Forecasting with the New-Keynesian Model: An Experiment with Canadian Data *

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Abstract

This paper documents the out-of-sample forecasting accuracy of the New Keynesian Model for Canadian data. We repeatedly estimate the model over samples of increasing lengths, forecasting out-of-sample one to four quarters ahead at each step. We then compare these forecasts with those arising from an unrestricted VAR using recent econometric tests. We show that the accuracy of the New Keynesian model's forecasts compares favourably to that of the benchmark. The principle of parsimony is invoked to explain these results.

1 Introduction

The New-Keynesian model is a workhorse of modern macroeconomic analysis. It is used widely to study the impact of various shocks on economic activity and thus inform the decisions of monetary policy makers worldwide. The model is attractive because it provides a coherent determination of the time paths of aggregate variables within a framework that features explicit optimizing behaviour on the part of households and firms, under the constraint that some nominal prices are 'sticky', i.e. that changes to these prices are costly.[1]

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[1] A detailed exposition of the model can be found in King (2000).

Recently, researchers have started to estimate the model's parameters using aggregate macroeconomic data and standard econometric techniques.[2] However, the model has seldom been used to generate out-of-sample forecasts; as a result, evidence on the quality of these forecasts remains scarce.

To contribute to this evidence, the present paper documents the out-of-sample forecasting properties of a standard version of the New Keynesian model, using Canadian aggregate data. Throughout we seek to replicate the restrictions and data availability of actual forecasting exercises. Specifically, we estimate the model on samples of increasing lengths, compute forecasts one-through-four quarters out at each step, and formally compare these forecasts with those arising from an unrestricted VAR using recent econometric tests of forecast accuracy.

Our preliminary results shows that the forecasting accuracy of the New Keynesian model compares favourably to that from the VAR benchmark. In particular, the model's forecasting ability for output and real money balances may actually surpass that of an unrestricted VAR, particulary as the forecasting horizon increases. However, the forecasting ability of the model for inflation tends to be poorer than the VAR benchmark. This occurs because a negative trend in inflation, clearly visible through the sample, is not captured by the present model. Further, our results also suggests that a combination of the two sets of forecasts may have superior forecasting power to that of the VAR benchmark alone.

The New Keynesian model, like any Dynamic Stochastic General Equilibrium (DSGE) model, can be interpreted as a VAR whose parameters are restricted by non-linear constraints arising from the model's structure. The experiment we conduct is thus one where the out-of-sample forecasting properties of a restricted model are compared to those from its unrestricted counterpart. The possibility that better accuracy be obtained by the restricted model is discussed in general terms in Clements and Hendry (1998), Chapter 12. It has also been demonstrated in practice for VARs in the Bayesian frameworks of Doan et al. (1984) (whose prior consisted of random walks for all variables) and Ingram and Whiteman (1994) (who used the Real Business Cycle model to generate the priors).

More recently, Del Negro and Schorfheide (2004) use a simple version

[2] Ireland (1997, 2001a, 2003, 2004) and Dib (2003a,b, forthcoming) estimate the parameters by maximum likelihood. Christiano et al. (forthcoming) estimate the parameters by minimizing the distance between the model's impulse responses following monetary policy shocks and the impulse responses computed with identified VARS. Smets and Wouters (2003) and Del Negro and Schorfheide (2004) use Bayesian techniques to compute the posterior distribution of the model's parameters, after specifying prior distributions.

of the New Keynesian model as priors for their Bayesian VAR estimation, and show that the forecasting properties of the resulting model compares favourably to that of completely unrestricted VARs. Further, Ireland (2004) shows that his hybrid RBC-VAR model has better forecasting performance than an unrestricted VAR for American data; in addition, Dolar and Moran (2002) verify that these results also hold for Canadian data. The present paper extends these analyses by employing a complex version of the New Keynesian model with capital accumulation, estimating it using Canadian data, and, using recent econometric tests due to Diebold and Mariano (1995) and Harvey et al. (1997, 1998), formally comparing its forecasting accuracy to that of the unrestricted benchmark.

The rest of the paper is organized as follows. Section 2 presents the model, which is representative of the New-Keynesian literature. Section 3 discusses the steps involved in the estimation of the model, while section 4 presents the estimation results over the longest sample we consider (1981:3 to 2004:1). Section 5 describes the forecasting experiment we conduct and presents our results about the accuracy of the model's forecasts. Section 6 assesses these results and concludes.

2 Model

The structure of the model is inspired by Dib (forthcoming) and Ireland (2003). The economy is populated by households, firms producing a final good, firms producing intermediate goods, and a monetary authority. The final good market is competitive: the firms take input prices as given, produce an homogenous good, and their output prices are perfectly flexible. The final good is divided between consumption and investment. There exists capital adjustment costs that restrict the extent to which the capital stock can be modified; these costs are borne by the households, who own the economy's capital. An array of intermediate goods serve as the inputs into the final good's production. By contrast to the final good sector, intermediate-good-producing firms operate under monopolistic competition; each firm produces a distinct good, for which it chooses the market price. However, nominal changes to the price these firms charge are restricted, following Calvo (1983), so that these prices are 'sticky'. Intermediate good production requires the capital and labour services, for which the firms act as price takers. Finally, the monetary authority manages a short-term nominal interest rate to respond to inflation, output, and money growth deviations.

2.1 Household

Households derive utility from consumption, c_t, real balances, M_t/P_t, and leisure $(1-h_t)$, where h_t represents worked hours. A representative household's expected lifetime utility is described as follows:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, M_t/P_t, h_t), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, and the single-period utility function is specified as:

$$u(\mathbf{.}) = \frac{\gamma z_t}{\gamma - 1} \log \left(c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}} \right) + \eta \log(1-h_t), \tag{2}$$

where γ and η are positive structural parameters, and z_t and b_t are serially correlated shocks. As shown by McCallum and Nelson (1999), the preference shock z_t resembles, in equilibrium, a shock to the IS curve of more traditional Keynesian analysis. On the other hand, b_t is interpreted as a shock to money demand. These two shocks evolve according to the following first-order autoregressive processes:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}, \tag{3}$$

and

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{4}$$

where $\rho_z, \rho_b \in (-1,1)$ and the serially uncorrelated innovations ε_{zt} and ε_{bt} are normally distributed with zero mean and standard deviations σ_z and σ_b, respectively.

The representative household enters period t with k_t units of physical capital, M_{t-1} units of nominal money balances, and B_{t-1} units of bonds. During period t, the household supplies labour and capital to the intermediate-good-producing firms, for which it receives total factor payment $R_{kt}k_t + W_t h_t$, where R_{kt} is the nominal rental rate for capital and W_t is the nominal wage. Further, the household receives a lump-sum transfer from the monetary authority, T_t, as well as dividend payments D_t from intermediate-good-producing firms. The household allocates these funds to consumption purchases c_t and investment in capital goods i_t (both priced at P_t), to money holdings M_t and to financial bonds B_t, which are priced at $1/R_t$ (R_t denotes the gross nominal interest rate between t and $t+1$). The following budget constraint therefore applies:

$$P_t\left(c_t + i_t\right) + M_t + B_t/R_t \leq R_{kt}k_t + W_t h_t + M_{t-1} + B_{t-1} + T_t + D_t. \tag{5}$$

Investment i_t increases the capital stock over time according to

$$k_{t+1} = (1-\delta)k_t + i_t - \Psi\left(k_{t+1}, k_t\right), \tag{6}$$

where $\delta \in (0,1)$ is the constant capital depreciation rate and $\Psi(.,.)$ is a capital-adjustment cost function specified as $\frac{\psi}{2}\left(\frac{k_{t+1}}{k_t}-1\right)^2 k_t$, where $\psi > 0$ is the capital-adjustment cost parameter. With this specification both total and marginal costs of adjusting capital are zero in the steady-state equilibrium.

The representative household chooses c_t, M_t, h_t, k_{t+1} and B_t in order to maximize expected lifetime utility (1) subject to the budget constraint (5) and the investment constraint (6). The first-order conditions for this problem are as follows:

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} = \lambda_{t;} \tag{7}$$

$$\frac{z_t b_t^{\frac{1}{\gamma}} (M_t/P_t)^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t\left(\frac{P_t\lambda_{t+1}}{P_{t+1}}\right); \tag{8}$$

$$\frac{\eta}{1-h_t} = \lambda_t \frac{W_t}{p_t}; \tag{9}$$

$$\psi\left(\frac{k_{t+1}}{k_t}-1\right)+1 = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{R_{kt+1}}{P_{t+1}} + 1 - \delta + \psi\left(\frac{k_{t+2}}{k_{t+1}}-1\right)\frac{k_{t+2}}{k_{t+1}}\right)\right]; \tag{10}$$

$$\frac{1}{R_t} = \beta E_t\left[\frac{P_t\lambda_{t+1}}{P_{t+1}}\right]; \tag{11}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint (5).

As shown by Ireland (1997) and Dib (forthcoming), combining conditions (7), (8) and (11) yields the following optimization-based money-demand equation:

$$\log(M_t/p_t) \simeq \log(c_t) - \gamma\log(r_t) + \log(b_t), \tag{12}$$

where $r_t = R_t - 1$ denotes the net nominal interest rate between t and $t+1$, γ is the interest elasticity of money demand, and b_t is a serially correlated money-demand shock.

2.2 The final goods-producing firm

The final good, Y_t, is produced by assembling a continuum of intermediate goods $y_{jt}, j \in (0,1)$ that are imperfect substitutes with a constant elasticity of substitution θ. The aggregation function is defined as

$$Y_t \leq \left(\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1. \tag{13}$$

The final good-producing firms behave competitively, maximizing profits and taking the market price of the final good P_t as well as the intermediate-good prices $p_{jt}, j \in (0,1)$ as given. The maximization problem of a representative, final good-producing firm is therefore

$$\max_{\{y_{jt}\}_{j=0}^1} \left[P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \right],$$

subject to (13). The resulting input demand function for the intermediate good j is

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} Y_t, \tag{14}$$

and represents the economy-wide demand for good j as a function of its relative price and of the economy's total output of final good Y_t. The no profit condition in the sector implies that the final-good price index P_t satisfies

$$P_t = \left(\int_0^1 p_{jt}{}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{15}$$

2.3 The intermediate goods-producing firm

The intermediate good-producing firm j uses capital and labor services k_{jt} and h_{jt} to produce y_{jt} units of its differentiated good, according to the following constant-returns-to-scale technology:

$$y_{jt} \leq k_{jt}^{\alpha} \left(A_t h_{jt} \right)^{1-\alpha}, \quad \alpha \in (0,1), \tag{16}$$

where A_t is an aggregate technology shock that is common to all intermediate good-producing firms. This shock follows a first-order autoregressive process, as follows:

$$\log A_t = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{17}$$

where $\rho_A \in (-1, 1)$ is an autoregressive coefficient, $A > 0$ is a constant, and ε_{At} is normally distributed with mean zero and standard deviation σ_A.

Each intermediate good-producing firm sells its output under monopolistic competition; the economy-wide demand for the good produced by producer j is given by (14). Further, following Calvo (1983), we assume that each firm is allowed to re-optimize its output price only at random intervals. Specifically, with probability ϕ the firm must charge the price that was in effect in the preceding period, indexed by the steady-state rate of inflation π; with probability $1-\phi$, the firm is free to re-optimize and choose a new price. This implies that on average the firm will not re-optimize for $1/(1-\phi)$ periods.[3]

At time t, if firm j receives the signal to reoptimize, it chooses a price $\widetilde{p}_{jt}$, as well as contingency plans for h_{jt+k}, k_{jt+k}, for all $k \geq 0$ that maximize its discounted, expected (real) total profit flows for the period of time where it will not be able to reoptimize its output price. The profit maximization problem is therefore the following:

$$\max_{\{k_{jt},h_{jt},\widetilde{p}_{jt}\}} E_0 \left[\sum_{k=0}^{\infty} (\beta\phi)^k \lambda_{t+k} D_{jt+k}/P_{t+k} \right],$$

where D_{jt+k}/P_{t+k}, the real profit flow at time $t+k$, is

$$D_{jt+k} = \widetilde{p}_{jt}\pi^k y_{jt+k} - R_{kt+k}k_{jt+k} - W_{t+k}h_{jt+k}. \tag{18}$$

The inclusion of ϕ^k in the expression reflects the fact that the probability $\widetilde{p}_{jt}$ remains in effect (including of indexation) at time $t+k$ is ϕ^k.

Profit maximization is undertaken subject to the demand for good j (14) and to the production function (16) (to which the Lagrangean multiplier $\xi_t > 0$ is associated). The first-order conditions for this optimization problem are:

$$(k_{jt}) \qquad \frac{R_{kt}}{P_t} = \alpha q_t \frac{y_{jt}}{k_{jt}}; \tag{19}$$

$$(h_{jt}) \qquad \frac{W_t}{P_t} = (1-\alpha) q_t \frac{y_{jt}}{k_{jt}}; \tag{20}$$

$$(\widetilde{p}_{jt}) \qquad \widetilde{p}_{jt} = \frac{\theta}{\theta-1} \frac{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{-\theta})^k \lambda_{t+k} Y_{t+k} q_{t+k} P_{t+k}^{\theta}}{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{1-\theta})^k \lambda_{t+k} Y_{t+k} P_{t+k}^{\theta-1}}; \tag{21}$$

where $q_t = \xi_t/\lambda_t$ is the real marginal cost of the firm.

Because of the symmetry in the demand they face for their good (14), all firms that are allowed to reoptimize choose the same price $\widetilde{p}_{jt}$, which we

[3] This follows Yun (1996). Alternatively, Christiano et al. (forthcoming) assume that when the re-optimization signal is not received, the price is increased by the *preceding period*'s rate of inflation. Finally, Smets and Wouters (2003) implement a flexible specification that nests the two cases, and estimates an additional ‘indexation’ parameter.

therefore denote $\widetilde{p}_t$. Considering the definition of the price index in (15) and the fact that at the economy's level, a fraction $1-\phi$ of intermediate-good producing firms do reoptimize, the aggregate price index P_t evolves according to

$$P_t^{1-\theta} = \phi P_{t-1}^{1-\theta} + (1-\phi)(\widetilde{p}_t \pi)^{1-\theta}. \tag{22}$$

Equations (19) and (20) state that firms choose their production inputs so that their costs equal their marginal product weighted by the real marginal cost. Equation (21) relates the optimal price to the expected future price of the final good and to the expected future real marginal costs. Taking a first-order approximation of this condition, of (22), and combining them leads us to derive the model's New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi} \widehat{q}_t, \tag{23}$$

which relates the present period's deviation of the inflation rate from its average ($\widehat{\pi}_t$) to the expectation of future rates as well as to today's marginal costs ($\widehat{q}_t$), an indicator of the strength of economic activity.[4]

2.4 The monetary authority

Following Ireland (2003) and Dib (forthcoming), we assume that monetary authorities conduct monetary policy by managing the short-term nominal interest rate, R_t, in response to deviations of inflation, $\pi_t = P_t/P_{t-1}$, output, Y_t, and money growth, $\mu_t = \overline{M}_t/\overline{M}_{t-1}$ from their target (or steady-state) levels. The interest rate reaction function is given by:

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(Y_t/Y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t), \tag{24}$$

where R, π, Y, and μ are the target (steady-state) values of R_t, π_t, Y_t, and μ_t, respectively. Further, v_t is a monetary policy shock that evolves according to

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}, \tag{25}$$

where $\rho_v \in [0,1)$ is an autoregressive coefficient and ε_{vt} is a zero-mean, serially uncorrelated shock with standard deviation σ_v.

The policy coefficients ϱ_π, ϱ_y, and ϱ_μ are chosen by the monetary authorities. When $\varrho_\pi > 0$, $\varrho_y > 0$, and $\varrho_\mu = 0$, monetary policy follows the

[4] Starting with Galí and Gertler (1999), a sizeable literature has employed single-equation econometric methods to estimate various specifications of the New Keynesian Philips Curve.

Taylor (1993) rule, in which the nominal interest rate increases in response to deviations of inflation and output from their steady-state values. In this case, a unique equilibrium exists only if ϱ_π is greater than unity. In contrast, under the rule (24), a unique equilibrium exists as long as the sum of ϱ_π and ϱ_μ exceeds one.

Two interpretations of the presence of money-growth rates in the rule (24) are possible. First, monetary policy can be described as following a modified Taylor (1993) rule that adjusts the short-term nominal interest rate in response to changes in money-growth as well as to deviations of inflation and output. The money-growth rate can be interpreted as an indicator of expected inflation or as a proxy for some omitted variables to which monetary policy should respond, such as the exchange rate or financial variables. Alternatively, Ireland (2003) points out that the central bank's monetary policy can be characterized as a combination policy that influences a linear combination of the interest rate and the money-growth rate to control inflation.

2.5 Symmetric equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions. Let $r_{kt} = R_{kt}/P_t$, $w_t = W_t/P_t$, and $m_t = M_t/P_t$ denote the real rental rate on capital services, the real wage, and real balances, respectively. The equilibrium of this economy consists in a sequence of allocations $\{Y_t, c_t, m_t, h_t, k_t\}_{t=0}^{\infty}$ a sequence of prices and co-state variables $\{w_t, r_{kt}, R_t, \pi_t, \lambda_t, q_t\}_{t=0}^{\infty}$ and the stochastic processes for technology, money demand, monetary policy, and preference shocks. These allocations, prices, and shocks are such that (i) households, final good-producing firms, and intermediate good-producing firms optimize, (ii) the monetary policy rule (24) is satisfied, and (iii) the following market-clearing conditions are satisfied:

$$k_t = \int_0^1 k_{jt}\, dj; \tag{26}$$

$$h_t = \int_0^1 h_{jt}\, dj; \tag{27}$$

$$M_t = \overline{M_t}; \tag{28}$$

$$B_t = 0; \tag{29}$$

$$Y_t = c_t + i_t. \tag{30}$$

Appendix A contains a detailed list of the model's equilibrium conditions.

Next, the steady-state of the system is computed, a first-order linear approximation of the equilibrium system around these steady-state values is formed, and using Blanchard and Kahn (1980)'s procedure for transforming this forward-looking model into a difference-equation system, the following state-space solution is obtained:

$$\widehat{s}_{t+1} = \mathbf{\Phi}_1 \widehat{s}_t + \mathbf{\Phi}_2 \, \varepsilon_{t+1}, \tag{31}$$

$$\hat{d}_t = \mathbf{\Phi}_3 \widehat{s}_t, \tag{32}$$

where $\widehat{s}_t$ is a vector of state variables that includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector ε_{t+1} contains the random innovations.[5] The elements of matrices $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$, and $\mathbf{\Phi}_3$ depend on the model's structural parameters. Therefore, the state-space solution in (31)–(32) can be used to estimate the underlying parameters of the model via a maximum likelihood procedure.

3 Calibration, Data, and Estimation

We set some of the model's structural parameters prior to estimation, because they are not identified or the data used contains only weak information about them. Specifically, the parameter η, denoting the weight on leisure in the utility function, is set to 1.35, which implies that the representative household spends roughly one third of its non-sleeping time in market activities (work). The share of capital in production, α, and the depreciation rate, δ, are assigned values of 0.33 and 0.025, respectively; these values are commonly used in the literature. The parameter θ, the degree of monopoly power in intermediate-goods markets, is set equal to 6, implying a steady-state price to marginal cost markup of 20%, which matches the values usually used in similar studies. Finally, both Ireland (2001a) and Dib (2003b) remark that it is difficult to estimate the coefficient ψ that governs the severity of the capital adjustment costs without using data on the capital stocks. We thus follow them and fix this parameter to a value of 15, the value used by Dib (2003b).

The remaining parameters are estimated using maximum likelihood. This requires us to select a subset of the control variables $\hat{d}_t$ in (32) for which

[5] For any stationary variable x_t, $\hat{x}_t = \log(x_t/x)$ is the deviation of x_t from its steady-state value x. In our specification, $\widehat{s}_t = \left(\hat{k}_t, \hat{m}_{t-1}, \hat{A}_t, \hat{b}_t, \widehat{v}_t, \hat{z}_t\right)'$, $\hat{d}_t = \left(\hat{\lambda}_t, \hat{q}_t, \hat{m}_t, \hat{y}_t, \hat{R}_t, \hat{r}_{kt}, \hat{c}_t, \hat{\pi}_t, \hat{w}_t, \hat{h}_t, \hat{\mu}_t\right)'$, and $\varepsilon_{t+1} = (\varepsilon_{At+1}, \varepsilon_{bt+1}, \varepsilon_{vt+1}, \varepsilon_{zt+1})'$. See Appendix A for a complete list of the steady-state conditions as well as the linearized equations.

data is available, as well as selecting the appropriate rows of $\mathbf{\Phi}_3$. Next, the likelihood of the sample $\{\hat{d}_t\}_{t=1}^T$ is computed recursively using the Kalman filter (Hamilton, 1994, Chap. 13). Finally, the parameter values that maximize the likelihood are found using standard numerical procedures.[6]

Since the model is driven by four shocks, we estimate the model using data for four series, which allows us to avoid problems of stochastic singularity. We use output, inflation, a short-term interest rate and real money balances. Output is measured by real, final domestic demand. Inflation is the gross rate of increase in GDP deflator. The nominal interest rate is the three-month Treasury Bill rate. Finally, real money balances are measured by dividing the M2 money stock by the GDP deflator. Further, output and real money balances are expressed in per-capita terms using the civilian population aged 16 and over.

As written, the model implies that all variables are stationary and fluctuate around a constant mean. In the data, however, the output and money data exhibit upward trends, while inflation and interest rates trend downwards. There are two ways to reconcile the absence of trends in the model with those observed in the data. First, we can render the data stationary before estimation by –among other possibilities– regressing the logarithm of each variable on a constant and a time trend and using the detrended series as the estimation data.[7] Alternatively, we can introduce secular growth in technology, and rewrite the model in terms of detrended variables; this implies that the steady-state growth in output and in real money balances will be equal to each other and to the growth in technology. In that manner, the estimation of the output and money trends is conducted within the structural model.[8] Note that this trend is not shared by inflation and interest rates, which means that the model imposes a no-trend hypothesis on these variables. The results presented in this preliminary version of the paper concern this second estimation method.[9]

[6]In addition to Dib (2003a,b, forthcoming) and Ireland (2003, 2004), this estimation method is also used by Bergin (2003) and Bouakez et al. (2002), among others. Appendix B provides some of the details about the estimation procedure. We employ the simplex algorithm, as implemented by *Matlab*.

[7]One might want to introduce a trend break at 1990:4 for real money balances, because $M2$ growth has experienced a definite break around that period.

[8]Details on the detrended model are available from the authors.

[9]It might be possible to rewrite the model by allowing for a trend in the inflation target of the monetary authorities, a trend that would be shared by the interest rate because of the Fisher relation.

4 Estimation Results (1981:3 to 1995:4)

4.1 Parameter Estimates

We first estimate the model's structural parameters over a sample running from 1981:3 through 1995:4. Tables 1 reports the maximum-likelihood estimates of the parameters, alongside their standard errors and t-statistics. Almost all of the estimated parameters are statistically significant ant economically meaningful. The estimate of the discount rate β is 0.988, which implies an annual steady-state real interest rate of just over 4 per cent. The estimates of b, the parameter determining the steady-state ratio of real balances to consumption, is 0.49; while that of γ, the constant elasticity of substitution between consumption and real balances, is about 0.06, similar to that estimated by Dib (2003a) for the Canadian economy. The estimate of ϕ, the probability of not adjusting prices in the next period, is 0.61. Thus, on average, firms keep their prices unchanged, except for indexation, for about two quarters and a half. This estimate is very close to those obtained in the closed-economy estimates of Dib (2003b).

The estimates of the monetary policy parameters are statistically significant, with the exception of ϱ_y. Specifically, The responses of monetary policy to inflation, output, and money growth (ϱ_π, ϱ_y, and ϱ_μ) are 01.01, 0.02, and 0.52, respectively.[10] The estimates of ρ_v and σ_v, the persistence coefficient and standard deviation of monetary policy shocks, are close to 0.27 and 0.007, respectively. Overall, the estimates of monetary policy parameters are similar to those previously estimated by Dib (2003b, forthcoming) for the Canadian economy. They indicate that, to achieve its objectives, the Canadian monetary authorities have responded strongly to inflation deviations, more modestly to money growth variations, and hardly (if at all) to output deviations from steady state.

The autoregressive coefficient estimates indicate that the technology, money demand, and preference shocks are relatively persistent, with the money demand shock being the most persistent ($\rho_z = 0.997$). The standard deviation estimates suggest that the the aggregate demand-side shocks (money demand and preferences) are the most volatile.

[10]There is no problem of indeterminate equilibrium as long as the sum of the estimated values of ϱ_π and ϱ_μ exceeds one.

4.2 Impulse response functions

Figures 1 to 4 display the response of the economy following the four types of exogenous shocks, for the estimated parameter values. The response of output is measured as a deviation from its steady-state value, whereas the responses of the three other variables are labeled as net annualized rates, in percentage points.

Figure 1 plots the economy's response to a monetary policy tightening. Following the monetary policy action, the nominal interest rate increases, and its return to its steady-state values is moderately fast (recall that the estimated serial correlation in monetary policy shocks, ρ_v, is fairly small.) Output, inflation and money growth by contrast, fall sharply on impact. Output and inflation return gradually to steady state, while money growth overshoots slightly in the following periods, converging back to steady state from above. This reflects the slow adjustment of prices, a result of the Calvo (1983) assumption. Notice that the negative, contemporaneous correlation between interest rates and money growth –the liquidity effect– is consistent with the evidence. More importantly, endogenous money helps to create a liquidity effect in the estimated models: an instantaneous increase in the short-term nominal interest rate is accompanied by a decrease in the money growth rate.

Figure 2 shows the economy's responses to a 1% shock to money demand (an increase in b_t), which exogenously increases the households' preference for money. The figure shows that this shock has only a small impact on the economy: output and inflation decrease on impact, but only slightly. Money growth increases sharply, however, to cover the increase in demand. Since the rule followed by monetary authorities include a response to money growth increases, the nominal interest rate increases slightly, which causes the slight output decrease.

The positive money-demand shock increases households' real money holdings, and as this shock fades away, the increase in the short term interest rates pulls real money holdings back to their initial values, which in turn leads money growth and inflation back to their initial, steady-state values. This result matches Poole's (1970) classic analysis, in which the monetary policy authority changes the short-term nominal interest rate to react to exogenous demand-side disturbances.

Figure 3 presents the effects of a 1 per cent positive technology shock. In response to the shock, output jumps up instantaneously, while the nominal interest rate and inflation fall below their steady-state levels. Money growth responds positively to the shock before falling below its steady-state level after two quarters. The deflationary pressure brought about by the shock leads to a sustained easing of monetary policy; recall the monetary policy rule in(24). This mechanism helps to magnify the increase in output, which peaks above its steady-state level three quarters after the shock. Therefore, the Bank's response helps the economy to adjust to the supply-side disturbances.

Finally, Figure 4 shows the impulse responses to a 1 per cent increase in the preference shock z_t; this is an exogenous shock to the household's marginal utility of consumption and real balances. In response to this shock, output, the nominal interest rate, inflation, and money growth jump immediately above their steady-state levels before returning gradually to those levels. Because the estimates of the preference autoregressive coefficient, ρ_z, are relatively large, the computed impulse responses are highly persistent. To control the impact of preference shocks on output and inflation, the Bank increases modestly but persistently the short-term nominal interest rate.

4.3 Volatility, autocorrelations, and variance decomposition

To continue our assessment of our estimated New Keynesian model, Table 2 reports the standard deviations, expressed in percentage terms, of output, real balances, inflation and the nominal interest rate as computed from the data and from the estimated model. In the data, output and real balances have standard deviations of 3.44 and 2.78 per cent, respectively. Inflation and the short-term nominal interest rate are less volatile; their standard deviations are less than 0.6 per cent. The table shows that the model (i) underpredicts the volatility of output, (ii) generates real balance- volatility close to that observed in the data, and (iii) slightly overpredicts the volatility of inflation and the nominal interest rate.

To understand which of the four shocks are driving the results, Table 3 decomposes the forecast-error variances of output, real balances, inflation and the nominal interest rate into the fractions that can be attributed to each of the shocks. The table shows that preference and technology shocks are the most important source of fluctuation in output, both in the short and long term. Monetary policy shocks also account for a smaller but significant fraction of output fluctuations in the short term. Monetary policy and technology shocks are the most important factors determining fluctuations in the inflation rate. Together, they account for around 80 per cent of fluctuations at the one-quarter-ahead horizon. Preference shocks do contribute to some of inflation volatility, particularly at loner horizons. The great majority of interest rate fluctuations are attributable to preferences shocks; the contribution of the other shocks, particulary that of monetary policy shocks, is not significant. Finally, technology and money demand shocks explain more than 90 per cent of the fluctuations in real money balances, while, once again, monetary policy shocks explain about 10 per cent of the short-term fluctuations.

5 Forecasting Accuracy of the Model

5.1 The Experiment

The results presented so far indicate that the estimated New Keynesian model (NK) matches reasonably well several features of the data, as well as providing a coherent explanation for how several types of shocks affect the economy.

There are no guarantee, however, that a a good performance in within-sample analysis can translate into superior forecasting performance.[11] In order to assess the forecasting properties of the NK model, we compute out-of-sample forecasts for the NK model a unrestricted VAR benchmark.[12] Specifically, we begin by estimating both models using data from 1981:3 through 1995:4. These estimates are used to produce forecasts one-though four-quarters-ahead, i.e. for 1996:1 to 1996:4, for the four variables used in the estimation (output, inflation, interest rates, and real money balances). Next, the sample is extended to 1996:1, the estimates are updated, and then used to produce another set of forecasts, for 1996:2 through 1997:1. Estimates and forecasts are updated in this manner until the end of the available sample; at this point, we have time series for one, two, three, and four-quarter-ahead forecasts spanning the range 1996:1 to 2004:1. These forecasts can then be compared to realized data over the same period.

Figure 5 to 7 illustrate the results. First, Figure 5 presents the forecasts arising from the new Keynesian model and realized data. The figure shows that the model provides what appears to be a relatively good characterization of output fluctuations, for both one-quarter-ahead and four-quarter-ahead forecasts. Further, the model forecasts the movements of real money balances reasonably. The model appears to have more difficulty, however, to forecasts future movements in inflation and interest rates. Recall that the

[11] Indeed, some of the factors that help producing a good fit within sample may be precisely the ones that lead to weaker forecasting performance.

[12] The VAR benchmark contains the same variables and is estimated with a constant, a trend, and two lags of each variable. Repeating the entire analysis using a VAR(1) does not change the key results of our analysis

model's structure imposes a zero-trend condition to these variables, which might be putting the model at a disadvantage.

Figures 6 and 7 compare the forecasting errors of the New Keynesian model (the solid line) with those arising from the VAR(2) benchmark (the dotted lines) for the case of one-quarter-ahead forecasts (Figure 6) and four-quarter-ahead forecasts (Figure 7). The two pictures paint a relatively pictures. In Figure 6, the VAR benchmark seems to produce the smaller forecasting errors at the one-quarter-ahead horizon, for output and interest rates, notably. In contrast, when four-quarter-ahead forecasts are studied (Figure 7), the NK model seems to outperform its benchmark, at least for output and real money balances. Throughout, the inflation forecasts appear to be very close one to the other, with a slight advantage to the VAR benchmark, particularly at the four-quarter-ahead horizon.

5.2 Formal Tests of Forecasting Accuracy

The graphical illustrations in Figure 6 and Figure 7 suggest that the NK model may have a slight advantage for forecasting output, while the VAR benchmark may forecast inflation and possibly interest rate slightly better. To ascertain whether the differences in forecasting are statistically significant, we start by using the test introduced in Diebold and Mariano (1995). Define the forecast errors arising from the New Keynesian model as $\{e_t^M\}_{t=1}^T$ and those from the VAR benchmark as $\{e_t^B\}_{t=1}^T$. Further, define a sequence of 'loss differentials' $\{l_t\}_{t=1}^T$ where $l_t = (e_t^B)^2 - (e_t^M)^2$. If the NK model is a better forecasting tool, one would expect that on average, the loss differentials l_t would be positive. Conversely, one would expect negative values if the VAR benchmark is superior. Following this intuition, the Diebold and Mariano (1995) test considers the null hypothesis $H_0 : E[l_t] = 0$; positive values of the statistic suggest that the forecasts from the New Keynesian model have lower mean-squared errors, while negative values favour the VAR benchmark. The test statistic (denoted DM) is asympotically normal and standard critical values are the relevant ones.[13] More recently, Harvey et al. (1997) have proposed a corrected Diebold and Mariano (1995) statistic in order to reduce size distortions that might be significant in small samples. The corrected statistic is compared to a Student's t distribution with $N-1$ degrees of freedom, where N is the number of forecasted data.

Table 5 presents the results of these tests. The first column reports the Mean Square Error (MSE) of the NK model, relative to that of the

[13] The statistic is computed as $DM = l/\hat{\sigma}(l)$ where l is the sample average of l_t and $\hat{\sigma}(l)$ is an (HAC-consistent) estimate of the standard deviation of l.

VAR benchmark. Values smaller than one therefore suggest a superior forecasting accuracy for the NK model, and values bigger that one suggest the converse. The table suggests first that the NK model may have better forecasting power for output and real money balances, however, this superior performance may not take hold until medium term forecasting horizons (one year) are studied. [14] Second, the table also suggests that as far as inflation and interest rates are concerned, it is the VAR benchmark that may be the superior forecasting vehicle.

In order to test whether these differences are statistically significant, the last two columns report the Diebold and Mariano (1995) and Harvey et al. (1997) statistics. The overall message of these tests is that there are only a few cases (most notably for real money balances, where the NK model has the advantage) where there is sufficient evidence to conclude that one model holds statistically significant, superior forecasting power compared to the other.

Even if the tests reported in Table 5 were to be conclusive, the forecasts from the lesser model may still contain some information not present in those from the first model; in such an instance, combining both forecasts would reduce further the forecasting errors.[15] A more stringent test of whether one model dominates another in forecasting would therefore be to test whether the second model contains *any* information not contained in the forecasts from the first model.

In this context, Granger and Newbold (1973) define the forecasts from one model as "conditionally efficient" when combining them with those from another model does not lead to an overall decrease in forecast accuracy. Chong and Hendry (1986) define the same situation as one where the first set of forecasts "encompass" those from the second model: there is no need to keep the second model's forecasts because the information they contain is encompassed by those of the first model. To implement this test, we follow Harvey et al. (1998), which propose test statistics similar to those in Diebold and Mariano (1995) and its Harvey et al. (1997) correction. The null hypothesis is that the NK model forecasts contains no information that isn't already contained in those from the VAR.[16]

[14]It remains to be researched whether this advantage to the NK model continues to apply to further forecasting horizons of interest to central bank forecasting, in the order of four to eight quarters.

[15]For example, the lesser model might outperform the first in specific times, such as when the economy is in recession.

[16]Specifically, we are interested in running the following regression:

$$e_t^B = \gamma(e_t^B - e_t^M) + \epsilon_t,$$

Table 6 presents the results. The first column presents the test statistic as proposed by Diebold and Mariano (1995) and the second the correction proposed by Harvey et al. (1997). Recall that high values of the test statistic reject the hypothesis that no value can be gained from using the NK forecasts when the VAR model is available. Similar to what was discussed from Table 5, the results in Table 6 suggest that the VAR forecasts may encompass those from the NK model for inflation and interest rates, but not for output and real money balances.

The comparison between the forecasting accuracy of the NK model and the VAR benchmark until this point has pitted a model where inflation and interest rates were restricted to hold no trends (the NK model) against one where such trends were present (the VAR benchmark). A better suited comparison might be between two models for which inflation and interest rates are restricted to contain no significant trends. Table 7 presents this comparison: the NK model is the same, but the VAR benchmark is restricted to hold no trends. The results change significantly: most notably, there is now much less evidence that the VAR benchmark can outperform the NK models when forecasting inflation and interest rates. In fact, the only evidence of statistically significant superior forecasting accuracy arises from real money balances, where the NK model possess the advantage. Finally, Table 8 presents the results of another experiment designed to lessen the disadvantage of the NK model: it reports the results of estimating the model of samples of fixed lengths (60 quarters) instead of estimating the models on samples of increasing lengths, keeping the starting point fixed. Once again, the result is that the evidence against the NK model is decreased significantly and again the only statistically significant results about superior forecasting accuracy advantage the NK model.

6 Discussion and Conclusion

Since the coming of age of the RBC model and the DSGE (dynamic stochastic general equilibrium) methodology in macroeconomics, researchers have identified several dimensions along which these models were at odds with features of the observed data, For example, Cogley and Nason (1995)

where e_t^B and e_t^M represent the forecasting errors from the VAR benchmark model and the NK model, respectively. The null hypothesis is $H_0 : \gamma = 0$. Under the null, the errors made by the VAR benchmark cannot be explained (and thus potentially reduced) by information arising from the NK model. Note that we could conversely test whether the NK forecasts encompass those from the VAR, i.e. whether there is *any* information in the VAR forecasts that is not present in the NK forecasts.

show that the simple RBC model, because of its weak internal propagation mechanism, cannot match the autocorrelation function of output or the impulse responses of Blanchard and Quah (1989). For their part, Chow and Kwan (1998) demonstrate that the same model, once translated into a VAR in employment, investment, and productivity, implies restrictions on that VAR that are strongly rejected by the data. Models that extend the simple RBC structure by introducing nominal rigidities and multiple sources of volatility, such as those in Ireland (1997) and Kim (2000), have also had problems replicating observed features of the data, like the strong autocorrelation properties of inflation or output.

In such a context, the evidence that structural models like the New Keynesian structure employed in this paper may display comparable or even better out-of-sample forecasting ability than unrestricted VARs may seem surprising.[17] Taken generally, this evidence suggests that restricted or parsimonious specifications, although at odds with some features of the data, may often outperform unrestricted alternatives in out-of-sample exercises. Clements and Hendry (1998, 1999) assess the validity of this conjecture. The main trade-off discussed is that of sampling variability (introduced in the unrestricted specification by the estimation of numerous parameters) versus inconsistency (introduced in parsimonious models by imposing possibly false restrictions). Their assessment is that, absent frequent structural breaks, parsimony is unlikely to deliver important improvements to forecasting ability. On the other hand, the presence of frequent structural breaks leaves open the potential for significant improvements from imposing some restrictions (among them over-differencing) and a better estimation of the deterministic elements of the model.

Overall, the preliminary results presented in this paper are very encouraging for researchers working with New Keynesian models. The statistical tests we report suggest that at a minimum, restricting a VAR by imposing the New Keynesian model may have no negative impact on its forecasting performance. In the case of output and real money balances, the model may in fact be the one with the superior forecasting accuracy.

More work remains to be accomplished for to assess the robustness of this conjecture. Most notably, the New Keynesian model might be extended to include a trend in the inflation target of the authorities, which would allow the model to better track the downward trend in inflation and interest rates

[17] As mentioned in the introduction, Ingram and Whiteman (1994) and DeJong, Ingram and Whiteman (2000) display such evidence. In an earlier paper, Ireland (1995) reports that, once translated into a bivariate VAR, the simple version of the permanent income theory is rejected within-sample but helps the model to better forecast out-of-sample.

over the last 20 years. Further, differencing data, rather than removing linear trends, to render them stationary might be studied.[18]

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[18] In the experiments of Stock and Watson (1999, 2002) the macroeconomic variables used where for the most part differenced. Ireland (2001b), however, reports in a formal comparison between estimating trend-stationary or difference-stationary RBC models that the better out-of-sample forecasting accuracy arises from the trend stationary specification.

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Table 1: Maximum-likelihood estimates and standard errors
(1981Q3 to 2004Q1)

Parameter	Estimate	Std. Deviation	t-statistic
β	0.988	0.002	537
γ	0.061	0.018	3.46
ϱ_{π}	1.005	0.219	4.30
ϱ_{μ}	0.516	0.139	3.73
ϱ_{y}	0.021	0.043	0.48
ρ_v	0.268	0.090	2.99
σ_v	0.007	0.001	6.22
ϕ	0.613	0.066	9.28
A	3.638	0.200	18.10
ρ_A	0.939	0.059	16.02
σ_A	0.015	0.003	5.90
b	0.488	0.071	6.89
ρ_b	0.9967	0.005	215.45
σ_b	0.011	0.001	9.95
ρ_z	0.878	0.050	17.73
σ_z	0.022	0.004	5.69
π^{ss}	1.012	0.003	352.23
γ_Y	1.002	0.002	537.51
LL	858.2181		

Note: LL is the maximum log-likelihood value.

Table 2: Volatility

Variable	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{R}_t$	$\hat{m}_t$
Data	3.44	0.60	0.48	2.78
Model	2.33	0.74	0.61	2.87

Table 3: Forecast-error variance decompositions

		Percentage owing to:			
Quarters	Variance in %	Tech.	Mon.dem.	Policy	Pref.
		A. Output			
1	0.0121	49.71	6.94	12.09	31.27
2	0.0248	58.18	3.84	7.29	30.68
4	0.0476	66.09	2.06	3.98	27.88
10	0.0904	75.56	1.10	2.10	21.24
		C. Inflation			
1	0.0023	31.52	17.20	48.61	2.66
2	0.0029	30.46	15.12	46.08	8.34
4	0.0034	28.56	13.03	40.47	17.94
10	0.0040	26.91	11.09	34.43	27.56
		D. Nominal interest rate			
1	0.0010	6.30	4.10	0.5	95.22
2	0.0017	2.72	2.78	2.5	94.25
4	0.0026	5.89	5.89	2.2	92.03
10	0.0038	10.72	10.72	1.5	87.84
		B. Real balances			
1	0.0125	41.25	41.83	7.93	9.00
2	0.0324	42.56	44.80	4.26	8.38
4	0.0757	42.70	48.42	1.98	6.91
10	0.1887	39.69	55.49	0.80	4.01

Table 4. The Forecasting Experiment (1996:1 - 2004:1)

Estimate	Forecast k periods ahead			
	$k=1$	$k=2$	$k=3$	$k=4$
$1981:3 \longrightarrow 1995:4$	$1996:1$	$1996:2$	$1996:3$	$1996:4$
$1981:3 \longrightarrow 1996:1$	$1996:2$	$1996:3$	$1996:4$	$1997:1$
$1981:3 \longrightarrow 1996:2$	$1996:3$	$1996:4$	$1997:1$	$1997:2$
$1981:3 \longrightarrow 1996:3$	$1996:4$	$1997:1$	$1997:2$	$1997:3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$1981:3 \longrightarrow 2003:2$	$2003:3$	$2003:4$	$2004:1$	$---$
$1981:3 \longrightarrow 2003:3$	$2003:4$	$2004:1$	$---$	$---$
$1981:3 \longrightarrow 2003:4$	$2004:1$	$---$	$---$	$---$

Table 5: Testing for Equality in Forecasting Accuracy: NK Model and VAR(2) Benchmark; 1996:1 - 2004:1

Variable	Relative MSE (NK Model)[a]	DM stat.[b] (p-value)	HLN stat.[c] (p-value)
		Forecasting One Period Ahead	
Output	1.58	-1.73(0.08)	-1.70(0.10)
Inflation	1.03	-0.21(0.83)	-0.21(0.84)
Interest Rate	1.82	-2.26(0.02)	-2.22(0.03)
Money	0.72	1.95(0.05)	1.92(0.06)
		Forecasting Two Periods Ahead	
Output	1.20	-0.54(0.60)	-0.51(0.62)
Inflation	1.07	-0.39(0.70)	-0.37(0.71)
Interest Rate	1.58	-1.28(0.20)	-1.22(0.23)
Money	0.59	2.31(0.02)	2.20(0.04)
		Forecasting Three Periods Ahead	
Output	0.72	0.63(0.53)	0.58(0.57)
Inflation	1.11	-0.51(0.62)	-0.46(0.65)
Interest Rate	1.49	-1.08(0.32)	-0.92(0.37)
Money	0.52	2.35(0.02)	2.16(0.04)
		Forecasting Four Periods Ahead	
Output	0.49	1.05(0.29)	0.93(0.36)
Inflation	1.08	-0.34(0.74)	-0.30(0.77)
Interest Rate	1.47	-0.86(0.39)	-0.76(0.45)
Money	0.47	2.19(0.03)	1.93(0.06)

[a] MSE (NK Model) / MSE (VAR Benchmark); values smaller than 1 therefore suggest superior performance by the NK model

[b] Test statistic from the Diebold and Mariano (1995) test. The null hypothesis if of equal forecasting accuracy between the two models. The statistic is asymptotically normal.

[c] Harvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N the number of forecasts.

Table 6: Forecast Encompassing: Does the NK model provide *any* information not contained in the VAR Benchmark?

Variable	DM stat.[a] (p-value)	HLN stat.[b] (p-value)
	Forecasting One Period Ahead	
Output	2.87 (0.00)	2.83 (0.00)
Inflation	0.55 (0.29)	0.54 (0.30)
Interest Rate	-1.03 (0.85)	-1.02 (0.16)
Money	2.25 (0.01)	2.25 (0.02)
	Forecasting Two Periods Ahead	
Output	2.81 (0.00)	2.68 (0.00)
Inflation	0.40 (0.35)	0.38 (0.35)
Interest Rate	-0.74 (0.77)	-0.71 (0.24)
Money	2.52 (0.01)	2.41 (0.01)
	Forecasting Three Periods Ahead	
Output	2.38 (0.01)	2.19 (0.02)
Inflation	0.31 (0.38)	0.29 (0.39)
Interest Rate	-0.67 (0.75)	-0.62 (0.27)
Money	2.74 (0.00)	2.52 (0.01)
	Forecasting Four Periods Ahead	
Output	2.04 (0.02)	1.80 (0.04)
Inflation	0.49 (0.31)	0.43 (0.33)
Interest Rate	-0.60 (0.73)	-0.53 (0.30)
Money	2.61 (0.00)	2.31 (0.01)

[a] Test statistic proposed by Harvey et al. (1998). The null hypothesis is that the forecasts from the NK model provide no information not already contained in those from the VAR benchmark.

[b] Harvey et al. (1997)'s correction. The statistic follows a t_{N-1} distribution

Table 7: Testing for Equality in Forecasting Accuracy: NK Model and VAR(2) Benchmark with no trend; 1996:1 - 2004:1

Variable	Relative MSE (NK Model)[a]	DM stat.[b] (p-value)	HLN stat.[c] (p-value)
	Forecasting One Period Ahead		
Output	1.68	-1.97(0.05)	-1.94(0.06)
Inflation	1.05	-0.36(0.72)	-0.35(0.73)
Interest Rate	1.32	-1.15(0.25)	-1.14(0.26)
Money	0.74	1.81(0.07)	1.78(0.08)
	Forecasting Two Periods Ahead		
Output	1.38	-1.05(0.30)	-1.00(0.33)
Inflation	1.12	-0.63(0.53)	-0.61(0.55)
Interest Rate	0.96	0.13(0.89)	0.13(0.90)
Money	0.64	1.89(0.06)	1.80(0.08)
	Forecasting Three Periods Ahead		
Output	0.90	0.28(0.77)	0.26(0.80)
Inflation	1.18	-0.86(0.40)	-0.77(0.45)
Interest Rate	0.79	0.60(0.55)	0.55(0.59)
Money	0.59	1.95(0.05)	1.79(0.08)
	Forecasting Four Periods Ahead		
Output	0.49	0.92(0.36)	0.81(0.43)
Inflation	1.08	-0.77(0.44)	-0.68(0.50)
Interest Rate	1.47	0.79(0.43)	0.70(0.49)
Money	0.47	2.34(0.02)	2.06(0.05)

[a] MSE (NK Model) / MSE (VAR Benchmark); values smaller than 1 therefore suggest superior performance by the NK model

[b] Test statistic from the Diebold and Mariano (1995) test. The null hypothesis if of equal forecasting accuracy between the two models. The statistic is asymptotically normal.

[c] Harvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N the number of forecasts.

Table 8: Testing for Equality in Forecasting Accuracy: NK Model and VAR(2) Benchmark with fixed window (60 quarters); 1997:1 - 2004:1

Variable	Relative MSE (NK Model)[a]	DM stat.[b] (p-value)	HLN stat.[c] (p-value)
	Forecasting One Period Ahead		
Output	1.60	-1.40(0.16)	-1.38(0.18)
Inflation	1.07	-0.38(0.70)	-0.37(0.71)
Interest Rate	0.70	1.92(0.05)	1.89(0.07)
Money	0.68	2.45(0.02)	2.40(0.02)
	Forecasting Two Periods Ahead		
Output	0.96	0.10(0.92)	0.10(0.93)
Inflation	1.07	-0.38(0.70)	-0.36(0.72)
Interest Rate	0.51	2.39(0.02)	2.26(0.03)
Money	0.54	2.55(0.01)	2.41(0.02)
	Forecasting Three Periods Ahead		
Output	0.65	1.16(0.25)	1.05(0.30)
Inflation	1.08	-0.36(0.72)	-0.32(0.75)
Interest Rate	0.49	1.90(0.06)	1.72(0.10)
Money	0.48	1.69(0.09)	1.53(0.14)
	Forecasting Four Periods Ahead		
Output	0.46	1.58(0.11)	1.37(0.18)
Inflation	1.02	-0.12(0.90)	-0.10(0.92)
Interest Rate	0.46	1.66(0.10)	1.43(0.16)
Money	0.42	1.26(0.21)	1.09(0.29)

[a]MSE (NK Model) / MSE (VAR Benchmark); values smaller than 1 therefore suggest superior performance by the NK model

[b]Test statistic from the Diebold and Mariano (1995) test. The null hypothesis if of equal forecasting accuracy between the two models. The statistic is asymptotically normal.

[c]Harvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N the number of forecasts.

Figure 1: A Monetary Policy Tightening
(Shock occurs at $t = 5$)

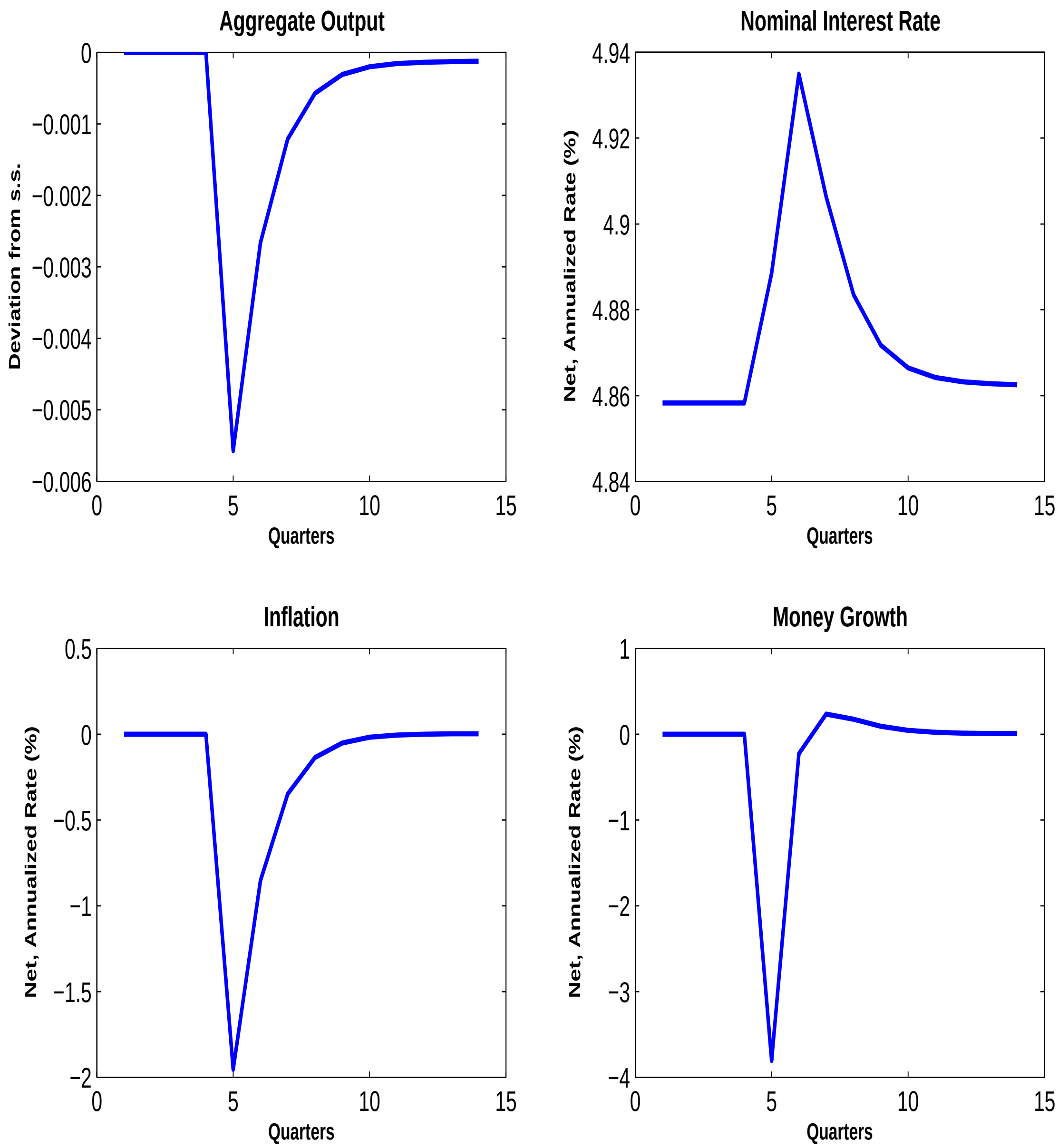


Figure 2: The Economy's Response to a Positive Money-Demand Shock

(Shock occurs at $t = 5$)

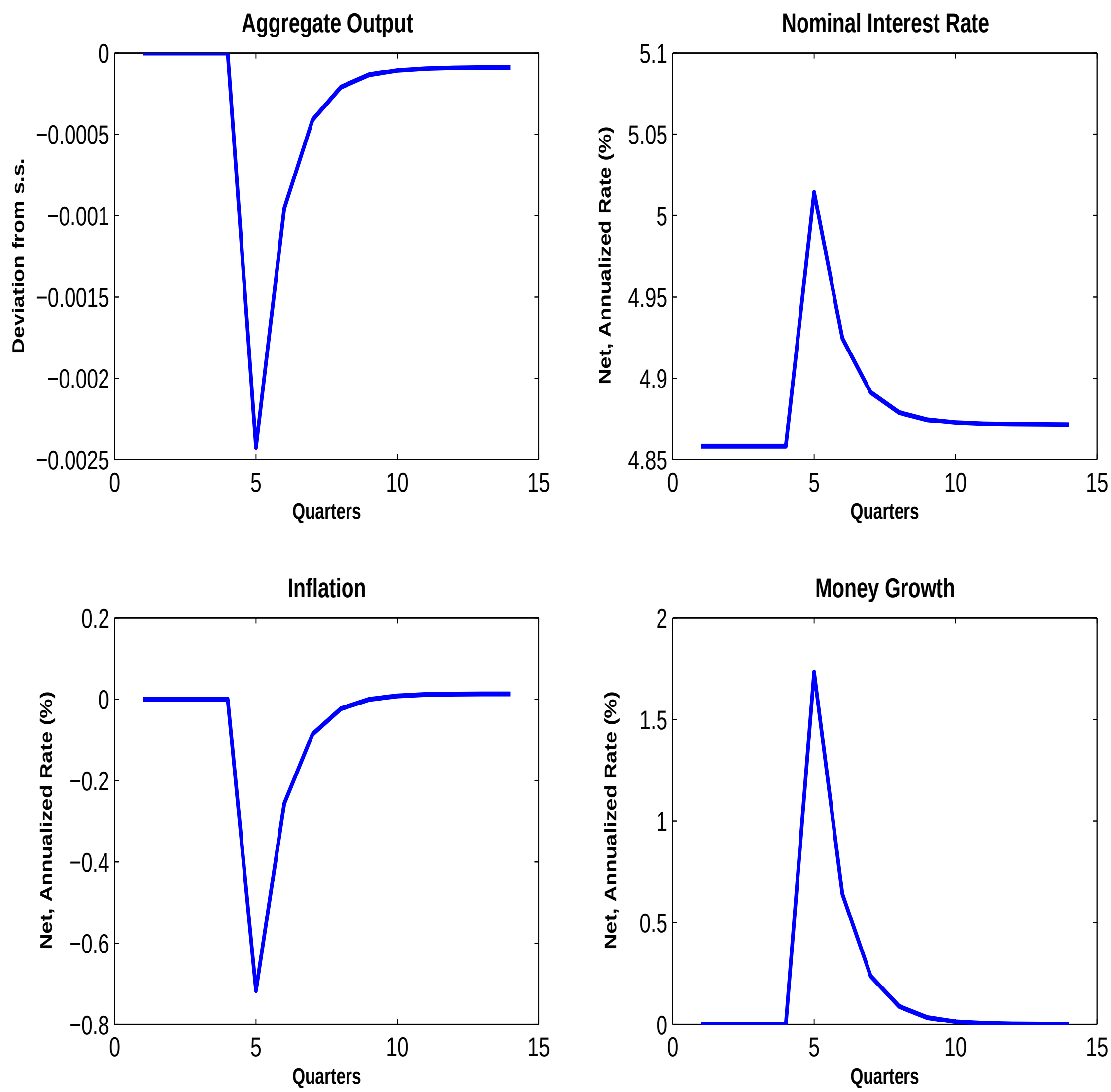


Figure 3: The Economy's Response to a Positive Technology Shock

(Shock occurs at $t = 5$)

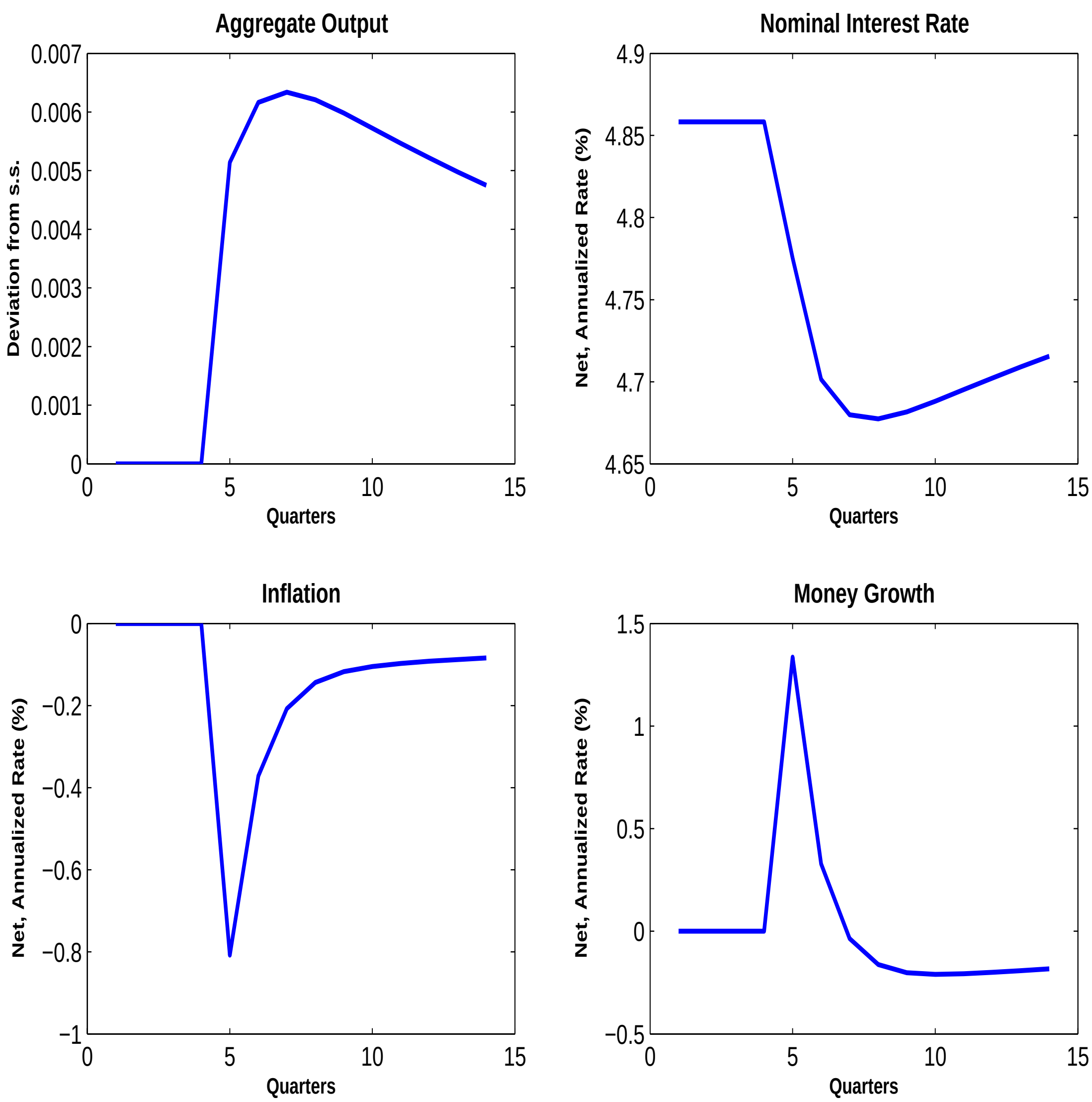


Figure 4: The Economy's Response to a Positive Preference Shock

(Shock occurs at $t = 5$)

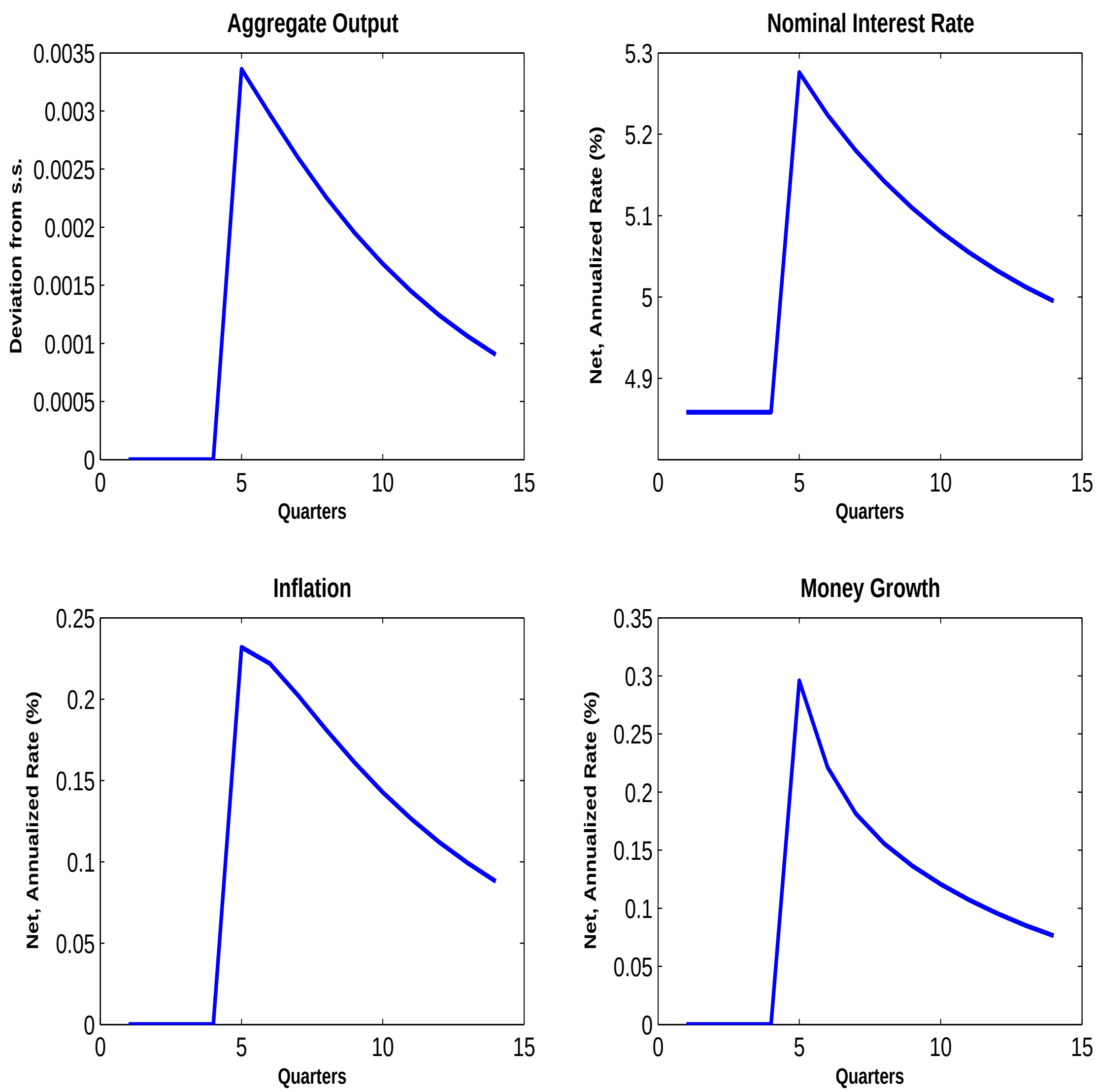


Figure 5. Actual Data and Forecasts from the New Keynesian Model

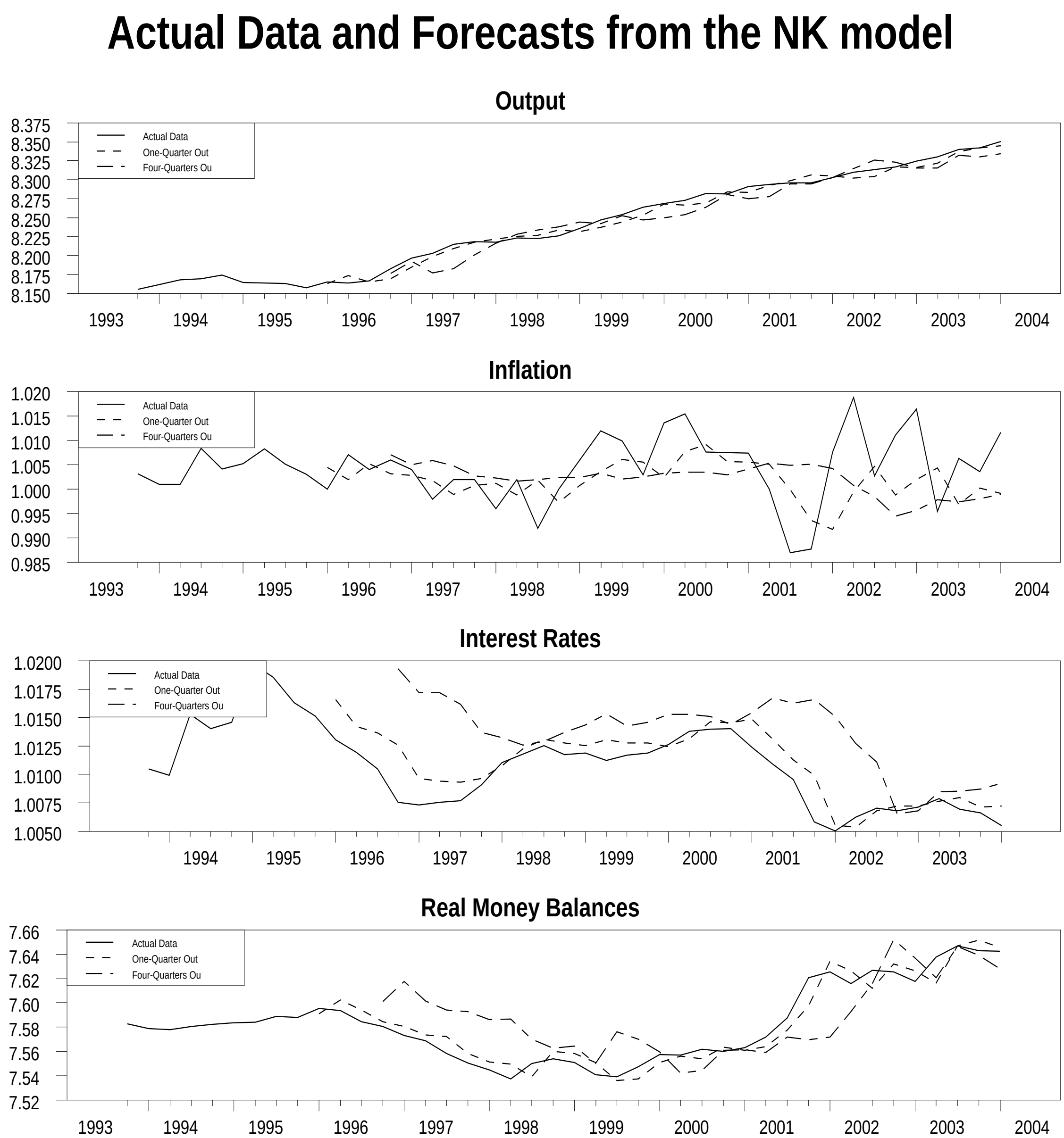


Figure 6. Forecast Errors: One-Quarter Ahead

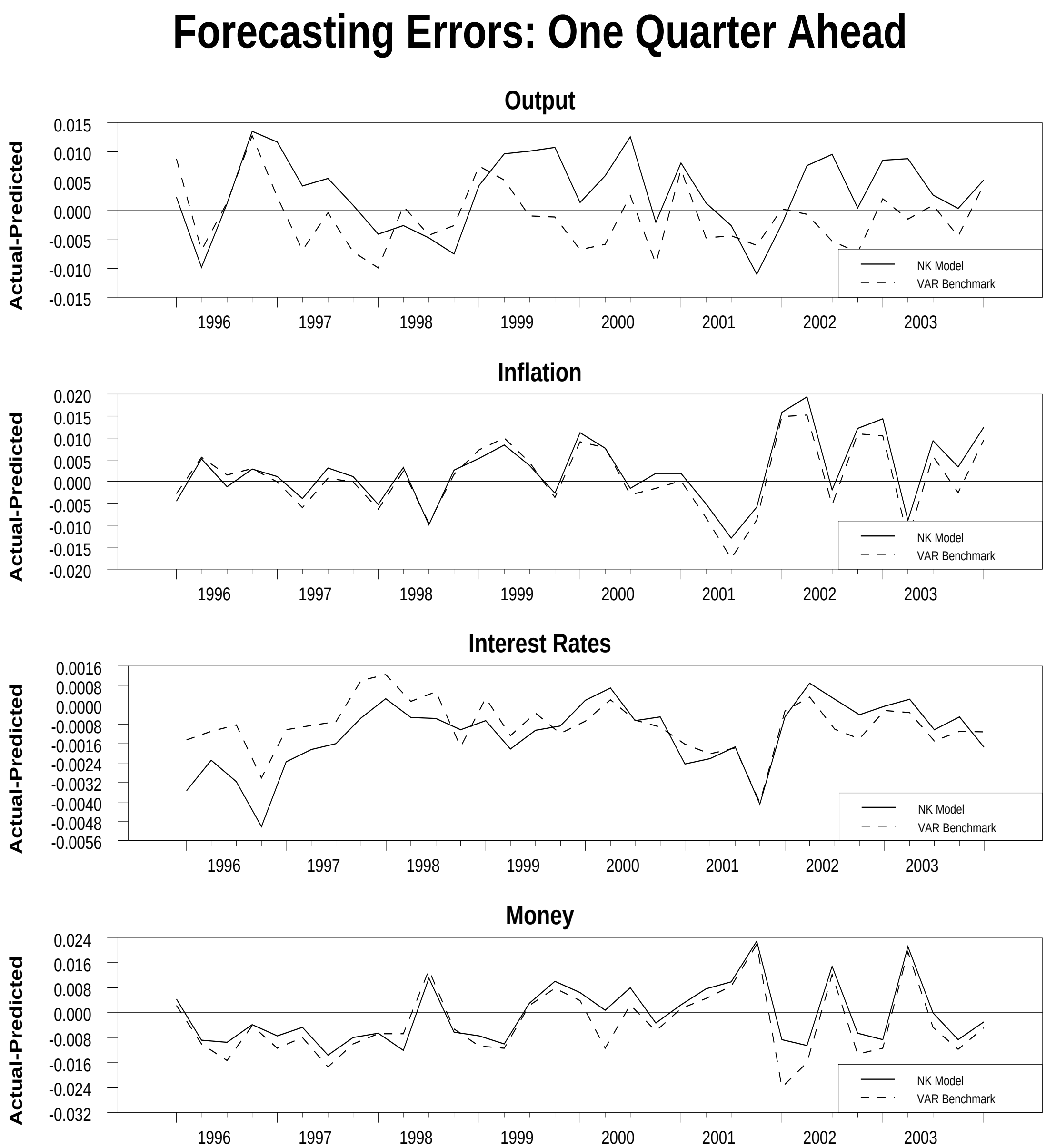
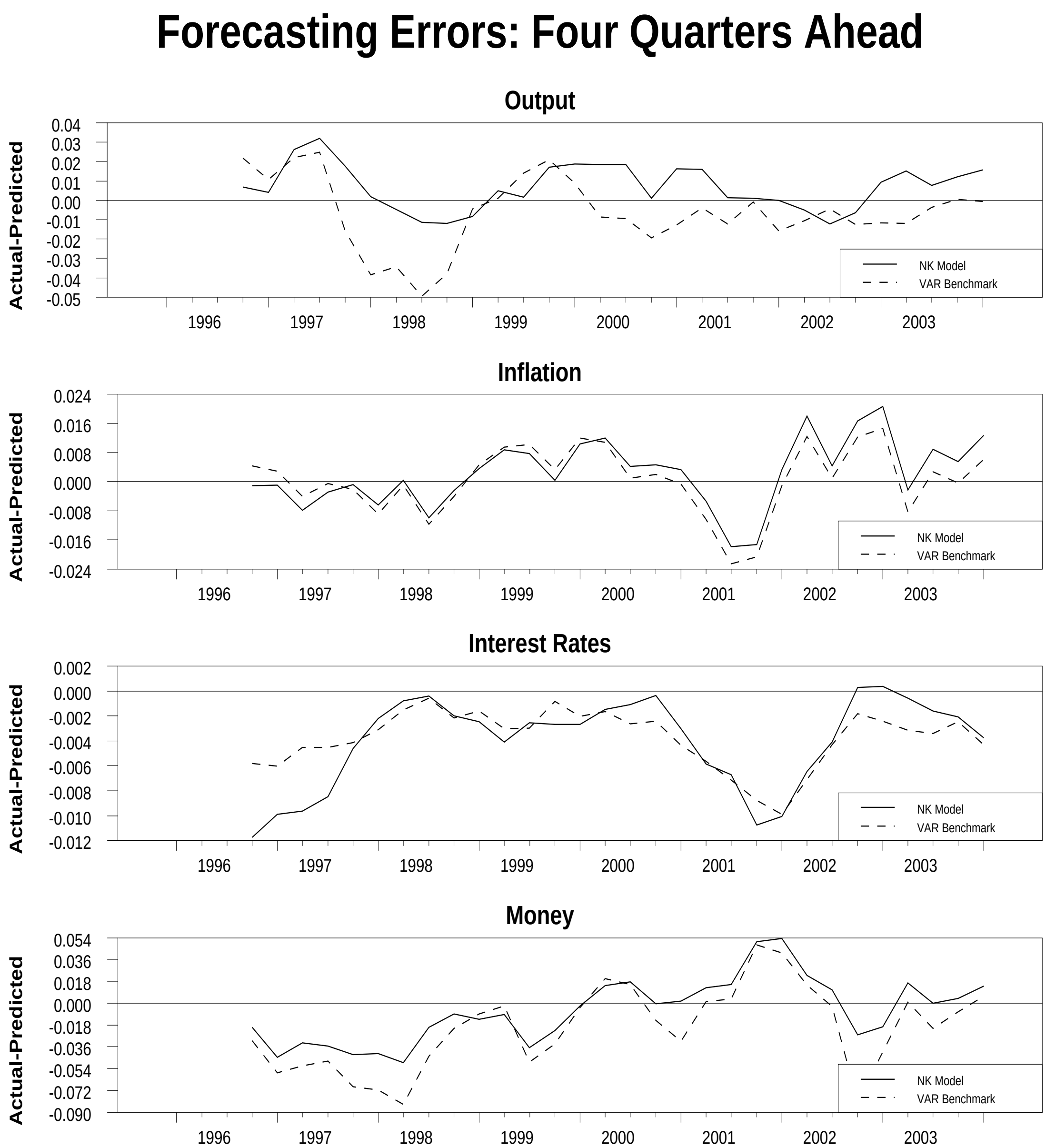


Figure 7. Forecast Errors: Four Quarters Ahead



A Solving the New Keynesian Model

A.1 The symmetric equilibrium

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \tag{A.1}$$

$$\frac{z_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right); \tag{A.2}$$

$$\frac{\eta}{1-h_t} = \lambda_t w_t; \tag{A.3}$$

$$\beta E_t \left[\lambda_{t+1} \left(r_{kt+1} + 1 - \delta + \psi \left(\frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}} - (\psi/2)(\frac{k_{t+2}}{k_{t+1}} - 1)^2 \right) \right]$$
$$= \lambda_t \left[1 + \psi \left(\frac{k_{t+1}}{k_t} - 1 \right) \right]; \tag{A.4}$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \right] \tag{A.5}$$

$$y_t = k_t^{\alpha} (A_t h_t)^{1-\alpha}; \tag{A.6}$$

$$\frac{\alpha y_t}{k_t} = q_t r_t; \tag{A.7}$$

$$\frac{(1-\alpha) y_t}{h_t} = q_t w_t; \tag{A.8}$$

$$\widetilde{p}_t = \frac{\theta \pi_t}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta \phi \pi^{-\theta})^k \lambda_{t+k} y_{t+k} q_{t+k} (\prod_{s=1}^{k} \pi_{t+s}^{\theta})}{E_t \sum_{k=0}^{\infty} (\beta \phi \pi^{1-\theta})^k \lambda_{t+k} y_{t+k} (\prod_{s=1}^{k} \pi_{t+s}^{\theta-1})}; \tag{A.9}$$

$$\pi_t^{1-\theta} = (1-\phi) \widetilde{p}_t^{1-\theta} + \phi \pi; \tag{A.10}$$

$$y_t = c_t + k_{t+1} - (1-\delta) k_t - (\psi/2)(\frac{k_{t+1}}{k_t} - 1)^2; \tag{A.11}$$

$$\mu_t = \frac{m_t \pi_t}{m_{t-1}}; \tag{A.12}$$

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t); \tag{A.13}$$

$$\log(A_t) = (1-\rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \tag{A.14}$$

$$\log(b_t) = (1-\rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{A.15}$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}; \tag{A.16}$$

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}. \tag{A.17}$$

A.2 Finding The Non-stochastic Steady-State

Setting all shocks to their mean, the economy converges to a steady state in which all variables in (A.1) to (A.17) are constant. Removing the time subscripts to denote the steady-state values of these variables, one is lead to the following system:

$$\mu = \pi; \tag{A.18}$$

$$R = \frac{\pi}{\beta}; \tag{A.19}$$

$$r_k = \frac{1}{\beta} - 1 + \delta; \tag{A.20}$$

$$q = \frac{\theta - 1}{\theta}; \tag{A.21}$$

$$\lambda c = \left[1 + b\left(\frac{\mu}{\mu - \beta}\right)^{\gamma - 1}\right]^{-1}; \tag{A.22}$$

$$\lambda m = \lambda c b\left(\frac{\mu}{\mu - \beta}\right)^{\gamma}; \tag{A.23}$$

$$\frac{k}{y} = \frac{\alpha q}{r_k}; \tag{A.24}$$

$$\frac{c}{y} = 1 - \delta\left(\frac{k}{y}\right); \tag{A.25}$$

$$wh\lambda = \frac{q(1-\alpha)(\lambda c)}{(c/y)}; \tag{A.26}$$

$$h = \frac{wh\lambda}{\eta + wh\lambda}; \tag{A.27}$$

$$y = hA\left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.28}$$

A.3 Linearized System

The next step in the solution is to compute a first-order approximation of (A.1) to (A.17) around the steady state. A hatted variable denotes the deviation relative to the steady-state value of the variable. The equations are divided between *Static* and *Dynamic* equations.

A.3.1 Static equations

$$\hat{y}_t + (\alpha - 1)\hat{h}_t = \alpha\hat{k}_t + (1-\alpha)\hat{A}_t; \quad (A.29)$$
$$\hat{\mu}_t - \hat{\pi}_t = \hat{m}_t - \hat{m_{t-1}}; \quad (A.30)$$
$$\hat{y}_t - \hat{r}_{kt} = \hat{k}_t - \hat{q}_t; \quad (A.31)$$
$$(-1 - (\gamma-1)\lambda c)\hat{c}_t = \gamma\hat{\lambda}_t + ((\gamma-1)\lambda((\mu-\beta)/\mu)m)\hat{m}_t$$
$$+(\lambda((\mu-\beta)/\mu)m)\hat{b}_t - \gamma\hat{z}_t; \quad (A.32)$$
$$(-\beta/(\mu-\beta))\hat{R}_t - (\lambda((\gamma-1)/\gamma)c)\hat{c}_t = \hat{\lambda}_t + (\lambda((\gamma-1)/\gamma)(\mu-\beta)/\mu)m) + 1/\gamma)\hat{m}_t$$
$$+((\lambda((\mu-\beta)/\mu)m - 1)/\gamma)\hat{b}_t - \hat{z}_t; \quad (A.33)$$
$$(h/(1-h))\hat{h}_t - \hat{w}_t = \hat{\lambda}_t; \quad (A.34)$$
$$\hat{y}_t - \hat{w}_t - \hat{h}_t = -\hat{q}_t; \quad (A.35)$$
$$\hat{R}_t - \varrho_\mu\hat{\mu}_t - \varrho_\pi\hat{\pi}_t - \varrho_y\hat{y}_t = \hat{v}_t; \quad (A.36)$$

These equations can be rewritten compactly in matrix form as

$$AZ_t = BX_t + CU_t$$

where A, B, and C are 8x8, 8x5, and 8X4 matrices, respectively and we have $Z_t = (\hat{y}_t, \hat{R}_t, \hat{r_{kt}}, \hat{c}_t, \hat{\pi}_t, \hat{w}_t, \hat{h}_t, \hat{\mu}_t)'$ (a vector of endogenous variables), $X_t = (\hat{k}_t, \hat{m_{t-1}}, \hat{\lambda}_t, \hat{q}_t, \hat{m}_t)'$ (a vector of state and co-state variables) and $U_t = (\hat{A}_t, \hat{b}_t, \hat{v}_t, \hat{z}_t)'$ (a vector of shock variables).

A.3.2 Dynamic equations

The dynamic equations are the following:

$$\beta\hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1-\beta\phi)(1-\phi)}{\phi}\hat{q}_t; \quad (A.37)$$
$$(\psi(\beta(1-\delta) - (1+\beta)))\hat{k}_{t+1} + (\beta(1+r_k-\delta))\hat{\lambda}_{t+1}$$
$$+\beta r_k\hat{r}_{kt+1} + (\beta\psi y/k)\hat{y}_{t+1} - (\beta\psi c/k)\hat{c}_{t+1} = -\psi\hat{k}_t + \hat{\lambda}_t; \quad (A.38)$$
$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{c}_t; \quad (A.39)$$
$$\hat{\lambda}_{t+1} - \hat{\pi}_{t+1} = \hat{\lambda}_t - \hat{R}_t; \quad (A.40)$$
$$\hat{m}_t = \hat{m}_t. \quad (A.41)$$

Again, this can be rewritten in matrix form as

$$DX_{t+1} + EZ_{t+1} = FX_t + GZ_t + HU_t$$

where D, E, F, G, and H are 5X5, 5X8, 5X5, 5X8, and 5X4 matrices, respectively.

Using the static and dynamic equations, we can solve the model using the methodology of Blanchard and Kahn (1980), which leads us to the following first-order state-space solution of the system:

$$\begin{aligned} \widehat{s}_{t+1} &= \mathbf{\Phi}_1 \widehat{s}_t + \mathbf{\Phi}_2 \varepsilon_{t+1}, \\ \hat{d}_t &= \mathbf{\Phi}_3 \widehat{s}_t, \end{aligned}$$

where the matrices $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$ and $\mathbf{\Phi}_3$ are functions of the structural parameters of the model.

B Estimating the Model

To be added

C Data

The model is estimated using data that spans the period 1981:3 to 2004:1. The data is taken from Statistics Canada's CANSIM database, for which we list the associated mnemonics. Output Y_t is *final domestic demand* [V1992068], of quarterly frequency and in chained 1987 dollars. We convert this series into per-capita terms using the population of age 15 and over.

The interest rate R_t is the three month treasury bill rate ([V122531]), a series of daily frequency, for which we take a quarterly average.

Finally, the money stock M_t is *M2* [B1630] which is of monthly frequency; we take a quarterly average and convert the resulting series into real, per-capita terms by dividing it with the GDP implicit price deflator ([D100465]) and the population age 15 and over.

All variables are then first logged and then detrended by regressing them against a linear trend, using the OLS residuals as the variables for estimation. There exists a break point in the trend for the $M2$ series at 1990:1. Consequently, we take that break in trend into account when computing the detrended series for real money balances.