# On Artificial Structural Unemployment under Monopolistic Competition with a Coordinated Market Restriction 

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#### Abstract

Above market clearing wages are shown to prevail as an outcome of a game in which employers possess and employees lack the ability to coordinate. It is established in a monopolistically competitive framework that it may be optimal for individual firms to coordinate and restrict entry of indirect competitors and thus increase profits by paying above market clearing wages as the higher wage bill need not outweigh the increase in profits due to entry restriction. Resulting unemployment is shown to be socially costly. The paper notes that a tax on revenue of the incumbent firms can be welfare improving. Finally, a new perspective is cast on the issue of the real wage volatility and the business cycle.


## 1 Introduction

The objective of this paper is to present a novel contribution to the understanding of the genesis of unemployment. The paper shows that unemployment can arise in an environment where coordination on the part of employers is feasible whereas employees lack the ability to synchronize their actions. Specifically, in an explicit monopolistic competition setting the paper shows that monopolistically competitive profit maximizing firms can choose to coordinate and pay above market clearing wages in order to restrict entry by potential indirect competitors as long as the elasticity of substitution between inputs in the production function is sufficiently large. The paper establishes that whenever such coordination takes place equilibrium unemployment arises as an outcome. Moreover, it is shown that the coordination itself can be enforced in a dynamic equilibrium setup with trigger strategies. Finally, the paper casts a new perspective on the phenomenon of real wage rigidity.

[^0]The issue of unemployment has been central to macroeconomics for many decades. The understanding of the origins of unemployment is based on two competing theories: the efficiency wage model of Solow [42], Shapiro and Stiglitz [38] and the search theoretic approach of Diamond [22] and Pissarides [34]. The two approaches extensively elaborated in the literature that has followed despite some mild criticism, like the bond critique of the efficiency wage model or vacancy unemployed comovement in the search models, have been relatively successful and have rightly become the centerpieces of modern macroeconomic thinking. Unquestionably, there is little doubt that both motivational wages as well as the frictional nature of the labor market are the key factors that lead to the existence of unemployment. Nevertheless, the two approaches in their purest forms do imply that the existence of unemployment is either unavoidable as job search takes time and efficient if the bargaining power is properly balanced or in fact plays a role of a disciplining device and indirectly enhances welfare. Agreeably the view that the presence of unemployment does not inhibit welfare beyond the loss due to existence of a purely physical constraint or even individually rational economic constraint must be expressed with significant reservation. Accordingly, the paper complements the existing literature and presents an alternative mechanism that leads to unemployment as an equilibrium outcome that is socially costly.

The paper in built on two conceptual ideas and several technical assumptions. Specifically, as it was noted earlier the paper assumes that there exists an asymmetry in the employers' and employees' abilities to coordinate and steer the market outcomes, i.e., the paper not only departs from the concept of walrasian markets, but it in fact gives the power to manage the labor market to a coordinated group of profit maximizing individuals. Moreover, the paper assumes that projects differ by type with some being more profitable in the ex ante sense than the others. On the technical part the paper assumes the presence of nonlinearities in the form of fixed costs. In addition, aggregate demand externality, following Blanchard and Kiyotaki [15], as an outcome of monopolistic competition is introduced into the model. Moreover, in the model the elasticity between different intermediate goods is assumed to be high consequently, the overall, economy wide, profits expand with the number of varieties available, but profits per firm, each delivering a single good, fall with the number of varieties of goods delivered to the market. Consequently, new entry and resulting indirect competition imposes a negative burden on the existing firms. Finally, labor is assumed to heterogenous in nature with each variety of an intermediate good requiring a specific type, skill, of labor in its production process.

This basic structure leads in a quite natural way to the existence of unemployment in equilibrium that is socially costly. The mechanism can be described as follows. Incumbent firms having an informational advantage over firms that consider entry into the market can coordinate and pay an above market clearing wage to individuals who possess skills of no value to the incumbent firms, but who are productive if hired by a new entrant. Clearly, the incumbent firms incur a direct loss as they hire unproductive workers. On the other hand by paying an above market clearing wage they restrict entry by new entrants as some of
them find it unprofitable to enter and pay the above market clearing wage. A smaller number of new entrants lowers the overall profits, but rises the profit per firm. Naturally the incumbent firms must balance the two effects: a rise in profits due to entry restriction and a rise in costs due to an extra wage bill paid out to unproductive workers. It is shown that the first effect can dominate and indeed incumbent firms can choose to coordinate and pay an above market clearing wage and restrict entry and thus increase their overall net profits. Naturally, in the simplest case the coordination suffers in the static framework from the free rider problem and in equilibrium it is enforced in a dynamic setting with trigger strategies. The coordinated behavior of incumbent firms leads to unemployment. Some agents are not hired in equilibrium even though there is a fundamental demand for their skills and moreover some agents of the type identical to those not hired actually are hired and receive wages in excess of their direct productivity. It is shown that unemployment is socially costly and be dealt with a proper tax policy on the incumbent firms. Moreover, the paper establishes that the strategic behavior of the incumbent firms casts new light on the issue of real wage volatility over the business cycle.

The paper is organized as follows. Section (2) outlines the basic model. The equilibrium properties of the model are developed in section (3). Section (4) studies the dynamic properties of the model. The last section concludes.

## 2 Model

The model is developed in an explicit general equilibrium framework and is based on the standard framework of monopolistic competition of Blanchard and Kiyotaki [15]. The exposition of the model starts with the presentation of a static version of the model followed by the description of a dynamic rendering of the model.

### 2.1 The Static Case

It is assumed that there is a single final consumption good consumed by all agents in the economy. In addition, there exist a set of intermediate goods that serve as productive inputs in the process of production of the final consumption good. The final consumption good is produced from intermediate goods with a standard CES technology

$$
\begin{equation*}
c=\left(\int_{0}^{n} c_{i}^{\gamma} d i\right)^{\frac{1}{\gamma}} . \tag{1}
\end{equation*}
$$

The market for the final consumption good is perfectly competitive. Contrary to most formulations $n$ is assumed to be an equilibrium variable rather than a fixed parameter and is determined by the number of potential entrants willing to produce.

There are in total $n_{1}+n_{k}$ intermediate goods available for production. However, not all of them are actually demanded. Specifically, it is assumed that the demands for a number $n_{1}$ of goods exist and are positive with probability $q>\frac{1}{2}$
and do not not exist with probability $1-q$. Similarly, the paper assumes that the demands for a number $n_{k}$ of goods exist and are positive with probability $q^{k}$ and do not exist with probability $1-q^{k}$. Under these assumptions the aggregate production function 1 can be rewritten as

$$
\begin{equation*}
c=\left(\int_{0}^{n_{1}+n_{k}} c_{i}^{\gamma} x_{i} d i\right)^{\frac{1}{\gamma}}, \tag{2}
\end{equation*}
$$

where $x_{i}$ is the indicator function and is equal to 1 if the actual realization if the demand for good $i$ is positive and is equal to 0 otherwise.

The production process of intermediate goods requires two inputs one of sector specific nature and the other of general nature. In particular, the production function of good $i$ takes the form

$$
\begin{equation*}
c_{i}=L_{i}^{\alpha} L_{G, i}^{1-\alpha}, \tag{3}
\end{equation*}
$$

where $L_{i}$ is the input of labor that is specific to the production of good $i$ and $L_{G, i}$ is the input of the general labor, equally fit in the production processes of all goods, used for production of good $i$. The markets for intermediate goods are assumed to be monopolistic with at most one producer serving the market for a single good and freely setting the quantity delivered to the market.

There exists a number $n_{1}+n_{k}$ of potential producers of the intermediate goods. The producers become the owners of the profit income that they earn if they choose to produce. The preferences of a given producer are represented with

$$
U(c)=\left\{\begin{array}{l}
c-\theta, \text { if production is undertaken }  \tag{4}\\
c, \text { if production is not undertaken }
\end{array}\right.
$$

where $\theta$ represents a fixed time invariant constant and $c$ is the value of real consumption of the final consumption good. In other words, it is assumed that producers if they choose to produce they receive a fixed disutility costs of $\theta$, but at the same time they become the owners of the profits and consequently can afford to purchase consumption $c$. On the other hand if a given producer chooses not to produce she receives no income and cannot purchase any consumption, but at the same time receives no disutility from production and her net utility is 0 . Naturally, this specific representation of the preferences implies an equilibrium cutoff strategy on the part of producers. In particular, if expected profits from production in a given sector exceed $p \theta$, where $p$ denotes the price of the final consumption good consumption, then production is undertaken. Otherwise the producer remains idle.

The economy is populated with a total supply of $L_{G}$ of general labor that can be used for production of any intermediate good. In addition, for any intermediate good $i$ there exists a total supply of $L_{S}$ of skill specific labor fit to produce the good and totally unproductive elsewhere, i.e., the gross supply of skilled labor is $\left(n_{1}+n_{k}\right) L_{S}$. Laborers of all types enjoy consumption derive no disutility from work and consequently devote all of their income to purchases of the final consumption good.

### 2.1.1 Equilibrium with no Market Restriction

The solution for the equilibrium does not involve any novel techniques. The only complication stems from the individual demands for the intermediate goods being stochastic. To save on notation it is convenient to denote $\frac{1}{1-\gamma(1-\alpha)}$ with $\mu_{1}$ and $\frac{\alpha \gamma}{1-\gamma(1-\alpha)}$ with $\mu_{2}$. Observe that if production is undertaken of all intermediate goods, the case in equilibrium, then the overall price level is given by

$$
\begin{equation*}
p=\left(n_{1} q p_{1}^{\frac{\gamma}{\gamma-1}}+n_{k} q^{k} p_{k}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}} \tag{5}
\end{equation*}
$$

where $p_{j}, j \in\{1, k\}$, denotes the price of goods, it is identical for all goods in equilibrium, for which the demand exists with probability $q^{j}$ ex ante and its current realization is positive. For notational convenience the paper indexes all sectors with the demand being positive with probability $q^{j}$ with a single index $j$.

By assumption, the demand for a single intermediate good $j$ is stochastic. In equilibrium due to the CES formulation it takes the form

$$
p_{j}=\left\{\begin{array}{c}
D^{1-\gamma} p^{\gamma} c_{j}^{\gamma-1} \text { with probability } q^{j}  \tag{6}\\
0 \text { with probability } 1-q^{j}
\end{array}, j \in\{1, k\}\right.
$$

where $D$ denotes the nominal value of the demand for the final consumption good. Expected profit maximization in sector $j$ leads to the following first order conditions

$$
\begin{align*}
q^{j} D^{1-\gamma} p^{\gamma} \gamma \alpha L_{j}^{\alpha \gamma-1} L_{G, j}^{\gamma(1-\alpha)} & =w_{j}  \tag{7}\\
q^{j} D^{1-\gamma} p^{\gamma} \gamma(1-\alpha) L_{j}^{\alpha \gamma} L_{G, j}^{\gamma(1-\alpha)-1} & =w .
\end{align*}
$$

Again, $j \in\{1, k\}$ and $w_{j}$ denotes the nominal wage of skilled labor in sector $j$ and $w$ the wage of general labor the same in all sectors. Conditions (7) imply in particular that $\frac{\alpha}{1-\alpha} \frac{L_{G, j}}{L_{j}}=\frac{w_{j}}{w}$. Moreover, profit maximization in the final consumption good producing sector implies that $\left(\frac{c_{j_{1}}}{c_{j_{2}}}\right)^{\gamma-1}=\frac{p_{j_{1}}}{p_{j_{2}}}$, which when combined with the preceding relationship and the fact that the supply of the specific labor is identical in all sectors implies that $\frac{w_{j_{1}}}{w_{j_{2}}}=\left(\frac{q^{j_{1}}}{q^{j_{2}}}\right)^{\mu_{1}}$. The very last expression in turn implies that in equilibrium there are only to possible values depending on the likelihood of the demand being positive of wages paid to specific labor. The two values are denoted with $w_{k}$ and $w_{1}$. Similarly, there are only two possible positive price levels for the intermediate goods. The two are denoted with $p_{k}$ and $p_{1}$ and are given by the standard formulae of markup over marginal cost, $j \in\{1, k\}$,

$$
\begin{equation*}
p_{j}=\frac{1}{q^{j} \gamma} \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} w_{j}^{\alpha} w^{1-\alpha} \tag{8}
\end{equation*}
$$

The market clearing condition for the general labor $L_{G}=n_{1} L_{G, 1}+n_{k} L_{G, k}$, together with the fact that in equilibrium $\frac{L_{G, k}}{L_{G, 1}}=\frac{w_{k}}{w_{1}}$ and the fact that $\frac{w_{k}}{w_{1}}=$
$\left(\frac{q^{k}}{q}\right)^{\mu_{1}}$ leads to the following employment levels in the respective sectors

$$
\begin{equation*}
L_{G, j}=\frac{q^{j \mu_{1}}}{n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}} L_{G}, j \in\{1, k\} \tag{9}
\end{equation*}
$$

Naturally, specific labor employed in each sector is equal to its supply, i.e., $L_{j}=L_{S}, j \in\{1, k\}$.

These standard relationships allow to establish that in equilibrium the following hold

$$
\begin{align*}
\omega & =\gamma(1-\alpha)\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\frac{1-\gamma}{\gamma}+\alpha} L_{S}^{\alpha} L_{G}^{-\alpha},  \tag{10}\\
\omega_{j} & =\gamma \alpha q^{j \mu_{1}}\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\frac{1-\gamma}{\gamma}-(1-\alpha)} L_{S}^{\alpha-1} L_{G}^{1-\alpha},
\end{align*}
$$

where $\omega=\frac{w}{p}$ and $\omega_{j}=\frac{w_{j}}{p}$ denote real wages paid to general labor and type $j$ labor. In addition, the real output takes the form

$$
\begin{equation*}
y=\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\frac{1-\gamma}{\gamma}+\alpha} L_{S}^{\alpha} L_{G}^{1-\alpha} . \tag{11}
\end{equation*}
$$

Furthermore, the expected real values of profits in the respective sectors can be expressed as

$$
\begin{equation*}
\pi_{j}=(1-\gamma) q^{j \mu_{1}}\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\frac{1-\gamma}{\gamma}-(1-\alpha)} L_{S}^{\alpha} L_{G}^{1-\alpha} . \tag{12}
\end{equation*}
$$

To close the model in a consistent way it suffices to assume that the disutility costs on the part of producers that arises from production is exactly equal to the expected profits in a type $k$ sector, i.e., to assume that

$$
\begin{equation*}
(1-\gamma) q^{k \mu_{1}}\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\frac{1-\gamma}{\gamma}-(1-\alpha)} L_{S}^{\alpha} L_{G}^{1-\alpha}=\theta \tag{13}
\end{equation*}
$$

Several immediate observations can be made. First of all, there is no unemployment as all factors are employed. In addition, as long as the condition (13) is met the analogous condition for firms that face less demand uncertainty holds with inequality and actually all enter their respective markets. Moreover, both output and the real wage of the general labor are increasing with the number of varieties whereas the expected profit in a single sector not necessarily so. Specifically, if the value of $\frac{1-\gamma}{\gamma}-(1-\alpha)$ is negative then the expected profits in a single market fall with the number of overall number of markets and clearly from the perspective of a single firm any additional entry is undesirable. On the contrary, exit of the existing firms is beneficial. It should be emphasized that in the model irrespective of the value of $\frac{1-\gamma}{\gamma}-(1-\alpha)$ the overall, economy wide profits, actually do increase with the number of varieties. In what follows it is assumed that the value of $\mu$ is negative, i.e.,

Assumption \#1: The model parameters satisfy

$$
\begin{equation*}
\mu=\frac{1-\gamma}{\gamma}-(1-\alpha)<0 \tag{14}
\end{equation*}
$$

i.e., real value of expected profits in a single firm decreases with the number of varieties.

### 2.1.2 Equilibrium with Market Restriction

The model as presented up to this point is very conventional and does not lead to any new results. However, the observation made in the last paragraph of the preceding subsection suggests the feasibility of the following strategy on the part of producers. As profits per firm do increase if the number of entrants falls it may be beneficial to restrict entry. How entry can be restricted? Note that by assumption type $k$ firms face an informational disadvantage and their expected profits are lower as the existence of the demands they deal with is more uncertain. Could it be then beneficial to coordinate and offer a wage to specific labor in sectors $k$ that exceeds the market clearing wage in order to drive some of the type $k$ firms out of the market and thus restrict the entry. Naturally, such a strategy increases the costs as firms that choose to follow it must pay an above market clearing wage to individuals whom they find totally unproductive. On the other hand if entry is indeed reduced then the overall profits can actually increase as there is less indirect competition. Which effect dominates? This subsection focuses on determining the trade-off. Any more detailed analysis requires that the conceptual assumptions needed must be made explicit. Henceforth the paper assumes that.

Assumption \#2: Firms have the ability to coordinate. In particular, a group of firms can choose to enter a given market and hire freely on that market. Workers do not have the ability to coordinate. In particular, workers cannot in a coordinated manner refuse the wage that is offered to them. They must maximize their individual utility and consequently take the wage that is being offered.

There are a total of $n_{1}$ firms that face positive demands with probability $q$ and a total of $n_{k}$ firms that confront positive demands with probability $q^{k}$. Assume that the $n_{1}$ of type 1 firms coordinate and commit, not just threaten, to enter type $k$ labor markets and hire exactly $(1-\phi) L_{S}$ workers on each market. Naturally, the type 1 firms will pay the real wage that is needed to entice $(1-\phi) L_{S}$ units on each market of type $k$ specific labor and presumably that wage will exceed the market clearing wage given by (10.) This commitment leaves the profit maximization problem of firms in sectors $k$ unchanged. The firms still take the new wage as given and choose their inputs. However, the wage paid to type $k$ labor, given that now type $k$ firms must compete with type 1 firms for type $k$ labor, is higher. In equilibrium, a higher than before real wage lowers profits of firms in sectors $k$ and some of them choose not to enter as it was assumed that the condition (13) held at the outset. Therefore, there will be labor with sector specific skills, type $k$, that is not hired at all as some type $k$ firms that demand type $k$ skills simply opt out of the market. Consequently, on any type $k$ labor market fraction $1-\phi$ of available labor is hired by type 1 firms that find type $k$ labor to be of no intrinsic value and the remainder, the fraction $\phi$, is either hired by the corresponding type $k$ firm that actually values that specific form of type $k$ labor and chooses to be active or remains unemployed.

Let $n_{k}^{*}$ be the actual number of type $k$ firms that choose to enter. Note that
$n_{k}^{*}$ can be as low as 0 and as high as $n_{k}$. Under the standing commitment of the $n_{1}$ type 1 firms the overall employment of type $k$ specific labor in type $k$ firms is given by $n_{k}^{*} \phi L_{S}$ the employment of type $k$ specific labor in type 1 firms is given by $n_{k}(1-\phi) L_{S}$, which leaves the unemployed pool of $\left(n_{k}-n_{k}^{*}\right) \phi L_{S}$. Can this pool actually arise?

As noted earlier the commitment on the part of the type 1 firms does not affect the first order conditions of type $k$ firms. Moreover, the first order conditions of type 1 firms are not affected either as the firms bear in effect a fixed cost in the form of the wage bill to workers whom they find unproductive. Accordingly, the two sets of the two first order conditions still hold as, $j \in\{1, k\}$,

$$
\begin{align*}
q^{j} D^{1-\gamma} p^{\gamma} \gamma \alpha L_{j}^{\alpha \gamma-1} L_{G, j}^{\gamma(1-\alpha)} & =w_{j}^{R}  \tag{15}\\
q^{j} D^{1-\gamma} p^{\gamma} \gamma(1-\alpha) L_{j}^{\alpha \gamma} L_{G, j}^{\gamma(1-\alpha)-1} & =w^{R} .
\end{align*}
$$

Similarly, the first order conditions from the final consumption good producing sector remain unchanged and in particular it is still true that $\left(\frac{c_{k}}{c_{1}}\right)^{\gamma-1}=\frac{p_{k}}{p_{1}}$. The solution for the equilibrium is similar to the previous one except that now it is necessary to account for the fact that not all type $k$ specific labor is employed in type $k$ sectors, while all of type 1 specific labor is employed in type 1 sectors, i.e., $L_{1}=L_{S}$ and $L_{k}=\phi L_{S}$. Consequently the ratio of general labor employed in a sector 1 firm to general labor employed in a sector $k$ firm is given by $\frac{L_{G, 1}}{L_{G, k}}=\left(\frac{q^{k}}{q}\right)^{\mu_{1}} \phi^{\mu_{2}}$ and in turn the levels of employment of general labor in a type 1 firm and a type $k$ firm take the form

$$
\begin{align*}
L_{G, 1} & =\frac{q^{\mu_{1}}}{n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}} L_{G}  \tag{16}\\
L_{G, k} & =\frac{q^{k \mu_{1}} \phi^{\mu_{2}}}{n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}} L_{G}
\end{align*}
$$

The level of output and the real reward to the general labor are given by

$$
\begin{align*}
y & =\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{1+\mu} L_{S}^{\alpha} L_{G}^{1-\alpha}  \tag{17}\\
\omega^{R} & =\gamma(1-\alpha)\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{1+\mu} L_{S}^{\alpha} L_{G}^{-\alpha} . \tag{18}
\end{align*}
$$

The real wages of the specific factors can be expressed as

$$
\begin{align*}
\omega_{1}^{R} & =\gamma \alpha q^{\mu_{1}}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu} L_{S}^{\alpha-1} L_{G}^{1-\alpha}  \tag{19}\\
\omega_{k}^{R} & =\gamma \alpha q^{k \mu_{1}} \phi^{\mu_{2}-1}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu} L_{S}^{\alpha-1} L_{G}^{1-\alpha}
\end{align*}
$$

Moreover, the real expected profits in a sector $k$ firm take the form

$$
\begin{equation*}
\pi_{k}^{R}=(1-\gamma) q^{k \mu_{1}} \phi^{\mu_{2}}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu} L_{S}^{\alpha} L_{G}^{1-\alpha} \tag{20}
\end{equation*}
$$

Note that free entry condition for type $k$ firms implies that in an interior equilibrium it must be

$$
\begin{equation*}
(1-\gamma) q^{k \mu_{1}} \phi^{\mu_{2}}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu} L_{S}^{\alpha} L_{G}^{1-\alpha}=\theta \tag{21}
\end{equation*}
$$

The real net profit of a type 1 firm takes a slightly more complicated shape as the type 1 firms commit to hire the fraction $(1-\phi)$ of type $k$ labor. There are in total $n_{1}$ of type 1 firms and the the number of type $k$ firms is $n_{k}$, therefore, each type 1 firm hires exactly $\frac{n_{k}}{n_{1}}(1-\phi) L_{S}$ units of type $k$ labor. The prevailing wage is given by (19). Therefore, in equilibrium the expected real net profits are

$$
\begin{equation*}
\pi_{1}^{R}=(1-\gamma) q^{\mu_{1}}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu} L_{S}^{\alpha} L_{G}^{1-\alpha}-\frac{n_{k}}{n_{1}}(1-\phi) L_{s} \omega_{k}^{R} \tag{22}
\end{equation*}
$$

Combining conditions (13) and (21) leads in particular to

$$
\begin{equation*}
\phi^{\mu_{2}}\left(n_{1} q^{\mu_{1}}+n_{k}^{*} q^{k \mu_{1}} \phi^{\mu_{2}}\right)^{\mu}=\left(n_{1} q^{\mu_{1}}+n_{k} q^{k \mu_{1}}\right)^{\mu} \tag{23}
\end{equation*}
$$

and allows to express the ratio of real net profits in a sector 1 firm as expressed by (22) net of disutility $\theta$ when firms in type 1 sectors choose to coordinate, $\pi_{1}^{R}-\theta$, to real net profits as expressed by (12) net of disutility $\theta$ when there is no coordination the part of type 1 firms, $\pi_{1}-\theta$. The ratio is given by

$$
\begin{equation*}
\frac{\pi_{1}^{R}-\theta}{\pi_{1}-\theta}=1+\frac{q^{\mu_{1}}}{q^{\mu_{1}}-q^{k \mu_{1}}}\left(\phi^{-\mu_{2}}-\frac{\alpha \gamma}{(1-\gamma)} \frac{n_{k}}{n_{1}} \frac{q^{k \mu_{1}}}{q^{\mu_{1}}} \frac{1-\phi}{\phi}-1\right) \tag{24}
\end{equation*}
$$

Not surprisingly the ratio takes the value of 1 when $\phi=1$, i.e., when there is no artificial push of the real wage above the market clearing level. When $\phi$ assumes values smaller than 1 there are two competing effects. First of all, there is positive effect $\phi^{-\mu_{2}}$ stemming from the profit increase due to market restriction and lesser entry. In addition, there is an additional effect that decreases the ratio as the wage bill to sectors $k$ slack workers becomes larger. Which of the two effects dominates? There is no definite answer and in fact the model parameters play a role. The derivative of (24) with respect to $\phi$ at 1 is equal to, up to a positive scaling constant, $-\mu_{2}+\frac{\alpha \gamma}{1-\gamma} \frac{n_{k}}{n_{1}} \frac{q^{k \mu_{1}}}{q^{\mu_{1}}}$ and it could actually be negative implying that starting from $\phi=1$ a marginal decrease in $\phi$ can actually increase per firm profit. Moreover, the expression (24) possesses actually a maximum. Let $\phi_{\arg \max }$ be the value of $\phi$ corresponding to the maximum. Furthermore, an interior solution to (21) is feasible as long as $\phi$ exceeds $\phi_{\text {min }}=\left(1+\frac{n_{k}}{n_{1}} \frac{q^{k \mu_{1}}}{q^{\mu_{1}}}\right)^{\frac{\mu}{\mu_{2}}}$. It turns out that it is possible to fix the parameters of the model to have the two things occur at the same time, i.e., to make the derivative negative at 1 and to have $\phi_{\text {min }}<\phi_{\arg \max }<1$. In other words, there exists a possibility of an interior maximum for the expression (24.) Moreover, in the subsequent sections that consider a fully dynamic model it is shown that indeed this may be the case. This section simply assumes that this happens to be. Figure (1) depicts the expression (24) as a function of $\phi$ over the relevant interval.

Before discussing strategic implications stemming from the existence of an interior maximum of (24) it is worthwhile to rationalize the existence in the first place. Fundamentally, the discrepancy, assumed, but realistic, in the informational contents of information sets between the incumbent firms and potential


Figure 1: The Dependance of the Ratio of Profits net of Disutility Costs $\theta$ with Restriction to Profits net of Disutility Costs $\theta$ without Restriction as a Function of $\phi . \alpha=\frac{1}{3}, \gamma=0.69, k=10, q=0.9, \theta=0.0559$.
entrants is the key culprit. Incumbent firms face positive demands with probability $q$ whereas potential new entrants with probability $q^{k}$. As a result the expected profits of the incumbent firms exceed those of the potential entrants by a discrete factor of $\left(\frac{q}{q^{k}}\right)^{\mu_{1}}$. If a coordinated market restriction occurs then two types of firms experience an indirect gain in profits due to aggregate demand externality as the number of varieties is lowered. However, the gain for the type 1 firms is proportional to $q^{\mu_{1}}$ whereas for the type $k$ firms to $q^{k \mu_{1}}$. In addition, type $k$ firms, potential entrants, experience a loss due to a larger wage bill as there is an artificially high wage being paid out to type $k$ labor and that loss is proportional to $q^{k \mu_{1}}$. At the same time type 1 firms do experience a loss as they commit themselves towards hiring individuals that are unproductive from their perspective. However, the loss is proportional to $q^{k \mu_{1}}$ as the extra wage bill is paid out to type $k$ labor. Consequently at the margin type $k$ firms experience a gain proportional to $q^{k \mu_{1}}$ and a loss that is proportional to $q^{k \mu_{1}}$. At the margin the two actually do balance as the free entry condition (21) must hold and type $k$ firms remain indifferent. On the other hand the gain for type 1 firms is proportional to $q^{\mu_{1}}$ whereas the loss is proportional to $q^{k \mu_{1}}$ and there is a natural wedge between the gain and the loss. It turns out that provided that some additional proportionality constants, the case in equilibrium, assume proper values the wedge can be actually positive and at the margin it is beneficial to type 1 firms to coordinate and restrict entry. The distribution of employment of type


Figure 2: The Actual Engagement of Type $k$ Labor.
$k$ labor is depicted in figure (2).
Naturally, in the current setup coordination on the part of type 1 firms increases their net profits, but cannot be supported as an equilibrium outcome as it is individually rational for each type 1 firm to deviate from cooperation and release its unproductive, type $k$, workers and thus save on the wage bill irrespective of the behavior of the remaining type 1 firms. Consequently, in the static context coordination and hence unemployment cannot occur. On the other hand it is straight forward to establish that in a simple repeated game the cooperation can be enforced with standard trigger strategies and unemployment can be an outcome. The following section studies the issue in a more detailed manner.

## 3 Dynamic Equilibrium

The variables $n_{1}$ and $n_{k}$ were assumed to be exogenously given and naturally in the static model time invariant. This section makes specific assumptions on the process that governs the evolutions of variables $n_{1}$ and $n_{k}$. Primarily, the process is chosen for its technical simplicity and is thought of as one resembling the life cycle of a single product. However, the results need not obtain for an arbitrary process. In particular, it is assumed that any type 1 good that is demanded with probability $q$ in period $t$ remains a type 1 good in period $t+1$ if the demand for the good is positive in period $t$. Moreover, a type 1 good demanded with probability $q$ in period $t$ turns into a type $k$ good in period
$t+1$ if the demand for the good does not exist in period $t$. A type $k$ good that is not demanded in a given period remains a type $k$ good in the following period. In equilibrium it may be the case that not all type $k$ firms choose to enter and undertake production. For technical simplicity, it is assumed that type $k$ firms that do not enter in a given period remain type $k$ in the following period. Finally, type $k$ firms that enter in a given period for which the demand turned out to be positive turn into type 1 firms in the following period. As it was noted it may be the case that not all type $k$ firms enter in a given period ${ }^{1}$. Let $\psi_{t} \in[0,1]$ be the fraction of type $k$ firms that choose to enter in period $t$. Mathematically, the process can be described as a Markov process and can be summarized with the following relationships

$$
\begin{align*}
n_{1}^{t+1} & =q n_{1}^{t}+\psi_{t} q^{k} n_{k}^{t}  \tag{25}\\
n_{k}^{t+1} & =(1-q) n_{1}^{t}+\psi_{t}\left(1-q^{k}\right) n_{k}^{t}+\left(1-\psi_{t}\right) n_{k}^{t}
\end{align*}
$$

The process is to capture a very basic notion that demands for specific goods can expire and that there is a positive chance that goods that were demanded in the past and are no longer demanded can be demanded again in the future. The agents in a given period are assumed to know the process (25), but are assumed to take the contemporaneous values of $n_{k}^{t}$ and $n_{1}^{t}$ as given when making decisions in period $t$. However, the agents do take into account that the equilibrium choices made a given period can influence future values of $n_{k}^{t+1}$ and $n_{1}^{t+1}$.

Let $V_{1}^{t}, V_{k, A}^{t}$, and $V_{k, I}^{t}$ denote the value of a type 1 firm, type $k$ firm that chooses to be active and the value of a type $k$ firm that refrains from entering a given market. Assuming that the future payoffs are discounted at the rate $\beta$ it is straightforward to establish the following dynamic relationships

$$
\begin{align*}
V_{1}^{t} & =\pi_{1}-\theta+q \beta V_{1}^{t+1}+(1-q) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right)  \tag{26}\\
V_{k, A}^{t} & =\pi_{k}-\theta+q^{k} \beta V_{1}^{t+1}+\left(1-q^{k}\right) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right) \\
V_{k, I}^{t} & =\beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right) .
\end{align*}
$$

The processes describing the evolution of the distribution of project types (25) as well as the values of different states (26) admit the existence of a steady state. Specifically, by choosing a proper value of $\theta$ it is possible to ensure that there exists a steady state in which the owners of type $k$ projects are just indifferent between entering and producing and staying inactive, i.e., a steady state with $V_{k, A}^{t}=V_{k, I}^{t}$.

The extension of the time horizon allows for the determination of the steady state values of $n_{1}=\frac{\psi q^{k}}{1-q+\psi q^{k}}, n_{k}=\frac{1-q}{1-q+\psi q^{k}}$, where the overall number of potential projects is normalized to 1 . Moreover, in the steady state where type $k$ firms are just indifferent between entering and staying idle the values of different projects take the form $V_{k, A}=0, V_{k, I}=0$, and $V_{1}=\frac{\pi_{1}-\theta}{1-q \beta}$. Finally,

[^1]the indifference between entering and not entering on the part of type $k$ firms requires that
\[

$$
\begin{equation*}
\theta-\pi_{k}=\frac{q^{k} \beta}{1-q \beta}\left(\pi_{1}-\theta\right) . \tag{27}
\end{equation*}
$$

\]

The condition (27) allows for an explicit determination of $\psi$ as a function of the parameters of the model and $\theta$ in particular. It is straightforward to verify that in equilibrium, as expected, $\psi$ is decreasing with $\theta$ and increasing with $\beta$. Moreover, as compared to the static case $\psi$ is higher as type $k$ firms are willing to tolerate a one period negative flow in utility in expectation of positive flows in subsequent periods. To simplify notation it is assumed in what follows that the parameters of the model are such that $\psi$ determined by the condition (27) is equal to 1 , i.e., all type $k$ firms are indifferent between entering and not and all enter.

The introduction of an explicit time dimension in the model has not brought any new insights if one concentrates on the nonrestricted case. The situation changes when coordination on the part of type 1 firms in a given period is allowed. Note that if coordination in period $t$ is permitted then coordination not only increases the current level of net profits as shown in the preceding section, but it also increases future profits as coordination and entry restriction in period $t$ influences the number of type 1 firms in periods $t+1$ and beyond and lowers future indirect competition. Naturally, it is assumed that if coordination occurs then coordinating firms do take into account their impact of their decisions on future distribution of project types and at the same time they treat the contemporaneous values of $n_{1}^{t}$ and $n_{k}^{t}$ as predetermined.

Observe that $\psi_{t}$ need not be equal to 1 whenever coordination occurs. The evolution of project types can be now described with (25) with $\psi_{t}$ being the result of an equilibrium strategy played by type $k$ firms, which in fact is to enter with probability $\psi_{t}$ and not to enter with probability $1-\psi_{t}$. The value functions for a single firm can now be described with (26) with the first expression replaced with

$$
\begin{equation*}
V_{1}^{t}=\pi_{1}-\theta-\frac{n_{k}^{t}}{n_{1}}\left(1-\phi_{t}\right) \omega_{k}^{R} L_{S}+q \beta V_{1}^{t+1}+(1-q) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right) \tag{28}
\end{equation*}
$$

Note that the values now depend also on $\phi_{t}$. Presumably, one should consider $\phi_{t}$ a choice variable on the part of coordinating type 1 firms and allow type 1 firms to choose $\phi_{t}$ (each period) to maximize the value of a type 1 firm. Indeed this is assumed in to be the case in the subsequent section. This section has a more restricted scope and aims only at showing feasibility of unemployment, i.e., it shows that type 1 firms may find it optimal to coordinate and to create unemployment rather than not without making any suggestion that the actual degree of coordination is the most desirable one on the part of type 1 firms.

In a steady state where type $k$ firms are just indifferent between entering and not entering it is necessarily the case that $V_{k, A}=V_{k, I}=0$. In addition, the value of a type 1 firm is given by $V_{1}=\frac{\pi_{1}^{R}-\theta}{1-q \beta}$. Furthermore, the indifference
between entering and not on the part of type $k$ firms implies that

$$
\begin{equation*}
\theta-\pi_{k}=\frac{q^{k} \beta}{1-q \beta}\left(\pi_{1}^{R}-\theta\right), \tag{29}
\end{equation*}
$$

which defines $\psi$ as a response to $\phi$ chosen by type 1 firms. Finally, the description of the steady state is completed with the steady state expression for $n_{1}$ as a function of $\psi$, i.e., $n_{1}=\frac{q^{k} \psi}{1-q+q^{k} \psi}$. Before proceeding further, observe that from the perspective a type 1 firm the likelihood of staying a type 1 firm in the subsequent period is independent of any specific actions the firm takes as it is simply equal to $q$. However, the continuation values do depend on the state variables and in particular if in period $t$ firms choose a small value of $\phi_{t}$ (a significant push of the real wage above the market clearing level) then there is little entry, $\psi_{t}$ is small, and the number of type 1 firms in the future is smaller as a smaller proportion of type $k$ firms turns into type 1 firms making future per firm profits larger. Clearly this additional gain, as compared to the static case, leads type 1 firms to restrict entry even further, i.e., to choose a smaller $\phi_{t}$ in order to limit indirect competition in the future. As argued earlier at the same time type $k$ firms are more reluctant to stay idle as the chance of becoming a type 1 firm is $q^{k}$ if production is undertaken and being active gives a chance for an increased continuation value. Other things equal this should lead to a more costly entry restriction on the part of type 1 firms and a larger $\phi_{t}$ selected in equilibrium. There are two competing effects with the former being dominant as the higher continuation values for type 1 firms are discounted with $q \beta$ and for type $k$ firms only with $q^{k} \beta$. Therefore, one can expect more entry restriction and more unemployment in the dynamic case than in the static case. Figure (3) presents the steady state dependence of $\psi$ on $\phi$ and the corresponding steady state value of the value of a type 1 firm $V_{1}$ when coordination takes place relative to the value with no coordination is present as a function of $\phi$. Naturally, as both graphs reveal there is range of the values of $\phi$ for which a steady state exist and more over there exists a range of values of $\phi$ for which coordination actually increases the steady state value of a type 1 firm. Naturally, the two graphs indicate that certain steady states with coordination and market restriction are feasible. Obviously, consistency requires that coordination itself be a part of equilibrium, i.e., rather be an outcome than assumed. This is attained by considering strategic interactions between type 1 firms in an infinite horizon game with cooperation enforced on the path with trigger strategies.

To close the exposition of the steady state analysis one must verify that the equilibrium that has been constructed can be supported by trigger strategies as the simple static solution suffers from a severe free rider problem. Before arguing that indeed the equilibrium can be in fact supported with trigger strategies it is necessary to address a methodological issue. In fact throughout the paper is organized around the notion of an infinite number of atomistic agents as a key building block. This is done for two technical reasons. First of all, the monopolists' pricing decisions can be reflected in a simple analytic form as a fixed markup over the marginal cost. In addition, the law of large numbers can


Figure 3: The Dependence of $\psi$ on $\phi$ and the Value $V_{1}$ on $\phi$ in the Steady State. $\phi$ is Treated as a Parameter of the Model. $\alpha=1 / 3.5, \gamma=0.9, \beta=0.95, q=0.85$, and $k=8$.
be applied in dealing with the evolution of project types over time. However, admittedly, the assumption of agents being atomistic makes a discussion of strategic interactions a conceptual challenge. Nevertheless, the paper keeps the initial assumption and treats the atomistic agents as a group that can interact strategically. Specifically, the paper assumes that agents can play the following strategies: a type 1 firm does hire $\frac{n_{k}}{n_{1}}(1-\phi) L_{S}$ units of type $k$ labor and pays an above market clearing wage as long as all type 1 firms do that; as soon as a single type 1 firm deviates then all type 1 firms switch to the punishment phase and pay market clearing wages in all future periods.

In verifying that a given steady state solution can indeed be supported with trigger strategies one encounters two technical problems. First of all, equilibrium values of $\phi$ and $\psi$ do depend on $\beta$ and $q$ and it is not possible to invoke a folk theorem for repeated games. Moreover, the distribution of project types does change whenever the punishment phase is triggered. Nevertheless, it is still trivial to verify that no agent wants to deviate if the economy happens to be off the path and the punishment phase has been triggered. The situation by far more intricate on the path. Nevertheless, it can be shown, detailed exposition in Appendix A, that there are values of $\phi$ for which the steady state can be supported with trigger strategies.

In this subsection it has been established that there exists, trigger strategy enforced, steady states with a coordinated market restriction and consequently unemployment. However, the degree of coordination has been treated as a


Figure 4: The Value Function of a Type 1 Firm
parameter. The following subsection relaxes the latter assumption and allows type 1 firms to consider $\phi$ a choice variable.

## 4 Dynamics

The purpose of this section is to analyze the responsiveness of the economy with coordinated market restriction to technology shocks. Specifically, the section focuses on identification of dynamics that are due purely to the existence of coordinated market restriction and are absent when the labor market functions efficiently. First the paper starts with an analysis of the technology shocks induced dynamics in an economy with no artificial unemployment and then it performs an analogous analysis in an economy with coordinated market restriction.

### 4.1 Economy with no Market Restriction

Recall that the set of Bellman equations tying the continuation values is given by the system of equations (26). Moreover, under the assumption ${ }^{2}$ that in the steady state all type $k$ firms are just indifferent between entering and not entering and all actually choose to enter, i.e., equation (27) is satisfied with $\psi$ equal to 1 the value function of a type 1 firm is presented in figure (4).

[^2]

Figure 5: Technology Shocks Induced Percentage Deviation of the Total Employment and the Average Wage Relative to the Respected Steady State Values.

As expected the value function decreases with $n_{1}$, i.e., the larger the number of indirect competitors the smaller the value of a single firm. Naturally, the cumulative value of all type 1 firms does increase with $n_{1}$, but at a rate smaller then the rate of increase of $n_{1}$ and consequently the value of a single firm falls with $n_{1}$.

The economy's responsiveness to technology shocks can be illustrated best in a simple comparative-static-like exercise. Assume that the economy is initially in the steady state and at time $t$ it is hit with a transitory technology shock. Specifically, assume that the production function at time $t$ becomes

$$
\begin{equation*}
c_{i}=A L_{i}^{\alpha} L_{G, i}^{1-\alpha}, \tag{30}
\end{equation*}
$$

and then returns to its original form given by (3) in period $t+1$. Naturally, this specific form implies that technological disturbances potentially influence the equilibrium at time $t$ and consequently the distribution of project types in the subsequent periods, but leave the functional form of the continuation values unaffected. Figure (5) present the evolution of total employment, both skilled and general, as a function of $A$ in period $t$ and the average real wage, given by

$$
\begin{equation*}
\bar{\omega}=\frac{\omega L_{G}+\omega_{1} n_{1} L_{S}+\omega_{k} n_{k}^{*} L_{S}}{L_{G}+n_{1} L_{S}+n_{k}^{*} L_{S}}, \tag{31}
\end{equation*}
$$

as a function of $A$, where $n_{k}^{*}=n_{k} \psi^{*}$ denotes the actual number of type $k$ project run at a given point in time.

Clearly, as the economy is assumed to be initially in the steady state in which all type $k$ firms are just indifferent between entering and staying idle and all choose to enter a positive shock does not increase employment and leads only to a wage increase. On the other hand a negative shock lowers both employment and the average wage as inferior technology drives some of type $k$ firms of the market (in equilibrium all type $k$ firms are still indifferent between entering and staying idle, but some of them stay out $\psi<1$.)

### 4.2 Economy with Market Restriction

It was shown earlier that steady state equilibria with a coordinated market restriction did exist. However, the degree of coordination, market restriction, $\phi$ was assumed to have been a parameter of the model. Naturally, to make the study of shock induced dynamics meaningful it is necessary to endogenize $\phi$. The most natural manner to proceed is to assume that type 1 firms choose $\phi$ to maximize the value of their firms. Naturally, this implies that the set of Bellman equations must be modified further with equation (28) replaced with
$V_{1}^{t}=\max _{\left\{\phi_{t}\right\}}\left\{\pi_{1}-\theta-\frac{n_{k}^{t}}{n_{1}}\left(1-\phi_{t}\right) \omega_{k}^{R} L_{S}+q \beta V_{1}^{t+1}+(1-q) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right)\right\}$.
In the process of maximization in (32) agents treat the contemporaneous state variable $n_{1}^{t}$ as given. However, agents do know that a specific choice of $\phi_{t}$ influences the fraction $\psi_{t}$ of type $k$ firms that choose to enter in period $t$ and consequently the distribution of future project types, i.e., agents rationally take into account the functional dependence of $\psi_{t}$ on $\phi_{t}$ as well as other variables. For notational simplicity let $\Pi^{1}\left(\phi_{t}, \psi_{t}, n_{1}^{t}\right)=\pi_{1}-\theta-\frac{n_{k}^{t}}{n_{1}}\left(1-\phi_{t}\right) \omega_{k}^{R} L_{S}$ be the real net value of the utility flow of a type 1 firm. In addition, let $\phi_{t}^{*}$ be the optimal choice made by type 1 firms and $\psi_{t}^{*}$ be the optimal reaction on the part of type $k$ firms at time $t$. Note, that in general $\psi_{t}$ depends on $\phi_{t}$ and on $n_{t}$. Moreover, in an interior steady state where restriction occurs the values $V_{k, A}$ and $V_{k, I}$ are both zero. Consequently, along the equilibrium path, with an interior solution, the following conditions must hold

$$
\begin{equation*}
\Pi_{\phi}^{1}+\Pi_{\psi}^{1} \frac{d \psi_{t}}{d \phi_{t}}+q \beta \frac{d V_{1}^{t+1}}{d n_{1}^{t+1}} q^{k} n_{t}^{k} \frac{\partial \psi_{t}}{\partial \phi_{t}}=0 \tag{33}
\end{equation*}
$$

i.e., at any point in time the choice of $\phi_{t}^{*}$ must be optimal. Naturally, all partial derivatives are evaluated at the optimum. Moreover, along the optimal path the equation (32) must naturally be satisfied, i.e.,

$$
\begin{equation*}
V_{1}^{t}=\Pi^{1}+q \beta V_{1}^{t+1} \tag{34}
\end{equation*}
$$

where the actual value of $n_{1}^{t+1}$ is induced with the choice of $\phi_{t}^{*}$. Moreover, differentiating (34) with respect to $n_{1}^{t}$ at the optimum along the equilibrium path and invoking the envelope property it is straightforward to show that

$$
\begin{equation*}
\frac{\partial V_{1}^{t}}{\partial n_{1}^{t}}=\Pi_{n_{1}}^{1}+\Pi_{\psi}^{1} \frac{\partial \psi_{t}}{\partial n_{1}}+q \beta \frac{d V_{1}^{t+1}}{d n_{1}^{t+1}}\left(q-q^{k} \psi_{t}+q^{k} n_{t}^{k} \frac{\partial \psi_{t}}{\partial n_{t}}\right) \tag{35}
\end{equation*}
$$

As assumed, along the equilibrium path type $k$ firms must actually be indifferent between entering and staying idle, i.e., as in an interior steady state $V_{k, A}=V_{k, I}$ the following condition

$$
\begin{equation*}
0=\Pi^{k}+q^{k} \beta V_{1}^{t+1}, \tag{36}
\end{equation*}
$$

where $\Pi^{k}=\pi_{k}-\theta$ is the net per period utility flow of a type $k$ firm, must hold. Naturally, in general the indifference condition (36) defines an implicit functional ${ }^{3}$ dependence of $\psi_{t}$ on $\phi_{t}$ and on $n_{t}$. Condition (36) when differentiated with respect to $\phi_{t}$ and then alternatively with respect to $n_{t}$ becomes

$$
\begin{align*}
& 0=\Pi_{\phi}^{k}+q^{k} \beta \frac{d V_{1}^{t+1}}{d n_{1}^{t+1}} q^{k} n_{t}^{k} \frac{\partial \psi_{t}}{\partial \phi_{t}}  \tag{37}\\
& 0=\Pi_{n_{1}}^{k}+\Pi_{\psi_{t}}^{k} \frac{\partial \psi_{t}}{\partial n_{t}}+q^{k} \beta \frac{d V_{1}^{t+1}}{d n_{1}^{t+1}}\left(q-q^{k} \psi_{t}+q^{k} n_{t}^{k} \frac{\partial \psi_{t}}{\partial n_{t}}\right) . \tag{38}
\end{align*}
$$

The relationships (36), (33), (34), (35), and (37) together with $V_{1}^{t}=V_{1}^{t+1}$ describe implicitly the equilibrium path along which it is equally beneficial to operate a type $k$ as to remain idle. Let $\phi^{*}$ and $\psi^{*}$ be the corresponding steady state values. In the steady state the numbers of type 1 project and type $k$ projects are given by $n_{1}=\frac{\psi^{*} q^{k}}{1-q+\psi^{*} q^{k}}, n_{k}=\frac{1-q}{1-q+\psi^{*} q^{k}}$. The relationship (34) together with the condition (36) allow to determine an analytic expression for $\psi^{*}$ as a function of $\phi^{*}, n_{1}$ and $n_{k}$. However, in the steady state the values of $n_{1}$ and $n_{k}$ do depend on $\psi^{*}$ itself. Unfortunately, the two relationships are nonlinear and without imposing further restrictions on the parameter values it is necessary to obtain the relationship between $\psi^{*}$ and $\phi^{*}$ with numerical techniques. Figure (??), dashed line, presents the dependence for a set of reference parameters.

Combining the steady state equivalents of (33),(35) and (36) with the steady state expressions for $n_{1}$ and $n_{k}$ permits to obtain a new relationship tying $\phi$ and $\psi$. Figure (6), solid lines, presents the relationship.

It is clear that indeed there are interior solutions in the steady state, i.e., there exists a $\phi$ different than 1 and the corresponding value of $\psi$ for which the discounted value of the utility flow of a type 1 firm is maximized. In other words, there exists steady states with type 1 firms restricting employment of the specific labor and creating unemployment. Naturally, it must be emphasized that the steady states may correspond to only to LOCAL rather than GLOBAL maxima ${ }^{4}$ in (32), nevertheless, they do exist.

In summary, in an interior steady state equilibrium type 1 firms commit to hire the proportion $1-\phi$ of type $k$ labor. The equilibrium wage paid out to employed type $k$ labor is equal to the markup of $\frac{1-\phi}{\phi}$ over the market clearing wage that prevails when $\phi=1$. Higher than the market clearing wage discourages type $k$ firms from entering and in fact only a fraction $\psi$ do enter and the remainder remain idle. Consequently, if the steady state number of type $k$

[^3]

Figure 6: The Dependence of $\phi$ on $\psi$ in an interior Steady State. $\alpha=1 / 3.5, \gamma=$ $0.9, \beta=0.95, q=0.85$, and $k=8$.
firms is equal to $n_{k}$ then the total amount of skilled specific labor of type $k$ is equal to $n_{k} L_{S}$ and the fraction $n_{k} \psi \phi L_{S}$ is hired by type $k$ firms, the fraction $n_{k}(1-\phi) L_{S}$ is hired by type 1 firms and the rest $n_{k}(1-\psi) \phi L_{S}$ remain unemployed. This unemployed pool is created artificially by a specific choice of $\phi$. It it is worth noting that some type $k$ firms choose not to operate and consequently create structural unemployment as some type $k$ labor is not hired even though there exists a fundamental demand for the skills of type $k$ unemployed labor. Naturally, in the steady state the unemployed pool evolves as some demands expire and other reaper in accordance with the process (25). However, at any point in time there are exactly $n_{k}(1-\psi) \phi L_{S}$ units of slack type $k$ labor.

In general the system of Bellman equations defining the continuation values admits multiplicity of equilibria. Accordingly, the actual paths that the economy follows do depend on both the initial position and the expectations held by both type 1 and type $k$ firms. The paper shuns away from the discussion of equilibrium selection and the role of expectations and limits its attention to a local analysis around a specific steady state. The value function of a type 1 firm in the neighborhood of an interior steady state with type $k$ firms being indifferent between entering and staying idle is depicted in figure (28).

Two important observations can be made. First of all the value function is locally decreasing. This is primarily due to the assumption that $\mu$ is negative. Moreover, perhaps not surprisingly the value of a type 1 firm is higher when type 1 firms choose to coordinate than steady state value of a type 1 firm when type 1 firms choose not to coordinate (the dashed line.) Figure (8) shows the


Figure 7: The Value of a Type 1 Firm in the Steady State. $\alpha=1 / 3.5, \gamma=$ $0.9, \beta=0.95, q=0.85$, and $k=8$.
equilibrium choices of $\phi$ and $\psi$ as functions of $n_{1}$. It needs to be reiterated that issues pertaining to multiplicity of equilibria, such as expectations and equilibrium selection, allow both $\phi$ and $\psi$ to be considered optimal only in the local sense. Nevertheless, the given choice of $\phi$ as a function of $n_{1}$ as shown in figure (8) is locally optimal and no agent has an incentive to deviate.

As described in the preceding subsection technology shocks do influence both the level of employment and the real wage. The impact of technology shocks in the case with type 1 firms choosing to coordinate can be described in an analogous comparative-static-like ${ }^{5}$ exercise. For that purpose assume that starting from the steady state the economy is hit with a technology shock at time $t$ with the production function becoming

$$
\begin{equation*}
c_{i}=A L_{i}^{\alpha} L_{G, i}^{1-\alpha} \tag{39}
\end{equation*}
$$

for one period and then returning to its normal form starting from period $t+1$ onwards. As before, this simple shock structure does not affect the functional forms of the continuation values and the economy's responsiveness to technology shocks can be analyzed easily. Figure () shows the level of total employment and the average real wage, given by,

$$
\begin{equation*}
\bar{\omega}_{R}=\frac{\omega^{R} L_{G}+\omega_{1}^{R} n_{1} L_{S}+\omega_{k}^{R} n_{k}^{*} \phi L_{S}+\omega_{k}^{R} n_{k}(1-\phi) L_{S}}{L_{G}+n_{1} L_{S}+n_{k}^{*} \phi L_{S}+n_{k}(1-\phi) L_{S}} \tag{40}
\end{equation*}
$$

[^4]

Figure 8: Optimal $\phi$ and $\psi$ as functions of $n_{1} . \alpha=1 / 3.5, \gamma=0.9, \beta=0.95, q=$ 0.85 , and $k=8$.
as a function of $A$, where $n_{k}^{*}=n_{k} \psi$ is the actual number of type $k$ firms that choose to enter.

## 5 Conclusions

The literature that deals with the issue of unemployment has been very vivid in the last quarter of the last century and the first several years of the new millennium. Two paradigms, search models and efficiency wage theories, that have shaped the thinking about the issue of unemployment have led to an immense increase in the understanding of the origins of unemployment. Moreover, the contributions of the last twenty five years not only have proven to be of conceptual value, but also have been used extensively in a wide range of policy tools. Nevertheless, it appears that there is strong demand for an alternative explanation of the presence of unemployment as the notion that unemployment can cause no welfare costs as long as the bargaining processes is properly balanced beyond those stemming from a purely physical constraint and the notion that unemployment serving as a as a credible deterrent can be actually welfare improving are seldom expressed unconditionally. This paper using several technical and two conceptual assumptions, one of them being the assumption of the ability to manage the market outcomes by a coordinated group of market players, aims at responding to the expressed demand.

The model presented in the paper shows that it may be optimal for an
individual firm, as long as others follow the suite, to offer wages that exceed the market clearing level even if workers to which the offer is made happen to be totally unproductive from the perspective of the firm and thus to create a disequilibrium in the labor market. The reason for that is a simple one and amounts to an observation that with higher than the market clearing wage it may be too costly for potential entrants who face an informational disadvantage to enter and to compete with the incumbent firms. Consequently, when entry is restricted by higher than the market clearing wages profits can actually be higher even though firms' costs are higher due to a larger wage bill. The cooperation between the incumbent firms is enforced in a dynamic setting with trigger strategies. The paper shows that the presence of unemployment is socially costly. It establishes that a revenue tax on the incumbent firms can spur competition and increase welfare. Finally, the paper adds a new insight into the debate on real wage volatility over the business cycle.

The paper should be viewed purely as a theoretical contribution. Its goal is to complement rather than to challenge the existing literature. The key results signal rather a feasibility of certain outcomes than an outright assertion that the described situation actually takes place.

## A Appendix

The purpose of this appendix it to show that agents may indeed find it optimal to stay on the path and continue cooperation rather then to deviate and trigger the punishment phase. As it was noted earlier the evaluation of the continuation values off the path can be potentially a formidable task for two reasons. First of all, the distribution of project types evolves from the initial steady state to the final steady state. Secondly, agents' expectations do influence the evolution process. Therefore, before any explicit calculation is made it is necessary to precisely define the evolution off the path.

Observe that once the punishment phase is triggered then there arises a possibility of a long run steady state in which at any point in time all type $k$ firms choose to enter, i.e., $\psi_{L R}=1$. Moreover, the approach path along which at any point in time all type $k$ firms enter, i.e., $\psi_{t}=1$ is also feasible. The following several lines of algebra prove this point formally.

Let the initial steady state be characterized with $\left(\phi_{0}, \psi_{0}\right)$ and the corresponding distribution of project types be given by $n_{1}^{0}=\frac{q^{k} \psi_{0}}{1-q+q^{k} \psi_{0}}$ and $n_{k}^{0}=\frac{1-q}{1-q+q^{k} \psi_{0}}$. Note that if the punishment phase is triggered at time 0 then the evolution of project types can be described with, under the assumption that $\psi_{t}=\psi_{0}$ at time 0 and $\psi_{t}=1$ at all future periods,

$$
\begin{align*}
n_{1}^{1} & =q n_{1}^{0}+q^{k}\left(1-n_{1}^{0}\right) \psi_{0}  \tag{41}\\
n_{1}^{t+1} & =q n_{1}^{t}+q^{k}\left(1-n_{1}^{t}\right)
\end{align*}
$$

Naturally, given that $0<q-q^{k}<1$ the process leads to the steady state with $n_{1}^{L R}=\frac{q^{k}}{1-q+q^{k}}$ and $n_{k}^{L R}=\frac{1-q}{1-q+q^{k}}$. Moreover, as $n_{1}^{1}=n_{1}^{0}<n_{1}^{L R}$ the approach to
the steady state occurs along the path at which at any point in time $n_{1}^{t}<n_{1}^{L R}$ and $n_{1}^{t}<n_{1}^{t+1}$ and consequently the following hold, as $\mu$ is a negative number,

$$
\begin{align*}
& \left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}>\left(q^{\mu_{1}} n_{1}^{L R}+q^{k \mu_{1}} n_{k}^{L R}\right)^{\mu},  \tag{42}\\
& \left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}>\left(q^{\mu_{1}} n_{1}^{t+1}+q^{k \mu_{1}} n_{k}^{t+1} \psi_{t+1}\right)^{\mu} \tag{43}
\end{align*}
$$

with $\psi_{t}=\psi_{0}$ at 0 and $\psi_{t}=1$ afterwards.
Clearly, the path is feasible. It remains to establish that the path is payoff consistent, i.e., it needs to be shown that the evolution of $\psi_{t}^{\prime} s$, with $\psi_{t}=\psi_{0}$ at 0 and $\psi_{t}=1$ thereafter, reflects optimality on the part of type $k$ firms. Naturally, the following dynamic equation must be satisfied once the punishment phase has been triggered

$$
\begin{aligned}
V_{1}^{t} & =(1-\gamma) q^{\mu_{1}}\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}-\theta+q \beta V_{1}^{t+1}+(1-q) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right)(44) \\
V_{k, A}^{t} & =(1-\gamma) q^{k \mu_{1}}\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}-\theta+q^{k} \beta V_{1}^{t+1}+\left(1-q^{k}\right) \beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right) \\
V_{k, I}^{t} & =\beta \max \left(V_{k, A}^{t+1}, V_{k, I}^{t+1}\right) .
\end{aligned}
$$

Note that in the long run, it is the case that $V_{1}>V_{k, I}=V_{k, A}$. Note that the path along which at all dates except the first one $\psi_{t}=1$ requires that $\forall t: V_{k, A}^{t} \geq V_{k, I}^{t}$. Assuming this to be true one reduces the system of dynamic equations (44) to

$$
\begin{align*}
V_{1}^{t} & =(1-\gamma) q^{\mu_{1}}\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}-\theta+q \beta V_{1}^{t+1}+(1-q) \beta V_{k, A}^{t+1}(45)  \tag{45}\\
V_{k, A}^{t} & =(1-\gamma) q^{k \mu_{1}}\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}-\theta+q^{k} \beta V_{1}^{t+1}+\left(1-q^{k}\right) \beta V_{k}^{t}\left(\frac{4}{4} \bar{\phi}\right) \\
V_{k, I}^{t} & =\beta V_{k, A}^{t+1} . \tag{47}
\end{align*}
$$

Now using (43) it is straight forward to show that $\forall t: V_{1}^{t} \geq V_{1}^{t+1}, V_{k, A}^{t} \geq V_{k, I}^{t+1}$ and $V_{k, I}^{t} \geq V_{k, I}^{t+1}$. Moreover, it must be

$$
\begin{equation*}
W^{t}=(1-\gamma)\left(q^{\mu_{1}}-q^{k \mu_{1}}\right)\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}+\beta\left(q-q^{k}\right) W^{t+1} \tag{48}
\end{equation*}
$$

where $W^{t}=V_{1}^{t}-V_{k, A}^{t}$. clearly $W^{t}$ must be positive for all $t$ as in the long run it is positive and (48) holds. Moreover, equation (48) implies as (43) holds that $\forall t: W^{t} \geq W^{t+1}$. Now, equation (46) and (47) imply that

$$
\begin{equation*}
V_{k, A}^{t}-V_{k, I}^{t}=(1-\gamma) q^{k \mu_{1}}\left(q^{\mu_{1}} n_{1}^{t}+q^{k \mu_{1}} n_{k}^{t} \psi_{t}\right)^{\mu}-\theta+q^{k} \beta W^{t+1} . \tag{49}
\end{equation*}
$$

Again (43) and the fact that $W^{t} \geq W^{t+1}$ for all $t$ imply that

$$
\begin{equation*}
\forall t: \quad V_{k, A}^{t}-V_{k, I}^{t} \geq V_{k, A}^{t+1}-V_{k, I}^{t+1} \tag{50}
\end{equation*}
$$

Now given that in the long run $V_{k, A}=V_{k, I}$ it must be that $\forall t: V_{k, A}^{t}-V_{k, I}^{t} \geq 0$. Hence, the solution to (45), (46) and (47) satisfies the initial assumption, i.e., there exists a payoff consistent path along which all type $k$ firms do enter.

Given the feasibility of a specific path off the path it is possible to assume that the economy will follow that specific path once the punishment phase is triggered. The values of type 1 firms for different degrees of coordination $\phi$ on the path and off the path are given in the picture below (9) clearly indicating that a number of steady states can be enforced with trigger strategies.


Figure 9: The Values of Staying on the Path and Deviating off the Path. $\alpha=$ $1 / 3.5, \gamma=0.9, \beta=0.95, q=0.85$, and $k=8$.

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[^1]:    ${ }^{1}$ It may be even the case that not all type 1 firms choose to enter. For expositional purposes this option is ignored. However, in all simulations presented in the text it is never the case that type 1 firms remain voluntarily idle.

[^2]:    ${ }^{2}$ This should be the case in the truly long run as one should not expect, given an option to retrain, a permanent skill mismatch.

[^3]:    ${ }^{3}$ It can actually be a correspondence.
    ${ }^{4}$ Some of them can even represent minima rather than maxima.

[^4]:    ${ }^{5}$ An obvious alternative would be to add a new state variable and model the value functions as functions of both $A$ and $n_{1}$, i.e., to set $V_{i}^{t}=V_{i}^{t}\left(A_{t}, n_{1}^{t}\right)$. The paper follow the simpler approach to economize on computational complexity.

