Inspecting the noisy mechanism: the stochastic growth model with partial information

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Abstract

We derive a framework which allows the modelling of partial information in dynamic general equilibrium. We apply this framework to a canonical real business cycle model and show that which variables are in households' information sets has a significant effect, both qualitatively and quantitatively, on the dynamics of the model.

1 Introduction

Underlying most dynamic general equilibrium (DGE) modelling is the assumption that agents can perfectly observe the state variables. In this paper we investigate the consequences of relaxing this assumption. We present a simple derivation, implemented as a software "toolkit" which augments the standard solution of a DGE model and allows the modeler to specify which variables are within households' information sets, and the degree to which they are measured with error.

In the absence of perfect measures of the state variables, we assume that agents use the Kalman filter to estimate the states from observable data. This requires us to extend the standard Kalman filter to cases where both the potentially unobservable state variables (e.g. capital) and observed variables (e.g. output) may depend on the dynamic choice variables (typically consumption). Here we build on work by Pearlman (1986, 1992) and Svennson and Woodford (2002, 2003, 2004) but our use of the method of undetermined coefficients makes our derivation more transparent.

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The "toolkit" we provide which implements this derivation can be applied to a wide range of DGE models. However we investigate the implications of partial information by taking the stochastic growth model as presented in Campbell (1995). We following Campbell's approach of using a simple model to carefully clarifying the mechanisms underlying the model's dynamics. In Campbell's model the two state variables are the capital stock and technology. Our basic assumption is that both of these are unobservable.

This is not a strong assumption. If the representative consumer really did know the capital stock, it seems surprising that there is such extensive debate on how it should be measured: capital stock estimates inferred from financial data (Hall, (2000), Laitner and Stolyarov, (2004)) differing from published data by anything up to a factor of three. There is an even more intense debate on the nature and size of technology shocks (see, for example, Gali (1999)).

We compare Campbell's model under full information (with technology and capital perfectly observable) with various degrees of partial information. Our main results are:

- Information on returns alone can replicate the full information case, but the resulting solution displays knife-edge stability
- In general, with arbitrarily small measurement error in returns, there is a stable limited information solution which differs in significant ways from the full information solution e.g. consumption optimally *falls* in response to a positive innovation to technology
- In contrast, data on output (or wages) brings the system arbitrarily close to the full information solution as measurement error falls to zero, so excessive noise in returns does not affect the economy's dynamics as long as output data is available
- There is no upper bound on the variance contribution of noise. However poorly measured the observed variables are, the explosive root in the capital stock means that it is always optimal for consumption to respond to the best estimate of capital.

When we calibrate measurement error to investigate if these effects are quantitatively significant and find that

- measurement error can account for around 10% of volatility in consumption.
- if capital is measured using a permanent inventory approach, a reasonable degree of measurement error in output can give sufficiently high variance in measured capital to approach the non-observability we assume

In Campbell's model there is a single exogenous process for technology. However some recent DGE models often contain multiple shocks, Altig et al (2003) containing no less than eight. We extend Campbell's model to include more shocks, and investigate the implications of agents being unable to observe them directly. We find that while the Kalman filter is very efficient at untangling the various shocks, measurement error can reduce this efficiency considerably and the combination can increase the volatility of consumption by up to 20% over the full information case, and significantly change the form of the impulse response functions.

The existing literature has two strands. The largest relates to the problem of setting monetary policy under imperfect information. Most such models (Svennson and Woodford (2002, 2004), Aoki (2003, 2006) look at the problem of asymmetric information when the monetary policymaker has partial information by the private sector is perfectly informed. Pearlman (1992) and Svennson and Woodford (2003) look at the case of symmetric partial information, where the private sector and the policymaker share the same information set. Our work shares with these studies the concept of "certainty equivalence": optimal consumption responds in the same way to the best estimates of the state variables as it would to the true state variables were there full information; and the autoregressive representation of the estimated states is identical to that of the true states under full information.

The second strand in the literature in which we place this paper investigates the implications of limited information not for a policy maker but for households. Kydland and Prescott (1982) consider the case where technology has two components, but only the aggregate is observable, and this is taken up by Bomfim (2003) who investigates the quantitative implications of this "permanent / transitory confusion". Both these models are constructed so that as soon as consumption decisions have taken place, the production process reveals the true value of the capital stock. Although both of the papers use the Kalman filter, since they are making inferences only about exogenous processes, they avoid the dependence of the Kalman filter on endogenous variables. Keen (2004) investigates the consequences of the private sector having partial information on the behaviour of the monetary policymaker and finds that this assumption can account for several business cycle features better than the standard model.

2 Solving dynamic general equilibrium models with partial information

If agents cannot observe the state variables without error, we assume that they apply the Kalman filter to the information available to them in order to derive efficient estimates of the states. Since in a DGE model the states in general depend on a dynamic choice variable, we need to modify the standard derivation.

2.1 A general system representation

Dynamic general equilibrium models can be written in the form We therefore write the system in a general form very close to that assumed in standard derivations of the Kalman Filter:

$$\xi_{t+1} = F_\xi \xi_t + F_c c_t + v_{t+1} \quad (1)$$
$$y_t = H'_\xi \xi_t + H_c c_t + w_t \quad (2)$$
$$E_t \Delta c_{t+1} = E_t D_y y_{t+1} + D_c y_t \quad (3)$$

The first block of equations describes the evolution of the r state variables ξ, which may or may not be observable; the second block describes the measurement process for the n measured variables, y. The third block describes the optimality conditions for c, the dynamic choice variables[1] .The structural innovations, v are assumed iid, with covariance matrix $Q = E\left[v_t v_t'\right]$[2].

For generality we can in principle allow the measurement errors w to be serially dependent by representing them as a vector autoregression of the form

$$w_{t+1} = \rho w_t + \omega_{t+1} \quad (4)$$

where ω is iid with covariance matrix $R = E\left[\omega_t \omega_t'\right]$. The two innovations ω_t and v_t may in principle be contemporaneously correlated but are assumed uncorrelated at all other leads and lags.

We write the system in this way to follow the standard derivation of the Kalman filter in Hamilton (1994) who writes the system as

$$\xi_{t+1} = F_\xi \xi_t + v_{t+1} \quad (5)$$
$$y_t = H'_\xi \xi_t + w_t \quad (6)$$

Comparing our system (1) to (3) with the standard Kalman filter problem shows the key difference between it and our model is that both equations depend on the dynamic choice variable, consumption.

The behaviour of consumption is crucial for the stability of the states. In standard Kalman Filter problems the states are fully exogenous (F_c is set to zero), and F_ξ therefore represents the state autoregressive matrix, which is usually assumed to have stable, or at worst borderline stable unit eigenvalues. In DGE models, due to the dynamics of the capital stock, F_ξ

[1] Although throughout we assume there is a single dynamic choice varialble, in principle c could represent a vector of such variables.

[2] The standard form of a DGE model (eg in McCallum (1998)) allows more complex dependencies such as the states depending on the true value of the measured variables $y_t - w_t$. It is straightforward to accomodate such dependencies in our framework.

will have a single explosive eigenvalue. The system can therefore only be stabilized by the behaviour of consumption. Under full information this stabilization follows directly from the standard rational expectations solution; with partial information things are less straightforward.

2.2 Full information solution

The full information case is a special case of the system in (1) to (3) with $H_\xi = I_r, H_c = 0, w_t = 0 \forall t$ implying $y_t = \xi_t$. We follow Campbell in deriving the full information solution by the method of undetermined coefficients. We conjecture a solution for optimal consumption

$$c_t = \eta' \xi_t \tag{7}$$

then substitute this into the state evolution equation (1) and the Euler equation (3) to obtain

$$\left(\eta' - D_\xi\right)\left(F_\xi + F_c \eta'\right) \xi_t = D_c \eta' \xi_t \tag{8}$$

which is a set of nonlinear equations for the elements of η that can be solved either analytically or numerically.

The solution to (8) will typically be non-unique, but stability of the rational expectations solution requires that, conditional upon the optimal solution of η, the reduced form autoregressive matrix of the states

$$G = F_\xi + F_c \eta' \tag{9}$$

has eigenvalues less than or equal to unity (there is a single unit eigenvalue if technology contains a random walk component).

2.3 Indirect observability

The full information solution can be replicated straightforwardly in the case that H_ξ, in the measurement equation, is a full rank $r \times r$ matrix, and $w_t = 0 \forall t$. In this case the state variables can be replaced in the state equation by setting $\xi_t = H_\xi^{-1} y_t - H_\xi^{-1} H_c c_t$ and can therefore be treated as known. The system can therefore be solved as in the previous section, with an implied static consumption function in terms of observables.

2.4 Partial information solution

For the general case we need to apply the Kalman Filter, allowing for the endogeneity of consumption to the filtering process.

Following Pearlman (1986) and Svennson and Woodford (2004) we conjecture is that under partial information optimal consumption will be certainty-equivalent:

$$c_t = \eta' \widehat{\xi}_t \tag{10}$$

where $\widehat{\xi}_t = E_t \xi_t$ is the optimal estimate of the current state vector given available information,[3] and η is identical to the coefficient vector derived for the full information case in (8).

We show in the appendix that conditional upon this consumption function, there is a convergent Kalman Filter recursion that ultimately results in the following autoregressive process for the estimated states:[4]

$$\widehat{\xi}_{t+1} = G\widehat{\xi}_t + \beta\varepsilon_{t+1} \tag{11}$$

where G is as defined in (9), and ε_t is the innovation to the measured variables, given by

$$\varepsilon_{t+1} \equiv y_{t+1} - E_t y_{t+1} \tag{12}$$

The matrix β is the solution to the following set of iterative matrix equations. Just as in the derivation of the standard filter, this iteration depends only on population parameters and is not a function of the data.

$$\beta_t = P_t J_t \left[J_t' P_t J_t\right]^{-1} \tag{13}$$

$$J_t' = \left[I_n - H_c \eta' \beta_t\right]^{-1} H_\xi' \tag{14}$$

$$P_{t+1} = F_\xi M_t F_\xi' + Q \tag{15}$$

$$M_t = \left[I_r - \beta_t J_t'\right] P_t \tag{16}$$

where

- β is the "Kalman Gain" matrix that extracts updated information on the unobservable states from observable innovations.[5]
- J_t captures the reduced- form dependence of observable innovations on the (unobservable) one-step ahead state forecast errors
- $P_{t+1} = E\left[\left(\xi_{t+1} - E_t\xi_{t+1}\right)\left(\xi_{t+1} - E_t\xi_{t+1}\right)'\right]$ is the covariance matrix

[3] Whenever we write the expectations operator E_t we mean expectations taken at time t given information available at time t. Where we write period $t's$ estimate of the states at t we write $\widehat{\xi}_t$, the standard Kalman filter literature uses $\widehat{\xi}_{t|t}$. For the forecast at time t of the states at period $t+1$ we write $E_t\widehat{\xi}_{t+1}$ instead of the standard $\widehat{\xi}_{t+1|t}$

[4] Note that we remove the measurement errors, w_t from (2) by incorporating them into the states (with an appropriate redefinition of the matrices in both (2) and (1) with the result that r, the dimension of the redefined states is given by the number of structural states plus n. This alone ensures that r is always strictly greater than n (thus ruling out indirect observability, as derived in the previous section, which requires $r = n$). This both simplifies the derivation, and easily accommodates both serial dependence of measurement errors, as in (4) and in principle some correlation with the structural innovations.

[5] The alternative, more common definition, is the Kalman gain for updating one-step ahead *forecasts* of the states, given by $K = G\beta$.

of the one-step ahead state forecast errors

- $M_t = E\left[\left(\xi_t - \widehat{\xi}_t\right)\left(\xi_t - \widehat{\xi}_t\right)'\right]$ is the covariance matrix of *current* "state" measurement errors (i.e., captures the uncertainty in current state estimates).[6]

The process for the estimated states (11) can be used to give an ARDL representation of the consumption function (10) in terms of the observables

$$c_t = A(L)\, y_t$$

where A is some polynomial in the lag operator L.

2.5 Outline derivation

The appendix gives a full description of the derivation, but we sketch the main features here. The general approach is standard, but there are several distinctive features of the solution that arise from the endogeneity of both states and measured variables to consumption.

Conditional upon initial estimates of the states in some period $t-1$,[7] a forecast is made of measured variables in period t, using (2),

$$E_{t-1}y_t = H_\xi' E_{t-1}\xi_t + H_c\eta' E_{t-1}\widehat{\xi}_t \tag{17}$$

where the second term captures the dependence of measured variables on consumption, after substituting from the consumption function, (10). After making the same substitution the true states in turn depend both on their own lagged value and on lagged estimates (using (1),

$$\xi_t = F_\xi \xi_{t-1} + F_c\eta'\widehat{\xi}_{t-1} + v_t \tag{18}$$

It follows, by applying the law of iterated expectations that the predicted values of measured variables in period t depend only on state estimates in period $t-1$:

$$E_{t-1}y_t = \left(H_\xi' + H_c\eta'\right) G\widehat{\xi}_{t-1} \tag{19}$$

where G is as defined in (9).

In period t the Kalman Filter updates the state estimates in response to the one-step ahead prediction error in y_t, given knowledge of the underlying structural relations. At this stage in the recursion the first complication enters compared to the standard Kalman Filter derivation. The prediction

[6] P_{t+1} is often denoted $P_{t+1|t}$, and using the same terminology, $M_t = P_{t|t}$

[7] In the appendix we discuss the issue of initial conditions, but for most cases we find a unique stable steady state so initial conditions are not a major concern.

error will be given by

$$\varepsilon_t \equiv y_t - E_{t-1}y_t = H'_\xi \left[\xi_{t-1} - E_{t-1}\xi_t\right] + H_c\eta' \left(\widehat{\xi}_t - E_{t-1}\widehat{\xi}_t\right) \tag{20}$$

The first element captures the (unobservable) impact of deviations of the true states from their predicted values. The second element (which is absent in the standard derivation) captures the response of consumption in period t to the observable innovation to the *estimated* states . But this in turn will depend on the innovation in y_t.[8] After allowing for this simultaneity,

$$\varepsilon_t \equiv y_t - E_{t-1}y_t = [I_n - H_c\eta'\beta_t]^{-1} H'_\xi \left[\xi_t - E_{t-1}\xi_t\right] = J'_t \left[\xi_t - E_{t-1}\xi_t\right] \tag{21}$$

giving the definition of J_t in (14). The left-hand side is a set of $n \times 1$ innovations which are a linear combination of $r \times 1$ (with $r > n$) unobservable state prediction errors. Thus the matrix J' and P_t, the current estimate of the covariance matrix of one-step ahead prediction errors for the states embody all that is known about the stochastic properties of the observable innovations. The Kalman gain matrix β_t can then be derived from (13) conditional upon a given estimate of P_t[9] and hence updated state estimates can be derived, as in (11).

The final stage of the recursion derives an updated estimate of the one step ahead prediction covariance matrix P_{t+1} given by the recursive formula in (15). The first term in this formula captures the impact of current state measurement error persisting into the next period, the second the impact of new structural shocks (thus even under perfect information P_{t+1} is non-zero).

Another distinctive feature of the endogenous Kalman filter is that, while forecasts of state variables allow for the impact of the consumption function in determining the autoregressive matrix of the estimated states this does *not* impact on one-step ahead uncertainty (since the marginal impact of today's consumption on tomorrow's states is known today even if current states are unknown). As a result the formula only allows for the direct impact of today's states on tomorrow's states, via the matrix F_ξ. As noted above, in the stochastic growth model, with dynamic efficiency, this matrix has a single explosive eigenvalue. This feature might be expected to lead to non-convergence of the Kalman recursion[10] but we show in the appendix that under the same very general conditions that admit a unique rational expectations solution under full information, the recursion has a unique

[8] Via an equation of the form of (11), but with β replaced by β_t

[9] In principle this requires the solution of two simultaneous matrix equations, (13) and (14) although in practice, in iterating towards steady-state values, the steady state can be achieved by defining β_t in terms of J_{t-1} or vice versa.

[10] Hamilton's (1994) proof of convergence relies on assumptions that would require F_ξ to have stable eigenvalues (ie, for the states to be stationary even if $\eta = 0$ or $H_c = 0$).

solution, and β_t, J_t, P_t and M_t converge to unique matrices β, J, P and M.

2.6 Discussion

If the Kalman Filter succeeds in replicating full information, M, the covariance matrix of state measurement errors estimates, defined in (16) is a matrix of zeros[11]. In this case P, the matrix of one-step ahead state forecast errors, is simply given by Q, the covariance matrix of the structural innovations. There may also be cases (we examine one in particular below) where $P = Q$, but P is *not* the stable steady state of the recursion for P_{t+1}. In such cases the replication of full information is a knife-edge result, that collapses with infinitesimally small measurement errors in (2).

Forecasts of the estimated states, given by (11) have an identical autoregressive representation to forecasts of the *true* states under full information: i.e., under both partial and complete information

$$E_t\xi_{t+1} = E_t\widehat{\xi}_{t+1} = G\widehat{\xi}_t$$

where, trivially, under full information, $\widehat{\xi}_t = \xi_t$. Since forecasts of neither estimated nor actual states depend on β, the Kalman Gain matrix, incomplete observability has no impact on optimal forecasts, which depend only on structural parameters. As a result the coefficient vector η in the conjectured consumption function under incomplete observability, (10) can be derived under the assumption of full information. Thus optimal consumption is certainty-equivalent, verifying our conjecture.

Despite this property of certainty equivalence, new dynamics arise for two reasons. First the covariance structure of identified innovations to the estimated states may differ radically from that of the true underlying shocks; and second, the identified innovations themselves, while white noise conditional upon the limited information set, are in reality complex lag polynomial functions of the underlying shocks.

While the property of certainty-equivalence implies that the estimated states follow an identical autoregressive process to that of the true states under full information, this does *not* imply that the covariance properties of the innovations to estimated states are identical to those of the structural innovations. Thus certainty equivalence does not imply that observable impulse responses will reveal true impulse responses (. We shall present some examples where the resulting differences are very significant.

The efficiency of the Kalman Filter ensures that ε_t, the observable innovations to measured variables, are white noise conditional upon t - dated information. However in reality they are not white noise, but a complex lag polynomial function of the true structural innovations (in which we now

[11] Which is, it should be stressed, quite distinct from the R, the covariance matrix of measurement errors in (2).

include any measurement error). Using (11), (12), (1) and (10) there is a minimal structural autoregressive representation of the estimated and actual states, given by

$$\begin{bmatrix} I-\beta H_c\eta' & -\beta H_\xi' \\ 0 & I \end{bmatrix}\begin{bmatrix} \widehat{\xi}_{t+1} \\ \xi_{t+1} \end{bmatrix} = \begin{bmatrix} [I-\beta J']\,G & 0 \\ F_c\eta' & F_\xi \end{bmatrix}\begin{bmatrix} \widehat{\xi}_t \\ \xi_t \end{bmatrix} + \begin{bmatrix} 0 \\ v_{t+1} \end{bmatrix} \tag{22}$$

so that that in underlying terms dynamic responses under incomplete observability are more complex than under full information, and all impulse responses are driven by the structural innovations[12]. Note also that the dynamics of the true states are also more complex than under full information, since they are affected by the lagged responses of consumption to estimated states.

Equation (13) shows that for an estimate of β to exist, the $n \times n$ matrix $J'PJ$ must be invertible. If this is not the case, there is some redundancy in the measurement equation. Given the definition of J_t in (14) the recursion for P_{t+1} in (15) (in which "new" error variance enters via Q, the covariance matrix of the structural shocks) and given that $Q = F_u S F_u'$ is of reduced rank, we show in the appendix that the invertibility condition requires the matrix $H_\xi' F_u$ to be of full row rank. Thus simply adding new measured variables may provide no new information if the additional variables do not respond differentially to the structural shocks.

On the other hand, if the row rank condition on the matrix $H_\xi' F_u$ is satisfied, there are s structural innovations (including measurement errors) and the number of measured variables, $n = s$, it may be possible for the Kalman Filter to replicate full information, even when there are fewer measured variables than states. In contrast with the simpler case of indirect observability, however, this result is conditional upon the Kalman Filter recursion having converged (hence strictly speaking requires and infinite history of past observations), and may not always be robust to arbitrarily small measurement error. The resulting consumption function in terms of observables is not static, as in the $n = r$ case, but is a finite order autoregressive distributed lag function $c_t = \varphi(L)y_t$.

While the explosive eigenvalue in the state dynamics does not prevent the Kalman Filter from converging, in one key respect it does change the nature of the resulting estimates. In the standard case, when the states are independently stationary (i.e., if F_ξ has stable eigenvalues) then the lower is the quality of the "signal" provided by the measured variables, the closer β

[12] The structural innovations are in time series terms "non-fundamental" innovations, ie we can write

$$\varepsilon_t = A(L)v_t$$

but some of the roots of $A(L)$ are within the unit circle, thus v_t can only be recovered from the future, rather than the history of ε_t.If these roots are replaced with their reciprocals ε_t is white noise.

gets to zero. In the limit if all diagonal elements of R go to infinity, β goes to zero and the optimal estimates of the state variables in this standard case are simply their unconditional means. However, this result does *not* hold when F_ξ has explosive eigenvalues - instead β tends to a finite non-zero limiting value. In the specific context of the stochastic growth model, this means that, however poor the information set, optimal consumption must always respond to it. The intuition for this result is that without such a response the capital stock will explode or contract without limit.

2.7 The nature of the impulse response functions

In what follows we examine the impulse response functions to structural shocks. Such impulse responses would not be identifiable in real time, but *would* become identifiable after a sufficient amount of time had elapsed, since the Kalman Filter implies "backward-smoothed" estimates of state variables that exploit future information (effectively, these systematically exploit the benefits of hindsight).

A structural innovation (which might could be measurement error) leads to innovations in the observed variables. The matrix β translates these innovations into changes in the estimates of the states, then the matrix $H'_\xi + H_c\eta$ from the measurement equation (2) updates the estimates of the measured variables conditional on optimal behaviour. The efficiency of the Kalman filter ensures these are equal to the observed values, giving an "adding up constraint"

$$\left(H'_\xi + H_c\eta\right)\beta\varepsilon_0 = \varepsilon_0 \tag{23}$$

Take the case where there is a unit innovation to a single observed variable, and two state variables, then $\left(H'_\xi + H_c\eta\right)$ is 1x2 and β is 2x1 and we can write, with

$$\begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = 1 \tag{24}$$

where the terms on the left-hand side are the effect on the observed variable of the change in the forecast of each state.

3 The stochastic growth model

Throughout the paper we use the canonical real business cycle model to illustrate the implications of . In this section we present an outline of this model, for full details see Campbell (1995). There is a representative

household which consumes, owns capital and supplies labour, and maximizes its expected utility

$$E_t \sum \beta^i \left[\log C_{t+i} + \theta \frac{(1 - N_{t+i})^{1-\gamma_n}}{1 - \gamma_n} \right] \tag{25}$$

subject to a budget constraint

$$W_t N_t + R_t K_t = C_t + I_t \tag{26}$$

where C is consumption, I investment, K capital, W the wage, R the return to capital, and a law of motion for capital

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{27}$$

Firms minimize costs

$$W_t N_t + R_t K_t \tag{28}$$

subject to a production function

$$Y_t = A_t^{\alpha} K_t^{1-\alpha} N_t^{\alpha} \tag{29}$$

where A is total factor productivity, the law of motion for capital where I is investment, and a resource constraint

$$Y_t = C_t + I_t \tag{30}$$

Campbell (1995) shows that when this system is detrended and linearized it can be written as the four equations

$$\begin{aligned} k_{t+1} &= \lambda_1 k_t + \lambda_2 (a_t + n_t) + (1 - \lambda_1 - \lambda_2) c_t && (31) \\ r_t &= \lambda_3 (a_t + n_t - k_t) && (32) \\ E_t \Delta c_{t+1} &= \lambda_3 E_t r_{t+1} && (33) \\ n_t &= \nu [(1 - \alpha) k_t + \alpha a_t - c_t] && (34) \end{aligned}$$

and a process for log technology

$$a_t = \phi a_{t-1} + u_t \tag{35}$$

where $u \sim N\left(0, \sigma_u^2\right)$ The parameters $\lambda_1, \lambda_2, \lambda_3$ and ν are functions of the underlying parameters $\theta, \gamma_n, \alpha, \beta$ and ϕ: definitions can be found in Campbell (1995).

In the context of the general representation, the state variables are the capital stock, k and the technology process a_t, and the first block of equations (1) consists of the capital evolution equation (31) and the process for technology (35). The measured variables y can be any of the variables in the system, including the states. The third block (3) consists only of the

Euler equation (33).

3.1 Partial information

Underlying our assumption of a representative agent are a large number of households that trade securities to insure against any idiosyncratic shocks. This trade means their consumption paths are perfectly correlated and we can think of a representative consumer. We assume all the households have the same information set. While households directly observe their own variables they do not have full knowledge of the other households in the economy so are, in general, unable to compute aggregate quantities and no securities are traded which reveal this information. Thus households only have knowledge of aggregate quantities only to the extent that we explicitly put these variables in their information set.

4 A single technology shock

In this section we take the stochastic growth model with fixed labour supply ($\nu = 0$ in (34)) and assume that the two state variables, the capital stock k and technology a are unobservable. Although our discussion in this section is mainly qualitative, for the impulse response functions we use the same calibration as in Campbell (1995)[13].

4.1 Information on returns only

4.1.1 No measurement error

If only returns are observable the measurement block (2) contains the single equation (32). For the moment we shall assume that returns are measured without error. We show in the appendix that, with only returns observable, one solution to the equations for the Kalman filter (2) to (3) replicates the full information case with $P = Q$ and $M = 0$. However this solution is knife-edge stable in two senses. Firstly, the resulting consumption function only gives a stable solution if the coefficients are exactly correct. In numerical terms, although the impulse response functions appear to be the same as in the full information case, if we run the model out for many periods (a few thousand is usually sufficient) consumption either explodes or collapses.

To understand this behaviour it is useful to consider the consumption function directly. From (10) this is given in terms of the estimated states as

$$c_t = \eta_k \hat{k}_t + \eta_a \hat{a}_t \tag{36}$$

We show in the appendix that there is an equivalent ARDL representation in terms of the observables

[13] $\delta = 0.025; \alpha = 0.667; \beta = 0.99; \phi = 0.9; g = 0.005$

$$c_t = \mu_1 r_t + \mu_2 r_{t-1} + \mu_3 c_{t-1} \tag{37}$$

with, crucially $\mu_1 > 0$ (in the full information case a positive innovation to technology increases consumption, and from (32) it must increase returns). The instability in the numerical solution arises as follows. Assume that, for whatever reason, η_{cr} is slightly above its true value. The household will consume slightly more than is optimal. This will mean that in the next period the capital stock is less than had the household behaved optimally, and the returns higher. So again the household will consume more than is optimal, further depressing the capital stock and so on. Because of the explosive eigenvalue, even the error in the calculation of the $\mu's$ resulting from machine precision lead to such instability.

The second form of knife-edge stability arises if we introduce arbitrarily small measurement error. Now, as long as the variance of the innovation to measurement error is non-zero, there is a finite probability that an observed innovation in the interest rate is due to measurement error. Given the full-information consumption function, the optimal response to an observed increase in the interest rate will be to increase consumption. However, if the observed innovation is in fact due to measurement error (remember, on impact the source of the true innovation cannot be identified) this will depress the capital stock and further raise the interest rate, resulting in higher consumption in the next period and a still higher interest rate. So the full-information solution will be unstable even if we allow the measurement error to become arbitrarily small[14].

4.1.2 Arbitrarily small measurement error

With arbitrarily small measurement error, the Kalman filter converges to a solution which has the characteristic that capital is not perfectly known and the forecast variance of technology is greater than the variance of the innovation to technology. A positive innovation to returns, in a partial information world, could be due to two things

1. A positive innovation in technology in this period
2. A negative innovation to technology in the previous period (we explain this below)
3. The previous period's estimate of capital being too high

Note that the certainty-equivalent response to the first of these is to increase consumption, and to the second and third is to reduce consumption.

[14] This instability is manifested in the Kalman filter iteration. We can show that the full information case $P = Q$ solves the Kalman equatioins (xx) to (xx). But the iteration will not converge to this value.

The response of consumption in the partial information case is therefore ambiguous. The Kalman filter works by using the unconditional properties of the shocks to optimally attribute weights to the three possibilities given the observed innovation to returns. These are translated into revised estimates for the states, and these in turn allow the household to form expectations over the future path of returns and make its optimal consumption decision.

Figure 1a show shows the response of consumption to a unit innovation to the true process for technology. First note that it differs dramatically from the full-information case: consumption responds negatively on impact. This might seem surprising, but this is the only way to ensure stability of the system. Agents cannot, at least on impact, distinguish whether an increase in the interest rate is due to measurement error or an innovation in technology. If they respond by increasing consumption, the system will be unstable as described above. Figure 1b shows the effect of the shock on the estimates of the states. Note that on impact the household increases its forecast of technology (a combination of effects 1 and 2) and lowers its estimate of capital (effect 3).

Given a shock at time $t = 0$, The household uses these estimates to form a forecast of period 1's interest rate

$$E_t\hat{r}_1 = \lambda_3 E_t\left(\hat{a}_1 - \hat{k}_1\right) = \lambda_3\left(\phi\hat{a}_0 - \lambda_1\hat{k}_0 - \lambda_2\hat{a}_0 - (1-\lambda_1-\lambda_2)\,c_0\right) \quad (38)$$

In period 1, given that no further shocks happen, the observed interest rate will be

$$r_1 = \lambda_3 E_t\left(a_1 - k_1\right) = \lambda_3\left(\phi a_0 - \lambda_1 k_0 - \lambda_2 a_0 - (1-\lambda_1-\lambda_2)\,c_0\right) \quad (39)$$

so, remembering that true capital is pre-determined, $k_0 = 0$ the observed innovation is

$$\varepsilon_1 = r_1 - E_t\hat{r}_1 = \lambda_3\left[(\phi-\lambda_2)(a_0-\hat{a}_0) + \lambda_1\hat{k}_0\right] \quad (40)$$

and using the adding up constraint (23).

$$a_t - k_t = \hat{a}_t - \hat{k}_t \quad (41)$$

we can write

$$\varepsilon_1 = -\lambda_3\left[(\phi-\lambda_2-\lambda_1)\,\hat{k}_0\right] \quad (42)$$

Since a positive observed innovation to returns always means capital has been underestimated, $\hat{k}_0 < k_0$ and since dynamic efficiency means that $\lambda_1 + \lambda_2 > 1$ so, given a true innovation to technology, the forecast is above the actual level of the interest rate in the next period $E_t\hat{r}_1 > r_1$ so when the next period comes, the observed innovation is negative $\varepsilon_1 < 0$. Given this, the household uses the Kalman filter to update its forecasts, reducing the estimate of technology and increasing the estimate of capital according to (41). This process continues in each subsequent period: the household uses its current estimates to form forecasts of the interest rate in the next period. The actual interest rate differing from this forecast brings new information. The system converges asymptotically back to the steady state.

The observed innovation is positive in the period of the structural innovation and negative thereafter. So an observed positive innovation in returns today could be the result of a negative shock to technology yesterday and means that today's estimate of technology should be reduced. The assumption of our impulse response functions is that all variable, states, estimated states and forecasts, start from the steady state. An innovation to returns is a sign that this initial estimate of technology might have been incorrect. This explains why the second effect listed above exists. The response of estimated technology is a weighted average of effects (1) and (2).

How does this behaviour change with the persistence of the technology shock? We can show there is a critical value $\phi^* = \frac{1}{\lambda_1 + \lambda_2}$ such that $E(\varepsilon_t, a_t) = 0$ so the innovation in the interest rate brings no instantaneous information about the level of technology: positive technology shocks are always interpreted as negative capital stock shocks.

If $\phi < \phi^*$(as it is in our calibration) a positive innovation to the interest rate means households increase their estimate of technology (the case described above), but if $\phi > \phi^*$ a positive technology shock leads to a *reduction* in household's estimate of technology. (42) shows that as ϕ increases so too, over things being equal, does the magnitude of the innovation in the period after the shock. So as ϕ increases, it becomes more likely that an observed innovation in returns today was due to a negative technology shock yesterday. When ϕ is above the critical value, effect (2) (which leads to a reduction in the estimate of technology) starts to dominate effect (1) and the estimate of technology falls in response to a positive innovation today.

4.2 Information on output only

4.2.1 No measurement error

If aggregate output is measured without error, it can be combined with knowledge of aggregate consumption and the resource constraint to deduce investment and hence, given the history of output, the capital stock. In this case

$$P = Q = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \tag{43}$$

and

$$\beta = QH_\xi \left[H_\xi' Q H_\xi\right]^{-1} = \left[\begin{array}{c} 0 \\ \frac{1}{\alpha} \end{array}\right] \tag{44}$$

then since the observed innovation $\varepsilon_t = \alpha u_t$ then $\beta\varepsilon_t$ is the true innovation and the process for the observed states (11) in the same as with full information.

4.2.2 Measurement error

What if output is now measured with error? The observed innovation in y is given from (12) by

$$\varepsilon_{t+1} = \beta\left(y_{t+1} - E_t\widehat{y}_{t+1}\right) = \xi_{t+1} - E_t\widehat{\xi}_{t+1} \tag{45}$$

where the state vector is in this case $\xi = \left[\begin{array}{ccc} k & a & \omega \end{array}\right]'$ with ω being the process for the measurement error.

With measurement error, a positive innovation in output could be due to:

1. A positive innovation in technology in this period
2. The previous period's estimate of technology being too low
3. The previous period's estimate of capital being too low
4. A positive innovation to the measurement error

The certainty-equivalent response to the first three of these is to increase consumption, and to the last is to keep consumption constant. So the response to an innovation in output will be unambiguously to increase consumption, but to increase it be less than in the full information case.

Figure 2a shows the response of consumption to a unit innovation to technology which is observed by agents as an innovation in output. Consumption initially responds by less than in the full information case since there is a possibility the innovation was due to measurement error to which the optimal response would be zero. This can be understood by looking at the estimated states on figure 2b. The observed innovation is apportioned between an increase in the estimate of technology (a combination of effects 1 and 2); an increase in the estimate of the capital stock (effect 3) and an increase in the estimate of the measurement error (effect 4). Note that the change in the estimate of the capital stock is relatively small, with output data there is much higher uncertainty about technology than capital.

The forecast value of output for the next period takes into account the possibility that the original innovation was due to measurement error. Given that the innovation was to technology, actual output will be above this forecast so the observed innovation will again be positive (note the contrast with the returns-only case). Again this innovation could be due to the four effects listed, and the Kalman filter updates the estimates of the states accordingly. While consumption under-responded on impact, later there is a prolonged over-response which brings the capital stock back to the steady state.

Two limiting cases are particularly interesting. Firstly, as the variance of the innovation to measurement error tends to zero, the solution approaches asymptotically that in the full-information case, in contrast to the returns-only case. Secondly, as the relative variance of the innovation to measurement error increases, so too does the unconditional variance of the measurement error and the Kalman filter assigns a higher weight on the observed innovation's resulting from measurement error.

One might expect that as the variance became very large, the optimal response of consumption would tend to zero as, in the limit, there is no useful information in output. However this is not the case. If the innovation to technology has a non-zero variance, there is always some probability that the observed innovation is due to an innovation in technology. Were consumption not to respond to such an innovation, the system would be unstable since the capital stock would then explode. So given a positive observed innovation, consumption must always increase however noisy is output. But since the impact effect of an observed innovation must be the same whatever its source, this means that as the variance of the measurement error increases, it accounts for more and more of the variance in the states. Table 1 shows this effect

Table 1: Variance contributions with only output data as noise in output increases

		Variance contributions	
$\frac{\sigma_\omega^2}{\sigma_u^2}$	$\frac{\sigma_c^2}{\sigma_c^2 \text{ (full inf)}}$	Technology	Meas error
1	1.02	98.9%	1.4%
10	1.11	89.1%	9.9%
50	1.36	69.4%	30.6%
100	1.61	55.9%	44.1%
1000	5.11	15.3%	84.7%
∞	∞	0%	100%

.

This behaviour arises from the explosive root in the equation for capital (56) and to understand this it is useful to look at an even simpler case. Instead of a persistent technology shock with optimal choice of consumption feeding back on the level of the capital stock, think of the case where capital is perturbed by a white noise shock, and it is measured with error

$$k_{t+1} = \lambda k_t + v_{t+1} \quad (46)$$

$$y_t = k_t + w_t \quad (47)$$

where $v \sim N(0, Q)$ and $w \sim N(0, R)$. The updating equation for the forecast of capital is then

$$\widehat{k}_{t|t} = \widehat{k}_{t|t-1} + \beta \left(y_t - \widehat{y}_{t|t-1} \right) \tag{48}$$

with, β, a scalar given by

$$\beta = \frac{E_{t-1} P_t}{E_{t-1} P_t + R} \tag{49}$$

We show in the appendix that

$$\begin{aligned} \lim_{R \to \infty} \frac{P}{R} &= 0, \lambda \leq 1 \\ &= \lambda^2 - 1, \lambda > 1 \end{aligned} \tag{50}$$

As the variance of the measurement error becomes large, the signal to noise ratio $\frac{Q}{R}$ tends to zero. If the process for capital is non-explosive ($\lambda \leq 1$) it is optimal for the forecast of capital not to respond to the observation, $\beta \to 0$. However if the process for capital is explosive, not responding to the signal leads to a much worse estimate for the capital stock since $var(y) = R$ but $var\left(\hat{k}\right) = \left(\lambda^2 - 1\right) R$ and, for λ close to 1, $0 < \lambda^2 - 1 < 1$. If the forecast didn't respond, it would inherit the explosive nature of the capital process and its variance would tend quickly to infinity. By responding this explosive behaviour is avoided.

4.3 Information on output and returns

4.3.1 No measurement error

If households can perfectly observe the returns r and output y the measurement equation is

$$\begin{bmatrix} y \\ r \end{bmatrix} = H_\xi \xi = \begin{bmatrix} 1-\alpha & \alpha \\ -\lambda_3 & \lambda_3 \end{bmatrix} \begin{bmatrix} k \\ a \end{bmatrix} \tag{51}$$

and as long as the matrix on the right-hand side is invertible (which will be the case as long as $\alpha \neq 0.5$) households indirectly observe k and a through our observations of y and r. This is the benchmark case in, for example, Mehra and Prescott (1982) when, even if capital and technology are not directly observable, they can be recovered.

4.3.2 Measurement error in returns only

Since we have shown above that output data alone is sufficient to replicate the full-information case, measurement error in returns has no effect as long as perfect output data is available.

4.3.3 Measurement error in output only

The response of consumption when output is measured with error is shown in figure 3a. The technology shock is now observed by the household as a positive innovation to output and returns. Compared with the case of only output data discussed in the previous section, information on returns allows the household to better distinguish between a measurement error and a technology innovation. This means that the response of consumption tends to be closer to the full information case. Figure 3b shows the response of the estimated states, and clearly the estimated measurement error is much smaller than in the output only case. Also note that the estimated capital stock falls very slightly on impact. This is because there is the possibility that the innovation in output was due to measurement error, and that in returns was due to a negative technology shock in the past.

As the variance of the measurement error increases relative to the technology shock, the response of consumption approaches the case when only returns are observable. Table 2 shows the unconditional variance of consumption as the variance of the measurement error increases.

Table 2: Variance contributions with output and returns observable as noise in output increases

		Variance contributions	
$\frac{\sigma_\omega^2}{\sigma_u^2}$	$\frac{\sigma_c^2}{\sigma_c^2 \text{ (full inf)}}$	Technology	Noise
1	1.01	98.5%	1.5%
10	1.11	89.0%	11.0%
50	1.39	79.8%	20.2%
100	1.55	81.6%	18.4%
1000	1.89	96.0%	4.0%
∞	1.96	100%	0%

Even very noisy output data can bring the response of consumption much closer to the full-information case than were there none. For example, if the variance of the measurement error in output is 100 times the standard deviation of the technology shock, the variance of consumption increases by half as much as it would were this data not available. In contrast to the case described in table 1, as the variance of output increases above a limit, the cost of responding to it is higher than the value of the information it brings, so the variance contribution falls and as measurement error becomes very large, the response of consumption tends to that if there were no output data at all.

So although section (xx) suggested that returns data could lead to optimal consumption behaving very differently from the full-information case, in fact even very noisy output data is used to mitigate this effect. The corollary to this is, if returns are very noisy but output data is available, the noise in returns does not affect optimal consumption very much.

4.4 Calibrating measurement error

Prescott (1986) asserts that a real business cycle model of the type we have described can match selected moments of the data closely if it is driven by an Solow residual with autoregressive parameter of 0.9 and innovation standard deviation of 0.7%.

Maintaining our assumption that the true states, capital and the technology process, are unobservable, we need to calibrate noise in the other variables of the model - output, returns, labour and the wage. Orphanides (2003) measures the variation between real time and final estimates of the output gap, and estimates that the error follows a first-order autoregressive process with persistence parameter 0.91 with the standard deviation of the shock equal to 0.93% per quarter. In earlier work, Orphanides (2002) using a wider sample period finds similar autoregressive parameters and, depending on the detrending method used, standard deviations from 1.4 to 2.8. Note the magnitude of these estimates - they are roughly the same as the standard deviations of the output gap itself.

The autoregressive nature of the process corresponds to subsequent revisions of the data improving on the initial estimate, until a final estimate is reached. although we don't model this process explicitly, the Kalman filter could be used to calculate improved estimates of past data based on later realizations of the observed variables.

Orphanides (1999) emphasizes that his estimates do not capture might be called fundamental measurement error, that final estimates of the output gap may be incorrect. So the values we should be understood be lower bounds on the actual, unobservable measurement error. In what follows we look at two cases, a low noise case with 0.9 and a high noise case of 2.2 (the approximate average of Orphanides' estimates).

The volatility of the stock market is however around 20% per annum (Campbell, Lo and McKinley (1992)) so a natural way to calibrate noise would be such that the total volatility of returns is close to this. However, as Campbell (1995) mentions, in the real business cycle model technology shocks have relatively little effect on returns, so to achieve this level of volatility requires a very large amount on measurement error and we show below that this means returns contain very little useful information.

4.5 Results

Table 3 shows standard deviations of consumption given various assumptions about information and the above calibration. The baseline case is the full information for which the standard deviation of consumption is 1.16% per quarter.

Table 3: standard deviations of consumption with measurement error

Information	Low error calibration		High error calibration	
	σ_c	Increase over full inf	σ_c	Increase over full inf
Full	1.16	-	1.16	-
r only	2.27	95.8%	2.27	95.8%
y only	1.20	3.5%	1.33	14.4%
r,y	1.20	3.5%	1.32	14.4%
r,y,k	1.20	3.45%	1.32	13.6%

If the only information available to the household is the stock market, the volatility of consumption almost doubles compared with the full information case. As shown in above, information on output allows the agent to get much closer the full information case. With the "low error" calibration, only observing output increases the volatility of consumption by 3.52% over the basecase, and in the "high error "calibration by 14.4%. Adding noisy information on returns gives almost no useful information over that already available in output.

4.6 The unobservability of the capital stock

Assuming capital is measured using an inventory approach, measurement errors cumulate up through the capital accumulation equation (27). If at time $t = 0$ we know the capital stock K_0 without error, we can solve this equation backwards to give

$$\tilde{K}_t = \sum_{j=0}^{t-1} (1-\delta)^j \, \tilde{I}_j + K_0 \tag{52}$$

If measured investment is $I_t = \tilde{I}_t + u_t$ where $u \sim N\left(0, \sigma_u^2\right)$ then the variance of capital around the true value,

$$var\left(K - \tilde{K}\right) = \sigma_K^2 = \sigma_I^2 \sum_{j=0}^{t-1} (1-\delta)^{2j} \tag{53}$$

As t becomes large this will tend to the limit

$$\sigma_k = \sigma_i \sqrt{\frac{1}{\delta\,(2-\delta)}} \tag{54}$$

which for our calibration gives the standard deviation in the capital stock to be approximately 5 times larger than the measurement error in investment.

To assess our assumption of the "non-observability" of the capital stock, we add capital to the agents information set, and calibrate its measurement error according to the calculation above (converting into logs). In the case where agents also have noisy information on output and returns (line 3) of the table, adding capital reduces the increase in the standard deviation of consumption above the full information case from 3.52% to 3.45% with the "low error" calibration, and from 13.9% to 13.6%. This shows that the assumption of non-observability does not have any significant effect on our conclusions.

5 An example with two technology shocks

The simplest case of a model with multiple shocks is where the process for aggregate technology a is made up of two separate components

$$\begin{aligned} a_t &= a_t^1 + a_t^2 \\ a_t^1 &= \phi^1 a_{t-1}^1 + u_t^1 \\ a_t^2 &= \phi^2 a_{t-1}^2 + u_t^2 \end{aligned}$$

where u^1 and u^2 are white noise errors independent of each other. Since the two components of technology have identical effects on the model economy, unless an agent can observe them directly it is not possible to distinguish them. This leads to so called "permanent - transitory confusion" which has been extensively discussed in the consumption literature, for example Quah (1990). We model this more complex process for technology using Campbell's model as before ,but now we allow labour supply to vary, taking $\nu = 2$.

Figure 4 shows the response of consumption to innovations in the two technology processes (with $\phi^1 = 0.9$ and $\phi^2 = 0.6$) given that output and returns are observable without error. On impact the response to the two shocks is the same; over time the agent learns to distinguish which one has actually occurred. If we now introduce noise in the measurement of output and returns, the agent responds less to the signal on impact. Table 4 shows the effects in terms of standard deviations of these two sorts of non-observability.

Table 4: Standard deviations of consumption with two tech shocks

	σ_c	Increase over full inf
Full information	1.50	
y,r - no error	1.51	1.2%
y,r - low error	1.56	4.0%
y,r - high error	1.69	13.3%

Bomfim (2001) presents a model similar to this. His agents observe capital without error, aggregate technology with error but not the individual components of technology. His striking headline result is that, under the baselines calibration, the presence of measurement error in aggregate technology, *reduces* the volatility of output "by around 13%". He explains this by noting "when the indicator, aggregate technology, is noisy, agents effectively discount all preliminary announcements by always attributing some fraction of each new reading to measurement error" (Bomfim 2001).

Looking just at the impact effects, this result does not seem implausible, however the unconditional variances are clearly higher with noise than without. Also, Bomfim has the second technology process as white noise. Given his calibration, 98.6% of the unconditional variance of consumption comes from the persistent technology shock so it is hard to understand how noise can result in a 14% change in the volatility of consumption, whatever the sign.

However Bomfim's model includes a strategic complementaries in output. Even if the strange result can be accounted for by this, it is important to note that it does not generalize and, in general, increased measurement error leads to increased volatility.

6 An example with multiple shocks

The DGE literature has considered a wide range of potential shocks. Altig et al (2003) present a model containing no less than eight different types of shocks. But the widespread assumption is that agents can observe the shocks directly. Although it is far beyond the scope of this paper to undertake a full comparison between Altig et al's model under full information under and under partial information (though the model could be solved using our toolkit), we investigate the realism of this assumption by adding some more shocks to Campbell's model and seeing how partial information affects the response of consumption. We follow Altig et al (2003) in modelling two technology shocks of difference persistence, a shock to the productivity of investment, a preference shock which affects the marginal substitutability between consumption and leisure and a shock to government spending.

The equations of the model with variable labour supply are modified to be

$$E_t \sum \beta^i \left[\chi_t \log c_{t+i} + \theta \frac{(1 - N_{t+i})^{1-\gamma_n}}{1 - \gamma_n} \right] \tag{55}$$

where χ is the preference shock and

$$K_{t+1} = (1 - \delta) K_t + \zeta_t I_t \tag{56}$$

where ζ is the shock to the efficiency of investment.

Government spending modifies the resource constraint

$$Y_t = C_t + G_t + I_t \tag{57}$$

where G_t is a stochastic process.

These shocks are assumed to follow first-order autoregressive processes in logs, with persistence ϕ_χ, ϕ_ζ and ϕ_G and innovation standard deviation $\sigma_\chi, \sigma_\zeta$ and σ_G respectively. We calibrate the standard deviations of the five shocks so that in the full-information case the contributions to the unconditional variance of consumption are approximately 40% from the persistent technology shock ($\phi_{z1} = 0.9$); 20% from the transitory technology shock ($\phi_{z2} = 0.3$); 10% from government spending shocks; 10% from preference shocks and 10% from shocks to the efficiency of investment.

Table 5: Standard deviations of consumption with multiple shocks

	σ_c	Increase over full inf
Full information	2.08	
y,r - no error	2.17	4.3%
y,r - low error	2.27	9.1%
y,r - high error	2.44	17.1%

Again, we can split the effect of these changes into that resulting from being unable to observe the shocks directly, and that resulting from noise. The requirement that agents infer what shocks have occurred from the observable variables increases the standard deviation of consumption by just 4% compared with the full information case. With noise calibrated at the "low" rate, this increases to 9% and with noise at the high rate this increases to 17%.

So in terms of unconditional standard deviations we find that, as with the simpler case of two technology shocks, partial information resulting in confusion of shocks seems less important than measurement error. This is a measure of the efficiency of the Kalman filter in distinguishing between different shocks.

However in terms of the true impulse response functions, as observed by the econometrician after the shock, there are more significant differences. Figure 5 shows the response of consumption to a true 1% innovation to technology under the "low error" calibration. Let's assume that this is what the econometrician observes in the data, but the modeler ignores partial information and uses some technique to estimate a full information model to best fit this response. Estimating preference parameters to minimize the deviation of the model's impulse response function from that observed gives $\gamma_c = 2.7$ and $\gamma_n = 9.2$ as opposed to the values of unity in the true model.

7 Conclusion

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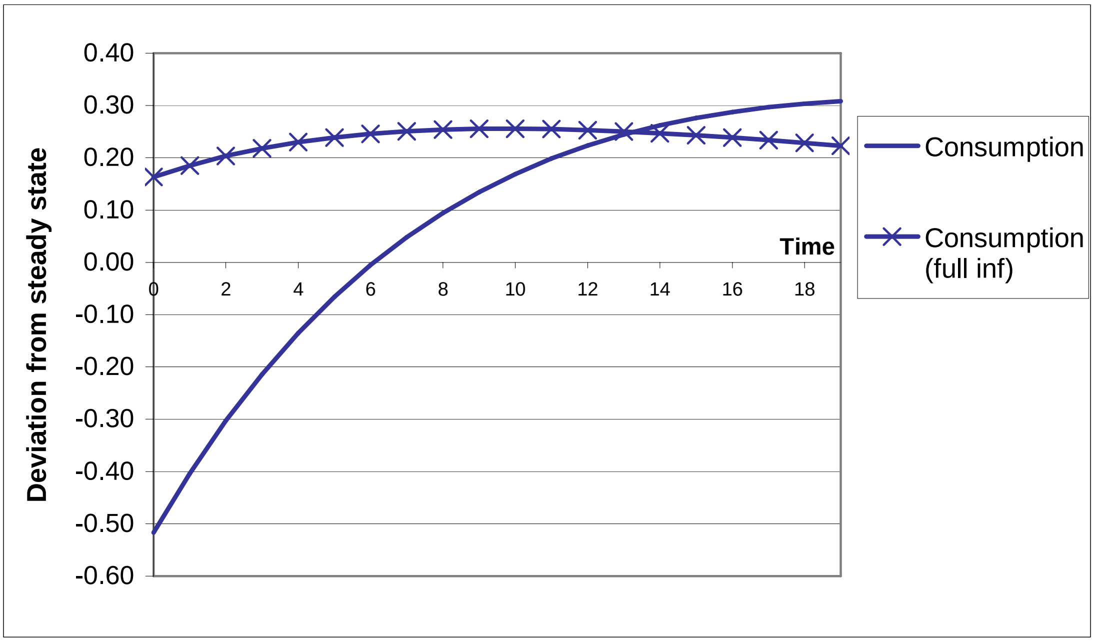
Figure 1: Only returns observable: arbitrarily small measurement error

1a: Response of consumption to technology shock



1b: Response of state estimates to technology shock

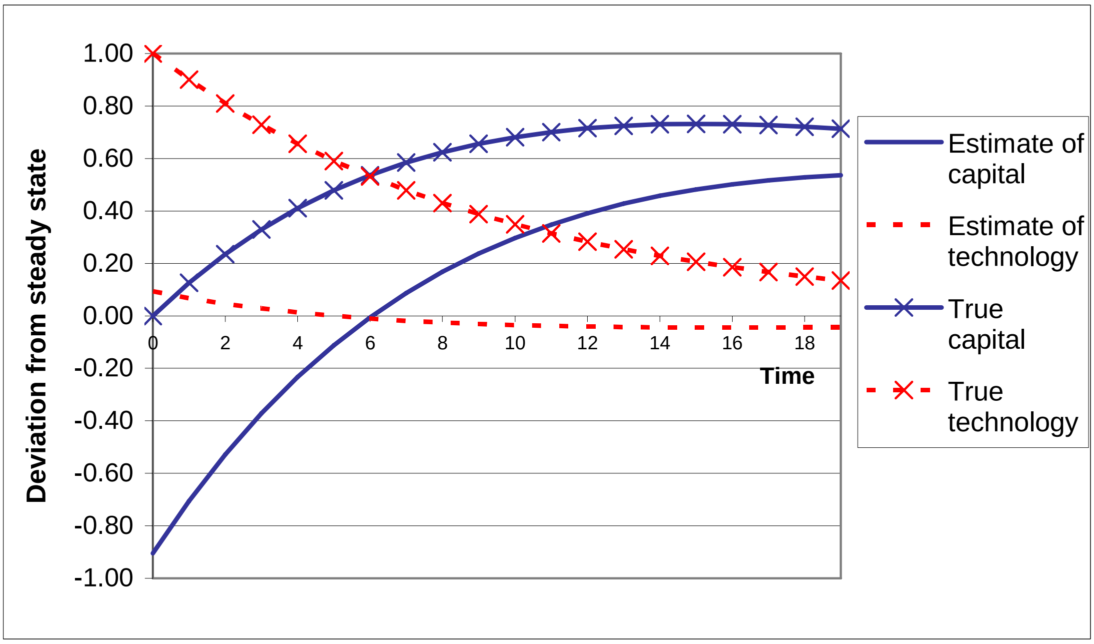
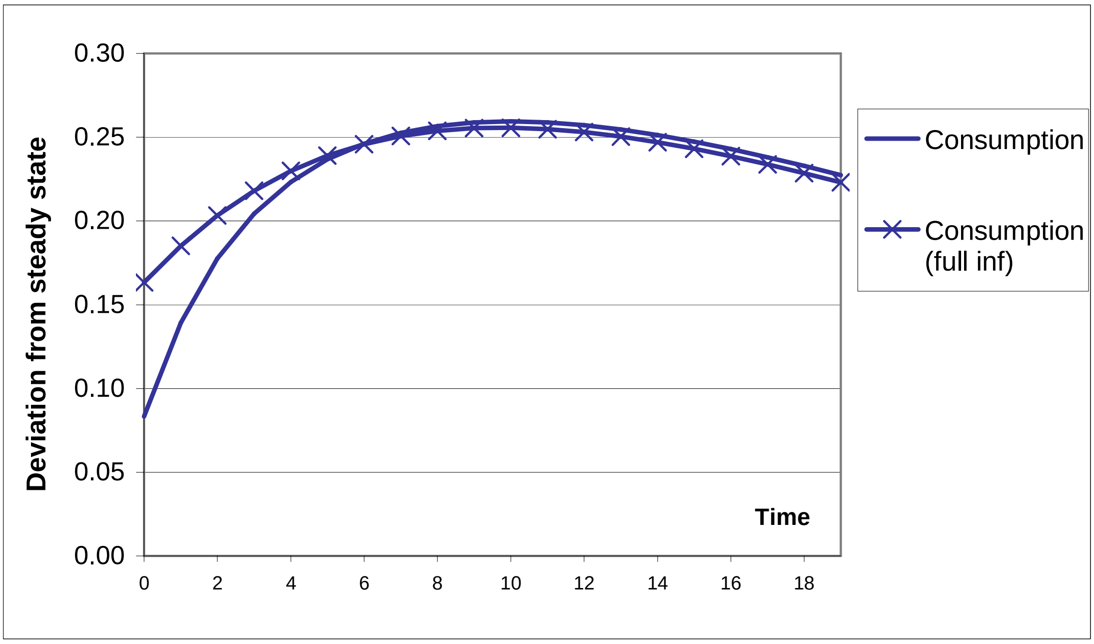


Figure 2: Only output observable, with error

2a: Response of consumption to technology shock



2b: Response of state estimates to technology shock

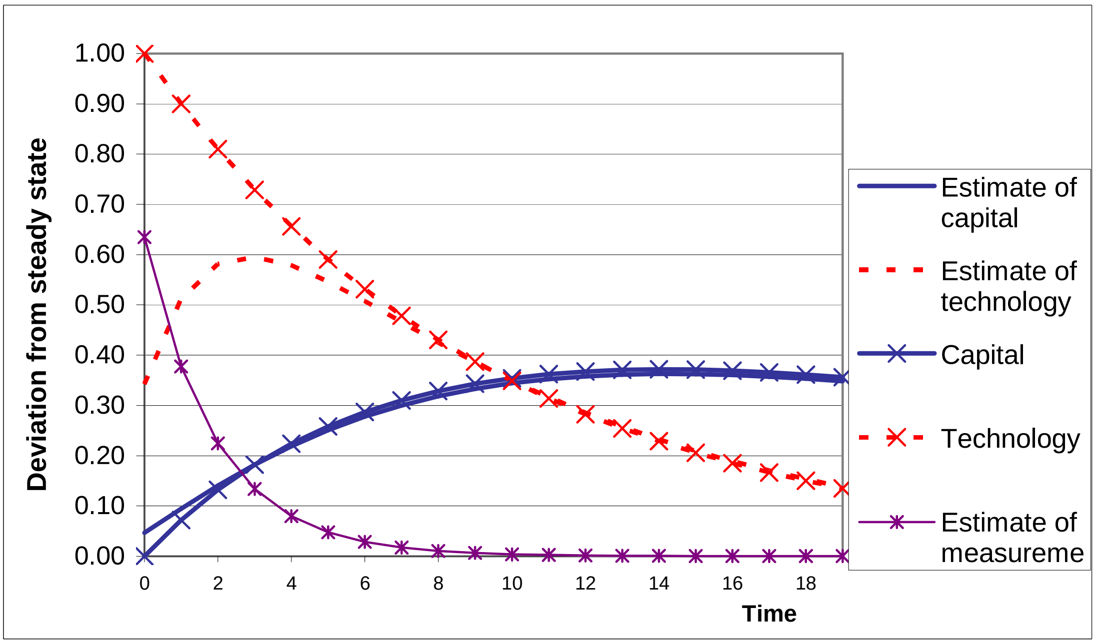
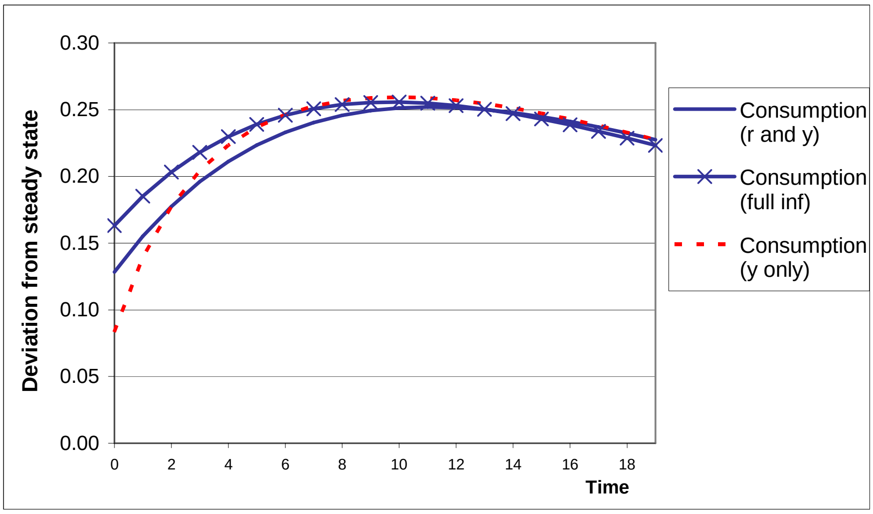
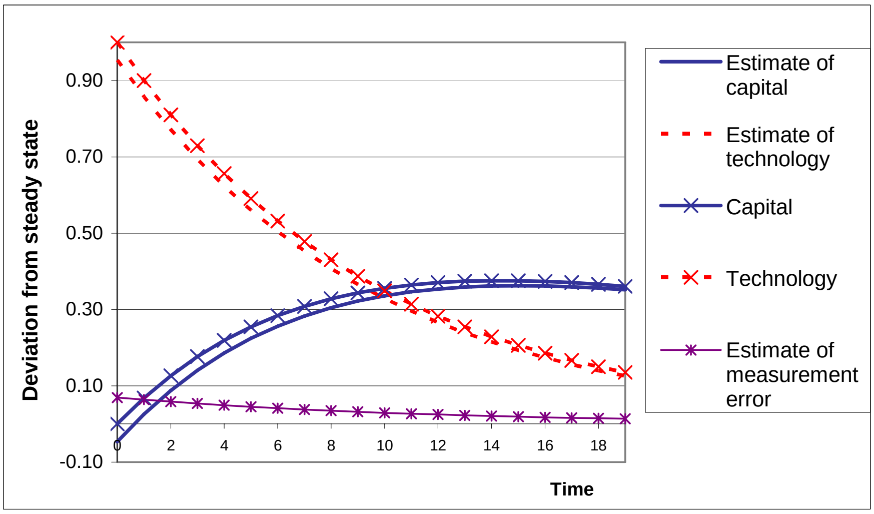


Figure 3: Output and returns observable, measurement error in output only

3a: Response of consumption to technology shock



3b: Response of state estimates to technology shock



3c: Sensitivity of consumption response to variance of measurement error

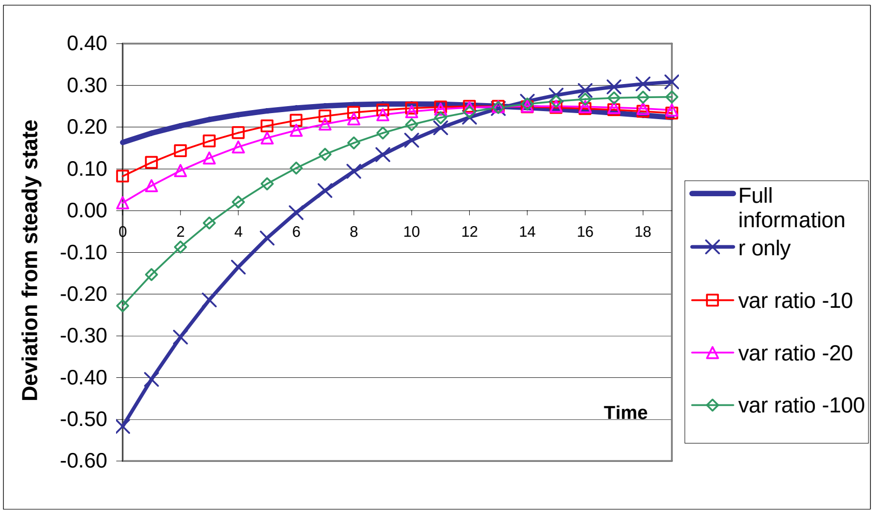


Figure 4: Response of consumption to innovation in persistent component of technology in model with two technology shocks

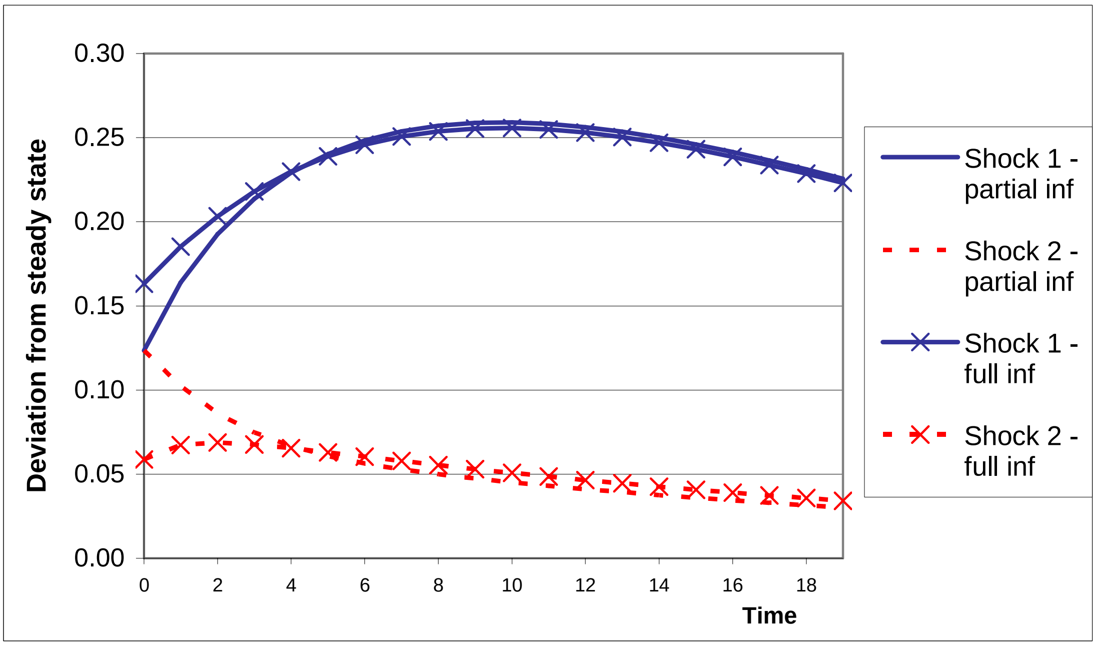


Figure 5: Response of consumption innovation in persistent component of technology in model with multiple shocks