

Is it possible to construct derivatives for the Paris residential market?

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Abstract

In this paper we address the issue of the robustness of the price level, mean, and variance estimates for two sets of repeat sales real estate price indices: the classical WRS method and a PCA factorial method, as elaborated in Baroni, Barthélémy and Mokrane (2007). Our work can be seen as an extension of Clapham, Englund, Quigley and Redfearn (2006), with the aim of helping to judge of the efficiency of such indices in designing real estate derivatives contracts. We use an extensive repeat sales database for the Paris (France) residential market. We describe the dataset used and compute the parameters (drift and volatility) of the indices produced over the period 1982-2005. The aim here is to test the sensitivity of these two indices to revision due to additional repeat-sales transactions information. Our analysis is conducted on the global Paris market and on sub-markets.

Our main conclusion is that the revision problem may cause serious concern for the stability of key parameters that are used as inputs in the pricing of derivatives contracts. The impact of index revision is important on the estimate of the index price level. This result is consistent with the finding of the existing literature for the US and Swedish markets. We also find that although the revision impact on the trend estimate can be important, the WRS method seems more robust and derivatives contracts such as swaps may be based on such indices. Finally, and this is probably the most promising result, revision influence on volatility estimates seems to be less stringent, and according to the robustness of the volatility estimate, the BBM factorial index seems to fare relatively better than the WRS index. Hence, we find that the factorial index could better sustain volatility based derivatives such as call or put options.

Introduction

Reestimation of the real estate indices is necessary when new information can change their level in the past. It can occur when the methods used to incorporate data changes or when new data have to be integrated in the past series. This last issue is particularly important for repeat sales indices. New sales on properties can let introduce a new data at the purchase period which was already considered in the index. This form of revision may have substantial consequences for investment and performance valuation, but also in real estate derivatives valuation.

Clapham et al. (2006) in a recent article have studied the impact on indices of including additional sales in the set of observations used in their construction. To measure this impact, they consider the index price for four indices: the Fisher Ideal Index derived from a series of cross-sectional hedonic regressions, a “naïve” repeat sales index, a more sophisticated repeat sales index which includes changes in the hedonic characteristics of properties, and a chained Fisher index constructed from a series of geometric averages of Paasche and Laspeyres indices. They observe that repeat sales indices are particularly exposed to the revision issue. Their study relies upon Swedish data on all sales of owner-occupied single-family dwellings during a 19-year period, 1981-1999. The detailed physical description of each dwelling allows verifying there are no changes in quality between sales. They use this dataset to measure the magnitudes of revision, defining revision as a change in estimated price levels that results from the re-measurement that occurs when new information is introduced in the form of additional sales. For the indices based on repeat sales, revision results from the addition of new paired-sales to the sample. Their conclusions are that the revision in the price level estimates is two to six times greater for the repeat-sales indices relative to the Fisher Ideal Index. Moreover the revision for the repeat-sales indices is asymmetric and downward revision is more prevalent than upward. They notice that most of the revision occurs in the first 10 quarterly estimates and price estimates become more stable thereafter. As derivatives products for home equity insurance and aggregate price futures would be largely impacted by revision if they

were based on repeat-sales indices, they conclude that the development of futures markets in housing prices would be better served by hedonic-based indices.

In this paper our purpose is to show that the revision issue affects differently two types of repeat-sales indices: a classical WRS index and a PCA repeat-sales index. Using a Paris housing dataset, we measure the impact of revision on prices, returns and volatility of these two indices. The analysis is driven on the global Paris market and its sub-markets.

Firstly, we consider how these two indices are built. The Paris dataset used for the empirical study is then described. Finally we present different measures of revision for these indices.

1 The Repeat-sales indices

The repeat-sales indices are computed from real transactions of properties for which the price and the date of initial acquisition and resale are available. Their main advantage consists in taking into account the unicity of each property even if specific characteristics are unavailable. We present below two different methodologies. The first one corresponds to the Case & Shiller classical methodology (WRS). The second one is a factorial method (BBM) using a Principal Component Analysis and is described in Baroni, Barthelemy and Mokrane (2007).

1.1 The WRS index

The method begins by stating that the price of say good i at date t is a function of four terms: the good's quality at date t , the value of the underlying global real estate index at date t , a random walk variable linked to good i at date t and an error term, here again linked to good i at date t (modeled as a white noise). Case and Shiller (1987) generalize the work of Bailey, Muth and Nourse (1963) and thus provide the first approach of repeat measures methods for construction of real estate indices. The main merit of this model based on repeat sales, is that it does not presuppose any mechanical form for the behavior of the underlying real estate index. Since 1987, the model has attracted a lot of attention and has given rise to a number of improvements or critics. See Baroni et al. (2005) for a

presentation of these improvements suggested in the literature along four issues (constant quality assumption, selection bias, revision issue, heteroscedasticity).

1.1.1 WRS framework

The Weighted Repeat Sales (WRS)¹ of Case and Shiller (1987) starts by introducing the intrinsic price of good i ($i = 1, \dots, n$), p_{it} at date t . By defining $P_{it} = \ln(p_{it})$ as the natural logarithm of the good's price, at t , and $I_t = \ln(i)$ as the property index at t , the model states the following:

$$P_{it} = I_t + H_{it} + N_{it} \quad (1)$$

where H_{it} is a Gaussian random walk that represents asset i 's own trend. By construction,

$$E[H_{it} - H_{i\tau}] = 0$$

$$E[H_{it} - H_{i\tau}]^2 = (t - \tau)\sigma_H^2$$

What's more, H_{it} is non correlated with I_t , for all i and t , N_{it} is a white noise, and represents the property market's imperfections. By assumption, $E[N_{it}] = 0$ and $E[N_{it}]^2 = \sigma_N^2$. What's more, N_{it} is uncorrelated with either I_t , or H_{jt} , for all j and all t , except when $i = j$ et $t = t'$.

The sale price of asset i , V_{it} is defined as the sum of the asset plus its quality. The difference in value for asset i between date t and τ , when one assumes that the asset's quality is unchanged², is written as the sum of three differences: for the index, for the trend of house i (random walk), and the property market's imperfections (white noise)³. So, the difference in value for asset i between date t and τ is the difference of the log of index plus $H_{it} - H_{i\tau} + N_{it} - N_{i\tau}$ which represents the idiosyncratic terms. How can then be estimated the real estate index using a repeat sales dataset?

¹ The principles of this method date back to Bailey, Muth and Nourse (1963).

² See Case and Shiller (1987).

³ $V_{it} - V_{i\tau} = P_{it} - P_{i\tau} = I_{it} - I_{i\tau} + H_{it} - H_{i\tau} + N_{it} - N_{i\tau}$.

1.1.2 Econometric Modeling

The change in the asset value can be written as following:

$$V_{it} - V_{i\tau} = [(1 \times I_t) + (-1 \times I_\tau)] + H_{it} - H_{i\tau} + N_{it} - N_{i\tau} \quad (2)$$

We should then notice that the dates t and τ are given in a theoretical point of view. In practice, it depends on the minimum observation time period in the dataset which will define the minimum slice. Then, how can the time be taken into account?

Time Intervals

The overall period of analysis may be sliced into S subperiods. The two dates, τ and t are observed in one of those subperiods:

| | | | | | | | | | | | |
|---------|---|---|--------|---|---|-----|-----|-------|-----|-------|-----|
| Periods | 1 | 2 | 3 | 4 | 5 | ... | s | $s+1$ | ... | $S-1$ | S |
| | | | | | | | | | | | |
| Time | 1 | | τ | | | | t | | | | T |

One refers to date (τ) for the acquisition date and to (t) as the resell date. By aggregating the observations (a buy or sell transaction) by subperiod, one may construct a discontinuous series. The interval $[\underline{t}_s, \bar{t}_s[$ represents the s^{th} subperiod. The discontinuity depends on the time length $\bar{t}_s - \underline{t}_s$ (what's more the quality of the resulting index will in fact depend on the number of observations n_s for each sub period s). Hence, when using S subperiods for every transaction i , the relationship given in (2) may be approximated by⁴:

$$V_{it} - V_{i\tau} = \sum_{s=1}^S \Phi_s D_{is} + H_{it} - H_{i\tau} + N_{it} - N_{i\tau} \quad (3)$$

where, for transaction i , the dummy D_{is} is -1 if the first sale date belongs to the s^{th} period, +1 if the resell date belongs to the s^{th} period and 0 in the other cases. Moreover Φ_s is the parameter to be estimated.

⁴ Note that by construction we will include a transaction in our analysis only if the sale and resale dates do not belong to the same subperiod.

Model Details

Denote by ε_i the error term associated to asset i . To estimate the price index, the following equation is used:

$$i \in \{1, \dots, n\}, \quad V_{it} - V_{ir} = \sum_{s=2}^S \Phi_s D_{is} + \varepsilon_i \quad (4)$$

Each value of the log price index is represented by a regression coefficient, Φ_s for the s^{th} period, except for the first value which is set to 0 as a normalisation. What's more the model is without a constant. Hence, for all $s = 2, \dots, S$, the value $\hat{\Phi}_s$ will be an estimator for I_s , i.e. an estimator of the logarithm of the period s price index.

With R the vector of log price returns, Φ the $S-1$ vector of parameter and D the matrix of the $S-1$ dummy variables, the model is thus simply:

$$R = D\Phi + \varepsilon, \text{ where } \varepsilon \sim N(\mathbf{0}, \Sigma) \quad (5)$$

The form of the variance-covariance matrix being quite particular, only the diagonal is non null.

Index Construction

The estimate of the index will depend on the way the time period is subdivided in sub periods⁵. We typically obtain period values for the WRS index (month, semester ...) depending on the value chosen for S ⁶. Whatever $s = 2, \dots, S$, $\hat{\Phi}_s$ will be the estimator for the log of the index I_s . We have then:

$$\widehat{\ln(p_{it} / p_{ir})} = \hat{\Phi}_s, \text{ if } t(i) \in [\underline{t}_s, \bar{t}_s[, \tau(i) \in [\underline{t}_1, \bar{t}_1[\quad (6)$$

⁵ Transactions i take place over T units of time (weeks, months, quarters...). It is therefore equivalent to either specify the number of periods S or the time length of the period in units (the smallest period being the one contained in the original transactions data).

⁶ To the comments made in *Footnote 5*, one may add that the index also depends on the nature of the returns initially observed or used in the estimate. This has to do with the way one constructs vector R in the model. Based on monthly transactions, one may construct returns for higher periods of time.

with $\tau(i)$ and $t(i)$ the acquisition and resell dates of asset i . This in turn gives the result for the index⁷:

$$\left\{ \begin{array}{l} \hat{i}_s = 100 e^{\Phi_s}, \forall s = 2, \dots, S \\ \text{with the initial reference being } i_1 = 100 \text{ (} 100 \times e^{\Phi_1}, \text{ with } \Phi_1 \text{ set to zero)} \end{array} \right.$$

1.2 The PCA factorial index

This index is a repeat sales index constructed from economic and financial variables (see Baroni, Barthél my, Mokrane, 2007). Real estate returns are computed from the repeat sales transactions and are associated with their corresponding returns for the economic and financial variables. Hence for each observation composed of two transactions of a same asset, a returns vector is elaborated. Then, each real estate returns is explained by the other returns, using a linear regression. The explicative variables of the most parsimonious model are the systematic factors. Finally, the index is constructed from the factors time series, which are determined by a Principal Component Analysis (PCA) as a combination of variables.

There are two steps in the procedure which are not situated in the same space: the first one in the transaction dimension and the second one in the time dimension.

1.2.1 The Transaction dimension: determination of the factors

The real estate price returns

On n repeat transactions, each observation is denoted i , the first transaction date $T_1(i)$, the purchase price $P_1(i)$, the second transaction date $T_2(i)$ and the resale price $P_2(i)$. From the prices one can deduce the real estate price return related to the observation i :

$$R_{re}(i) = \frac{P_2(i)}{P_1(i)} \quad (7)$$

To be able to compare these returns, let us fix a reference period p whose value is expressed in days. $R_{re}^p(i)$ the p period price return rate is defined as:

$$R_{re}^p(i) = (R_{re}(i))^{\frac{p}{T_2(i) - T_1(i)}} \quad (8)$$

The p price return in logarithm is denoted $LnR_{re}^p(i)$. It can be expressed as p times the price return in logarithm for one day.

⁷ Recall $I_t = \ln(i_t)$.

$$LnR_{re}^p(i) = \ln \left[R_{re}^p(i) \right] = \ln \left[\left(R_{re}(i) \right)^{\frac{p}{T_2(i) - T_1(i)}} \right] = p \times \ln \left[\left(R_{re}(i) \right)^{\frac{1}{T_2(i) - T_1(i)}} \right] = p \times LnR_{re}^1(i) \quad (9)$$

The corresponding price returns

For the economic and financial variables, for each observation, the price return is computed on the same time period as the one used for the real estate price return. The information is extracted from the time series of those variables.

For all $j = 1, \dots, k$ and for all $t = 1, \dots, T$, let us denote $X_j(t)$, the value of the j^{th} variable at time t . For each transaction i , the corresponding price return for all the k variables can be computed for the period that covers $T_1(i)$ to $T_2(i)$. For these two periods, the variables values are denoted respectively $X_j[T_1(i)]$ and $X_j[T_2(i)]$ and the corresponding price return for the variable j is:

$$R_j(i) = \frac{X_j[T_2(i)]}{X_j[T_1(i)]} \quad (10)$$

To compare all those returns, $R_j^p(i)$ is defined as the corresponding price return for variable j calculated for the period related to transaction i :

$$R_j^p(i) = \left(R_j(i) \right)^{\frac{p}{T_2(i) - T_1(i)}} = \left(\frac{X_j[T_2(i)]}{X_j[T_1(i)]} \right)^{\frac{p}{T_2(i) - T_1(i)}} \quad (11)$$

The corresponding p price return in logarithm is denoted $LnR_j^p(i)$. As mentioned for the real estate price returns, it can be expressed as p times the corresponding price return in logarithm for one day:

$$LnR_j^p(i) = p \times LnR_j^1(i) \quad (12)$$

The factor construction

The relationship between the real estate period price returns $R_{re}^p(i)$ and those of the explanatory variables is:

$$R_{re}^p(i) = b \prod_{j=1}^k \left(R_j^p(i) \right)^{\gamma_j} \quad (13)$$

where k is the number of variables in the structural relation. Thus, for the price return in logarithm, the following linear relation is established

$$LnR_{re}^p(i) = \delta + \sum_{j=1}^k \gamma_j LnR_j^p(i) \quad (14)$$

where

- $LnR_{re}^p(i)$ is the logarithm of the period p real estate price return for transaction i ,

- $LnR_j^p(i)$ are for each variable j the logarithm of the corresponding period price return,
- $\delta = \ln(b)$.

As the k variables may be collinearly linked the factorial base is changed by using a PCA on the k variables. k linearly independent variables are then obtained. For each transaction i , we have:

$$\forall \alpha = 1, \dots, k, \quad LnFR_\alpha^p(i) = \sum_{j=1}^k u_{\alpha j} LnR_j^p(i) \quad (15)$$

where

- $LnFR_\alpha^p(i)$ is, for transaction i , the period p equivalent price return for factor α ,
- $u_{\alpha j}$ is the weight of the variable j in the factor α . $u_{\alpha j}$ is normalized and $\forall \alpha \neq \beta$, $u_\alpha \perp u_\beta$.

The relationship between the real estate returns and the equivalent factors returns are :

$$LnR_{re}^p(i) = \delta + \sum_{\alpha=1}^k \beta_\alpha LnFR_\alpha^p(i) \quad (16)$$

By adding an error term, the regression model is the following:

$$\forall i = 1, \dots, n, \quad LnR_{re}^p(i) = \delta + \sum_{\alpha=1}^k \beta_\alpha LnFR_\alpha^p(i) + \varepsilon(i) \quad (17)$$

where $\forall i = 1, \dots, n, E[\varepsilon(i)] = 0, V[\varepsilon(i)] = \sigma_i^2$. By using (15) the previous regression model becomes:

$$\forall i = 1, \dots, n, \quad LnR_{re}^p(i) = \delta + \sum_{\alpha=1}^k \beta_\alpha \left(\sum_{j=1}^k u_{\alpha j} LnR_j^p(i) \right) + \varepsilon(i) \quad (18)$$

And for transaction i , the following estimated price return is:

$$\forall i = 1, \dots, n, \quad \widehat{R}_{re}^p(i) = \exp \left[\widehat{\delta} + \sum_{\alpha=1}^k \widehat{\beta}_\alpha \ln \left(\frac{F_\alpha[T_2(i)]}{F_\alpha[T_1(i)]} \right) \right] = \exp \left[\widehat{\delta} + \sum_{\alpha=1}^k \widehat{\beta}_\alpha LnFR_\alpha^p(i) \right] \quad (19)$$

1.2.2 The time series dimension: the index

In the time series dimension, the k factor indices $F_\alpha(t)$ can be established from the series of returns $FR_\alpha(t)$ and $\forall t = 2, \dots, T$,

$$\forall \alpha = 1, \dots, k, \quad \ln(FR_\alpha(t)) = \ln \left(\frac{F_\alpha(t)}{F_\alpha(t-1)} \right) = \sum_{j=1}^k u_{\alpha j} \ln \left(\frac{X_j(t)}{X_j(t-1)} \right) \quad (20)$$

which gives

$$\forall \alpha = 1, \dots, k, \quad FR_\alpha(t) = \prod_{j=1}^k \left(\frac{X_j(t)}{X_j(t-1)} \right)^{u_{\alpha j}} \quad (21)$$

and then

$$\forall t = 2, \dots, T, F_{\alpha}(t) = F_{\alpha}(t-1) \times FR_{\alpha}(t), \text{ with } F_{\alpha}(1) = 100 \quad (22)$$

In the time series dimension, the asset price return can be constructed by using the p time series variables $F_{\alpha}(t)$,

$$\forall t = 2, \dots, T, \widehat{LnR}_{re}(t) = \hat{\delta} + \sum_{\alpha=1}^k \hat{\beta}_{\alpha} \ln\left(\frac{F_{\alpha}(t)}{F_{\alpha}(t-1)}\right) \quad (23)$$

$$\forall t = 2, \dots, T, \widehat{R}_{re}(t) = \exp\left(\hat{\delta} + \sum_{\alpha=1}^k \hat{\beta}_{\alpha} \ln\left(\frac{F_{\alpha}(t)}{F_{\alpha}(t-1)}\right)\right) = e^{\hat{\delta}} \times \prod_{\alpha=1}^k \left(\frac{F_{\alpha}(t)}{F_{\alpha}(t-1)}\right)^{\hat{\beta}_{\alpha}} \quad (24)$$

The parameter t is expressed in the unit chosen for the index time period p which can be: the year, the semester, the quarter, etc... Finally the PCA factorial repeat sales index is generated using the following equation:

$$\forall t = 2, \dots, T, \widehat{\text{Index}}(t) = \widehat{\text{Index}}(t-1) \times \widehat{R}_{re}(t), \text{ with } \text{Index}(1) = 100 \quad (25)$$

1.3 Estimation of the indices' trend and volatility

This estimation method is based on the results provided by the Law of Large Numbers (LLN): the mean of random variables⁸ converges to the expectation. Then the mean of a great number of random variables realizations is calculated. Before doing this estimation, the random variable on one single sample (for us, one path for the dynamic of the portfolio stochastic process) must be defined.

A single path

Firstly, let us consider the equation (26). This equation implies that, if we consider date t belonging to the investment period $[0, T]$, the time t distribution for price P_t is a log-normal distribution⁹:

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_t \text{ with } W_t \longrightarrow N(0, t) \quad (26)$$

The Brownian component W_t is defined by:

⁸ Different hypothesis are needed for this result and depending on the kind of LLN the convergence differs.

⁹ See Hull for a description of asset modelling.

$$W_t - W_{t-1} = \xi_t \text{ with } \xi_t \longrightarrow N(0,1) \quad (27)$$

To estimate the mean and variance, the return is estimated for each t as:

$$\hat{R}_t = \frac{\hat{P}_t}{\hat{P}_{t-1}}, \quad t = 1, \dots, T \quad (28)$$

and the logarithm of the return as:

$$\widehat{\ln R}_t = \ln(\hat{R}_t) = \ln\left(\frac{\hat{P}_t}{\hat{P}_{t-1}}\right), \quad t = 1, \dots, T \quad (29)$$

Then the standard following formulas are used to estimate the trend and the volatility of the process based on the estimated price index. The expected mean of logs is:

$$\hat{m} = \frac{1}{N} \sum_{t=1}^N \ln\left(\frac{\hat{P}_t}{\hat{P}_{t-1}}\right) \quad (30)$$

The return volatility is:

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{t=1}^N \left[\ln\left(\frac{\hat{P}_t}{\hat{P}_{t-1}}\right) - \hat{m} \right]^2 \quad (31)$$

The Brownian's trend is then estimated as follows:

$$\hat{\mu} = \hat{m} + \frac{1}{2} \hat{\sigma}^2 \quad (32)$$

In order to evaluate with precision these three estimators $\hat{\mu}$, $\hat{\sigma}$ and \hat{P}_T , bootstrap simulation methods can be used (the analytical form of the variances of these estimators is not trivial). It consists in considering the empirical distribution of the estimator built with different estimations based on pseudo-random samples which replace the observed sample¹⁰.

With K pseudo-random samples, K estimations can be computed.

¹⁰ see for instance Spanos (1999) p. 597-600.

- for the price at time T : $(\hat{P}_T^{(1)}, \dots, \hat{P}_T^{(k)}, \dots, \hat{P}_T^{(K)})$
- for the trend: $(\hat{\mu}^{(1)}, \dots, \hat{\mu}^{(k)}, \dots, \hat{\mu}^{(K)})$
- for the standard deviation: $(\hat{\sigma}^{(1)}, \dots, \hat{\sigma}^{(k)}, \dots, \hat{\sigma}^{(K)})$

The empirical distribution is an estimation of the true distribution of the estimator and the moments (as the variance for instance) can be computed from this distribution.

2 The database

2.1 Description

To compare empirically the impact of revision on these two repeat sales indices, we have constructed a database for Paris with two sources: the real estate price returns for each transaction and the corresponding computed returns from economic and financial variables.

The real estate price returns are extracted from the CD-Bien database which lists all real estate transactions written in front of a notary for Paris. From this database, we extracted 138 861 transactions for which we had the information on both the initial price and date (posted 1st of January 1982) at which the properties had been bought (date 1, T_1) as well as the price and date (date 2, T_2) for the following resale¹¹. To every observation in the database, we associate the holding period (*duration*) which corresponds to the difference $T_2 - T_1$ expressed in days. These repeat measures transactions represent around 25% of the total number of transactions.

The economic and financial returns needed for the PCA factorial index are calculated from a series of indicators that one a priori believes to have some form of explanatory power of price changes. Nine variables were selected based on two criteria. The first one required that potential factors have a clear economic interpretation and presupposed links with real estate markets. The second was the

¹¹ We have to note important features concerning the database's structure: we only observe those transactions whose second transaction has taken place after 1993. Moreover, the percentage of transactions registered in the database is increasing over time. These two characteristics imply that the number of observations doubles between 2000 and 2005.

availability of the data on the period running from 1st of January 1982. The indices selected to serve as factors were thus constructed with base 100 at the start of 1982. They are the following:

1. *Savings as a percentage of disposable income*: Datastream France gross household saving index
2. *Consumer price index*: the INSEE (the French National Statistical office) consumer price index
3. *Rents as measured by the OLAP residential reletting index*¹²
4. *Long-term interest rate*: Datastream France zero coupon ten-year bond rate
5. *Short-term interest rate*¹³: Datastream France zero coupon one-year bond rate
6. *Demographic index*: the INSEE regional population index for Ile-de-France
7. *Listed real estate*: Datastream index dedicated to the stocks of the largest listed real estate companies in France
8. *Unemployment*: the INSEE index for France (in rate)
9. *MSCI equity market index*: the MSCI index for stocks listed in France.

The whole set of rates was then transformed into periodical returns.

Figure 1 represents the distribution of transaction dates for repeat sales in the CD-Bien database (*date1*, *date2*, and the total number of transactions). As can be seen, starting in January 1982, the database does contain an increasing number of acquisitions for which we do have a resell price and date. However, it does not contain any resell date prior to June 1994. This particular feature of the CD-Bien database may potentially induce a form of bias in the sense that the proportion of long holding period transactions is over-represented in the data.

¹²

The OLAP (OLAP stands for Observatoire des Loyers de l'Agglomération Parisienne) residential reletting index is based on a large sample of apartments that are regularly surveyed for which new lettings are systematically documented in order to produce the Paris and close suburban areas rent index.

¹³

For long-term and short-term interest rates, the series are calculated by applying the rates to a basis of 100 as of January 1982. The applied rate corresponds to the average of the day-to-day rates for the ten-year or one-year bonds and for a given period (month).

2.2 Estimation of the indices and their parameters

2.2.1 For the whole Paris market

Figure 2 illustrates the evolution of the above-mentioned indices since 1982 for Paris residential. As it is noticed in Baroni, Barthélémy, Mokrane (2007), the drawings of these two indices are very similar.

Table 1 shows that μ estimates are similar for the two indices, whatever the periodicity. We notice that μ is increasing from 2000 to 2002 and it is due to the strong increase of the prices during these two years (see *Figure 2*). The volatility estimates are also similar for the two indices when the periodicity is annual, but they are more and more different when the periodicity is increasing. Moreover the volatility decreases when the periodicity is increasing, and it is more noticeable for the BBM index.

| | | From 1981 to | | | | | |
|------------------|------------|---------------|------------------|----------------|------------------|----------------|------------------|
| | | End of 2000 | | End of 2001 | | End of 2002 | |
| | | <u>Mu</u> | <u>Std. Dev.</u> | <u>Average</u> | <u>Std. Dev.</u> | <u>Average</u> | <u>Std. Dev.</u> |
| Annual | WRS | 0.0550 | 0.0757 | 0.0564 | 0.0754 | 0.0588 | 0.0751 |
| | BBM | 0.0579 | 0.0751 | 0.0562 | 0.0724 | 0.0615 | 0.0755 |
| Half-year | WRS | 0.0548 | 0.0595 | 0.0563 | 0.0593 | 0.0590 | 0.0587 |
| | BBM | 0.0557 | 0.0558 | 0.0540 | 0.0541 | 0.0591 | 0.0571 |
| Quarter | WRS | 0.0530 | 0.0542 | 0.0546 | 0.0534 | 0.0578 | 0.0531 |
| | BBM | 0.0580 | 0.0445 | 0.0535 | 0.0426 | 0.0553 | 0.0459 |
| Every two months | WRS | 0.0538 | 0.0614 | 0.0553 | 0.0603 | 0.0577 | 0.0577 |
| | BBM | 0.0581 | 0.0387 | 0.0555 | 0.0413 | 0.0531 | 0.0413 |

Table 1: Trend and volatility estimates

Table 2 reports the variance of the estimators as well as the part of the error with a 95% confident interval for an annual estimation (1981-2000). For instance, for the volatility estimation of the WRS index, the standard error of the estimator is estimated to 0.00092. Relatively to the standard estimation of 0.0758, twice this value gives a relative error of 2.42%. For this sample, the BBM index estimators seem more precise than the WRS index' ones.

| Estimator | WRS | | | BBM | | |
|----------------|------------------------|------------------------|---|------------------------|------------------------|---|
| | Mean of the estimators | S.E. of the estimators | Relative error according to a 95% CI (in %) | Mean of the estimators | S.E. of the estimators | Relative error according to a 95% CI (in %) |
| $\hat{\mu}$ | 0.0550 | 0.00052 | 1.89 | 0.0579 | 0.0004 | 1.34 |
| $\hat{\sigma}$ | 0.0758 | 0.00092 | 2.42 | 0.0752 | 0.0007 | 1.92 |
| \hat{P}_T | 269.38 | 2.7098 | 2.01 | 284.62 | 2.0623 | 1.45 |

Table 2: Estimators' variance

2.2.2 For sub-markets

Paris is divided in 20 different districts called “arrondissements”¹⁴. In the central districts (1-9th “arrondissement”) the blocks of the apartments are heterogeneous, in contrast with the outlying districts (10-20th “arrondissement”). Firstly, we have estimated the WRS and BBM indices for the two groups. The estimators of the trend and the volatility for these sub-indices are similar as for the whole market. Due to the heterogeneity, it is noticeable that the central districts offer a higher return with more volatility than the outlying districts. Secondly we have determined the same estimators for each “arrondissement” (see *Table 3a* and *3b*).

¹⁴

A map of Paris and a short description of each arrondissement is available at: http://www.intransit-international.com/housing_paris_arrondissement_tour.html

| | From 1981 to end of 2002 | | | |
|--------------------|--------------------------|------------------|----------------|------------------|
| | WRS | | BBM | |
| | <u>Average</u> | <u>Std. Dev.</u> | <u>Average</u> | <u>Std. Dev.</u> |
| Paris | 0.0588 | 0.0751 | 0.0615 | 0.0755 |
| Central districts | 0.0640 | 0.0785 | 0.0680 | 0.0774 |
| Outlying Districts | 0.0576 | 0.0752 | 0.0600 | 0.0753 |
| Arr. 1 | 0,0643 | 0,1068 | 0,0633 | 0,0848 |
| 2 | 0,0670 | 0,0860 | 0,0698 | 0,0764 |
| 3 | 0,0680 | 0,0847 | 0,0700 | 0,0817 |
| 4 | 0,0676 | 0,0933 | 0,0701 | 0,0852 |
| 5 | 0,0635 | 0,0847 | 0,0645 | 0,0658 |
| 6 | 0,0721 | 0,0895 | 0,0753 | 0,0854 |
| 7 | 0,0634 | 0,0967 | 0,0685 | 0,0846 |
| 8 | 0,0558 | 0,1114 | 0,0631 | 0,0960 |
| 9 | 0,0625 | 0,0908 | 0,0665 | 0,0751 |
| 10 | 0,0629 | 0,0772 | 0,0628 | 0,0769 |
| 11 | 0,0602 | 0,0837 | 0,0646 | 0,0754 |
| 12 | 0,0592 | 0,0795 | 0,0614 | 0,0790 |
| 13 | 0,0567 | 0,0766 | 0,0561 | 0,0693 |
| 14 | 0,0627 | 0,0763 | 0,0636 | 0,0718 |
| 15 | 0,0563 | 0,0754 | 0,0584 | 0,0732 |
| 16 | 0,0518 | 0,0828 | 0,0596 | 0,0873 |
| 17 | 0,0607 | 0,0848 | 0,0611 | 0,0764 |
| 18 | 0,0569 | 0,0830 | 0,0602 | 0,0814 |
| 19 | 0,0568 | 0,0809 | 0,0589 | 0,0801 |
| 20 | 0,0574 | 0,0849 | 0,0557 | 0,0730 |

Table 3a: Annual trend and volatility estimates

| | From 1981 to end of 2002 | | | |
|--------------------|--------------------------|------------------|----------------|------------------|
| | WRS | | BBM | |
| | <u>Average</u> | <u>Std. Dev.</u> | <u>Average</u> | <u>Std. Dev.</u> |
| Paris | 0.0578 | 0.0531 | 0.0553 | 0.0459 |
| Central districts | 0.0630 | 0.0754 | 0.0636 | 0.0489 |
| Outlying Districts | 0.0570 | 0.0548 | 0.0566 | 0.0440 |
| Arr. 1 | 0,0865 | 0,2608 | 0,0592 | 0,0692 |
| 2 | 0,1407 | 0,3678 | 0,0683 | 0,0456 |
| 3 | 0,0894 | 0,2369 | 0,0651 | 0,0561 |
| 4 | 0,0911 | 0,2381 | 0,0652 | 0,0513 |
| 5 | 0,0853 | 0,1913 | 0,0608 | 0,0416 |
| 6 | 0,0897 | 0,1731 | 0,0712 | 0,0495 |
| 7 | 0,0741 | 0,2047 | 0,0615 | 0,0561 |
| 8 | 0,0819 | 0,2685 | 0,0550 | 0,0657 |
| 9 | 0,0743 | 0,1871 | 0,0631 | 0,0467 |
| 10 | 0,0723 | 0,1323 | 0,0630 | 0,0444 |
| 11 | 0,0645 | 0,1261 | 0,0596 | 0,0462 |
| 12 | 0,0628 | 0,1187 | 0,0574 | 0,0467 |
| 13 | 0,0624 | 0,1273 | 0,0527 | 0,0418 |
| 14 | 0,0732 | 0,1337 | 0,0603 | 0,0470 |
| 15 | 0,0554 | 0,0885 | 0,0550 | 0,0450 |
| 16 | 0,0572 | 0,1446 | 0,0525 | 0,0551 |
| 17 | 0,0636 | 0,0928 | 0,0585 | 0,0453 |
| 18 | 0,0594 | 0,1033 | 0,0560 | 0,0475 |
| 19 | 0,0655 | 0,1298 | 0,0576 | 0,0469 |
| 20 | 0,0666 | 0,1047 | 0,0542 | 0,0423 |

Table 3b: Annual trend and volatility estimates computed from quarterly indices

As expected, the parameters by district are various and depend on the typology of the apartments in each district. For annual estimates, the average trend for each district seems to be correctly estimated compared with the average for the whole Paris. On the contrary, for the volatility, only the estimators by districts for BBM seem relevant with those of the whole Paris. For quarterly estimates, as the number of observations per period is decreasing (see *Table 3b*), the estimator of the volatility is biased and over-estimated. Consequently, the trend is over-estimated as well and the estimators for the WRS index by districts become very different from those of the whole Paris. At the opposite and due to the index methodology, the estimators for the BBM are not biased and look much more stable.

Table 4 gives the relative error of the estimators as determined in *Table 2* for the central districts, the outlying districts and then for each “arrondissement”.

| | N | WRS | | | BBM | | |
|--------------------|-------|-------------|----------------|-------------|-------------|----------------|-------------|
| | | $\hat{\mu}$ | $\hat{\sigma}$ | \hat{P}_T | $\hat{\mu}$ | $\hat{\sigma}$ | \hat{P}_T |
| Paris | 73142 | 1.89 | 2.42 | 2.01 | 1.34 | 1.92 | 1.45 |
| Central districts | 14038 | 3.73 | 7.81 | 4.22 | 2.99 | 4.75 | 3.57 |
| Outlying districts | 59104 | 2.10 | 2.64 | 2.20 | 1.53 | 2.16 | 1.60 |
| Arr. 1 | 664 | 16.20 | 29.35 | 16.34 | 14.15 | 24.83 | 14.97 |
| 2 | 1 069 | 18.62 | 31.48 | 19.50 | 10.87 | 16.46 | 12.38 |
| 3 | 1 788 | 10.05 | 22.15 | 12.61 | 9.14 | 15.56 | 10.93 |
| 4 | 1 238 | 13.67 | 25.35 | 17.07 | 8.87 | 15.56 | 11.00 |
| 5 | 1 969 | 8.50 | 20.57 | 9.83 | 6.57 | 14.06 | 7.37 |
| 6 | 1 461 | 9.83 | 22.25 | 13.69 | 7.44 | 11.39 | 9.98 |
| 7 | 1 828 | 10.05 | 23.60 | 12.78 | 8.09 | 13.29 | 10.01 |
| 8 | 1 145 | 17.00 | 27.22 | 21.09 | 11.78 | 18.20 | 13.55 |
| 9 | 2 876 | 8.79 | 20.41 | 10.44 | 6.34 | 9.29 | 7.20 |
| 10 | 4 137 | 8.21 | 16.01 | 8.94 | 6.36 | 8.25 | 6.94 |
| 11 | 7 037 | 5.74 | 14.00 | 6.43 | 4.40 | 6.58 | 4.83 |
| 12 | 4 035 | 6.56 | 14.87 | 7.29 | 5.24 | 8.06 | 5.69 |
| 13 | 3 565 | 9.69 | 16.81 | 8.85 | 6.29 | 8.57 | 6.24 |
| 14 | 3 630 | 9.99 | 18.18 | 9.92 | 6.34 | 9.12 | 6.89 |
| 15 | 6 957 | 5.83 | 8.48 | 5.55 | 4.23 | 7.27 | 4.08 |
| 16 | 4 934 | 6.54 | 11.49 | 6.49 | 5.27 | 7.58 | 5.34 |
| 17 | 6 622 | 7.05 | 9.41 | 7.32 | 4.27 | 6.26 | 4.61 |
| 18 | 8 968 | 6.10 | 7.12 | 6.29 | 3.96 | 5.10 | 4.10 |
| 19 | 4 187 | 8.46 | 12.21 | 8.64 | 6.11 | 6.11 | 6.04 |
| 20 | 5 032 | 7.52 | 16.22 | 7.23 | 5.82 | 6.53 | 5.99 |

Table 4a: Relative error on estimation 1982-2000

| | N | WRS | | | BBM | | |
|--------------------|--------|-------------|----------------|-------------|-------------|----------------|-------------|
| | | $\hat{\mu}$ | $\hat{\sigma}$ | \hat{P}_T | $\hat{\mu}$ | $\hat{\sigma}$ | \hat{P}_T |
| Paris | 145974 | 1.13 | 1.90 | 1.22 | 0.71 | 1.07 | 1.21 |
| Central districts | 25786 | 2.11 | 5.63 | 4.02 | 1.53 | 2.64 | 2.86 |
| Outlying districts | 118783 | 1.13 | 1.52 | 1.91 | 0.72 | 1.24 | 1.29 |
| Arr. 1 | 1 405 | 9.05 | 24.72 | 15.51 | 8.17 | 15.97 | 13.83 |
| 2 | 2 117 | 8.11 | 19.57 | 15.09 | 5.66 | 10.34 | 10.78 |
| 3 | 3 385 | 6.29 | 15.64 | 11.81 | 4.46 | 7.83 | 8.35 |
| 4 | 2 349 | 7.01 | 18.61 | 12.89 | 4.76 | 8.84 | 8.58 |
| 5 | 3 662 | 5.97 | 15.71 | 9.77 | 3.57 | 8.48 | 6.34 |
| 6 | 2 822 | 6.34 | 18.48 | 11.86 | 4.71 | 7.42 | 9.13 |
| 7 | 3 507 | 5.99 | 16.92 | 11.65 | 4.31 | 9.10 | 8.09 |
| 8 | 2 339 | 8.83 | 26.95 | 18.82 | 6.18 | 13.26 | 10.99 |
| 9 | 5 605 | 4.53 | 13.76 | 8.96 | 3.41 | 5.58 | 6.61 |
| 10 | 8 050 | 4.09 | 8.47 | 7.63 | 2.65 | 4.32 | 4.98 |
| 11 | 13 755 | 2.57 | 10.78 | 4.76 | 2.14 | 3.44 | 4.07 |
| 12 | 8 163 | 3.42 | 8.88 | 6.11 | 2.52 | 4.78 | 4.61 |
| 13 | 7 013 | 4.62 | 10.39 | 7.06 | 3.28 | 4.95 | 5.66 |
| 14 | 7 304 | 4.94 | 10.97 | 7.84 | 3.13 | 5.44 | 5.46 |
| 15 | 13 988 | 3.19 | 6.22 | 4.73 | 3.13 | 3.41 | 3.33 |
| 16 | 10 210 | 6.34 | 18.48 | 11.86 | 2.11 | 5.30 | 4.16 |
| 17 | 13 228 | 3.90 | 6.63 | 6.49 | 2.46 | 3.10 | 4.38 |
| 18 | 17 819 | 2.59 | 5.02 | 4.75 | 1.88 | 2.85 | 3.42 |
| 19 | 8 746 | 3.73 | 7.96 | 6.58 | 3.50 | 4.51 | 5.92 |
| 20 | 10 507 | 4.15 | 10.41 | 6.48 | 2.77 | 4.20 | 4.80 |

Table 4b: Relative error on estimation 1982-2005

Relative errors are always smaller for the BBM index than for the WRS index. When the period is larger, the errors are weaker. The estimators are obviously better when the number of observations is greater. We can deduce the estimators are less precise for the “arrondissements” than for the whole Paris. Moreover, it is noticeable that for a similar number of observations, the districts where the housing is more homogeneous reveal a smaller volatility.

3 The impact of revision

Let us consider now the impact of introducing new data in the index construction. The repeat-sales indices are subject to permanent revision by construction, because of the first transaction when it takes place at a date situated before the last period of the index. Our purpose consists of measuring the effect of new data on the level of index (price), but also on its trend and volatility.

To study the impact of the revision, the estimation period cannot include all the observations. The smallest considered period is 1981-2000 and the revision is established until the period 1981-2003,

which leads to around 50% more observations¹⁵. It is not possible to take smaller period because the number of new observations would be too large.

3.1 Impact of revision on the index price

Firstly, like Clapham et al. (2006), we define revision as a change in estimated price levels that result from the remeasurement of the indices when additional sales are introduced. We define an estimate of the price level in period t using information from an initial period 1 to the current period τ as $P(t,1,\tau)$, where $\tau \geq t$. Revision corresponds to the process of evolving of the estimated index from the initial estimate $P(t,1,t)$ to a current estimate $P(t,1,T)$, as data are extended beyond the initial period t . The time series evolution of the index estimates from initial to current, that is, for $\tau = t, t+1, \dots, T$ represents the revision path.

Figure 3 illustrates for the two indices this revision path. The revision is considered quarterly from 2001:1 for 12 quarters. We represent successively the index revision in absolute price level, in price level relative to previous estimate, and the cumulative change from initial to current estimates. The graphs reveal revision is generally more important for the BBM index than for the WRS index, and the WRS index converges more rapidly to a maximum cumulative revision¹⁶. These results are confirmed if the periodicity of the indices and revisions is changed (see *Table 5* to *8*). As observed by Clapp and Giaccoto (1999) and Clapham et al. (2006), downward revisions of the WRS index are more common than upward revisions in the late periods. It is not so clear in the early periods. This is not at all the case for the BBM index where downward and upward revisions may occur and where cumulative revisions are systematically upward.

¹⁵

For instance, if the dataset ends in December 2005, taking 3 years of revision implies that the estimation period stops in December 2002. So the estimation is realized for observations from 1981 to 2002. Adding one more year leads to the estimation of the 1981-2003 index. From this estimation the 1981-2002 index is extracted. The same method is then used for the other revisions.

¹⁶

Note that the BBM index takes into account all the factors computed by the PCA. We have verified that if only the first factors are used, the impact of revision is higher. In fact, the more factors are used, the weakest is the cumulative effect.

The amount of cumulative revision in average shows a large difference between the two indices (+0.07% for the WRS and -0.79% for the BBM). In itself, the revision is rather small for the WRS and at least significantly less important than observed by Clapham et al. (2006).

If we suppose now that a future contract exists and is collateralized to one of these two indices, we can say, for instance, that futures markets could tolerate a maximum of 1% revision in one quarter and a 1.5% cumulative revision to the initial estimate. *Table 7* and *8* give how often the indices violate these criteria. We can observe that the WRS index is about to respect this criteria (1 occurrence of limit exceeded in *Table 7* and no occurrences in *Table 8*). Here also the WRS seems to be much better than the BBM.

However, even the WRS index may be problematic as underlying of an insurance product or a future. Consider for example an owner that purchases an apartment today at 300 000 euros. Let us suppose that this owner sells the apartment after one year at a loss of 0.25% (750 euros) and without any move in the annual index. After revision, if the index has changed downward of 0.5% or more (*Table 8* shows this can occur with certain frequency), the loss has doubled at least to 1500 euros. Consequently, in reality, we can conclude the impact of revision will be significant for the WRS index. For the BBM index, the impact would be so large that it invalidates its use.

3.2 Impact of revision on the index trend

Now, we consider the impact of revision on the indices' trend, as computed in section 1.3. For example, such revision may impact real estate swaps the underlying index of which would be one of our indices. Such OTC swaps already exist in UK and their recent improvement could mean they are a good answer to investors' needs.

As for price levels, we represent in *Figure 4* the revision process for a quarterly periodicity from 2000:1 to 2002:4. Logically, the results are similar to those obtained for price level. Although the BBM index seems to be more sensitive to revision, it is noticeable that the impact of revision in itself is pretty weak for both indices. Even for the BBM index it does not exceed 0.2 bps. *Tables 9* to *12*

confirm the relative robustness of the two indices to revision of the trend. However, on average, the BBM index is about five times more sensitive than the WRS index if we consider the final trend estimate compared to the initial estimate (see *Table 10*). According to *Tables 11* and *12*, they show that the BBM index exceeds higher limits more frequently.

We can conclude that the revision issue would not have to prevent these indices to be underlying of swaps, from a technical point of view. For instance, an investor could be interested in swapping the return on a stock index like CAC40 against a return on housing repeat-sales index, in order to acquire real estate risk from a property investor. In this case, the WRS index seems to be a little more appropriate than the BBM index.

3.3 Impact of revision on the index volatility

Finally, the impact of revision on the indices' volatility is considered using the same measurement than for the price level and the trend. This path of revision is particularly important for considering options with a repeat-sales index as underlying. The recent creation of such derivatives by the Chicago Mercantile Exchange shows these products are expected by the market. However their success is linked to the representativity of underlying indices but also to their robustness to revision. If the option pricing is too much sensitive to index revision, the investors will not be able to use such products to hedge their positions

Figure 5 represents the path of revision of the two indices volatility in absolute term, in relative term compared with the previous estimate, and in cumulative term. *Tables 13* to *16* give the measurement of the impact according to different periodicity. As expected (see Ghysels et al. 1995), the volatility is sensitive to additional transactions and to the relative volume of transactions compared to those present in the dataset before the revision. If t is after 2002, the impact of revision is much smaller and this is more explicit for the BBM index than for the WRS. We can also notice in *Table 13* that the revision is lower in average for the BBM index than for the WRS, even if the standard deviation remains relatively high for the two indices.

To appreciate if the level of volatility revision is sufficiently low to construct options on such indices, let us take an example using a Black & Scholes valuation. Suppose the price level of the index is 312.6 in 2002:1 and an investor buy a 1 year-maturity call at a strike of 325. If the short-term interest rate is 3.25%, the call premium would be priced at 8.42 for a volatility of 7.32%. If the revision change on volatility is 1.5%, the impact on the premium value will be 1%. In this example, the revision seems to have a relatively weak impact on the call value. In fact, the valuation model is much more sensitive to the difference in the volatility initial estimate due to the methodology of the index. As shown in *Figure 5*, the initial estimate of volatility is significantly different for the WRS index and the BBM index. Moreover, the importance of the revision error must be compared with the error due to the estimation method (see above 2.2). Considering *Table 2*, the relative error of the annual estimate of the volatility for the WRS index is around 2.63%. It confirms the relative impact of revision on this parameter.

To be more precise on the impact of revision, the same analysis is driven on the Paris sub-markets. *Table 17* gives the results for annual revision on the three parameters above. For the prices and the returns the impact of revision is generally higher for the districts than for the whole Paris. This comment is more acute for WRS than for BBM, especially for the prices. The reason may be that the estimator's variance is greater for the sub-markets than for the whole market (see *Table 4*). According to the impact of revision on the volatility, it is often lower for the sub-markets than for the whole Paris, even if the estimators' variance is higher. The interpretation may be that the risk measured by the volatility is more stable during the time for the district than for the whole Paris.

Conclusion

We tried to determine if the impact of revision generated by addition of new data could be a serious obstacle to create derivatives on repeat-sales indices. To measure empirically this impact, we considered two different indices computed for the Paris housing market: the classical WRS index and

a PCA factorial index, both elaborated from the same dataset. We focused on the paths of revision for the price level, the trend and the volatility of these indices.

Our conclusions are consistent with those present in the literature for the US or the Swedish market according to the price level of the indices. Their sensitivity to revisions makes difficult the improvement of insurance contracts to hedge price variations. Moreover, the variance of the index level estimator is large enough to reject the use of these indices even without considering the revision issue (the range is estimated at 17.35 and relatively to an index of 269.38 this is important).

Then we considered the impact of revisions on the trend of the indices. The results are very similar to those obtained for the price level, but the “lack” of stability may not be sufficient to prevent the development of derivatives products as swaps on indices. In this perspective, the WRS index seems to offer a better stability than the PCA factorial index.

Finally, we examined to which extent revisions impact the measure of volatility of these indices. The sensitivity is real for both indices, but it seems to be less important in average for the factorial index. According to possible options on real estate index, our results suggest that this sensitivity would not be a substantial obstacle. This sensitivity is comparable to the error induced by the estimation method. So, the main obstacle lies in the confidence the investors can have in the measurement itself, independently of the revision effects. Various periodicities or different methodologies can lead to different measurements of volatility and it could be the main obstacle to construct these derivatives.

APPENDIX

1. Figures

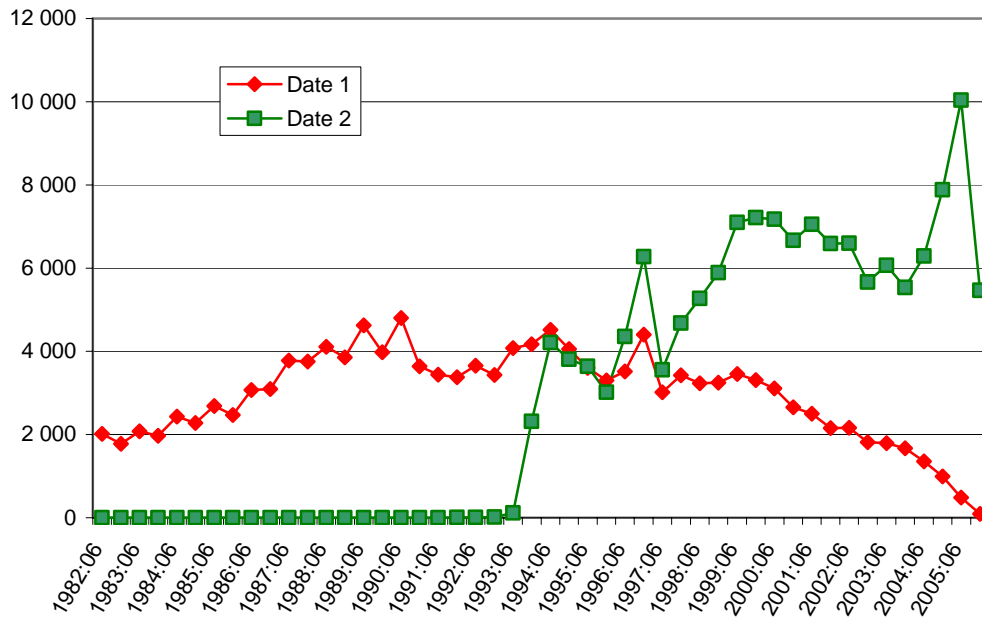


Figure 1 : Number of observations per semester - 1982:06 to 2005:12

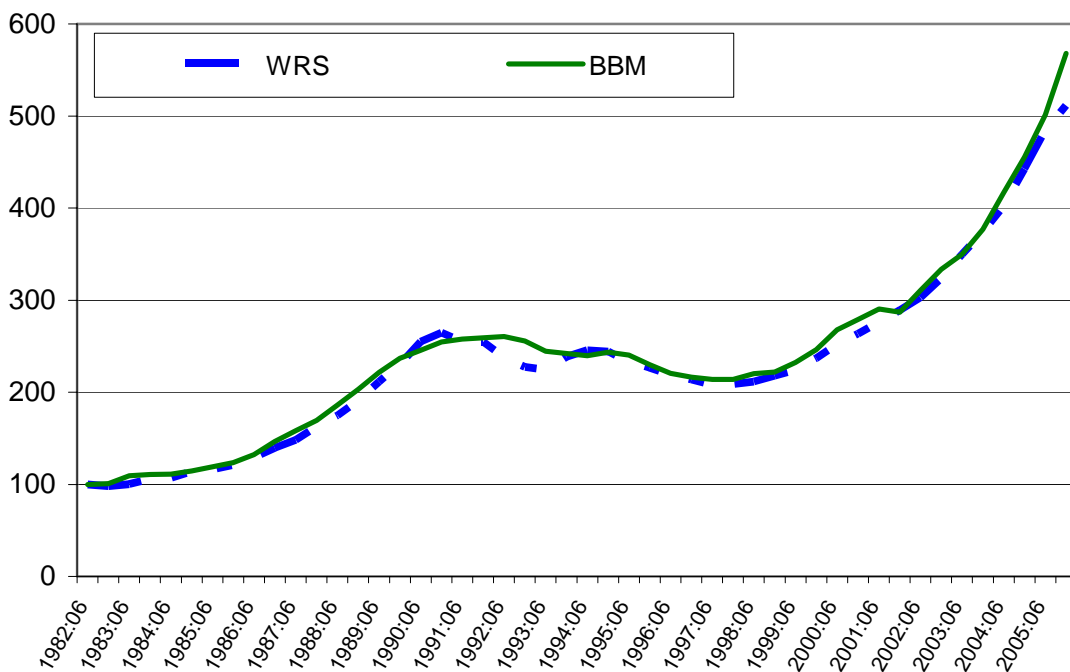
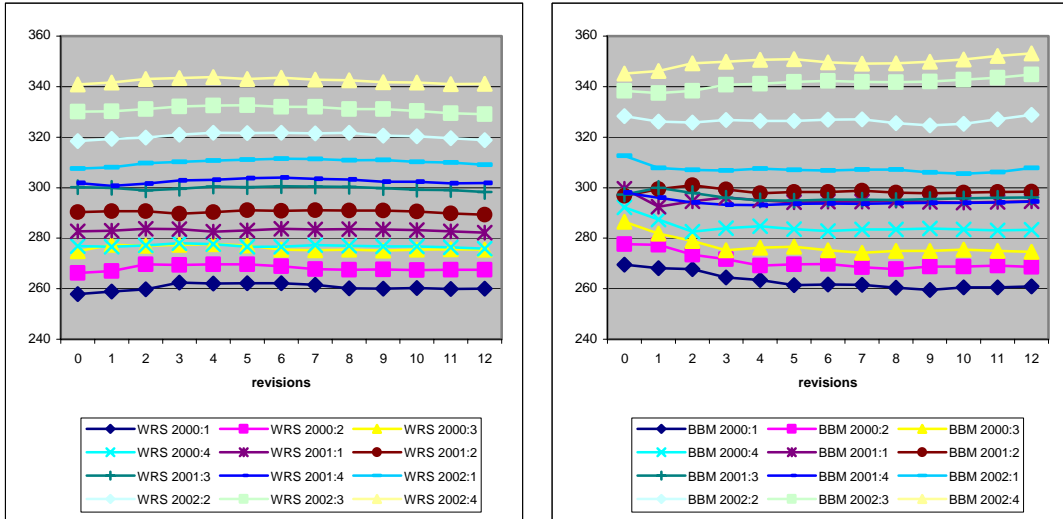
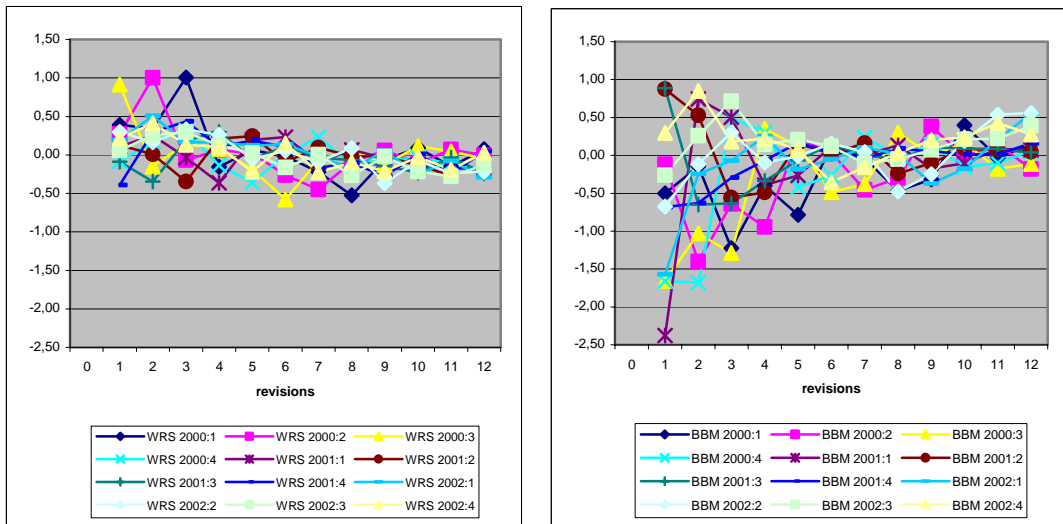


Figure 2: Two repeat sales indices for Paris

Change at quarter $s = 100 \times [P(t,1,s)]$; $t = 2001:1, \dots, 2002:4$; $s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [P(t,1,s) / P(t,1,s-1) - 1]$; $t = 2001:1, \dots, 2002:4$; $s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [P(t,1,t+12) / P(t,1,t) - 1]$; $t = 2001:1, \dots, 2002:4$; $s = t+1, \dots, t+12$

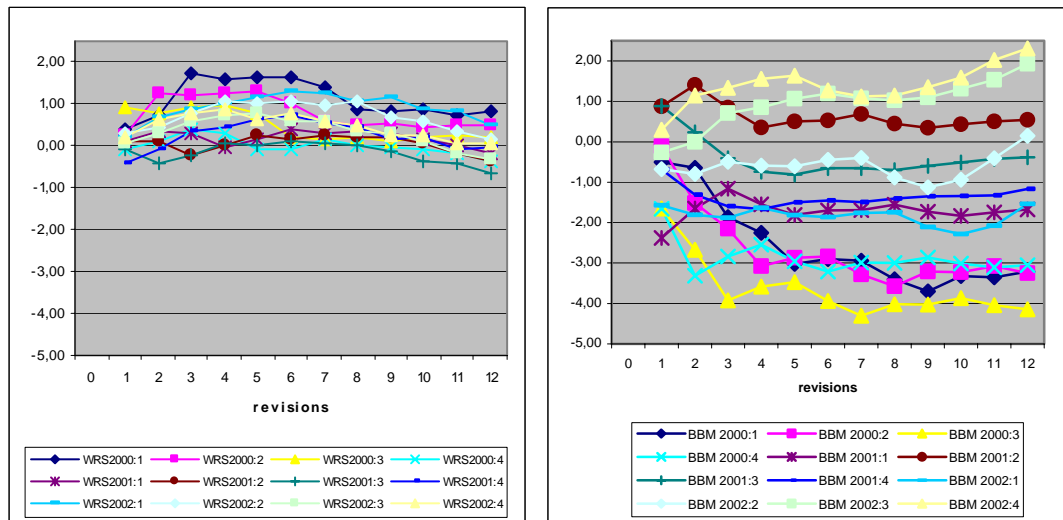
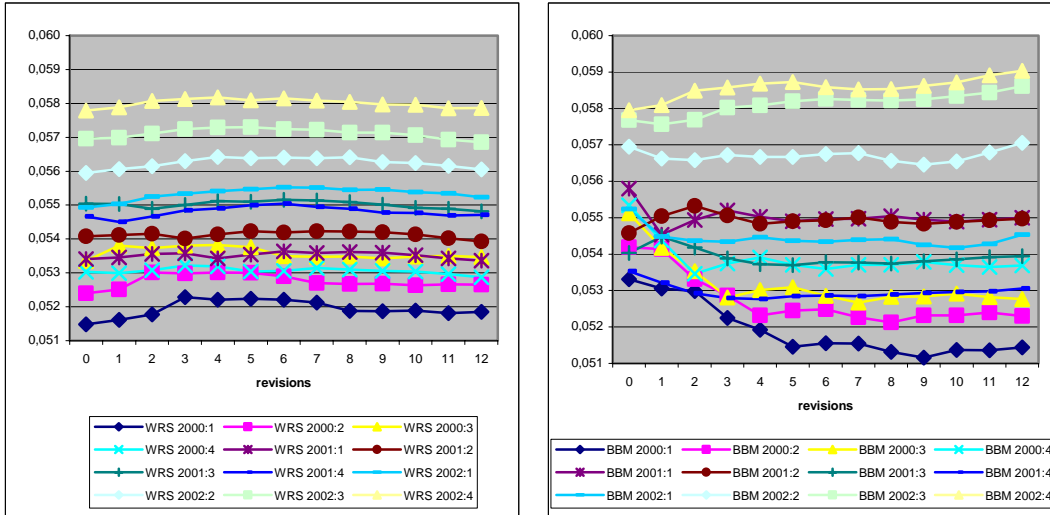
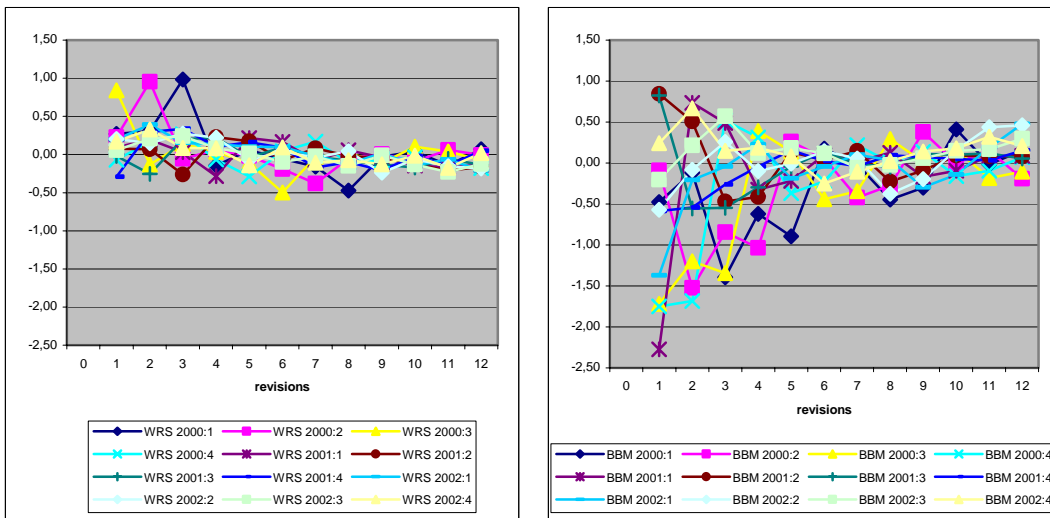


Figure 3 : Estimated path of revision of price level, 2000 :1 to 2002 :4 for 12 quarters (absolute level, relative and cumulative)

Change at quarter $s = 100 \times [\mu(t, 1, s)]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [\mu(t, 1, s) / \mu(t, 1, s-1) - 1]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [\mu(t, 1, t+12) / \mu(t, 1, t) - 1]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$

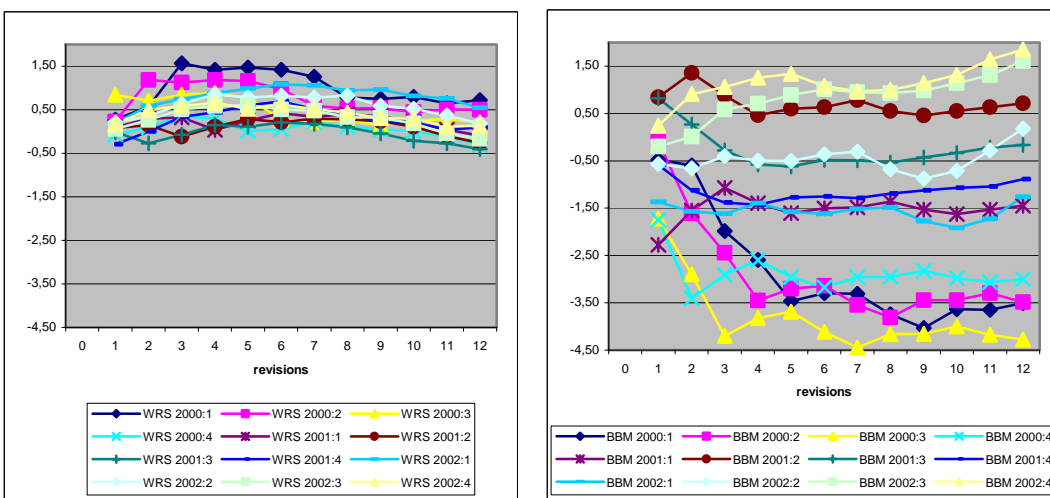
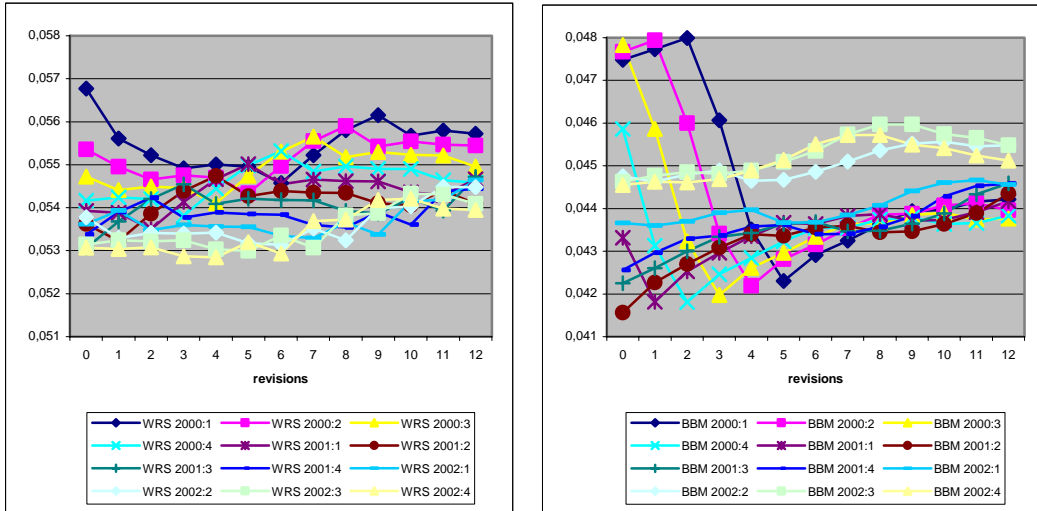
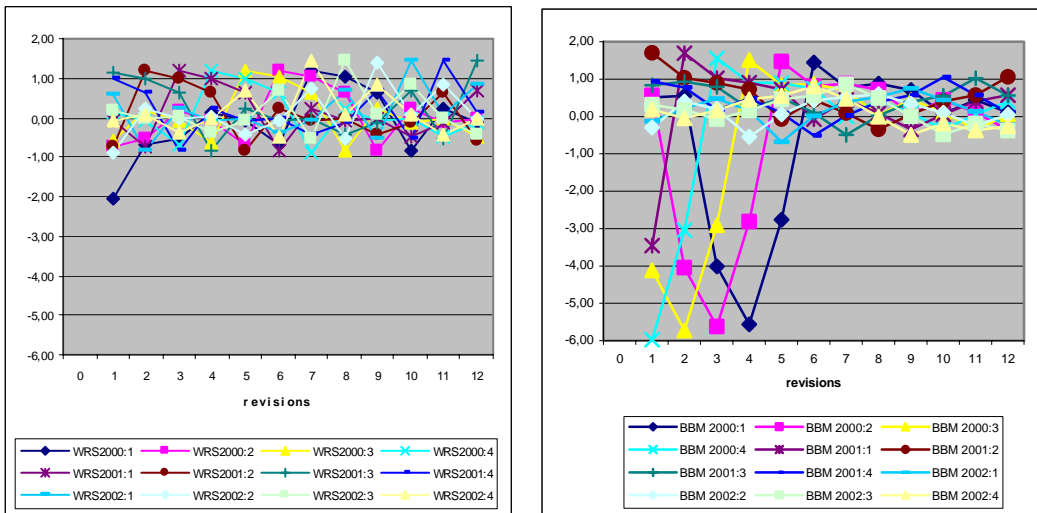


Figure 4 : Estimated path of revision of indices' trend, 2000 :1 to 2002 :4 for 12 quarters (absolute level, relative and cumulative)

Change at quarter $s = 100 \times [\sigma(t, 1, s)]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [\sigma(t, 1, s) / \sigma(t, 1, s-1) - 1]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$



Percent change at quarter $s = 100 \times [\sigma(t, 1, t+12) / \sigma(t, 1, t) - 1]; t = 2001:1, \dots, 2002:4; s = t+1, \dots, t+12$

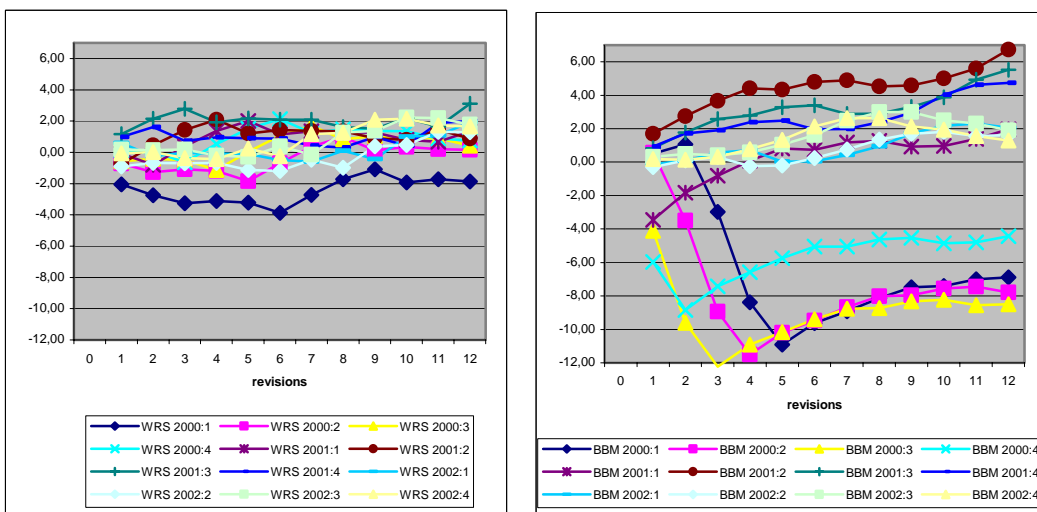


Figure 5 : Estimated path of revision of indices' volatility, 2000 :1 to 2002 :4 for 12 quarters (absolute level, relative and cumulative)

2. Tables

| | | Percent change in Price level estimate relative to previous estimate | | | | | | | |
|------------------|-----|--|-----------|------------------|-----------|-------------------|-----------|-----------------|-----------|
| | | All revisions | | Early revisions* | | Middle revisions* | | Late revisions* | |
| | | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. |
| Annual | WRS | 0.05 | 0.54 | 0.75 | 0.27 | -0.16 | 0.07 | -0.43 | 0.18 |
| | BBM | -0.11 | 0.88 | -0.32 | 1.34 | 0.18 | 0.48 | -0.20 | 0.39 |
| Half-year | WRS | 0.11 | 0.39 | 0.54 | 0.34 | -0.02 | 0.14 | -0.19 | 0.13 |
| | BBM | -0.08 | 0.62 | -0.33 | 0.91 | -0.02 | 0.31 | 0.11 | 0.34 |
| Quarter | WRS | 0.00 | 0.24 | 0.17 | 0.29 | -0.06 | 0.18 | -0.11 | 0.12 |
| | BBM | -0.09 | 0.50 | -0.31 | 0.76 | -0.06 | 0.23 | 0.09 | 0.21 |
| Every two months | WRS | -0.03 | 0.22 | 0.07 | 0.25 | -0.08 | 0.19 | -0.08 | 0.15 |
| | BBM | -0.07 | 0.36 | -0.21 | 0.54 | -0.04 | 0.21 | 0.05 | 0.18 |
| Average | WRS | 0.03 | 0.37 | 0.38 | 0.29 | -0.02 | 0.24 | -0.20 | 0.15 |
| | BBM | -0.09 | 0.62 | -0.29 | 0.94 | 0.01 | 0.32 | 0.01 | 0.30 |

* 'Early' is defined as the 1/3 periods, 'Middle' as the second 1/3 periods and 'Late' as the last 1/3 periods.

Table 5. Price revision: period-by-period

| | | Percent change in final Price level estimates relative to initial estimate | | | |
|------------------|-----|--|-------|------|-----------|
| | | Average | Min | Max | Std. Dev. |
| Annual | WRS | 0.16 | -0.19 | 0.34 | 0.25 |
| | BBM | -0.35 | -1.38 | 0.36 | 0.74 |
| Half-year | WRS | 0.65 | 0.14 | 1.00 | 0.30 |
| | BBM | -0.49 | -2.22 | 1.15 | 1.30 |
| Quarter | WRS | 0.03 | -0.64 | 0.82 | 0.41 |
| | BBM | -1.12 | -4.15 | 2.31 | 2.01 |
| Every two Months | WRS | -0.57 | -1.55 | 0.72 | 0.66 |
| | BBM | -1.18 | -3.44 | 2.39 | 1.79 |
| Average | WRS | 0.07 | -0.56 | 0.72 | 0.43 |
| | BBM | -0.79 | -2.80 | 1.55 | 1.54 |

Table 6. Price Revision: Cumulative Change from initial to Current Estimates

| | | Frequency that revision exceeds some limit | | | | | | |
|------------------|-----|--|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% |
| Annual | WRS | 89 | 56 | 33 | 11 | 0 | 0 | 0 |
| | BBM | 89 | 89 | 44 | 22 | 11 | 0 | 0 |
| Half-year | WRS | 67 | 44 | 14 | 6 | 0 | 0 | 0 |
| | BBM | 78 | 53 | 28 | 6 | 6 | 3 | 0 |
| Quarter | WRS | 64 | 23 | 4 | 1 | 0 | 0 | 0 |
| | BBM | 74 | 42 | 20 | 6 | 3 | 1 | 0 |
| Every two months | WRS | 60 | 22 | 3 | 0 | 0 | 0 | 0 |
| | BBM | 65 | 32 | 14 | 3 | 1 | 0 | 0 |
| Average | WRS | 70 | 36 | 14 | 4 | 0 | 0 | 0 |
| | BBM | 77 | 54 | 26 | 9 | 5 | 1 | 0 |

Table 7. Price Revision Limits

| | | Frequency that revision exceeds some limit | | | | | | | |
|------------------|-----|--|-------|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% | 4.00% |
| Annual | WRS | 78 | 44 | 22 | 0 | 0 | 0 | 0 | 0 |
| | BBM | 89 | 56 | 56 | 11 | 0 | 0 | 0 | 0 |
| Half-year | WRS | 97 | 89 | 36 | 8 | 0 | 0 | 0 | 0 |
| | BBM | 97 | 83 | 42 | 33 | 22 | 8 | 0 | 0 |
| Quarter | WRS | 62 | 40 | 14 | 3 | 0 | 0 | 0 | 0 |
| | BBM | 97 | 88 | 70 | 53 | 33 | 27 | 19 | 3 |
| Every two months | WRS | 80 | 59 | 20 | 5 | 2 | 0 | 0 | 0 |
| | BBM | 92 | 83 | 73 | 46 | 38 | 26 | 13 | 0 |
| | WRS | 79 | 58 | 23 | 4 | 0 | 0 | 0 | 0 |
| | BBM | 94 | 79 | 61 | 39 | 26 | 17 | 11 | 3 |

Table 8. Price Cumulative Revision Limits

| | | Percent change in trend level estimate relative to previous estimate | | | | | | | |
|-----------------|-----|--|-----------|-----------------|-----------|------------------|-----------|----------------|-----------|
| | | All revisions | | Early revisions | | Middle revisions | | Late revisions | |
| | | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. |
| Annual | WRS | 0.17 | 0.49 | 0.81 | 0.17 | -0.04 | 0.06 | -0.28 | 0.15 |
| | BBM | -0.03 | 0.84 | -0.29 | 1.27 | 0.26 | 0.46 | -0.07 | 0.35 |
| Half-year | WRS | 0.12 | 0.36 | 0.53 | 0.32 | -0.01 | 0.12 | -0.15 | 0.12 |
| | BBM | -0.06 | 0.62 | -0.33 | 0.92 | 0.02 | 0.28 | 0.12 | 0.32 |
| Quarter | WRS | 0.01 | 0.21 | 0.16 | 0.26 | -0.04 | 0.15 | -0.09 | 0.09 |
| | BBM | -0.10 | 0.50 | -0.32 | 0.75 | -0.05 | 0.23 | 0.08 | 0.19 |
| Every two years | WRS | -0.04 | 0.19 | 0.04 | 0.23 | -0.08 | 0.18 | -0.08 | 0.13 |
| | BBM | -0.07 | 0.35 | -0.22 | 0.52 | -0.03 | 0.21 | 0.04 | 0.15 |
| Average | WRS | 0.06 | 0.33 | 0.38 | 0.25 | 0.01 | 0.21 | -0.15 | 0.12 |
| | BBM | -0.07 | 0.60 | -0.29 | 0.91 | 0.05 | 0.31 | 0.04 | 0.27 |

Table 9. Trend revision: period-by-period

| | | Percent change in final trend level estimates relative to initial estimate | | | |
|-----------------|-----|--|-------|------|-----------|
| | | Average | Min | Max | Std. Dev. |
| Annual | WRS | 0.50 | 0.26 | 0.64 | 0.17 |
| | BBM | -0.11 | -1.16 | 0.53 | 0.75 |
| Half-year | WRS | 0.74 | 0.34 | 0.97 | 0.24 |
| | BBM | -0.39 | -2.23 | 1.01 | 1.28 |
| Quarter | WRS | 0.10 | -0.40 | 0.72 | 0.34 |
| | BBM | -1.14 | -4.28 | 1.85 | 1.98 |
| Every two years | WRS | -0.73 | -1.57 | 0.53 | 0.54 |
| | BBM | -1.22 | -3.61 | 1.90 | 1.81 |
| Average | WRS | 0.15 | -0.34 | 0.72 | 0.35 |
| | BBM | -0.71 | -2.82 | 1.33 | 1.54 |

Table 10. Trend Revision: Cumulative Change from initial to Current Estimates

| | | Frequency that revision exceeds some limit | | | | | | |
|------------------|-----|--|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% |
| Annual | WRS | 56 | 56 | 33 | 11 | 0 | 0 | 0 |
| | BBM | 100 | 56 | 33 | 22 | 11 | 0 | 0 |
| Half-year | WRS | 67 | 33 | 14 | 3 | 0 | 0 | 0 |
| | BBM | 75 | 50 | 25 | 6 | 6 | 3 | 0 |
| Quarter | WRS | 55 | 13 | 2 | 0 | 0 | 0 | 0 |
| | BBM | 72 | 38 | 17 | 7 | 3 | 1 | 0 |
| Every two months | WRS | 58 | 18 | 2 | 0 | 0 | 0 | 0 |
| | BBM | 60 | 28 | 13 | 3 | 1 | 0 | 0 |
| Average | WRS | 59 | 30 | 13 | 3 | 0 | 0 | 0 |
| | BBM | 92 | 70 | 50 | 16 | 4 | 0 | 0 |

Table 11. Trend Revision Limits

| | | Frequency that revision exceeds some limit | | | | | | | |
|-----------------|-----|--|-------|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% | 4.00% |
| Annual | WRS | 100 | 89 | 22 | 0 | 0 | 0 | 0 | 0 |
| | BBM | 78 | 67 | 44 | 11 | 0 | 0 | 0 | 0 |
| Half-year | WRS | 100 | 94 | 33 | 3 | 0 | 0 | 0 | 0 |
| | BBM | 94 | 72 | 39 | 33 | 19 | 8 | 0 | 0 |
| Quarter | WRS | 65 | 40 | 8 | 1 | 0 | 0 | 0 | 0 |
| | BBM | 95 | 85 | 64 | 44 | 29 | 28 | 21 | 6 |
| Every two years | WRS | 73 | 55 | 15 | 2 | 0 | 0 | 0 | 0 |
| | BBM | 93 | 82 | 62 | 43 | 34 | 26 | 20 | 0 |
| Average | WRS | 84 | 69 | 20 | 1 | 0 | 0 | 0 | 0 |
| | BBM | 90 | 79 | 58 | 42 | 28 | 18 | 12 | 5 |

Table 12. Trend Cumulative Revision Limits

| Percent change in volatility level estimate relative to previous estimate | | | | | | | | | |
|---|-----|---------------|-----------|-----------------|-----------|------------------|-----------|----------------|-----------|
| | | All revisions | | Early revisions | | Middle revisions | | Late revisions | |
| | | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. | Average | Std. Dev. |
| Annual | WRS | 1.20 | 0.37 | 1.66 | 0.25 | 1.66 | 0.25 | 0.87 | 0.15 |
| | BBM | 0.80 | 0.80 | 0.78 | 1.31 | 0.81 | 0.38 | 0.80 | 0.29 |
| Half-year | WRS | 0.47 | 0.49 | 0.82 | 0.61 | 0.82 | 0.61 | 0.29 | 0.30 |
| | BBM | 0.35 | 1.07 | -0.02 | 1.70 | 0.61 | 0.52 | 0.47 | 0.30 |
| Quarter | WRS | 0.09 | 0.65 | 0.00 | 0.69 | 0.00 | 0.69 | 0.12 | 0.59 |
| | BBM | -0.01 | 1.38 | -0.56 | 2.16 | 0.34 | 0.64 | 0.18 | 0.40 |
| Every two years | WRS | -0.16 | 0.94 | -0.33 | 1.13 | -0.33 | 1.13 | -0.10 | 0.73 |
| | BBM | -0.11 | 1.35 | -0.65 | 2.03 | 0.19 | 0.87 | 0.14 | 0.40 |
| Average | WRS | 0.40 | 0.65 | 0.54 | 0.74 | 0.23 | 0.66 | 0.30 | 0.50 |
| | BBM | 0.26 | 1.18 | -0.11 | 1.83 | 0.48 | 0.63 | 0.40 | 0.35 |

Table 13. Volatility revision: period-by-period

| Percent change in volatility level estimates relative to initial estimate | | | | | |
|---|-----|---------|--------|------|-----------|
| | | Average | Min | Max | Std. Dev. |
| Annual | WRS | 3.65 | 3.26 | 4.23 | 0.42 |
| | BBM | 2.40 | 1.07 | 3.40 | 0.98 |
| Half-year | WRS | 2.87 | 1.65 | 5.47 | 1.28 |
| | BBM | 2.12 | -1.15 | 5.21 | 2.19 |
| Quarter | WRS | 1.12 | -1.86 | 3.11 | 1.18 |
| | BBM | -0.15 | -8.50 | 6.72 | 5.12 |
| Every two Years | WRS | -2.93 | -9.44 | 0.32 | 2.72 |
| | BBM | -1.83 | -13.08 | 7.52 | 7.31 |
| Average | WRS | 1.18 | -1.60 | 3.28 | 1.63 |
| | BBM | 0.64 | -5.42 | 5.71 | 4.62 |

Table 14. Volatility Revision: Cumulative Change from initial to Current Estimates

| | | Frequency that revision exceeds some limit | | | | | | |
|-----------------|-----|--|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% |
| Annual | WRS | 100 | 100 | 100 | 78 | 22 | 0 | 0 |
| | BBM | 100 | 100 | 78 | 56 | 11 | 0 | 0 |
| Half-year | WRS | 78 | 58 | 36 | 11 | 3 | 3 | 0 |
| | BBM | 92 | 78 | 56 | 28 | 19 | 6 | 3 |
| Quarter | WRS | 74 | 58 | 49 | 12 | 1 | 1 | 0 |
| | BBM | 84 | 69 | 44 | 16 | 11 | 8 | 6 |
| Every two years | WRS | 97 | 82 | 67 | 24 | 11 | 2 | 0 |
| | BBM | 86 | 66 | 37 | 19 | 10 | 8 | 6 |
| Average | WRS | 87 | 74 | 63 | 31 | 9 | 1 | 0 |
| | BBM | 91 | 78 | 54 | 30 | 13 | 5 | 4 |

Table 15. Volatility Revision Limits

| | | Frequency that revision exceeds some limit | | | | | | | |
|-----------------|-----|--|-------|-------|-------|-------|-------|-------|-------|
| | | 0.10% | 0.25% | 0.50% | 1.00% | 1.50% | 2.00% | 3.00% | 4.00% |
| Annual | WRS | 100 | 100 | 100 | 89 | 67 | 56 | 44 | 11 |
| | BBM | 89 | 89 | 89 | 56 | 44 | 22 | 11 | 0 |
| Half-year | WRS | 100 | 92 | 89 | 56 | 47 | 22 | 19 | 11 |
| | BBM | 92 | 83 | 75 | 61 | 42 | 31 | 28 | 11 |
| Quarter | WRS | 86 | 72 | 51 | 27 | 15 | 6 | 3 | 0 |
| | BBM | 92 | 88 | 77 | 72 | 60 | 52 | 44 | 40 |
| Every two years | WRS | 92 | 85 | 72 | 59 | 48 | 40 | 30 | 17 |
| | BBM | 93 | 86 | 79 | 73 | 65 | 56 | 50 | 44 |
| | WRS | 95 | 87 | 78 | 58 | 44 | 31 | 24 | 10 |
| | BBM | 92 | 86 | 80 | 65 | 53 | 40 | 33 | 24 |

Table 16. Volatility Cumulative Revision Limits

| | WRS | | | BBM | | |
|-----------|-------|--------|------------|-------|--------|------------|
| | Price | Return | Volatility | Price | Return | Volatility |
| Paris | 0,16 | 0,5 | 3,65 | -0,35 | -0,11 | 2,4 |
| arr 1-9 | -0,6 | -0,21 | 2,55 | -0,71 | -0,34 | 1,94 |
| arr 10-20 | 0,25 | 0,6 | 3,7 | -0,21 | -0,02 | 2,35 |
| arr 1 | -3,67 | -4,31 | -6,86 | 1,84 | 0,31 | 5,77 |
| arr 2 | -0,76 | -0,93 | -1,87 | -0,49 | 0,27 | 7,11 |
| arr 3 | -0,24 | 0,02 | 1,13 | 0,83 | 0,89 | 1,07 |
| arr 4 | -0,52 | -0,52 | -0,79 | -0,54 | -0,42 | -0,29 |
| arr 5 | 1,38 | 0,87 | -2,70 | -1,07 | -1,04 | -2,30 |
| arr 6 | -1,40 | -1,02 | -0,34 | -1,77 | -1,49 | -3,78 |
| arr 7 | 0,56 | 0,23 | -1,27 | -0,67 | -0,30 | 2,08 |
| arr 8 | -5,20 | -2,78 | 12,19 | 0,46 | 0,37 | -1,68 |
| arr 9 | 1,01 | 1,11 | 1,19 | -0,20 | 0,24 | 4,26 |
| arr 10 | -1,53 | -0,95 | 3,70 | 0,64 | 0,77 | 2,94 |
| arr 11 | 1,90 | 1,74 | 0,79 | 0,63 | 0,60 | 1,40 |
| arr 12 | 0,49 | 0,54 | 1,15 | 0,04 | 0,26 | 2,76 |
| arr 13 | 1,15 | 1,24 | 1,93 | 0,13 | 0,08 | 1,71 |
| arr 14 | 0,32 | 0,48 | 2,29 | -0,23 | 0,04 | 2,74 |
| arr 15 | -0,91 | -0,60 | 2,38 | -1,46 | -1,11 | 2,47 |
| arr 16 | 2,88 | 2,80 | -0,29 | 0,15 | 0,20 | -1,24 |
| arr 17 | 0,49 | 0,75 | 2,94 | -0,29 | -0,14 | 1,62 |
| arr 18 | 0,46 | 0,77 | 2,89 | 0,17 | 0,22 | 1,90 |
| arr 19 | 1,56 | 2,13 | 4,14 | 0,79 | 1,04 | 2,98 |
| arr 20 | -2,05 | -1,33 | 5,34 | -1,46 | -1,23 | 3,04 |

Table 17: Cumulative Revision effect (mean of 3 revisions, annual index)

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