

Multi-Sectoral Cascading and Price Dynamics - A Bayesian Econometric Evaluation

ALEJANDRO JUSTINIANO

Federal Reserve Board

MICHAEL KUMHOF

International Monetary Fund

FEDERICO RAVENNA

University of California, Santa Cruz

February 14, 2006

Abstract

Recent evidence by Bils and Klenow (2004) and Klenow and Kryvtsov (2004) shows that the average price duration for US CPI-basket goods is in the order of one to two quarters, challenging the monetary business cycle research to try and explain how short price durations can nevertheless generate a large degree of aggregate inflation persistence. We empirically test the relevance of a cascading structure of production as an explanation for short price durations and large aggregate inflation persistence. The final good is produced through a chain of intermediate goods, which undergo several processing stages. At each stage the price is set in nominal terms, and can be adjusted only at random intervals. Though each individual price is adjusted frequently, because the final good price embeds the intermediate price movements, it will turn out to have a large degree of stickiness. We estimate the model using Bayesian techniques to evaluate the relative role of indexation, pricing contracts length, and cascading production structure in the US postwar data. The estimation shows that short pricing contracts within the standard Calvo pricing mechanism are compatible with large inflation persistence, and inflation indexation turns out to play a much less relevant role - in other words, it ends up being a reduced-form model for the cascading production structure.

Keywords: Inflation Inertia; Monetary Policy; Bayesian Estimation

1 Introduction

The recent literature on dynamic stochastic general equilibrium models of the business cycle has focused on finding optimizing microfoundations for two important features observed in US data: a large degree of inflation inertia, and a persistent impact on output of monetary policy shocks. Both these features have proven challenging for the most popular modeling setups, namely the New Keynesian paradigm, requiring the introduction of exogenous inflation indexation in the firm's pricing policies (justified as quasi-optimal rule of thumb behaviour, for example, in Gali and Gertler, 1999) and relatively long duration of pricing policies in models with nominal price rigidity (see Christiano, Eichenbaum and Evans, 2005). Recent evidence by Bils and Klenow (2004) and Klenow and Kryvtsov (2004) shows instead that the average price duration for US CPI-basket goods is in the order of one to two quarters. The inflation dynamics literature has produced alternative, and increasingly sophisticated models of optimal pricing to try and explain how short price durations can nevertheless generate a large degree of aggregate inflation persistence (Golosov and Lucas, 2003, Mankiw and Reis, 2002, Mackowiak and Wiederholt, 2004).

This paper evaluates a mechanism first explored by Basu (1995) and recently investigated by Dotsey and King (2005) and Huang and Liu (2001 and 2004) as an explanation for short price durations and large aggregate inflation persistence: a cascading structure of production. We assume the final good is produced through a chain of intermediate goods, which undergo several processing stages. At each stage the price is set in nominal terms, and can be adjusted only at random intervals, consistently with the Calvo model of staggered price adjustment. Though in each sector the price is adjusted frequently, because the final goods price embeds the price movements of all intermediate goods, it will turn out to have a large degree of stickiness. The same results can be obtained through an input-output production structure, where firms' production is horizontally rather than vertically integrated.

We estimate the model using Bayesian techniques to evaluate the relative roles of indexation, pricing contract length, and cascading production structure in the US postwar data. We show that once the cascading production structure is introduced, US data favor a specification with short pricing contracts. Within the standard Calvo pricing mechanism, short individual price duration is therefore compatible with large inflation persistence. Inflation indexation turns out to play a much less relevant role than previously estimated - in other words, it ends up being a reduced-form model for the cascading production structure.

2 The Model

The economy consists of a continuum of measure one of households indexed by $i \in [0, 1]$, a continuum of firms indexed by $j \in [0, 1]$, a continuum of financial intermediaries indexed by $z \in [0, 1]$, and a government.

2.1 Households

Household i maximizes lifetime utility, which depends on his per capita consumption $C_t(i)$, leisure $1 - L_t(i)$ (where 1 is the fixed time endowment and $L_t(i)$ is labor supply), and real money balances $M_t(i)/P_t$ (where $M_t(i)$ is nominal money and P_t is the aggregate price index):

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left\{ S_t^c (1 - v) \log(H_t(i)) - S_t^L \psi \frac{L_t(i)^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} + \frac{a}{1 - \epsilon} \left(\frac{M_t(i)}{P_t} \right)^{1-\epsilon} \right\} . \quad (1)$$

Throughout, shocks are denoted by S_t^x , where x is the variable subject to the shock. Households exhibit external habit persistence with respect to C_t^i , with habit parameter ν :

$$H_t(i) = C_t(i) - \nu C_{t-1} . \quad (2)$$

Consumption C_t^i is a CES aggregator over individual varieties $c_t(i, j)$, with time-varying elasticity of substitution $\sigma_t > 1$,

$$C_t(i) = \left(\int_0^1 c_t(i, j)^{\frac{\sigma_t-1}{\sigma_t}} dj \right)^{\frac{\sigma_t}{\sigma_t-1}} , \quad (3)$$

and the aggregate price index P_t is the consumption based price index associated with this consumption aggregator,

$$P_t = \left(\int_0^1 P_t(j)^{1-\sigma_t} dj \right)^{\frac{1}{1-\sigma_t}} . \quad (4)$$

Households accumulate capital according to

$$K_{t+1}(i) = (1 - \Delta)K_t(i) + I_t(i) . \quad (5)$$

We assume that demand for investment goods takes the same CES form as demand for consumption goods, equation (3), which implies identical demand functions for goods varieties j .

In addition to capital, households accumulate money and one period nominal government bonds $B_t(i)$ with gross nominal return i_t .¹ Their income consists of nominal wage income $W_t(i)L_t(i)$, nominal returns to utilized capital $R_t^k x_t K_t(i)$, where x_t is the rate of capital utilization, and lump-sum profit redistributions

¹All financial interest rates and inflation rates, but not rates of return to capital, are expressed in gross terms.

from firms and intermediaries $\int_0^1 \Pi_t(i, j) dj$ and $\int_0^1 \Pi_t(i, z) dz$. Expenditure consists of consumption spending $P_t C_t(i)$, investment spending $P_t I_t(i)(1 + S_t^I)$, where S_t^I is an investment shock, the cost of utilizing capital at a rate different from 100% $P_t a(x_t) K_t(i)$, where $\bar{x} = 1$ and $a(1) = 0$, lump-sum taxation $P_t \tau_t$, quadratic capital and investment adjustment costs, and quadratic costs of deviating from the economywide average labor supply (more on this below). The budget constraint is therefore

$$\begin{aligned}
B_t(i) = & (1 + i_{t-1})B_{t-1}(i) + M_{t-1}(i) - M_t(i) & (6) \\
& + W_t(i)L_t(i) + R_t^k x_t K_t(i) - P_t a(x_t) K_t(i) \\
& + \int_0^1 \Pi_t(i, j) dj + \int_0^1 \Pi_t^z(i, z) dz - P_t \tau_t(i) \\
& - P_t C_t(i) - P_t I_t(i)(1 + S_t^I) \\
& - P_t \frac{\theta_k}{2} K_t(i) \left(\frac{I_t(i)}{K_t(i)} - \Delta \right)^2 - P_t \frac{\theta_i}{2} K_t(i) \left(\frac{I_t(i)}{K_t(i)} - \frac{I_{t-1}}{K_{t-1}} \right)^2 \\
& - W_t \frac{\phi_w}{2} \frac{(L_t(i) - L_t)^2}{L_t} .
\end{aligned}$$

We assume complete contingent claims markets for labor income, and identical initial endowments of capital, bonds and money. Then all optimality conditions will be the same across households, except for labor supply. We therefore drop the index i . The multiplier for the budget constraint (6) is denoted by λ_t/P_t , and the multiplier of the capital accumulation equation (5) is $\lambda_t q_t$, where q_t is Tobin's q . Then the first-order conditions for $c_t(j)$, B_t , C_t , I_t , K_{t+1} , and x_t are as follows:

$$c_t(j) = C_t \left(\frac{P_t(j)}{P_t} \right)^{-\sigma_t} , \quad (7)$$

$$\lambda_t = \beta i_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) , \quad (8)$$

$$\frac{S_t^c(1-v)}{H_t} = \lambda_t , \quad (9)$$

$$q_t = 1 + \theta_k \left(\frac{I_t}{K_t} - \Delta \right) + \theta_i \left(\frac{I_t}{K_t} - \frac{I_{t-1}}{K_{t-1}} \right) + S_t^I , \quad (10)$$

$$\begin{aligned}
\lambda_t q_t = & \beta E_t \lambda_{t+1} [q_{t+1}(1 - \Delta) + r_{t+1}^k] & (11) \\
& + \theta_k \left(\frac{I_{t+1}}{K_{t+1}} - \Delta S_{t+1}^I \right) \frac{I_{t+1}}{K_{t+1}} + \theta_i \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I_t}{K_t} \right) \frac{I_{t+1}}{K_{t+1}} \\
& - \frac{\theta_k}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \Delta S_{t+1}^I \right)^2 - \frac{\theta_i}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I_t}{K_t} \right)^2 \Big] ,
\end{aligned}$$

$$\hat{r}_t^k = \epsilon \hat{x}_t , \quad (12)$$

where $\epsilon = \overline{a''(x)/a'(x)} > 0$. We will return to the household's wage setting problem at a later point, as we will be able to exploit analogies with firms' price setting. Full derivations of all first-order conditions in the paper, their transformation into a stationary system through normalization by technology and the inflation target, and their linearization, are presented in a separate Technical Appendix (available on request).

2.2 Firms

Each firm j sells a distinct product variety. Heterogeneity in price setting decisions and therefore in demand for individual products arises because each firm receives its price changing opportunities at different, random points in time. We first describe the cost minimization problem and then move on to profit maximization.

2.2.1 Cost Minimization

The production function for variety j is Cobb-Douglas in labor $\ell_t(j)$, capital $k_t(j)$ and intermediate goods $N_t(j)$:

$$y_t(j) = \left[(S_t^y \ell_t(j))^{1-\alpha} k_t(j)^\alpha \right]^{1-\eta} N_t(j)^\eta, \quad (13)$$

where

$$\ell_t(j) = \left(\int_0^1 L_t(i, j)^{\frac{\sigma_t^w - 1}{\sigma_t^w}} di \right)^{\frac{\sigma_t^w}{\sigma_t^w - 1}}, \quad (14)$$

$$k_t(j) = \left(\int_0^1 k_t(z, j)^{\frac{\sigma_t^k - 1}{\sigma_t^k}} dz \right)^{\frac{\sigma_t^k}{\sigma_t^k - 1}}, \quad (15)$$

$$N_t(j) = \left(\int_0^1 N_t(x, j)^{\frac{\sigma_t - 1}{\sigma_t}} dz \right)^{\frac{\sigma_t}{\sigma_t - 1}}, \quad (16)$$

where the last three equations state that each firm employs a CES aggregate of different labor, capital and intermediates varieties supplied by different households, financial intermediaries and firms. Let w_t be the aggregate real wage (the cost of hiring the aggregate (14)), and u_t the aggregate user cost of capital (the cost of hiring the aggregate (15)). These are determined in competitive factor markets and discussed in more detail below. Intermediate goods are simply an aggregate of final output varieties $y_t(j)$, and are demanded by each firm subject to a CES technology (16) that is identical to household and investor demand for final output. Then the real marginal cost corresponding to (13) is

$$mc_t = A \left(\frac{w_t}{S_t^y} \right)^{(1-\alpha)(1-\eta)} (u_t)^{\alpha(1-\eta)}, \quad (17)$$

where $A = (\alpha(1-\eta))^{-(\alpha(1-\eta))} ((1-\alpha)(1-\eta))^{-((1-\alpha)(1-\eta))} \eta^{-\eta}$. Introducing multiple layers of cascading subject to market power at every level tends to make

economic profits a very sizeable component of final GDP, while correspondingly reducing the shares of labor and capital. We therefore introduce fixed costs of production, following Altig et al. (2005), such that in steady state a firm makes zero economic profits. Formally, this is modeled as net output after fixed costs being equal to $y_t(j) - \chi S_t^y$, with χ calibrated such that steady state economic profits are zero. Technology S_t^y is stochastic and consists of both i.i.d. shocks to the level of technology and of highly persistent shocks to the growth rate of technology:

$$S_t^y = S_{t-1}^y g_t \quad , \quad (18)$$

$$g_t = g_t^{gr} g_t^{iid} \quad ,$$

$$\ln g_t^{gr} = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1}^{gr} + \hat{\varepsilon}_t^{gr} \quad ,$$

$$\ln g_t^{iid} = \hat{\varepsilon}_t^{iid} \quad .$$

Let $\tilde{Y}_t = \int_0^1 y_t(j) dj$, $\ell_t = \int_0^1 \ell_t(j) dj$, and $k_t = \int_0^1 k_t(j) dj$. Given that factor markets are competitive so that all firms face identical costs of hiring aggregates of capital and labor (14) and (15), we can derive the following aggregate input demand conditions:

$$\ell_t = (1 - \alpha)(1 - \eta) \frac{mc_t}{w_t} \tilde{Y}_t \quad , \quad (19)$$

$$k_t = \alpha(1 - \eta) \frac{mc_t}{u_t} \tilde{Y}_t \quad , \quad (20)$$

$$N_t = \eta mc_t \tilde{Y}_t \quad . \quad (21)$$

2.2.2 Profit Maximization

Following Calvo (1983) it is assumed that each firm receives price changing opportunities that follow a geometric distribution. Therefore the probability $(1 - \delta)$ of a firm's receiving a new opportunity is independent of how long ago it was last able to change its price. It is also independent across firms, so that it is straightforward to determine the aggregate distribution of prices. Each firm maximizes the present discounted value of real profits. The first two determinants of profits are real revenue $P_t(j)y_t(j)/P_t$ and real marginal cost $mc_t y_t(j)$. In each case demand is given by

$$y_t(j) = Y_t \left(\frac{P_t(j)}{P_t} \right)^{-\sigma_t} \quad , \quad (22)$$

which follows directly from consumer demand functions (7) and identical demands from investors and government (see below). Two key features of our model concern first the manner in which firms set their prices when they receive an opportunity to do so, and the cost (through excessively large or small demand) of setting prices far away from prevailing average market prices P_t . To model the latter, we assume that firms face a small quadratic cost Φ_t of

deviating from the output level of its average competitor, meaning the firm that charges the current market average price. The cost is therefore

$$\Phi_t = \frac{\phi}{2} Y_t \left(\frac{y_t(j) - Y_t}{Y_t} \right)^2 . \quad (23)$$

The term Y_t in front of the quadratic term serves as a scale factor. As for price setting, we assume that when a firm j gets an opportunity to decide on its pricing policy, it chooses both its current price level $V_t(j)$ and the gross rate $v_t(j)$ at which it will update its price from today onwards until the time it is next allowed to change its policy. At any time $t + k$ when the time t policy is still in force, its price is therefore

$$P_{t+k}(j) = V_t(j)(v_t(j))^k . \quad (24)$$

As for the possibility of introducing even more general price paths, it seems natural to focus on equilibria characterized by a constant expected long-run growth rate of the nominal anchor.² The model can then be solved by linearizing around that growth path, in which case it is sufficient to allow firms to specify their pricing policies up to the growth rate of their price path. This permits the use of conventional solution methods, which makes quantitative analysis much more straightforward.

Firms discount profits expected in period $t + k$ by the k -period ahead real intertemporal marginal rate of substitution and by δ^k , the probability that their period t pricing policy will still be in force k periods from t . They take into account aggregate demand for their output (22). The firm specific index j can be dropped in what follows because all firms that receive a price changing opportunity at time t will behave identically. Their profit maximization problem is therefore

$$\begin{aligned} \text{Max}_{V_t, v_t} E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} & \left[\left(\frac{V_t (v_t)^k}{P_{t+k}} \right)^{1-\sigma_t} Y_{t+k} \right. \\ & \left. - mc_{t+k} \left(\frac{V_t (v_t)^k}{P_{t+k}} \right)^{-\sigma_t} Y_{t+k} - \frac{\phi}{2} Y_{t+k} \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right)^2 \right] . \end{aligned} \quad (25)$$

We define the front-loading term for price setting, the ratio of a new price setter's first period price to the market average price, as $p_t \equiv V_t/P_t$, cumulative aggregate inflation as $\Pi_{t,k} \equiv \prod_{j=1}^k \pi_{t+j}$ for $k \geq 1$ ($\equiv 1$ for $k = 0$), and the mark-up term as $\mu_t = \frac{\sigma_t}{\sigma_t - 1}$. Then the firm's first order conditions for the choice of its initial price level V_t and its inflation updating rate v_t are

$$p_t = \mu_t \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(mc_{t+k} + \phi \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right) \right)}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{(v_t)^k}{\Pi_{t,k}} \right)} , \quad (26)$$

²This includes both a constant steady state growth rate of the nominal anchor and a unit root in that growth rate, as in this paper.

$$p_t = \mu_t \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \left(mc_{t+k} + \phi \left(\frac{y_{t+k}(j) - Y_{t+k}}{Y_{t+k}} \right) \right)}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \left(\frac{(v_t)^k}{\Pi_{t,k}} \right)} . \quad (27)$$

The intuition for this result becomes much clearer once these conditions are log-linearized and combined with the log-linearization of the aggregate price index. As this is algebraically very involved, the details are presented in the Technical Appendix. We discuss the key equations here. They replace the traditional one-equation New Keynesian Phillips curve with a three-equation system in $\hat{\pi}_t$, \hat{v}_t and an inertial variable $\hat{\psi}_t$:

$$E_t \hat{\pi}_{t+1} = \hat{\pi}_t \left(\frac{2}{\beta} - \delta \right) + \hat{v}_t ((1 - \delta)(1 + \delta)) + \hat{\psi}_t \left(\delta(1 + \delta) - \frac{2}{\beta} \right) \quad (28)$$

$$- \frac{2(1 - \delta)(1 - \delta\beta)}{(\delta\beta)(1 + \phi\bar{\mu}\bar{\sigma})} (\widehat{mc}_t + \hat{\mu}_t) + \frac{(1 - \delta)}{(1 + \phi\bar{\mu}\bar{\sigma})} (E_t \hat{\mu}_{t+1} - \hat{\mu}_t) ,$$

$$E_t \hat{v}_{t+1} = \hat{v}_t + \frac{(1 - \delta\beta)^2}{(\delta\beta)^2} \frac{\delta}{1 - \delta} \hat{\psi}_t - \frac{(1 - \delta\beta)^2}{(\delta\beta)^2} \frac{\delta}{1 - \delta} \hat{\pi}_t \quad (29)$$

$$+ \frac{(1 - \delta\beta)^2}{(\delta\beta)^2 (1 + \phi\bar{\mu}\bar{\sigma})} (\widehat{mc}_t + \hat{\mu}_t) ,$$

$$\hat{\psi}_t = \delta \hat{\psi}_{t-1} + (1 - \delta) \hat{v}_{t-1} - \hat{\varepsilon}_t^{\pi^*} . \quad (30)$$

Equations (28) and (29) show the evolution of the two forward-looking variables, $\hat{\pi}_t$ and \hat{v}_t . The most notable feature is the presence of the term $(1 + \phi\bar{\mu}\bar{\sigma})$ in the denominator of the terms multiplying marginal cost. It results from the upward-sloping firm-level marginal cost curve, and as long as $\phi > 0$ it makes prices less sensitive to changes in marginal cost. Note that both the steepness of the marginal cost curve ϕ and the elasticity of the demand curve $\bar{\sigma}$ affect this term. Equation (30) is, in deviation form and allowing for permanent changes in the inflation target $\hat{\varepsilon}_t^{\pi^*}$, the weighted average of all those past firm-specific inflation rates \hat{v}_t that are still in force between periods $t - 1$ and t , and which therefore enter into period t aggregate inflation. This term is inertial, and the degree of inertia depends directly on δ and therefore on the average contract length.

The following key equation follows from the differencing and log-linearization of the aggregate price index:

$$\hat{\pi}_t = \frac{1 - \delta}{\delta} \hat{p}_t + \hat{\psi}_t . \quad (31)$$

The two components of this equation reflect the two main sources of aggregate inflation inertia in response to shocks. The first term \hat{p}_t represents inflation caused by significant instantaneous price changes (relative to the aggregate price level) of new price setters, so called ‘front loading’. Note that in a Calvo-Yun model this is the only term driving inflation. But in our case the quadratic cost

term means that significant front loading can be very costly, because it generally causes big deviations from industry average output during part of the duration of a pricing policy. New price setters will therefore respond as much as possible through changes in their updating rates \hat{v}_t . But these only slowly feed through to aggregate inflation via $\hat{\psi}_t$, which initially mainly reflects the continuing effects of price updating decisions made before the current realization of shocks. The result is that past inflation, by (31) and (30), becomes a key determinant of current inflation.

In our sensitivity analysis we will report not only the fit of our model, but also that of a Calvo (1983) model with Yun (1996) indexation to steady state inflation, augmented as in the baseline case by firm-specific marginal cost and sticky user costs. That model, in our case with markup shocks, gives rise to the following one-equation representation of the inflation process, the New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \frac{((1 - \delta\beta)(1 - \delta))}{\delta(1 + \phi\bar{\mu}\bar{\sigma})} \widehat{mc}_t + \frac{(1 - \delta)}{\delta(1 + \phi\bar{\mu}\bar{\sigma})} (\hat{\mu}_t - \delta\beta\hat{\mu}_{t+1}) \quad . \quad (32)$$

This equation can be directly derived from (28), (29) and (30) by setting $\hat{v}_t = \hat{\psi}_t = 0$. In other words, a firm in our model is always free to behave exactly like a Calvo-Yun price setter by front-loading all its price changes into the current price. However, this is generally far from optimal, especially if the processes driving inflation are highly persistent. And for aggregate inflation dynamics, as is well known, this kind of price setting implies very little inflation inertia and persistence.

2.3 Household Wage Setting

Every firm j must use composite labor (14), a CES aggregate with elasticity of substitution σ_t^w of the labor varieties supplied by different households. Firms' costs minimization, aggregated over all firms, yields demands

$$L_t(i) = L_t \left(\frac{W_t(i)}{W_t} \right)^{-\sigma_t^w} \quad , \quad (33)$$

where the aggregate nominal wage is given by

$$W_t = \left(\int_0^1 (W_t(i))^{1-\sigma_t^w} di \right)^{\frac{1}{1-\sigma_t^w}} \quad . \quad (34)$$

The term driving wage inflation is the log-difference between the marginal rate of substitution between consumption and leisure and the real wage. The marginal rate of substitution is given by

$$mrs_t = \frac{S_t^L \psi L_t(i)^{\frac{1}{\gamma}}}{\lambda_t} \quad . \quad (35)$$

Household nominal wage setting can then be shown to follow the same pattern as the price setting discussed in the previous subsection. With an appropriate change of notation, and after replacing \widehat{mc}_t with $\widehat{mrs}_t - \widehat{w}_t$, it leads to an identical set of equations to (28)-(31) above. The reader is referred to the Technical Appendix for details.

2.4 Financial Intermediaries

We assume that all capital is intermediated by a continuum of intermediaries indexed by $z \in [0, 1]$. These agents are competitive in their input market, renting capital K_t from households at rental rate r_t^k . On the other hand, they are monopolistically competitive in their output market, lending capital varieties $k_t(z)$ to firms at rental rates $u_t(z)$. This gives rise to sluggish user costs of capital, which interact in the model with sticky wages to produce stickiness in marginal cost.

Every firm j must use composite capital, a CES aggregate with elasticity of substitution σ^k of the varieties supplied by different intermediaries. Firms' costs minimization yields demands

$$k_t(z) = k_t \left(\frac{u_t(z)}{u_t} \right)^{-\sigma^k}, \quad (36)$$

where the overall user cost to firms is given by

$$u_t = \left(\int_0^1 (u_t(z))^{1-\sigma^k} dz \right)^{\frac{1}{1-\sigma^k}}. \quad (37)$$

The profit maximization problem of the intermediary follows the same pattern as firms' problem. We define the gross intermediation spread as $s_t = u_t/r_t^k$ and the gross rate of change of user cost as $\pi_t^k = u_t/u_{t-1}$. With an appropriate change of notation and after replacing \widehat{mc}_t with $-\widehat{s}_t$, we obtain an identical set of equations to (28)-(31) above. The Technical Appendix contains the details.

2.5 Government

We assume that there is an exogenous stochastic process for government spending GOV_t

$$GOV_t = S_t^{gov} \overline{GOV}, \quad (38)$$

with demands for individual varieties having the same form as consumption demand for varieties (7). The government's fiscal policy is assumed to be Ricardian, with the government budget balanced period by period through lump-sum taxes τ_t , and with an initial stock of government bonds of zero. The budget constraint is therefore

$$\tau_t + \frac{M_t - M_{t-1}}{P_t} = GOV_t. \quad (39)$$

We assume that the central bank pursues an interest rate rule for its policy instrument i_t . Its quarterly inflation target π_t^* is assumed to follow a unit root process:

$$\pi_t^* = \pi_{t-1}^* \varepsilon_t^{\pi^*} . \quad (40)$$

The year-on-year inflation rate is denoted as $\pi_{4,t} = \pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}$. The current year-on-year inflation target is simply the annualized quarter-on-quarter inflation target, $\pi_{4,t}^* = (\pi_t^*)^4$. Finally, the steady state gross real interest rate is given by $1/\beta_g$, where $\beta_g = \beta/\bar{g}$. Then we have

$$i_t^4 = [i_{t-1}^4]^{\xi^{int}} [\beta_g^{-4} \pi_{4,t}]^{1-\xi^{int}} \left[\frac{\pi_{4,t+1}}{\pi_{4,t}^*} \right]^{\xi^{\pi}} S_t^{int} , \quad (41)$$

where S_t^{int} is an autocorrelated monetary policy shock. A *government policy* is defined as a set of stochastic processes $\{i_s, \pi_s^*, \tau_s\}_{s=t}^{\infty}$ such that, given stochastic processes $\{P_s, S_s^{int}\}_{s=t}^{\infty}$, the conditions (39) and (41) hold for all $s \geq t$.

2.6 Equilibrium

An *allocation* is given by a list of stochastic processes $\{B_s, M_s, C_s, I_s, L_s, K_s, k_s, Y_s, L_t(i, j), k_t(z, j), i, j, z \in [0, 1]\}_{s=t}^{\infty}$. A *price system* is a list of stochastic processes $\{P_s, W_s, R_s^k, U_s\}_{s=t}^{\infty}$. *Shock processes* are a list of stochastic processes $\{S_s^c, S_s^L, S_s^{inv}, S_s^{gov}, S_s^{int}, \mu_s, \mu_s^w, S_s^y, \pi_s^*\}_{s=t}^{\infty}$. Then the equilibrium is defined as follows:

An equilibrium is an allocation, a price system, a government policy and shock processes such that

(a) *given the government policy, the price system, shock processes, the restrictions on wage setting, and the sequence $\{L_s\}_{s=t}^{\infty}$, the allocation and the sequences $\{V_s^w(i), v_s^w(i), i \in [0, 1]\}_{s=t}^{\infty}$ solve households' utility maximization problem,*

(b) *given the government policy, the price system, shock processes, the restrictions on price setting, and the sequence $\{Y_s\}_{s=t}^{\infty}$, the allocation and the sequences $\{V_s(j), v_s(j), j \in [0, 1]\}_{s=t}^{\infty}$ solve firms' cost minimization and profit maximization problem,*

(c) *given the government policy, the price system, shock processes, the restrictions on setting user costs, and the sequence $\{k_s\}_{s=t}^{\infty}$, the sequences $\{V_s^k(z), v_s^k(z), z \in [0, 1]\}_{s=t}^{\infty}$ solve intermediaries' profit maximization problem,*

(d) *the goods market clears at all times,*

$$\begin{aligned} \text{Aggregate Output} & : Y_t = \left(\int_0^1 y_t(j)^{\frac{\sigma_t-1}{\sigma_t}} dj \right)^{\frac{\sigma_t}{\sigma_t-1}} , & (42) \\ \text{Auxiliary Variable} & : \tilde{Y}_t = \int_0^1 y_t(j) dj , \\ \text{Goods Clearing} & : y_t(j) = c_t(j) + I_t(j) + GOV_t(j) + N_t(j) \quad \forall j , \\ \text{Aggregate Goods Clearing:} & Y_t = C_t + I_t + GOV_t + N_t . \end{aligned}$$

(e) the labor market clears at all times,

$$\ell_t = \int_0^1 \left[\left(\int_0^1 L_t(i, j) \frac{\sigma_t^{w-1}}{\sigma_t^{w-1}} di \right)^{\frac{\sigma_t^w}{\sigma_t^w-1}} \right] dj \quad , \quad (43)$$

(f) the market for capital clears at all times,

$$\begin{aligned} \text{Firm Demand for Capital} & : k_t = \int_0^1 \left[\left(\int_0^1 k_t(z, j) \frac{\sigma_t^{k-1}}{\sigma_t^{k-1}} dz \right)^{\frac{\sigma_t^k}{\sigma_t^k-1}} \right] dj \quad , \\ \text{Intermediary Supply/Demand of Capital} & : \tilde{k}_t = \int_0^1 \int_0^1 \tilde{k}_t(z, j) dz dj \quad , \\ \text{Capital Market Clearing I} & : k_t(z, j) = \tilde{k}_t(z, j) \quad \forall z, j \quad , \\ \text{Household Capital Stock} & : K_t = \int_0^1 \int_0^1 K_t(z, j) dz dj \quad , \\ \text{Capital Market Clearing II} & : \tilde{k}_t(z, j) = x_t K_t(z, j) \quad \forall z, j \quad . \end{aligned} \quad (44)$$

(g) the bond market clears at all times,

$$B_t = 0 \quad . \quad (45)$$

Outside of steady state it will generally be true that $\tilde{Y}_t \neq Y_t$ and $x_t K_t \neq k_t$.³ It is however straightforward to show that $\overline{\tilde{Y}} = \overline{Y}$, $\widehat{\tilde{Y}} = \widehat{Y}$, $\bar{x}\bar{K} = \bar{k}$, and $\hat{x} + \hat{K} = \hat{k}$, so that log-linearization that assumes equality between these aggregates is valid.

³This does not concern us for labor because we do not track an aggregate labor *supply* variable.

3 Estimation Methodology, Priors, and Calibration

3.1 Estimation Methodology

The model above model is log-linearized and then estimated in two steps in DYNARE-MATLAB. In the first step, we compute the posterior mode using an optimization routine (CSMINWEL) developed by Chris Sims. Using the mode as a starting point, we then use the Metropolis-Hasting (MH) algorithm to construct the posterior distributions of the model and the marginal likelihood.⁴ We choose as our baseline case a particular combination of structural model features and priors for parameters, and use the parameter estimates for this case to construct impulse responses. Sensitivity analysis will be performed by either restricting certain parameters or shocks, or by removing some features of the structural model, and by comparing the marginal likelihood to that of the baseline case.

3.2 Calibration of Parameters that Determine the Steady State

The list of those model parameters that pin down the steady state are listed in the top panel of Table 1. We set the annual steady-state rate of productivity growth to 1.7 percent, the average over our sample. The rate of productivity growth and quarterly discount rate β together pin down the equilibrium real interest rate in the model. Given productivity growth of 1.7 percent, we set the discount rate at 0.999 to generate an equilibrium annual real interest rate of 2.1 percent. The quarterly depreciation rate on capital is assumed to be 0.025, implying an annual depreciation rate of 10 percent. The elasticities of substitution among goods, labor inputs and capital inputs are assumed to be 5.35, 7.25 and 11.00 respectively, resulting in markups of 23%, 16% and 10%. These assumptions are combined with a share of capital in valued added of 0.28, while we will experiment with a range of assumptions for the share of profits prior to fixed costs, which is driven by the parameter η . Government is assumed to absorb 18 percent of GDP in steady state. Most of these values are similar to what have been employed in other DSGE models of the US economy—see Juillard, Karam, Laxton and Pesenti (2005) and Bayoumi, Laxton and Pesenti (2004).

⁴For one estimation run the whole process takes anywhere from 6-8 hours to complete using a Pentium 4 processor (3.0 GHz) on a personal computer with 1GB of RAM. DYNARE includes a number of debugging features to determine if the optimization routines have truly found the optimum and if enough draws have been executed for the posterior distributions to be accurate.

3.3 Specification of the Stochastic Processes

Table 2 reports the specifications of the stochastic processes for the 10 structural shocks in the model.⁵ Following Juillard, Karam, Laxton and Pesenti (2005) we classify shocks as demand and supply shocks depending on the short-run covariance they generate between inflation and real GDP. Shocks that raise demand by more than supply and cause inflation to rise in the short run are classified as demand shocks, while shocks that produce a negative covariance between inflation and GDP are classified as supply shocks. Based on this classification system, shocks to government absorption, the Fed funds rate, the inflation target, consumption, and investment, $[\widehat{s}_t^{gov}, \widehat{s}_t^{int}, \widehat{\pi}_t^*, \widehat{s}_t^c, \widehat{s}_t^{inv}]$, are all classified as demand shocks. In the case of the shock to the inflation target we assume that it follows a unit root, to account for permanent historical shifts in long-term inflation expectations. In all other cases we allow these shocks to be serially correlated. Shocks to wage and price markups as well as labor supply shocks, $[\widehat{\mu}_t^w, \widehat{\mu}_t, \widehat{s}_t^L]$, are classified as supply shocks. Labor supply shocks are assumed to be serially correlated, while both markup shocks have zero serial correlation.

The remaining 2 shocks determine the growth rate of productivity (\widehat{g}_t) and are split into 2 components, \widehat{g}_t^{gr} and \widehat{g}_t^{iid} . The first component \widehat{g}_t^{gr} is assumed to be serially correlated ($\widehat{g}_t^{gr} = \rho_{gr}\widehat{g}_{t-1}^{gr} + \widehat{\varepsilon}_t^{gr}$), while the second component is assumed to be white noise ($\widehat{g}_t^{iid} = \widehat{\varepsilon}_t^{iid}$). The classification of the \widehat{g}_t^{iid} shock is simple because increases in its value make output rise and inflation fall. However, the classification of the \widehat{g}_t^{gr} shock as a demand or supply shock is more difficult. Interestingly, when shocks to this component are highly serially correlated they generate responses that are indistinguishable from what many professional forecasters would characterize as shocks to consumer and business confidence in that they result in sustained increases in aggregate demand and a temporary, but persistent, increase in inflation and hours worked.

3.4 Prior Distributions

Our assumptions about the prior distributions can be grouped into two categories: (1) parameters for which we have relatively strong priors based on our reading of existing empirical evidence and their implications for macroeconomic dynamics, and (2) parameters where we have fairly diffuse priors. Broadly speaking, parameters in the former group include the core structural parameters that influence, for example, the lags in the monetary transmission mechanism, while parameters in the latter category include the parameters that characterize the stochastic processes (i.e., the variances of the shocks and the degree of persistence in the shock processes). Our strategy is to estimate the model with a base-case set of priors and then to report results based on plausible alternatives.

The first, fourth and fifth columns of Table 3 report our assumptions about the prior distributions for the 12 structural core parameters of the model. On

⁵In their model of the US economy, Smets and Wouters (2004) also allow for ten structural shocks, six of which are specified as first-order stochastic processes and four of which are assumed to be white noise.

the household side this includes the habit-persistence parameter $[v]$, the Frisch elasticity of labor supply $[\gamma]$, the adjustment cost parameters on capital and investment $[\theta_k, \theta_i]$. There are six parameters characterizing pricing policies, the three parameters that determine the duration of pricing policies in the markets for goods, labor and capital $[\delta, \delta_w, \delta_k]$ and the three quadratic cost parameters that determine the steepness of the marginal cost⁶ curve for prices, wages, and user costs $[\phi, \phi_w, \phi_k]$. Finally we have the two parameters of the interest rate reaction function $[\xi_{int}, \xi_\pi]$. The fourth column reports the type of distribution we assume (Beta, Normal, Inverted Gamma). Following standard conventions we will be using Beta distributions for parameters that fall between zero and one, inverted gamma (invg) distributions for parameters that need to be constrained to be greater than zero and normal (norm) distributions in other cases. The first column of each table reports our priors for the means of each parameter and the value in the fifth column represents a measure of uncertainty in our prior beliefs about the mean (measured as a standard error). The second and third columns report the posterior means of the parameters, and 90% confidence intervals that are based on 40,000 replications of the Metropolis-Hastings algorithm. The assumptions about and results for the remaining parameters are reported in a similar format in Tables 4 and 5.

3.4.1 Priors about Structural Parameters (Table 3)

Habit Persistence in Consumption $[v]$: We set the prior at 0.90 as high values are required to generate realistic lags in the monetary transmission mechanism and hump-shaped consumption dynamics—see Bayoumi, Laxton and Pesenti (2004) for a discussion of the role of habit persistence in generating hump-shaped consumption dynamics in response to interest rate shocks. This prior is somewhat higher than other studies such as Boldrin, Christiano and Fisher (2001), who use a value of 0.7.

Frisch Elasticity of Labor Supply $[\gamma]$: We set the prior at 0.50. Pencavel (1986) reports that most microeconomic estimates of the Frisch elasticity are between 0 and 0.45, and our calibration is at the upper end of that range, in line with much of the business cycle literature.⁷

Adjustment Costs on Changing Capital and Investment $[\theta_k, \theta_i]$: We set priors equal to 5 and 50 for θ_k and θ_i . These assumptions are based on analyzing the simulation properties of the model. The data do not seem to have much to say about these parameters other than that they cannot be zero or very large. This is not uncommon.

Duration of Pricing Policies $[\delta, \delta_w, \delta_k]$: The duration of pricing policies is $(1/(1 - \delta))$. In the base case we set the prior equal to a three quarters duration for prices, wages and user costs, therefore the priors equal 0.66 for $[\delta, \delta_w, \delta_k]$.

⁶Or the marginal rate of substitution minus the real wage (for wages), or minus the gross intermediation spread (for user costs).

⁷As discussed by Chang and Kim (2005), a very low Frisch elasticity makes it difficult to explain cyclical fluctuations in hours worked, and they present a heterogeneous agent model in which aggregate labor supply is considerably more elastic than individual labor supply.

This is based on our reading of the empirical literature for the US and on the results cited in ECB (2005). In the US, consumer prices are re-set on average (slightly faster than) every two quarters, while the average for producer prices is four quarters. As our model does not distinguish between the two, it seems reasonable to choose an intermediate prior of three quarters. The priors for wages and user costs are set to the same value, but at least for user costs we will consider alternatives in the sensitivity analysis.

Steepness of Marginal Cost Curve [ϕ, ϕ_w, ϕ_k]: Simulation experiments with the model suggest that plausible values for these parameters might fall between 0.50 and 2.0. In our base case we set the prior at 1.0. Our sensitivity analysis includes a case where all three of these parameters are restricted to be zero. There are significant interactions between these adjustment cost parameters and the duration parameters that will be explained below.

Interest Rate Reaction Function [ξ_{int}, ξ_π]: We impose prior means of 0.5 to be consistent with previous work, but we make these priors diffuse to allow them to be influenced significantly by the data.

3.4.2 Priors about Structural Shocks (Tables 4-5)

Persistence parameters for the structural shocks [$\rho_{gov}, \rho_{inv}, \rho_c, \rho_{int}, \rho_{gr}, \rho_L, \rho_\mu, \rho_{\mu^w}$]: Table 4 reports the assumptions about the priors for these parameters. With the exception of the shocks to the markups and the autocorrelated productivity shocks we set the prior means equal to 0.85 with a fairly diffuse prior standard deviation of 0.10. For the two markup shocks we impose zero serial correlation. These priors are consistent with other studies such as Smets and Wouters (2004) and Juillard, Karam, Laxton and Pesenti (2005).

We treat the prior on the serial correlation parameter for the productivity shocks differently. Here, we utilize a tight prior so that the model can generate highly persistent movements in the growth rate relative to its long-run steady state. As mentioned earlier, this is necessary to explain some facts in our sample (persistent upward revisions in expectations of medium-term growth prospects), but it is also more consistent with the data over the last century in the United States and other countries, where productivity growth has departed from its long-term average growth rate for as long as decades in many cases. Obviously, there will not be a lot of information in our short sample for estimating this parameter, and not surprisingly, the data will be silent on the matter as it should be.⁸ We are considering adding expectations of long-term productivity growth to the list of observable variables to help identify this parameter, but have not attempted to do so at this point.

Structural shocks standard errors [$\sigma_{\hat{\varepsilon}^{gov}}, \sigma_{\hat{\varepsilon}^{inv}}, \sigma_{\hat{\varepsilon}^c}, \sigma_{\hat{\varepsilon}^{int}}, \sigma_{\hat{\varepsilon}^{\pi^*}}, \sigma_{\hat{\varepsilon}^{iid}}, \sigma_{\hat{\varepsilon}^{gr}}, \sigma_{\hat{\varepsilon}^L}, \sigma_{\hat{\varepsilon}^\mu}, \sigma_{\hat{\varepsilon}^{\mu^w}}$]: Table 5 reports our assumptions about the priors for these parameters. The strategy here was to develop rough priors of the means by looking at the model's impulse response functions, conditional on all the other

⁸Provided the researcher can provide sensible priors, Bayesian techniques offer a major advantage over other system estimators such as maximum likelihood, which in small samples can often allow key parameters such as this one to wander off in nonsensical directions.

priors, and then to form a diffuse prior around this mean in order to let the data adjust the parameters in a way that improves the overall fit of the model. The specific values for these priors are not intuitive, as they require a very detailed knowledge of the structure of the model. Consequently, the reader might be well-advised to turn to the model's IRFs (which are based on the model's posterior distribution) to interpret how important each one of these shocks is.

4 Estimation Results

4.1 Parameter Estimates

To be completed.

4.2 Impulse Response Functions

To be completed.

In Figure 1 we display the impulse response to a 50 basis points contractionary monetary policy shock. This illustrates the substantial contribution of cascading to structural inflation inertia. In this case we have calibrated the contract length of price, wage and user cost contracts to be uniformly low at only 2 quarters, while the share of intermediates in production is high (but realistic) at $\eta = 0.75$. We observe that despite the low degree of nominal rigidities inflation is very inertial, and a contractionary shock has real effects normally only associated with much longer contract lengths.

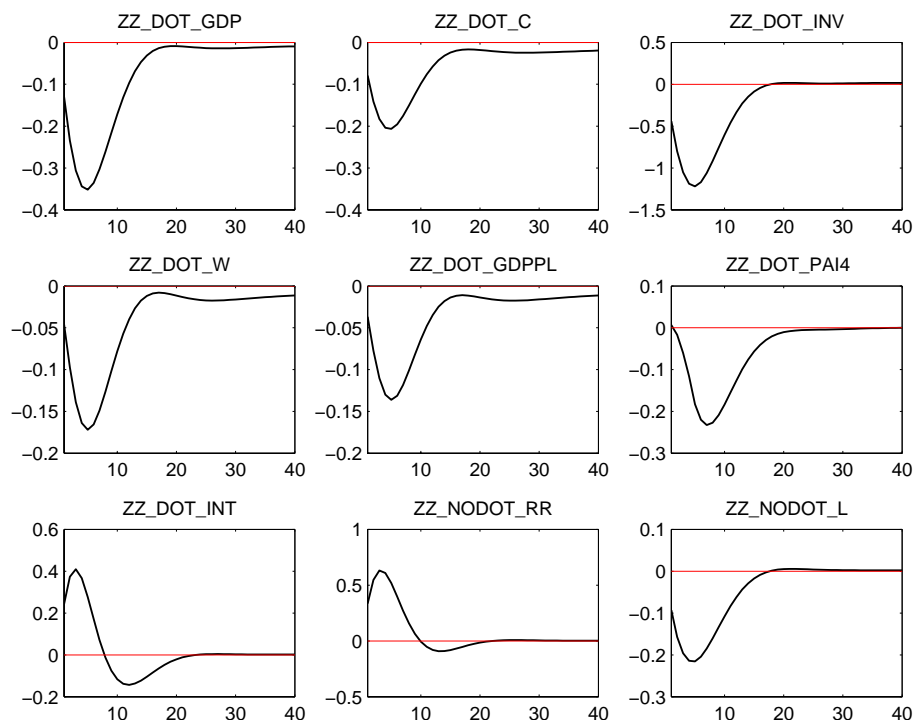


Figure 1

5 Conclusion

To be completed.

References

- [1] Bilal, M. and Klenow, P. (2004), “Some Evidence on the Importance of Sticky Prices”, *Journal of Political Economy*, **112**, 947-985.
- [2] Christiano, L.J., Eichenbaum, M. and Evans, C. (2005), “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *Journal of Political Economy*, **113**(1), 1-45.
- [3] Galí, J. and Gertler, M. (1999), “Inflation Dynamics: A Structural Econometric Analysis”, *Journal of Monetary Economics*, **44**(2), 195-222.
- [4] Golosov, M. and Lucas, R.E. (2003), “Menu Costs and Phillips Curves”, NBER Working Papers, No. 10187.
- [5] Klenow, P. and Kryvtsov, O. (2004), “State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?”, Working Paper.
- [6] Mankiw, G. N. and Reis, R. (2002), “Sticky Information or Sticky Prices: A Proposal to replace the New-Keynesian Phillips Curve”, *Quarterly Journal of Economics*, **117**, 1295-1328.

Table 1: Assumptions About Parameters and Steady-State Ratios

Parameters:	Value
Discount Rate β	0.999
Share of Capital in Value Added α	0.28
Capital Depreciation Rate Δ	0.025
Share of Government Spending in Steady State Output ω_g	0.18
Steady State Quarterly Growth Rate \bar{g}	$(1.017)^{\frac{1}{4}}$
Elasticity of Substitution among Goods in Steady State $\bar{\sigma}$	5.35
Elasticity of Substitution among Labor Inputs in Steady State $\bar{\sigma}_w$	7.25
Elasticity of Substitution among Capital Inputs $\bar{\sigma}_k$	11.00
Steady-State Ratios:	
Labor's Income Share	0.59
Consumption-to-GDP Ratio	0.62
Investment-to-GDP Ratio	0.20
Government Spending-to-GDP Ratio	0.18
Price Markup $\bar{\sigma}/(\bar{\sigma} - 1)$	1.23
Wage Markup $\bar{\sigma}_w/(\bar{\sigma}_w - 1)$	1.16
User Cost Markup $\bar{\sigma}_k/(\bar{\sigma}_k - 1)$	1.10

Table 2: Specification of the Stochastic Processes

Assumptions about the Shocks	Stochastic Processes
Total Factor Productivity	$\widehat{g}_t = \widehat{g}_t^{gr} + \widehat{g}_t^{iid}$
Demand Shocks	
Government Absorption	$\widehat{s}_t^{gov} = \rho_{gov} \widehat{s}_{t-1}^{gov} + \widehat{\varepsilon}_t^{gov}$
Investment	$\widehat{s}_t^{inv} = \rho_{inv} \widehat{s}_{t-1}^{inv} + \widehat{\varepsilon}_t^{inv}$
Marginal Utility of Consumption	$\widehat{s}_t^c = \rho_c \widehat{s}_{t-1}^c + \widehat{\varepsilon}_t^c$
Monetary Policy Reaction Function	$\widehat{s}_t^{int} = \rho_{int} \widehat{s}_{t-1}^{int} + \widehat{\varepsilon}_t^{int}$
Inflation Target	$\widehat{\pi}_t^* = \widehat{\pi}_{t-1}^* + \widehat{\varepsilon}_t^{\pi^*}$
Autocorrelated Growth Shocks	$\widehat{g}_t^{gr} = \rho_{gr} \widehat{g}_{t-1}^{gr} + \widehat{\varepsilon}_t^{gr}$
Supply Shocks	
Price Markup	$\widehat{\mu}_t = \widehat{\varepsilon}_t^\mu$
Wage Markup	$\widehat{\mu}_t^w = \widehat{\varepsilon}_t^{\mu^w}$
Marginal Disutility of Labor	$\widehat{s}_t^L = \rho_L \widehat{s}_{t-1}^L + \widehat{\varepsilon}_t^L$
I.i.d. Growth Shocks	$\widehat{g}_t^{iid} = \widehat{\varepsilon}_t^{iid}$

Table 3: Estimation Results
Parameters

	Prior	Mean Estimate	90% Interval	Density	Std
v	0.90			<i>Beta</i>	0.10
γ	0.50			<i>Normal</i>	0.10
δ	0.66			<i>Beta</i>	0.10
δ_w	0.66			<i>Beta</i>	0.10
δ_k	0.66			<i>Beta</i>	0.10
ϕ	1.00			<i>Normal</i>	0.20
ϕ_w	1.00			<i>Normal</i>	0.20
ϕ_k	1.00			<i>Normal</i>	0.20
θ_k	5.00			<i>Normal</i>	1.00
θ_i	50.00			<i>Normal</i>	10.00
ξ_{int}	0.50			<i>Normal</i>	0.20
ξ_π	0.50			<i>Normal</i>	0.20

Table 4: Estimation Results Continued
Parameters

	Prior	Estimate	90% Interval	Density	Std
ρ_{gov}	0.85			<i>Beta</i>	0.10
ρ_{inv}	0.85			<i>Beta</i>	0.10
ρ_c	0.85			<i>Beta</i>	0.10
ρ_{int}	0.85			<i>Beta</i>	0.10
ρ_{gr}	0.95			<i>Beta</i>	0.01
ρ_L	0.85			<i>Beta</i>	0.10

Table 5: Estimation Results Continued
Standard Deviation of Shocks

	Prior	Estimate	90% Interval	Density	std
$\sigma_{\hat{\varepsilon}^{gov}}$	0.025			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{inv}}$	0.2000			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^c}$	0.0250			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{int}}$	0.0100			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{\pi^*}}$	0.0010			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{iid}}$	0.0010			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{gr}}$	0.2000			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^L}$	0.0050			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^\mu}$	0.0250			<i>invg</i>	<i>inf</i>
$\sigma_{\hat{\varepsilon}^{\mu^w}}$	0.0250			<i>invg</i>	<i>inf</i>