

# Do multivariate filters have better revision properties?

## An empirical analysis <sup>1</sup>

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## Abstract

The output gap plays a crucial role in thinking of many central banks yet real time measurements undergo substantial revisions as more data become available (Orphanides (2001), Orphanides and van Norden (forthcoming)). Some central banks, such as the Bank of Canada and the Reserve Bank of New Zealand, augment the Hodrick and Prescott (1997) filter with conditioning structural information to circumvent the impact of revisions to the output gap estimates. In this paper, we use a state space Kalman filter framework to examine whether the augmented (“*multivariate filters*”) achieve this objective. We use the New Zealand data, since the Reserve Bank of New Zealand is one of the central banks, which uses a “multivariate filter” and publishes the estimates from this filter. We empirically show that the multivariate filters are not any better in real-time because of the uncertainty about trends in the augmenting variables. The addition of structural equations increase the number of signal equations, but at the same time adds more unobserved trend/equilibrium variables to the system. We find that how these additional trends/equilibrium values are treated matters a lot, and hence increase the uncertainty around the estimates. In addition, the revisions from these models can be as large as a univariate Hodrick-Prescott filter. Hence, the criticism of the real time output gap estimates remain to be a very valid one, despite the central banks’ attempt to overcome it by means of additional information.

**Keywords:** Output gap, Filtering, New Zealand  
*JEL classification:* C32, E32

# 1 Introduction

The output gap is a very powerful concept in thinking and models of many inflation targeting central banks.<sup>1</sup> However, the output gap is unobservable, due to the unobserved nature of ‘potential output’. The recent literature, Orphanides (2001), Orphanides and van Norden (2002) and Orphanides and van Norden (forthcoming), have shown that the difficulty of estimating the output gap in real time compromise its usefulness as an indicator of inflation in real time, despite being a good variable to use in ex-post analysis.<sup>2</sup>

Filters are widely used in extracting the trends, and hence estimating the cyclical component of time series, such as output gap. Any filter, except a linear one, suffer from the end-point problem in real time. Some part of the problem is due to data revisions. More importantly, however, the main source of the end-point problem is not knowing the future. As more and more information (or data) become available, the view on the trend/cycle for a particular point in time is revised and these revisions can be substantial. Some central banks, such as the Reserve Bank of New Zealand, Bank of Canada, Central Bank of Czech Republic and Reserve Bank of Australia for example, augment a simple Hodrick and Prescott (1997) filter with some additional structural information to overcome the end-point problem. These augmented versions of the HP filter are often referred to as “multivariate” (MV hereafter) filters. The basic idea behind this is the following: Since the univariate filters have difficulty in distinguishing trend from cycle at the end of the sample in real time, using a(some) structural equation(s), which has some information about trend or cycle, can enable the filter to do a better job. Hence one would expect the output gap estimates from such filters to get revised less.

In this paper we empirically assess the claims that multivariate filter lead to better revision properties or reduce the end-point uncertainty in estimating output gap. We use the New Zealand data as the Reserve Bank of New Zealand is one of the central banks which uses this technique and publishes the estimates from it.<sup>3</sup>

We estimate the multivariate filter, identical to the one used by the Reserve Bank of New Zealand, in state-space with the Kalman filter. The flexible nature of the state-space

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<sup>1</sup>Monetary Policy Statements and Inflation reports by the central banks refer to the output gap concept often.

<sup>2</sup>A recent conference is devoted to the issue of real time monetary policy making. See [www.bundesbank.de](http://www.bundesbank.de) for the papers at the conference

<sup>3</sup>See Reserve Bank of New Zealand’s Monetary Policy Statements, on [www.rbnz.govt.nz](http://www.rbnz.govt.nz)

models enable us to assess the importance of the main assumptions embedded in the MV filter. We find that the multivariate filters do not lead to any better or more reliable estimates of the output gap. The additional information used do contain additional equilibrium values, parameters and residuals to be estimated. We found that the additional information does not overcome the problems in real time, because of unobserved trends in these additional variables. This increases the uncertainty further. Our findings indicate that the concerns of the real-time literature are valid and needs to be addressed.

The remainder of the paper is structured as follows. Section 2 discusses the existing literature on the estimation of output gap and the real time problems. Section 3 introduces the MV filter and discusses the problems associated with the MV filter. Section 4 presents the results. Section 5 looks at the real time properties of the MV filter and compares them to a HP filter. Section 6 concludes.

## **2 Literature review**

The output gap is defined as deviations of actual output from potential output, where potential output is the level of output that is consistent with productive capacity of the economy. Yet this ‘potential output’ concept is not observed. Like many other central banks, the Reserve Bank of New Zealand also relies on this “unobserved measure” in its policy making. Black (1997) show that in the Forecasting and the Policy System (FPS), the Reserve Bank of New Zealand’s main macroeconomic model, domestic inflation is determined within a Phillips curve framework using the output gap.

However, what seems a perfectly good predictor of inflation in ex-post, may not work in real time. Orphanides and van Norden (forthcoming) look at the real time inflation predicting power of about 10 different output gap estimates. They conclude that the relationship is very good in ex-post, but not in real time. Similar conclusions have been found for other countries. For Australia, for example, Robinson and van Zyl (2003) found that the real time output gaps, based on the Reserve Bank of Australia filters, are not good forecaster of inflation. Cayen and van Norden (2004) found similar results for Canada. On the other hand, Graff (2004) state that the Reserve Bank of New Zealand’s output gap measure does predict the non-tradable inflation well, despite the other problems associated with the methodology.

Because the output gap concept is not observable, economists try to estimate it. As it is the case in many other areas of economics, there are more than one ways of estimating the output gap, yet probably no one knows which estimate is the correct one, especially in real time. It is quite a standard practice in economic modelling to distinguish the long-run properties of a system from its fluctuations around the long-run trend. There are various approaches to do this, but what is interesting is that most of these can be conveniently represented within a state-space model.

Apart from the mechanical filters such as the HP, ARMA and linear filters, unobserved components models provide an alternative framework for estimating the output gap. Unobserved components models decompose the observed output into its trend and cycle components. Watson (1986), Harvey (1985), Clark (1987) and Harvey and Jaeger (1993) are examples of this approach. Scott (2000) replicated the Harvey and Jaeger (1993) model for New Zealand.

Kuttner (1994) adds a Phillips curve to the univariate model of Watson and Gerlach and Smets (1997) models modify Harvey (1985) and Clark (1987), by adding a Phillips curve equation.

For New Zealand, Scott (2000) estimates a common cycles model *a la* Harvey and Jaeger (1993). Claus (2003) estimates a structural vector autoregression to estimate the output gap. However, the main model the Reserve Bank of New Zealand uses for the output gap estimation is the so-called the multivariate (MV) filter Conway and Hunt (1997).

MV filter is an alternative way of estimating unobserved potential output, proposed by, Laxton and Tetlow (1992).<sup>4</sup> This method stems from use of the standard HP filter, augmented by some relevant economic information. It has been used by the Central Banks of Canada and New Zealand to estimate potential output (see Buttler (1996) and Conway and Hunt (1997)). OECD (2000) uses the same approach to estimate the NAIRU. Recently Benes and N'Diaye (2004) uses a similar approach to estimate the potential output and NAIRU jointly for the Central Bank of Czech Republic.

Boone (2000) extends on Harvey (1985) and puts the MV filter in state space. This is done in two steps. Firstly, the minimisation problem is written as a state space model. Secondly, restrictions are imposed on the variances of the equations of the state space

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<sup>4</sup>See St Amant and van Norden (1997) for a critique of the this approach.

model, to reproduce the balance between the elements of the minimisation programme.

### 3 Multivariate filter approach

Laxton and Tetlow (1992) argue that the simple HP filter can be augmented with residuals coming from structural relationships. Hence minimising this new “set of problem” would be better than a mechanical HP filter. Because the HP is an augmented HP filter, or its starting point is an HP filter, let us start with the conventional HP filter and put it in state-space.

The Hodrick and Prescott (1997) filter gives an estimate of the unobserved potential output as the solution to the following minimisation problem:

$$\min_{y_t^*} \sum (y_t - y_t^*)^2 + \lambda_1 (\Delta^2 y_t^*)^2 \quad (1)$$

where  $y$  is the observed output,  $y^*$  is the unobserved potential output,  $\lambda_1$  is the ratio between the variances of the cyclical and the trend components of the output. This problem is of course invariant to a homothetic transformation; therefore what matters is the ratio. Hodrick and Prescott (1997) suggest some parameterisation of  $\lambda_1$  depending on the frequency of data and used 1600 for the quarterly US output. Harvey and Jaeger (1993) find the Hodrick and Prescott parameterisation to be appropriate for the US real output. Razzak (1997) uses a time-varying smoothing parameter, depending on whether the shocks were supply side or demand side at a certain time.

The MV filter seeks to estimate the unobserved variable as the solution to the minimisation problem below.

$$\min_{y_t^*} \sum (y_t - y_t^*)^2 + \lambda_1 (\Delta^2 y_t^*)^2 + \lambda_2 \zeta_{1,t}^2 + \lambda_3 \zeta_{2,t}^2 \quad (2)$$

where the first two squared terms are the Hodrick-Prescott filter. The third and the fourth terms are additional residuals that go into the minimisation problem, on top of the HP filter. If  $\lambda_2$  and  $\lambda_3$  are zero, this is an HP filter. In the augmented version of the filter, the residuals  $\zeta_{i,t}$  come from other estimated economic relationships:

With  $\lambda_1$  to  $\lambda_3$  given, the residuals in the MV filter used by Reserve Bank of New Zealand uses the following two structural equations (see Conway and Hunt (1997)): The

Okun's law and a link between capacity utilisation survey data and the output:

$$\bar{u}_t = -\beta(y_t - y_t^*) + \zeta_{1,t} \quad (3)$$

$$\bar{cu}_t = \gamma(y_t - y_t^*) + \zeta_{2,t} \quad (4)$$

where  $y$  is the output and  $y^*$  the potential output,  $u$  is the unemployment rate and  $cu$  the capacity utilisation rate. The equilibrium values for unemployment rate and the capacity utilisation are  $u^*$  and  $cu^*$  respectively.

The terms  $\lambda_2$  and  $\lambda_3$  give the balance between the HP filter and the economic information embodied in the additional equation. A high value for corresponds to a better fit of the economic relationship, and an unobserved variable that can depart significantly from the observed variable.<sup>5</sup> As for the simple HP filter, the smoothing constants  $\lambda_1$  and  $\lambda_2$  and  $\lambda_3$  reflect the weights attached to different elements of the minimisation problem. The estimated unobserved variable is not only a simple moving average going through the observed series, but is also modelled to give a better fit to the economic relationship. The MV filter can also be reproduced by a Kalman filter, following a similar methodology. In essence the problem is simply to add the additional structural equation in the set of measurement equations:

So, if we write the MV filter in state space, we get the following three measurement equations:

$$y_t = y_t^* + e_t^y \quad (5)$$

$$\bar{u}_t = -\beta(y_t - y_t^*) + e_t^u \quad (6)$$

$$\bar{cu}_t = \gamma(y_t - y_t^*) + e_t^{cu} \quad (7)$$

where  $\bar{u}$  and  $\bar{cu}$  are the deviations of unemployment rate and capacity utilisation from their trend/equilibrium respectively.

There are two unobserved equations, potential output, growth rate of potential output

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<sup>5</sup>Kuttner (1994) for example uses the Phillips curve in estimating potential output.

$$y_t^* = y_{t-1}^* + g_{t-1} \quad (8)$$

$$g_t^* = g_{t-1}^* + e_t^{g^*} \quad (9)$$

If we put this in state space, the measurement equations become

$$\begin{pmatrix} 1 & 0 & 0 \\ \beta & 1 & 0 \\ -\gamma & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \bar{u}_t \\ \bar{cu}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \beta & 0 \\ -\gamma & 0 \end{pmatrix} \begin{pmatrix} y_t^* \\ g_t^* \end{pmatrix} + \begin{pmatrix} e_t^y \\ e_t^u \\ e_t^{cu} \end{pmatrix} \quad (10)$$

where  $y_t$  is the log of real output,  $u_t$  is the unemployment rate,  $cu_t$  is the capacity utilisation deviation from its sample average,  $y_t^*$  is the potential output,  $g_t$  is the growth rate of the potential output,  $\bar{u}_t$  is the equilibrium unemployment rate.

State equations can be collected in the following matrix.

$$\begin{pmatrix} y_t^* \\ g_t^* \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ g_{t-1}^* \end{pmatrix} + \begin{pmatrix} e_t^{y^*} \\ e_t^{g^*} \end{pmatrix} \\ \Rightarrow Y_t = Z_t Y_{t-1} + \vartheta_t \quad (11)$$

with  $(e_t^y \ e_t^u \ e_t^{cu} \ e_t^{y^*} \ e_t^{g^*})' \sim N(0, H)$

$$H = \begin{pmatrix} \sigma_{e^y}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{e^u}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{e^{cu}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{e^y}^2 / \lambda_1 \end{pmatrix} \quad (12)$$

The above approach has one very important assumption: The structural information used are in ‘gap’ forms. In other words, The trend and equilibrium values in unemployment rate and capacity utilisation are assumed to be known with certainty when calculating the deviations of these variables from their respective equilibrium/trend values. Therefore, inferring one unobserved variable (potential output) with the another unobserved variable, which assumed to be known with certainty would hide the true real time properties of the filter used. Hence, we allow these additional trends to be estimated within



the system. This is saying, that, the policy maker needs to estimate them jointly and will probably make mistakes in all these trends in real time. So, we change the model by allowing the trends to be estimated as unobserved state variables:

$$y_t = y_t^* + e_t^y \quad (13)$$

$$u_t = u_t^* + \beta(y_t - y_t^*) + e_t^u \quad (14)$$

$$cu_t = cu_t^* + \gamma(y_t - y_t^*) + e_t^{cu} \quad (15)$$

There are for unobserved equations, potential output, growth rate of potential output, the equilibrium unemployment rate and equilibrium capacity utilisation rate.

$$y_t^* = y_{t-1}^* + g_{t-1} \quad (16)$$

$$g_t^* = g_{t-1}^* + e_t^{g^*} \quad (17)$$

$$u_t^* = u_{t-1}^* + e_t^{u^*} \quad (18)$$

We can put these in state-space, in order to be able to implement the Kalman filter in the following fashion:

Measurement equations

$$\begin{pmatrix} 1 & 0 & 0 \\ \beta & 1 & 0 \\ -\gamma & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ u_t \\ cu_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \beta & 0 & 1 & 0 \\ -\gamma & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t^* \\ g_t^* \\ u_t^* \\ cu_t^* \end{pmatrix} + \begin{pmatrix} e_t^y \\ e_t^u \\ e_t^{cu} \end{pmatrix} \quad (19)$$

where  $y_t$  is the log of real output,  $u_t$  is the unemployment rate,  $cu_t$  is the capacity utilisation deviation from its sample average,  $y_t^*$  is the potential output,  $g_t$  is the growth rate of the potential output,  $u_t^*$  and  $cu_t^*$  are the trend unemployment and capacity utilisation rates respectively.

State equations can be collected in the following matrix.

$$\begin{pmatrix} y_t^* \\ g_t^* \\ u_t^* \\ cu_t^* \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ g_{t-1}^* \\ u_{t-1}^* \\ cu_{t-1}^* \end{pmatrix} + \begin{pmatrix} e_t^{y^*} \\ e_t^{g^*} \\ e_t^{u^*} \\ e_t^{cu^*} \end{pmatrix}$$

$$\Rightarrow Y_t = Z_t Y_{t-1} + \vartheta_t \quad (20)$$

with  $(e_t^y \ e_t^u \ e_t^{cu} \ e_t^{y^*} \ e_t^{g^*} \ e_t^{u^*} \ e_t^{cu^*})' \sim N(0, H)$

$$H = \begin{pmatrix} \sigma_{e^y}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{e^u}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{e^{cu}}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{e^y}^2/\lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{e^u}^2/\lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{e^{cu}}^2/\lambda_3 \end{pmatrix} \quad (21)$$

Once we allow all trends to be ‘unobservable’, and the parameters to be estimated, then we can be sure that the results would probably be sensitive to the following:

1. the parameters e.g. what sort of output gap a 1 per cent deviation of unemployment from equilibrium is thought to correspond to
2. the equilibrium values in these structural equations, i.e. equilibrium capacity utilisation rate and unemployment rate,
3. how the different bits of information are weighted in the minimisation, hyperparameters.

The classic argument in favour of the augmenting equations is that these variables are not revised. In New Zealand, the capacity utilisation and unemployment data are not revised. But although their level values are not revised by the statistician, the “gap terms” which enter into the minimisation are sensitive to how their trends are treated. It has been shown that the ex-post errors of a gap concept such as output gap, unemployment gap do not mostly come from the revisions to the data, they come from getting the trend wrong in real time. For example, if the equilibrium in capacity utilisation is not assumed to be known, then its deviation from its equilibrium would suffer from the real time problem. Hence, assuming some exogenous trend values in these variables may be problematic.

The Conway and Hunt (1997) state that their MV filter determines the trend unemployment rate by means of an HP filter. Although the unemployment rate is not revised by the statistician, the gap measured by means of an HP filter it still has very large revisions ex-post. Hence what goes into the minimisation problem in real time can be very different than what should really go, which is the ex-post value. <sup>6</sup>

Finally, the MV filter has a ‘stiffener’ assumption at the end of the sample. That is, there is an explicit assumption about the trend growth rate for the last few observations in the sample. This is to overcome the end point problem. A filter will struggle to tell us how potential output has evolved at the end of the sample, so we add an exogenous guess and put some weight on it. This mitigates the end point problem, but insofar as our guess is wrong it will add an additional error.

## 4 Results

We start with a baseline model, where our baseline model is a replication of the official MV filter of the Reserve Bank of New Zealand<sup>7</sup> The reason for doing so is that we want to have that as our benchmark, so we can compare the additional models to the benchmark. In this model, we imposed the MV filter equilibrium values and coefficients. The coefficients imposed are the Okun (1962)’s law and the capacity utilisation coefficients, which are -0.33 and 1 respectively. The first coefficient  $\beta$  is the so-called Okun’s coefficient, which maps the changes in output gap to the changes in unemployment from its equilibrium. The coefficient suggests that for every 1 per cent deviation of output from its potential, the unemployment deviates from its natural rate by 0.33 per cent, which is a relationship of 1 to 3. This coefficient is very similar to the original estimates of Okun (1962) and slightly different than the 1 to 2 relationship found by Scott (2000). The other coefficient,  $\gamma$ , is the relationship between the capacity utilisation and the output gap. The calibrated coefficient of 1 means that a 1 per cent output gap is associated with 1 per cent capacity

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<sup>6</sup>Recently the Reserve Bank of New Zealand started augmenting the unemployment equation with a survey measure of skill shortages to solve the endpoint problem in estimating the trend unemployment rate. The skill shortage data is a weighted average of skilled and unskilled labour shortage survey by the Quarterly Survey of Business Opinion. However, this doesn’t totally solve the problem, because the equilibrium value of the skill shortages survey has to be inferred. In words, this would be inferring one latent variable with another latent variable

<sup>7</sup>Throughout this analysis, we referred to the “filtered” or one sided estimates from the final vintages as the “real time”. Although this is ‘quasi-real time, it has been shown that the revisions to the data is a very small part of the total revisions.

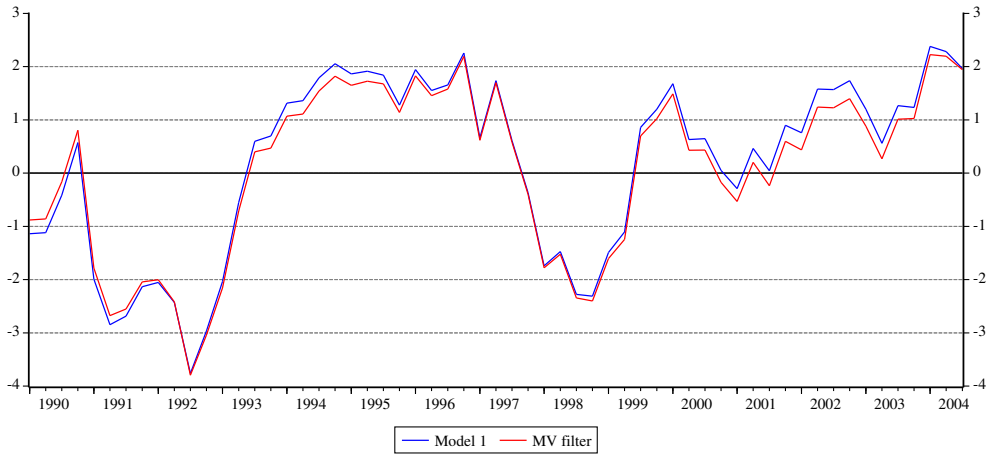


Figure 1: MV filter and the replicated MV filter with the Kalman filter

utilisation gap. For the trend unemployment and the equilibrium capacity utilisation, we use the values from the official MV filter.<sup>8</sup> So, all coefficients and equilibrium values we use in this benchmark model are same as the official MV filter. Table 1 shows below shows the descriptive statistics of the official MV filter and the one we replicated here, which we call MV-Kalman Filter (MV-KF).

**Table 1** MV and MV-KF

	Mean	Median	Std Dev	Min	Max
MV	0.0372	0.379	1.443	-3.793	2.266
MV-KF	0.0510	0.461	1.519	-3.760	2.378

The similarity in the movements and profiles of the two estimates, made us comfortable assuming that what we do is a good description of the MV filter in a different framework.

When we compare the revision properties of the MV-KF to the HP filter, to see if it is helping at the end of the sample. It is clear that the revision properties of the MV-KF filter are much better. Table 2 below shows this, although not by much.

<sup>8</sup>The equilibrium unemployment was measured by means of an HP filter until 2002, and since then augmented with skill shortage survey series. Capacity utilisation equilibrium is 88 per cent until 1993 and 89 per cent thereafter.

**Table 2** Revisions to MV-KF and HP

	MAE	Median	Std Dev	Min	Max
MV-KF	1.05	0.77	1.39	-3.82	2.20
HP	1.37	-0.31	1.00	-5.51	2.34

#### 4.1 Do parameters, hyperparameters matter?

Having been convinced that we have been able to replicate the official MV filter, and established that it improves over HP filter, we now go and examine whether the superiority of the MV filter over HP is sensitive to the assumptions in the MV filter. Table 3 below compares our version of the MV filter (MV-KF) with two other versions of the same model.

**Table 3** Estimates from Different Models

Parameter	MV filter	MV-KF1	MV-KF2
$\beta$	-0.33	-0.268	-0.31
	(na)	(0.163)	(0.038)
$\gamma$	1	0.75	0.56
	(na)	(0.08)	(0.12)
<i>Log-likelihood</i>	634.77	637.77	686.74

First column on Table 3 is the replicated MV filter. The parameters do not have standard errors in this column, as they are calibrated to the parameters used in official MV filter. Second column, MV-KF2 estimates the structural equations parameters, instead of calibrating them. One major change in this model is the change in the coefficient that determines how big a capacity utilisation gap we get for a 1 percent output gap. The estimated coefficient turns out to be 0.75, which implies a 1 per cent output gap is associated with a capacity utilisation deviation of 0.75 per cent from its trend. This is statistically significantly different from the 1:1 ratio calibrated into the MV filter. The estimated Okun's law coefficient is also different, -0.26. This indicated that a 1 per cent output gap implies unemployment to be 0.26 percent below equilibrium. In this model, the hyperparameters are calibrated to the official figures. In third column, MV-KF3, we still impose the trend values in the additional information to the filter but we estimate the

coefficients and hyperparameters. The hyperparameters are close to the official figures but the parameters in the structural equations are now very different. The coefficients in capacity utilisation equation and Okun's law are 0.56 and -0.31 respectively. Although the latter parameter is close to the calibrated one, the former is half the size of the calibrated one.

What do the resulting trend growth rate and the output gap estimates look like from these different models? The resulting output gap and the potential growth rates from these 3 different models and the MV<sup>9</sup> filter are compared in Table 4. The trend growth rates are slightly higher in the MV-KF models. In terms of the output gap, the MV filter falls in the middle compared with the others.

**Table 4** Trend growth rate and output gap estimates

	MV-KF	MV	MV-KF-2	MV-KF3
$g_t^*$	3.84	3.61	3.76	3.84
$gap$	1.96	1.94	2.14	1.75

It is obvious that small changes to the way the MV filter is treated can make some changes to the final output gap estimate. The resulting output gap can be 1.75 per cent or 2.14 per cent for example. Although this difference seems small, at times, they may diverge significantly.

## 4.2 Other trends do matter

Our analysis in the previous subsection has one important missing aspects: We assumed the trends are known with certainty. We used the equilibrium/trend values used by the MV filter in the models above. In fact, these are unobserved variables too, which the Kalman filter approach allows us to estimate. However, as soon as we start doing so, we face some problems about the treatment of the unobserved variables. For example, is the capacity utilisation equilibrium constant or time varying? Does it have a structural break in a constant trend as assumed by the MV filter? These kinds of differences in the treatment of the unobserved trend values can affect the final output gap estimates. The approach we take in this section is the following: We try all the possible modelling of the

<sup>9</sup>The MV filter values are taken from the December 2004 projection round of the Reserve Bank of New Zealand.

trend values, within some sensible boundaries. For example, we will allow the capacity utilisation trend to be time varying, but will not allow it to be extremely flexible.

The results of these models are summarised in Table 5. The first column shows the results from the first model, MV-KF4, estimates the capacity utilisation equilibrium as a constant with a mean shift by the end of the economic reforms in 1992. We estimate the mean shift by a dummy variable in the capacity utilisation equation. This is to test the rather ad-hoc level shift assumption in the MV filter. The equilibrium capacity utilisation for the pre 1992 period is estimated at 86 per cent, as oppose to 88 per cent in the MV filter (the last column). The dummy for the level shift in 1992 adds about 3.5 per cent to the initial capacity utilisation equilibrium of 86 per cent, making the current equilibrium level just below 90 per cent, against the 89 per cent of the MV. The coefficients  $\alpha$  and  $\beta$  are estimated 0.7 and -0.3 respectively (similar to MV-KF2 in the previous section).

The second column (MV-KF5) differs from the previous one only in the following way: Coefficients and weights are calibrated to the MV filter ones, to see if the constant capacity utilisation equilibrium would change. It turns out that, the result is not different from MV-KF4. So a freely estimated 1992-break gives us a constant (post-1992) equilibrium value quite similar to the 89 percent calibrated in the MV filter. The model in the third column (MV-KF6), allows the unemployment trend to be an unobserved time varying variable inside the model. This gives an equilibrium unemployment rate of 4.49 per cent. In this model coefficients are imposed from the previous model and the weights are estimated.

MV-KF7 is the most flexible of all the models we estimates here, where all trend/equilibrium values are estimated. It allows the capacity utilisation and unemployment trends to be unobserved. The model gives a capacity utilisation equilibrium of 88.8 per and a trend unemployment of 4.68 per cent for the current quarter. In this model, the equilibrium unemployment rate is a latent variable and is assumed to follow a random walk. As we noted in the previous section, currently MV filter uses the skill shortages to infer the trend unemployment rate. Here, we can introduce the trend unemployment as a new latent variable.

Table 5 also shows the output gap estimated by these new set of models. In addition, it also shows the estimated trend growth rate for from each of these models. As far as the potential growth is concerned, these models all show a growth rate of around 3.8 per

**Table 5** Estimates from the Models

	MV-KF 4	MV-KF-5	MV-KF-6	MV-KF 7	MV
$cu^*$	86.7	87	87.3	NA	88
$cu_{dummy}$	2.99	2.4	2.21	NA	1
$cu^* + dummy$	89.7	89.4	89.5	NA	89
$cu_t^*$	NA	NA	NA	88.8	NA
$u_t^*$	NA	NA	4.49	4.68	4.47
$g_t$	3.84	3.80	3.83	3.80	3.61
$gap$	1.68	1.62	1.66	2.24	1.96
$Log - likelihood$	617	704.36	624.70	600.37	NA

cent for the current quarter.

We find that output gap estimates from different models have a very similar pattern. However, the gap they would give for a point in time can differ substantially. This is a function of the co-movement of the three different information. Different models weight and interpret the various bits of conditioning information differently. If the bits of conditioning information ever totally disagreed, the difference between the various models would be considerably larger. The main reason for this is the way the trends/equilibrium values are estimates or treated. For example, for the capacity utilisation, most models give slightly higher trend values, just below 90 per cent, for the current period. On the other hand, the time varying trend (model 7) follows the MV filtered trend rather closely. For the trend unemployment rate, all filters, except the HP filter, yield to a trend value of just below 5 per cent. Models 6 and 7 have relatively higher estimated value compared with the MV filter's assumptions. These two figures indicate that, there is also uncertainty associated with the trend values of these additional information that the MV filter uses.

Even though the additional information is used to nail down the estimates at the end of the sample, the additional information brings additional problems: How the trend/equilibrium values in these additional information is treated does matter.

## 5 Revision Properties

Since the central banks use the output gap data in real time, the revision properties of the estimates that come out of this multivariate approach is very important. Orphanides and van Norden (forthcoming) and others have all shown that no output gap model can



escape from substantial revisions in ex-post. Hence, using the output gap in real time can lead to wrong policy conclusions.<sup>10</sup>

In the previous section we showed that the output gap estimates can differ, depending on how one uses the extra information. One can get different gap estimates at the end of the sample, depending on whether he/she estimates or calibrates the parameters, hyper-parameters or other trends. In this section, we will look at the revision properties of the different models we estimated above.

So far we have shown that the way the trends, coefficients are treated can lead to different output gap estimates ex-post. How about the real time performance of different models? How do they perform? The beginning of the previous chapter showed that the official MV filter had better revision properties than the HP filter. If the MV filter is really superior to an HP filter, in terms of better revisions, this should not only hold one calibrated version of the model and it should be insensitive to different ways of estimating the model

However, that was not the true comparison, as the MV filter was assuming that the trends in other variables were known. A better comparison is when the other trends are also estimated, as one does not know what they are. In this section, we will look at the revision properties of the models we presented above.

Previously, Twaddle (2002) argued that the revisions to the real time output gap estimates of the MV filter are not as bad as an HP filter, but still very high. However, as we argued above, the MV filter assumes no uncertainty in parameters or the equilibrium values of the conditioning variables. Once we allow uncertainty around these, the revisions can be substantially larger.

Table 6 shows the descriptive statistics of the revision to the models presented above<sup>11</sup>. We start with the HP filter and descriptions of the real time output gap, ex-post output gap and revisions to the real time output gap from the HP filter. We also report the AR(1), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE).

The first block in table 6, HP filter, shows that the revisions to the real time HP filter estimates can be revised substantially. The RMSE for the HP filter is 1.67 and MAE

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<sup>10</sup>See Orphanides (2001) and Nelson and Nikolov (2003) for an explanation of great inflation by the output gap mis-measurement

<sup>11</sup>Table 6 uses the statistics for the full sample. Of course, at the end of the sample, the real time and the ex-post figures are the same. Therefore, the figures on Table 6 understate the severity of the numbers. If we exclude the last 2 years' data, the figures get worse for the filters.

**Table 6** Revision Properties

	Revisions	Real-time	Ex-post
<b>HP</b>			
Mean	0.32	0.32	0.00
Max	4.51	4.07	2.67
Min	-2.34	-3.06	-4.30
Std Dev	1.65	1.55	1.45
Revisions	AR(1)	RMSE	MAE
	0.971	1.67	1.27
<b>Model 1</b>			
Mean	0.04	0.09	0.05
Max	3.82	4.31	2.37
Min	-2.20	-3.60	-3.76
Std Dev	1.38	2.19	1.51
Revisions	AR(1)	RMSE	MAE
	0.957	1.38	1.05
<b>Model 2</b>			
Mean	0.05	0.10	0.05
Max	3.84	4.40	2.52
Min	-2.14	-3.65	-4.07
Std Dev	1.45	2.28	1.61
Revisions	AR(1)	RMSE	MAE
	0.963	1.44	1.11
<b>Model 3</b>			
Mean	0.07	0.25	0.18
Max	2.50	3.17	3.21
Min	-1.90	-3.90	-4.44
Std Dev	1.01	1.90	1.65
Revisions	AR(1)	RMSE	MAE
	0.901	1.01	0.82
<b>Model 4</b>			
Mean	0.04	0.23	0.18
Max	2.68	2.99	3.55
Min	-2.23	-4.19	-4.39
Std Dev	1.05	1.87	1.65
Revisions	AR(1)	RMSE	MAE
	0.888	1.05	0.84
<b>Model 5</b>			
Mean	0.02	0.02	0.00
Max	4.52	4.45	2.61
Min	-2.28	-3.95	-4.10
Std Dev	1.66	2.23	1.51
Revisions	AR(1)	RMSE	MAE
	0.964	1.65	1.25
<b>Model 6</b>			
Mean	0.02	0.02	0.00
Max	4.53	4.45	2.63
Min	-2.27	-3.93	-4.13
Std Dev	1.67	2.22	1.51
Revisions	AR(1)	RMSE	MAE
	0.965	1.66	1.26
<b>Model 7</b>			
Mean	0.58	0.00	0.00
Max	5.33	5.93	2.96
Min	-2.27	-3.64	-4.77
Std Dev	1.86	2.71	1.92
Revisions	AR(1)	RMSE	MAE
	0.977	1.94	1.39

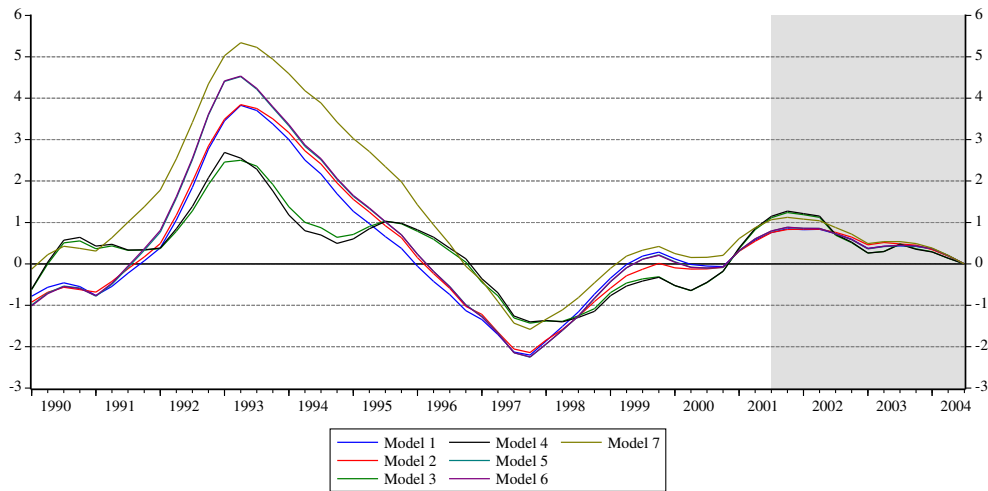


Figure 2: Revisions to different models

is 1.27. The revisions to the HP filter has an AR(1) coefficient of 0.971, which is very persistent.

The Models 1 to 4 seem to improve from an HP filter, in terms of smaller RMSE and MAEs. Furthermore the AR(1) coefficients in the revisions from these models are also smaller. However, as we argued throughout this paper, this comes from the assumptions in other trend variables in the MV filter. When we allow other trends to be unobservable (as in Model 5, 6 and 7), the superiority of the MV filter disappears. The revisions get larger and they become as persistent as the HP filter. The AR(1) coefficient in Models 5, 6 and 7 are 0.964, 0.965 and 0.977 respectively, where the last one being higher than the HP filter in fact.

The Figure 2 below shows the revisions to the different models estimates. It shows that the revisions get larger, as we relax the MV filter assumptions and estimate the coefficients and other trends. The revisions from the Model 7 for example is as bad as that of the HP filter.

## 6 Conclusions

The output gap concept plays a crucial role in thinking and behaviour of many inflation targeting central banks. Yet, output gap is difficult to measure, due to the unobserved

nature of potential output. The real time estimates of the output gap can be subject to large revisions, due to end-point problems associated with filtering techniques. Some central banks, including the Bank of Canada, Reserve Bank of New Zealand use what is called “multivariate” filters to mitigate this problem. In this paper we tested whether the multivariate filters (or at least one class of them) achieves this objective. We found that the models where the trend and coefficient values are imposed exogenously, the revision properties are better, compared with an HP filter. However, as one treats these unobserved trends as additional latent variables in the system, the real time problem gets worse. The latter is a much more realistic claim, as one can mis-measure every single trend/equilibrium value involved. For example, in the multivariate filter, one can make real time errors on capacity utilisation equilibrium, NAIRU and potential output. As a result, the revisions would become worse. We also found that estimates of the gap can heavily depend on how these trends and coefficients are treated. This makes us conclude that the Orphanides criticism of the output gap estimates is a very valid one, despite the central banks’ attempt to mitigate the problem. This illustrates the importance of working with non-gap based approach.

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**Appendix 1: The Kalman filter and the smoothed estimates** Many dynamic models can be written and estimated in state-space form. The Kalman filter is the algorithm that generates the minimum mean square error forecasts for a given model in state space. If the errors are assumed to be Gaussian, the filter can then compute the log-likelihood function of the model. This enables the parameters to be estimated by using maximum likelihood methods.

For simplicity let us consider a measurement equation that has no fixed coefficients:

$$Y_t = \Gamma_{t-1} + \varepsilon_t \quad (22)$$

where  $Y_t$  is a vector of measured variables,  $\Gamma_t$  is the state vector of unobserved variables,  $X_t$  is a matrix of parameters and  $\varepsilon_t \sim N(0, H)$ . The state equation is given as:

$$\Gamma_t = \Gamma_{t-1} + \eta_t \quad (23)$$

where  $\eta_t \sim N(0, Q)$ .<sup>12</sup>

Let  $\gamma_t$  be the optimal estimator of  $\Gamma_t$  based on the observations up to and including  $Y_t$ ,  $\gamma_{t|t-1}$  the estimator based on the information available in  $t-1$  and  $\gamma_{t|T}$ , and  $\gamma_{t|T}$  the estimator based on the whole sample. We define the covariance matrix  $P$  of the state variable as follows:

$$P_{t-1} = E((\Gamma_{t-1} - \gamma_{t-1})(\Gamma_{t-1} - \gamma_{t-1})') \quad (24)$$

The predicted estimate of the state variable in period  $t$  is defined as the optimal estimator based on information up to the period  $t-1$ , which is given by:

$$\gamma_{t|t-1} = \gamma_{t-1} \quad (25)$$

while the covariance matrix of the estimator is:

$$P_{t-1} = E((\Gamma_t - \gamma_{t|t-1})(\Gamma_t - \gamma_{t|t-1})') = P_{t-1} + Q \quad (26)$$

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<sup>12</sup> $Q$  and  $H$  are referred as the hyperparameters of the model, to distinguish them from the other parameters.

The filtered estimate of the state variable in period  $t$  is defined as the optimal estimator based on information up to period  $t$  and is derived from the updating formula of the Kalman filter:<sup>13</sup>

$$\gamma_t = \gamma_{t|t-1} + P_{t|t-1} X_t' (X_t P_{t|t-1} X_t' + H)^{-1} (Y_t - X_t a_{t|t-1}) \quad (27)$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} X_t' (X_t P_{t|t-1} X_t' + H)^{-1} X_t P_{t|t-1} \quad (28)$$

The smoothed estimate of the state variable in period  $t$  is defined as the optimal estimator based on the whole set of information, i.e. on information up to period  $T$  (the last point of the sample). It is computed backwards from the last value of the earlier estimate  $\gamma_{T|T} = \gamma_T$ ,  $P_{T|T} = P_T$  with the following updating relations:

$$\gamma_{t|T} = \gamma_t + P_t^* (\gamma_{t+1|T} + \gamma_t) \quad (29)$$

$$P_{t|T} = P_t + P_t^* (P_{t+1|T} + P_{t+1|t}) P_t^{*'} \quad (30)$$

where  $P_t^* = P_t P_{t+1|t}^{-1}$ .

Depending on the problem studied one can be interested in any one of those three estimates. In our particular case, looking at smoothed values is more appropriate, as the point is not to use the Kalman filter to produce forecasts but to give the most accurate information about the path followed by the time-varying coefficients. Therefore it is more informative to use the full data set to derive each value of the state variables.

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<sup>13</sup>This estimator minimises the mean squared errors when the expectation is taken over all the variables in the information set rather than being conditional on a particular set of values. (See Harvey (1989) for a detailed discussion). Thus the conditional mean estimator,  $\gamma_t$ , is the minimum mean squared estimator of  $\Gamma_t$ . This estimator is unconditionally unbiased and the unconditional covariance matrix of the estimator is the  $P_t$  matrix given by the Kalman filter.