Comparative analysis and estimation of mathematical methods of market risk valuation in application to Russian stock market.

Andrey M. Boyarshinov

Rapid development of **risk management** as a new kind of philosophy of strategic control in Russian financial business has been progressing since the middle of the last decade of 20th century. It was caused by an influence of a wide variety of factors and tendencies which changed an approach to risk control radically. In particular, these factors include globalization of world economy, deregulation processes, development of derivatives markets, interconnection of financial markets and risks, informational and technological development and so on.

In this work a problem of evaluating market risks is considered. This problem became relevant during last years and was caused by a rapid increase in numbers of financial markets participants. Researches in a field of risk valuation have been conducted in the world for decades, but in our country – for only a few years.

Distinctive features of Russian stock market are short period if existence (in comparison with European markets), instability (even over such a short period), and strong dependence on political situation.

There is a wide variety of methods which allow fulfilling quantitative and qualitative estimation of market risk. Most commonly used qualitative methods are different methods of changeability estimation (in particular, volatility estimation), expert assessment, and sensitivity measures. Qualitative methods are in general represented by a wide specter of Value-At-Risk models used all around the world.

Apart from the above mentioned problem of risk estimation, there are two more which are strongly connected to it: these are problem of conformity of estimate and problem of estimating loss probability. For those problems there is also a multitude of solutions part of which is considered in this paper.

1. Problem description

In process of development of decision support system “Prognoz.Risk” the following problems were encountered and solved:

- Analyze existing methods of risk valuation and estimation [1-4] and determine their efficiency in application to Russian stock market;
- Make a comparative analysis of methods of estimating risk measures accuracy [5];
- Implement most accurate methods (on the basis of results of above mentioned analysis) as calculation algorithms in the decision support system.
- Implement algorithms of verification [6] (quality and efficiency estimation) of risk measures calculation methods and test implemented risk measure calculation algorithms.
- Evaluate efficiency of risk measures calculation algorithms.

2. Value-at-Risk models

2.1. Equal-weights historical simulation.

Historical simulation is one of the simplest methods and does not require much theoretical research [1-3].

Portfolio value changes are calculated using historical sample of portfolio risk factors changes applied to current (on calculation date) values of risk factors. Thus a sample of scenery portfolio values is obtained from which a sample of portfolio value changes is derived (changes are relative to current portfolio value). Obtained sample is then sorted and VaR estimate is taken as a change in portfolio value which is exceeded in scenery sample no more than required number of times, for example no more than 5% of all scenery values.

2.1. Weighted historical simulation (exponential weights).

Difference between this method and the previous one (equal-weights simulation) is that each case in portfolio value change sample is assigned a certain weight which depends on how far in the past this case is from calculation date [6]. Weighted historical simulation gives a VaR estimate depending more on the most recent observations and thus allows better risk estimation in case of frequent portfolio value fluctuations.

Advantages of historical simulation methods:

1. No assumptions required on risk factors distributions;
2. Easiness of full portfolio revaluation based on scenery data;
3. Intuitive simplicity and obviousness.

Disadvantages:

1. Past does not always provide a good model for the future;
2. High probability of obtaining an erroneous result in case of insufficient sample size.

2.3. Value-at-Risk: variance-covariance method.

Variance-covariance method uses historical sample of portfolio risk factors values just as historical simulation method but it is based upon an assumption that risk factors logarithmic yields are normally-distributed [2,4]. VaR estimate then equals to sample quantile of portfolio yield volatility. Portfolio yield volatility is calculated with consideration of covariance between portfolio risk factors yields.

2.4. Variance-covariance method: exponentially weighted covariance.

Specifics of this method lie in stronger influence of most recent observations on covariance matrix [4,6]. This model uses specific method of calculating time-dependent covariance matrix. Such matrix Σ_t is calculated as follows. $\tilde{\Sigma}_{t+1} = \frac{1-\lambda}{1-\lambda^T} \sum_{s=0}^{T-1} \lambda^s r_{t-s} r_{t-s}^T = \lambda \tilde{\Sigma}_t + (1-\lambda) r_t r_t^T$.

Here λ is a factor of weight decay ($0 < \lambda < 1$).

Advantages of variance-covariance methods:

1. Comparatively easy to implement;
2. Less historical data is required than for historical simulation methods;
3. Acceptable precision and accuracy in most cases.

Disadvantages:

1. Low quality of estimating securities with nonlinear dependence of price on risk factors.
2. Assumption of logarithmic distribution of risk factor yields is not always correct.
3. Ignoring risk of extreme events which can lead to significant losses in portfolio value.

2.6. Constant correlation GARCH methods.

Generalized autoregressive conditional heteroskedasticity model is a generalization of weighted covariance model [6]. Simplest and most frequently used GARCH(1,1) model is given by

$$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \beta \sigma_t^2 ,$$

where ω, α, and β are model parameters (actually, parameter matrices).

To reduce number of parameters of generalized model different modifications are used which impose additional constraints on model. In constant-correlation GARCH model, assumption is used that off-diagonal elements of covariance matrix are given by $\sigma_{ij,t+1} = \rho_{ij} \sigma_{i,t} \sigma_{j,t}$, where correlation coefficients ρ_{ij} are time-independent. Diagonal elements $\sigma_{i,t}$ can be calculated for example with GARCH(1,1).

2.7. Monte-Carlo method.

Monte-Carlo methods are based upon assumption about a kind of statistical distribution of portfolio risk factors and are generally similar to historical simulation methods. Difference between them lies in different samples used for calculation. Monte-Carlo methods use statistically generated samples based upon risk factors distribution which is usually based on preliminary observations. It is needed to say that sample used for Monte-Carlo method is usually tenths and even hundreds of times larger than sample used for historical simulation.

The rest of VaR calculation process is similar to that of historical simulation method.

Advantages of Monte-Carlo methods:
1. Calculation use real portfolio risk factors (risk factors yields) distribution;
2. Easiness of implementation of almost any kind of distribution;
3. Intuitive simplicity and obviousness.

Disadvantages:
1. Calculation process is very resource-dependent and may take a significant time;
2. Insufficient sample size of observations used to derive distribution kind and parameters may result in an incorrect VaR estimate.

3. Models evaluation methods

To ascertain that models used in risk estimation are precise and accurate, quality and efficiency measures are required to be calculated. Measures used in current work include:

- Average uncovered losses to VaR ratio;
- Average unused reserves value;
- Maximum loss to VaR Ratio;
- Multiple to obtain coverage;
- Binary loss function (namely, number of exceptions);
- Color zone (based on BCBS recommendations).

4. Calculation base

For implementation of above mentioned models, algorithms of risk valuation have been developed on the basis of Value-at-Risk methodic as the most widely used. Algorithms are implemented using software means of specialized decision support system “Prognoz.Risk” based upon “Prognoz” analytical suite.

For the purpose of backtesting [1] of developed algorithms, market risk measures were calculated using open data from Russian stock market (MICEX). For mentioned risk measures figures of quality and effectiveness were calculated.

Portfolio used for calculation is purely theoretical. Portfolio consists of ten high-liquid Russian equities (‘blue chips’) and a dozen of Russian bonds (Eurobonds, GKO-OFZ and a few liquid corporate bonds). Volumes of positions on all securities were adjusted to make each position market value roughly the same.

5. Market risk control system “Prognoz.Risk”

Decision support system “Prognoz.Risk” is designed to fulfill the following basic functions:

- Consolidation of information required for market risk management process in data storage;
- Calculation of portfolios and positions structure on the basis of initial information;
- Calculation of market values for positions and portfolios;
- Calculation of risk measures for objects from single positions to complex portfolios;
- Risk measures verification (backtesting) on the base of historical data;
- Stress-testing and scenery analysis of portfolios and positions;

- Portfolio optimization based on yield/risk criteria;
- Building reports of any required structure and quick data analysis implemented in report builder.

Decision support system is developed on the base of "Prognoz 3" Analytical Suite and supports multi-user work. System functions as a flexible framework which can be configured to suite needs of even the most sophisticated customers.

6. Description of results.

For all models mentioned above, verification (backtesting) procedures were completed and color zone (on the basis of instructions by Basle committee on Banking Supervision) was determined for each model. Results are presented in graphical form (fig.1 – fig.7) and explained below.

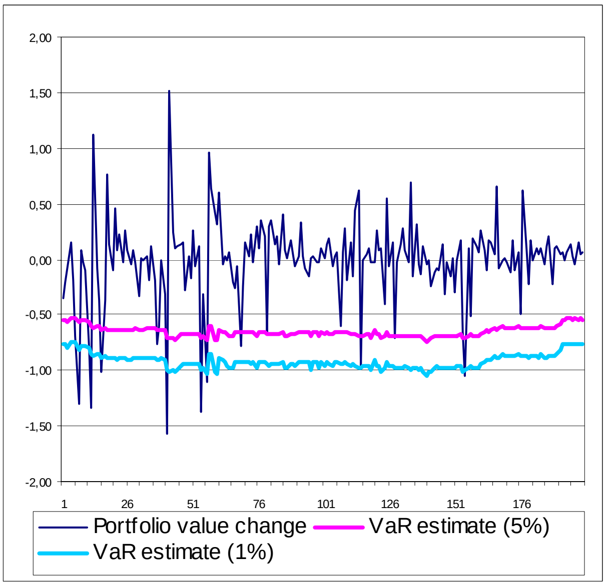


fig. 1. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for GARCH (1,1) model

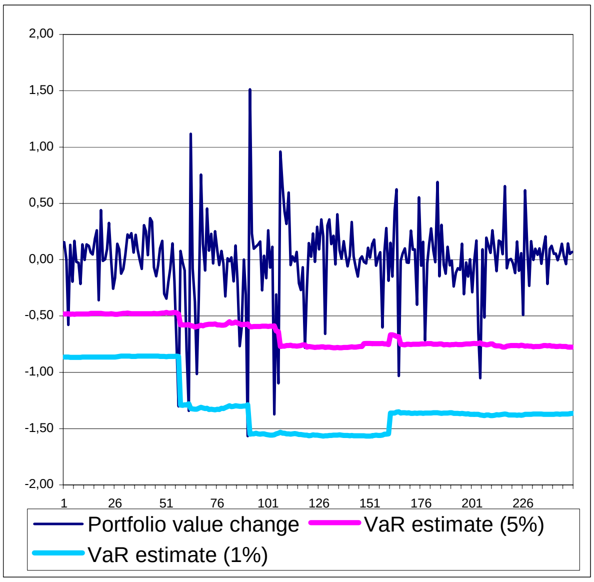


fig. 2. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for historical simulation model

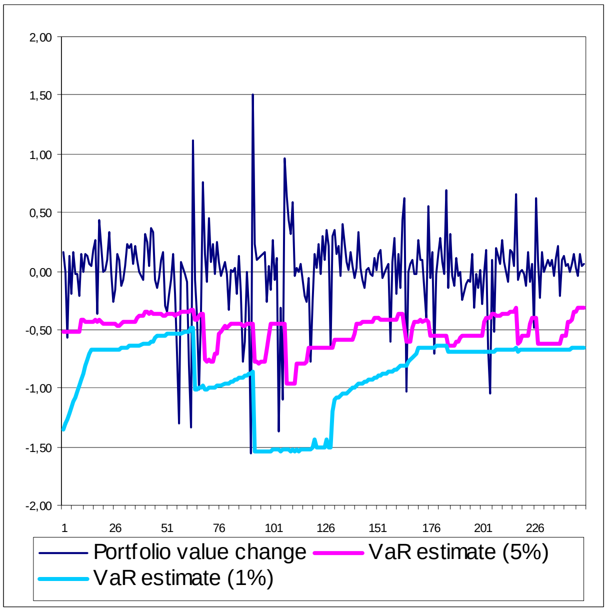


fig. 3. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for weighted historical simulation model (decay 0.97)

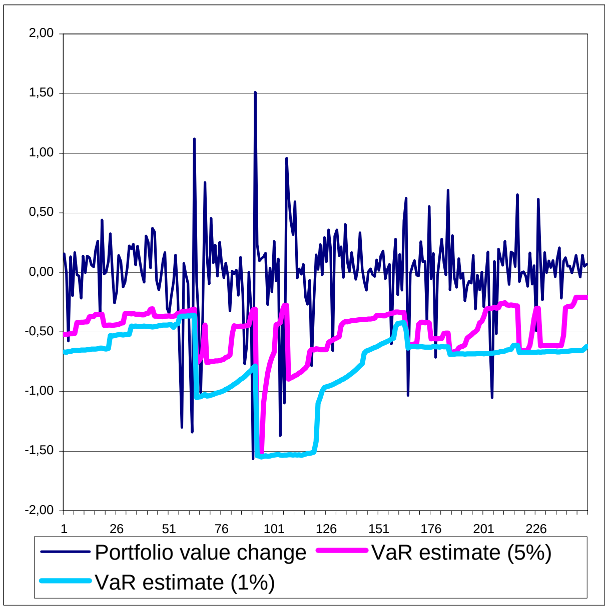


fig. 4. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for weighted historical simulation model (decay 0.94)

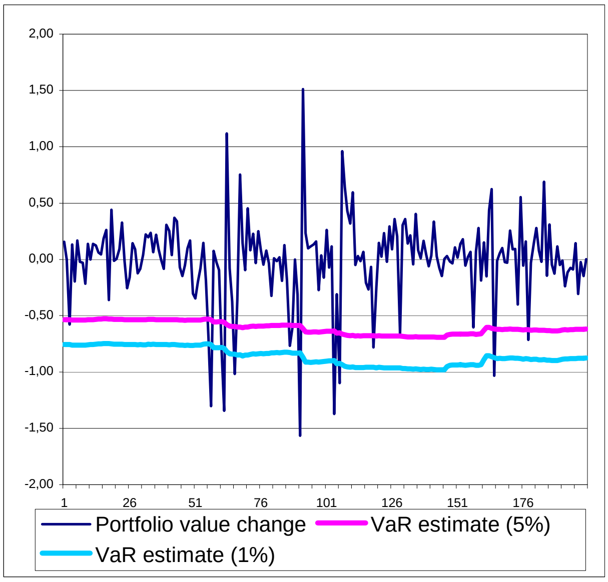


fig. 5. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for variance-covariance model

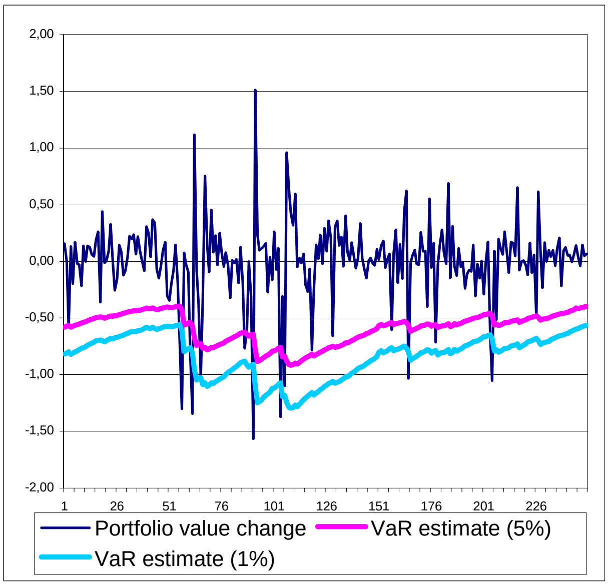


fig. 6. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for variance-covariance model (decay 0.97)

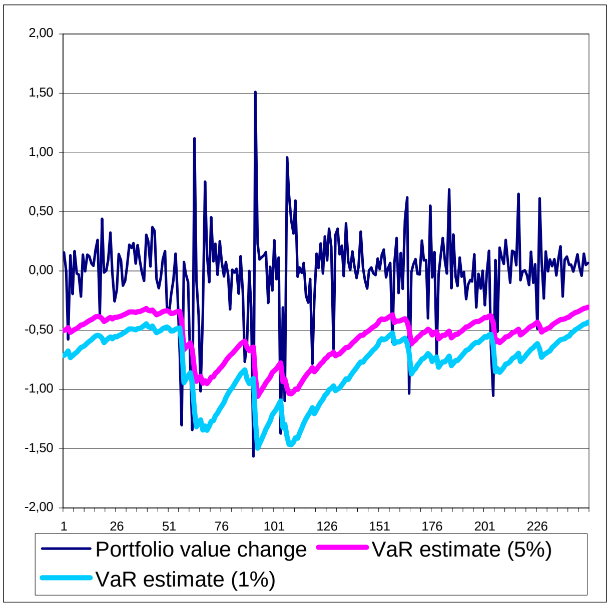


fig. 7. Relative portfolio value change and VaR estimates at 1% and 5% significance levels for weighted historical simulation model (decay 0.94)

Color zones for models (based on number of exceptions and significance level):

Model	**Color zone (5%)**	**Color zone (1%)**
Historical simulation	Green	Green
Weighted historical simulation (0.97)	Yellow	Yellow
Weighted historical simulation (0.94)	Yellow	Yellow
Variance-covariance model	Green	Yellow
Weighted variance-covariance model (0.97)	Green	Green
Weighted variance-covariance model (0.94)	Green	Yellow
GARCH(1,1) model	Green	Yellow

Apart from color zone which is only a qualitative index, comparative quantitative indices were calculated.

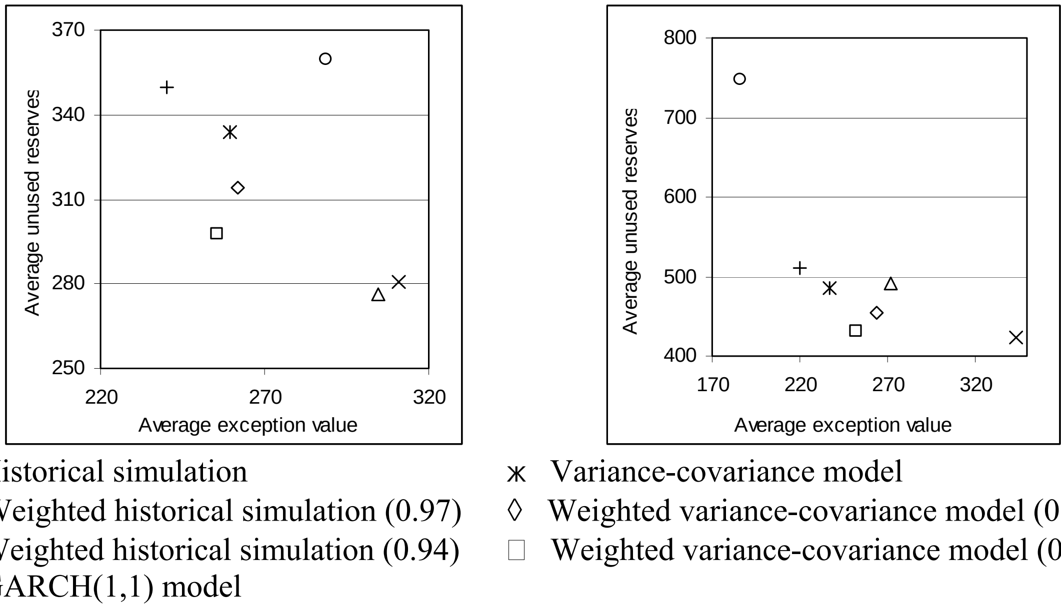


Fig. 8. Average unused reserves and average exception value figures at 5% (left) and 1% (right) significance levels for different Value-at-Risk models.

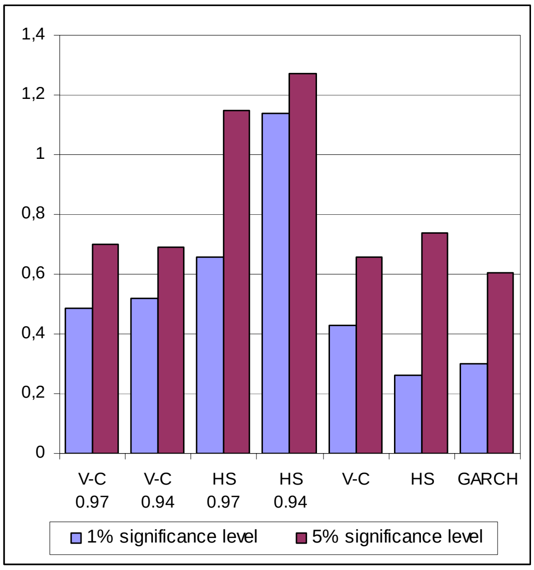
Diagrams above (fig. 8) show relation between average exception value (uncovered loss) and average unused reserves (unused part of funds for covering unexpected losses). From left graph following conclusions can be drawn:

1. Models have a wide spread of values giving a good choice of different opportunities. Weighted historical simulation models together show the lowest value of average unused reserves, but highest uncovered losses. Plain historical simulation model has the least number of exceptions but average uncovered losses and average unused reserves values are rather high.
2. GARCH(1,1) model allows minimizing uncovered losses in comparison with other models but the amount of coverage (which cannot be used for any other purpose) is significant.
3. All variance-covariance models give roughly the same average exception value but average unused reserves value increase with increase in decay coefficient (in unweighted model it equals to 1).

Right graph (1% significance level) allows following conclusions:

1. Historical simulation models show increasing value of average unused reserves and decreasing value of average exception value with increase in decay coefficient (it is also noteworthy that plain historical simulation model has much less exception cases than other two which lie in yellow zone).
2. Variance-covariance models show not so different results, but only one of them (weighted, with decay coefficient equal to 0.97) lies in green zone.
3. GARCH(1,1) model does show good results for a non-risky investor (low average exception value) but also lies in yellow zone.

Apart from comparative statistics, quantitative statistics were evaluated for each method at different significance levels as shown in fig. 9 (V-C – variance-covariance models, HS – historical simulation).



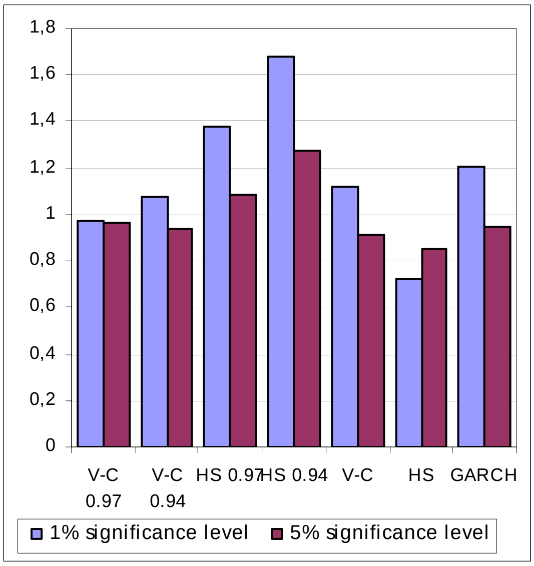


Fig. 9. Average uncovered loss to VaR ratio (left) and multiple to obtain coverage (right) for different Value-at-Risk models.

Left part of fig. 9 shows average values of uncovered loss to VaR ratios for each of the models described above. The lower this value is the closer is VaR estimate given by the model to reality.

At 5% significance level only two models stand out having this value approximately twice higher than all other models – these are both weighted historical simulation models. Other models have this index at approximately 0.6 – 0.7.

At 1% significance level the lowest value of this index which is 0.26 is observed from historical simulation model. Average uncovered loss to VaR ratio for most of described models does not exceed 0.5. Again, highest values are observed from weighted historical simulation models.

Multiple to obtain coverage which is presented on the right side of fig. 9 tells how close VaR estimates derived from models are to required significance level. The closer this index value to 1 the better VaR estimate corresponds to required significance level. If value of multiple to obtain coverage is lower than 1 it means that model under examination is over-conservative and VaR estimate can be lowered without harming the significance level of the model. If multiple is significantly higher than 1 then model gives low VaR estimate and should be used with caution.

It is also noteworthy that meaning of multiple to obtain coverage has been changed in this paper from original [] to better reflect recommendations of BCBS. Exact changes are mentioned further in the conclusions section.

7. Conclusions.

7.1. Results and recommendations.

Research described in this paper allowed formulating following results:

At 5% significance level good results can be obtained from variance-covariance models (both exponentially weighted and unweighted) as they give best result based on criteria of risk/reserves. Lowest number of exception cases is given by unweighted historical simulation, but this model also gives a significant amount of unused reserves which marks it as highly conservative.

Monte-Carlo method, as expected, yields results very close to results of historical simulation method (tested only at 5% significance level because of serious time-consumption) but behaves better when sample size is not enough for historical simulation method to work properly. Alco calculations with Monte-Carlo method are very time-consuming and take up tens times more time.

Exponentially weighted historical simulation models do not yield acceptable results.

For 1% significance level only unweighted historical simulation and weighted variance-covariance methods (weight decay 0.97) successfully pass backtesting procedure. Just as for 5% significance level, historical simulation shows strong conservatism. Although average exception for this method is higher than for most other methods, its number of exceptions is significantly lower.

Exponentially weighted (0.97) variance-covariance method shows good tracking of portfolio value changes at both 1% and 5% significance levels.

GARCH(1,1) model which relies on modeling of covariance matrix requires double historical sample in comparison to historical simulation or variance-covariance methods and shows acceptable results combined with good calculation speed.

Based on the results of this work recommendations can be formulated for using risk-estimation models in different cases.

- VaR estimation using historical simulation method should be used when investor takes a conservative strategy and tends toward minimizing market risks.
- Exponentially weighted historical simulation methods may give a good VaR estimate on stable markets where significant fluctuations are rare occasions.
- Variance-covariance methods are good for aggressive investors who are interested in minimum size of their reserves. These methods provide good tracking of portfolio value changes. It is also needed to mention here that exception cases often lead to serious uncovered losses.
- Evaluating VaR value using GARCH(1,1) models proved to be something between 'conservative' historical simulation methods and 'aggressive' variance-covariance models. Such method ofrisk-estimating can be recommended to balanced or cautious investors.
- Monte-Carlo method require some preliminary work to determine the underlying distribution of portfolio values (or portfolio value yields, whichever is better) and very long time to calculate a sample which should usually be tens and even sometimes hundreds of times larger than sample for historical simulation method.

7.2. Algorithms modifications.

While working on this paper, some adjustments have been made to Value-at-Risk estimation algorithms. So far, no mention of such adjustments was found in sources available to author of this paper.

Smoothing historical VaR

In some cases of use of historical simulation for estimating VaR, pointer to position in calculation sample can actually fall between two positions, for example when sample size is 250 and significance level is 5% pointer will be at 12.5 which is exactly between 12th and 13th elements. In this work such problem was solved by applying a linear interpolation of values to values of sample elements to the 'left' and 'right' of pointer.

In normal historical simulation results of this smoothing are often hardly visible. In case of applying interpolation to weighted historical simulation, plotting VaR estimates over time results in a much smoother curve.

Instead of simple linear interpolation another method can be used as well but in this paper only linear interpolation has been applied to calculations.

Correcting marginally-adequate VaR

There can always be cases when some VaR calculation models can not be verified successfully (due to different circumstances and market conditions) and appear in yellow zone. There is a way to 'fix' such models. It is based on backtesting results – multiple to obtain coverage. The principal idea is that each VaR estimate should be corrected by this multiplier derived from past backtesting results. Of course, this multiplier never stays constant and should thus be corrected. Each backtesting takes up a significant amount of time, thus it is recommended to acquire time interval over which the multiplier should remain constant empirically, based upon available calculation resources.

To further increase precision of VaR estimates calculation, multiplier can be obtained with help of GARCH models, but such calculations require large samples of historical data and take significantly more time.

8. References.

1. Risk Management. A Practical Guide. RiskMetrics Group, 1999. www.riskmetrics.com.
2. RiskMetrics: Technical Document, 4th edition. http://www.riskmetrics.com.
3. Jorge Mina, Jerry Yi Xiao. Return to RiskMetrics: the Evolution of a Standard. RiskMetrics, April 2001. http://www.riskmetrics.com/.
4. An improved methodology for measuring VaR. Peter Zangari. RiskMetrics Monitor: Second quarter 1996.
5. James Engel, Marianne Gizycki. Conservatism, Accuracy and Efficiency: Comparing Value-at-Risk Models. March, 1999
6. Supervisory Framework for the Use of "Backtesting" in Conjunction with the Internal Models Approach to Market Risk Capital Requirements. Basle Committee on Banking Supervision. January, 1996. http://www.bis.org/.