Monetary Policy under Balance Sheet Uncertainty

Saki Bigio
Universidad del Pacifico
and Central Reserve Bank of Peru

Marco Vega
LSE
and Central Reserve Bank of Peru

Draft: February 2005
Preliminary Version - Comments welcomed

Abstract

A group of developing countries bear high rates of financial dollarisation. Under this circumstance, monetary-policy makers are uncertain about the presence and scale of potentially harmful effects that might appear because of balance sheet mismatches arising from high and unexpected depreciations of the domestic currency.

We build a setup whereby central bankers have two competing models in mind. Model A is a standard model for a small open economy whereas Model B has a built-in non-linear balance sheet effect. Whether the balance sheet mismatch problem exists or not, a Bayesian optimization procedure that assigns a positive probability to Model B, perpetuates model-indeterminacy. This happens because the optimal Bayesian regulator does not allow sizeable exchange rate swings (dirty floating), and therefore blurs the information to distinguish among models. We call this effect the Balance Sheet Trap.

We show that, given the presence of the Balance Sheet Trap, introducing the learning dynamics into the central banker's problem is optimal. Thus, we argue that intentional policy experimentation is highly desirable since it provides for an escape to the Balance Sheet Trap.

JEL Classifications: C11, C61, E52

Key Words: Balance Sheet Effect, Model Uncertainty, Monetary Policy, Policy Experimentation

1 Introduction

A country is financially dollarised whenever its inhabitants hold assets and liabilities denominated in both; domestic and foreign currency. In turn, the existence of financial dollarisation creates uncertainty about the presence and scale of potential harmful effects arising from mismatches on the budget constraint of agents.

The harmful after-effects result from high and unexpected exchange rate depreciations. There is a group of empirical and theoretical works[1] that has documented contractionary effects of exchange rate depreciations due to imbalances in the asset-liability positions of economic agents. This occurs because non-tradable agents hold assets denominated in domestic currency but liabilities denominated in US Dollars. Without insurance to cover financial losses arising from the sudden mismatch in balance sheets, agents might enter default.

On the other hand, high unexpected depreciations might be beneficial to agents that produce some fraction of their income in US Dollars[2]. The sudden shift in the value of the nominal exchange rate translates to a sudden shift in the real exchange rate and therefore produces beneficial wealth and competitiveness effects.

Both types of potential outcomes are not distinct though. It all depends on which type of agent is affected most and how the sequence of effects unfolds. Moreover, it is very likely that the effects will offset one another depending on the magnitude of the shock. Precisely, this is the difficulty faced by policy makers in financially dollarised economies: being uncertain about the latent overall effects underlying sizeable exchange rate movements.

The problem for the policy makers is therefore not trivial. As Goodhart (2005) puts it, a central bank is mostly concerned about three facets of stability: Domestic price stability, external stability of the value of the currency, and overall stability in the financial system. Failure to prevent or dampen the sources and effects of instability are costly to society.

Uncertainty about the potential financial instability underlying high rates of financial dollarisation is therefore a major policy issue. However it has not been taken into account in the literature even though it is essential for optimal policy analysis in small, financially dollarised economies. The aim of this paper is precisely this, to tackle model uncertainty through the presence of two rival models that deliver opposite effects from unexpected exchange rate movements. Just as in a real world environment, the policy maker does not know which of the two models drives the true data generating process.

If policy makers miss the true model, then the outcomes in terms of the data generating process might render higher losses compared to informationally efficient outcomes. Thus, according to Brainard (1967), policy makers will tend to optimize in a Bayesian fashion depending on a priori beliefs about the validity of the two models at hand.

The problem with a Bayesian optimization procedure is that model prior probabilities will finally depend on the policies behind. Since the distinction among models will only be clear once large swings in the exchange rate are observed, a policy that does not allow them will

---

[1] Notably Krugman (1999), Calvo and Reinhart (2002), Céspedes et.al (2004) and Frankel (2005).

[2] As exporters do in their regular business or agents holding assets in that currency.

unintentionally render model indeterminacy. If model indeterminacy is caused by a fear of floating policy we will say the economy faces a Balance Sheet trap.

In the paper we propose that if an economy faces a Balance Sheet trap, social welfare is suboptimal since model certainty allows for correct policy decisions. In that sense, we argue that intentional policy experimentation as proposed by Cogley et.al (2005) may be desirable. This experimentation will imply a monetary policy rule that deviates for the optimal Bayesian because it will also weight the fact that exchange rate swings will provide information on what the true model is. In this manner, policy experimentation will provide an escape to the trap.

Up to this point, the main goal of this paper is to highlight that a Monetary Policy that faces Balance Sheet Uncertainty should involve intentional policy experimentation. For this reason, in this initial version we keep the core models unrealistically simple (and static): because we want to emphasize the benefits of policy experimentation in a non-linear world. Thus, the only dynamics in this version of the paper are those of the learning mechanisms.

Regarding future versions of this paper, we have many tasks to still do. First, we have purposely left the models as simple and as static as possible, precisely to capture the dynamic benefits of policy experimentation. We still have work to do in terms of setting our experiments with richer model dynamics capturing balance sheet effects. Having a better-suited structure will allow us to take the model to real data. Additionally, we are aware of the fact that proper loss function needs to be supported by this structure. We should be capable of collapsing this loss into a single Bayesian loss function.

The paper continues as follows: In Section 2, a block of exercises solve optimal policies under the unrealistic assumption that the central banks knows with certainty either of the two models A or B. Then in Section 3, we solve a second block of models, in which the central banks uses the wrong model. In Section 4 we deal with model uncertainty and present the optimal Bayesian policies. Section 5 refers to the Balance sheet trap whereas in Section 6 we explore the benefits of policy experimentation once the economy falls into the trap. We introduce learning dynamics and solve for the dynamic problem in this section. Finally, we conclude in Section 6.

2   Monetary Policy under Model Certainty

{SEC2}

We start with an ideal case of certainty about the model that drives the path of an small open economy. This case will serve as a benchmark for comparison to other informational assumptions about uncertainty detailed in next sections. We depict two models, Model A describes a standard mechanism of a small open economy whereby exchange rate depreciations are expansionary. Model B describes an economy governed by a non-linear exchange rate effect that works in the opposite direction than that of Model A.

2.1 Model A: Standard model of the economy

This model is characterized by three equations which are perfectly known by policy makers. The model describes the simple transmission mechanism of a small open economy[3].

$$\pi_t = \gamma_1 y_t + \gamma_2 \delta_t + \varepsilon_{\pi,t} \qquad (1) \quad \{PC\}$$

$$\delta_t = -i_t + \varepsilon_{\delta,t} \qquad (2) \quad \{UIP\}$$

$$y_t = \theta_1 i_t + \theta_2 \delta_t + \varepsilon_{y,t} \qquad (3) \quad \{GAP\}$$

Equation [1] describes the behaviour of π_t (in deviations from its target rate) as a function of the output gap (y_t), a cost-push factor determined by the exchange rate depreciation[4] rate δ_t, and an iid disturbance ($\varepsilon_{\pi,t}$). Here, the parameters γ_1 and γ_2 are both positive.

Next, the depreciation of the currency is inversely affected by the position of the monetary policy instrument relative to some neutral rate[5]. Variable δ_t is also affected by the shock $\varepsilon_{\delta,t}$ which is known to be an AR(1) process of the form

$$\varepsilon_t^{\delta} = \rho \varepsilon_{t-1}^{\delta} + \mu_t \qquad (4) \quad \{AR1\}$$

with a gaussian noise μ_t. This means that, up to some extent, the policy instrument can offset exchange rate shocks.

Last, equation [3] describes how the output gap is determined as a function of the policy instrument, the depreciation rate and a shock. Monetary policy can restrain the output gap via $\theta_1 \leq 0$ while in this economy, depreciations are expansionary ($\theta_2 \geq 0$).

There is no intrinsic persistence in any of the main driving equations but the process for the exchange rate shocks. This means that monetary policy only reacts to those shocks. In order to properly characterize monetary policy, we introduce a standard optimality criterion: The central bank sets the instrument i_t to minimize an evenly weighted loss function

$$L = E_t \sum_{s=t} \beta^{s-t} \left(\pi_s^2 + y_s^2 \right) \qquad (5) \quad \{LOSS\}$$

---

[3] The standard lagged endogenous and expectational variables featuring dynamic, rational expectation neo-Keynesian models are omitted in this first version of the paper.

[4] The nominal exchange rate is given by the domestic price of the US Dollar and a depreciation (δ_t) of the currency means a increase in this variable.

[5] This neutral rate can be an exogenous comparable world interest rate or a domestic neutral interest rate.

against the backdrop of the small open economy defined by equations [1-3]. The solution procedure here is done in a discretionary fashion, namely, the central banks optimizes period by period. As a result, given the information at any given time, the setting of the instrument is as follows (See Appendix):

$$i_t = \Lambda \varepsilon^\delta_{t-1} \qquad \qquad (6) \quad \{\texttt{RULE}\}$$

When the exchange rate shock unexpectedly hits, it propagates instantaneously to inflation via both the cost-push factor and the expansionary effect on the output gap. This shock is expected to have persistence, so in order to dampen further future loss-making deviations of inflation and the output gap, the interest rate is raised ($\Lambda > 0$).

2.2 Model B: Introducing contractionary depreciations (Balance Sheet Effect)

In this case, the inflation and the exchange rate equation behave in the same way as Model A. However, the evolution of the output gap is driven by a function that behaves non-linearly to exchange rate depreciations. For small exchange rate fluctuations the behavior is fairly linear and similar to that of model A. Namely, exchange rate depreciations increase the output gap for standard competitiveness reasons. The important change occurs when the shocks are large. In this situation, a large exchange rate depreciation is indeed bad for the economy given that it may lead to a financial collapse with real consequences.

Therefore model B captures the financial fragility consequences of exchange rate fluctuations common in the policy debate of financially dollarised economies. The output gap equation in this model is now

$$y_t = \theta_1 i_t + \theta_2 \delta_t + \theta_3 \delta_t^2 + \varepsilon_{y,t} \qquad \qquad (7) \quad \{\texttt{GAPD}\}$$

Here, the parameter θ_3 is negative. Hence, a large enough variation in δ_t lowers the output gap. Facing model B, what would an optimizing central bank do against an otherwise same exchange rate shock as in model A? A large enough shock depresses the output gap but boosts inflation. If the interest rate is increased to ameliorate future exchange rates then the output gap cost is high, otherwise the inflationary cost is high.

In this case, there is no explicit form for the optimal solution given that the problem is not linear-quadratic anymore. However, the solution is characterised in Figure 1 where we depict interest rate response and the resulting nominal exchange rate depreciation resulting from exchange rate shocks. The continuous lines represent the certainty solution under Model A whereas the discontinuous plots show the solution under Model B.

The optimal solution implies a smoother exchange rate equilibrium response but a greater interest rate response to shocks. However, when the size of the exchange rate shock is small

5

in absolute value, both types of model solutions are virtually unrecognizable from each other. Another interesting insight of the solution to Model B is the underlying asymmetric optimal rule. Large positive shocks elicit a stronger response on interest rates than large negative shocks of the same absolute size[6]. This result is broadly in line with asymmetric rule proposals outlined in recent literature related to Balance Sheet affected economies such us Lahiri and Vegh (2001) and Morón and Winkelried (2005). Furthermore, the optimal rule for this model allows for greater appreciations than depreciations for shocks of the same absolute magnitude.
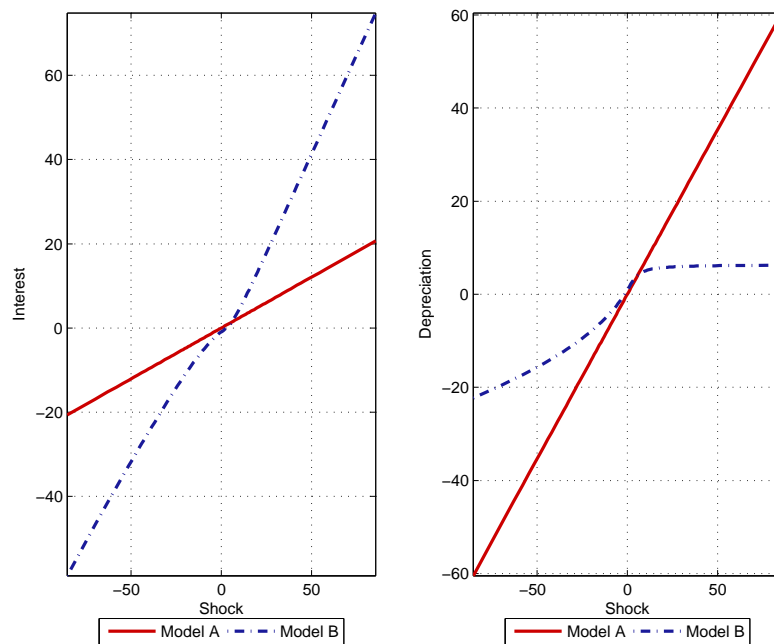

Figure 1: Optimal Rule Comparison - Model A vs. Model B   {Fig1}

3   Misspecification and Policy Outcomes

{SEC3}

This section performs the exercise of model misspecification. In this case, the central bank can mistakenly believe that the economy accords to one model when in fact the true model is quite different. As time evolves, there is no learning mechanism in place and so, the central bank remains ever ignorant about the true model. The outcome of this extreme exercise is obviously not optimal. However, the measured degree of sub-optimality serves also as a benchmark for the more interesting case where learning is in place.

The first type of misspecification assumes that the central bank mistakenly believes that the economy is financially dollarized and that there are balance sheet effects inducing

---

[6] In real world situations, it is not the interest rate but other type of instruments such us FX interventions that might react in this fashion.

contractionary exchange rate depreciations[7]. The second exercise considers a central bank that neglects the presence of a balance-sheet problem and thus, thinks that exchange rate depreciations are always expansionary.

The exercise consists in simulating the economy under the respective true model but solving for the policy instrument under either central bank misspecification. In Table 3 we report the standard deviations as ratios to the standard deviations obtained by simulating Model A with exact certainty.

The exercise reveals that the volatility of the output gap is significantly greater when the true model has a balance sheet mechanism. For this reason, the central bank's loss is increased significantly solely with the presence of this channel. More importantly, the exercise reveals that neglecting the balance sheet effect is much more harmful than mistakenly assuming its existence. Hence, this asymmetry in the loss will bias the monetary policy to behave as if the true model does have a balance sheet effect.

Thus, in a first instance, we find a rationale to the fear of balance sheet effect. Moreover, it is this characteristic that will bias a Bayesian policy towards behaving as the balance sheet policy. We enter the Bayesian grounds to solving the problem in the following section.

4 Monetary Policy under Model Uncertainty (The Bayesian Policy)

{SEC4}

In this section we introduce the fact that central bankers are uncertain about which the actual model driving the economy. Therefore, they assign a probability p_t to model A and a probability $(1 - p_t)$ to model B. This probability evolves according to the odds ratio, i.e, a combination of model fit and their shocks and an initial prior belief p_0. Given no parameter uncertainty within each model, and simply taking into account model uncertainty the expected loss function for period t becomes

$$L_t = p_t L_{t|A} + (1 - p_t) L_{t|B} \tag{8}$$

{BLOSS}

Where $L_{t|A}$ and $L_{t|B}$ represent the losses that would occur conditional on central bank reacting as if model A or B were true. Both loss functions can be represented in terms of the instrument rate i_t and the observed lagged exchange rate disturbance $\varepsilon_{\delta,t}$ (See Appendix for details). The expected loss function in [8] has all parameters which are allowed to vary in time if we introduce parameter uncertainty, otherwise, they remain constant over time.

The first-order condition for optimality, conditional on the parameter values is:

---

[7] Conversely, it thinks that exchange rate appreciations are expansionary

$$p_t \left[2(A_{1t}^2 + B_{1t}^2) i_t + 2(A_{1t}A_{2t} + B_{1t}B_{2t}) \rho \varepsilon_{t-1}^{\delta} \right] + \qquad (9) \quad \{\text{BFOC}\}$$

$$(1 - p_t^1) \left[4\lambda_{4t} i_t^3 + 3\lambda_{3t} i_t^2 + 2\lambda_{2t} i_t + \lambda_{1t} \right] = 0$$

which is weighted average of two factors representing the optimal condition under each model alone. If the probability for model A at time t is small, then policy instrument resulting in [9] looks very similar to the one obtained in the exact certainty solution to model B. In fact, regrouping [9] allows us to write the equation as a 3rd degree polynomial form similar to the one that solves model B with certainty (See the Appendix for further details).
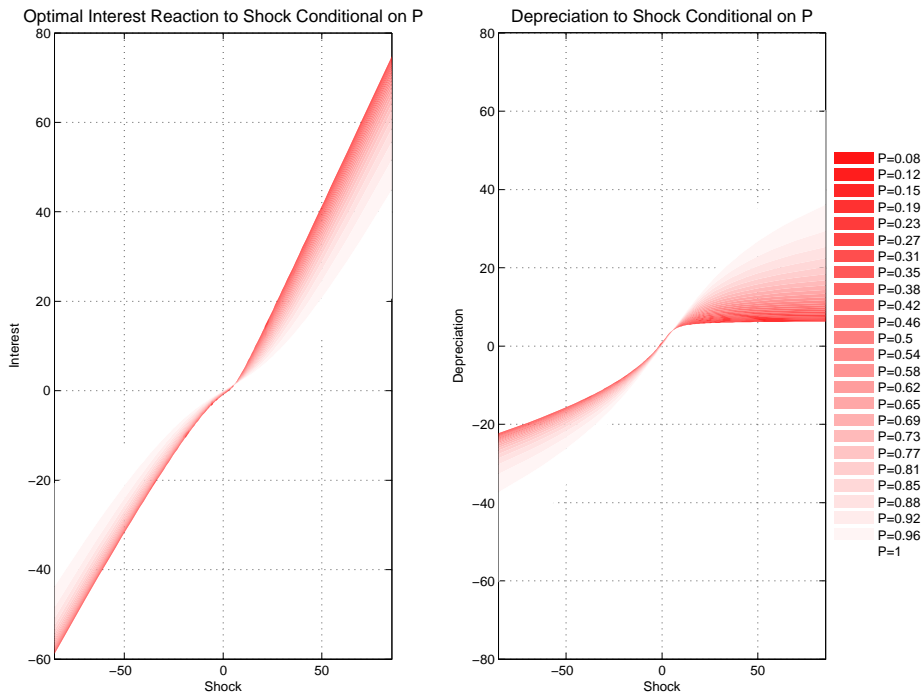

Figure 2: Optimal Rule (Bayesian) Depending on Probabilities {Fig2}

In the left panel of Figure 2 we show the optimal reaction of the interest for shocks of different size. This figures represent fan charts. For each probability p_t a colour is assigned. As the areas in the graphs gain colour density, the probability leans towards the model with Balance Sheet (tends to 0). One can observe the asymmetry in terms of the response given probabilities. Each colour is equally spaced in terms of the probability, though the denser colours are tight together in terms of the response. This means that for low model probabilities, the optimal response is very similar. For high probabilities, marginal changes make greater differences in terms of the optimal response. Something similar happens with the expected depreciation (found in the right panel) since it is a linear combination of the movements in the interest rate.

The updating of model A probability in the Bayesian approach depends on the odd's ratio

$$p_t = p\left(M^A|Data\right) = \frac{p\left(Data|M^A\right)p\left(M^A\right)}{p\left(Data|M^B\right)p\left(M^B\right) + p\left(Data|M^A\right)p\left(M^A\right)} \qquad (10) \quad \{\text{ODDS}\}$$

Following Cogley et.al (2005) this ratio can be transformed dynamically into:

$$p_t = \frac{p_{t-1}p\left(Data_t|Data_{t-1},M^A\right)}{p_{t-1}p\left(Data_t|Data_{t-1},M^A\right) + (1-p_{t-1})p\left(Data_t|Data_{t-1},M^B\right)} \qquad (11) \quad \{\text{ODDSU}\}$$

The intuition behind this ratio is that the probability that the policy maker will assign to a given model for being true will depend on a weighted average (with weights given by the previous periods model probabilities) of the likelihood of the data at period t conditional to each model and the data. It is key to perform simulations: we will show how this ratio evolves in time in the following section. We will again pay attention to this dynamics later in Section 6 once we introduce the learning dynamics into the Policy Maker's problem.

4.1 Model Comparisons

We need to simulate the model in order to compare the outcomes under uncertainty and Bayesian updating of probabilities. The baseline parameter calibration used in these simulations is given in Table 1. The parameters are calibrated in order to replicate the behaviour of quarterly data. The simulations presented in figures 3 and ?? are carried out for 25 periods and averaged over 100 replications of the same exercise.

$\beta = 0.995$	$\gamma_2 = 0.15$
$\theta_1 = -0.5$	$\sigma_\mu = 10$
$\theta_2 = 0.15$	$\sigma_y = 4$
$\theta_3 = -0.05$	$\sigma_\pi = 4$
$\gamma_1 = 0.3$	$\rho = 0.9$

4.2 Evaluation of Central Bank's Losses

We now discuss the outcome of the simulations carried out using the core calibration to compare the characteristics underlying each policy for both, when the true models is A or Model B. The initial prior for the Bayesian Policy is **0.5**. Probabilities are updated using [11]. In the top panel of Table 1 we depict the volatilities of all the variables in the models relative to Model A volatilities under perfect certainty. At a first glance, one can compare the outcomes of Models A and B. When comparing the volatility of the interest rates among the models with certainty we find that the model with a balance sheet effect has at least **4** times more volatility. The variance of the depreciation rate is $(\frac{3}{4})$ of that of the model without the balance

9

sheet effect. It is clear that in this context, the variance of output and inflation will be greater. In particular, output is **12** times more volatile while for the inflation rate the figures show the **double** of the volatility. When the real model does not have the balance sheet channel but the central bank believes it does, the volatility of output is increased by 150% whereas inflation is maintained almost at the same level. In the table [reference] we can also find the implications of missing what the right model is. In terms of losses, even though both policies imply perfect information on what the true model is, the loss is reduced more than **8** times if the balance sheet mechanism is not present.

Given that, for this calibration, the worst possible case is to misbelieve the presence of the Balance Sheet effect, it is clear how the optimal Bayesian policy will have a bias to act as if the true model were B. Missing what the true model is if the real model is B could be disastrous.

The Bayesian policy seems to be very efficient for both models. One can observe that, since probabilities are updated (and in this case we assume no parameter uncertainty) the results are similar to those found with full model certainty as opposed to the results with misspecification.

MOMENT COMPARISON AMONG MODELS WITH PARAMETER CERTAINTY

Relative Variance
(as % of the model without Balance Sheet Effect)

Variable	Output	Inflation	Depreciation	Interest Rate
(when the true model is Model A)				
Policy with Certainty	100%	100%	100%	100%
Policy with Misbeleives	212%	101%	83%	412%
Bayesian Policy	99%	105%	110%	137%
(when the true model is Model B)				
Policy with Certainty	957%	160%	83%	412%
Policy with Misbeleives	1572%	225%	100%	100%
Bayesian Policy	9923%	1587%	124%	632%

Welfare Comparison
(as % of the model without Balance Sheet Effect)

(when the true model is Model A)	
Policy with Certainty	100%
Policy with Misbeleives	165%
Bayesian Policy	114%
(when the true model is Model B)	
Policy with Certainty	717%
Policy with Misbeleives	1189%
Bayesian Policy	728%

Table 1: Moment Comparison

5   Balance Sheet Indeterminacy: The Balance Sheet Trap

{SEC5}

5.1   Evolution of Model Probabilities for both models

In the two panels of figure 3 we plot the evolution of model probabilities when the true models are Model A and Model B respectively. We find the evolution of Model probabilities given

distinct initial prior probabilities for each model. We observe that regardless of the true model, the odds ratio seems to stabilize for any initial prior probability. It seems as if the initial prior is indeed important since after 25 periods it is still determinant of the final level that is reached by the odds ratio. For both models, what the true model is relevant for the direction of the path for several of the probabilities. For both cases, regardless of the starting point, probabilities seem to converge to a given probability which is closer to the corresponding to the real model but still, not close as to render identical policies. Though, it is worth noting how for several threshold priors, the probabilities seem to change direction in the other direction.
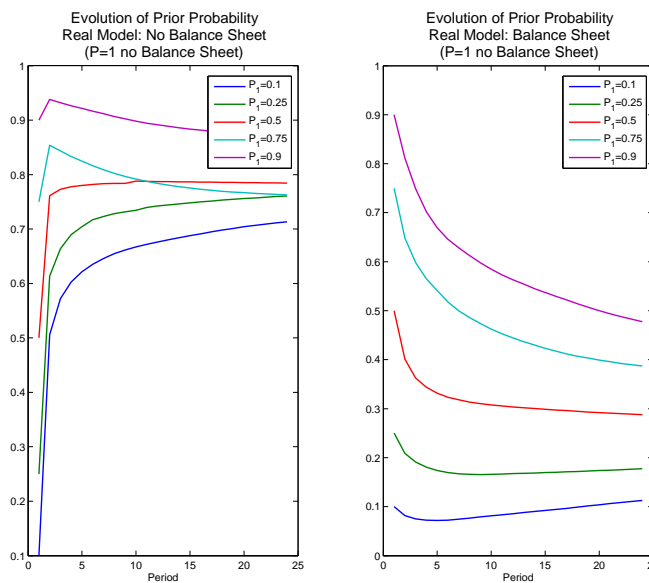

Figure 3: Evolution of the odds ratio for different priors (200 simulations)

It seems as if underlying the simulations, there is some convergence point. At this point we can elucidate what we mean by Balance Sheet Trap: if the true model is Model A (no balance sheet trap) the fact that there is a bias towards acting as if the real Model is B, does not allow for great swings in the exchange rate. In that sense are indistinguishable. The same happens when the true model is B.

Even though we argued that Bayesian policies are optimal under model uncertainty in a static world, it is a dynamically suboptimal policy. The problem is that the policymaker is not incorporating into his decision the fact that greater swings in the exchange rate allow identification. If this were the case, is it probable that the convergence in probability could be achieved at a point closer to the true model. In the short run, experimentation policies may not be loss optimizing but they are all over the long run since they will tend to stabilize they priors at a more certain point.

5.2 Digression: The relevance of Parameter Uncertainty

Parameter uncertainty is fundamental. We can show how this issue may indeed distort the results of the evolution of model probabilities. If central banks know what the true model is after several periods, the data will sort out the real model. Since a wrong model has a constant bias by definition, it is clear that it will retard the learning process.

The two plots in Figures 4 capture precisely this feature when the true model is A and B respectively. The colored areas represent the evolution of the prior probabilities with parameter certainty starting from a 0.5 prior probability. The dashed lines show the same sequence but in this case parameters are reestimated every period. The upper panel in Figure 4 shows that the prior with parameter uncertainty converges to its true value. What is actually happening is that the prior is collapsing but since the parameter regarding the balance sheet is equal to zero, Model B collapses to Model A when it is true.

When the true model is Model B, the learning process is retarded as previously discussed. We observe this in the low panel.

Though qualitatively distinct, we want to argue that parameter uncertainty does not bring significantly distinct results to the main argument of this paper.

6 Endogenizing the Learning Dynamics: an Escape to the Trap

{SEC6}

In the previous sections we find how the learning dynamics of this model can lead to what we can call *balance sheet trap*. In this section we provide an escape to the trap. In order to do so, we develop an exercise in the spirit of Cogley et.al (2005). The idea is to answer if policy experimentation is desirable, in the sense that it not only provides an escape to the trap but a significant reduction in the central bank loss.

So, we incorporate, the fact that policy decisions will affect model certainty in the future. Relying in the core-Bayesian model, we now re-define the problem, in order to incorporate the dynamics of the Prior probability into the central banker's problem. This problem can be re-formulated as:

$$L \;=\; E_t \left[\sum_{s=t} \beta^{s-t} (\pi_s^2 + y_s^2 \mid P_s) \right]$$
$$\text{s.t. } 11$$

Which may be also expressed as the following Bellman equation

$$V(\varepsilon_{\delta,t}, P_t) = \min_{\{i_t\}} E[(\pi_t(i_t))^2 + (y_t(i_t)/P_t)^2] + \beta E[P_t V(\varepsilon_t^\delta, P_{t+1}^1) + (1-P_t)E[V(\varepsilon_t^\delta, P_{t+1}^2)] \quad (12) \quad \{\text{VALUE}\}$$

12

We solve this equation numerically as explained in the appendix A.3. The value function for this problem is

If we take slices from the Model Probability axis in Figure 5 we can observe the quadrature of the loss function. The greater in magnitude the shocks, the greater the loss. Additionally, we can also see how for the same shock, as probabilities tend to the model with Balance Sheet, the loss function grows decreasingly (i.e. the Loss Function for this problem is concave in probability).

The evolution of model probabilities should be faster with policy experimentation. We show that this is true regardless of which is the true model. In figures 6 and **??** we show the evolution of model probabilities when the true model does not include a Balance Sheet Mechanism and when it does. The solid areas represent the evolution of probabilities in the general Bayesian Policy and the dashed lines show the same evolution when policy experimentation is introduced. From the figures we can see how policy experimentation increases the speed of learning.

6.1   Loss Evaluation in the Context of Policy Experimentation

[Section to be Completed after Revision]

7   Conclusions: Policy Experimentation and More to Come

{SEC7}

This paper has a strong policy recommendation: Monetary Policy Makers should take into account their own learning when they are uncertain about the presence of a Balance Sheet effect. In this paper we argue that there are chances that a dollarized economy may fall in what we call as a Balance Sheet Trap. For the baseline calibration, an efficient way out of the trap is policy experimentation which yields benefits in terms of reduced losses.

As mentioned in the introduction, we will add a richer model dynamics to better match the data. This could let us reverse engineer the policies carried out in dollarized economies. We could then build an idea of how model probabilities have evolved over time and capture trends. We could then argue if countries have fallen into Balance Sheet traps or not.

On the other hand, several issues are still worth exploring further on. For instance the benefits from Policy Experimentation may reduce loss substantially if agents rationally endorse what the policymakers will do. If they are aware of the fact that policymakers will allow greater swings in the exchange rate, perhaps this becomes a sufficient incentive to use hedging mechanisms and develop the capital market in such a manner that the Balance Sheet effect is reduced. Obviously, this would complicate significantly the study. Central banks would have to estimate parameters that are endogenous (perhaps a micro-founded DSGE structure) and incorporate this endogeneity into the dynamic problem. We have the feeling that in this context, results would be very similar to the ones found in Chang and Velasco (2004).

References

Armas, A. and F. Grippa(2005), Targeting Inflation in a Dollarized Economy: The Peruvian Experience, IADB Working Paper.

Brainard, W. (1967), Uncertainty and the Effectiveness of Policy, *American Economic Review*, 57, Papers and Proceedings, 411-425.

Calvo, G. and C. Reinhart(2002), Fear of Floating, *Quarterly Journal of Economics*, 117, 379-408.

Calvo, G. and C. Reinhart(2002), Fear of Floating, *Quarterly Journal of Economics*, 117, 379-408.

Cavallo, M., K. Kisselev, F. Perri and N. Roubini(2004), Exchange Rate Overshooting and the Costs of Floating, Mimeo, New York University.

Céspedes, L., R.Chang and A. Velasco(2002), Balance Sheets and Exchange Rate Policy, *American Economic Review* 94 (September) pp. 1183–1193

Chang R. and A. Velasco(2004), Monetary Policy and the Currency Denomination of Debt: A Tale of Two Equilibria, *National Bureau of Economic Research* 10827

Cogley, T. R. Colacito and T. Sargent(2005), Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas, Working Paper.

Frankel J.(2005), Contractionary Currency Crashes in Developing Countries, *IMF Staff Papers*, 52, No. 2.

Goodhart C.(2005), The Future of Central Banking, FMG Special Paper No. 162.

Krugman, P.(1999), Balance Sheets Effects, the Transfer Problem and Financial Crises, in Isard P., A. Razin and A.K. Rose, eds., *International Finance and Financial Crises: Essays in Honour of Robert P. Flood, Jr.*, (New York: Kluwer Academic Publishers), pp. 31-44.

Lahiri A. and C. and Vegh(2001),Living with a Fear of Floating: An Optimal Policy Perspective, NBER Working Paper 8391.

Morón E. and D. Winkelried(2005), Monetary Policy Rules for Financially Vulnerable Economies, *Journal of Development Economics*, 76, 23-51.

A Appendix

A.1 Solution to the Model Without Balance Sheet

$$\min_{\{i_t\}} E \sum_t (\pi_t)^2 + (y_t)^2$$

we have for period t:

$$
\begin{aligned}
&(\gamma_1 y_t + \gamma_2 \delta_t + \varepsilon_t^\pi)^2 + (\theta_1 i_t + \theta_2 \delta_t + \varepsilon_t^y)^2 \\
= \quad & (\gamma_1(\theta_1 i_t + \theta_2(-i_t + \varepsilon_t^\delta) + \varepsilon_t^y) + \gamma_2(-i_t + \varepsilon_t^\delta) + \varepsilon_t^\pi)^2 + (\theta_1 i_t + \theta_2(-i_t + \varepsilon_t^\delta) + \varepsilon_t^y)^2 \\
= \quad & ((\gamma_1(\theta_1 - \theta_2) - \gamma_2)i_t + (\gamma_1\theta_2 + \gamma_2)\varepsilon_t^\delta + \gamma_1\varepsilon_t^y + \varepsilon_t^\pi)^2 + ((\theta_1 - \theta_2)i_t + \theta_2\varepsilon_t^\delta + \varepsilon_t^y)^2
\end{aligned}
$$

using the variable redefinitions from

$$
\begin{aligned}
A_1 &= (\gamma_1(\theta_1 - \theta_2) - \gamma_2) = (\gamma_1 B_1 - \gamma_2) < 0 \\
A_2 &= (\gamma_1\theta_2 + \gamma_2) = (\gamma_1 B_2 + \gamma_2) > 0 \\
B_1 &= (\theta_1 - \theta_2) < 0 \\
B_2 &= \theta_2 > 0
\end{aligned}
$$

so the static problem is:

$$
\begin{aligned}
&\max_{\{i_t\}} E [(A_1 i_t + A_2 \varepsilon_t^\delta + \gamma_1 \varepsilon_t^y + \varepsilon_t^\pi)^2 + (B_1 i_t + B_2 \varepsilon_t^\delta + \varepsilon_t^y)^2] \\
= \quad & \max_{\{i_t\}} E [(A_1 i_t)^2 + (A_2 \varepsilon_t^\delta)^2 + (\gamma_1 \varepsilon_t^y)^2 + (\varepsilon_t^\pi)^2 + ... \\
& ...2A_1 A_2 i_t \varepsilon_t^\delta + 2A_1 \gamma_1 i_t \varepsilon_t^y + 2A_1 i_t \varepsilon_t^\pi + 2A_2 \gamma_1 \varepsilon_t^\delta \varepsilon_t^y + 2A_2 \varepsilon_t^\delta \varepsilon_t^\pi + 2\gamma_1 \varepsilon_t^\delta \varepsilon_t^y ... \\
& ...(B_1 i_t)^2 + (B_2 \varepsilon_t^\delta)^2 + (\varepsilon_t^y)^2 ... \\
& 2B_1 B_2 i_t \varepsilon_t^\delta + 2B_1 i_t \varepsilon_t^y + 2B_2 \varepsilon_t^\delta \varepsilon_t^y]
\end{aligned}
$$

or regrouping and taking expectations:

$$
\begin{aligned}
= \quad & \max_{\{i_t\}} E [(A_1^2 + B_1^2)i_t^2 + 2(A_1 A_2 + B_1 B_2)i_t \rho_{t-1} \varepsilon_{t-1}^\delta] \\
& + (A_2^2 + B_2^2)(\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}) + \gamma_1^2 \sigma_t^{2y} + \sigma_t^{2\pi}
\end{aligned}
$$

15

assuming the following about the stochastic processes:

$$\varepsilon_t^\delta = \rho\varepsilon_{t-1}^\delta + \mu_t; \mu_t \sim N(0, \sigma_\mu^2)$$
$$\varepsilon_t^y \sim N(0, \sigma_y^2)$$
$$\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$$

we have that the first order condition takes the following form:

$$A_1^2 i_t + A_1 A_2 \rho\varepsilon_{t-1}^\delta + B_1^2 i_t + B_1 B_2 \rho\varepsilon_{t-1}^\delta = 0$$
$$\frac{-(A_1 A_2 + B_1 B_2)}{(A_1^2 + B_1^2)} E_t(\varepsilon_t^\delta) = i_t$$
$$\frac{-(A_1 A_2 + B_1 B_2)}{(A_1^2 + B_1^2)} \rho\varepsilon_{t-1}^\delta = i_t$$

$$\frac{-(A_1 A_2 + B_1 B_2)}{(A_1^2 + B_1^2)} \rho\varepsilon_{t-1}^\delta = i_t$$

where

$$A_1 = (\gamma_1(\theta_1 - \theta_2) - \gamma_2) = (\gamma_1 B_1 - \gamma_2)$$
$$A_2 = (\gamma_1 \theta_2 + \gamma_2) = (\gamma_1 B_2 + \gamma_2)$$
$$B_1 = (\theta_1 - \theta_2)$$
$$B_2 = \theta_2$$

This optimal rule minimizes the problem as long as:
$A_1^2 + B_1^2 > 0$;always true if it follows that

$$A_1^2 \neq 0$$
$$(\gamma_1 B_1 - \gamma_2)^2 \neq 0$$
$$B_1 \neq \frac{\gamma_2}{\gamma_1}$$
$$\theta_1 - \theta_2 \neq \frac{\gamma_2}{\gamma_1}$$

and

$$\theta_1 \neq \theta_2$$

conditions that are, both, always true given the sign restrictions.on the parameters. The central bank will always react to the shock for parameters:

$$A_1 A_2 \neq 0$$

which given the previous restriction only implies that:

$$\theta_2 \neq -\frac{\gamma_2}{\gamma_1}$$

which is guaranteed given the sign restrictions on the parameters. Thus, under this assumption, a central bank will always stabilize the exchange rate via its interest rate to optimize his problem.

A.2 Solution to the Model with Balance Sheet

Objective Function:

$$\min_{\{i_t\}} E \sum_t (\pi_t)^2 + (y_t)^2$$

were for period t we have:

$$
\begin{aligned}
& (\gamma_1 y_t + \gamma_2 \delta_t + \varepsilon_t^\pi)^2 + (\theta_1 i_t + \theta_2 \delta_t + \theta_3 \delta_t^\psi + \varepsilon_t^y)^2 \\
= \ & (\gamma_1(\theta_1 i_t + \theta_2(-i_t + \varepsilon_t^\delta) + \theta_3(-i_t + \varepsilon_t^\delta)^\psi + \varepsilon_t^y) + \gamma_2(-i_t + \varepsilon_t^\delta) + \varepsilon_t^\pi)^2 \ldots \\
& \ldots + (\theta_1 i_t + \theta_2(-i_t + \varepsilon_t^\delta) + \theta_3(-i_t + \varepsilon_t^\delta)^\psi + \varepsilon_t^y)^2 \\
= \ & ((\gamma_1(\theta_1 - \theta_2) - \gamma_2)i_t + (\gamma_1\theta_2 + \gamma_2)\varepsilon_t^\delta + \theta_3(-i_t + \varepsilon_t^\delta)^\psi + \gamma_1\varepsilon_t^y + \varepsilon_t^\pi)^2 \ldots \\
& \ldots + ((\theta_1 - \theta_2)i_t + \theta_3(-i_t + \varepsilon_t^\delta)^\psi + \theta_2\varepsilon_t^\delta + \varepsilon_t^y)^2
\end{aligned}
$$

Redifing variables:

$$
\begin{aligned}
A_1 &= (\gamma_1(\theta_1 - \theta_2) - \gamma_2) = (\gamma_1 B_1 - \gamma_2) \\
A_2 &= (\gamma_1\theta_2 + \gamma_2) = (\gamma_1 B_2 + \gamma_2) \\
B_1 &= (\theta_1 - \theta_2) \\
B_2 &= \theta_2 \\
g(i_t, \varepsilon_t^\delta) &= \theta_3(-i_t + \varepsilon_t^\delta)^\psi
\end{aligned}
$$

17

so the static problem is:

$$\max_{\{i_t\}} E_t[(A_1 i_t + A_2 \varepsilon_t^\delta + \gamma_1 g(i_t, \varepsilon_t^\delta) + \gamma_1 \varepsilon_t^y + \varepsilon_t^\pi)^2 + (B_1 i_t + g(i_t, \varepsilon_t^\delta) + B_2 \varepsilon_t^\delta + \varepsilon_t^y)^2]$$

$$= \max_{\{i_t\}} E_t[(A_1 i_t)^2 + (A_2 \varepsilon_t^\delta)^2 + (\gamma_1 g(i_t, \varepsilon_t^\delta))^2 + (\gamma_1 \varepsilon_t^y)^2 + (\varepsilon_t^\pi)^2 + ...$$

$$...2A_1 A_2 i_t \varepsilon_t^\delta + 2A_1 \gamma_1 i_t \varepsilon_t^y + 2A_1 i_t \varepsilon_t^\pi + 2A_2 \gamma_1 \varepsilon_t^\delta \varepsilon_t^y + 2A_2 \varepsilon_t^\delta \varepsilon_t^\pi + 2\gamma_1 \varepsilon_t^\delta \varepsilon_t^y ...$$

$$...2A_1 \gamma_1 g(i_t, \varepsilon_t^\delta) i_t + 2A_2 \gamma_1 g(i_t, \varepsilon_t^\delta) \varepsilon_t^\delta + 2\gamma_1^2 g(i_t, \varepsilon_t^\delta) \varepsilon_t^y + 2\gamma_1 g(i_t, \varepsilon_t^\delta) \varepsilon_t^\pi$$

$$...(B_1 i_t)^2 + (B_2 \varepsilon_t^\delta)^2 + (\varepsilon_t^y)^2 + g(i_t, \varepsilon_t^\delta)^2 ...$$

$$...2B_1 B_2 i_t \varepsilon_t^\delta + 2B_1 i_t \varepsilon_t^y + 2B_2 \varepsilon_t^\delta \varepsilon_t^y ...$$

$$...2B_1 g(i_t, \varepsilon_t^\delta) i_t + 2B_2 g(i_t, \varepsilon_t^\delta) \varepsilon_t^\delta + 2g(i_t, \varepsilon_t^\delta) \varepsilon_t^y]$$

assuming the following about the stochastic processes:

$$\varepsilon_t^\delta = \rho \varepsilon_{t-1}^\delta + \mu_t; \mu_t \tilde{} N(0, \sigma_\mu^2)$$
$$\varepsilon_t^y \tilde{} N(0, \sigma_y^2)$$
$$\varepsilon_t^\pi \tilde{} N(0, \sigma_\pi^2)$$

we have that the first order condition takes the following form:

$$0 = (A_1^2 + B_1^2) i_t + \frac{(\gamma_1^2 + 1)}{2} \frac{\partial E_t g(i_t, \varepsilon_t^\delta)^2}{\partial i} + (A_1 \gamma_1 + B_1) \left(g(i_t, \varepsilon_t^\delta) + i_t \frac{\partial E_t g(i_t, \varepsilon_t^\delta)}{\partial i} \right)$$

$$... + (A_2 \gamma_1 + B_2) \left(\frac{\partial E_t g(i_t, \varepsilon_t^\delta) \varepsilon_t^\delta}{\partial i} \right) + (A_1 A_2 + B_1 B_2) \rho \varepsilon_{t-1}^\delta$$

for $\psi = 0$ we have:

$$E_t g(i_t, \varepsilon_t^\delta)^2 = E_t \theta_3^2 (-i_t + \varepsilon_t^\delta)^4$$

$$= \theta_3^2 (i_t^4 - 4i_t^3 \rho \varepsilon_{t-1}^\delta + 6i_t^2 (\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}) - 4i_t (3\rho \varepsilon_{t-1}^\delta \sigma_\mu^2 + \rho^3 \varepsilon_{t-1}^{3\delta}) + E_t \left(\varepsilon_t^\delta \right)^4)$$

18

and

$$\mathop{E}_{t} g(i_t, \varepsilon_t^{\delta}) = \theta_3(i_t^2 - 2i_t \rho \varepsilon_{t-1}^{\delta} + (\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}))$$

$$\mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})\varepsilon_t^{\delta} = \theta_3(i_t^2 \rho \varepsilon_{t-1}^{\delta} - 2i_t(\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}))$$

so taking the derivatives with respect to i_t we have that:

$$\frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})^2}{\partial i} = \theta_3^2(4i_t^3 - 12\rho \varepsilon_{t-1}^{\delta} i_t^2 + 12(\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta})i_t - 4(3\rho \varepsilon_{t-1}^{\delta}\sigma_\mu^2 + \rho^3 \varepsilon_{t-1}^{3\delta}))$$

$$\frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})\varepsilon_t^{\delta}}{\partial i} = 2\theta_3(i\rho \varepsilon_{t-1}^{\delta} - (\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}))$$

$$\frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})}{\partial i} = 2\theta_3(i - \rho \varepsilon_{t-1}^{\delta})$$

The minimizing condition becomes:

$$
\begin{aligned}
0 = & \; (A_1^2 + B_1^2)i_t + \left(\frac{\gamma_1^2 + 1}{2}\right)\left(\frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})^2}{\partial i}\right) + (A_1 A_2 + B_1 B_2)\rho \varepsilon_{t-1}^{\delta} \\
& + (A_1 \gamma_1 + B_1)\left(g(i_t, \varepsilon_t^{\delta}) + i_t \frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})}{\partial i}\right) + (A_2 \gamma_1 + B_2)\frac{\partial \mathop{E}_{t} g(i_t, \varepsilon_t^{\delta})\varepsilon_t^{\delta}}{\partial i} \\
= & \; (A_1^2 + B_1^2)i_t + 2\theta_3^2(\gamma_1^2 + 1)(i_t^3 - 3\rho \varepsilon_{t-1}^{\delta} i_t^2 + 3(\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta})i_t - (3\rho \varepsilon_{t-1}^{\delta}\sigma_\mu^2 + \rho^3 \varepsilon_{t-1}^{3\delta})) \\
& + (A_1 A_2 + B_1 B_2)\rho \varepsilon_{t-1}^{\delta} + (A_1 \gamma_1 + B_1)\left(\theta_3(i_t^2 - 2i_t\rho \varepsilon_{t-1}^{\delta} + (\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta})) + 2\theta_3(i_t^2 - i_t\rho \varepsilon_{t-1}^{\delta})\right) \\
& + 2\theta_3(A_2 \gamma_1 + B_2)(i\rho \varepsilon_{t-1}^{\delta} - (\sigma_\mu^2 + \rho^2 \varepsilon_{t-1}^{2\delta}))
\end{aligned}
$$

regrouping in terms of i_t :

$$
\begin{aligned}
0 \; = \; & 2\theta_3^2(\gamma_1^2+1)i_t^3\ldots \\
& +(-6\theta_3^2(\gamma_1^2+1)\rho\varepsilon_{t-1}^\delta + 3\theta_3(A_1\gamma_1+B_1))i_t^2\ldots \\
& +((A_1^2+B_1^2)+6\theta_3^2(\gamma_1^2+1)(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}). \\
& +2\theta_3(-2(A_1\gamma_1+B_1)+(A_2\gamma_1+B_2))\rho\varepsilon_{t-1}^\delta)i_t\ldots \\
& -2\theta_3^2\left(\gamma_1^2+1\right)\rho^3\varepsilon_{t-1}^{3\delta}+(-6\theta_3^2\left(\gamma_1^2+1\right)\sigma_\mu^2+(A_1A_2+B_1B_2))\rho\varepsilon_{t-1}^\delta\ldots \\
& +\theta_3(A_1\gamma_1+B_1-2(A_2\gamma_1+B_2))(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta})
\end{aligned}
$$

Checking

$$
\begin{aligned}
C_1 \; &= \; 2\theta_3^2(\gamma_1^2+1) > 0 \\
C_2 \; &= \; \theta_3(A_1\gamma_1+B_1) < 0 \\
C_3 \; &= \; \theta_3(A_2\gamma_1+B_2) > 0 \\
D_1 \; &= \; 3(C_2-C_1\rho\varepsilon_{t-1}^\delta) \\
E_1 \; &= \; (A_1^2+B_1^2)+3C_1(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta})+2(-2C_2+C_3)\rho\varepsilon_{t-1}^\delta \\
F_1 \; &= \; -C_1(\rho^3\varepsilon_{t-1}^{3\delta}+3\rho\varepsilon_{t-1}^\delta\sigma_\mu^2)+(A_1A_2+B_1B_2)\rho\varepsilon_{t-1}^\delta+(C_2-2C_3)(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta})
\end{aligned}
$$

so the minimization condition may be re-written as

$$
0 = i_t^3 + a_2 i_t^2 + a_1 i_t + a_0
$$

where

$$
\begin{aligned}
a_2 \; &= \; \frac{D_1}{C_1} \\
a_1 \; &= \; \frac{E_1}{C_1} \\
a0 \; &= \; \frac{F_1}{C_1}
\end{aligned}
$$

When $\theta_3 = 0$.we have the solution to the model with no Balance Sheet.

The conditions for the solution to be a local minimum imply that::

$$
0 < 3i_t^2 + 2a_2 i_t + a_1
$$

When $a_2^2 - 12a_1 > 0$, and, the minimizing root should fall out of the following interval:

$$i_t \notin \left[-\frac{1}{3}a_2 - \frac{\sqrt{a_2^2 - 12a_1}}{6}, -\frac{1}{3}a_2 + \frac{\sqrt{a_2^2 - 12a_1}}{6} \right]$$

otherwise there is only one real root which minimizes the problem and we know this because this polynomial is strictly increasing at its extremes. We know from the solution of the cubic equation that in order to have two or more real roots, and hence evaluate the optimal solution we need to have $d \geq 0$; where d is the polynomial discriminant of the cubic equation. The discriminant takes the form of:

$$
\begin{aligned}
d &= Q^3 + R^2 \\
Q &= \left(\frac{3a_1 - a_2^2}{9} \right)^3 \\
R &= \left(\frac{9a_2a_1 - 27a_0 - 2a_2^3}{54} \right)^2
\end{aligned}
$$

The general solution to the cubic polynomium is:

$$
\begin{aligned}
&-\frac{1}{3}a_2 + (S+T) \\
&-\frac{1}{3}a_2 + \frac{1}{2}(S+T) + \frac{1}{2}\iota\sqrt{3}(S-T) \\
&-\frac{1}{3}a_2 + \frac{1}{2}(S+T) - \frac{1}{2}\iota\sqrt{3}(S-T)
\end{aligned}
$$

where

$$
\begin{aligned}
S &= \sqrt[3]{R + \sqrt[2]{D}} \\
T &= \sqrt[3]{R - \sqrt[2]{D}}
\end{aligned}
$$

If $d > 0$, because we know the function is strictly increasing at its extremes we can say that the solution to the problem is: $i_t = -\frac{1}{3}a_2 + (S+T)$. Otherwise we have to evaluate the roots against the restriction and the remaining two would have to be evaluated in the original function. To have $d < 0$ we need at least to have that $3a_1 < a_2^2$. This implies that $E_1 < \frac{D_1^2}{C_1}$ which yields a very complicated set of parameter rules that guarantee a unique solution for any shock size. We thus leave the general solution to the problem to concentrate the attention in the economics of the problem.

Aside to the analytical solution, we may evaluate the three roots in the expected objective function. In order to so expand the expected objective function:

$$\underset{t}{E}g(i_t,\varepsilon_t^\delta)^2 = \underset{t}{E}\theta_3^2(-i_t+\varepsilon_t^\delta)^4$$

$$= \theta_3^2(i_t^4 - 4i_t^3\rho\varepsilon_{t-1}^\delta + 6i_t^2(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}) - 4i_t(3\rho\varepsilon_{t-1}^\delta\sigma_\mu^2+\rho^3\varepsilon_{t-1}^{3\delta}) + \underset{t}{E}\left(\varepsilon_t^\delta\right)^4)$$

and

$$\underset{t}{E}g(i_t,\varepsilon_t^\delta) = \theta_3(i_t^2 - 2i_t\rho\varepsilon_{t-1}^\delta + (\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}))$$

$$\underset{t}{E}g(i_t,\varepsilon_t^\delta)\varepsilon_t^\delta = \theta_3(i_t^2\rho\varepsilon_{t-1}^\delta - 2i_t(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}))$$

$$(A_1^2+B_1^2)i_t^2 + (A_2^2+B_2^2)(\rho^2\varepsilon_{t-1}^{2\delta}) + 2(A_1A_2+B_1B_2)i_t\rho\varepsilon_{t-1}^\delta...$$
$$+(\gamma^2+1)*\underset{t}{E}g(i_t,\varepsilon_t^\delta)^2 + 2(A_1\gamma_1+B_1)i_t\underset{t}{E}g(i_t,\varepsilon_t^\delta)...$$
$$+2(A_2\gamma_2+B_2)\underset{t}{E}g(i_t,\varepsilon_t^\delta)\varepsilon_t^\delta$$

replacing the expected values and grouping the constants we have:

$$(A_1^2+B_1^2)i_t^2 + (A_2^2+B_2^2)(\rho^2\varepsilon_{t-1}^{2\delta}) + 2(A_1A_2+B_1B_2)i_t\rho\varepsilon_{t-1}^\delta$$
$$+(\gamma^2+1)\theta_3^2(i_t^4 - 4i_t^3\rho\varepsilon_{t-1}^\delta + 6i_t^2(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}) - 4i_t(3\rho\varepsilon_{t-1}^\delta\sigma_\mu^2+\rho^3\varepsilon_{t-1}^{3\delta}) + (6\rho^2\varepsilon_{t-1}^{2\delta}\sigma_\mu^2+\rho^4\varepsilon_{t-1}^{4\delta}))$$
$$+2\theta_3(A_1\gamma_1+B_1)(i_t^3 - 2i_t^2\rho\varepsilon_{t-1}^\delta + i_t(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta})) + 2\theta_3(A_2\gamma_1+B_2)(i_t^2\rho\varepsilon_{t-1}^\delta - 2i_t(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}))$$
$$+K$$

Which regrouped yield:

$$\theta_3^2(\gamma^2+1)i_t^4$$
$$2(-2\theta_3^2(\gamma^2+1)\rho\varepsilon_{t-1}^\delta + \theta_3(A_1\gamma_1+B_1))i_t^3$$
$$((A_1^2+B_1^2) + 6\theta_3^2(\gamma^2+1)(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}) + 2(-2\theta_3(A_1\gamma_1+B_1)+\theta_3(A_2\gamma_1+B_2))\rho\varepsilon_{t-1}^\delta)i_t^2$$
$$2((A_1A_2+B_1B_2)\rho\varepsilon_{t-1}^\delta - 2\theta_3^2(3\rho\varepsilon_{t-1}^\delta\sigma_\mu^2+\rho^3\varepsilon_{t-1}^{3\delta})(\gamma^2+1)$$
$$+(\theta_3(A_1\gamma_1+B_1) - 2\theta_3(A_2\gamma_1+B_2))(\sigma_\mu^2+\rho^2\varepsilon_{t-1}^{2\delta}))i_t$$
$$(A_2^2+B_2^2)(\rho^2\varepsilon_{t-1}^{2\delta}) + \theta_3^2(\gamma^2+1)(6\rho^2\varepsilon_{t-1}^{2\delta}\sigma_\mu^2+\rho^4\varepsilon_{t-1}^{4\delta})$$

or in terms of the replaced variables:

$$\frac{C_1}{2} i_t^4$$
$$\frac{2}{3} D_1 i_t^3$$
$$(E_1) i_t^2$$
$$2F_1 i_t$$
$$(A_2^2 + B_2^2)(\rho^2 \varepsilon_{t-1}^{2\delta}) + \frac{C_1}{2}(6\rho^2 \varepsilon_{t-1}^{2\delta} \sigma_\mu^2 + \rho^4 \varepsilon_{t-1}^{4\delta})$$

or a simple quartic equation:

$$\lambda_4 i_t^4 + \lambda_3 i_t^3 + \lambda_2 i_t^2 + \lambda_1 i_t + \lambda_0$$

where:

$$\lambda_4 = \frac{C_1}{2}$$
$$\lambda_3 = \frac{2}{3} D_1$$
$$\lambda_2 = E_1$$
$$\lambda_1 = 2F_1$$
$$\lambda_0 = (A_2^2 + B_2^2)(\rho^2 \varepsilon_{t-1}^{2\delta}) + \frac{C_1}{2}(6\rho^2 \varepsilon_{t-1}^{2\delta} \sigma_\mu^2 + \rho^4 \varepsilon_{t-1}^{4\delta})$$

A.3 Solving the uncertainty case with Bayesian approach

{APP3}

The loss function

$$P_t \left((A_{1t}^2 + B_{1t}^2) i_t^2 + 2(A_{1t}A_{2t} + B_{1t}B_{2t}) i_t \rho \varepsilon_{\delta,t-1} \right)$$
$$+ (1 - P_t)(\lambda_{4t} i_t^4 + \lambda_{3t} i_t^3 + \lambda_{2t} i_t^2 + \lambda_{1t} i_{2t})$$
$$+ (P_t)((A_2^2 + B_2^2)(\sigma_\mu^2 + \rho^2 \varepsilon_{\delta,t-1}^2) + \gamma_1^2 \sigma_{y,t}^2 + \sigma_{\pi,t}^2) + (1 - P_t)\lambda_{0t}$$

the polynomial solution

23

$$
\begin{aligned}
0 &= 4\chi_4 i_t^3 + 3\chi_3 i_t^2 + 2\chi_2 i_t + \chi_1 \\
\chi_4 &= (1 - P_t^1)\lambda_{4t} \\
\chi_3 &= (1 - P_t^1)\lambda_{3t} \\
\chi_2 &= (P_t^1)(A_{1t}^2 + B_{1t}^2) + (1 - P_t^1)\lambda_{2t} \\
\chi_1 &= (P_t^1)2(A_{1t}A_{2t} + B_{1t}B_{2t})\rho\varepsilon_{t-1}^\delta + (1 - P_t^1)\lambda_{1t} \\
\chi_0 &= P_t^1((A_2^2 + B_2^2)(\sigma_\mu^2 + \rho^2\varepsilon_{t-1}^{2\delta}) + \gamma_1^2\sigma_t^{2y} + \sigma_t^{2\pi}) + (1 - P_t^1)\lambda_{0t}
\end{aligned}
$$

A.4   Solving the Bellman Equation

The Bellman equation for this problem is:

$$
\begin{aligned}
V(\varepsilon_t^\delta, P_t^1) &= \min_{\{i_t\}} \underset{t}{E}[(\pi_t(i_t))^2 + (y_t(i_t)/P_t^1)^2]\ldots \\
&\quad \ldots + \beta \underset{t}{E}[(P_t^1)V(\varepsilon_t^\delta, P_{t+1}^1) + (1 - P_t^1)\underset{t}{E}[V(\varepsilon_t^\delta, P_{t+1}^2)]
\end{aligned}
$$

where probabilities P_t^1 implies the probability at period t of believing that the true model is model 1 (i.e. no balance sheet) The discounted future value function is the interesting. It is composed by a the waged sum of to future value functions. The first of these is the expected value function given the fact that if the real model is the no balance sheet model there is an expected future probability of believing that the real model is model one (P_{t+1}^1). The second part of the equation is very similar, though the probability P_{t+1}^2 is now equal to the expected probability of believing that the true model is model 1 once we take as given that the true model is model two. Thus given this formulation, the solution of this problem takes into account, no only the current loss, as the static problems, but also the learning process taking into account the fact that the their is an expected evolution for the model prior that will finally depend on what the true model is:

The procedure for the numerical solution of this problem is:

1. First discretize the interest parity shock space using (Tauchen). We uniformly descretize the model prior space and finally the interest rate space.

2. Use an initial guess for the Value Function.

3. Loop over all model prior probabilities and the depreciation shocks;

4. Then, use the vectorization of the interest rate space and compute the contemporaneous component of the Bellman equation (we can do this since the model is static)

5. step5 For the discretization of the aggregate demand shock, we compute both, the expected model prior if the true model is 1 and if the true model is 2.

6. Compute Value Function for both models expected model prior probabilities with the use of the initial guess

7. Once the loops are done, take the output value function as the initial guess.
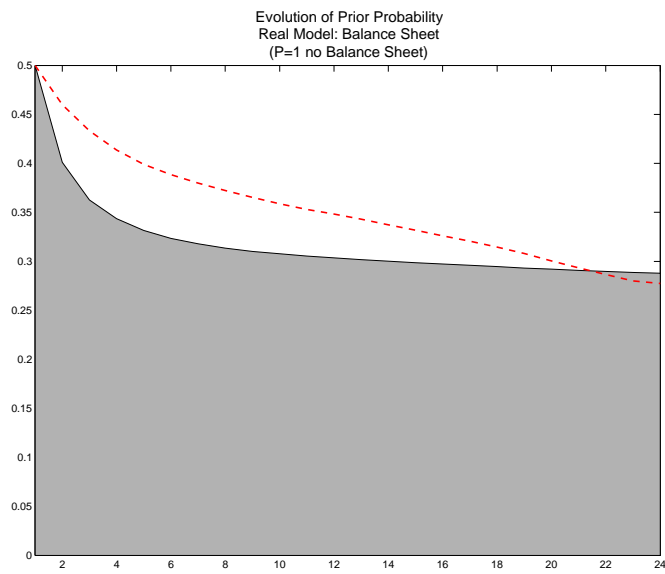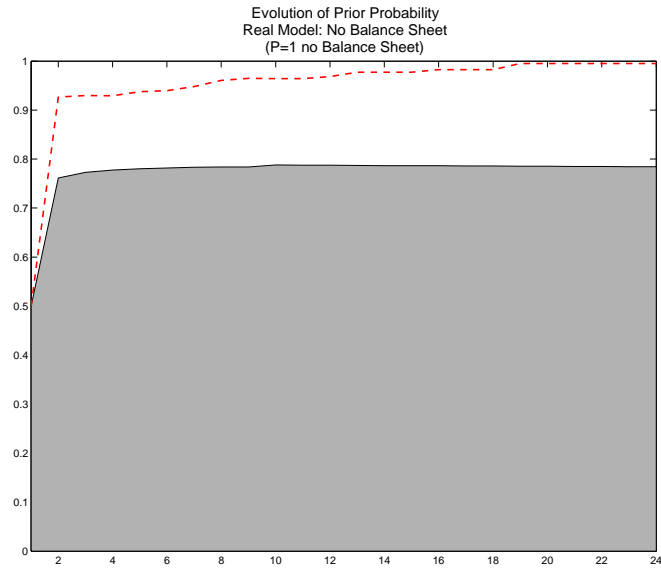
8. We repeat steps (3)-(7) until convergence.

Evolution of Prior Probability
Real Model: No Balance Sheet
(P=1 no Balance Sheet)

Evolution of Prior Probability
Real Model: Balance Sheet
(P=1 no Balance Sheet)

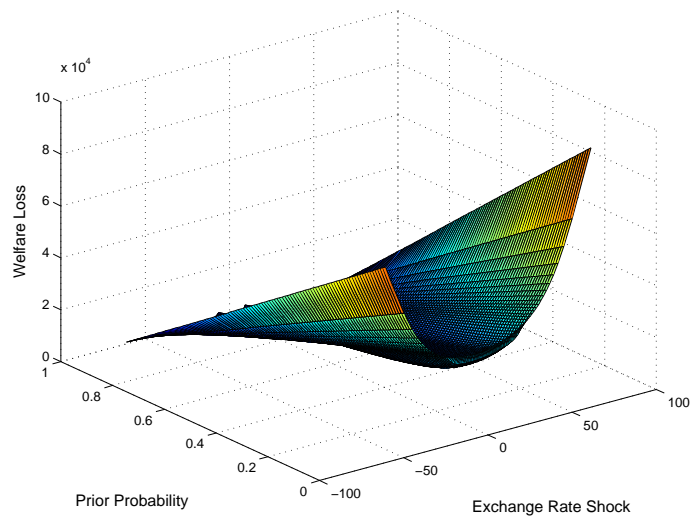Figure 4: nbs learning vs. no learning
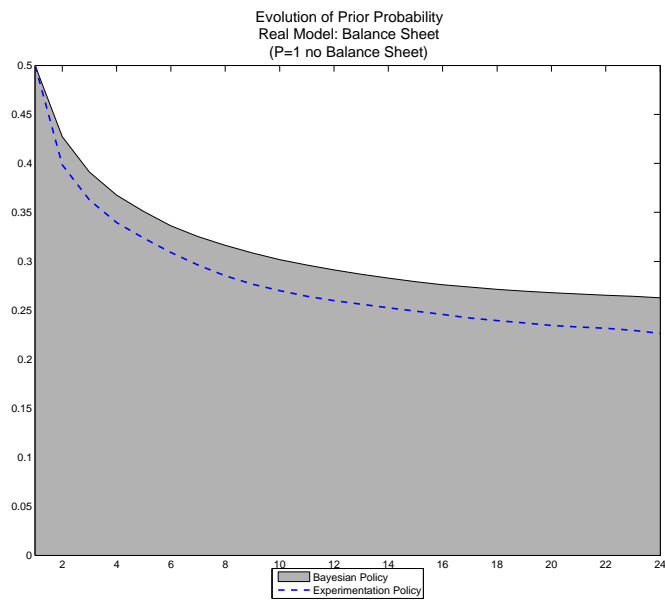
{Fig4}

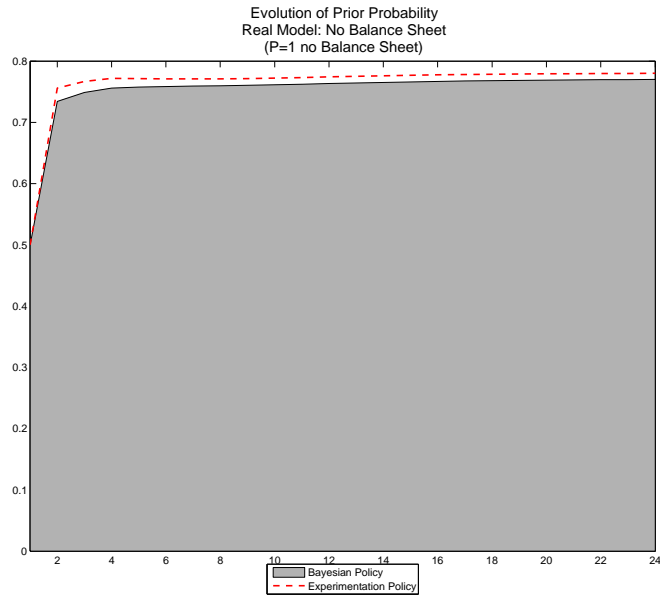Figure 5: Value Function Map

{Fig7}

27

Figure 6: Evolution of Model Prior when Real Model is No Balance Sheet

{Fig10a}

28

MOMENT COMPARISON – EXPERIMENTATION

Relative Variance
(as % of the model without Balance Sheet Effect)

Variable	Output	Inflation	Depreciation	Interest Rate
(when the true model is Model A)				
Policy with Certainty	100%	100%	100%	100%
Bayesian Policy	113%	110%	108%	128%
Policy Experimentation	1110%	197%	109%	93%
(when the true model is Model B)				
Policy with Certainty	2167%	283%	78%	419%
Bayesian Policy	2153%	284%	81%	498%
Policy Experimentation	6125%	197%	109%	109%

Loss Comparison
(as % of the model without Balance Sheet Effect)

(when the true model is Model A)	
Policy with Certainty	100%
Bayesian Policy	113%
Policy Experimentation	108%
(when the true model is Model B)	
Policy with Certainty	895%
Bayesian Policy	994%
Policy Experimentation	3179%