0816/
$\alpha_\zeta$	Technological learning index (or progress ratio) for ZETA	/.2485/
$c_{0\mu}$	Scaling parameter for cost curve of MIU technology	/.045/
$c_{1\mu}$	Exponent parameter for cost curve of MIU technology	/.2.15/
$c_{0\zeta}$	Scaling parameter for cost curve of ZETA technology	/.180/

$c_{1\zeta}$	Exponent parameter for cost curve of ZETA technology	/1/
$\rho$	Initial rate of social time preference per year	/03/
A	Level of total factor productivity	/01685/
$\sigma$	CO2-equivalent emissions-GNP ratio	/272/
$\delta_i$	Depreciation rate for technology $i$	/10/
$\gamma$	Capital elasticity in production function	/30/
$\varepsilon$	Technological inertia	/05/
$M_{AT}(0)$	Concentration in atmosphere 1990 (b.t.c.)	/735/
$M_{UP}(0)$	Concentration in upper strata 1990 (b.t.c.)	/781/
$M_{LO}(0)$	Concentration in lower strata 1990 (b.t.c.)	/19230/
a11	Carbon cycle transition matrix	/66616/
a12	Carbon cycle transition matrix	/33384/
a21	Carbon cycle transition matrix	/27607/
a22	Carbon cycle transition matrix	/60897/
a23	Carbon cycle transition matrix	/11496/
a32	Carbon cycle transition matrix	/00422/
a33	Carbon cycle transition matrix	/99578/
$T_{LO}(0)$	1985 lower strat. temp change (C) from 1900	/06/
$T(0)$	1985 atmospheric temp change (C) from 1900	/43/
$K(0)$	1990 value capital trillion 1990 US dollars	/47/
$\kappa_1$	Speed of adjustment parameter for atm. temperature	/226/
CS	Equilibrium atm temp increase for CO2 doubling (deg C)	/2.8/
$\kappa_2$	Coefficient of heat loss from atm to deep oceans	/44/
$\kappa_3$	Coefficient of heat gain by deep oceans	/02/
d1	Damage coeff 1	/00071/
d2	Damage coeff 2	/00242/
d3	Exponent of damage function	/2/

## Appendix B: Learning rates and unit cost functions

In the stylized learning mechanism, a commonly used learning-by-doing (LBD) component for each technology  $i$  is

$$C_i = c_{0i} [K_i(t)]^{-\alpha_i}, \quad i = \mu, \zeta \quad (\text{A5}')$$

where  $c_{0i}$  is a normalization parameter and  $\alpha_i$  is the learning elasticity index. Every doubling of installed capacity ( $K_i(t)$ ) reduces the technology costs ( $C_i$ ) by a factor of  $2^{-\alpha_i}$ , which is also called “progress ratio” (PR). The complementary “learning rate” (LR) is  $1 - PR = 1 - 2^{-\alpha_i}$ , which gives the percentage reduction in the capital investment costs of newly installed capacity for every doubling of cumulative capacity (c.f., Anderson, 1999). And, the unit cost function for each technology  $i$  (in the form of a generalized power function as a dual to the Cobb-Douglas production) can be estimated as

$$\ln \left[ \frac{COST_i(t)}{GDP(t)} \right] = \ln c_{0i} + c_{i_i} \ln [i(t)] - \alpha_i \ln K_i(t), \quad i = \mu, \zeta. \quad (A5'')$$

## References

- Alley, R.B. et al., 2003, Abrupt climate change, *Science* 299, 2005.
- Barro, R. J., 1990, Government spending in a simple model of endogenous growth, *Journal of Political Economy* 98, 103-125.
- Barro, R. J. and Sala-i-Martin, X, 2004, *Economic growth* (2nd edition), MIT Press, Cambridge, MA.
- Anderson, D., 1999, Technical progress and pollution abatement: an economic review of selected technologies and practices, Imperial College Working Paper, London.
- Arrow, K.J., 1962, The economic implications of learning by doing, *Review of Economic Studies* 29, 155-173.
- Campbell, J.Y., 1996, Understanding risk and return, *Journal of Political Economy* 104, 298-345.
- Corsetti, G., 1997, A portfolio approach to endogenous growth: equilibrium and optimal policy, *Journal of Economic Dynamics and Control* 21, 1627-1644.
- Cubasch, U. et al., 2001, in *Climate change 2001: the scientific basis, summary for policy makers*, J.T. Houghton et al., (eds.), Cambridge University Press, UK.
- Clark, L. and Weyant, J., 2002, Modeling induced technical change – an overview,” Working Paper, Stanford University.
- Dessai, S., Lu, X. and Hulme, M., 2003, Development of a simple probabilistic climate scenario generator for impact and adaptation assessment, In: *Proceedings of the Royal Meteorological Society 2003 conference*, UK.
- Gollier, C, Jullien B, and Treich, N, 2000, Scientific progress and irreversibility: an economic interpretation of the precautionary principle, *Journal of Public Economics* 75, 229-253.
- Iman, R.L. and Conover W.J., 1982, A distribution-free approach to inducing rank correlation among input variables, *Communications Statist.* B11(3), 311-334.
- IPCC, 2001, *Third Assessment Report: the scientific basis*, R.T. Watson and the Core Writing Team (eds.), Cambridge University Press, UK.



- IPCC, 2000, Special Report Emission Scenarios, N. Nakicenovic and R. Swart (eds.), Cambridge University Press, UK.
- Kelly, D.L and Kolstad, C.D., 1999, Bayesian learning, growth, and pollution, *Journal of Economic Dynamics and Control* 23, 491-518.
- Kim, S.-R., Keller, K., and Bradford, D.F., 2004, Optimal technological portfolios for climate change policy under uncertainty, The 10<sup>th</sup> International Conference for the Society for Computational Economics, July 8-10, 2004, University of Amsterdam, The Netherlands.
- Kolstad, C.D., 1996, Learning and stock effect in environmental regulation: the case of greenhouse gas emissions, *Journal of Environmental Economics and Management* 31, 1-18.
- Malliari, A.G. and Brock, W.A., 1982, *Stochastic methods in economics and finance*, North-Holland, Amsterdam.
- Manne, A. and Richels, R., 1992, *Buying greenhouse insurance*, MIT Press, Cambridge.
- Mastrandrea, M. and Schneider, S.H., 2004, Probabilistic integrated assessment of “dangerous” climate change, *Science*, 304.
- McDonald A. and Schratzenholzer, L., 2001, Learning rates for energy technologies, *Energy Policy* 29, 255-261.
- Morgan M.G. and Henrion, M., 1990, *Uncertainty: a guide to dealing with uncertainty in quantitative risk and policy Analysis*, Cambridge University Press, Cambridge, UK.
- Nakicenovic, N., Gruber, A., and McDonald, A., eds., 1998, *Global energy perspectives*, Cambridge University Press, Cambridge, UK.
- Nordhaus, W.D., 1994, *Managing the global commons*, MIT Press, Cambridge, MA.
- Nordhaus, W.D. and Boyer, J., 2000, *Warming the world: economic models of global warming*, MIT Press, Cambridge, MA.
- Nordhaus, W.D. and Popp, D., 1997, What is the value of scientific knowledge?: an application to global warming using the PRICE model,” *The Energy Journal* 18, 1-45.
- Nordhaus, W.D, Modeling induced innovation in climate-change policy, in: A, Grubler, N. Nakicenovic, W.D. Nordhaus (Eds.), *Technological Change and the Environment, Resources for the Future*, Washington, D.C., 2002, 182-209.
- Obstfeld, M., 1994, Risk-taking, global diversification, and growth, *American Economic Review* 84, 1310-1329.
- Oppenheimer, M. 1998, Global warming and the stability of Western Antarctic Ice Sheet, *Nature* 393, 322.
- Pacala, S. and Socolow, R., 2004, Stabilization wedges: solving the climate problem for the next 50 years with current technologies, *Science* 305, 968-972.
- Petit, J.R. et al., 1997, Four climate cycles in vostok ice core, *Nature*, 387, 359.
- Pizer, W.A., 1999, The optimal choice of climate change policy in the presence of uncertainty, *Resource and Energy Economics*, 21, 255-287.
- Popp, D., 2004, ENTICE, endogenous technological change in the DICE model of global warming, *Journal of Environmental Economics and Management* 48, 742-968.

- Sims, R.E. et al., 2003, Carbon emission and mitigation cost comparisons between fossil fuel, nuclear and renewable energy sources for electricity generation, *Energy Policy*, 31, 1315-1326.
- Romer, D., 1996, *Advanced macroeconomics*, The McGraw-Hill Companies, Inc.
- Romer, P., 1986, Increasing returns and long run growth, *Journal of Political Economy* 94, 1002-1037.
- Stott, P.A. and Kettleborough, J.A., 2002, Origins and estimates of uncertainty in predictions of 21<sup>st</sup> century temperature rise, *Nature* 416, 723-726.
- Turnovsky, S.J. 1995, *Methods of macroeconomic dynamics*, MIT Press, Cambridge, MA.
- Ulph, A. and Ulph, D., 1997, Global warming, irreversibility, and learning, *The Economic Journal* 107, 636-650.
- Webster, M. et al., 2003, Uncertainty analysis of climate change and policy response, *Climatic Change*, 61, 295-320.
- Weyant, J.P., 1997, Technological change and climate policy modeling.” In: *Proceedings of the paper presented at the IIASA Workshop on Induced Technological Change and the Environment*, Laxenberg, 26-27 June.