Asset Price Volatilities and Trading Volumes in Dynamically Complete Markets with Heterogeneous Agents

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Abstract

Apart from the risk premium of equity over bonds, volatility of asset prices and trading volumes are two aspects of the already developed general equilibrium asset pricing theory that fail to show any resemblance with real data. The assumption of agent homogeneity has been relaxed in a number of studies where agents face uninsurable income shocks but fail to provide a consistent explanation that addresses all issues. This study assumes a dynamically complete asset market with two states and two assets, where the agents are allowed to be heterogeneous either in terms of their risk preferences, or in terms of their beliefs for the probabilities of the exogenous shock. The assumption that the (logarithm of the) aggregate dividend follows a mean reverting process with an iid state dependent shock, helps us construct a non-trivial equilibrium in which the endogenously determined wealth distribution across agents is part of the economy's state vector. Qualitative results show that this kind of setup works in the right direction in all issues addressed, while quantitative results are produced using a projection method as well as a variant of the Krusell and Smith (1997) method.

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1 Introduction

The standard representative agent model, as exemplified by the Lucas (1978) tree model, even though the paradigm for a framework to study asset prices, is known to fail in a number of predictions. Apart from the Equity Premium Puzzle [see for example Mehra and Prescott (1985) and Campbell (1999)], the agent homogeneity assumption stands as an obstacle in explaining asset price volatilities as well as trading volumes. As Hansen and Jagannathan (1991) show, the discrepancy between the volatilities of the asset prices and that of the aggregate consumption growth, is impossible to explain with a sensible set of preference parameters. As trading is concerned, things are even worse as the homogeneous agent models are not the only ones that predict no-trade equilibria. In particular, Judd, Kubler and Schmedders (2003a) in an extended Lucas (1978) model, that incorporates agent heterogeneity in terms of preferences, show that general equilibrium restrictions rule out trading in long lived assets. Further, Constantinides and Duffie (1996) construct an incomplete market setting with uninsurable income shocks that results also into a no-trade equilibrium.

While common intuition dictates that the agent heterogeneity is most probably the element required to explain endogenously, volatility and volume, there are two issues that remain widely unresolved. Firstly, what type of heterogeneity is mostly responsible in each case and secondly, through what avenues? Further, given the evidence that the cross-sectional variation in consumption growth goes a long way in explaining the risk premia observed, as shown by Brav, Constantinides and Geczy (2002), can the agent heterogeneity produce similar variation in marginal rates of intertemporal substitutions across agents?

The heterogeneous agent literature has mostly concentrated on ex-post agent heterogeneity coming from completely or partially uninsurable income shocks, like in Constantinides and Duffie (1996), Mankiw (1986), Heaton and Lucas (1996) and Krusell and Smith (1997). In their paper Constantinides and Duffie need to assume persistent income shocks so that they avoid close to complete risk sharing by the agents. However, they go to the other extreme allowing for no risk sharing with a no-trade equilibrium. Building on the same framework Storesletten, Telmer and Yaron (2001) present an overlapping generations model with idiosyncratic income shocks where there is some trade, and therefore partial risk sharing, where the interesting pricing implications come from the way agents accumulate or deccumulate assets as they age. The main problem of this framework is that it is not able to address all issues at the same time. On the one end of the spectrum is the risk premium and the volatility puzzles while on the other end is the issue of trading, with the deciding factor being, how much risk sharing is allowed.

This study concentrates on ex-ante agent heterogeneity, in a dynamically complete market setting, either in terms of risk preferences, as in Dumas (1989), Wang (1996) and Chan and Kogan (2002), or in terms of beliefs as in Abel (1989) and Jouini and Napp (2004). In particular we use standard time and state separable preferences with power utility, where the agents are allowed to have different risk aversion parameters. Alternatively, the agents are allowed to have their own beliefs as to the probabilities of the exogenous shock, which they do not update.

The model is constructed within the Lucas (1978) framework, while we try to built weakly stationary recursive equilibria that requires the wealth distribution in order to predict the evolution of the system. This is achieved in our complete asset markets, by assuming that the (logarithm of the) dividend follows a mean reverting process that is driven by a time-independent and state-dependent exogenous shock. For the case of heterogeneity in beliefs, an interesting evolution of the wealth distribution, that affects trading and prices, is achieved even with the standard Markovian process for the dividends. However, for the case of risk preference heterogeneity, the results of Judd, Kubler and Schmedders (2003a) apply. The mechanism through which the wealth distribution evolves in a non-trivial way (meaning being dependent only on the exogenous shock) is different in the two cases considered. In general, the hedging demands of the agents depend on their risk aversion parameter, their beliefs about the probabilities of the exogenous shock, their relative wealth and the aggregate risk that they face. When the agents have different beliefs for the probabilities of the states, in equilibrium they disagree on their consumption growth rates. This results in a non-trivial evolution of their relative wealth levels which has significant qualitative as well as quantitative implications both in terms of trading as well as asset prices. When agents have different risk aversion parameters obviously make different investment decisions. However, the essential element here is that the risk that they face each period changes with the current level of the dividend, which in turn implies continuous adjustments in their portfolios.

A major issue in general equilibrium models with heterogeneous agents is the way the equilibrium is approximated, when the wealth distribution (or alternatively the distribution of asset holdings) plays a role in the evolution of the system [see Wang (1994), Duffie et al. (1994) and Kubler and Polemarchakis (2004)]. Since closed form solutions are not available, many methods have been developed to approximate closely the evolution of the system. See for example Judd, Kubler and Schmedders (2003b) for projection and perturbation methods, Rios-Rull (1997), Krusell and Smith (1997, 1998) or den Haan (1994) for methods that use bounded rationality and Heaton and Lucas (1996) for a discretized space method, just to mention a few.

An endogenous state space normally arises in an incomplete market setting, but as we show here it is not necessary. Our complete market setting as well as the assumption of power utility helps us in solving the agent's value function, as well the decision rules, in a semi-closed form and expressing the unknown part of the solution in terms of an assumed equilibrium pricing kernel and the law of motion of the state vector. A method, therefore is developed where projection methods are used in order to approximate the pricing kernel and the law of motion. The advantages of our method are mainly three. First, the number of unknown functions that are required to approximate, decrease considerably from the case where the individual Euler equations are used together with the market clearing conditions. Secondly, with the semi-closed form solution of the value function, its shape is ensured to be preserved. Last but not least, with the way the individual decision rules are constructed, they are smooth in the state vector, and they can be used when assuming frequent trading.

A second method is also used to approximate our equilibrium. It is a variant of the Krusell and Smith (1997) method, which is extended to incorporate heterogeneous agents and long lived assets. This method however cannot be used for many assets of many different types of agents. With two assets and two agents (or types of agents) the state vector is the wealth of the first agent (or the wealth distribution of the first type), and the current dividend level. The functions that need to be approximated are hence, the two price functions as well as the law of motion.

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