**INSTITUT FÜR WIRTSCHAFTSPOLITIK
AN DER UNIVERSITÄT ZU KÖLN**

Product and Process Innovations in a Horizontally Differentiated Product Market

Pia Weiß
Institute for Economic Policy

IWP–Discussion Papers No. 2001/7
November 2001

Product and Process Innovations in a Horizontally Differentiated Product Market

Pia Weiß
Institute for Economic Policy*

Abstract

For a market of horizontal product differentiation, the paper examines the effects of the level of competition on the firm's decision between a product and process innovation. When firms have to choose between the two types of innovation, it is demonstrated that both firms undertake the product innovation when the competition is intense. For intermediate levels of competition, the firms choose different investment projects, and for less intense competition, the firms pursue cost–reducing innovations. When firms may undertake both innovations, they decide to undertake a mixture of the innovations depending on the innovation cost structure. Again, the firms are willing to incur higher costs into product innovations, when the competition is initially intense.
JEL–Classification: L13
Keywords: Product innovation; Process innovation; Horizontal product differentiation

1 Introduction

The interrelation of the market structure of an industry and the R&D investment has received much attention over decades.[1] Earlier works were mostly concerned with investments in R&D in homogeneous product markets (for an excellent survey see e.g. Reinganum, 1992). Within this strand of literature, the effect of the degree of competition on the incentives to innovate were widely discussed. In his seminal contribution, Schumpeter (1942) argued in favour of the monopoly while Arrow (1962) established the reverse proposition. Only recently, this question has been addressed in a framework of a differentiated product market.[2]

* The author wishes to thank Stephanie Rosenkranz, Walter Elberfeld as well as the participants of the research colloquium of Prof. von Weizsäcker for helpful comments on an earlier version. The usual disclaimer applies. Financial aid from the State of North–Rhine Westfalia is gratefully acknowledged.

1 There are many aspects of market structure. Here, it is used as a synonym for the degree of competition.

2 Bester and Petrakis (1993) e.g. for cost–reducing innovations in a model of horizontal product differentiation. Among others, Bonanno and Haworth (1998) and Boone (2001) consider process innovations in a model of vertical product differentiation. See e.g. Greenstein and Ramey (1998), Shaked and Sutton (1990), and Rosenkranz (1995) for a model of vertically differentiated products and product innovations.

Most contributions have focused on either a product or a process innovation or else have left unspecified, which particular type of innovation is studied. However, it seems to be more realistic to consider situations, in which a firm has the choice at least between a product and a process innovation. This framework is a prerequisite to understand which factors let firms undertake R&D projects aiming at a product or at a process innovation. The principal purpose of the present paper is to enquire into the role of the intensity of competition on the firms' decision to engage in a product *or* process innovation. To my notion, only two papers have addressed this question so far: Bonanno and Haworth (1998) and Rosenkranz (1996). Bonanno and Haworth (1998) consider a market of vertical product differentiation. The incentive of one firm to innovate are compared for a low degree of (Cournot) competition and a more intense (Bertrand) competition. They demonstrate that the high quality firm is apt to choose the product innovation in a less intense state of (Cournot) competition, while it is inclined to pursue the process innovation in a more dense (Bertrand) competition. For the low quality firm, the reverse result appears.

The paper of Rosenkranz (1996) considers a duopoly in a horizontally differentiated product market with Cournot competition. In her contribution, the firms may carry out both, process and product innovations, at the same time. She studies the influence of the consumers' willingness to pay in the sense of the market size as well as the firms' decision to form research joint ventures on the firms' research portfolio. In addition, the welfare effects of R&D–cooperations are considered. The main result of Rosenkranz related to the present work is that the firms will always pursue both, product and process innovations at the same time.

The present paper is based on a Hotelling (1929)–type duopoly model with Bertrand competition. In this framework, the intensity of competition can be measured by the distance of the variety's location on the variety line. Here, the possibility arises to consistently study all possible degrees of competition from a local monopoly to perfect competition. The question of the effects of the degree of competition on the firms' investment pattern is studied in three successive steps. First, it is assumed that firms have to decide between a product and a process innovation, where the investment costs are negligible. Here, it can be demonstrated that (1) both firms pursue process innovations, when the level of competition is low, (2) the firm choose different types of innovations in an intermediate intensity of competition, and (3) both firms engage in product innovations when competition is intense. Subsequently, non–negligible investment costs are considered. It will be shown that under reasonable conditions the results obtained under negligible investment costs continue to hold. Finally, a setup is studied, in which firms may carry out process and product innovations at the same time. Restrictions on the investment costs functions can be established, so that firms indeed carry out a mixture of product and process innovations. If those restrictions are not met, firms will undertake only one type of innovation or none at all. In addition, it can be shown that the optimal product innovation positively depends on the willingness to pay for a variety differing from the most preferred one and positively on the initial degree of competition.

The paper is organised as follows: Section 2 introduces the main assumptions concerning the timing, the information and the equilibrium concept of the model. Subsequently, section 3 characterises the Nash equilibrium in prices for all possible combinations of the two types of innovation. Then, section 4 introduces the framework, in which both firms can choose between a process and a product innovation and the innovation costs are negligible. Section 5 assumes strictly positive investment costs and establishes conditions under which the innovation pattern for negligible innovation costs re–emerges. Section 6 studies the firms' decision when both types of innovation can be pursued at the same time and section 7 provides a summary.

2 The Basic Framework

The firms' investment problem is addressed in a two–stage non–cooperative game of perfect information. Consider a market of a horizontally differentiated consumption good. There is a continuum of potential varieties ϕ with total mass one, i.e. $\phi \in [0,1]$. In the initial situation, two firms A and B are operating in the market. They produce specific varieties ϕ_i, $i = A, B$. The actually produced varieties can be illustrated by points ϕ_A and ϕ_B on the unit line. Without loss of generality, it is assumed that $\phi_A \leq \phi_B$.

At the beginning of the first stage, firms decide on their research projects. They can pursue a process or a project innovation. The former aims at reducing the production costs, whereas the latter introduces a new variety, while the old one is abandoned. In the remainder of this stage, the projects are carried out and completed at the end of the first stage.[3] For simplicity, it is posited that neither project is subject to uncertainty. At the begin of the second stage, the firms observe the opponent's investment decision. They simultaneously choose prices and realise profits and the game ends.

As usual, the Nash equilibria (NE) are determined by applying the concept of subgame perfection. Therefore, the second–stage price equilibrium is determined first. Subsequently the first–stage investment problem is analysed. Here, three different scenarios are considered: (1) Firms can undertake only one research project, so that they have to decide between the two types. The investment costs I are supposed to be negligible, i.e. $I = 0$ is imposed. (2) Again, it is posited that the research projects are mutually excluding. The investment costs are identical for both projects, but strictly positive. This situation may arise when firms have a certain investment budget to spend. When the research projects have a fixed or minimum scale, they have to choose between the two types of innovation. This case may be associated to short–term investment projects. (3) Firms may undertake process and product innovations at the same time. They also determine the scale of each investment. As firms have a greater choice to specify the investment plans, this case may serve as an example for long–term research projects.

[3] In the first stage, all innovation activities necessary to either offer a new variety or to produce more efficiently are carried out. Therefore, one may think of the first stage as comprising several time periods, during which the old varieties are produced using the old technology.

3 The Price Equilibrium

There is a continuum of consumers with total mass one. They have different preferences regarding the most preferred variety. Let $\chi \in [0,1]$ denote the consumers' most favoured variety and let χ be uniformly distributed. It is assumed that the consumers' preferences can be represented by

$$U(\chi, \phi_i, p_i) = r - t(\chi - \phi_i)^2 - p_i, \tag{1}$$

where p_i denotes the price of variety ϕ_i. The term $t(\chi - \phi_i)^2$ is the disutility, which a consumer experiences by not purchasing his favoured variety. The parameter t can be interpreted as an consumer's willingness to pay sub–ideal varieties. Then, the parameter r is the reservation price for the individual's most preferred variety. Hence, if $p_i > r$ even the consumer favouring the variety ϕ_i is not willing to buy it. Henceforward, it is assumed that r is sufficiently large, so that every consumer purchases one or the other variety, i.e. the market is fully covered.[4]

Consumer $\bar{\chi}$ is indifferent between buying variety ϕ_A and ϕ_B if both options yield the same utility. From equation (1), the indifferent consumer can be identified with [5]

$$\bar{\chi} = \frac{\phi_A + \phi_B}{2} + \frac{p_B - p_A}{2t(\phi_B - \phi_A)}.$$

With $\phi_A \leq \phi_B$ and the market is fully covered, consumers $\chi < \bar{\chi}$ purchase variety ϕ_A while consumers $\chi \geq \bar{\chi}$ buy variety ϕ_B. Accordingly, the firms' demand functions are given by

$$\begin{aligned} d_A &= \bar{\chi} = \frac{\phi_A + \phi_B}{2} + \frac{p_B - p_A}{2t(\phi_B - \phi_A)}, \\ d_B &= 1 - \bar{\chi} = \frac{2 - (\phi_A + \phi_B)}{2} + \frac{p_A - p_B}{2t(\phi_B - \phi_A)}. \end{aligned}$$

The production technology is assumed to exhibit constant returns to scale and to be independent of the particular variety offered, so that the unit costs c_i are constant. To characterise the price equilibrium for all possible investment decisions, unit costs may diverge indicating that firms apply different technologies. Then, the firms' profit function can be written as $\pi_i = (p_i - c_i)d_i$. As usual, the NE in prices is a pair of functions depending on the parameters of the model, i.e. the cost and the market structure:

$$p_A^*(c_A, c_B, \phi_A, \phi_B) = \frac{1}{3}\left(2c_A + c_B + t(\phi_B - \phi_A)(2 + \phi_A + \phi_B)\right), \tag{2}$$

$$p_B^*(c_A, c_B, \phi_A, \phi_B) = \frac{1}{3}\left(c_A + 2c_B + t(\phi_B - \phi_A)(4 - \phi_A - \phi_B)\right). \tag{3}$$

The price functions have the following properties: The price of both varieties is negatively correlated with the intensity of competition, i.e. with the degree of product differentiation

4 Peitz (1999) pointed out that the consumers' demand may also be restricted by their income. He demonstrated that a NE may fail to exist in such situations. Clearly, the non–existence problem is also relevant for the present model. Accordingly, a sufficiently large income is supposed as well.

5 Due to the assumption that the reservation price r is sufficiently large, so that the entire market is covered, the indifferent consumer realises a positive utility.

$\phi_B - \phi_A$. In the extreme case of identical varieties ($\phi_A = \phi_B$), the price functions are identical to those obtained in a standard Bertrand game. In this situation, the firms charge at unit costs if both firms use identical technologies. Otherwise, the firm employing the less efficient technology drops out and the more efficient firms supplies the entire market.

With the equilibrium prices given in (2) and (3), the equilibrium profits net of the investment costs can be determined with

$$\pi_A^*(\Delta k, \phi_A, \phi_B) = \frac{1}{18t(\phi_B - \phi_A)} \left(\Delta k + t(\phi_B - \phi_A)(2 + \phi_A + \phi_B)\right)^2 - I_A, \tag{4}$$

$$\pi_B^*(\Delta k, \phi_A, \phi_B) = \frac{1}{18t(\phi_B - \phi_A)} \left(-\Delta k + t(\phi_B - \phi_A)(4 - \phi_A - \phi_B)\right)^2 - I_B, \tag{5}$$

where $\Delta k \equiv c_B - c_A$. These profit functions define the payoffs for all possible investment activities undertaken in the first stage.[6]

4 Negligible Investment Costs

To yield comparable results for the three alternative scenarios, the situation is regarded to be identical at the beginning of the first stage. In particular, it is assumed that both firms initially use identical technologies, i.e. $c_A = c_B = c$. For notational convenience, it is additionally posited that history led to a symmetric market structure: at the begin of the first stage, firm $B's$ variety is $\phi_B = 1 - \phi_A$. Then, the assumption of $\phi_A \leq \phi_B$ implies that ϕ_A takes values in the interval $[0, 1/2]$. Due to the symmetry assumption, a particular value of ϕ_A also contains information on the intensity of competition. Clearly, the lower the degree of product differentiation, the higher is the intensity of competition. As the former is $(1 - 2\phi_A)$ in the symmetric setup, the intensity of competition is negatively correlated with ϕ_A.

In the scenario studied in this section, firms can undertake only one investment project. As the investment costs are regarded to be zero ($I_A = I_B = 0$), the option 'do not invest at all' is not relevant here (see the next section). As a consequence, the firms will either invest in a process or in a product innovation. When aiming at the former option, the firm produces with a new technology in the second stage, which reduces the unit costs by a fixed amount of δ. If a firm undertakes a product innovation, it is in the position to offer the profit maximising variety by the end of the first stage. The old variety is abandoned. Due to the assumption that the market is initially fully covered, a firm cannot gain a local monopoly position. Hence, product innovations are always non–drastic in the present model (cf. Rosenkranz (1996)). Both projects are alike in the sense that neither the outcome nor the time of completion are uncertain. The only distinguishing characteristics of the two projects is the result itself.

Given that the investment costs are negligible, there are four potential equilibria:

---

[6] Note that the firm using the less efficient technology drops out whenever they initially produce identical varieties ($\phi_A = \phi_B$). The remaining firm receives a positive profit. This can directly be verified from the profit functions. Let $c_B < c_A$. Then, firm A exits the market and firm B's profit is derived with $(-c_B + t\phi_B(4 - \phi_B))^2/18t\phi_B$.

(1) (np,np): Both firms undertake the product innovation. They produce the profit maximising varieties ϕ_A^* and ϕ_B^* employing the old technology, i.e. $\Delta k = 0$.

(2) (np,cr): Firm A produces a new variety ϕ_A^* using the old technology while firm B introduces a new technology and supplies the old variety $1-\phi_A$. Therefore, $\Delta k = -\delta < 0$.

(3) (cr,np): Firm A introduces a new technology and firm B produces a new variety ϕ_B^*, so that $\Delta k = \delta > 0$.

(4) (cr,cr): Both firms implement a new technology. Since the cost reduction achieved is identical for both firms, $\Delta k = 0$ holds true.

In general, the profit maximising variety ϕ_i^* may depend on the opponent's investment decision. To determine these optimal varieties, consider first firm A's situation in case (1) or (2). As both firms behave rationally and anticipate the second–stage outcome, firm A's optimal choice ϕ_A^* has to satisfy

$$\frac{d\pi_A^*}{d\phi_A} = \frac{d_A}{3(\phi_B - \phi_A)}\left(\Delta k + t(\phi_B - \phi_A)(\phi_B - 2 - 3\phi_A)\right) \leq 0. \tag{6}$$

In case (1), firm B aims at introducing a new variety as well. Both firms employ the old technology, so that $\Delta k = 0$. Since $\phi_A \leq \phi_B$, (6) holds with strict inequality for all $\phi_A^* \in [0,1/2]$. Therefore, firm A will offer $\phi_A^* = 0$ in the second stage. Consider now case (2). Firm B implements a more efficient technology, whereas firm A continues to use the old one. The efficiency gain of firm A is $\Delta k = -\delta < 0$. Again, there is no ϕ_A^* in the relevant range, so that (6) is satisfied with equality. Firm A chooses $\phi_A^* = 0$. Due to the symmetry of the model, firm B will produce $\phi_B^* = 1$, when introducing a new variety. Hence, for the second stage, the firms' optimal product choice is to seek the maximal degree of differentiation.[7]

Given the optimal choices for the new varieties ϕ_A^* and ϕ_B^*, the payoffs realised in each potential equilibrium can be specified. Using $\phi_A^* = 0$, $\phi_B^* = 1$, and the information on the cost structure given in the four cases above, it can easily be verified from equations (4) and (5) that the payoffs are symmetric in the following sense: in case (1) and (4) the opponents realise identical profits. Firm A's and B's profit from investing into a process (product) innovation are equivalent if the rival decides to undertake a product (process) innovation (case (2) and (3)). Table 1 summarises the net–payoffs in a schematic manner. In general, the payoffs π^i are functions of the cost reduction achieved by a process innovation δ and of the variety initially produced by firm A.[8] As the value of ϕ_A is also a measure of

[7] As it is well–known, there are two opposing effects on the optimal product choice in this class of models. One induces the firm to move to the centre of the line. This increases the demand as long as the market is not fully covered. The second one causes the firm to choose products towards the end of the line. This increases the degree of product differentiation and thereby enables the firms to charge higher prices. As we presumed that the market is fully covered at the begin of the first stage, the demand effect is irrelevant.

[8] As the initial market structure was supposed to be symmetric, the old variety of firm B is fully determined by ϕ_A.

		firm B	
		np	cr
firm A	np	(π^A, π^A)	(π^B, π^C)
	cr	(π^C, π^B)	(π^D, π^D)

Table 1: The payoff matrix for negligible investment costs

the intensity of competition, the payoffs π^i are correlated with the degree of competition prevailing in the initial situation.

To analyse the effect of competition on the firms' innovative activity, it is therefore determined for which values of ϕ_A the four cases are a Nash equilibrium (NE). The appendix proves the following proposition

Proposition 1. *Given $\delta < \min\{t(6\sqrt{2}-7)/4, 6t\}$, there exists a $\underline{\phi}_A(\delta)$, and a $\overline{\phi}_A(\delta)$, with $\underline{\phi}_A(\delta) < \overline{\phi}_A(\delta)$ and $\underline{\phi}_A, \overline{\phi}_A \in (0, 1/2)$, so that*

(1) (np, np) is a NE for $\phi_A \geq \overline{\phi}_A$

(2) (np, cr) and (cr, np) are NEs for $\underline{\phi}_A \leq \phi_A < \overline{\phi}_A$

(3) (cr, cr) is a NE for $\phi_A < \underline{\phi}_A$

The proposition states that both firms will undertake a product innovation if the initial degree of competition is sufficiently high. In contrast, both firms will pursue the process innovation if the initial level of competition is sufficiently low. For intermediate degrees of competition, the firms pursue different types of innovation.

For an intuitive explanation, consider the extreme case of $\phi_A = 1/2$. In this situation, the firms produce identical varieties. They charge a price identical to the unit costs and the profits of both firms are zero. As a process innovation yields identical cost reductions, firms continue to receive zero profits after both introduced a more efficient technology. On the other hand, a product innovation guarantees a strictly positive profit for both firms. Consequently, it is profitable for one firm to undertake a product innovation independent of the rival's actions. In this situation, case (1) states that the increase in the rival's profits from developing a new variety is higher than the one resulting from a cost reduction given the efficiency gain is comparable small. Case (1) verifies that this result holds true for ϕ_A in the neighbourhood of $\phi_A = 1/2$, i.e. for a sufficiently high level of competition.

With a lower initial degree of competition, the possibility of charging prices, which exceed the unit cost, increases. In those situations, case (2) confirms that it is still profitable for one firm to pursue a product innovation independent of the rival's decision. However, the initial level of competition is low enough, so that the rival's gain from introducing a new technology is higher as compared to the one realised by offering a new variety.

Finally, consider the case of $\phi_A = 0$, in which the initial level of competition attains its minimum. As the maximal product differentiation is also the optimal one, a new variety cannot be introduced. Firms can only introduce a more efficient technology. As investment costs are zero, it is true that the equilibrium (cr, cr) and an equilibrium, in which neither firm undertakes any research project, yields identical profits. However, the option 'do not invest at all' is not credible, since a firm is strictly better off by developing a new technology given the rival does not pursue a research project. Hence, the firms are in a prisoner's dilemma situation. Then, case (3) states that the prisoner's dilemma arises in the neighbourhood of $\phi_A = 0$.

Not surprisingly, our results differ from those found by Bonanno and Haworth (1998). This can be attributed to the different markets considered. Bonanno and Haworth (1998) address the question for a market of vertically differentiated products. Here, the market positions of the firms is not identical. Therefore, the results depend on whether the quality leader or the quality follower innovates. In the model presented here, the firms are exactly alike, so that neither firm has an advantage over the other.

Remark 4.1. In the present model, the second–stage product differentiation is always optimal in the sense that the firm produces the profit maximising variety given the rival's actions. Since the optimal variety ϕ_i^* is always the one located at the extreme end of the variety line, the product choice is independent of the variety initially produced. As a consequence, the firms can achieve a marginal or a large product innovation at the same negligible investment costs. A more convincing assumption would e.g. be that only a limited product innovation is feasible given the fixed and negligible investment budget. However, imposing the more plausible assumption leads to an increasingly inconvenient notation without altering the results. Accordingly, the fact that the firms are able to develop the profit maximising variety may be seen as a pure notational convenience.

5 Non–negligible Investment Costs

In the previous section, it was assumed for simplicity that the investment costs are zero. In this situation, firms always undertake one or the other research project. Clearly, this situation will not generally be observed with strictly positive investment costs. Therefore, this section investigates, under which circumstances proposition 1 continues to hold with positive investment costs. Accordingly, the setup considered here is the same as the one in the previous section. The investment costs are assumed to be strictly positive ($I_i > 0$), identical for product and process innovations, and identical for both firms ($I_A = I_B$).

Firstly, observe that firms now have an additional option: not to invest at all. Consequently, there are 5 potential NE in addition to the ones described in the previous section:

(1) (ni, ni): Neither firm undertakes a research project. This will typically be observed when the investment costs are extremely high.

(2) (ni, np) and (np, ni): One firm carries out the product innovation and offers the profit maximizing variety ϕ_i^* in the second stage. The other firm does not invest. As both firms use the old technology, Δk equals zero.

(3) (ni, cr) and (cr, ni): One firm implements a new technology. The rival chooses to do nothing and continues to use the old technology. Therefore, Δk equals $-\delta$ and δ when firm B and firm A is the innovating one.

		firm B		
		ni	np	cr
	ni	(π^I, π^I)	(π^E, π^F)	(π^G, π^H)
firm A	np	(π^F, π^E)	(π^A, π^A)	(π^B, π^C)
	cr	(π^H, π^G)	(π^C, π^B)	(π^D, π^D)

Table 2: The payoff matrix for non–negligible investment costs

Since the investment costs are fixed, the firms' decision on the profit maximising variety when undertaking a product innovation remains unchanged. Therefore, firms seek to maximise the degree of product differentiation, i.e. they choose $\phi_A^* = 0$ and $\phi_B^* = 1$ respectively. With this information and the ones given in the description of the cases, the net payoffs for the potential NE can be determined from equations (4) and (5). Table 2 illustrates the potential outcomes of the investment game, where all payoffs are net of investment costs. Once again, the payoffs prove to be symmetric, so that only the upper or the lower triangle of the matrix need to be considered henceforward.

As in the previous section, the net payoffs will generally be functions of the efficiency gain δ and the old variety ϕ_A. In addition, the payoffs will typically depend on the investment costs as well. Hence, one would naturally presume that (ni, ni), where neither firm undertakes a research project, is a NE for extremely high investment costs. As this outcome is uninteresting, it is assumed that the investment costs are such that $\pi^F, \pi^H > \pi^I$. This condition is sufficient to rule out (ni, ni) as a NE.

Note also, that the payoff matrix presented in table 1 reappears in the lower left part of table 2. Consequently, the NE's discussed in the previous section remain potential outcomes of the investment game. In fact, the following proposition states the sufficient conditions, under which the results presented in proposition 1 continue to hold for strictly positive investment costs.

Proposition 2. *There exist parameters δ_{AC}, δ_{BD}, δ_{DG}, δ_{DG} and functions $\underline{\phi}_A(\delta)$, $\overline{\phi}_A(\delta)$, I_{AE}, I_{DG}, and I_{BG}, so that for*

$$I < \min\{I_{AE}(\underline{\phi}_A), I_{DG}(\underline{\phi}_A), I_{BG}(\overline{\phi}_A)\}, \tag{7}$$

$$\delta \in \delta_{AC} \cap \delta_{BD} \cap \delta_{DG} \cap \delta_{BG}, \tag{8}$$

proposition 1 holds true for positive investment costs.

Proof. See appendix. □

In fact, an investment environment identical to the one in the previous section is generated by conditions (7) and (8). Therefore, as long as these conditions are satisfied, the innovation pattern is described by proposition 1 even though the investment costs are strictly positive. Clearly, for investments costs sufficiently above the ones specified in equation (7), multiple equilibria and corner solutions may arise, i.e. NE's in which one of the firms does not invest at all.

To illustrate the results, a numerical example is presented. Consider the following parameter specification: $t = 1$ and $\delta = 0.2$.

Case 1. When investment costs are positive, the threshold values are derived with $\underline{\phi}_A = 0.23$ and $\overline{\phi}_A = 0.30$. The restrictions on the efficiency gain δ are found to be $\delta_{AE} = 0.37$, $\delta_{BD} = 6$, $\delta_{DG} = 0.67$, and $\delta_{BG} = 0.36$. Condition (8) can be specified with $\delta \in [0, 0.36]$. Hence, an efficiency gain of $\delta = 0.2$ satisfies condition (8). For this cost reduction, the upper limits of the investment cost are obtained with $I_{AE}(\underline{\phi}_A) = 0.05$, $I_{DG}(\underline{\phi}_A) = 0.06$, and $I_{BG}(\overline{\phi}_A) = 0.09$, so that condition (7) requires $I \in [0, 0.05)$. The boundaries still seem to be strong. However, the lower limit for the investment costs, for which the firms will choose not to invest rather than to pursue a process innovation ($\pi^I > \pi^H$) for $\phi_A = 0$, equals 0.07. This this condition is a prerequisite for the situation, in which neither firm undertakes an investment, to be a NE.

6 Non–excluding Investment Projects

So far, the firms' situation when deciding on the research projects was characterised by two properties: (1) firms could only carry out one of them and (2) the investment costs were independent of the scale of the research projects.[9] Both may be an accurate description of the firm's options under certain circumstances. One may associated these properties with short–term investment projects following a well–known technological path, where only minor problems have to be solved. On the other hand, long–term investments are typically more flexible regarding the scale of the projects and the investment costs. When a decision on the project scale is possible, the firms are in the position to pursue both types of innovation. Therefore, this section relaxes the assumption of mutually excluding investment projects as well as the one of fixed–scale projects.

In particular, the initial situation is presupposed to be the same as described above: both firms use identical technologies ($c_A = c_B = c$) and the market structure is symmetric ($\phi_B = 1 - \phi_A$). Firms decide on the characteristics of their research projects. Firm i chooses a cost reduction $\Delta c_i \in [0, c]$ and the scale $\Delta\phi_i \in [0, \phi_i]$ of the product innovation to be obtained. Let $G(\Delta c_i)$ and $K(\Delta\phi_i)$ be the costs associated. The investment cost are supposed to be increasing functions of the project scale, where the costs rise the more the larger the scale is: $G'(\cdot) > 0$, $G''(\cdot) > 0$, $K'(\cdot) > 0$, and $K'' > 0$.[10] Clearly, when firms not invest into a more efficient technology or a new variety, the respective investment costs are zero: $G(0) = 0$ and $K(0) = 0$.

As in the previous sections, the firms will aim at a higher degree of product differentiation. Doing so decreases the competition and enables both firms to charge higher prices at a given technological level. Therefore, firm A's new variety is given by $\phi_A^* = \phi_A - \Delta\phi_A$, whereas firm B's new variety is determined with $\phi_B^* = 1 - \phi_A + \Delta\phi_B$.

Restricting attention to symmetric solutions, i.e. where $\Delta c_A = \Delta c_B = \Delta c$ and $\Delta\phi_A = \Delta\phi_B = \Delta\phi$, the following result is derived:

9 In particular, this applies to the product innovation. Investment costs were the same whether the firm aimed at a marginal change of the product characteristics or the profit maximising ones.

10 The fact that the costs are separable implies that there are no externalities from pursuing both research projects.

Proposition 3. *There exists a symmetric NE, where* $(\Delta c^*, \Delta\phi^*)$ *are determined by:*

$$\Delta c^* = \begin{cases} 0 & \textit{if} \quad G'(0) \geq 1/3, \\ \{\Delta c \in [0,c] | G'(\Delta c) = 1/3\} & \textit{otherwise}, \end{cases} \tag{9}$$

$$\Delta\phi^* = \begin{cases} 0 & \textit{if} \quad K'(0) \geq 2t(2-\theta)/9, \\ \{\Delta\phi \in [0,\phi_A] | K'(\Delta\phi) = 2t(2-2\Delta\phi-\theta)/9\} & \textit{otherwise}, \end{cases} \tag{10}$$

with $\theta = 1 - 2\phi_A$.

Proof. See Appendix. □

The second lines of equations (9) and (10) specify the interior solutions of the investment game. Given that the firms invest into technology improvements, it can be seen in equation (9) that the cost reduction is independent of the initial variety and, hence, of the degree of competition. The efficiency gain is solely determined by the properties of the investment cost function $G(\cdot)$. From the second line of equation (10), $d\Delta\phi/d\theta = -(2t/9)/(K'' + 4t/9) < 0$ ensues. This reveals that the size of the product innovation is negatively correlated to the degree of product differentiation. This is a result of the firm's inherent inclination to escape competition. If the level of competition is initially high the firms has a high willingness to pay for relaxing competition.

The upper lines of equations (9) and (10) show the circumstances under which corner solutions appear, i.e. when Δc, $\Delta\phi$ are zero. For both innovation types, firms will not undertake a research project in the respective line when the first unit of the innovation is too costly. In particular, the firms will prefer the old technology, whenever $G'(0) \geq 1/3$. The upper limit for the investment cost of the first unit are again independent of the degree of competition. In the case of product innovations, the firms continue to produce the old variety when $K'(0) \geq 2t(1-\theta)/9$. Here, firms are prepared to incur higher investment costs for the first unit when competition is more intense. This resembles the findings of the previous sections: product innovations are more profitable when competition is high.

It can also be seen in (10) that the optimal change of the product characteristics increases in the parameter t. This parameter is the consumers' willingness to pay for varieties differing from the most preferred one. Hence, the optimal product innovation $\Delta\phi_i$ is positively correlated to the willingness to pay for sub–ideal varieties. In contrast, equation (9) shows that the optimal degree of process innovation is independent of the consumers willingness to pay for sub–ideal varieties.

Before relating the results to the literature, they are summarised.

- The firms will generally not invest when the investment costs for the first unit are too high.
- When firms undertake a process innovation, the scale of the project is independent of the degree of competition.
- Firms are generally prepared to incur higher costs for product innovations when the competition is initially more intense.

– The willingness to pay (t) for varieties different from the ideal one and the optimal degree of product innovation are inversely related.[11]

– Depending on the specific investment cost function, the symmetric NE may be a situation in which

  (1) no investments are pursued,

  (2) only process innovations

  (3) or only product innovations are carried out,

  (4) both types of innovations are undertaken.

Rosenkranz (1996) addressed a similar question for horizontally differentiated product market. Dissimilar from the present model, she studies a model where consumers have a preference for product variety. From this difference in the setup ensue the diverging results concerning the optimal cost structure of the new technology. In Rosenkranz the cost structure depends on the chosen variety whereas it has been demonstrated for the present model that the optimal degree of cost reduction is independent of the initial variety. In addition, in Rosenkranz's contribution corner solutions are not analysed. In the model presented here, firms may decide to pursue only one type of innovation or may as well carry out none of them depending on the investment costs functions. The reason for the diverging results lies in a slight difference of the setup. The model presented here presumes that both firms are already operating in the market when they decide on the research projects. As a consequence, there is a status quo defined by the pre–investment situation. Then, the profitability of an investment project depends also on the status quo. In particular, the after–innovation net profits have to be higher than the pre–innovation profits to undertake a specific investment. As the latter are strictly positive in the present model, the firms will not undertake every research project yielding a positive after–innovation net profit. In the setup of Rosenkranz, the firms start producing only after the innovation. Hence, the firms will pursue every research project generating a positive net payoff since there is no status quo. Presumably, corner solutions would arise even in Rosenkranz (1996) contribution, when firms were active before the investment game starts or if R&D costs were prohibitively high.

7 Summary

In a model of horizontally differentiated products, it was analysed how the degree of competition affects the innovation pattern. A two–stage non–cooperative game was employed, where firms decide on their research activities in the first period and simultaneously choose prices in the second one.

[11] A low value for the parameter t indicates a high willingness to pay for sub–ideal varieties. This can be seen by setting t equal to zero in the individuals' utility function. Then, the individuals do not care about differences in the varieties and are prepared to pay the same price for all possible varieties.

Three situations for the investment game were considered. In the first, firms have to choose between a product and a process innovation. The investment costs were supposed to be zero. Here, it has been demonstrated that both firms develop a new variety when competition is initially intense. In contrast, they will both pursue the process innovation in an environment of low competition. For intermediate levels of competition the firms undertake different innovation types. The second case reexamined this setting for strictly positive investment costs. Conditions were established, under which the just described innovation pattern emerges even with positive investment costs. Finally, a situation was considered, in which firms choose the scale of the research projects. It has been shown that corner solutions may arise, i.e. firms do not invest into a specific type of innovation if the first unit is too costly. When investment costs for both innovations types are moderate, they will undertake process and product innovations at the same time. It has also been demonstrated that the scale of the a product innovation and the willingness to pay for the first unit are the higher, the more intense the initial degree of competition is.

All specifications presented here assumed that innovation activities are not subject to uncertainty. In particular, the completion date and the result of a research project are highly uncertain in reality. Naturally, one would wish to model the innovation activity as a stochastic process. However, the results stated in proposition 1 and 2 depend more on the assumption that the investment costs are identical for both innovation types. Probably, equivalent results can be obtained as long as the stochastic process describing the innovation processes is the same for both types.

Apart from incorporating uncertainty into the analysis, the model can be extended in different respects. One would e.g. wish to study the effect of the market size on the innovation pattern. This question was subject of the Rosenkranz (1996) contribution for vertical product differentiation. Furthermore, one may be interested in the influence of the preference parameters on the innovation activities.

Finally, one may wish to consider the present model in a market of vertical product differentiation. Cremer and Thisse (1991) demonstrate that models of horizontal differentiation satisfying certain conditions can be regarded as special cases of the Mussa and Rosen (1978) type model of vertical product differentiation. As the model presented here satisfies those conditions, the derived results will carry over to the Mussa and Rosen (1978) type models.

References

Arrow, K. (1962): “Economic welfare and the allocation of resources for inventions”. In: R. Nelson (ed.), “The Rate and Direction on Inventive Activity”, Princeton University Press, Princeton, pp. 609–626.

Bester, H. and E. Petrakis (1993): “The Incentives for Cost Reduction in a Differentiated Industry”. *International Journal of Industrial Organization*, vol. 11, pp. 519–534.

Bonanno, G. and B. Haworth (1998): “Intensity of competition and the choice between product and process innovation". *International Journal of Industrial Organization*, vol. 16, pp. 495–510.

Boone, J. (2001): "Intensity of Competition and the Incentive to Innovate". *International Journal of Industrial Organization*, vol. 19, pp. 705–726.

Cremer, H. and J.-F. Thisse (1991): "Location models of horizontal differentiation: A special case of vertical differentiation models". *The Journal of Industrial Economics*, vol. 39, pp. 383–390.

Greenstein, S. and G. Ramey (1998): "Market structure, innovtion and verticl product differentiation". *International Journal of Industrial Organization*, vol. 16, pp. 285–311.

Hotelling, H. (1929): "Stability in Competition". *Economic Journal*, vol. 39, pp. 41–57.

Mussa, M. and S. Rosen (1978): "Monopoly and Product Quality". *Journal of Economic Theory*, vol. 18, pp. 301–317.

Peitz, M. (1999): "A Difficulty with the Address Models of Product Differentiation". *Economic Theory*, vol. 14, pp. 717–727.

Reinganum, J. F. (1992): *The Timing of Innovations: Research, Development, and Diffusion*, vol. 1, chap. 14. North–Holland, Amsterdam et al., pp. 849–908.

Rosenkranz, S. (1995): "Innovation and Cooperation under Vertical Product Differentiation". *International Journal of Industrial Organization*, vol. 13, pp. 1–22.

Rosenkranz, S. (1996): "Simultaneous choice of process and product innovation". CEPR Working Paper No. 1321. (forthcoming in Journal of Economic Behavior and Organization 2002).

Schumpeter, J. A. (1942): *Capitalism, Socialism and Democracy*. Harper & Row, New York.

Shaked, A. and J. Sutton (1990): "Multiproduct Firms and Market Structure". *RAND Journal of Economics*, vol. 21, pp. 45–62.