Demographic Uncertainty and Labour Market Imperfections in a Small Open Economy*

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Abstract

This paper extends Gertler's (1999) tractable overlapping generations model by allowing for imperfect labour markets and distortionary taxation. Furthermore, we allow for stochastic variation in demographic structure. The model is then used to study demographic change in a small open economy of Finland. The simulations highlight a key role played by labour market imperfections in determining a fiscal burden of ageing in defined benefit pensions systems. Higher labour market imperfections lead into considerably stronger responses of labour supply and taxes on ageing. Thus, imperfections magnify the problem associated with fiscal sustainability in ageing society. Stochastic simulations suggest that lengthening of working time has rather minor impact on alleviating the fiscal burden of ageing: Only a small fraction of stochastic variation in endogenously determined contribution rate is explained by the stochastic variation in the length of working time. Variation in fertility rate is clearly much more important.

1 Introduction

According to demographic trends, the old-age dependency ratio will almost double in Finland during the next 30 years. This generates a formidable burden on the pension system and long term fiscal balance. Several calculations detecting direct costs of ageing on fiscal balance indicate that pension expenditures share to GDP could raise roughly by 5 percentage points if the current pension system would prevail[1]. This would mean that contribution rates would need to rise several percentage points in order to fund the system.

Demographic aging will also lead to an increased expenditure on social welfare and health care. Some estimates show that social services to elderly population would double during the next 30 years. To the extent that these services remain publicly funded and efficiency gains remain moderate, the growth in the proportion of elderly people will further increase the tax burden of the working-age population and the public sector's share of the national economy. In addition, in Finland, additional tax burden on labour would be particularly detrimental for economic growth, given that tax burden on labour is already now at a very high level in

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[1] See for instance Lassila (2005)

international standards. The same applies to the size of the public sector in general. In addition to direct effects on fiscal balances, population ageing will have a number of critical impacts also on economic activity. The most important is certainly the effect on labour market.

To harness rather complicated and long-lasting effects of aging in the open economy like Finland, a comprehensive macroeconomic framework is needed. It should use key pension parameters to map the main features of the pension system. In addition, the effect of ageing on budgets and possible feedback affecting economic activity from other fiscal policy measures should be taken into account. In order to achieve this, the Bank of Finland's new dynamic general equilibrium macroeconomic model *Aino* has been enhanced by allowing for demographic transition arising from ageing and by adapting the model to track the key features of the current pension schemes.

Aino depicts the Finnish economy as dynamically optimizing small open economy with an internationally given real interest rate and non-stochastic balanced growth path. At balanced growth path, economic growth is determined by exogenously given growth of labour saving technology and population. Accumulation of financial assets and physical capital reflect optimal intertemporal decisions of households and firms. Optimal consumption and labour supply decision are based on Gertler (1999) tractable overlapping generation model, and extended to allow distortionary taxes and time varying retirement and death probabilities. Pensions are conditioned on aggregate wage level and on demographic trend. Finally, given that Finnish pension system is partially funded, we consider the funded part of the pension system as contractual saving (assets accumulated by the pension fund) and the PAYG part as a transfer from workers to pensioners. These transfers are financed by collecting pension contributions from the firms and the workers.

The results from various simulations highlight the key role played by taxation in the assessment of the economic costs of an ageing population. When the responses of labour supply, real wages and hence private consumption to higher taxation are consistently accounted for, the economy settles at a level of taxation clearly above that generally estimated in mechanical sustainability calculations. Even if the effective retirement age were to increase as expected, the burden from pension payments alone would cause the tax rate to rise to a level above that witnessed in the worst years of recession in the mid-1990s in Finland. In the light of the model calculations, the efficiency losses induced by higher taxation due to demographic change thus appear considerable.

In order to quantify the effects of uncertainty associated with demographic projection, we use stochastic simulations. Stochastic simulations suggest that there is considerable uncertainty associated to economy's reaction of demographic shocks in the long run. The results suggest also that lengthening of working time has rather minor impact on alleviating the fiscal burden of ageing: Only a small fraction of the stochastic variation in endogenously determined contribution rate is explained by the stochastic variation in working period. Variation in fertility rate is clearly much more important.

Rest of the paper is organised as follows. Section 2 provides a brief overview to aging projections and pension system in Finland. Section 3 discusses the model, including a description of the pension system in the model. Section 4-6 discusses the results from various policy experiments including a sensitivity analysis and stochastic simulations. Section 7 concludes.

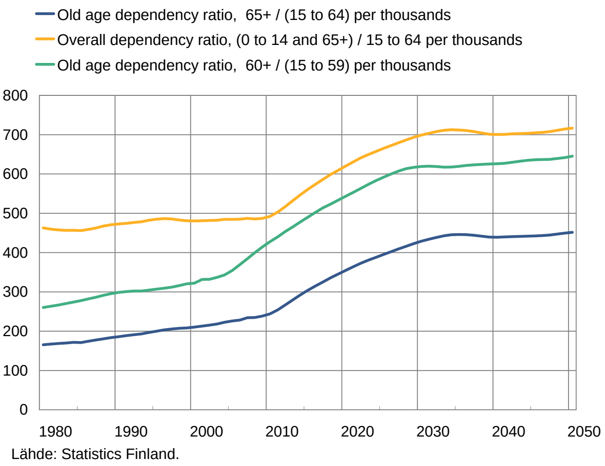

Figure 1: Dependency ratios

2 Aging and pension system in Finland

2.1 Aging - long-run projections

Changes in the projected population structure in Finland up to 2050 are highlighted in this section. Demographic trends and underlying assumptions are also discussed using illustrations from key age cohorts identified by the model structure. Finally, average pension scheme parameters are constructed on the basis of population projections broken down by relevant age cohort.

Population ageing is increasing the economic burden on the working age population. According to the latest population prognoses, the fastest growth in the old age dependency ratio will take place in the 2020s and 2030s, when the baby boom generation born after the second world war retires. After that, the ratio will continue to grow at a slower pace. The overall dependency ratio gives a somewhat less gloomy picture, since, the decreasing number of children resulting from the low fertility rate to some extent offsets the increase in life expectancy and the growth in the number of retirees (1). The overall dependency ratio is projected to level off by the 2030's. Immigration is forecast to remain insignificant, at only about 5000 people a year on net basis. Finally, according to Statistics Finland estimates, the life expectancy for men will grow from the current figure of 75.1 year to 82.1 year by 2040. For women the corresponding figures are 81.8 and 86.8 years.

2.2 An overview of the pension system

The pension scheme in Finland is defined benefit in the sense that pensions paid are not directly dependent on contributions workers have made to employment pension schemes or/and the yield of pension funds. Instead, rather than benefits changing, it has been the contribution rates that have moved in response to possible shortfalls in the balance. If there has been a danger of the agreed funding rate not being achieved, the level of contributions has been raised. The second general feature of the Finnish system is the negligible role of private and occupational pension schemes of private firms. Nearly all old age pensions are provided by employment pension institutions or national pension institutions closely controlled by the state.

The latest reform, effective from the beginning of 2005, moved the pension system in a more actuarial direction. With the introduction of a flexible retirement age of between 63 and 68, the level of pensions is being linked more explicitly than before with exit age and thus with contributions made by the insured. Furthermore, starting from 2009, changes in life expectancy will have an effect on pension levels. In addition, pension indexation has been changed so that all pensions will be indexed using a weight of 0.8 for living costs and 0.2 for wages where before a so called midway index was applied up to age 65. This will contribute to a comparative erosion in the value of pensions in compared to wage level. Contributions are collected from employers and employees. Currently, on average 16.8% is collected from employers and 4.6 % from the employees. Further changes in contribution rates have been agreed to be shared equally (50-50) between employers and employees.

Due to the benefit based nature of the Finnish employment pension system, long-term aggregate pension expenditure can be approximated simply from demographic factors and from the parameters determining average pension benefits. The most important factors are (naturally) demographic trends and the average age at which people start to draw pensions - which together determine the total number of pensioners. The average level of pensions is affected by wage levels and the pension index and therefore consumption prices and productivity developments. In addition, changes in life expectancy will have an effect on level of benefits. In addition, the replacement rate became dependent on exit age.

3 The model

3.1 General features

Aino depicts the Finnish economy as dynamically optimizing small open economy with an internationally given real interest rate and non-stochastic balanced growth path. At balanced growth path, economic growth is determined by exogenously given growth of labour saving technology and population. Accumulation of financial assets and physical capital reflect optimal intertemporal decisions of households and firms. A special attention has been devoted in the modelling of optimal consumption and labour supply decision such that the demographic change can be dealt with, yet maintaining an analytical tractability.

Households' saving decisions, and thus accumulation of financial assets, are influenced by households' desire to smooth consumption over time. Individuals are expected to have finite lives which consist of two distinct periods. We label the households living in these two different periods as 'workers' and 'retirees', as in Gertler's (1999).[2] In order to capture changing labour supply incentives of elderly, we assume that the 'retirees' participate in the labour markets. However, in comparison to workers, their labour efficiency is lower. Lower labour efficiency can capture issues such as part-time work and possibly lower productivity. In more general, elastic labour supply allows demographic change to feed into adjustment of capital and investment throught capital-labour substitution effect.

[2] Also Keushnigg (2004) has analysed the ageing in Austria using a variant of the Gertler (1999) model.

The likelihood that the worker may lose part of his labour income due to retirement, induces her to discount the future income stream at higher rate than otherwise. This reduces consumption and increases saving. In this sense, workers save for a rainy-day and retirement. This view is consistent with the study of Gourinchas and Parker (2002), who find that empirically observed saving patterns are in accordance with forward-looking optimizing behavior in a life-cycle setup with income uncertainty. Their study suggest that precautionary saving motive in early life implies that between 60-70 percent of non-pension wealth is due to precautionary saving. Finally, the planning horizon of pensioners is shorter than workers' due to the constant periodic probability of death. Therefore, in the model, pensioners' propensity to consume out of wealth is greater than that of the working-age population. Gourinchas and Parker (2002) estimate marginal propensity to consume out of liquid assets of 6-7 percent to retirees.

Individuals receive transfers from both the central government as well as from pension funds, following the general features of the social security system of Finland. In order to maintain analytical tractability and ease computational burden of the simulations, however, pensions are related to prevailing aggregate wage level, and not an individual characteristics. Other transfers are treated as lump sum. Given that Finnish pension system is partially funded, we consider the funded part of the pension system as contractual saving (assets accumulated by the pension fund) and the PAYG part as a transfer from workers to pensioners. These transfers are financed by collecting pension contributions from the firms and the working age as discussed above.

Supply side (production structure) is based on CES -production technology with factor augmentation in the underlying technological progresses and nominal and real rigidities. The model is closed by fiscal rules. Given that we separate between central government and the pension fund, one fiscal rule determines the pension fund's long-term net lending rate, while the other determines the central government 'debt ratio'.

3.2 Demographics

Consumers are assumed to be borne as workers.Conditional on being a worker in the current period, the probability of remaining one in the next period is ω_t, while the probability of retiring is $1 - \omega_t$. . These transition probabilities are independent on individuals' employment tenure, so that average tenure of working is $\frac{1}{1-\omega_t}$. In order to allow for non-stationary demographic structure, we subindex the probabilities by t. Once an individual has retired she is facing a periodic probability of death $(1-\gamma_t)$. Given that the survival probability γ_t is assumed to be independent of retirement tenure, but that it may depend on calendar time, the average retirement period at each point of time is $\frac{1}{1-\gamma_t}$.

Let N_t^w denote the stock of worker alive at time t. We then assume that $\left(1 - \omega_{t+1} + n_{t+1}^w\right)$ new workers are born in $t + 1$. This implies that the working age population grows at the gross rate of $1 + n_t^w$:

$$N_{t+1}^w = (1 - \omega_{t+1} + n_{t+1}^w)N_t^w + \omega_{t+1}N_t^w = \hat{N}_{t+1}^w N_t^w \tag{3.1}$$

where $\hat{N}_{t+1}^w = 1 + n_{t+1}^w$. Given constant probabilities of retirement and death and that cohorts are large, retiree population (N_t^r) evolves according to

$$N_{t+1}^r = (1 - \omega_{t+1})N_t^w + \gamma_{t+1}N_t^r \tag{3.2}$$

where N_t^r refers to stock of retiree population at time t. With some manipulations, it can be shown that the retiree to worker ratio $\varphi_t = \frac{N_t^r}{N_t^w}$ evolves according to

$$\varphi_t \equiv \frac{N_t^r}{N_t^w} = \frac{1-\omega_t}{\hat{N}_t^w} + \gamma_t \frac{\varphi_{t-1}}{\hat{N}_t^w} \tag{3.3}$$

Defining a stock of whole population as $N_t = N_t^w + N_t^r$, we can express the growth rate of whole population as a function of retiree to worker ratio and growth rate of working age population as follows:

$$\hat{N}_t = \frac{(1+\varphi_t)\, N_t^w}{\left(1+\varphi_{t-1}\right) N_{t-1}^w} = \frac{(1+\varphi_t)}{\left(1+\varphi_{t-1}\right)} \hat{N}_t^w \tag{3.4}$$

Notice also that retiree population grows with gross rate

$$\hat{N}_{t+1}^r = (1-\omega_{t+1})\varphi_t^{-1} + \gamma_{t+1} \tag{3.5}$$

In the steady state the demographic change has ended and so we have that

$$\varphi = \frac{1-\omega}{\hat{N}-\gamma} \tag{3.6}$$

$$\hat{N} = \hat{N}^w = \hat{N}^r \tag{3.7}$$

In contrast to large scale overlapping generations models, such as Auerbach-Kottlikoff (1987) we do not follow individual cohorts within the two age groups. This limits our ability to model the demographic change and pension system in very detailed manner. We also need to abstract from many other potential sources of heterogeneity in consumption and labour supply behavior. However, we can still specify the retirement and death probabilities as well as the growth rate of working age population in such a way that demographic transition can be captured at reasonable accuracy in the aggregate level. Similarly, linking pensions to demographics, we can roughly mimic the associated trends in pensions and public expenditures. Finally, allowing deterministic trends in retirement and death probabilities, allows us to generate demographic "shocks" that feed into dependency ratio gradually, rather than instantly. Deterministic trends in retirement and death probabilities have been consistently dealt with when deriving the first order conditions, as discussed below.

3.3 Households

3.3.1 Preferences

Households' preferences are expressed recursively using the constant elasticity aggregator

$$V_t = [\{u(C_t, l_t)\}^{\rho_c} + \beta \{E_t(V_{t+1})^{\mu}\}^{\frac{\rho_c}{\mu}}]^{\frac{1}{\rho_c}} \tag{3.8}$$

V_t is the value function β gives subjective time preference. The parameter $\rho_c < 1$ captures intertemporal subsitution. A special case of $\mu = 1$ applied here, corresponds to a type of risk neutrality, where the agents are indifferent regarding risk, but still maintaining a non-trivial preference for the time at which consumption occurs (cf.

Farmer (1990)).[3] This special case is analytically tractable, since it generates linear decision rules even with (idiosyncratic) risk to income, asset return and length of life. In addition to risk neutrality and recursive structure of the preferences, we assume that individuals enjoy utility from consumption C_t as well as leisure according to following utility functional

$$u(C_t, l_t) = C_t^v (1 - l_t)^{1-v} \tag{3.9}$$

Taking into account the two distinct periods of life as well as retiring and death probabilities, the preferences of the households can be summarised as follows

$$V_t^z = \left\{ [(C_t^z)^v \, (1 - l_t^z)^{1-v}]^{\rho_c} + \beta^z [E_t(V_{t+1}|z)]^{\rho_c} \right\}^{\frac{1}{\rho_c}} \tag{3.10}$$

where

$$E_t(V_{t+1}|w) = \omega_t V_{t+1}^w + (1 - \omega_t) V_{t+1}^r, \;\; \beta^w = \beta \tag{3.11}$$

$$E_t(V_{t+1}|r) = V_{t+1}^r, \;\; \beta_t^r = \beta\gamma_t. \tag{3.12}$$

$z = w, r$ indicates whether the individual is worker or retired with obvious notation. C_t^z is consumption and $1 - l_t^z$ denotes leisure. Thus, l_t^z denotes the fraction of time allocated to work. v is the elasticity of the period utility with respect to consumption, while ρ_c captures the intertemporal curvature of the preferences. The willingness to smooth consumption over time implies a finite (constant) intertemporal elasticity of substitution $\sigma = 1/(1 - \rho_c)$. The retirees effective discount factor β^r is adjusted to take into account periodic probability of death.

We assume perfect annuities market in order to eliminate the impact of uncertainty about time of death: Remaining wealth that retirees hold at the time of death are invested in mutual fund which in turn invests them in available financial assets at each period of time. Those surviving to the following period receive a return that is proportional to his contribution to the fund. For instance, if R_t is the gross return per unit invested by the fund, the gross return for a surviving retirees is R_t/γ_t at time t.

Workers, in turn, faces a potential risk of a decline in wage income. However, since individual's preferences are over the mean of next period's value function, only a desire the smooth consumption over time affects on consumption pattern in the face of idiosyncratic income risk. Thus, a worker simply forms a certainty equivalent of his random utility as shown in equation.[4](3.11).

Both retirees and workers consume and save out of income derived from financial assets, labour and transfers received from the public sector. Given specific assumptions regarding preferences and population dynamics, there is no need to keep track on that how assets and consumption are distributed among retirees and workers. Since marginal propensities to consume are the same for each individual within the two groups, we can simply aggregate by summing across individuals within the

[3] Since ρ_c is bounded above by 1, it follows that the risk-neutral decision maker prefer late resolution of uncertainty (for details see Kreps and Porteus (1978)).

[4] Assumption of risk-neutrality is important. For instance, analysis of welfare effects of social security reforms are importantly affected by the consideration of risk. Mandatory social security system imposes an implicit tax on the households so that there is a reduction in expected life-cycle income (due to social security contributions). However, if social security system reduces the variance of life-cycle income by pooling the income risk between young and old generation, there is potentially a trade-off between a reduction in expected life-cycle income and the variance: Reduction of welfare due to mandatory social security would then be lower for risk-averse households than for risk-neutral ones.

groups. However, since marginal propensities to consume out of wealth differ between the two groups, we must keep track on how financial assets are distributed between workers and retirees: aggregate private consumption, which is a sum of consumption of workers and retirees will depend upon evolution of this wealth distribution.

3.3.2 Retirees

A retiree born at time j and retired at time k, and who survive at least until $t+1$ solves the maximisation problem

$$\max_{C_t^{rjk}, l_t^{rjk}} V_t^{rjk} = \left\{ [\left(C_t^{rjk}\right)^v (1-l_t^{rjk})^{1-v}]^{\rho_c} + \beta\gamma_t [E_t(V_{t+1}^{rjk})]^{\rho_c} \right\}^{\frac{1}{\rho_c}}$$

s.t.

$$A_{t+1}^{rjk} = \frac{1}{\gamma_t} R_t A_t^{rjk} + W_t(1-\mathfrak{t}_t^{RS})\xi l_t^{rjk} + \mathcal{T}_t^{rjk} - P_t^c C_t^{rjk} \tag{3.13}$$

where R_t denotes after tax gross rate of return of financial assets A_t^{rjk}, $\mathcal{T}_t^{rjk}$ denotes pensions and $\xi < 1$ is labour efficiency of retirees with respect to workers[5]. P_t^c is a price index of consumption to be determined later on. From the first order condition for labour, we can first derive a standard labour supply condition:

$$1 - l_t^{rjk} = \frac{1-v}{v} \frac{P_t^c C_t^{rjk}}{(1-\mathfrak{t}_t^{RS}) W_t \xi} \tag{3.14}$$

Solving the retiree's maximization problem with respect to consumption, using (3.14) and then aggregating over retirees results into following aggregate consumption equation:

$$P_t^c C_t^r = \epsilon_t \pi_t [R_t A_t^r + \mathcal{H}_t^r + \mathcal{S}_t^r] \tag{3.15}$$

$\mathcal{H}_t^r$ and $\mathcal{S}_t^r$ denote discounted after tax values of labour income and pensions and $\epsilon_t\pi_t$ is retirees marginal propensity to consume out of wealth. More specifically

$$\mathcal{H}_t^r = (1-\mathfrak{t}_t^{RS}) W_t \xi L_t^r + \frac{\mathcal{H}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1}/\gamma_{t+1}} \tag{3.16}$$

$$\mathcal{S}_t^r = \mathcal{T}_t^r + \frac{\mathcal{S}_{t+1}^r}{\hat{N}_{t+1}^r R_{t+1}/\gamma_{t+1}}. \tag{3.17}$$

Since the total social security payments (pensions) are distributed equally among retirees gross growth rate of retirees $\hat{N}_{t+1}^r$ enters into the discount factor. Discount factor of human wealth is similarly augmented $\hat{N}_{t+1}^r$. Notice that in the steady state $\hat{N}_t^r = \hat{N}$. Retirees marginal propensity to consume out of wealth $\epsilon_t\pi_t$ evolves according to following non-linear difference equation:

$$\epsilon_t\pi_t = 1 - \left(\frac{W_t/P_t^c}{W_{t+1}/P_{t+1}^c} \frac{(1-\mathfrak{t}_t^{RS})}{(1-\mathfrak{t}_{t+1}^{RS})} \right)^{\frac{(1-v)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{R_{t+1}}{\hat{P}_{t+1}^c} \frac{\gamma_t}{\gamma_{t+1}} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\epsilon_t\pi_t\gamma_{t+1}}{\epsilon_{t+1}\pi_{t+1}} \tag{3.18}$$

[5] Parameter ξ could also capture a difference in working hours supplied between retirees and workers.

where $\hat{P}^c_{t+1} \equiv P^c_{t+1}/P^c_t$. The retirees marginal propensity to consume varies with real interest rate $R_{t+1}/\hat{P}^c_{t+1}$ as well as with expected changes in real net wage income. Due to the fact that survival probability can vary over calendar time, it influences on retirees effective discount rate and introduces additional dynamics into marginal propensity to consume equation.

As in standard Yaari (1965) and Blanchard (1985) models, likelihood of death $(1-\gamma_t)$ in (3.18) raises the retirees' marginal propensity to consume. This can be seen easily by considering a limiting case of logarithmic preferences ($\sigma \to 1$) and when survival probability is constant. In this case

$$\epsilon\pi = 1 - \beta\gamma \tag{3.19}$$

3.3.3 Workers

As regards to workers, their decision problem reads as

$$\max_{C^{ws}_t, l^{ws}_t} V^{ws}_t = \left\{ [(C^{ws}_t)^{v} (1 - l^{ws}_t)^{1-v}]^{\rho_c} + \beta [E_t(V^{ws}_{t+1})]^{\rho_c} \right\}^{\frac{1}{\rho_c}} \tag{3.20}$$

s.t.

$$A^{ws}_{t+1} = R_t A^{ws}_t + (1 - \mathfrak{t}^{WS}_t - \mathfrak{t}^{WP}_t) W_t l^{ws}_t + \mathcal{T}^{ws}_t - P^C_t C^{ws}_t \tag{3.21}$$

$\mathcal{T}^{ws}_t$ denotes financial transfers to working age, $\mathfrak{t}^{WS}_t$ is worker's labour income tax rate and $\mathfrak{t}^{WP}_t$ is pension contribution rate. First order condition for labour yields standard labour supply condition

$$1 - l^{ws}_t = \frac{\frac{1-v}{v} P^c_t C^{ws}_t}{(1 - \mathfrak{t}^{WS}_t - \mathfrak{t}^{WP}_t) W_t} \tag{3.22}$$

Intertemporal maximisation in turn gives rise to a rather complicated Euler equation, but once more, consumption plan by workers aggregates to

$$P^c_t C^w_t = \pi_t [R_t A^w_t + \mathcal{H}^w_t + \mathcal{S}^w_t] \tag{3.23}$$

π_t is worker's marginal propensity to consume and $\mathcal{H}^w_t$ and $\mathcal{S}^w_t$ denote human and social security wealth correspondingly. Marginal propensity to consume out of wealth is a non-linear first order difference equation, which takes a form

$$\pi_t = 1 - \left(\frac{\left(1 - \mathfrak{t}^{WS}_t - \mathfrak{t}^{WP}_t\right) W_t / P^c_t}{W_{t+1}/P^c_{t+1}} \right)^{\frac{(1-\nu)\rho_c}{1-\rho_c}} \beta^{\frac{1}{1-\rho_c}} \left(\frac{\Omega_{t+1} R_{t+1}}{\hat{P}^c_{t+1}} \right)^{\frac{\rho_c}{1-\rho_c}} \frac{\pi_t}{\pi_{t+1}} \tag{3.24}$$

where $\mathfrak{t}^{WS}_t$ is statutory tax rate on wage income of the workers, $\mathfrak{t}^{WP}_t$ is pension contribution rate and Ω_{t+1} is the factor that weights the gross real return $R_{t+1}/\hat{P}^c_{t+1}$. This factor evolves according to

$$\Omega_{t+1} = \omega_t \left(\frac{1}{(1 - \mathfrak{t}^{WS}_{t+1} - \mathfrak{t}^{WP}_{t+1})} \right)^{1-\upsilon} + (1 - \omega_t) \epsilon_{t+1}^{-\frac{1-\rho_c}{\rho_c}} \left(\frac{1}{\xi \left(1 - \mathfrak{t}^{RS}_{t+1}\right)} \right)^{1-\upsilon} \tag{3.25}$$

where $\mathfrak{t}^{RS}_{t+1}$ is statutory tax rate paid by the retirees and $\epsilon_{t+1} > 1$ is a ratio of marginal propensity to consume of the retirees to that of the workers.

$\mathcal{H}_t^w$ in (3.23) is a discounted sum of the wage bill of workers (in net terms) and $\mathcal{S}_t^w$ is the sum across workers alive at t of the capitalised value of social security (in net terms). Both of these measures take into account corresponding discounted values at the time of retirement. Formally,

$$\begin{aligned}\mathcal{H}_t^w &= \frac{\omega_t \left(\frac{1}{(1-\mathfrak{t}_{t+1}^{WS}-\mathfrak{t}_{t+1}^{WP})}\right)^{1-\upsilon}\mathcal{H}_{t+1}^w}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^w} + \left(1-\mathfrak{t}_t^{WS}-\mathfrak{t}_t^{WP}\right)W_t L_t^w \qquad (3.26)\\ &+\frac{(1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho_c}{\rho_c}}\left(\frac{1}{\xi\left(1-\mathfrak{t}_{t+1}^{RS}\right)}\right)^{1-\upsilon}\varphi_{t+1}^{-1}\mathcal{H}_{t+1}^{r(t+1)}}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^r}\end{aligned}$$

$$\begin{aligned}\mathcal{S}_t^w &= \mathcal{T}_t^w + \frac{\omega_t\left(\frac{1}{(1-\mathfrak{t}_{t+1}^{WS}-\mathfrak{t}_{t+1}^{WP})}\right)^{1-\upsilon}\mathcal{S}_{t+1}^w}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^w} \qquad (3.27)\\ &+\frac{(1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}}\left(\frac{1}{\xi\left(1-\mathfrak{t}_{t+1}^{RS}\right)}\right)^{1-\upsilon}\varphi_{t+1}^{-1}\mathcal{S}_{t+1}^{r(t+1)}}{R_{t+1}\Omega_{t+1}\hat{N}_{t+1}^r}\end{aligned}$$

$\mathcal{H}_{t+1}^{r(t+1)}$ measures the aggregate value of human wealth for the working retiree who retired at time $t+1$, but was still working at time t. Similarly, $\mathcal{S}_{t+1}^{rj(t+1)}$ measures the value of total social security for the retiree, who retired at time $t+1$, but was still working at time t. A factor $\hat{N}_{t+1}^w$ augments the discount rate of the capitalised value of social security for the workers because with finite lives, a share of total social security entitlements of those currently alive declines over time as working age population grows. By similar argument, $\hat{N}_{t+1}^w$ enters into the discount factor of human wealth. Moreover, notice that in the limiting case of logarithmic preferences ($\sigma \to 1$) marginal propensity to consume is constant, and it depends only on discount rate β:

$$\pi = 1-\beta \qquad (3.28)$$

Enlarged discount rate due to presence of $\Omega_{t+1} > 1$ in the denominator of (3.26)-(3.27) means that workers value the human wealth and social security less relative to infinite horizon case. This in turn has a tendency to reduce working age individual's consumption and increase saving. Importantly, distortionary taxes similarly increase the workers' discount factor. This factor is also useful when assessing the importance of various channels of making the model depart from Ricardian equivalence assumption. Both distortinary taxes and finite lenght of life makes Ω large and correspondingly make the model "less Ricardian". This can be seen most easily by looking at the steady state value of Ω in the special case where retirees and workers face the same tax rate $\mathfrak{t}$. Then,

$$\Omega = \left(\frac{1}{\xi(1-\mathfrak{t})}\right)^{1-\upsilon}\left[\omega + (1-\omega)\epsilon^{-\frac{1-\rho_c}{\rho_c}}\right] \qquad (3.29)$$

3.4 Distribution of wealth and aggregate consumption

Workers and retirees different marginal propensities to consume are reflected in the rate at which the two groups accumulate financial assets. Aggregate consumption then depends on how financial assets are distributed among the two groups. Consequently, we need a state equation for distribution of wealth. Let $\lambda^r_{t+1} \equiv \frac{A^r_{t+1}}{A_{t+1}}$ be a share of financial assets held by the retirees and let $1 - \lambda^r_{t+1} \equiv \frac{A^w_{t+1}}{A_{t+1}}$ be the share of financial assets held by the workers. It can be shown that retirees share of financial wealth evolves according to:

$$\lambda^r_{t+1} = (1 - \frac{\epsilon_t \pi_t}{\nu}) \frac{R_t \lambda^f_t A_t}{A_{t+1}} + \frac{(1 - \tau^{RS}_t)\xi W_t N^r_t + \mathcal{T}^r_t - \frac{\epsilon_t \pi_t}{\nu}(\mathcal{S}^r_t + \mathcal{H}^r_t)}{A_{t+1}/\omega_t} + \frac{(1 - \omega_t)}{\omega_t} \tag{3.30}$$

Aggregate private consumption can then be obtained simply by summing up (3.15) and (3.23), using $\lambda^f_{t+1} \equiv \frac{A^r_{t+1}}{A_{t+1}}$ and remembering that all the assets are eventually held by the domestic consumers:

$$P^c_t C^H_t = \pi_t \left(\left[\left(1 - \lambda^r_t\right) R_t A_t + \mathcal{H}^w_t + \mathcal{S}^w_t \right] + \epsilon_t [\lambda^r_t R_t A_t + \mathcal{H}^r_t + \mathcal{S}^r_t] \right) \tag{3.31}$$

Equation for aggregate consumptions shows that transfers influence markedly on the evolution of the distribution of wealth, which in turn influences on aggregate consumption. Labour income taxes influence on consumption directly via the measures of human wealth and income transfers, but also indirectly through its effect on labour supply and distribution of assets between retirees and workers.

3.5 Assets

There are different financial assets available for consumers : domestic government bonds A^S_t, foreign bonds A^W_t and stocks issued by the domestic firms A^F_t. In addition, it is assumed that all the assets accumulated by the pension fund A^p_t are hold by the domestic consumers.

The domestic one period bonds pay a nominal return r_t, while the gross return of stocks is determined according to the profits of the firms in the model. Foreign bonds pay a return r^F_t, which is exogenously given. Arbitrage condition equates ex ante returns of domestic and foreign bonds yielding to a standard Uncovered Interest Rate Parity (UIP) condition. The share price is the nominal price (ex-dividend) of a unit of equity in period t. The factor defining the gross return of stocks is the profits of the firms Π^D_t in the model. This gross return is defined as follows

$$1 + r^D_t = [A^F_{t+1} + (1 - \mathfrak{t}^K_t)\Pi^D_t]/A^F_t \tag{3.32}$$

where $\mathfrak{t}^K_t$ denotes corporate tax rate. Optimal consumption plans can be combined with the arbitrage equation for holding different assets. This gives the two equations that relate after tax interest rates to each other

$$r^D_t = r^S_t(1 - \tau^S_t) + \daleth_t \tag{3.33}$$

$$1 + r^S_t = \left(1 + r^F_t\right) \frac{S_{t+1}}{S_t} \tag{3.34}$$

S_t is nominal exchange rate, r^S_t denotes domestic short-term nominal interest rate and r^F_t denotes corresponding foreign short term interest rate. $\mathfrak{t}^S_t$ is tax rate at source. The latter is a standard UIP condition. In addition to this, we assume an exogenously determined risk-premium $\daleth_t$ between domestic bonds and stocks issued by the domestic firms.

3.6 Labour Markets

The model features nominal wage rigidity in a form of Calvo pricing so that only a fraction of workers can re-set their wage in each period. For those not being able to optimise in period t, the wage is adjusted using the steady state growth rate of wages. This steady state growth rate is denoted by $w\bar{g}$ and it equals steady state productivity growth plus inflation. Behavior of aggregate nominal wages is then characterised by the following two equations:

$$W_t^* = \frac{\frac{(1-v)}{v} P_t^c C_t^w / \left(1 - \mathfrak{t}_t^{WS} - \mathfrak{t}_t^{WP}\right)}{[N_t^w - L_t^w]} \tag{3.35}$$

$$W_t = \frac{(1-q)\,\beta w\bar{g}}{(1+\beta(1-q)^2 w\bar{g}^2)} E_t W_{t+1} + \frac{(1-q)\,w\bar{g}}{(1+\beta(1-q)^2 w\bar{g}^2)} W_{t-1} + \frac{q(1-(1-q)\beta w\bar{g}^2)}{(1+\beta(1-q)^2 w\bar{g}^2)} W_t^* \tag{3.36}$$

where $P_t^c C_t^w$ is consumption of workers, N_t^w is worker population and L_t^w denotes labour demand of workers $q \in (0,1)$ is exogenous (Calvo) probability that determines how often randomly chosen worker is allowed to re-set her wage. $\mathfrak{t}_t^{WS}$ denotes labour income tax rate of working age population and $\mathfrak{t}_t^{WP}$ denotes pension contribution rate. Equation for optimal wage W_t^* is directly derived from the aggregate version of worker's labour supply decision.[6]

Given that workers' and retirees' labour efficiency differ, we define aggregate effective labour supply index L_t as

$$L_t = L_t^w + \xi L_t^r \tag{3.38}$$

Here $\xi \in (0,1)$ denotes the relative efficiency of a unit of retirees' labour. Labour demand for workers L_t^w is derived from (3.38) by assuming that retirees are always on their labour supply curve at prevailing wage (W), and that the domestic intermediate goods producer is always on its labour demand curve. In solving the steady state version of the model, the labour demand/supply indices presented above are made stationary by scaling them with N_t, while wages are scaled by labour augmenting technical change Λ_t^L and *numeraire* price level P_t, to be determined later on.

[6] Imperfect, non-competitive features for the labour markets can be introduced by assuming that each intermediate goods firm uses CES combination of differentiated types of workers. Their labour demand index is given by

$$L_t^w = \left[\int_0^1 L_t^w(j)^{\rho_L} dj\right]^{\frac{1}{\rho_L}}$$

where $L_t^w(j)$ denotes the demand of type j worker. Cost minimisation implies that the demand of worker type j depends upon relative wage and aggregate labour demand index as follows:

$$L_t^w(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\eta} L_t^w \tag{3.37}$$

where $\eta = \frac{1}{1+\rho_L}$ is elasticity of substitution among differentiated labour inputs. $W_t(j)$ denotes wage paid to worker type j and the wage index W is defined as

$$W_t = \left[\int_0^1 W_t(j)^{\frac{\rho_L}{\rho_L+1}} dj\right]^{\frac{1+\rho_L}{\rho_L}}$$

W^* then becomes $W_t^* = \frac{1}{\rho_L} mrs_t^w$, where mrs_t^w is marginal rate of substitution between consumption and leisure and ρ_L^{-1} is wage mark-up.

3.7 Public sector

The general government (public sector) is divided into two sectors, labeled as a state (central government) and pension funds. The state collects taxes from labour income at rate $\mathfrak{t}_t^{WS}$, $\mathfrak{t}_t^{RS}$, from capital gains at rate $\mathfrak{t}_t^{K}$ and from consumption at rate $\mathfrak{t}_t^{C}$. The state consumes C_t^S and pays both taxable and non-taxable income transfers to workers and to retirees. In the budget constraint the total transfers are denoted by $\mathcal{T}_t^S$. In addition, the state issues one period government bonds A_t^S that pays a nominal return r_t. In each period, the following budget constraint holds

$$
\begin{aligned}
&-\left(A_t^S - A_{t-1}^S\right) \text{ (net lending)} \\
&= \mathfrak{t}_t^{WS} W_t L_t^w + \mathfrak{t}_t^{RS} \xi W_t L_t^r \text{ (income tax revenues)} \\
&+ \mathfrak{t}_t^{K} \Pi_t \text{ (corporate income tax revenues)} \\
&+ \mathfrak{t}_t^{C} P_t^C C_t^F \text{ (indirect taxes)} \\
&+ \mathfrak{t}_t^{FS} W_t L_t \text{ (firms' social security contributions)} \\
&- P_t^C C_t^S \text{ (government consumption)} \\
&- \mathcal{T}_t^S \text{ (total net transfers)} \\
&- r_t A_{t-1}^S \text{ (interest payments)}
\end{aligned} \tag{3.39}
$$

3.8 Fiscal policy rule and fiscal response - a little detour

In order generate realistic response of the model to government spending shock, fiscal policy rule, together with the assumptions on price and wage imperfections are of crucial importance. Standard DSGE models build on New Keynesian tradition typically predict a strong negative response of private consumption to government spending shocks. This is typically due to the negative wealth effect which induces households increase their labour supply and cut private consumption. By an large, however, the empirical literature shows that government spending shocks have a positive or, at least, only moderate negative effect on private consumption[7].

In the Aino model, negative wealth effect of government consumption is reduced at least by two factors. First, when the government spending shock is modelled through increase in public purchases of private goods, assumption of imperfect competition generates aggregate demand externality. Subsequent increase in output and profits then dampens the negative wealth effect. Second, non-Ricardian households and relatively high discounting due to finite lives, makes the current consumption track current disposable income by more than in the standard "Ricardian economy". Consequently, both imperfections and finite lives of the households potentially dampen the strong negative wealth effect of government spending.

Beside the above mentioned features, the short and medium run impact of, say, government spending shock on private consumption in the model with non-Ricardian features crucially depend on how much the real wages react on fiscal impact and the degree of debt financing[8]. The degree of debt financing, in turn, is controlled by the fiscal rules in the models like this.

[7] Fata´s and Mihov 2001, Mountford and Uhlig 2001, Canzoneri et al. 2002, Perotti 2002, Gali et al. 2004.

[8] Real wage response is important because more negative response of real wage strengtheness a negative wealth effect of a government spending shock. Empirically, it has been suggested that the real-wage response to government spending shocks is rather small (Fata´s and Mihov 2001).

Typically, the models like this are closed either by a tax rule or by a lump sum transfer rule. We use a rather general form of a tax rule that stabilizes the evolution of government debt through labour income tax. Formally, the tax rule takes a following partial adjustment format[9]:

$$\tau_t^{ws} = \tau_{t-1}^{ws} - \theta_1(\tau_{t-1}^{ws} - \bar{\tau}^{ws}) + \theta_2(A_t^S - \bar{A}^S)/Y_t \tag{3.40}$$

The tax rule has two attractors $\bar{\tau}$ and $\bar{A}^S/Y_t$ towards which the tax, and consequently the debt to output ratio are stabilised. The rule has a feature by which fiscal spending shock generates realistic paths for debt to output ratio and labour income taxes as observed in the Finnish data. Benchmark values for the parameters θ_1 and θ_2 have been calibrated to 0.3 and 0.1 respectively. $\bar{\tau}$ and $\bar{A}^S/Y$ has been set such that the public debt to output ratio reach wanted steady state values. Because taxes are distortionary in the model, this way of closing the model, however, decreases the impact multiplier of government spending shock.

In order to illustrate this, figure 2 shows how the model reacts to government spending shock, here modelled as a shock to lump sum transfers to the households. In a benchmark simulation (red, solid line), aggregate consumption reacts mildly positively at the beginning but is quickly cut back so that after a one year private consumption is already negative relative to its steady state value. Positive impact of private consumption at the beginning is solely due to the fact that it succeeds stimulating only the retirees consumption. Their increase consumption by 0.8 % as a response to one percentage point increase in government spending. Worker's reaction to spending shock is negative from the beginning, as real wage increase is not enough to compensate expected surge in taxes.

Dynamic response paths look very different when we assume a very slow fiscal adjustment (blue, dashed line). Slow fiscal adjustment increases markedly the degree of debt financing, thus generating less tax distortions through the labour markets. Furthermore, given that workers (and retirees) discount future income streams at higher rate than at which government can borrow, fiscal policy that postpones taxes into the future boosts up consumption in the short-run. This makes the model clearly less Ricardian. Both these features are clearly illustrated in a moderate and prolonged positive response of workers' consumption to fiscal stimulus. The fact that retirees' consumption response is less affected by a slower fiscal adjustment shows how the distorting effect of taxes through the labour markets play a crucial role in determining the effectivenes of fiscal stimulus to the economy. Finally, in both simulations an investment crowding out is visible, yet rather moderate in the latter case.

3.9 Statutory pension fund

There are several motivations to consider the pension fund(s) separate from the central government. First, when the pension scheme is defined benefit and partly funded, we should consider the funded part of the pension system as contractual saving, as opposed to discretionary saving, while the PAYG part should be considered as a direct transfer from young generation (workers) to old generation (pensioners)[10].

[9] See for instance Railavo (2004) for the discussion on alternative fiscal policy rules and their stability properties.

[10] In Finland, approximately 25% of the pensions are funded.

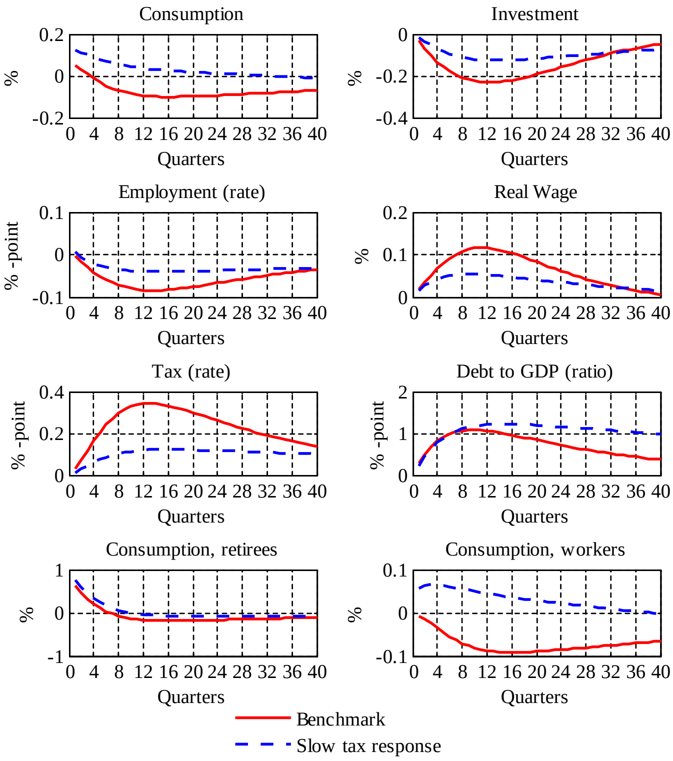

Figure 2: Dynamic responses to a government spending shock (transfers).
Note: This figure depicts the dynamic responses of the economy to a persistent government spending shock modelled as a lump sum transfer to households. The parameter governing the degree of persistence of a shock is set equal to 0.85. Both workers and retirees receive a transfer that is proportional to their relative shares in the population. Dynamic responses are depicted as deviations from the steady state, either in percentage or in percentage point terms as appropriate. Demographic structure is kept unchanged along the transition paths. Slow fiscal adjustment refers to the case where $\theta_1 = 0.5$ and $\theta_2 = 0.05$.

In Finland, where approximately 25% of the pensions are funded both features are quantitatively important

Second, pension contributions are considered, at least partly, taxes. Analogously with the previous section, this means that the way in which increasing fiscal burden of aging is financed along demographic transition path is of crucial importance as regards to labour market responses of the economy to aging. The trade-off is clearly as to what degree demographic transition is financed by changing the pension fund's asset position or by changing the contribution rates.[11]

Accordingly, we thus assume that the pension funds in the economy are administrated separately from the central goverment. The fund collects pension contributions from the private sector[12] as well as distributes pensions to retirees $\mathcal{T}_t^{PR}$. Pension funds accumulate financial assets A_t^P. In each period, therefore, the following flow budget constraint holds for the pension fund:

$$\begin{aligned} &-(A_t^P - A_{t-1}^P) \text{ (net lending)} \\ &= \mathfrak{t}_t^P W_t L_t \text{ (social security contributions of employer and employee)} \\ &- \mathcal{T}_t^{PR} \text{ (total transfers paid to retirees)} \\ &- r_t A_{t-1}^P \text{ (interest payments)} \end{aligned} \tag{3.41}$$

where $\mathfrak{t}_t^P = \mathfrak{t}_t^{FP} + \mathfrak{t}_t^{WP}$ is overall pension contribution rate, consisting of employer and employee contributions. Finally $\mathcal{T}_t^{PR}$ denote pensions and other transfers from pension funds to retirees.

3.10 Contribution rule

Inter-generational distribution of fiscal burdens related to ageing is not actively managed by institutional controls built into current pension schemes. According to Fenge and Werding (2003) "it merely just happens". Consequently, inter-generational distribution is hardly an issue when devising the contribution rule for the model like this. At the same time, however, the contribution rule should be made flexible, yet realistic enough to study alternative demographich scenarios and make policy experiments. As discussed earlier, in Finland the contribution rate has been adjusted so as to maintain the pension fund's fiscal balance. In fact, the pension funds net lending relative to aggregate wage sum has been rather stable during the last 25 years (sample mean is 0.07). Given this, the contribution rule should stabilise the net lending to aggregate wage sum to some predetermined level in the long run while maintaining a close but not a perfect relationship between contribution rate and ratio of pensions to wage sum . Consequently, we write a simple "net lending" rule

[11] Third important consideration is that pension funds hold the savings for the households that are completely illiquid. Households are thus not able to borrow, or at least only a very limited amount, against their savings accumulated in the pension funds. This means that households do not see pensions accumulated to the pension funds as perfect substitutes for more liquid forms of saving, such as bonds and equity. This is supported by the findings from empirical literature according to which a growth in partially funded pension schemes does boost personal saving, but not one-to-one. In the model's economy pension fund's assets are considered as part of the household' financial wealth that yield the same return as government bonds. Thus the current version of the model does not address the issue related to imperfect substitutability between private savings and contractual saving to the pension fund.

[12] It may also consume C_t^P and invests I_t^P and received small transfers $\mathcal{T}_t^{SP}$ from the state in each period. These are small items in the national accounts however, so we ignore them.

that stabilises a net lending to wage sum ratio to some prespecified target. Formally

$$\mathfrak{t}_t^P = \mathfrak{t}_{t-1}^P + \theta_3(\frac{(A_t^P - A_{t-1}^P)}{W_t L_t} + \bar{A}^P) \tag{3.42}$$

where $\bar{A}^P$ is the target rate for net lending to wage sum and θ_3 is a adjustment parameter. This form is flexible enough so that alternative policy experiments can be generated. For instance setting $\bar{A}^P$ to zero and θ_3 large implies pure PAYG system. In our benchmark simulations, we set $\theta = 0.15$ and $\bar{A}^P = 0.07$.

3.11 Pension expenditures

Allowing idiosyncratic history dependence in social security[13] and pension payments would make the model perhaps more realistic, but at the same time, we would loose analytical tractability. However, we can link the model's pension expenditures/transfers into the demographic structure and aggregate wages, by writing that

$$\mathcal{T}_t^R = \mu_t N_t^r W_t \tag{3.43}$$

where $\mu_t = \bar{e}_t / \bar{W}$ is average pension rate evaluated at initial steady state level of aggregate wages $\bar{W}$. Since in the model wage rate W_t is endogenous, we obtain projections for pension expenditures once we set a deterministic path to average pension rate μ_t. Total pension expenditures $\mathcal{T}_t^R$ are thus linked to average wages and number of pensioners in the model. Making use of our demographic assumption, we can express pension expenditures per capita in terms of dependency ratio, wages and pension rate:

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{N_t^r}{N_t} W_t \tag{3.44}$$

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{\varphi_t}{1+\varphi_t} W_t \tag{3.45}$$

Since W_t is endogenous the pension expenditures have a tendency to react too rapidly to changes in the aggregate wage. In order to ease the problem we use an alternative formulation where

$$\frac{\mathcal{T}_t^R}{N_t} = \mu_t \frac{\varphi_t}{1+\varphi_t} W_t^P \tag{3.46}$$

$$W_t^P = \alpha_p W_{t-1}^P + (1-\alpha_p) W_t \tag{3.47}$$

W_t^P is now a pension wage, which reacts only slowly to changes in aggregate wage during the demographic transition. The speed at which pension wage reacts to aggregate wage is governed by autoregressive parameter α_p. With this formulation, we can roughly mimic the dependence of pension expenditures per capita on aggregate wage levels.

[13] Regarding transfer to working age, we keep them independent of demographic structure. Similarly we assume no changes in government consumption, although in more realistic scenarios, aging may have implications to these aswell.

3.12 Production sector

The Aino model builds on the tradition of a neoclassical growth model. The technical progress is labour augmenting in the balanced growth path. Persistent, but temporary, growth and slump periods may exist due to shifts in capital-augmenting technical change.

The supply side is based on a single intermediate good that can be used in the production of final goods. Producers of this intermediate goods combine capital and labour using constant-elasticity-of- substitution (CES) production function and operate in monopolistic product markets. Prices of final goods are sticky in the form of Calvo-pricing. Domestic producers of intermediate products purchase their capital inputs (capital services) in a competitive capital market (from companies providing capital services) in which capital is freely for sale and transferable for use by other companies. Building up new capital generates cost — adjustment costs — in the form of lost capital stock.

Domestic intermediate goods are combined with the imported intermediate goods to produce final goods of three kinds: consumption goods, capital goods and exported goods. The production function — or, rather, aggregator — is CES. The production of final goods differ in terms of elasticity of substitution. All three types of final producers operate in competitive product markets in which they take the market price for their products as given in their own decision-making.

Finally, nominal import prices are assumed to be sticky in a manner corresponding to domestic intermediate goods prices. It is also assumed that, in the short term, exchange rate pass-through to import prices is incomplete. This is due to the fact that a fixed fraction of importing companies price their products in the local currency.

3.12.1 Domestic intermediate goods producer

The domestic composite intermediate good, Y_t, is produced according to the following constant elasticity of substitution (CES) production function that combine a continuum of individual goods $Y_t(j)$ ($j \in [0,1]$) (Dixit and Stiglitz 1977):

$$Y_t = \left[\int_0^1 Y_t(j)^{-\rho^z} dj \right]^{\frac{1}{-\rho^z}} .$$

The parameter $\rho^z \in [-1, \infty)$ determines the elasticity of substitution $1/(1+\rho^z)$. The cost minimization implies the following conditional demand function for the individual good j

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1}{1+\rho^z}} Y_t$$

and the price index for the composite domestic intermediate good

$$P_t = \left[\int_0^1 P_t(j)^{\frac{\rho^z}{1+\rho^z}} dj \right]^{\frac{1+\rho^z}{\rho^z}}$$

Domestic intermediate goods, $Y_t(j)$, are produced by producers who face monopolistic competition. The production technology and the factor augmenting technical trends are exogenously given. The production function is of the CES type and it takes a specific form of constant-returns-to-scale[14].

$$Y_t(j) = \left[\delta \left(\Lambda_t^K K_t\right)^{-\rho} + (1-\delta)\left(\Lambda_t^L L_t^F\right)^{-\rho}\right]^{-1/\rho}.$$

The factors of production include homogenous capital services, K_t, and labour input L_t^F. Λ_t^K and Λ_t^L denote time-varying[15] capital and labour-augmenting technical progress respectively. The elasticity of technical substitution is given by $1/(1+\rho)$, where ρ is the substitution parameter and δ share paremeter in production function. The technical change is labour-augmenting, Λ_t^L, in the balanced growth path.

The cost minimization implies the following real marginal costs

$$\frac{MC_t(j)}{P_t(j)} = \left[\delta^{\frac{1}{1+\rho}}\left(\frac{R_t}{\Lambda_t^K P_t(j)}\right)^{\frac{\rho}{1+\rho}} + (1-\delta)^{\frac{1}{1+\rho}}\left(\frac{W_t^F}{\Lambda_t^L P_t(j)}\right)^{\frac{\rho}{1+\rho}}\right]^{\frac{1+\rho}{\rho}},$$

where R_t denotes the nominal rental price of capital services and $W_t^F = (1 + \mathfrak{t}_t^{FP} + \mathfrak{t}_t^{FS})W_t$ represents nominal labour costs[16]. In the steady-state, prices, $P(j)$, are determined by mark-up, $\Upsilon (= -\frac{1}{\rho^z})$ over the marginal costs

$$P(j) = \Upsilon MC(j)$$

The first order conditions with respect to capital services and labour are given by

$$\begin{aligned} r_t - p_t &= \log\delta - \log(\Upsilon) - \rho\log\Lambda_t^K + (1+\rho)(y_t - k_t) \\ w_t^F - p_t &= \log(1-\delta) - \log(\Upsilon) - \rho\log\Lambda_t^L + (1+\rho)(y_t - l_t). \end{aligned}$$

Due to the monopolistic competition in the market for output, the slope of the demand curve, $\upsilon \equiv \log(\Upsilon)$, enters into both first order conditions.

The dynamics of the price level $P_t(j)$ of producer j arises from the assumption that a firm changes its price level when it receives a random "price-change signal" . A constant probability of receiving a price change signal is given by $1-\zeta$ ($\zeta \in [0,1]$). Since there is continuum of intermediate producers, $1-\zeta$ also represents the share of producers that have received such a signal and, consequently, got an opportunity to change their prices. The average time between price changes is given by $1/(1-\zeta)$. Solving the first order condition and linearizing in a standard way delivers following aggregate pricing equation for the intermediate goods producer

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \frac{(1-\zeta)(1-\zeta\beta)}{\zeta}\left[\upsilon + mc_t - p_t\right].$$

Inflation is determined by the expected inflation and log of markup[17] υ over the real marginal costs $mc_t - p_t$.

[14] Jalava, Pohjola, Ripatti and Vilmunen (2006) provide evidence that this may be reasonable approximation for the Finnish post-WWII data.

[15] See Ripatti and Vilmunen (2001) for further discussion about their properties and estimates using aggregate Finnish data.

[16] $\mathfrak{t}_t^{FP}$ and $\mathfrak{t}_t^{FS}$ denote firms pension and social security contributions respectively.

[17] In a more flexible version of the model we allow for time-varying markup. This, however, does not play a significant role in the study of pension issues.

3.12.2 Capital rental firms

Capital is a homogenous factor of production that is owned by a firm that rents capital to producers of domestic intermediate goods. It operates under perfect competition. Physical capital accumulation generates real adjustment costs in the form of lost capital stock. Capital accumulation is given by

$$K_t^p = I_t - \mathcal{S}\left(K_t^p, K_{t-1}^p, K_{t-2}^p\right) + K_{t-1}^p \left(1 - \delta_K\right) \tag{3.48}$$

where $\mathcal{S}(\cdot)$ denotes the adjustment costs of physical capital stock and δ_K is capital depreciation factor. The capital rental firm maximizes its expected discounted profits

$$\max_{\{I_t\}} E_t \sum_{s=0}^{\infty} M_{t,t+s} \Pi_{t+s}^K$$

subject to capital accumulation equation (3.48) and the definition of capital services[18], $K_t = K_{t-1}^p$. Its momentary profits are given by

$$\begin{aligned}
\Pi_t^K =& R_t K_t - P_t^I I_t \\
=& R_t K_{t-1}^p - P_t^I \left(K_t^p + \mathcal{S}_t(K_t^p, K_{t-1}^p, K_{t-2}^p) - K_{t-1}^p \left(1 - \delta_K\right)\right)
\end{aligned}$$

The price index of investment goods, P_t^I, is the price index of the domestic investment good retailer and R_t denotes rental rate of capital. The future profits are discounted using the nominal stochastic discount factor (pricing kernel) $M_{t,t+s} = \beta^s U'(C_{t+s}) P_t^C / [U'(C_t) P_{t+s}^C]$ (P_t^C refers to price index of composite consumer goods). The first order condition with respect to capital stock K_t^p is given as follows

$$\begin{aligned}
&- P_t^I E_t \left[1 + \mathcal{S}_t'(K_t^p, K_{t-1}^p, K_{t-2}^p)\right] \\
&+ E_t M_{t,t+1} \left\{R_{t+1} - P_{t+1}^I \left[\mathcal{S}_{t+1}'(K_{t+1}^p, K_t^p, K_{t-1}^p) - \left(1 - \delta^K\right)\right]\right\} \\
&- E_t M_{t,t+2} \left[P_{t+2}^I \mathcal{S}_{t+2}'(K_{t+2}^p, K_{t+1}^p, K_t^p)\right] = 0.
\end{aligned}$$

Due to the end-of-period timing of physical capital stock, the accumulated physical capital is in use in the following period. Hence, it is the expected following period's rental rate R_{t+1} that governs the current period investment decision. Adjustment cost function is quadratic in changes of the physical capital stock follows:

$$\mathcal{S}_t(.) = \frac{\gamma_1}{2} \frac{\left(\Delta K_t^p - \gamma_2 \Delta K_{t-1}^p\right)^2}{K_{t-1}^p}.$$

Formulation of adjustment costs allow for hump-shaped responses of investments to various shocks. The usual 'investment equation' can be obtained by substituting the parametric version of the adjustment costs into the first order condition.

[18] For simplicity we assume that capital services obtains a lagged value of physical capital stock. In a more general case by Ripatti and Vilmunen (2004) capital services depends also on the endogenous utilization rate. This extension does alter the results only in the business cycle frequencies and is, consequently, beyond the interest of this study.

3.12.3 Production of final goods

The economy is inhabited by two retailers that produce final goods (consumer goods and capital goods) combining domestic intermediate input — produced by the intermediate goods producers — and imported goods and services. They operate under perfect competition. The demand for the retailers' output is given by the private sector's and general government's consumption and investments. The output of the consumption goods retailer consist of the private consumption, and public purchases of market goods, $C_t^T \equiv C_t^H + C_t^{SF}$. The capital goods retailer face similar demand by private sector and public sector investments, $I_t^T \equiv I_t + I_t^S$. The production technology is CES for both consumption and investment retailer:

$$Q_t^j = \left[\delta^j \left(\Lambda_t^{Yj} Y_t^j\right)^{-\rho^j} + (1-\delta^j)\left(\Lambda_t^{Mj} M_t^j\right)^{-\rho^j}\right]^{-1/\rho^j}, \quad j = I, C^T$$

δ^j is respective share parameter, and ρ^j respective substitution parameter ($\sigma^j = 1/(1+\rho^j)$. M^j denotes imports and Y^j denotes domestic intermediate good. The time-varying factor augmenting technical progresses, Λ_t^{Yj} and Λ_t^{Mj} , reflect changes in the preferences related to domestic and imported intermediate goods.

Cost minimization results following price indices

$$P_t^j = (1-\mathfrak{t}_t^c)^{-1}\left[(\delta^j)^{\frac{1}{1+\rho^j}}\left(\frac{P_t}{\Lambda_t^{Yj}}\right)^{\frac{\rho^j}{1+\rho^j}} + (1-\delta^j)^{\frac{1}{1+\rho^j}}\left(\frac{P_t^{Mj}}{\Lambda_t^{Mj}}\right)^{\frac{\rho^j}{1+\rho^j}}\right]^{\frac{\rho^j+1}{\rho^j}}$$

and the conditional factor demands

$$Y_t^j = \left(\delta^j\right)^{1+\rho j}\left(\Lambda_t^{Yj}\right)^{\frac{-\rho^j}{1+\rho^j}}\left(\frac{P_t}{(1-\mathfrak{t}_t^c)P_t^j}\right)^{\frac{-1}{1+\rho^j}} Q_t^j$$

$$M_t^j = (1-\delta^j)^{1+\rho^j}\left(\Lambda_t^{Mj}\right)^{\frac{\rho^j}{1+\rho^j}}\left(\frac{P_t^{Mj}}{(1-\mathfrak{t}_t^c)P_t^C}\right)^{\frac{-1}{1+\rho^j}} Q_t^j.$$

The consumption goods retailer pays the indirect taxes, $\mathfrak{t}_t^C$. Hence, the tax base for indirect taxes consist of private consumption and government purchases. No indirect taxes are levied on the investment goods.

The elasticity of substitution between imported consumption goods and domestic intermediate products has been estimated by Ripatti and Vilmunen (2004). Using the cointegration techniques in the estimation they find that $\hat{\rho}^C = -0.7731$ (with standard error 0.049) implying elasticity of substitution 4.4. δ^C has been calibrated to 0.87. The estimation of the elasticity of substitution between imported investment goods and domestic intermediate goods suggest a value 2.2, which is given by the estimate of $\hat{\rho}^I = -0.538$ (with the standard error 0.183). This means that the factors are gross-substitutes. The calibrated value of the share parameter δ^I is 0.67.

Exporter is a firm that combines domestic intermediate input, Y_t^X, and imported raw materials, M_t^R to produce export good, X_t in competitive markets. Technology and preferences are identical to those of the retailers. Ripatti and Vilmunen (2004) assume that the imported-raw-materials-augmenting technical change may contain a deterministic linear time trend. This trend captures the structural change in the input usage of exports. Estimate for the elasticity of substitution is 0.45, implying that $\rho^X = 1.217$ (with standard error 0.378). Not surprisingly, the point estimate suggest that the imported raw materials and the domestic intermediate input are gross-complements in the production of exported goods and services. The calibrated value of the share parameter δ^X is 0.51.

3.12.4 Importing Firms

Imported goods and services are used by the retailers and the exporter in the *Aino* economy. They combine imported and domestically produced intermediate goods to produce final consumption, capital and exported goods. The consumer goods and services (including 5 per cent of imported energy) are used by the consumption goods retailer, capital goods and services are used by the capital goods retailer, and the exporter uses energy and intermediate goods in producing exported goods. Each of these retailers operate under perfect competition in their output markets. A model for import prices of imports by main use, ie for the retailer sector is derived by following the approach derived by Betts and Devereux (1996) and (2000) applied to Finnish aggregate import data by Freystätter (2003). A fraction of importers price their product in local (Finnish) currency and rest of importers in producer (in their own) currency. Their pricing contains identical frictions in the form of Calvo (1983b), ie they may change their price only in the case of receiving a random price-change-signal. Their marginal costs are identical too. The aggregation of the pricing behaviour over these two types of importers yields an import price Euler equation where import prices depend on expected future import price inflation, current and expected future changes in foreign exchange rates and on the real marginal costs of the importers. This generates incomplete exchange rate pass through in the model.[19]

3.13 Market Equilibrium

All the markets are in equilibrium at each point of time. The capital goods market is in the equilibrium if the supply of capital services by the capital rental firm equals to the demand for capital services by intermediate goods producers. Similarly the labour markets are in equilibrium when the demand of labour equals its supply $L_t^s = L_t^D$. In the intermediate goods sector the demand for intermediate goods by the retailers and exporter equals total supply:

$$Y_t^C + Y_t^I + Y_t^X = Y_t.$$

Markets for final goods clear when

$$\begin{aligned} C_t^S + C_t^H &= C_t^T \\ I_t^G + I_t &= I_t^T \\ \left(\frac{P_t^X}{S_t P_t^W}\right)^{-\rho^W} M_t^W &= X_t, \end{aligned}$$

where P_t^W is the aggregated export price of competing economies and M_t^W is aggregate imports of export markets. When market clearing conditions hold, the workers' and pensioners' budget constraints (3.13), (3.21), the general government

[19] Given that incomplete exchange rate pass through is not essential for the ageing simulations provided in this paper, we abstract from explicit derivation. Derivation is provided in Ripatti and Viertola (2004).

budget constraint (3.39) and pension fund's budget constraint 3.41 imply the following equation for the accumulation of foreign assets[20]

$$S_t A_t^W = (1 + r_t^F) S_t A_{t-1}^W + \underbrace{P_t^X X_t - P_t^{MR} M_t^R - P_t^{MC} M_t^C - P_t^{MI} M_t^I}_{\text{trade balance}}$$

where the lower line defines trade balance. The current account is given by $S_t(A_t^W - A_{t-1}^W)$ and the factor income account by $r_t^F S_t A_t^W$.

4 Policy designs

4.1 Model's calibration - initial steady state

In order to set up the model ready for demographic simulations and various policy experiments, we assume that the economy is initially in the steady state: The economy is growing at balanced growth path and demographic structure remains unchanged. The model's key parameters are then calibrated such as to reflect the main features of the Finnish economy. In particular, we adjust the key parameters such that "great ratios", factor shares, participation rates, demographic structure as well as fiscal variables reflect the situation in the Finnish economy on average during the last 10 years. The parameters of the production functions have been estimated using cointegration methods (Johansen 1995), while the parameters related to the capital stock's adjustment costs, depreciation function and import prices are estimated using GMM[21]. Tables (1)- (5) characterize the calibration of some of the key parameters and resulting values of the key endogenous variables.

In order to fit the participation rates observed we have set the relative efficiency of retirees to be roughly 30%. Intra-temporal elasticity of substitution has been set to 0.8 and inter-temporal elasticity of substitution is 0.45. The public-finance literature has a tendency to use value well below unity for the inter-temporal elasticity of substitution, while the RBC literature prefers large value of σ. Typically, smaller values of σ makes the economy react more strongly to fiscal stimulus (table 1).

Reflecting a calibrated value of intra-temporal elasticity of substitution, the value for the Frisch elasticity of labour supply for the workers is at the range of international microeconomic studies, which report the values from $0.15 - 0.32$.[22] Also, Kuismanen (2005a, 2005b) has estimated compensated labour supply elasticities using Finnish Labor Force survey data. Depending on the data sample and the methods used, his estimates range from 0.08 to 0.30. Distribution of wealth between working age and elderly seems reasonable in light of the demographic structure: 17 percent of the model economy's total financial wealth is owned by the retirees. Ω is additional factor that multiplies the worker's discount factor. This additional discount factor is clearly higher than for instance those obtained in Gertler (1999). The difference is mainly explained by the distorting nature of taxes. Similarly, retirees marginal propensity to consume is 'only' 60% higher than that of the workers.

The transition dynamics of wages and prices are heavily influenced by the Calvo parameters. We assume that average expected time of price changes is 5 quarters, while wages changes occur on average only every 8 quarters. In comparison to

[20] We have simplified this by ignoring a transfer account.

[21] For detailed description of estimation strategies, see Ripatti and Vilmunen (2004).

[22] Bayomi, Laxton and Pesenti (2004) uses the value 0.33 in the standard calibration of GEM to Euro area.

Variable	Explanation	Value
β	subjective discount factor, parameter	0.993
v	elasticity of period utility with respect to consumption	0.80
σ	Inter-temporal elasticity of substitution, parameter	0.45
ξ	Labour efficiency of retirees, parameter	0.30
ϵ	Relative marginal propensity to consume, variable	1.59
π	Worker's marginal propensity to consume, variable	0.019
λ^F	Distribution of financial asset wealth, variable	0.17
Ω	Additional discounting factor	1.10
ν^w	Frisch elasticity of labour supply (workers), variable	0.26
ν^r	Frisch elasticity of labour supply (pensioners), variable	0.23

Table 1: Calibration of Key Demand Side Parameters

relevant literature, wage changes occur seldom, but the values reflect the Finnish centralised wage setting system and the fact that centralised wage contracts are usually set for 1-2 years time in Finland (table 2). Similarly the Calvo parameter for prices is on high side.[23]

Variable	Explanation	Value
$\frac{1}{\zeta^w}$	Average expected time of wage changes	8.0 Q
$\frac{1}{1-\zeta^{PY}}$	Average expected time of price changes	5.1 Q

Table 2: Calibration of Calvo Parameters

Regarding parameters affecting most strongly to the pension system and demographic structure, we assume that individuals work on average 43 years and stay in retirement about 15 years. Annual growth rate of working age population is set to 0.18% per annum (table 3). The steady state retiree worker ratio amounts then to 33 percent, being in accordance with the recent data (60+/15-59). Pension rate has been calibrated to 0.55, which, given endogenous average wage and demographic structure means that pension expenditures amounts to 20% percent of wage sum also roughly in accordance with the recent data. Finally, pension contribution rate to employees is 3.9 percent, while the employees contribution rate amounts to 15.4 percent. Overall pensio contribution rate thus amounts to 19.3 percent.

Variable	Explanation	Value
$\frac{1}{1-\gamma}$	Expected time of retirement, in years, parameter	14.8
$\frac{1}{1-\omega}$	Expected time of working, in years, parameter	43.1
$\hat{N}^w$	Annual growth rate of working age %, parameter	0.18
φ	Old age dependency ratio, variable	0.33
τ^{WP}	Pension contribution rate of the workers %, implicit	3.9
τ^{FP}	Pension contribution rate of the firms %, parameter	15.4
$\mathcal{T}^R$	Pensions, % of wage sum, variable	20.5
μ	Pension rate	0.55

Table 3: Calibration of Key Parameters of the Pension System

[23] Vilmunen (2005) has studied price dynamics in the Finnish economy using a microlevel data on different items in consumer price index. He finds that firms change their prices on average every quarter.

Regarding public sector (excluding pension fund) it is assumed that the steady state debt to output ratio is 60 percent, while total public consumption amounts to 20.8 percent of output (table 4). Total social security transfers to workers are 6.4 percent of output. Finally labour income tax rate is calculated endogenously so as to satisfy the state's budget constraint, while the rest of the tax parameters have been set exogenously reflecting the current tax system in Finland. Retirees labour income tax rate is calibrated to be slightly lower than that of working age. This captures roughly the fact that the Finnish tax system is highly progressive.

Variable	**Explanation**	**Value**
κ	Fiscal rule adjustment, parameter	0.1
a^S	Debt (% of output), variable	60.0
τ^{WS}	Income tax rate % of workers, implicit	30.2
τ^{PS}	Income tax rate % of retirees, implicit	$.9 \times \tau^{WS}$
τ^K	Corporate tax rate %, parameter	19.2
τ^C	Indirect tax rate %, parameter	21.0
τ^{FS}	Firm's social security contribution rate, parameter	4.0
c^S	Public consumption (% of output), variable	20.8
$\mathcal{T}^{SW}$	Total transfers to workers (% of output), variable	6.4

Table 4: Calibration of Key Parameters of the State

Regarding supply side of the economy, we assume that the real interest rate is about 2.4 percent per annum (table 5). The economy grows at balanced growth path at the rate of 2.3 percent per annum, reflecting the labour saving technical change and the steady state growth rate of population. Given small size of Finland in the Euro area, a feedback from the Finnish economy to euro area level is very modest. Thus, a reasonable approximation is that the euro area policy rate is exogenous for the Finnish economy and foreign exchange rates are fixed[24]. Therefore, the model assumes that $\Delta S_t = 0$, and foreign nominal interest rate determines the domestic nominal interest rate up to UIP condition. At balanced growth path, inflation is set to 2.0%.

Price mark-up and share price to equity ratio are roughly in accordance with empirical evidence[25], while capital share parameter and elasticity of substitution between capital and labour have been estimated from historical data by Ripatti and Vilmunen (2001). Fiscal rule adjustment parameter κ is in line with other studies (see for instance Railavo (2002)) and capital depreciation rate is set to 8 % per annum.

Finally, table (6) summarises "great ratios", factor shares, net foreign asset position as well as employment rates at initial equilibrium. Comparison to the recent data shows that the model fits reasonable well to the recent data. The model's current calibration however has some difficulties to capture relatively low investment and private consumption share observed in the data.

[24] In the data we approximate the currency basket S_t according to export weights of the following countries: Germany, Italy, UK, USA, Sweden and Japan. See Ripatti and Viertola (2004) for details.

[25] Kilponen and Santavirta (2005) estimate from the microdata that a percentage share of operating profits to value of gross output in Finnish manufacturing firms has been roughly 8 % during the last decade.

Variable	Explanation	Value
$\bar{R}^F/\hat{P}^c$	Real interest rate, p.a., variable	2.4
$\widehat{Y}$	Output growth rate, p.a., variable	2.3
$\hat{\Lambda}_L$	Labour saving technical change p.a., parameter	2.08
δ^K	Capital depreciation rate, parameter	0.08
Υ	Price mark-up, parameter	1.08
$\bar{A}^F/\bar{\Pi}$	Price to equity ratio, variable	15.0
δ	capital share parameter	0.1
ρ	El. of substitution between capital and labour	0.72

Table 5: Calibration of Key Parameters in the Supply Side

Variable	Explanation	The data 1995-2005	Steady State
c^H	Private consumption share,%	51.3	54.0
i	Private investment rate,%	15.8	18.6
x	Export share,%	39.8	34.0
m	Import share,%	31.1	34.0
ls	Labour share,%	49.5	51.5
k/l	Capital intensity	4.8	4.3
a^w	Net foreign assets, %	−42.5	−23.3
l	Employment rate, %	58.6	55.4
l^w	Workers	56.6	53.8
l^r	Retirees	2.1	5.2

Shares calculated in nominal terms except for capital intensity

Table 6: Great Ratios, Factor Shares and Participation rates

4.2 Policy experiments

In order to facilitate rather complex general equilibrium effects associated with aging and different assumption that can be made regarding pension parameters, we design alternative demographic scenarios and policy experiments. The financial implications of ageing population are treated as shocks to the initial state of unchanged demographic age structure and steady pace of population growth. We performed five different simulations altogether, as given by in table 7.

The shock simulating population ageing incorporated both the higher life expectancy projected in the population forecast and the slowdown in the growth of the working-age population. Higher life expectancy was simulated by increasing the expected period of retirement by 5 years (table 7, column I) and then supplementing the shock with longer expected working time of 2.5 years, while taking into account slow down in the growth rate of working age population (table 7, column III). Experiment II is a pure demographic shock. Features inherent in the pension scheme, ie deferred retirement in response to accelerated accrual and a lower replacement rate, were built into the demographic effects by lowering the pension replacement rate by 10 percentage points (column IV). The analysis was completed by an equilibrium calculation where the replacement rate was endogenously defined while the rate of employee contribution kept unchanged, at its initial steady state level (column V).

Table 7: Demographics shocks relative to initial steady state

	Experiment				
Change	**I**	**II**	**III**	**IV**	**V**
Expected period of retirement, years	5.0	5.0	5.0	5.0	5.0
Growth rate of workers, % annually		−0.5	−0.5	−0.5	−0.5
Expected working time, years			2.5	2.5	2.5
Pension replacement rate, % -points				−10.0	μ^*

*endogenous

5 Long-run effects of ageing

5.1 Increasing life expectancy

Increase in life expectancy projected in the demographic forecast for Finland would alone have a pronounced effect on the long-term equilibrium of the economy. Extension of the period of retirement by five years, as assumed in the calculation, would imply an increase in the old-age dependency ratio by 11 percentage points. This would lead into 6.3% -point increase in pension expenditures per wage sum. Tax burden on labour would raise over 3.7% -point and be reflected in falling employment rates. In total contribution rate would go up by 6.7 percentage point. (table 8, column I). Given age cohorts of the same size and a stable fertility rate, higher life expectancy alone would impose a considerable burden on the economy. The falling employment rates are attributable to decline in the working-age population and higher tax burden. Both of these factors are reflected in rising real wages in the long term. Consumption response, when measured as a share of output reflects the household's different marginal propensities to consume. Retirees own a larger share of economy's financial wealth and they are more willing to consume it than the workers. Mirror effect of this is that capital share is higher in the aged economy.

5.2 Declining fertility rate

When considering that the age cohorts entering the labour market are smaller than those withdrawing from it, the old-age dependency ratio would increase by almost 16 percentage points (table 8, column II). Tax rate and employment responses would also be pronounced, in that the pension contribution rate would increase by 8.8 percentage points and the income tax rate by 3.4 percentage points. The employment rate would be down almost 6 percentage points reflecting a marked increase in real wage. Private consumption share would be up by 6.2 percentage points relative to the initial state of constant population growth. The major increase in tax rates is not only related to the pension expenditures, but also to the strong response of workers employment rates. This reduces tax bases, even if retirees respond positively. Retirees positive labour supply response attributes to longer retirement period and to the fact that their after tax real wage responds far less than after tax real wage of workers.

Table 8: Long Term Effects of Ageing

Variable	Explanation	Final steady-state, change I	II	III	IV	V
φ	Old-age dependency ratio**	10.9	15.5	13.0	13.0	13.9
t^P	Contribution rate**	6.7	8.8	7.0	1.3	a)0.0
	employee	1.4	1.8	1.4	.3	a)0.0
	employer	5.4	7.0	5.6	1.0	a)0.0
t^{WS}	Labour income tax rate**	2.3	5.0	3.4	0	-0.7
$\frac{\mathcal{T}^R}{WL}$	Pension expenditure/wage sum**	5.3	8.0	6.7	2.0	1.0
w	c)Real wage*	5.4	5.3	6.0	9.5	10.3
$(1-t)w$	c)After-tax real wage*	-0.38	-5.6	-1.6	9.5	11.5
c^H	Private consumption share*	5.1	6.2	6.4	6.8	13.6
λ	Wealth distribution**	3.4	6.1	5.2	5.5	5.6
l	Employment rate**	-3.7	-5.7	-4.7	-3.3	-3.0
l^w	Workers	-4.3	-6.2	-5.2	-4.3	-4.2
l^r	Retirees	2.1	1.8	1.6	3.5	3.7
k	Capital share*	21.0	23.5	22.4	21.1	21.0
a^w	Net foreign assets**	4.8	21.1	27.1	51.2	56.7
μ	Pension replacement rate**	0	0	0	-10.0	b)-12.0
y	Output, efficiency units	-1.7	-5.5	-3.4	-0.5	0.12

* Percentage change relative to initial steady state

** Change, percentage points relative to initial steady state

a) Contribution rate kept fixed at initial steady state, b) endogenous, c) in efficiency units

5.3 Pension reform and lower replacement rate

It has been estimated that the pension reform will extend labour force participation by an average of 2–3 years. This alone would clearly ease the burden of an ageing population (table 8, column III). The need to increase the contribution would be reduced by 1.8 percentage points. Similarly, a need to raise the labour income tax rate would decrease almost as much when compared to pure demographic shock. This would, in turn, be reflected in smaller employment losses in the long term.

The decline in the average replacement induced by the rules of pension indexation also has a considerable effect on the sustainability of the pension scheme. This effect was measured by the assumption that the ratio of average pension to average wage would decline by 10 percentage points, i.e. clearly less than suggested by the calculation discussed above. This would considerably lower the need for raising the contribution rate in response to population ageing in the long term. The labour income tax rate could be kept at current level, and contribution rates would need to be raised only by 1.3% -point. (8, column IV). Pension expenditure would raise 4.7 percentage points less when compared to the figure returned by the previous simulation (column III).

According to the results of the equilibrium calculation based on a rate of contribution we find that pensions would, in the long term, have to decline from their current levels by 12% -point in relation to average wages (table 8, column V). This would ensure that the ratio of pension expenditure would increase only by 1% point relative to wage sum in response to ageing of the population. However, real wages would still be increased by 10%, while employment rate would fall by 3 percentage point. Once more, retirees incentives to work would be improved, primarily because

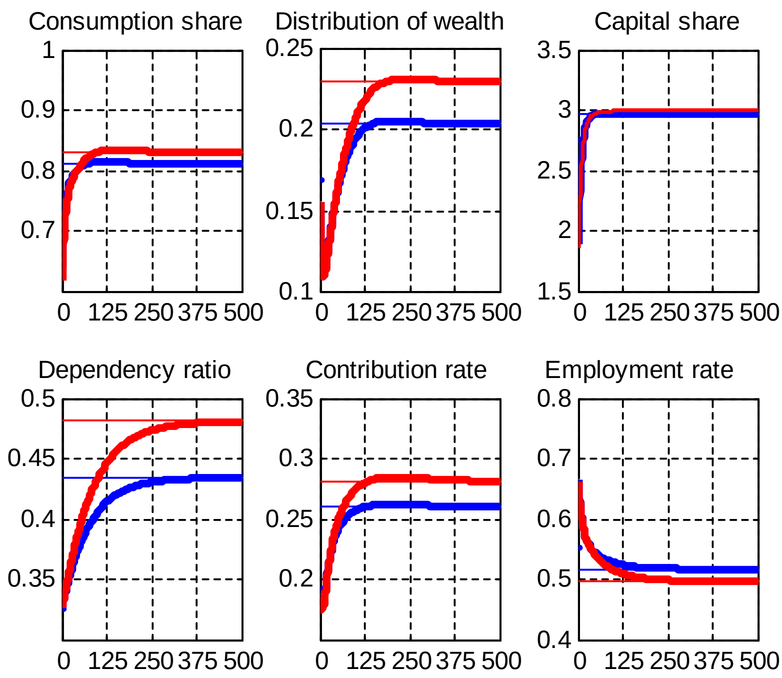

Figure 3: Pure demographic shocks and transition.
Blue line corresponds to experiment I and red line (lighter grey) to experiment II. Periods are quarters.

of the wealth effect, when compared to previous experiment. Finally output would remain practically intact in this case.

6 Dynamic effects

Demographic change generates rather complicated transitional dynamics in the life-cycle model. Dynamic optimisation and a perfect foresight assumption implies that the transition paths of the macroeconomic variables reflect the way in which optimising agents prepare for new demographic situation. These adjustment paths are further complicated by our assumption that demographic structure is changing over time. The adjustment paths can typically contain over- or undershooting of the forward looking variables, even though the transitional path of the demographic shocks is a smooth, very slowly evolving process. One such variable is consumption, which typically makes initially a jump after which it starts adjusting to the new long-run steady state. This is reflected in over- or undershooting of employment similarly.

Figure **??** shows how the model economy responds to pure anticipated shock to retirement age (I) and to the shock where fertility rate drops as described in experiment II.[26] The dynamic analysis reveals that demographic transition materialises very slowly and the economy's transition towards new steady state takes decades.

After a temporary drop in distribution of wealth in favour of workers, retirees eventually hold a larger share of the economy's financial wealth. The worker's tax burden on labour follows a similar pattern. Aggregate consumption reacts in advance

[26] All the simulations were done by *DYNARE* version *3.051*.

to demographic change and starts stabilising well before the demographic change is over. Beyond the response of aggregate consumption, there are opposite responses of retirees and workers. While the workers save for retirement by dropping their consumption, and also because of higher tax burden, retirees respond positively. Retirees are also now more numerous as shown by a gradual rise in dependency ratio. Lower fertility rate in addition to increasing life expectancy gradually increases the dependency ratio and leads into further increase in pension contribution rate. This effect is also associated with slightly higher consumption share and lower employment rate[27].

The figure 4 illustrates how extensions of working life and lower replacement rate alleviates the effects of a pure demographic shock. Consumption share is practically intact (red (lighter gray) line), while a need to increase contribution rates gradually ease along with less pronounced increase in the dependency ratio. Capital share increases initially more under extension of working life but eventually returns very close to the same level as with pure demographic shock. Longer working time means that there is a roughly 1% -point improvement in employment rate when compared to pure demographic shock. Extending the simulation by taking into account also a lower pension replacement rate shows that consumption share may now fall in the short and medium term. A drop in the replacement rate reduces pensioners' permanent income, with a subsequent temporary fall in consumption levels. As the need to raise contribution rate gradually ease, employment improves rather quickly, while consumption bounces back only after a several decades. Mirroring this, there is also a temporary fall in capital share and a permanent fall in distribution of wealth held by the retirees.

6.1 Sensitivity analysis

6.1.1 Subsitution parameters

The results presented are naturally subject to uncertainty regarding calibrated and estimated parameter values of the model, exogenous assumption on external environment of the economy and the demographic projections. In this section, we are primarily interested in sensitivity of the results with respect to intertemporal substitution (σ), elasticity of perio utility with respect to consumption (ν), as well as imperfection of the labour markets. These are the parameters that guide the dynamic and long run response of consumption, real wages and employment, which in turn influence on that how tax burden on labour evolves as a response to demographic ageing.

Figure 5 shows how the relationship between tax burden on labour, real wages and dependency ratio changes along different values of intertemporal substitution σ when the economy faces a pure demographic shock. A higher value of intertemporal substitution mitigates the effects of pure demographic shock on taxes, while the opposite is true for lower values of σ. The dynamic path for contribution rate is also influenced by the intertemporal substitution parameter. The real wage response is similarly affected by different values of intertemporal substitution. For high values of σ, real wage response is somewhat smoother, but in the long-run real wage hike

[27] A higher capital share and consumption share in the long run are compatible with Blanchard-Yaari asset market equilibrium condition. In essence, the condition delivers a positive relationship between consumption and capital stock as required by the current account balance in an open economy.

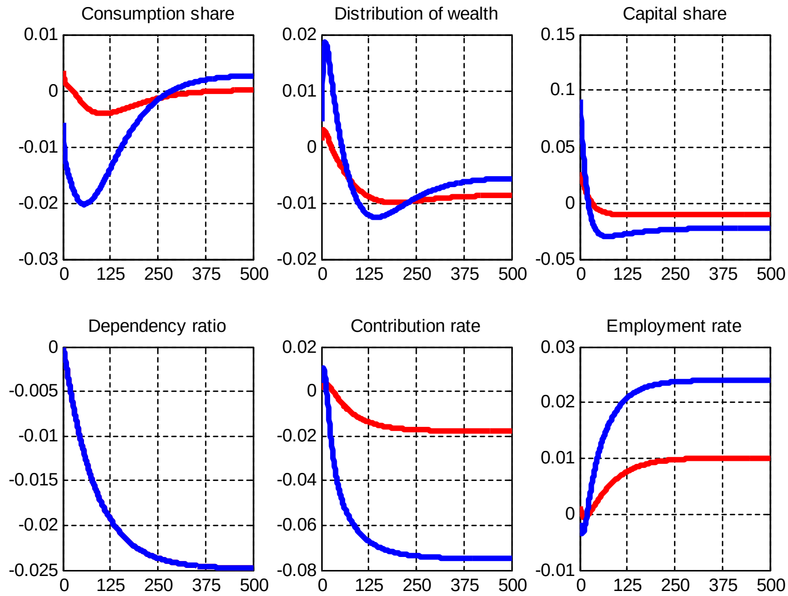

Figure 4: Extensions of working life and lower replacement rate ease fiscal burden of ageing.
Red line (lighter grey) corresponds to experiment III while blue line corresponds to experiment IV. Baseline is a pure demographic shock. Periods are quarters

is more pronounced, when compared to standard simulation.

Intratemporal substitution affects primarily on long-run response of the economy to demographic ageing as well as initial steady state. Sensitivity analysis in figure (6) shows how higher intratemporal substitution lead into somewhat smaller response of contribution rate which in turn is associated with slightly smaller negative response of employment.

6.1.2 Labour market imperfections

The main channel by which pension systems affect demand for labour is that employers view pension contributions, particularly in a pure PAYG systems, as a form of payroll tax. Ceteris paribus, then, the firms substitute labour with capital investment as tax burden on labour increases. Additional effect is due to possible imperfections. When the product markets are non-competitive, as in our model, the employers can pass the increased burden of their pension contribution to consumers via product prices. Similarly, when the labour markets are non-competitive, and employers are obliged to contribute to the pension system, they view such contribution as extra taxes and pass it to nominal wages. Higher nominal wages then increase the firm's marginal costs fueling also into higher production prices in the long-run. The final effect is that consumption is cut down.

We illustrate this by exploring the sensitivity of benchmark results to the assumption on the level of wage mark-up. In standard simulations, we assumed that labour markets are competitive in the sense there is no mark-up over marginal rate of substitution between leisure and consumption in the long-run. Extending the

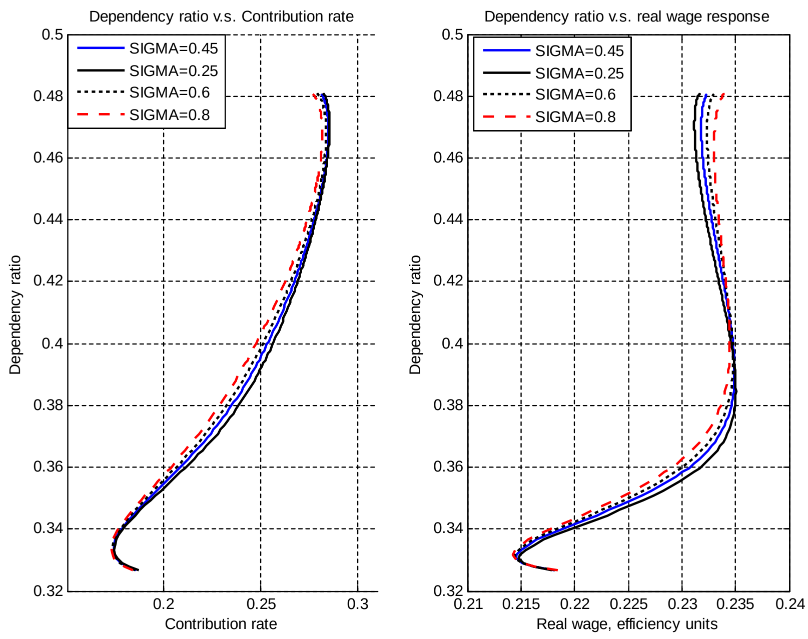

Figure 5: Pure demographic shock and sensitivity of contribution rate and real wage on intertemporal substitution parameter

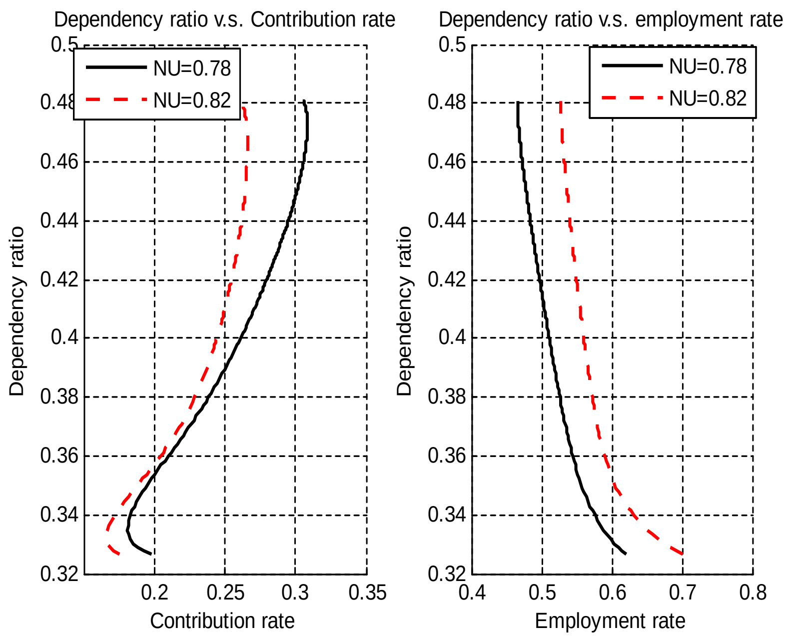

Figure 6: Pure demographic shock and sensitivity of contribution rate and employment to intratemporal subsitution.

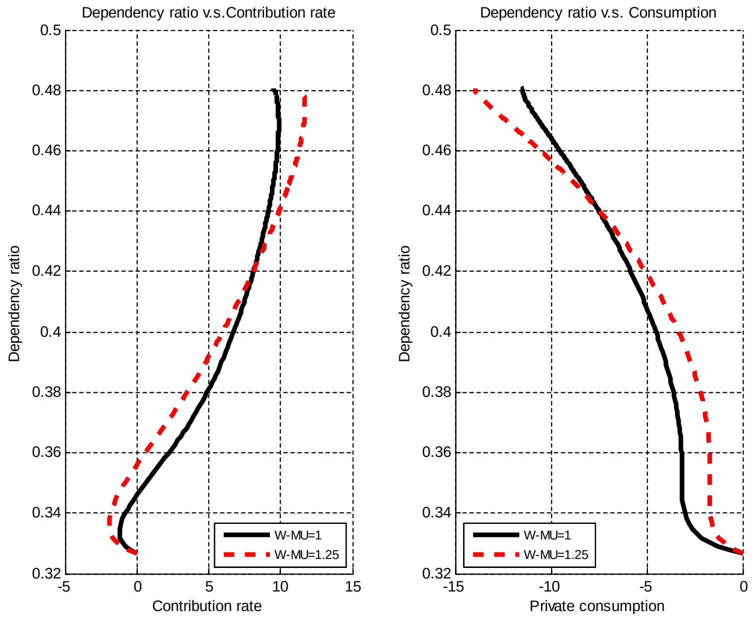

Figure 7: Sensitivity of the contribution rate and private consumption on labour market imperfections.
Contribution rate and private consumption are here expressed in percentage point and percentage difference relative to initial steady state correspondingly. W-MU refers to wage mark-up.

model to allow for the mark-up is straightforward (see footnote 6). Departing from an assumption of competitive labour markets yields the result that is summarised in figure (7). When the labour markets are imperfect in the sense that workers have pricing power over nominal wages, it turns out that contribution rates need to raise more, than under perfect competition. This is contrasted with the fact that relative to the initial steady state, private consumption is lowered more when the labour markets are imperfect. In comparison to standard simulation, a larger drop in aggregate consumption increases tax burden on labour and creates an additional distortion to the economy. General equilibrium response requires that overall taxation increases along with increased contribution rates.

6.2 Demographic uncertainty

Population scenarios and ageing reflect projected changes in fertility rates and longevity. These population scenarios are however subject to much uncertainty. OECD (2005) has reported that in the case of Finland, overall uncertainty surrounding longevity and fertility rate projections suggests that the dependency ratio will fluctuate ±10 percentage points around the official scenario.[28] Rather similar results were obtained in the project "Uncertain Population of Europe" (UPE) funded by the EU Commission. In this project, stochastic population forecasts were generated by simulating 3,000 alternative paths of future development for age-specific

[28] Assessing the robustness of demographic projections in OECD countries, Frederic Gonand, Economics Deparment Working Papers, No. 464, OECD)

mortality, fertility and net migration for different European countries. These different scenarios were then combined into a population scenarios using the classical cohort-component book-keeping equations. Based on the resulting age specific cohort tables from different simulations, the table 9 summarises some statics related to old-age dependency ratio. This helps us to anchor the degree of uncertainty related to old-age dependency ratio in Finland and thus quantify the degree of uncertainty related to demographic structure. The table suggests that risks in making large errors in population forecasts increase after 2020, making the confidence limits grow rather fast (see also Alho (1998)). By 2050 the standard error suggest that 90% confidence intervals are $(0.51, 0.83)$, suggesting a rather large uncertainty bounds around the mean projection.

Year	**Min**	**Mean**	**Max**	**S.E.**
2010	0.40	0.42	0.43	0.005
2020	0.48	0.54	0.61	0.019
2030	0.49	0.64	0.74	0.043
2040	0.41	0.65	0.94	0.071
2050	0.36	0.67	1.08	0.099

Table 9: Summary statistics from stochastic population simulations (based on own calculations from http://www.stat.fi/tup/euupe/)

In order to analyse the demographic uncertainty in the model, we take take the experiment III as a baseline (see table 7), but allow stochastic variation in retirement probability (ω_t), survival probablity (γ_t) and worker's growth rate $\hat{N}_t$. Allowing for stochastic variation around the deterministic trends in these variables generates deterministic paths, combined with stochastic variation, for the model's endogenous variables[29]. We are not able to generate the magnitudes of the uncertainty reported in table 9, as those would require implausibly large standard errors for the demographic shocks.

Nevertheless, table 10 summarises the results for the model's selected variables when the uncertainty on retirement rate, survival probability and worker's growth rate is calibrated such that to generate a standard error of 0.04 for the old-age dependency ratio[30]. The table however points to a sizeable uncertainty on economic outcomes of the model, once uncertainty on demographic stucture is accounted for.

Stochastic simulations allows us also to quantify the relative importance of different demographic shocks to economic outcomes. This can be done by looking at (asymptotic) variance decomposition as generated by the stochastic simulations of the model. Although the results from variance decomposition depends on the assumed standard errors of the shocks, columns 5-7 in table 10 clearly suggest that lengthening the working time has rather minor impact on alleviating the fiscal burden of ageing: Only 6 percent of the stochastic variation in the contribution rate is explained by the variation in working period. Stochastic variation in worker's growth rate, and thus in fertility rate is clearly much more important.

[29] DYNARE's functions *forecast* and *stoch_simul* allow to mix deterministic and stochastic shocks.We are indebted to Michel Juillard for developing these functions.

[30] Stochastic assumptions about retirement probability and survival probability imply that expected working period has a standard error of 5.5 years while the retirement period has a standard error of 5.2 years. Population growth rate has a standard error of 1.5 % in annual terms.

Variable	Explanation	Mean	Standard error	Variance decomposition		
				ω	γ	$\hat{N}_t^W$
φ	Dependency ratio	0.48	0.044	3.5	17.9	78.6
$\mathfrak{t}^P$	Contribution rate	0.29	0.025	6.0	31.3	62.8
$\mathfrak{t}^{WS}$	Labour income tax rate	0.37	0.023	4.7	25.1	70.2
$\frac{\mathcal{T}^R}{WL}$	Pension exp/wage sum	0.29	0.023	3.2	16.3	80.5
y	Output, efficiency units	0.19	0.007	5.1	26.9	68.1
L/N	Employment rate	0.49	0.016	3.8	20.1	76.1
ω	Retirement probability	0.9942	0.0007			
γ	Survival probability	0.9871	0.0035			
$\hat{N}_t^W$	Growth rate of workers	0.9992	0.0037			

a) All the moments are evaluated at final steady state

Table 10: Asymptotic moments from stochastic model simulations for selected model's variables

7 Conclusions

Sensitivity analyses show that tax effects may easily be over or underestimated when considering the general equilibrium effects of ageing in the life cycle model. This is especially related to the assumptions on inter-temporal elasticity of consumption ie consumers' willingness to adjust consumption behaviour over time and assumptions that can be made about possible imperfections in the labour markets. Lower values of inter-temporal substitution lead into a stronger tax responses of the economy to ageing. Similarly, if labour market imperfections are important, ageing will have more pronounced effects in terms of tax burden on labour and consumption. By contrast, changes in periodic utility of consumption involves clearly less uncertainty in these analyses. The uncertainty inherent in demographic forecasts themselves becomes also significant when the time horizon of the analysis extends far into the future.

These uncertainties notwithstanding, the analysis highlights the key role played by taxation in the assessment of the costs of an ageing population. When the responses of labour supply, wages and hence private consumption to higher taxation are consistently accounted for, the economy will, in the long term, settle at a level of taxation clearly above that generally estimated in mechanical sustainability calculations. It should be noted that even if the effective retirement age were to increase as expected, the burden from pension payments alone would cause the tax rate to rise to a level above that witnessed in the worst years of recession in the mid-1990s. In the light of the model calculations, the efficiency losses induced by demographic change thus appear considerable.

Taxation will increase especially under the assumption that the real value of pensions remains unchanged relative to wage developments. If the replacement rate falls, as seems likely under the current pension regime, there would be a much smaller increase in taxation in response to growing expenditure on pensions. The average decline in the level of pensions relative to wage developments, in turn, raises several concerns.

In Finland where supplementary pension funds are of minor importance by international comparison there is a risk that the falling purchasing power of pensions relative to wage developments will exert pressure on other social security schemes. The employee pension scheme, which appears to be financially sustainable, may thus, in practice, generate costs to local and central government in the form of higher expenditure on income support and other benefits. Voluntary provision for retirement is also likely to step up.

Another concern is related to the measures necessary for preparing for a slower growth rate for pensions relative to general earnings growth. The pension reform debate has focused strongly on the accelerated rate of accrual applied to newly awarded pensions, which encourages workers to remain longer in the labour force. One would think, however, that indexation of pensions alone would provide an incentive to longer working life, considering that it will reduce pensioners' relative standard of living, especially in later life. Furthermore, the stochastic simulations suggest that lengthening the working time has rather minor impact on alleviating the fiscal burden of ageing: Only a minor fraction of the stochastic variation in the contribution rate is explained by the variation in the length of working time. A variation in worker's growth rate, and thus in fertility rate, is clearly much more important. These concerns and results suggest that the debate over pensions and ageing should be broadened to consider economic policy issues wider than those raised by the mechanical sustainability calculations.

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A Appendices

A.1 Calculating pension replacement ratio

While there are good reasons to argue approximate one-to-one interdependency between average wage and average pension benefits only for those retiring over a particular period, the level of pensions for those who have already retired is not directly influenced by wage developments. This complicates projections for aggregate pension spending. For already retired population, if labour productivity grows, pension levels will grow at a slower pace than wages since the wage weight in the pension index is only 0.2. Therefore, in order to trace aggregate pension expenditure we split the population of pensioners into two groups: number of new pensioners in the period under consideration (workers exiting workforce) and number of those retired earlier.

This led us to a recursive equation for pension expenditure where, at the starting point, the number of new pensioners is same as the size of the age cohort we assume to have exited the workforce and taken retirement. This increases each year cumulatively with the size of the population in the age cohort taking into account the assumed mortality in this growing number of pensioners.

The number of old pensioners decreases as age cohorts assumed to have retired before die out. The following chart (8), where we assume exit age of 60 years, indicates that when 2004 is taken as the starting point, the original old pensioners will have disappeared by the start of 2040s.

Average replacement ratio was then constructed as follows. First, average pension of those pensioners retired before year 2004 (old pensioners) was determined simply by the pension level at the starting point adjusted by changes in the pension index. Second, the average pension of those retiring later (new pensioners) constitutes of two components: pensions of those retiring in the year considered and pensions of those new pensioners who had retired earlier. The replacement rate of those retiring each year is assumed to remain unchanged in relation to average wage (66 per cent, the ratio prevailed in 2003) except that it was adjusted with the life expectancy rate starting from year 2009. Pension of those retired earlier is adjusted with pension index. These three type of average pension indicators were weighted together by sizes of relevant age cohorts.

In the following chart, (9) the pension-wage ratios are calculated by assuming that inflation rate is 2 %, productivity growth (=real wage growth) 1.75 % and the number of those retiring each year same as the number of 60-years-olds. This together with unfavourable pension indexation and decreasing life expectancy rate will lower the pension-wage ratio in a considerable amount. In aggregate, pension-wage-ratio would decrease from the current level of about 60 per cent to about 40 per cent in 2040.

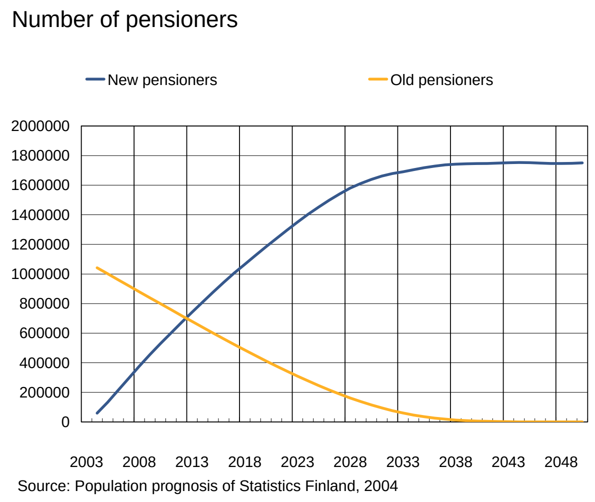

Figure 8: Projections for pensioners

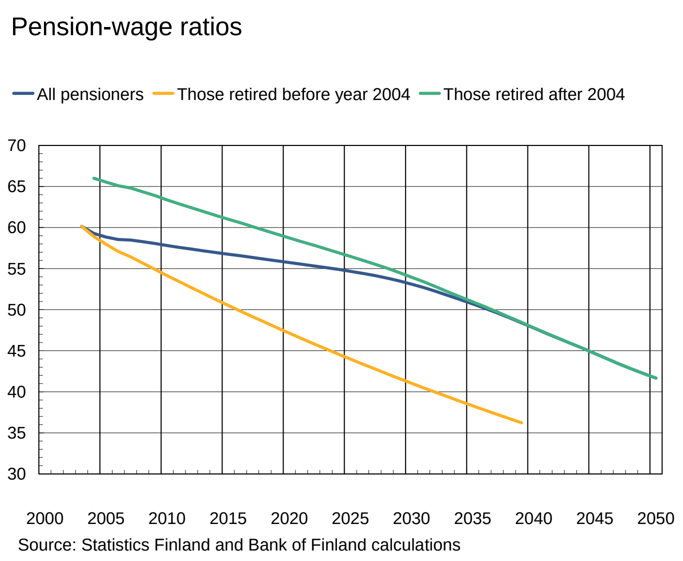

Figure 9: Projections for average pension-wage ratios