THE FUTURES PRICING PUZZLE

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JEL Classification Codes: D58, D74, D91, G12, G24

Key Words: Contango, Expectations, Normal Backwardations

This draft: November 8, 2004.

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ABSTRACT

This paper models commodity futures in a rational expectations equilibrium specifically (i) incorporating the conflict of interests between Hedgers (Producers-Consumers) and Speculators and (ii) superimposing constraints to immunize the real sector of the economy from shocks of excessive futures contracting. We extend the framework of Newbery and Stiglitz (1981), Anderson and Danthine (1983) and Britto (1984) to attribute the conflicting and puzzling results in the empirical literature to the presence of *multiple* equilibria ranked in a pecking order of *decreasing pareto-efficiency*. Thus, we caution empirical researchers on making inferences on data embedded with moving equilibria, as it can render their analysis of asset pricing mechanism incomprehensible. Finally, we rationalize the imposition of position limits by policy makers to help steer the equilibria to *pareto-inferior* ones, which make the real sector of the economy more resilient to shocks from the financial sector.

I. INTRODUCTION

Futures trading plays a vital role in society as it reallocates risk, reduces price volatility, offers liquidity, leads to price discovery, and enhances social welfare (see Turnovsky, 1983; Francis, 2000; and Goss, 2000). However, futures pricing has intrigued both academics and managers since the seminal work of John Maynard Keynes on Normal Backwardation in 1930. Keynes postulated that the futures discount/premium over expected spot price compensates a speculator for bearing price risk of a commodity.[1] The futures pricing mechanism is a function of hedgers' net position and their level of risk aversion. When supply and demand of futures by equally risk averse hedgers offset each other, there is no need for a discount or premium. Divergence of futures pricing from expected spot prices are contingent on the difference of risk aversion between short and long hedgers. Speculators are traders who are prepared to bear risk, in return for which they expect to earn an appropriate risk premium. As a result, speculators will buy [sell] futures contracts only if they expect

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[1] Keynes (1930) did not strictly distinguish between discount and premium, and referred to them both as *Normal Backwardation*. However, contemporary literature distinguishes both, and classifies *Normal Backwardation* and *Contango* as situations where futures trade below and above expected spot prices, respectively.

prices to increase [decrease]. Hedgers, on the other hand, are prepared to pay a premium to lay off unwanted risk onto speculators.

If hedgers, in aggregate, are short (or long hedgers are more risk averse than short hedgers), then speculators are net long, and in this case the futures will trade at a discount to expected spot price. This situation is referred to as *Normal Backwardation*. In contrast, if hedgers are net long (or long hedgers are more risk averse than short hedgers), speculators will be net short, and futures will trade at a premium to expected spot price. This situation is known as *Contango*. Thus, the futures discount/premium, according to Keynes (1930), serves as an insurance mechanism to transfer price risk to speculators. However, empirical investigation of the Normal Backwardation theory of Keynes (1930) reveals conflicting results, as Houthakker (1957) finds support for it in the corn, cotton and wheat futures, while Rockwell (1967) contradicts it using a broader data set.[2] Telser (1958) also contradicts the findings of Houthakker (1957), and Gray (1961) finds results consistent with those of Telser (1958).

Modern finance rationalizes the pricing discrepancy between futures and spot prices using (i) Capital Asset Pricing Model (CAPM)/ Consumption Capital Asset Pricing Model (C-CAPM) (Kolb, 1996; Breeden, 1980; and Jagannathan, 1985); or (ii) Hedging-Pressure Hypothesis (Hirshleifer, 1988); or (iii) General Equilibrium Theory (Anderson and Danthine, 1983; Britto, 1984; Young and Boyle, 1989; and Francis, 2000). However, the empirical investigation of these asset pricing theories is mired with conflicting results as:

(i)   Dusak (1973) finds zero systematic risk and zero returns on corn, soybean and wheat futures, while Bodie and Rosansky (1980) and Fama and French (1987) find contradictory positive returns supporting Normal Backwardation;

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[2]   It should be noted that the empirical results cited from the literature are in the context of Keynes (1930).

(ii)   Carter, Rausser and Schmitz (1983) find supporting evidence for CAPM and Hedging Pressure Theories contradicting Dusak (1973). However, Baxter, Conine and Tamarkin (1985) critique them relying on the suggestions of Marcus (1984) for the choice of market portfolio and find no risk premiums for corn, soybean and wheat futures. Ehrhardt, Jordan and Walking (1987) also find no risk premium on the above commodities using the Arbitrage Pricing Theory (APT);

(iii)   Raynauld and Tessier (1984) evaluate risk-premiums in corn, oat and wheat, but deduce that they do not support Normal Backwardation as the premiums do not benefit hedgers or speculators. Chang (1985) disagrees with the above study and finds that the speculator makes profits in commodities using non-parametric methods for determining risk premiums. Hartzmark (1987) disagrees with Chang (1985) as he finds that futures trading is profitable to hedgers but not to speculators. Thus, there are no risk-premiums to be earned, and he concludes against Normal Backwardation. Park (1985) examines currency, metal and plywood futures, and finds support for Normal Backwardation for metals and not for the others;

(iv)   Kolb (1992) studies 29 commodities and financial futures, and finds that only 7 of them support Normal Backwardation, while the remaining do not. Bessembinder (1992) also finds that Normal Backwardation is not exhibited in financial and metal futures, but exists in agricultural and foreign currency futures. Finally, Miffre (2000) analyzes 19 commodities and financial futures incorporating time variations in futures risk premiums to conclude in favor of Normal Backwardation.[3]

        Thus, the above studies have arrived at inconsistent results in spite of testing data sets of different commodities (and financial futures) over different time periods and employing extremely sophisticated statistical techniques. This discrepancy in results is also observed for some of the studies such as Dusak (1973), Carter, Rausser and Schmitz (1983), Chang (1985),

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[3]   In a recent paper Miffre (2003) refines her empirical methodology (using both CAPM and APT) for 26 commodity and financial futures to detect (i) Normal Backwardation for metal and financial futures and (ii) the Expectations Hypothesis for the agricultural commodities.

Baxter, Conine and Tamarkin (1985) and Ehrhardt, Jordan and Walking (1987), who have tested the same commodities (corn, soybeans and wheat) over roughly the same time period. This discrepancy between theoretical assertions and empirical behavior is a puzzle.[4] The key issues of concern to academics, practitioners and policy-makers in this area are as follows: *First*: Is there something missing in the theory? *Second*: Can we get such disparate results even when Normal Backwardation is the norm for commodities as demonstrated by Anderson and Danthine (1983) and Britto (1984)? *Third*: Are there any policy implications to be derived from this investigation?

The response to the first issue is in affirmative, as theoretical asset pricing models ignore the *natural* constraints inhibiting futures contracting by agents in the economy. For instance, in a world where there are two hedgers (commodity producer and consumer) and a speculator, their freedom to contract futures is curtailed as follows: The commodity producer is restricted to shorting an amount of futures he/she is able to produce in the worst state of the economy. The consumer is restricted by the aggregate demand-supply equilibrium condition in the spot market. Finally, the speculator is restricted by the aggregate demand-supply condition in the futures market. Ignoring these *natural* constraints can lead to erroneous results, and allow gyrations from the financial sector to permeate the real sector of the economy. Our assertion on limiting futures contracting is corroborated by Rolfo (1980) and Lee (2003).[5] It should be noted that apart from the *natural* constraints, policy makers impose additional constraints in the form of *position limits*, which are strictly enforced by futures exchanges such as the Chicago Board of Trade (CBOT), Chicago Mercantile Exchange (CME) etc.

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[4] Malliaris and Stein (1999) echo the same concern in their investigation of futures pricing from an efficiency perspective, which is interrelated with our methodology of rational expectations equilibrium described below (see also Sheffrin, 1996; and Bray, 1992).

[5] Rolfo (1980) is of the view that limited employment of the futures market is superior to a full short hedge under production variability. This is also supported by Lee (2003), who explicates the intrinsic difference between hedging and speculating as follows:

"In a textbook world, hedging means reducing a portfolio's net exposure to risk, and speculating means allowing a portfolio's exposure to risk. With the former, expected returns are muted and, with the latter, expected returns are larger."

The main purpose of this paper is to respond to the two issues posed earlier. That is, to rationalize the (i) puzzling behavior of futures prices in the empirical literature to the presence of *multiple* equilibria, ranked in the *decreasing* order of *pareto-efficiency*, (ii) imposition of additional constraints (by policy-makers) in the form of *position limits* to maneuver the multiple equilibria to *pareto-inferior* ones, which do not make the real sector of the economy vulnerable.

We model commodity futures in a rational expectations equilibrium by (i) incorporating the conflict of interest between hedgers (such as producers-consumers) and speculators and (ii) superimposing *natural* futures contracting constraints to prevent shocks (of excessive futures contracting) from impacting on the real sector of the economy.[6] We demonstrate how the end-user (consumer) of the commodity can strictly enforce that agents in the economy such as producers adhere to the futures contracting constraints. Our model is an extension of the framework of Newbery and Stiglitz (1981), Anderson and Danthine (1983) and Britto (1984), where random shocks of production (or yield risks) emanating from the supply side impact on the equilibrium pricing of the commodity on the demand side leading to price risks. This has credence in the real world as agricultural commodities are subject to the fluctuations of weather on the supply side, giving rise to changes in prices on the demand side.

Our approach using rational expectations equilibrium has a strong following in the academic and policy communities (see Sheffrin, 1996), while conflict of interest between competing economic agents is crucial for deriving their *supply* and *demand* side relationships and consequently their equilibrium parameters.

This paper is organized as follows: The modeling of futures contract along with its solution is explicated in Sections II and III, while Section IV provides some concluding remarks.

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6 Maddock and Carter (1982) define rational expectations as "the application of the principle of rational behavior to the acquisition and processing of information and to the formation of expectations." Bray (1992) explicates it further by classifying rational expectations equilibrium as "self-fulfilling," as economic agents form correct expectations given the pricing model and information.

II. MODEL DEVELOPMENT

For simplicity and mathematical tractability, we assume a one-period economy with two goods and three types of agents. There are n_P identical Producers (**P**), n_C identical Consumers (**C**) and n_S identical Speculators (**S**) each endowed respectively with e_P, e_C and e_S units of the Good 1, which is the *numeraire* good in our economy. Good 2 is a *perishable* good produced solely for the Consumers by the Producers.[7] The production process used for Good 2 is subject to random shocks ($\tilde{\xi}$) stemming from exogenous forces such as weather. The distribution of $\tilde{\xi}$ is known to all agents. Each producer converts x units of Good 1 into $\tilde{y}$ units of Good 2 using the production function $g(x, \tilde{\xi})$, where $\tilde{y} = g(x, \tilde{\xi})$. Furthermore, $g(0, \tilde{\xi}) = 0$, $\frac{\delta g}{\delta x} = g_1 > 0, \frac{\delta g}{\delta \xi} = g_2 > 0$, and $\frac{\delta^2 g}{\delta x^2} = g_{11} < 0$. The decision on the amount (x) of input to be used for the production of Good 2 is made in the beginning of the period, while the output of the production process (Good 2) is available at the end of the period. Futures contracting is initiated at the beginning of the period, while its settlement takes place at the end.[8] The demand for the perishable Good 2 is termed as $D(\tilde{p}, e_C)$, where $\tilde{p}$ is its stochastic price in the spot market, while e_C is the income (endowment) of the consumer. Since this demand stems only from the consumers, all that is produced is purchased for consumption at the end of the period. Good 2 is defined by the sign of the covariance between the two risks emanating from the optimal production yield ($\tilde{y}^*$) and the spot price ($\tilde{p}$) (see Hirshleifer, 1975). If the sign of this covariance is positive [negative], Good 2 is construed as normal [inferior], otherwise it is considered to be an intermediate commodity (see Rolfo, 1980; Anderson and Danthine, 1983,

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[7] The assumption of perishability of Good 2 is not crucial to our analysis. It is relaxed in Section III.b. to illustrate the invariance of the quality of our results.

[8] It should be noted that our one period model resembles that of a forward contract. This is because differences between futures and forward prices for short-term contracts with settlement dates less than nine months tend to be very small. That is, the daily marking to market process appears to have little effect on the setting of futures and forward prices. Moreover, if the underlying asset's returns are not highly correlated with interest rate changes, then the marking to market effects are small even for longer-term futures. Only for longer-term futures contracts on interest-sensitive assets will the marking to market costs be significant. Because of this, it is a common practice in the literature to analyze futures contracts as if they were forwards. For details see Ritchken (1996).

and Britto, 1984). All agents are risk averse and maximize their respective concave and differentiable Von Neumann-Morgenstern utility functions denoted by $U_P(\cdot)$, $U_C(\cdot)$, $U_S(\cdot)$.

II.a. The Commodity Producer:

The goal of each of the n_P Producers is to optimally select the amount (x) of Good 1 to be used in the production process and the amount (q_P) of Good 2 to be pre-sold in the futures market (at a unit price f) in order to maximize expected utility of consumption. That is,

Max. $E_0 \{ U_P(\tilde{c}_P)\}$

   (*in* c_P, x, q_P)

subject to the budget constraint

$$\tilde{c}_P = e_P + [\tilde{p} (\tilde{y}) - x] + q_P (f - \tilde{p}) = (e_P - x) + \tilde{p} (g(x, \tilde{\xi}) - q_P) + q_P (f) \tag{1}$$

where $E_0\{\cdot\}$ is the expectation operator at time 0, $\tilde{c}_P$ is the consumption of Producer at t = 1, while the remaining notations have the same meaning as stated earlier.

The budget constraint at t = 1 (Equation 1) incorporates consumption of Good 1 (stemming from residual of endowment net of input to production process (e_P – x)) along with the proceeds of selling Good 2 (denominated in the numeraire Good 1) via: (i) Futures Market (involving q_P units at a fixed price f) and (ii) Spot Market (involving residual units of output ($\tilde{y}$ – q_P) at the prevailing stochastic price ($\tilde{p}$)).

The objective function of each of the Producers can be rewritten as:

Max. $E_0 \{U_P[(e_P - x) + \tilde{p} (g(x, \tilde{\xi}) - q_P) + q_P (f)]\}$

   (*in* x, q_P)

The First Order Necessary Conditions (FONCs or Euler Equations) are evaluated as follows:

(i) At the margin, Producers will use an optimal amount x* of Good 1, which yields net benefit *at least* equal to zero. This results in the optimal yield (production level) $\tilde{y}^* = g^*(x^*, \tilde{\xi})$ given as follows:

$$\frac{E_0 [(U_P'(\tilde{c}_P))[\tilde{p} (g^*_1(x^*, \tilde{\xi}))]]}{E_0(U_P'(\tilde{c}_P))} - 1 \geq 0$$

$$\Rightarrow \left\{ \frac{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))\, E_0(\tilde{p}\,(g*_1(x*, \tilde{\xi})\,)) + Cov_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}),\, \tilde{p}\,(g*_1(x*, \tilde{\xi})\,))}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} \right\} - 1 \geq 0$$

$$\Rightarrow E_0(\tilde{p}\,(\tilde{y}*)) + \frac{Cov_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}),\, \tilde{p}\,(\tilde{y}*))}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} \geq 1 \qquad (2)^9$$

If the Producers have adequate initial endowments $(e_\mathbf{P})$ then the above equation will hold as an equality.

$$\Rightarrow E_0(\tilde{p}\,(\tilde{y}*)) + \frac{Cov_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}),\, \tilde{p}\,(\tilde{y}*))}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} = 1 \qquad (2a)$$

(ii)   At the margin, the Producers will sell forward $q_\mathbf{P}$ units of Good 2, which yield net benefits *at least* equal to zero. This yields optimal price of futures (f) given as follows:

$$f \geq \left\{ \frac{E_0\,(U_\mathbf{P}'(\tilde{c}_\mathbf{P})\,\tilde{p})}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} \right\} = \left\{ \frac{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))E_0(\tilde{p}) + Cov_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}),\, \tilde{p})}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} \right\}$$

$$= E_0(\tilde{p}) + \frac{Cov_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}),\, \tilde{p})}{E_0(U_\mathbf{P}'(\tilde{c}_\mathbf{P}))} \qquad (3)$$

The above equation represents the *supply* side relationship of $q_\mathbf{P}$ units of output pre-sold (at a price) f by the Producer, where the equality [strict inequality] sign is applicable when the natural constraint (described by Equation 10 in Section II.d. below) is non-binding [binding].

Thus, maximization of each Producer's objective requires the following:

(i)   The stochastic budget constraints (at t = 1), as depicted in Equation (1), be satisfied.

(ii)   The simplified FONCs (Euler Equations), i.e., Equations (2a) and (3), be satisfied.

(iii) The second order conditions for a maximum are satisfied.[10]

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9 The algebraic simplification of the above expression exploits the well known property of expectation of product of two random variables equals the product of their expectations in addition to the covariance between them (see Mood, Graybill and Boes, 1974).

10 We do not attempt to verify these conditions, as it can be shown that maximizing a concave and differentiable objective function (such as a von Neumann-Morgenstern utility function) with linear constraints gives a negative definite bordered Hessian matrix.

II.b.   The Consumer:

The goal of each of the n_C Consumers is to optimally select the amount of (q_C) Good 2 to pre-purchase in the futures market in order to maximize expected utility of consumption. That is,

Max. $E_0\{U_C(\tilde{c}_C)\}$

$(in\ c_C, q_C)$

subject to the budget constraint

$$\tilde{c}_C = e_C - (D(\tilde{p}, e_C))(\tilde{p}) + q_C(\tilde{p} - f) = e_C - \tilde{p}[D(\tilde{p}, e_C) - q_C] - f(q_C) \qquad (4)$$

where $\tilde{c}_C$ is the consumption of Consumer at $t = 1$, while the remaining notations have the same meaning as stated earlier.

The budget constraint at $t = 1$ (Equation 4) incorporates consumption utilizing endowment (e_C) in the numeraire Good 1 to pay for Good 2 purchased via: (i) Futures Market (involving q_C units at a fixed price of f) and (ii) Spot Market (involving residual demand units of $[D(\tilde{p}, e_C) - q_C]$ at the stochastic spot price ($\tilde{p}$)).

The objective function of each of the Consumers can be rewritten as

Max. $E_0\{U_C[e_C - (D(\tilde{p}, e_C))(\tilde{p}) + q_C(\tilde{p} - f)]\}$

$(in\ q_C)$

The FONC (Euler Equation) is evaluated as follows:

At the margin, the Consumer will pre-purchase q_C units of Good 2, which yield net benefits *at least* equal to zero. This yields optimal price of futures (f) given as follows:

$$f \leq \left\{\frac{E_0(U_C'(\tilde{c}_C)\tilde{p})}{E_0(U_C'(\tilde{c}_C))}\right\} = E_0(\tilde{p}) + \frac{Cov_0(U_C'(\tilde{c}_C), \tilde{p})}{E_0(U_C'(\tilde{c}_C))} \qquad (5)$$

The above equation represents the *demand* side relationship for q_C units of output pre-purchased at a price f by the Consumer, where the equality [strict inequality] sign is applicable when the natural constraint (described by Equation 9 in Section II.d. below) is non-binding [binding].

Thus, maximization of the Consumer's objective requires:

(i)   The stochastic budget constraint in period t = 1 represented by Equation (4) be satisfied.

(ii)   The simplified FONC, Euler Equation, i.e., Equation (5) be satisfied.

II.c.   The Speculator:

The goal of each of the n_S Speculators is to optimally select the amount (q_S) of Good 2 to pre-purchase in the futures market in order to maximize expected utility of consumption. That is,

Max. $E_0\{ U_S(\tilde{c}_S)\}$

(*in* c_S, q_S)

subject to the budget constraint

$$\tilde{c}_S = e_S + q_S\,(\tilde{p} - f) \tag{6}$$

where $\tilde{c}_S$ is the consumption of Speculator at t = 1, while the remaining notations have the same meaning as stated earlier.

The budget constraint at t = 1 (Equation 6) incorporates consumption utilizing endowment (e_S) in the numeraire Good 1 along with net-payoffs in the Futures Market in Good 2 (involving q_S units at the stochastic profit margin of ($\tilde{p} - f$)).

The objective function of each of the Speculators can be rewritten as:

Max. $E_0\{ U_S[e_S + q_S\,(\tilde{p} - f)]\}$

(*in* q_S)

The FONC (Euler Equation) is evaluated as follows:

At the margin, each of the Speculators will pre-purchase q_S units of the commodity, which yield net benefits *at least* equal to zero. This again yields optimal price of futures (f) given as follows:

$$f \leq \left\{\frac{E_0(U_S'(\tilde{c}_S)\,\tilde{p})}{E_0(U_S'(\tilde{c}_S))}\right\} = E_0(\tilde{p}) + \frac{Cov_0(U_S'(\tilde{c}_S),\,\tilde{p})}{E_0(U_S'(\tilde{c}_S))} \tag{7}$$

The above equation represents the *demand* side relationship for q_S units of output pre-purchased at a price f by the Speculator, where the equality [strict inequality] sign is

applicable when the natural constraint (described by Equation 11 in Section II.d. below) is non-binding [binding].

Thus, maximization of the Speculator's objective requires:

(i) The stochastic budget constraint in period $t = 1$ represented by Equation (6) be satisfied.

(ii) The simplified FONC, Euler Equation, i.e., Equation (7) be satisfied.

II.d. The *Natural* Futures Contracting Constraints:

(i) For the real sector of the economy to be in equilibrium, the aggregate demand must equal the *optimal* aggregate supply:

$$\Rightarrow n_C \, [D(\tilde{p}, e_C)] = n_P \, [g*(x*, \tilde{\xi})] = n_P \, [\tilde{y}*]$$

$$\Rightarrow D(\tilde{p}, e_C) = \frac{n_P}{n_C} \, [g*(x*, \tilde{\xi})] = \frac{n_P}{n_C} \, [\tilde{y}*] \qquad (8)$$

The above equation *endogenously* yields the stochastic pricing distribution of Good 2, i.e., $(\tilde{p})$ from the distribution of the random shock $(\tilde{\xi})$. This condition is equivalent to the information on the covariance between the stochastic variables, $\tilde{p}$ and $\tilde{y}*$. That is, on the classification of Good 2 as a Normal, Intermediate or Inferior commodity.

(ii) For the real sector of the economy to be immune from shocks of excessive financial contracting:

Consumers commit themselves to the minimum value of their *exogenous* demand function in the worst state of the economy, i.e., $\text{Min}.[D(\tilde{p}, e_C)] = \frac{n_P}{n_C} \{\text{Min}.[\tilde{y}*]\}$, using Equation (8).

$$\Rightarrow \frac{n_P}{n_C} \{\text{Min}[\tilde{y}*]\} \geq q_C > 0 \qquad (9)$$

Likewise, Producers refrain from entering into futures obligations (q_P) more than what they can deliver in the worst state of the economy, i.e., $\{\text{Min}.[\tilde{y}*]\}$.

$$\Rightarrow \{\text{Min}[\tilde{y}*]\} \geq q_P > 0. \qquad (10)$$

Since Consumers ultimately bear the brunt of any cost overruns in the real sector of the economy, they can (in the context of our model) strictly enforce the above conduct on

the Producers by refusing to enter into any offsetting futures position if the upper bound of Equation (10) is violated. This may seriously impact the demand for futures leading to a drop in its price (f) and impair the social welfare of errant Producers.

(iii)   For the financial sector of the economy to be in equilibrium:

Futures contracts negotiated by the suppliers (Producers) must equal that demanded by Consumers and Speculators.

$$\Rightarrow n_P \, q_P = n_C \, q_C + n_S \, q_S \tag{11}$$

III. MODEL SOLUTIONS

A Rational Expectations Equilibrium (REE) is defined as one where all agents in the economy are knowledgeable of the following:

(i)   Values of the random shocks $(\tilde{\xi})$ of the production process and its probability distribution, as it is an *exogenous* parameter of the model,

(ii)   Optimal input (x*) and yield of production process $(\tilde{y}*)$, as it is *endogenously* determined in a *unique* solution using Equation (2a),

(iii)   Demand function of Consumers for Good 2 $[D(\tilde{p}, e_C)]$, as it is also an *exogenous* parameter of the model, and

(iv)   Spot price $(\tilde{p})$ of Good 2 along with its probability distribution function, as it is *endogenously* determined in a *unique* solution using Equation (8).

This framework explicates the futures pricing puzzle and sheds light on the role of policy-makers as elaborated below.

III.a. Key Result

Theorem:

The model solutions entail *at most ten* equilibria ranked in the pecking order of decreasing *pareto-efficiency*. These equilibria range from the most efficient and least probable one (where the natural futures contracting constraints are violated) to the remaining *nine* (where the constraints are strictly binding). Normal Backwardation (in the sense of

Keynes, 1930) is still the norm in all these equilibria for strictly Normal or Inferior commodities. *The above nine equilibria on the lower rung of pareto-efficiency resolve the futures pricing puzzle.* Imposition of more restrictive constraints of position limits (by policy-makers) steers the equilibria to more manageable but *pareto-inferior* ones.

Proof:

The model solutions are ranked in the decreasing order of pareto-efficiency from the least restrictive equilibrium without any constraints to *six* with a single constraint and last *three* with two constraints. The pareto-ranking of the equilibria stems from the fact that welfare of agents in an unconstrained optimization model is higher than that in a constrained one. *Therefore, as more constraints are added to the model, the equilibria obtained decreases in pareto-efficiency.*

The Highest Ranked (and Least Probable) Equilibrium

This signifies an equilibrium, which makes the real sector of the economy most vulnerable to shocks from the financial sector. This equilibrium is evaluated by initially assuming the absence of *natural* futures contracting constraints. We superimpose the supply-demand constraint (Equation 11) on the respective pricing functions of various agents derived in Sections II.a-c. Since the equilibrium in this special case involves *four endogenous* variables (f, q_P, q_C, q_S), four *independent* Equations (3), (5), (7) and (11) are sufficient to yield a *unique* solution. We thus consolidate Equations (3), (5) and (7) in the form described below:

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_P{'}(\tilde{c}_P), \tilde{p})}{E_0(U_P{'}(\tilde{c}_P))} = \frac{Cov_0(U_C{'}(\tilde{c}_C), \tilde{p})}{E_0(U_C{'}(\tilde{c}_C))} = \frac{Cov_0(U_S{'}(\tilde{c}_S), \tilde{p})}{E_0(U_S{'}(\tilde{c}_S))} \qquad (12)$$

Here the marginal utility of each agent adjusts in such a way that no agent is able to extract any economic surplus from the other. Deviation of futures price from expected spot price is given in terms of a covariance term (of marginal utility of stochastic consumption with price risk) divided by expectation of marginal utility of consumption. The stochastic consumption variable of all agents is impacted distinctly by the joint yield and price risks as

described by Equations (1), (4) and (6) respectively. However, the imposition of strict equality of futures pricing on *all three* agents (in Equation (12)) invalidates the above equilibrium in most cases, and leads to a violation of Equations (9) or/and (10).

The Mid-Ranked Equilibria

In general, if one *natural* constraint is *strictly* binding, then one Futures Pricing Equations (3), (5) or (7) becomes non-binding, i.e., holds strictly as an inequality in *at most six* equilibria. Here, the economic surplus is extricated by the agent whose futures pricing equation holds as a strict inequality. Since this subcase involves three *endogenous* variables (f, q_P or q_C, q_S), three *independent* Equations [two from (3), (5) or (7) and one from (11)] are sufficient to yield a *unique* solution.

To elaborate the above point further:

(i)   If the *natural* constraint on Producer is binding then $q_P = \{Min[\tilde{y}^*]\}$.

$\Rightarrow n_P \{Min[\tilde{y}^*]\} = n_C q_C + n_S q_S$ (using Equation (11))

We thus solve for the *endogenous* variables (f, q_C, q_S) using the above conditions and the following equations.

Equilibrium **PC**:

Here, the futures pricing is determined by both Producers and Consumers, while the economic surplus is retained by the Speculator. That is,

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_P'(\tilde{c}_P), \tilde{p})}{E_0(U_P'(\tilde{c}_P))} = \frac{Cov_0(U_C'(\tilde{c}_C), \tilde{p})}{E_0(U_C'(\tilde{c}_C))} \text{ and}$$

$$f - E_0(\tilde{p}) < \frac{Cov_0(U_S'(\tilde{c}_S), \tilde{p})}{E_0(U_S'(\tilde{c}_S))} \tag{13a}$$

Equilibrium **PS**:

Here, the futures pricing is determined by both Producers and Speculators, while the economic surplus is retained by the Consumer. That is,

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_P'(\tilde{c}_P), \tilde{p})}{E_0(U_P'(\tilde{c}_P))} = \frac{Cov_0(U_S'(\tilde{c}_S), \tilde{p})}{E_0(U_S'(\tilde{c}_S))} \text{ and}$$

$$f - E_0(\tilde{p}) < \frac{Cov_0(U_C'(\tilde{c}_C), \tilde{p})}{E_0(U_C'(\tilde{c}_C))} \tag{13b}$$

Equilibrium **CS**:

Here, the futures pricing is determined by both Consumers and Speculators, while the economic surplus is retained by the Producer. That is,

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_C'(\tilde{c}_C), \tilde{p})}{E_0(U_C'(\tilde{c}_C))} = \frac{Cov_0(U_S'(\tilde{c}_S), \tilde{p})}{E_0(U_S'(\tilde{c}_S))} \text{ and}$$

$$f - E_0(\tilde{p}) > \frac{Cov_0(U_P'(\tilde{c}_P), \tilde{p})}{E_0(U_P'(\tilde{c}_P))} \tag{13c}$$

(ii)   If the *natural* constraint on the Consumer is binding then $q_C = \frac{n_P}{n_C} \{Min[\tilde{y}^*]\}$.

$\Rightarrow n_P \ q_P = n_P \{Min[\tilde{y}^*]\} + n_S \ q_S$ (using Equation (11))

$\Rightarrow n_P \{q_P - \{Min[\tilde{y}^*]\}\} = n_S \ q_S$

$\because q_P$ is restrained by Equation (10), i.e., $q_P \le \{Min[\tilde{y}^*]\} \Rightarrow q_S \le 0$.

Here too, we derive at most three equilibria (**PC'**, **PS'**, **CS'**) by solving for the *endogenous* variables (f, q_P, q_S) using the above conditions and the Equations similar to (13a-c).

The Lower-Ranked Equilibria

Finally, if the above mid-level equilibrium **PC** or **PC'** is infeasible, then we investigate the feasibility of the equilibria where the futures pricing function is determined by the Producer alone or the Consumer alone. Likewise, if the mid-level equilibrium **PS** or **PS'** is infeasible, then we investigate the feasibility of the equilibria where the futures pricing function is determined by the Producer alone or the Speculator alone. Similarly, if the mid-level equilibrium **CS** or **CS'** is infeasible, then we investigate the feasibility of the equilibria

where the futures pricing function is determined by the Consumer alone or the Speculator alone.

Thus, if the natural constraints on both Producers and Consumers are binding, i.e., $q_P = \{Min[\tilde{y}^*]\}$ and $q_C = \dfrac{n_P}{n_C} \{Min[\tilde{y}^*]\} \Rightarrow q_S = 0$ (Using Equation (11)). We again derive *at most* *three* more equilibria by using the above conditions and the respective pricing functions of any *one* agent (while those of the remaining two hold as strict inequalities) as described below. Here too the economic surplus is extricated by the agents whose futures pricing conditions hold as strict inequalities.

Equilibrium **P**:

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_P{}'(\tilde{c}_P), \tilde{p})}{E_0(U_P{}'(\tilde{c}_P))} ,$$

$$f - E_0(\tilde{p}) < \frac{Cov_0(U_C{}'(\tilde{c}_C), \tilde{p})}{E_0(U_C{}'(\tilde{c}_C))} \quad \text{and}$$

$$f - E_0(\tilde{p}) < \frac{Cov_0(U_S{}'(\tilde{c}_S), \tilde{p})}{E_0(U_S{}'(\tilde{c}_S))} \tag{14a}$$

Equilibrium **C**:

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_C{}'(\tilde{c}_C), \tilde{p})}{E_0(U_C{}'(\tilde{c}_C))} ,$$

$$f - E_0(\tilde{p}) > \frac{Cov_0(U_P{}'(\tilde{c}_P), \tilde{p})}{E_0(U_P{}'(\tilde{c}_P))} \quad \text{and}$$

$$f - E_0(\tilde{p}) < \frac{Cov_0(U_S{}'(\tilde{c}_S), \tilde{p})}{E_0(U_S{}'(\tilde{c}_S))} \tag{14b}$$

Equilibrium **S**:

$$f - E_0(\tilde{p}) = \frac{Cov_0(U_S{}'(\tilde{c}_S), \tilde{p})}{E_0(U_S{}'(\tilde{c}_S))} ,$$

$$f - E_0(\tilde{p}) > \frac{\text{Cov}_0(U_P{}'(\tilde{c}_P), \tilde{p})}{E_0(U_P{}'(\tilde{c}_P))} \text{ and}$$

$$f - E_0(\tilde{p}) < \frac{\text{Cov}_0(U_C{}'(\tilde{c}_C), \tilde{p})}{E_0(U_C{}'(\tilde{c}_C))} \tag{14c}$$

Thus, the presence of the middle and lower ranked *nine* equilibria in a dynamic setting has the capacity of rendering empirical analysis incomprehensible. This rationalizes the futures pricing puzzle.

Our general results hold true even when normal backwardation is the norm. This is because the sign of the fraction on the right hand sides (of futures pricing conditions with covariance terms) in Equations (12), (13a-c), (14a-c) are negative [positive] for strictly normal [inferior] Good 2. This is ascribed to the fact that futures contracting involve trade-off between risk and return. When the covariance between yield and price risks is highly positive [negative] for strictly normal [inferior] Good 2, hedgers (Producers-Consumers) reduce [increase] risk by committing to a futures price at a discount [premium] to expected spot in accordance with the insurance perspective of futures, as articulated in Newbery and Stiglitz (1981), Anderson and Danthine (1983) and Britto (1984).

Policy makers are apprehensive of spillovers from the financial sector of the economy into the real sector. They impose *position limits* on the participants especially the speculators. Imposition of any additional constraint in our framework, say on speculators, is reflected on the other agents (hedgers) through the aggregate supply-demand relationship of financial futures (Equation 11). This helps steer the equilibria from either the (i) mid-ranked ones (on the scale of pareto-efficiency) to lower-ranked ones or (ii) lower-ranked ones to still more restrictive, and thus more *pareto-inferior* ones.

Q.E.D.

III.b. Extension of Model to Storable Commodities

Our assumption of perishability of Good 2 (in Sections II and III.a.) helps make the model more tractable in a one period world. This assumption can be relaxed (with the

addition of storage) by extending our model to a two period one (as described in Anderson and Danthine, 1983) to evaluate the futures pricing inequality for a Storage Operator (**O**) in a manner similar to the above sections. This yields *at most* 14 equilibria of which 13 *pareto-inferior* ones (**PCS**, **PSO**, **CSO**, **PC**, **PS**, **PO**, **CS**, **CO**, **SO**, **P**, **C**, **S**, **O**) rationalize the futures pricing puzzle. Thus, the addition of storage does not affect the quality of our result despite its impact on the distribution of the spot price (Deaton and Laroque, 1992; and Chambers and Bailey, 1996).

IV. CONCLUDING REMARKS

The pricing of futures has baffled both academics and managers for decades. This issue has resulted in researchers questioning whether normal backwardation (in the context of Keynes, 1930) is really the norm for commodities. This paper, however, takes a completely different stance on this vital issue of asset pricing. We model the conflict of interest between economic agents superimposing futures contracting constraints to attribute the conflicting and puzzling results stemming from the empirical literature to the presence of *multiple equilibria* (ranked in a decreasing order of pareto-efficiency) even when normal backwardation holds true. Thus, we caution empirical investigators on drawing inferences of time series data containing moving equilibria, as it can make analysis of the asset pricing mechanism incomprehensible.

Our paper has profound implications for policy makers concerned about the economic stability. They are advised to take into consideration restrictions of the pricing model in terms of position limits on the number of futures contracts to be traded. Furthermore, policy-makers need to be aware of the real-world complications and implementation details of a successful hedging strategy using commodity futures. It is essential that they define limits to bets that can be taken in a clear and unambiguous way. They should set up procedures for ensuring that limits are strictly followed. It is also particularly important that policy makers monitor risks very closely when derivatives are used. This is because derivatives can be used for hedging as well as speculating. Without close monitoring, it is impossible to track if

derivatives trader responsible for hedging risk has switched to a speculator and exposed not only himself but also the economy to increased risk.

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