



Federal Reserve Bank of Chicago

Information Acquisition in Financial Markets: a Correction

Gadi Barlevy and Pietro Veronesi

WP 2007-06

Information Acquisition in Financial Markets: a Correction

Gadi Barlevy
Federal Reserve Bank of Chicago
Chicago, IL 60604

Pietro Veronesi
Graduate School of Business
University of Chicago
Chicago, IL 60637

April 24, 2007

In our 2000 paper “Information Acquisition in Financial Markets,” we argued that contrary to the conventional wisdom set forth in Grossman and Stiglitz (1980), it was theoretically possible that as more traders in financial markets acquire information, equilibrium prices would change in such a way that it became more difficult for remaining agents to infer the fundamentals from prices. We presented a model we thought demonstrated this claim. However, as was subsequently pointed out to us by Christophe Chamley, the expression we used for the value of information in that paper (expression 3.5) was incorrect. As demonstrated by Chamley (2007), using the correct expression for the value of learning reveals that learning is a strategic substitute in that model.

This leaves the question of whether the problem lies with our example or with the argument that when traders acquire information they can exacerbate the identification problem of remaining agents. In this note, we show that the argument we advanced is correct, although it requires that the fundamental value of the asset be correlated with noise trade. Since this feature was absent from our original model, it was incapable of generating complementarities, as Chamley (2007) reports.

To illustrate this point, we begin with a special case of the model we used in our 2000 paper to provide the simplest example of why equilibrium prices may not become more informative as some agents acquire information. For this special case, we can verify that complementarities only arise if the fundamental value of the asset is correlated with noise trade. We then use this insight to revisit our 2000 paper and show that by allowing the fundamentals and noise to be correlated, we can capture the tension we tried to model, namely that as more traders acquire information they make it more difficult for remaining agents to discriminate between low fundamentals and a negatively skewed distribution of supply shocks. We close with remarks on some of the work on complementarity in information acquisition that was written since our 2000 paper was published.

1 The 2×2 Case

We use the same notation as in our 2000 paper. Briefly, agents must choose to allocate their wealth between money and an asset that pays a random amount $\tilde{\theta}$ per share. There is a unit mass of risk-neutral agents who can observe $\tilde{\theta}$ if they pay a cost c . The demand of the informed traders is denoted $x^I(\tilde{\theta}, P)$, and of the uninformed is denoted $x^U(P)$. Traders can spend at most their initial endowment, equal to one unit of money, and cannot sell assets short. Demand for the asset by noise traders is $\frac{w}{P} - \tilde{x}$ for some positive constant w , where \tilde{x} is a random variable.

We begin with the simplest relevant case, where both $\tilde{\theta}$ and \tilde{x} assume only two possible values: $\tilde{\theta} \in \{\underline{\theta}, \bar{\theta}\}$ and $x \in \{x_0, x_1\}$. We will refer to the four states of the world as ω_1 through ω_4 , with respective probabilities π_1 through π_4 , using the following convention:

$$\begin{array}{c|cc} \bar{\theta} & x_0 & x_1 \\ \hline & \omega_1 & \omega_2 \\ \hline \underline{\theta} & \omega_3 & \omega_4 \end{array} \qquad \begin{array}{c|cc} \bar{\theta} & x_0 & x_1 \\ \hline & \pi_1 & \pi_2 \\ \hline \underline{\theta} & \pi_3 & \pi_4 \end{array}$$

Suppose first that $z = 0$, so all traders are uninformed. In this case, prices cannot depend on $\tilde{\theta}$. Generically, the price at x_0 will be different from x_1 , so the information set of an uninformed agent given prices will be given by

$$\Omega_0 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}. \quad (1)$$

Next, consider a particular value $z^* \in (0, 1)$ and suppose the number of informed traders $z = z^*$. If uninformed traders value information more when $z = z^*$, prices cannot fully reveal $\tilde{\theta}$. Hence, there must be at least two states in which the same price prevails. We verify in a technical appendix that the only two states in which the same price could arise are ω_2 and ω_3 . Thus, suppose x_0 and x_1 are such that when $z = z^*$, there exists an equilibrium for which $P(\omega_2) = P(\omega_3)$. If such values of x_0 and x_1 exist, the information set of an uninformed agent would be given by

$$\Omega_{z^*} = \{\{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_4\}\}. \quad (2)$$

Comparing Ω_{z^*} to Ω_0 , note that neither partition is a finer partitioning of the other. Hence, there is no sense in which uninformed agents can be said to be inherently more informed when there are z^* informed traders than when there no informed traders. In other words, equilibrium prices do not become more informative as more traders acquire information — they simply convey different information. This stands in contrast to what Grossman and Stiglitz (1980) obtain in their model, where prices necessarily become more informative in some well-defined sense as more agents become informed (specifically, prices are more informative in the Blackwell sense).

The fact that prices may not become more informative as more agents acquire information does not ensure that uninformed agents will value information more when facing the information set Ω_{z^*} than when facing the information set Ω_0 . To determine whether such a scenario is possible, we need to manually check each of the finitely many candidate equilibria for the 2×2 case. Going through the various cases reveals that this scenario is indeed possible, but only if x and θ are correlated (the detailed calculations are delegated to an appendix). The intuition for this is that since agents can infer \tilde{x} from prices when $z = 0$, the fact that \tilde{x} is correlated implies prices are already quite

informative about $\tilde{\theta}$ even when agents are all uninformed. Once some agents become informed, prices will depend on both $\tilde{\theta}$ and \tilde{x} , and it may be harder to infer $\tilde{\theta}$ from prices.

To illustrate this possibility, consider the following example. Set $w = 10$, $\underline{\theta} = 5$, $\bar{\theta} = 10$, and $x_0 = 0.5$. The final parameter, x_1 , is chosen specifically to give rise to equilibrium prices which are not fully revealing. In particular, let us first choose a particular fraction of informed traders z^* at which we want prices to not be fully revealing. We arbitrarily set $z^* = 0.1$, and then go about searching for an equilibrium in which $P(\omega_2) = P(\omega_3)$ when $z = z^*$. The equilibrium we construct will involve traders buying the asset at this joint price. Hence, the combined amount of wealth that informed and uninformed traders will spend on the asset should equal 1 in state ω_2 and $1 - z^*$ in state ω_3 . The market-clearing price in these respective states will be given by

$$P(\omega_2) = \frac{w+1}{x_1+1} \quad (3)$$

$$P(\omega_3) = \frac{w+1-z^*}{x_0+1} \quad (4)$$

These two expressions will be equal if

$$x_1 = \frac{w+1}{w+1-z^*}(x_0+1) - 1 = 0.51. \quad (5)$$

This is the value we assign to x_1 . Finally, we assume the following distribution over possible states:

π_1	π_2	=	0.05	0.50
π_3	π_4		0.40	0.05

Given these parameters, there is a unique equilibrium when $z = 0$. In this equilibrium, uninformed agents hold money when $x = x_0$ and the asset when $x = x_1$. Market clearing prices are given by

$$P(\omega_1) = P(\omega_3) = \frac{w}{x_0+1} = 6.67 \quad (6)$$

$$P(\omega_2) = P(\omega_4) = \frac{w+1}{x_1+1} = 7.27 \quad (7)$$

Since $E(\tilde{\theta}|P(\cdot, \cdot) = 6.67) = 5.56$ and $E(\tilde{\theta}|P(\cdot, \cdot) = 7.27) = 9.55$, uninformed traders will indeed prefer not to buy the asset at the first price but to buy it at the second. The value of purchasing information is equal to the expected value of avoiding the mistakes this policy commits in states ω_1 and ω_4 . The value of information is thus

$$\pi_1 \left(\frac{\bar{\theta}}{P(\omega_1)} - 1 \right) + \pi_4 \left(1 - \frac{\underline{\theta}}{P(\omega_4)} \right) = 0.04 \quad (8)$$

Next, consider the case where $z = z^*$. Informed traders buy the asset in states ω_1 and ω_2 and do not buy it in states ω_3 and ω_4 . Since equilibrium prices must fully reveal states ω_1 and ω_4 , uninformed traders will also buy the asset in state ω_1 and avoid it in state ω_4 . We conjecture an equilibrium where in the remaining two states ω_2 and ω_3 prices are given by (3) and (4), which by construction are equal, and uninformed traders strictly prefer to buy the asset at this common price. That is, we conjecture an equilibrium in which demand schedules and prices are as follows:

$x^I(\tilde{\theta}, P)$	$x^U(P)$	$P(\tilde{x}, \tilde{\theta})$
$\begin{array}{cc} x_0 & x_1 \\ \bar{\theta} & \begin{array}{ c c } \hline \text{buy} & \text{buy} \\ \hline \end{array} \\ \underline{\theta} & \begin{array}{ c c } \hline \text{don't buy} & \text{don't buy} \\ \hline \end{array} \end{array}$	$\begin{array}{cc} x_0 & x_1 \\ \bar{\theta} & \begin{array}{ c c } \hline \text{buy} & \text{buy} \\ \hline \end{array} \\ \underline{\theta} & \begin{array}{ c c } \hline \text{buy} & \text{don't buy} \\ \hline \end{array} \end{array}$	$\begin{array}{cc} x_0 & x_1 \\ \bar{\theta} & \begin{array}{ c c } \hline \frac{w+1}{x_0+1} = 7.33 & \frac{w+1}{x_1+1} = 7.27 \\ \hline \end{array} \\ \underline{\theta} & \begin{array}{ c c } \hline \frac{w+1-z^*}{x_0+1} = 7.27 & \frac{w}{x_1+1} = 6.61 \\ \hline \end{array} \end{array}$

(9)

To confirm that the demand schedule for uninformed traders is optimal, note that

$$E(\theta|P(\cdot, \cdot) = 7.27) = \frac{\pi_2 \bar{\theta} + \pi_3 \underline{\theta}}{\pi_2 + \pi_3} = 7.78$$

so the expected payout on the asset exceeds its price, and hence the asset is more valuable than money. The value of information in this case is equal to the expected value of avoiding the mistake of buying an overvalued asset in state ω_3 . Hence, the value of information is equal to

$$\pi_3 \left(1 - \frac{\underline{\theta}}{P(\omega_3)} \right) = 0.12 \quad (10)$$

Comparing (8) and (10), it is clear that uninformed agents value becoming informed more when $z = z^*$ than when $z = 0$, confirming the possibility of learning complementarities.

As a final remark, the complementarities in the above example can generate multiple equilibria with different amounts of information being acquired. Suppose the cost of information c is exactly equal the value of information when $z = z^*$ in (10). Under this assumption, there are two possible equilibrium fractions of informed agents, $z = 0$ and $z = z^*$. In the latter, agents are indifferent between becoming informed and not. For any other value of z , equilibrium prices must be fully revealing, i.e. $\Omega_z = \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}\}$, implying no agent will want to purchase information.

2 Revisiting Barlevy and Veronesi (2000)

Although the 2×2 case is convenient for demonstrating why prices need not become more informative as more agents acquire information, the example it produces seems rather knife-edge. The example requires that $P(\omega_2) = P(\omega_3)$, a condition that only happens when $z = z^*$ and this only because we choose a particular value for x_1 . For all other values of $z \notin \{0, z^*\}$, prices are fully revealing. It also implies demand for the asset by uninformed traders is increasing in the price of the asset when $z = 0$. We now construct an example in which prices are never fully revealing, where the value of information is increasing in z over an open range rather than at a particular value, and demand for the asset by uninformed traders is always decreasing in its price.

We construct this example by revisiting the model we laid out in our original 2000 paper. In that paper, we attempted to generate an example in which as more agents became informed, remaining agents became worse off because they had greater difficulty distinguishing whether prices were low because informed traders learned that fundamentals were unfavorable and drove prices down or because the distribution of supply placed more mass on high realizations of \tilde{x} . Since we used an incorrect expression for the value of information, we were unaware that we were attempting to model this in a way that does not in fact generate complementarities in information acquisition. Based on our analysis of the 2×2 case, one possible way to generate this tension is to allow \tilde{x} and $\tilde{\theta}$ to be correlated. We now provide a numerical example which shows that introducing correlation into our model does in fact yield complementarities in information acquisition.

Let $w = 1.1$. The payoff on the asset $\tilde{\theta}$ is equal to either 1 or 0.9 with equal probability. We next describe the distribution for \tilde{x} . As a first step, note that the unconditional expectation of $\tilde{\theta}$ is given

by $\frac{1}{2}(\underline{\theta} + \bar{\theta})$. Let x^* denote the value of \tilde{x} for which the market clearing price when half of the wealth of rational traders is allocated to buying the asset would equal this value. That is, x^* solves

$$\frac{w + 1/2}{x^* + 1} = \frac{1}{2}(\underline{\theta} + \bar{\theta}).$$

Given our other parameter choices, $x^* = 0.68$. Let λ denote a positive constant that is arbitrarily close to 0 and strictly less than $\frac{1}{2}$. We assume \tilde{x} is distributed as a step function that depends on $\tilde{\theta}$:

$$f(x|\tilde{\theta} = \underline{\theta}) = \begin{cases} (1 - \lambda)/x^* & \text{if } x \in [0, x^*] \\ \lambda/x^* & \text{if } x \in (x^*, 2x^*] \end{cases}$$

$$f(x|\tilde{\theta} = \bar{\theta}) = \begin{cases} \lambda/x^* & \text{if } x \in [0, x^*] \\ (1 - \lambda)/x^* & \text{if } x \in (x^*, 2x^*] \end{cases}$$

As in the 2×2 case, $\tilde{\theta}$ and \tilde{x} are assumed to be positively correlated.¹ Note that \tilde{x} is positively skewed when $\tilde{\theta} = \underline{\theta}$ and negatively skewed when $\tilde{\theta} = \bar{\theta}$. In our original paper, we attempted to construct an example in which skewness makes it more difficult to read fundamentals from prices, since a skewed distribution makes certain price more likely. This is still true in our present example, except we now assume \tilde{x} is skewed *in opposite directions* depending on $\tilde{\theta}$, whereas before we tried to generate an example in which \tilde{x} was skewed *in the same direction* for both values of $\tilde{\theta}$.

Given the parameterization above, the following set of demand and price functions constitute an equilibrium, as shown formally in the technical appendix.

(1) The demand schedule of informed traders is given by

$$x^I(\tilde{\theta}, P) \in \begin{cases} 0 & \text{if } P > \tilde{\theta} \\ \left[0, \frac{1}{P}\right] & \text{if } P = \tilde{\theta} \\ \frac{1}{P} & \text{if } P < \tilde{\theta} \end{cases}$$

(2) The demand of uninformed traders is the same for all values of z , and is given by

$$x^U(P) \in \begin{cases} 0 & \text{if } P > \frac{\underline{\theta} + \bar{\theta}}{2} \\ \left[0, \frac{1}{P}\right] & \text{if } P = \frac{\underline{\theta} + \bar{\theta}}{2} \\ \frac{1}{P} & \text{if } P < \frac{\underline{\theta} + \bar{\theta}}{2} \end{cases}$$

(3) The equilibrium price function is given by

$$P(x, \bar{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if } x+1 \leq \frac{w}{\bar{\theta}} \\ \bar{\theta} & \text{if } x+1 \in \left(\frac{w}{\bar{\theta}}, \frac{w+z}{\bar{\theta}}\right) \\ \frac{w+z}{x+1} & \text{if } x+1 \in \left[\frac{w+z}{\bar{\theta}}, \frac{2(w+z)}{\underline{\theta} + \bar{\theta}}\right] \\ \frac{\underline{\theta} + \bar{\theta}}{2} & \text{if } x+1 \in \left(\frac{2(w+z)}{\underline{\theta} + \bar{\theta}}, \frac{2(w+1)}{\underline{\theta} + \bar{\theta}}\right) \\ \frac{w+1}{x+1} & \text{if } x+1 \geq \frac{2(w+1)}{\underline{\theta} + \bar{\theta}} \end{cases}$$

$$P(x, \underline{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if } x+1 \leq \frac{2w}{\underline{\theta} + \bar{\theta}} \\ \frac{\underline{\theta} + \bar{\theta}}{2} & \text{if } x+1 \in \left(\frac{2w}{\underline{\theta} + \bar{\theta}}, \frac{2(w+1-z)}{\underline{\theta} + \bar{\theta}}\right) \\ \frac{w+1-z}{x+1} & \text{if } x+1 \in \left[\frac{2(w+1-z)}{\underline{\theta} + \bar{\theta}}, \frac{w+1-z}{\underline{\theta}}\right] \\ \underline{\theta} & \text{if } x+1 \in \left(\frac{w+1-z}{\underline{\theta}}, \frac{w+1}{\underline{\theta}}\right) \\ \frac{w+1}{x+1} & \text{if } x+1 \geq \frac{w+1}{\underline{\theta}} \end{cases}$$

¹One way to motivate this assumption is that if a high realization of $\tilde{\theta}$ reflected favorable economic conditions, returns to other investments are more likely to also be high. As a result, some agents who would have held on to the asset in less favorable economic conditions might prefer to liquidate the asset when conditions are favorable to allow them to take advantage of profitable private investment opportunities. For the investors whose decisions we study, the higher selloff among such agents would be viewed as “noise” trading.

This is the same price function that appeared in our original paper (and which is depicted in Figure 1 of that paper), where the cutoff P_z^* we describe there corresponds to $\frac{\underline{\theta} + \bar{\theta}}{2}$. Our parameterization implies $2x^* > \frac{w+1}{\underline{\theta}} - 1$, which ensures no price in the interval $(\underline{\theta}, \bar{\theta})$ is fully revealing when $\lambda > 0$.

Following Chamley (2007), the proper expression for the value of information in this case is

$$g(z) = \frac{1}{2} \int_{\frac{2w}{\underline{\theta} + \bar{\theta}} - 1}^{\frac{w+1-z}{\underline{\theta}} - 1} \left[1 - \frac{\underline{\theta}}{P(x, \underline{\theta})} \right] f(x | \tilde{\theta} = \underline{\theta}) dx + \frac{1}{2} \int_{\frac{w+z}{\bar{\theta}} - 1}^{\frac{2(w+z)}{\underline{\theta} + \bar{\theta}} - 1} \left[\frac{\bar{\theta}}{P(x, \bar{\theta})} - 1 \right] f(x | \tilde{\theta} = \bar{\theta}) dx \quad (11)$$

To generate an example in which the prices in $(\underline{\theta}, \bar{\theta})$ are not revealing, λ must be strictly positive. However, it is easier to gain intuition from the limiting case in which $\lambda = 0$. Figure 1 illustrates $g(z)$ for $\lambda = 0$, and shows it is constant for $z \in [0, 0.5)$, increases at $z = 0.5$, and ultimately declines.

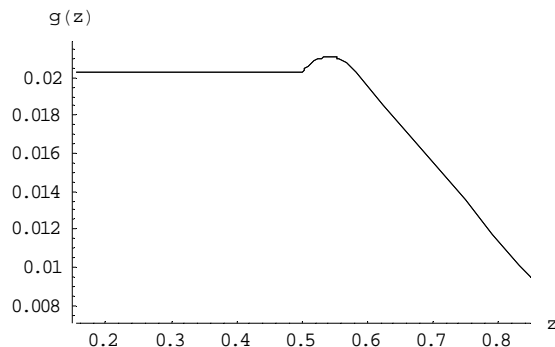


Figure 1: The function $g(z)$ for $\lambda = 0$

In an appendix, we provide a formal argument for why $g(z)$ is increasing just above $z = 0.5$. The intuition is as follows. When $z < 0.5$, uninformed traders choose the correct action for any price $P \neq \frac{\underline{\theta} + \bar{\theta}}{2}$. This is because, in equilibrium, prices above $\frac{\underline{\theta} + \bar{\theta}}{2}$ only occur when the fundamentals are bad and prices below $\frac{\underline{\theta} + \bar{\theta}}{2}$ only occur when the fundamentals are good. Changing z but leaving it below 0.5 thus has no effect on the prices that occur with positive probability, so $g(z)$ is invariant to changes in z . But once z rises above 0.5, prices in an open neighborhood of $\frac{\underline{\theta} + \bar{\theta}}{2}$ become possible in both states of the world, making it harder to read $\tilde{\theta}$ from prices. Consider a trader who resolves that if he is uninformed, he will buy the asset if its price is below $\frac{\underline{\theta} + \bar{\theta}}{2}$ but not if its price is greater than or equal to $\frac{\underline{\theta} + \bar{\theta}}{2}$. That is, we resolve the agent's indifference at $\frac{\underline{\theta} + \bar{\theta}}{2}$ in a particular way, which we can do without loss of generality. When $z < 0.5$, the only mistake an uninformed agent can commit is to not purchase the asset when $\tilde{\theta} = \bar{\theta}$, this if the price is equal to $\frac{\underline{\theta} + \bar{\theta}}{2}$. As z rises to just above 0.5, the agent will still occasionally fail to purchase the asset when $\tilde{\theta} = \bar{\theta}$. But now at least part of the time he would fail to purchase the asset at prices above $\frac{\underline{\theta} + \bar{\theta}}{2}$. Since the asset is more expensive, the loss from not having purchased it are smaller. On its own, this would lower the incentive to acquire information. But it is also the case that as z rises to just above 0.5, there will be a positive probability that the agent will buy the asset when $\tilde{\theta} = \underline{\theta}$ because informed traders in this state may drive the price to below $\frac{\underline{\theta} + \bar{\theta}}{2}$. An uninformed trader will thus commit mistakes that he would not have committed when $z < 0.5$, since prices are less informative. This is precisely the intuition we tried to capture in our previous paper. Under our assumptions, the fact that traders are more likely

to commit mistakes more than offsets the fact that previous mistakes the agent committed become less costly, and the value of information increases.

If we allow λ to be small but positive, the essential features of the above example remain unchanged. In this case, $g(z)$ will be decreasing over the interval $[0, 0.5)$, but will continue to increase at 0.5. In contrast to the 2×2 case, no price in the interval $(\underline{\theta}, \bar{\theta})$ fully reveals the true value of $\tilde{\theta}$, demand for the asset among uninformed traders is decreasing in its price, and the value of information will be increasing over a range of z rather than at a single value.

3 Subsequent Literature

Since our original paper was published, several other papers have been written that generate complementarity in acquisition information in financial markets, including Chamley (2006), Veldkamp (2006), and Ganguli and Yang (2006). The mechanisms generating complementarity in these papers are different from ours, and are thus of independent interest from the mechanism we originally conjectured but only now truly established. Of these, the closest in spirit to ours is Ganguli and Yang (2006). They assume informed agents receive distinct signals rather than identical signals as in our framework, and find that there exist equilibria in which as more agents learn, prices become more sensitive to the variable which agents have less precise information on. This is different from the mechanism we construct, but shares with our mechanism the feature that it may become harder to read fundamentals from prices as more agents acquire information. More recently, Hellwig and Veldkamp (2007) derive conditions for complementarity in information acquisition in generalized games, although they rule out publicly observable prices which are central to our mechanism.

References

Barlevy, Gadi and Pietro Veronesi, 2000. "Information Acquisition in Financial Markets" *Review of Economic Studies*, 67(1), p79-90.

Chamley, Christophe, 2006. "Complementarities in Information Acquisition with Short-Term Trades" *Mimeo* (first version: 2005)

Chamley, Christophe, 2007. "Strategic Substitutability in 'Information Acquisition in Financial Markets'" *Mimeo*.

Ganguli, Jayant and Liyan Yang, 2006. "Supply Signals, Complementarities, and Multiplicity in Asset Prices and Information Acquisition" *Mimeo*.

Hellwig, Christian and Laura Veldkamp, 2007. "Knowing What Others Know: Coordination Motives in Information Acquisition" *Mimeo*.

Veldkamp, Laura, 2006. "Media Frenzies in Markets for Financial Information" *American Economic Review*, 96(3), p577-601.

Technical Appendix for ‘Information Acquisition in Financial Markets: a Correction’

This appendix derives several results discussed but not formally derived in our note. The first section is devoted to analyzing the 2×2 case. Here we show that some prices must be fully revealing when some agents are uninformed, and that a necessary condition for information acquisition to be a strategic complement is that \tilde{x} and $\tilde{\theta}$ be correlated. The second section verifies that the demand schedules and price function we propose when we describe a revised version of 2000 model in which \tilde{x} and $\tilde{\theta}$ are correlated indeed constitute an equilibrium. The third section provides a rigorous analysis of why $g(z)$ in that version must be increasing in z at $z = 0.5$.

1. Analysis of the 2×2 Case

Let $\tilde{x} \in \{x_0, x_1\}$ and $\tilde{\theta} \in \{\underline{\theta}, \bar{\theta}\}$. In addition, we restrict w so that

$$\underline{\theta} < \frac{w}{x_1 + 1} < \frac{w + 1}{x_0 + 1} < \bar{\theta}.$$

This ensures that all equilibrium prices will be confined to the interval $(\underline{\theta}, \bar{\theta})$. We allow \tilde{x} and $\tilde{\theta}$ to be correlated, so the joint distribution of \tilde{x} and $\tilde{\theta}$ is any arbitrary probability matrix

$$\begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} \begin{array}{cc} x_0 & x_1 \\ \hline \pi_1 & \pi_2 \\ \hline \pi_3 & \pi_4 \end{array}$$

We derive two results mentioned in the text. First, we show that prices in states $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$ must be fully revealing. Second, we show that a necessary condition for information acquisition to be a strategic complement is that \tilde{x} and $\tilde{\theta}$ be correlated, i.e. that $\pi_1/\pi_3 \neq \pi_2/\pi_4$.

Since informed traders are worse off when others are informed (prices are pushed towards fundamentals, so the gains from buying the asset when $\tilde{\theta} = \bar{\theta}$ are lower), a necessary condition for learning to be a strategic complement is that uninformed traders also be worse off when there are more informed traders. We now consider all possible equilibria to see whether any can be consistent with this condition. We then examine when these can generate complementarity in information acquisition.

Note first that when $z = 0$, uninformed traders must undertake the same decision in equilibrium for a given realization of \tilde{x} regardless of the realization of $\tilde{\theta}$. Generically, then, we have four candidates for the demand schedules of uninformed traders when $z = 0$:

$$\begin{array}{cc} \text{Case I: } \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} \begin{array}{cc} x_0 & x_1 \\ \hline \text{buy} & \text{buy} \\ \hline \text{buy} & \text{buy} \end{array} & \text{Case II: } \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} \begin{array}{cc} x_0 & x_1 \\ \hline \text{don't buy} & \text{don't buy} \\ \hline \text{don't buy} & \text{don't buy} \end{array} \\ \\ \text{Case III: } \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} \begin{array}{cc} x_0 & x_1 \\ \hline \text{don't buy} & \text{buy} \\ \hline \text{don't buy} & \text{buy} \end{array} & \text{Case IV: } \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} \begin{array}{cc} x_0 & x_1 \\ \hline \text{buy} & \text{don't buy} \\ \hline \text{buy} & \text{don't buy} \end{array} \end{array}$$

Next, we consider the case where $z > 0$. We first argue that prices at $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$ must be fully revealing. For suppose there existed another pair $(x', \theta') \neq (x_0, \bar{\theta})$ such that $P(x', \theta') = P(x_0, \bar{\theta})$. The demand of uninformed traders, x^U , must be the same in both states. Demand for the asset by informed traders in state $(x_0, \bar{\theta})$ must equal $\frac{1}{P}$. The market clearing price $P(x_0, \bar{\theta})$ must then equal

$$\frac{w + z + (1 - z)\omega^U}{x_0 + 1} \quad (\text{A1})$$

where ω^U is the amount of wealth per trader spent on the asset by uninformed traders. Consider each of the remaining states. At $(x_1, \bar{\theta})$, demand of the informed is the same, and so the market clearing price is equal to $\frac{w + z + (1 - z)\omega^U}{x_1 + 1}$, which differs from (A1) given $x_0 \neq x_1$. At $(x_0, \underline{\theta})$, informed traders do not buy the asset, so the market clearing price is equal to $\frac{w + (1 - z)\omega^U}{x_0 + 1}$ which again cannot equal (A1) given $z > 0$. Finally, At $(x_1, \underline{\theta})$, informed traders do not buy the asset, and the market clearing price is equal to $\frac{w + (1 - z)\omega^U}{x_1 + 1}$ which is smaller than (A1) given $x_1 > x_0$ and $z > 0$. A similar argument can be used to rule out the possibility that there exists another state in which the price is identical to $P(x_1, \underline{\theta})$.

If the uninformed are to be worse off, it must be that $(x_1, \bar{\theta})$ and $(x_0, \underline{\theta})$ yield the same price. Otherwise the uninformed become fully informed and never make mistakes, whereas they do commit mistakes when $z = 0$. In other words, the only case in which complementarities can occur is if the information set of an uninformed trader changes from $\{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}$ to $\{\{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_4\}\}$ as some agents become informed.

We now work through the four candidate demand schedules for uninformed traders when $z = 0$:

Case I: Uninformed traders always buy when $z = 0$.

There are two candidate equilibria, depending on what uninformed traders do when $z > 0$:

(i) Uninformed traders continue to buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	buy	buy	buy	buy
$\underline{\theta}$	buy	buy	buy	don't buy

The implied market clearing prices $P(x, \theta)$ are given by

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$
$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$	$\frac{w+1-z}{x_0+1}$	$\frac{w}{x_1+1}$

In this case, agents are just as well off in $(x_0, \bar{\theta})$ and $(x_1, \bar{\theta})$. They are better off in $(x_1, \underline{\theta})$ since they no longer make a mistake. Although they continue to buy the asset in state $(x_0, \underline{\theta})$, they pay less for the asset and hence are better off. Moving to a higher z cannot make uninformed traders worse off.

(ii) Uninformed traders stop buying the assets in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	buy	buy	buy	don't buy
$\underline{\theta}$	buy	buy	don't buy	don't buy

The implied market clearing prices $P(x, \theta)$ are given by

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$	$\frac{w+1}{x_0+1}$	$\frac{w+z}{x_1+1}$
$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$	$\frac{w}{x_0+1}$	$\frac{w}{x_1+1}$

In this case, agents are just as well off in $(x_0, \bar{\theta})$. They are better off in $(x_0, \underline{\theta})$ and $(x_1, \underline{\theta})$ since they no longer make a mistake as they do when $z = 0$. But they are worse off in state $(x_1, \bar{\theta})$ since now they stop purchasing the asset even though it yields a high payoff. So this situation may be compatible with complementarities (although below we show it is not).

Case II: Uninformed traders never buy when $z = 0$.

There are two candidate equilibria, depending on what uninformed traders do when $z > 0$:

(i) Uninformed traders switch to buying the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	don't buy	don't buy	buy	buy
$\underline{\theta}$	don't buy	don't buy	buy	don't buy

The implied market clearing prices $P(x, \theta)$ are given by

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	$\frac{w}{x_0+1}$	$\frac{w}{x_1+1}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$
$\underline{\theta}$	$\frac{w}{x_0+1}$	$\frac{w}{x_1+1}$	$\frac{w+1-z}{x_0+1}$	$\frac{w}{x_1+1}$

Traders are just as well off at $(x_1, \underline{\theta})$. They are better off at $(x_1, \bar{\theta})$ and $(x_0, \bar{\theta})$ since they no longer make a mistake as they do when $z = 0$. But they are worse off in state $(x_0, \underline{\theta})$ since they now purchase an overvalued asset. So this case could be consistent with complementarities (although below we show it is not).

(ii) Uninformed traders continue not to buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

	$z = 0$		$z > 0$	
	x_0	x_1	x_0	x_1
$\bar{\theta}$	don't buy	don't buy	buy	don't buy
$\underline{\theta}$	don't buy	don't buy	don't buy	don't buy

The implied market clearing prices $P(x, \theta)$ are given by

$$\begin{array}{cc}
 z = 0 & z > 0 \\
 \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w}{x_0+1} & \frac{w}{x_1+1} \\ \frac{w}{x_0+1} & \frac{w}{x_1+1} \end{array} & \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w+1}{x_0+1} & \frac{w+z}{x_1+1} \\ \frac{w}{x_0+1} & \frac{w}{x_1+1} \end{array}
 \end{array}$$

Traders are just as well off at $(x_0, \underline{\theta})$ and $(x_1, \underline{\theta})$. In state $(x_1, \bar{\theta})$ they are just as well off, since they don't buy the asset (although note that the price changes). They are better off at $(x_0, \bar{\theta})$ since they no longer make a mistake. Moving to a higher z cannot make uninformed traders worse off.

Case III: Uninformed traders buy if x is low but not if x is high when $z = 0$.

There are two candidate equilibria, depending on what uninformed traders do when $z > 0$:

(i) Uninformed traders buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

$$\begin{array}{cc}
 z = 0 & z > 0 \\
 \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \text{don't buy} & \text{buy} \\ \text{don't buy} & \text{buy} \end{array} & \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \text{buy} & \text{buy} \\ \text{buy} & \text{don't buy} \end{array}
 \end{array}$$

The implied market clearing prices $P(x, \theta)$ are given by

$$\begin{array}{cc}
 z = 0 & z > 0 \\
 \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w}{x_0+1} & \frac{w+1}{x_1+1} \\ \frac{w}{x_0+1} & \frac{w+1}{x_1+1} \end{array} & \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w+1}{x_0+1} & \frac{w+1}{x_1+1} \\ \frac{w+1-z}{x_0+1} & \frac{w}{x_1+1} \end{array}
 \end{array}$$

Agents are just as well off in $(x_1, \bar{\theta})$. They are better off in $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$ since they don't make mistakes. But they are worse off in $(x_0, \underline{\theta})$ since they make a mistake. So this could be consistent with complementarities.

(ii) Uninformed traders do not buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

$$\begin{array}{cc}
 z = 0 & z > 0 \\
 \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \text{don't buy} & \text{buy} \\ \text{don't buy} & \text{buy} \end{array} & \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \text{buy} & \text{don't buy} \\ \text{don't buy} & \text{don't buy} \end{array}
 \end{array}$$

The implied market clearing prices $P(x, \theta)$ are given by

$$\begin{array}{cc}
 z = 0 & z > 0 \\
 \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w}{x_0+1} & \frac{w+1}{x_1+1} \\ \frac{w}{x_0+1} & \frac{w+1}{x_1+1} \end{array} & \begin{array}{c} \bar{\theta} \\ \underline{\theta} \end{array} & \begin{array}{cc} x_0 & x_1 \\ \frac{w+1}{x_0+1} & \frac{w+z}{x_1+1} \\ \frac{w}{x_0+1} & \frac{w}{x_1+1} \end{array}
 \end{array}$$

Agents are just as well off in $(x_0, \underline{\theta})$. They are better off in $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$ since they don't make mistakes. But they are worse off in state $(x_1, \bar{\theta})$ since they make a mistake. So this could be consistent with complementarities.

Case IV: Uninformed traders buy if x is high but not if x is low when $z = 0$.

There are two candidate equilibria, depending on what uninformed traders do when $z > 0$:

(i) Uninformed traders buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

$z = 0$	$z > 0$																		
<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	buy	don't buy	$\underline{\theta}$	buy	don't buy	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">buy</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	buy	buy	$\underline{\theta}$	buy	don't buy
	x_0	x_1																	
$\bar{\theta}$	buy	don't buy																	
$\underline{\theta}$	buy	don't buy																	
	x_0	x_1																	
$\bar{\theta}$	buy	buy																	
$\underline{\theta}$	buy	don't buy																	

The implied market clearing prices $P(x, \theta)$ are given by

$z = 0$	$z > 0$																		
<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$	$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_1+1}$</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1-z}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$	$\underline{\theta}$	$\frac{w+1-z}{x_0+1}$	$\frac{w}{x_1+1}$
	x_0	x_1																	
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$																	
$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$																	
	x_0	x_1																	
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+1}{x_1+1}$																	
$\underline{\theta}$	$\frac{w+1-z}{x_0+1}$	$\frac{w}{x_1+1}$																	

In this case, agents are just as well off in $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$. They are better off in $(x_0, \bar{\theta})$ since they no longer commit a mistake. Although they continue to make a mistake in $(x_0, \underline{\theta})$, they pay less for the overvalued asset, and so are better off. Moving to a higher z cannot make uninformed traders worse off.

(ii) Uninformed traders do not buy the asset in states $\{\omega_2, \omega_3\}$ after some traders become informed. Then we have

$z = 0$	$z > 0$																		
<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	buy	don't buy	$\underline{\theta}$	buy	don't buy	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> <td style="padding: 2px 10px; text-align: center;">don't buy</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	buy	don't buy	$\underline{\theta}$	don't buy	don't buy
	x_0	x_1																	
$\bar{\theta}$	buy	don't buy																	
$\underline{\theta}$	buy	don't buy																	
	x_0	x_1																	
$\bar{\theta}$	buy	don't buy																	
$\underline{\theta}$	don't buy	don't buy																	

The implied market clearing prices $P(x, \theta)$ are given by

$z = 0$	$z > 0$																		
<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$	$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px; text-align: center;">x_0</td> <td style="padding: 2px 10px; text-align: center;">x_1</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\bar{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+1}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w+z}{x_1+1}$</td> </tr> <tr> <td style="padding: 2px 10px; text-align: center;">$\underline{\theta}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_0+1}$</td> <td style="padding: 2px 10px; text-align: center;">$\frac{w}{x_1+1}$</td> </tr> </table>		x_0	x_1	$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+z}{x_1+1}$	$\underline{\theta}$	$\frac{w}{x_0+1}$	$\frac{w}{x_1+1}$
	x_0	x_1																	
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$																	
$\underline{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w}{x_1+1}$																	
	x_0	x_1																	
$\bar{\theta}$	$\frac{w+1}{x_0+1}$	$\frac{w+z}{x_1+1}$																	
$\underline{\theta}$	$\frac{w}{x_0+1}$	$\frac{w}{x_1+1}$																	

In this case, agents are just as well off in $(x_0, \bar{\theta})$ and $(x_1, \underline{\theta})$. They are better off in $(x_0, \underline{\theta})$ since they no longer commit a mistake. They are equally well off in $(x_1, \bar{\theta})$ since they don't buy (although prices change). Moving to a higher z cannot make uninformed traders worse off.

From the eight cases above, only four are potentially compatible with complementarities: I(ii), II(i), III(i), and III(ii). We consider each of these cases in turn.

Case I(ii): When $z = 0$, there are two prices in equilibrium: $\frac{w+1}{x_0+1}$ and $\frac{w+1}{x_1+1}$. The beliefs of uninformed traders are given by

$$\Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w+1}{x_0+1}\right) = \frac{\Pr(x_0, \bar{\theta})}{\Pr(x_0, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_1}{\pi_1 + \pi_3}$$

$$\Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w+1}{x_1+1}\right) = \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_1, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_4}$$

If uninformed traders prefer to buy the asset at all prices, then $P \leq E(\theta|P(\cdot, \cdot) = P)$ for all P . That is,

$$\begin{aligned}\frac{w+1}{x_0+1} &\leq \frac{\pi_1}{\pi_1+\pi_3}\bar{\theta} + \frac{\pi_3}{\pi_1+\pi_3}\underline{\theta} \\ \frac{w+1}{x_1+1} &\leq \frac{\pi_2}{\pi_2+\pi_4}\bar{\theta} + \frac{\pi_4}{\pi_2+\pi_4}\underline{\theta}\end{aligned}$$

The gain from becoming informed is given by

$$\pi_3 \left(1 - \underline{\theta} \frac{x_0+1}{w+1}\right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1+1}{w+1}\right)$$

Next, when $z > 0$, there are three prices in equilibrium: $\frac{w+1}{x_0+1}$, $\frac{w}{x_1+1}$, and a single price equal to $\frac{w+z}{x_1+1} = \frac{w}{x_0+1} = P_z^*$, which requires that

$$x_1 = \frac{w+z}{w}(x_0+1) - 1$$

for a particular value of z . The beliefs of an uninformed trader at P_z^* are given by

$$\Pr(\bar{\theta} | P(x, \theta) = P_z^*) = \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_3}$$

If traders prefer not to buy when the price is equal to P_z^* , then it must be the case that

$$P_z^* = \frac{w+z}{x_1+1} \geq \frac{\pi_2}{\pi_2+\pi_3}\bar{\theta} + \frac{\pi_3}{\pi_2+\pi_3}\underline{\theta}$$

which upon dividing through by P_z^* and rearranging yields

$$\pi_3 \left(1 - \frac{\underline{\theta}}{P_z^*}\right) \geq \pi_2 \left(\frac{\bar{\theta}}{P_z^*} - 1\right) \quad (\text{A2})$$

The gain from becoming informed is given by

$$\pi_2 \left(\bar{\theta} \frac{x_1+1}{w+z} - 1\right)$$

If uninformed traders gain more when $z > 0$ than when $z = 0$, then it must be true that

$$\begin{aligned}\pi_2 \left(\frac{\bar{\theta}}{P_z^*} - 1\right) &> \pi_3 \left(1 - \underline{\theta} \frac{x_0+1}{w+1}\right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1+1}{w+1}\right) \\ &> \pi_3 \left(1 - \underline{\theta} \frac{x_0+1}{w}\right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1+1}{w+1}\right) \\ &= \pi_3 \left(1 - \frac{\underline{\theta}}{P_z^*}\right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1+1}{w+1}\right)\end{aligned} \quad (\text{A3})$$

where the last step uses the fact that the common price $P_z^* = \frac{w}{x_0+1}$. Since $\pi_4 \left(1 - \underline{\theta} \frac{x_1+1}{w+1}\right) \geq 0$, conditions (A2) and (A3) are incompatible, so complementarities are not possible in this case.

Case II(i): When $z = 0$, there are two prices in equilibrium: $\frac{w}{x_0 + 1}$ and $\frac{w}{x_1 + 1}$. The beliefs of uninformed traders are given by

$$\begin{aligned}\Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w}{x_0 + 1}\right) &= \frac{\Pr(x_0, \bar{\theta})}{\Pr(x_0, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_1}{\pi_1 + \pi_3} \\ \Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w}{x_1 + 1}\right) &= \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_1, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_4}\end{aligned}$$

If uninformed traders prefer not to buy the asset at all prices, then $P \geq E(\theta \mid P(\cdot, \cdot) = P)$ for all P . That is,

$$\begin{aligned}\frac{w}{x_1 + 1} &\geq \frac{\pi_1}{\pi_1 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_1 + \pi_3} \underline{\theta} \\ \frac{w}{x_0 + 1} &\geq \frac{\pi_2}{\pi_2 + \pi_4} \bar{\theta} + \frac{\pi_4}{\pi_2 + \pi_4} \underline{\theta}\end{aligned}$$

The gain from becoming informed is given by

$$\pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_2 \left(\bar{\theta} \frac{x_1 + 1}{w} - 1 \right)$$

When $z > 0$, there are three prices in equilibrium: $\frac{w + 1}{x_0 + 1}$, $\frac{w}{x_1 + 1}$, and a single price equal to $\frac{w + 1}{x_1 + 1} = \frac{w + 1 - z}{x_0 + 1} = P_z^*$, so

$$x_1 = \frac{w + 1}{w + 1 - z} (x_0 + 1) - 1$$

The beliefs of an uninformed trader at P_z^* are given by

$$\Pr(\bar{\theta} \mid P(x, \theta) = P_z^*) = \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_3}$$

If traders want to buy at this price, then it must be the case that

$$P_z^* = \frac{w + 1}{x_1 + 1} \leq \frac{\pi_2}{\pi_2 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_2 + \pi_3} \underline{\theta}$$

which upon dividing through by P_z^* and rearranging yields

$$\pi_3 \left(1 - \frac{\underline{\theta}}{P_z^*} \right) \leq \pi_2 \left(\frac{\bar{\theta}}{P_z^*} - 1 \right) \quad (\text{A4})$$

The gain from becoming informed is given by

$$\pi_3 \left(1 - \underline{\theta} \frac{x_0 + 1}{w + 1 - z} \right)$$

If uninformed traders gain more when $z > 0$ than when $z = 0$, then it must be true that

$$\begin{aligned}\pi_3 \left(1 - \frac{\underline{\theta}}{P_z^*} \right) &> \pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_2 \left(\bar{\theta} \frac{x_1 + 1}{w} - 1 \right) \\ &> \pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_2 \left(\bar{\theta} \frac{x_1 + 1}{w + 1} - 1 \right) \\ &= \pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_2 \left(\frac{\bar{\theta}}{P_z^*} - 1 \right)\end{aligned} \quad (\text{A5})$$

Since $\pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) \geq 0$, conditions (A4) and (A5) are incompatible, so complementarities are not possible in this case.

Case III(i): When $z = 0$, there are two prices in equilibrium: $\frac{w}{x_0 + 1}$ and $\frac{w + 1}{x_1 + 1}$. The beliefs of uninformed traders are given by

$$\begin{aligned} \Pr \left(\bar{\theta} \mid P(x, \theta) = \frac{w}{x_0 + 1} \right) &= \frac{\Pr(x_0, \bar{\theta})}{\Pr(x_0, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_1}{\pi_1 + \pi_3} \\ \Pr \left(\bar{\theta} \mid P(x, \theta) = \frac{w + 1}{x_1 + 1} \right) &= \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_1, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_4} \end{aligned}$$

If uninformed traders buy the asset at price $\frac{w + 1}{x_1 + 1}$ but not at price $\frac{w}{x_0 + 1}$, it must be the case that $P \geq E \left(\theta \mid P(\cdot, \cdot) = \frac{w}{x_0 + 1} \right)$ and $P \leq E \left(\theta \mid P(\cdot, \cdot) = \frac{w + 1}{x_1 + 1} \right)$. That is,

$$\begin{aligned} \frac{w}{x_0 + 1} &\geq \frac{\pi_1}{\pi_1 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_1 + \pi_3} \underline{\theta} \\ \frac{w + 1}{x_1 + 1} &\leq \frac{\pi_2}{\pi_2 + \pi_4} \bar{\theta} + \frac{\pi_4}{\pi_2 + \pi_4} \underline{\theta} \end{aligned}$$

The gain from becoming informed is given by

$$\pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1 + 1}{w + 1} \right)$$

When $z > 0$, there are three prices in equilibrium: $\frac{w + 1}{x_0 + 1}$, $\frac{w}{x_1 + 1}$, and a single price equal to $\frac{w + 1}{x_1 + 1} = \frac{w + 1 - z}{x_0 + 1} = P_z^*$, so

$$x_1 = \frac{w + 1}{w + 1 - z} (x_0 + 1) - 1$$

The beliefs of an uninformed trader at P_z^* are given by

$$\Pr \left(\bar{\theta} \mid P(x, \theta) = P_z^* \right) = \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_3}$$

If uninformed traders want to buy at this price, then it must be the case that

$$P_z^* = \frac{w + 1}{x_1 + 1} \leq \frac{\pi_2}{\pi_2 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_2 + \pi_3} \underline{\theta}$$

The gain from becoming informed is given by

$$\pi_3 \left(1 - \underline{\theta} \frac{x_0 + 1}{w + 1 - z} \right)$$

Information acquisition is a strategic complement if

$$\pi_3 \left(1 - \underline{\theta} \frac{x_0 + 1}{w + 1 - z} \right) > \pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1 + 1}{w + 1} \right)$$

Since it is always possible to drive π_1 and π_4 to zero and π_3 to 1, it is always possible to generate complementarities with this type of equilibrium.

We now prove that complementarities can only occur if $\pi_1/\pi_3 \neq \pi_2/\pi_4$. First, note that when $z = 0$, prices are given by $\frac{w}{x_0 + 1}$ and $\frac{w + 1}{x_1 + 1}$. Since we choose x_0 and x_1 to equate $\frac{w + 1}{x_1 + 1}$ and $\frac{w + 1 - z}{x_0 + 1}$, it follows that

$$\frac{w + 1}{x_1 + 1} = \frac{w + 1 - z}{x_0 + 1} > \frac{w}{x_0 + 1}$$

provided $z < 1$. This implies that when $z = 0$, prices must be higher when supply is equal to x_1 than when it is equal to x_0 .

Now, suppose $\pi_1/\pi_3 = \pi_2/\pi_4$. Then we have

$$\frac{\pi_1}{\pi_1 + \pi_3} = \frac{\pi_2}{\pi_2 + \pi_4}$$

But the conditions for the demand schedule of the uninformed to be optimal when $z = 0$ require that

$$\begin{aligned} \frac{w}{x_0 + 1} &\geq \frac{\pi_1}{\pi_1 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_1 + \pi_3} \underline{\theta} \\ &= \frac{\pi_2}{\pi_2 + \pi_4} \bar{\theta} + \frac{\pi_2}{\pi_2 + \pi_4} \underline{\theta} \\ &\geq \frac{w + 1}{x_1 + 1} \end{aligned}$$

which contradicts the fact that $\frac{w + 1}{x_1 + 1} > \frac{w}{x_0 + 1}$ as we just demonstrated. It follows that $\pi_1/\pi_3 \neq \pi_2/\pi_4$.

Case III(ii): When $z = 0$, there are two prices in equilibrium: $\frac{w}{x_0 + 1}$ and $\frac{w + 1}{x_1 + 1}$. The beliefs of uninformed traders are given by

$$\begin{aligned} \Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w}{x_0 + 1}\right) &= \frac{\Pr(x_0, \bar{\theta})}{\Pr(x_0, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_1}{\pi_1 + \pi_3} \\ \Pr\left(\bar{\theta} \mid P(x, \theta) = \frac{w + 1}{x_1 + 1}\right) &= \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_1, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_4} \end{aligned}$$

If uninformed traders buy the asset at price $\frac{w + 1}{x_1 + 1}$ but not at price $\frac{w}{x_0 + 1}$, it must be the case that $P \geq E\left(\theta \mid P(\cdot, \cdot) = \frac{w}{x_0 + 1}\right)$ and $P \leq E\left(\theta \mid P(\cdot, \cdot) = \frac{w + 1}{x_1 + 1}\right)$. That is,

$$\begin{aligned} \frac{w}{x_0 + 1} &\geq \frac{\pi_1}{\pi_1 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_1 + \pi_3} \underline{\theta} \\ \frac{w + 1}{x_1 + 1} &\leq \frac{\pi_2}{\pi_2 + \pi_4} \bar{\theta} + \frac{\pi_4}{\pi_2 + \pi_4} \underline{\theta} \end{aligned}$$

The gain from becoming informed is given by

$$\pi_1 \left(\bar{\theta} \frac{x_0 + 1}{w} - 1 \right) + \pi_4 \left(1 - \underline{\theta} \frac{x_1 + 1}{w + 1} \right)$$

When $z > 0$, there are three prices in equilibrium: $\frac{w+1}{x_0+1}$, $\frac{w}{x_1+1}$, and a single price equal to $\frac{w+z}{x_1+1} = \frac{w}{x_0+1} = P_z^*$, so

$$x_1 = \frac{w+z}{w}(x_0+1) - 1$$

The beliefs of an uninformed trader at P_z^* are given by

$$\Pr(\bar{\theta} \mid P(x, \theta) = P_z^*) = \frac{\Pr(x_1, \bar{\theta})}{\Pr(x_1, \bar{\theta}) + \Pr(x_0, \underline{\theta})} = \frac{\pi_2}{\pi_2 + \pi_3}$$

If uninformed traders do not want to buy at this price, then it must be the case that

$$P_z^* = \frac{w+z}{x_1+1} \geq \frac{\pi_2}{\pi_2 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_2 + \pi_3} \underline{\theta}$$

The gain from becoming informed is given by

$$\pi_2 \left(\bar{\theta} \frac{x_1+1}{w+z} - 1 \right)$$

Information acquisition is a strategic complement if

$$\pi_2 \left(\bar{\theta} \frac{x_1+1}{w+z} - 1 \right) > \pi_1 \left(\bar{\theta} \frac{x_0+1}{w} - 1 \right) + \pi_4 \left(1 - \frac{\theta}{w+1} \right)$$

Since it is always possible to drive π_1 and π_4 to zero and π_2 to 1, it is always possible to generate complementarities with this type of equilibrium.

We now prove that complementarities can only occur if $\pi_1/\pi_3 \neq \pi_2/\pi_4$. First, note that when $z = 0$, prices are given by $\frac{w}{x_0+1}$ and $\frac{w+1}{x_1+1}$. Since we choose x_0 and x_1 to equate $\frac{w+z}{x_1+1}$ and $\frac{w}{x_0+1}$, it follows that

$$\frac{w}{x_0+1} = \frac{w+z}{x_1+1} < \frac{w+1}{x_1+1}$$

provided $z < 1$. This implies that when $z = 0$, prices must be higher when supply is equal to x_1 than when it is equal to x_0 .

Now, suppose $\pi_1/\pi_3 = \pi_2/\pi_4$. Then we have

$$\frac{\pi_1}{\pi_1 + \pi_3} = \frac{\pi_2}{\pi_2 + \pi_4}$$

But the conditions for the demand schedule of the uninformed to be optimal when $z = 0$ require that

$$\begin{aligned} \frac{w}{x_0+1} &\geq \frac{\pi_1}{\pi_1 + \pi_3} \bar{\theta} + \frac{\pi_3}{\pi_1 + \pi_3} \underline{\theta} \\ &= \frac{\pi_2}{\pi_2 + \pi_4} \bar{\theta} + \frac{\pi_2}{\pi_2 + \pi_4} \underline{\theta} \\ &\geq \frac{w+1}{x_1+1} \end{aligned}$$

which contradicts the fact that $\frac{w+1}{x_1+1} > \frac{w}{x_0+1}$ as we just demonstrated. It follows that $\pi_1/\pi_3 \neq \pi_2/\pi_4$.

To summarize, the only possible cases in which the value of information is greater when $z > 0$ than when $z = 0$ require that $\pi_1/\pi_3 \neq \pi_2/\pi_4$, i.e. \tilde{x} and $\tilde{\theta}$ must be correlated.

2. Verifying the Equilibrium for Continuously Distributed \tilde{x}

We now turn to the case where $\theta \in \{\underline{\theta}, \bar{\theta}\}$ with equal probability and

$$f(x|\tilde{\theta} = \underline{\theta}) = \begin{cases} (1-\lambda)/x^* & \text{if } x \in [0, x^*] \\ \lambda/x^* & \text{if } x \in (x^*, 2x^*] \end{cases}$$

$$f(x|\tilde{\theta} = \bar{\theta}) = \begin{cases} \lambda/x^* & \text{if } x \in [0, x^*] \\ (1-\lambda)/x^* & \text{if } x \in (x^*, 2x^*] \end{cases}$$

We wish to confirm that the demand schedules and equilibrium price functions in our note constitute an equilibrium.

Given the price function $P(x, \theta)$ provided in the text, we compute the beliefs over $\tilde{\theta}$ for all prices $P \in (\underline{\theta}, \bar{\theta})$ and confirm that the conjectured demand schedules in the text are optimal. For remaining prices, it is easy to see that demand is optimal. When prices are equal to $\underline{\theta}$ and $\bar{\theta}$, the state will be fully revealed when $z > 0$, and agents should be indifferent between the asset and money. For prices outside this range, beliefs do not matter; it will be dominant to either buy the asset or hold money.

For all $z \in [0, 1]$, under our maintained price function $P(x, \theta)$, any price $P \in \left(\underline{\theta}, \frac{\underline{\theta} + \bar{\theta}}{2}\right) \cup \left(\frac{\underline{\theta} + \bar{\theta}}{2}, \bar{\theta}\right)$ will occur at exactly two realizations, $(\underline{\theta}, x_0(P))$ and $(\bar{\theta}, x_1(P))$, where $x_0(P)$ and $x_1(P)$ denote the respective values of x at which the price function assumes a value of P . By Bayes' rule, the probability that $\tilde{\theta} = \bar{\theta}$ given any price $P \in \left(\underline{\theta}, \frac{\underline{\theta} + \bar{\theta}}{2}\right) \cup \left(\frac{\underline{\theta} + \bar{\theta}}{2}, \bar{\theta}\right)$ is given by

$$\Pr(\theta = \bar{\theta} | P(\cdot, \cdot) = P) = \frac{\frac{1}{2}f(x_1(P)|\tilde{\theta} = \bar{\theta})}{\frac{1}{2}f(x_0(P)|\tilde{\theta} = \underline{\theta}) + \frac{1}{2}f(x_1(P)|\tilde{\theta} = \bar{\theta})}$$

Consider first the case where $0 \leq z \leq \frac{1}{2}$. In this case, for $P \in \left(\frac{\underline{\theta} + \bar{\theta}}{2}, \bar{\theta}\right)$, it must be the case that $x_0(P) \leq x_1(P) < x^*$. Hence, for any $P \in \left(\frac{\underline{\theta} + \bar{\theta}}{2}, \bar{\theta}\right)$,

$$\Pr(\theta = \bar{\theta} | P(\cdot, \cdot) = P) = \frac{\frac{1}{2} \frac{\lambda}{x^*}}{\frac{1}{2} \frac{\lambda}{x^*} + \frac{1}{2} \frac{1-\lambda}{x^*}} = \lambda$$

and hence $E(\theta = \bar{\theta} | P(\cdot, \cdot) = P) = \lambda\bar{\theta} + (1-\lambda)\underline{\theta} < \frac{1}{2}(\bar{\theta} + \underline{\theta}) < P$ given $\lambda < \frac{1}{2}$. Hence, the expected value of the asset is lower than the price, and so uninformed traders should optimally refrain from buying the asset. By a similar argument, for $P \in \left(\underline{\theta}, \frac{\underline{\theta} + \bar{\theta}}{2}\right)$, $x^* < x_0(P) \leq x_1(P)$, and so

$$\Pr(\theta = \bar{\theta} | P(\cdot, \cdot) = P) = \frac{\frac{1}{2} \frac{1-\lambda}{x^*}}{\frac{1}{2} \frac{1-\lambda}{x^*} + \frac{1}{2} \frac{\lambda}{x^*}} = 1 - \lambda$$

in which case $E(\theta = \bar{\theta} | P(\cdot, \cdot) = P) = (1-\lambda)\bar{\theta} + \lambda\underline{\theta} > \frac{1}{2}(\bar{\theta} + \underline{\theta}) > P$ and so uninformed traders should optimally buy the asset.

Lastly, if $P = \frac{\underline{\theta} + \bar{\theta}}{2}$, the fact that $f(x|\tilde{\theta} = \bar{\theta}) = f(2x^* - x|\tilde{\theta} = \underline{\theta})$ implies

$$\int_{\frac{\underline{\theta} + \bar{\theta}}{2} - 1}^{\frac{2(w+1)}{\underline{\theta} + \bar{\theta}} - 1} f(x|\tilde{\theta} = \bar{\theta}) dx = \int_{\frac{2w}{\underline{\theta} + \bar{\theta}} - 1}^{\frac{2(w+1-z)}{\underline{\theta} + \bar{\theta}} - 1} f(x|\tilde{\theta} = \underline{\theta}) dx$$

which in turn implies

$$\Pr\left(\theta = \bar{\theta} \mid P(\cdot, \cdot) = \frac{\bar{\theta} + \underline{\theta}}{2}\right) = \frac{1}{2}$$

and so uninformed agents should be indifferent between buying the asset and not buying. Hence, for all prices $P \in (\underline{\theta}, \bar{\theta})$, our conjectured demand is optimal.

Next, we consider the case where $\frac{1}{2} < z \leq 1$. In this case, for a price $P \in \left(\frac{\underline{\theta} + \bar{\theta}}{2}, \bar{\theta}\right)$, the ordering can be either $x_0(P) < x_1(P) \leq x^*$ or $x_0(P) < x^* \leq x_1(P)$. The probability that $\tilde{\theta} = \bar{\theta}$ in these two respective cases is given by

$$\begin{aligned} \Pr(\theta = \bar{\theta} \mid P(\cdot, \cdot) = P) &= \frac{\frac{1}{2} \frac{1-\lambda}{x^*}}{\frac{1}{2} \frac{1-\lambda}{x^*} + \frac{1}{2} \frac{\lambda}{x^*}} = 1 - \lambda \\ \Pr(\theta = \bar{\theta} \mid P(\cdot, \cdot) = P) &= \frac{\frac{1}{2} \frac{1-\lambda}{x^*}}{\frac{1}{2} \frac{1-\lambda}{x^*} + \frac{1}{2} \frac{1-\lambda}{x^*}} = \frac{1}{2} \end{aligned}$$

In either case, the expected value $E(\theta = \bar{\theta} \mid P(\cdot, \cdot) = P) \leq \frac{1}{2}(\bar{\theta} + \underline{\theta}) < P$, so not purchasing the asset remains the optimal course of action at such a price. By a similar argument, for a price $P \in \left(\underline{\theta}, \frac{\underline{\theta} + \bar{\theta}}{2}\right)$, the conditional expectation $E(\theta = \bar{\theta} \mid P(\cdot, \cdot) = P) \geq \frac{1}{2}(\bar{\theta} + \underline{\theta}) > P$, so purchasing the asset remains the optimal course of action. Finally, by symmetry it remains true that

$$\Pr\left(\theta = \bar{\theta} \mid P(\cdot, \cdot) = \frac{\bar{\theta} + \underline{\theta}}{2}\right) = \frac{1}{2}$$

so agents are indifferent about buying the asset at this price. Again, for all prices $P \in (\underline{\theta}, \bar{\theta})$, our conjectured demand is optimal.

The last step is to confirm that prices clear markets. The market clearing condition is given by

$$zx^I(\tilde{\theta}, P) + (1-z)x^U(P) + \frac{w}{P} - \tilde{x} = 1$$

or alternatively

$$P(x, \theta) = \frac{w + z\omega^I + (1-z)\omega^U}{x + 1} \tag{A6}$$

where ω^I is the amount of wealth per trader spent on the asset by informed traders, and ω^U is analogously defined for unemployed workers. Using expression for $P(x, \theta)$ and the demand schedule for both types of traders confirms that the proposed price function is consistent with the market clearing price in (A6).

3. Properties of $g(z)$ at $z = 0.5$

In our note, we plot $g(z)$ against z for several numerical values and show that it has an upward sloping region. In this last section, we provide a graphical-based argument for why $g(z)$ must be increasing at $z = 0.5$.

Figure A1 illustrates the equilibrium price function $P(x, \theta)$ for $z = 0.5 - \varepsilon$ and $z = 0.5 + \varepsilon$ when ε is small. The heavy lines in each panel correspond to prices that occur with positive probability, while the thin lines correspond to prices that occur with zero probability when $\lambda = 0$ (or with very small probability if $\lambda > 0$). Formally, the only prices that occur with positive probability density are given by the set $\{P(x, \underline{\theta}) \mid x \leq x^*\} \cup \{P(x, \bar{\theta}) \mid x \geq x^*\}$.

The arrows in the figure are meant to represent the effect of increasing z on the equilibrium price function. In particular, an increase in z shifts the upper branch of $P(x, \bar{\theta})$ to the right, and the lower branch of $P(x, \underline{\theta})$ to the left. As evident from the figure, changes in z when $z < 0.5$ will only affect the equilibrium price function in a region that occurs with zero probability. For this reason, changes in z have no effect on the value of information $g(z)$ so long as $z < 0.5$. By contrast, when $z > 0.5$, changes in z affect the price function in a region that occurs with positive probability.

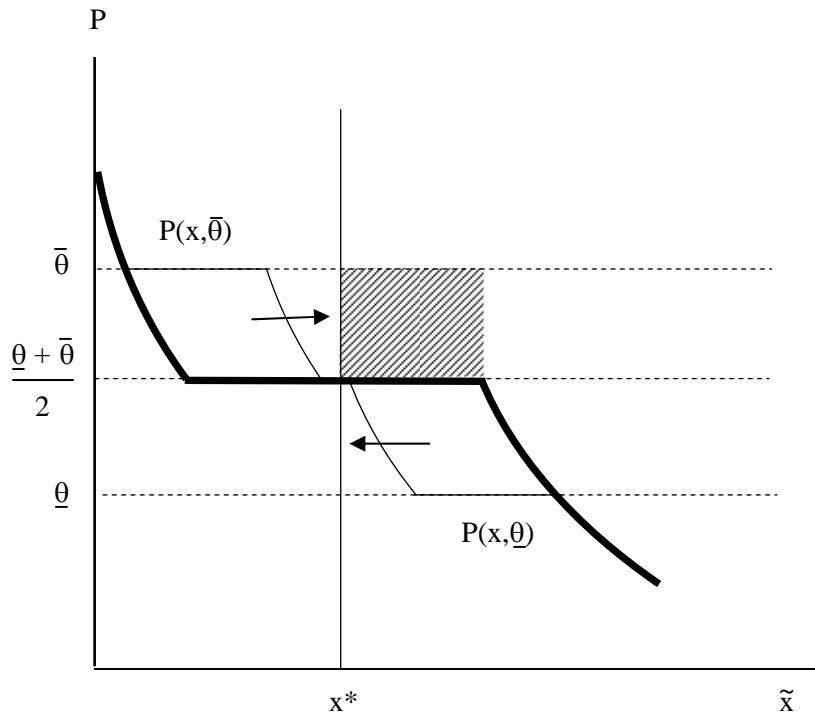
To understand why an increase in z just above 0.5 will lead to an increase in $g(z)$, we can use the fact that for $z \approx 0.5$, $g(z)$ is approximately proportional to the area between the equilibrium price and either $\underline{\theta}$ or $\bar{\theta}$, depending on whether an uninformed trader buys the asset or not, for prices in which an uninformed trader takes the wrong action. This area corresponds to the shaded area in the figure, which assumes an uninformed trader avoids buying the asset when indifferent, i.e. at a price of $\frac{\underline{\theta} + \bar{\theta}}{2}$. When $z < 0.5$, the only mistake this trader commits is not buying the asset when it is valuable. The cost is therefore proportional to the shaded rectangle in the left panel. As z rises to just above 0.5, we shave off a corner from this rectangle. This reflects the fact that when the trader fails to buy the asset, the price of the asset will be higher given z is higher, at least for some realizations of x . Learning the true value of $\tilde{\theta}$ in order to purchase the asset when $\tilde{\theta}$ is high is thus less valuable. At the same time, as z rises to just above 0.5, we also add the area of the trapezoid to the left of x^* . This reflects the fact that the trader might now purchase the asset when its fundamentals are low, specifically as informed prices drive down the price of the asset when $\tilde{\theta} = \underline{\theta}$. When $z < 0.5$, the trader would not have committed such a mistake. Since the area of the trapezoid exceeds the area of the triangle we subtract from the original cost, the total area must rise.

Formally, the implication of Figure A1 is that for $\lambda = 0$, the fall in the cost of mistakes a trader already committed is a second order effect when $z = 0.5$, while the increase in the probability of making a mistake is a first order effect at this value. Hence, $g(z)$ must increase as z rises in this region. We can see this analytically by differentiating the expression for $g(z)$:

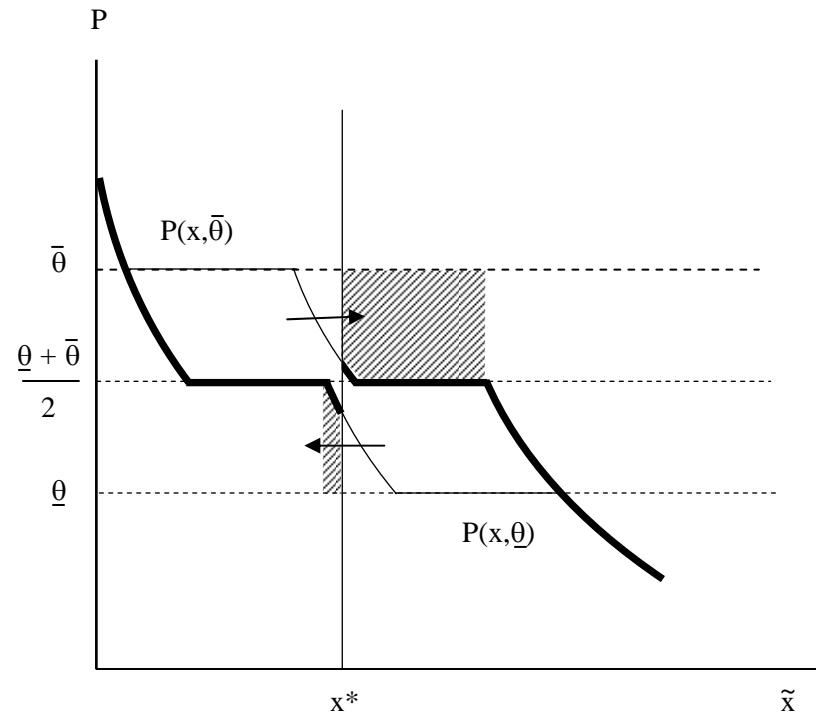
$$\begin{aligned}
g'(z) &= \frac{\bar{\theta} - \underline{\theta}}{(\underline{\theta} + \bar{\theta})^2} f(x^* | \tilde{\theta} = \bar{\theta}) \\
&+ \frac{1}{2} \int_{\frac{w+z}{\bar{\theta}} - 1}^{\frac{2(w+z)}{\underline{\theta} + \bar{\theta}} - 1} \bar{\theta} \frac{x+1}{(w+z)^2} f(x | \tilde{\theta} = \bar{\theta}) dx \\
&- \frac{1}{2} \int_{\frac{2(w+1-z)}{\underline{\theta} + \bar{\theta}} - 1}^{\frac{w+1-z}{\underline{\theta}} - 1} \underline{\theta} \frac{(x+1)}{(w+1-z)^2} f(x | \tilde{\theta} = \underline{\theta}) dx
\end{aligned} \tag{A7}$$

Setting $z = 0.5$, the limits of integration $\frac{2(w+z)}{\underline{\theta} + \bar{\theta}} - 1$ and $\frac{2(w+1-z)}{\underline{\theta} + \bar{\theta}} - 1$ in (A7) both collapse to x^* . When $\lambda = 0$, it follows that $f(x | \tilde{\theta} = \underline{\theta}) = 0$ for $x > x^*$, $f(x | \tilde{\theta} = \bar{\theta}) = 0$ for $x < x^*$, so the value of the two integrals in (A7) collapse to zero, confirming these terms are second order for z around

0.5. As for the remaining term, the fact that $\lim_{x \rightarrow x^{*+}} f(x|\tilde{\theta} = \bar{\theta}) = 1/x^*$ implies that as we take the limit from the right, this expression is equal to $\frac{\bar{\theta} - \underline{\theta}}{x^* (\underline{\theta} + \bar{\theta})^2}$, confirming that the first order effect of increasing z is unambiguously positive.



$z = 0.5 - \varepsilon$



$z = 0.5 + \varepsilon$

Figure A1

Working Paper Series

A series of research studies on regional economic issues relating to the Seventh Federal Reserve District, and on financial and economic topics.

Standing Facilities and Interbank Borrowing: Evidence from the Federal Reserve's New Discount Window <i>Craig Furfine</i>	WP-04-01
Netting, Financial Contracts, and Banks: The Economic Implications <i>William J. Bergman, Robert R. Bliss, Christian A. Johnson and George G. Kaufman</i>	WP-04-02
Real Effects of Bank Competition <i>Nicola Cetorelli</i>	WP-04-03
Finance as a Barrier To Entry: Bank Competition and Industry Structure in Local U.S. Markets? <i>Nicola Cetorelli and Philip E. Strahan</i>	WP-04-04
The Dynamics of Work and Debt <i>Jeffrey R. Campbell and Zvi Hercowitz</i>	WP-04-05
Fiscal Policy in the Aftermath of 9/11 <i>Jonas Fisher and Martin Eichenbaum</i>	WP-04-06
Merger Momentum and Investor Sentiment: The Stock Market Reaction To Merger Announcements <i>Richard J. Rosen</i>	WP-04-07
Earnings Inequality and the Business Cycle <i>Gadi Barlevy and Daniel Tsiddon</i>	WP-04-08
Platform Competition in Two-Sided Markets: The Case of Payment Networks <i>Sujit Chakravorti and Roberto Roson</i>	WP-04-09
Nominal Debt as a Burden on Monetary Policy <i>Javier Díaz-Giménez, Giorgia Giovannetti, Ramon Marimon, and Pedro Teles</i>	WP-04-10
On the Timing of Innovation in Stochastic Schumpeterian Growth Models <i>Gadi Barlevy</i>	WP-04-11
Policy Externalities: How US Antidumping Affects Japanese Exports to the EU <i>Chad P. Bown and Meredith A. Crowley</i>	WP-04-12
Sibling Similarities, Differences and Economic Inequality <i>Bhashkar Mazumder</i>	WP-04-13
Determinants of Business Cycle Comovement: A Robust Analysis <i>Marianne Baxter and Michael A. Kouparitsas</i>	WP-04-14
The Occupational Assimilation of Hispanics in the U.S.: Evidence from Panel Data <i>Maude Toussaint-Comeau</i>	WP-04-15

Working Paper Series *(continued)*

Reading, Writing, and Raisinets ¹ : Are School Finances Contributing to Children's Obesity? <i>Patricia M. Anderson and Kristin F. Butcher</i>	WP-04-16
Learning by Observing: Information Spillovers in the Execution and Valuation of Commercial Bank M&As <i>Gayle DeLong and Robert DeYoung</i>	WP-04-17
Prospects for Immigrant-Native Wealth Assimilation: Evidence from Financial Market Participation <i>Una Okonkwo Osili and Anna Paulson</i>	WP-04-18
Individuals and Institutions: Evidence from International Migrants in the U.S. <i>Una Okonkwo Osili and Anna Paulson</i>	WP-04-19
Are Technology Improvements Contractionary? <i>Susanto Basu, John Fernald and Miles Kimball</i>	WP-04-20
The Minimum Wage, Restaurant Prices and Labor Market Structure <i>Daniel Aaronson, Eric French and James MacDonald</i>	WP-04-21
Betcha can't acquire just one: merger programs and compensation <i>Richard J. Rosen</i>	WP-04-22
Not Working: Demographic Changes, Policy Changes, and the Distribution of Weeks (Not) Worked <i>Lisa Barrow and Kristin F. Butcher</i>	WP-04-23
The Role of Collateralized Household Debt in Macroeconomic Stabilization <i>Jeffrey R. Campbell and Zvi Hercowitz</i>	WP-04-24
Advertising and Pricing at Multiple-Output Firms: Evidence from U.S. Thrift Institutions <i>Robert DeYoung and Evren Örs</i>	WP-04-25
Monetary Policy with State Contingent Interest Rates <i>Bernardino Adão, Isabel Correia and Pedro Teles</i>	WP-04-26
Comparing location decisions of domestic and foreign auto supplier plants <i>Thomas Klier, Paul Ma and Daniel P. McMillen</i>	WP-04-27
China's export growth and US trade policy <i>Chad P. Bown and Meredith A. Crowley</i>	WP-04-28
Where do manufacturing firms locate their Headquarters? <i>J. Vernon Henderson and Yukako Ono</i>	WP-04-29
Monetary Policy with Single Instrument Feedback Rules <i>Bernardino Adão, Isabel Correia and Pedro Teles</i>	WP-04-30

Working Paper Series *(continued)*

Firm-Specific Capital, Nominal Rigidities and the Business Cycle <i>David Altig, Lawrence J. Christiano, Martin Eichenbaum and Jesper Linde</i>	WP-05-01
Do Returns to Schooling Differ by Race and Ethnicity? <i>Lisa Barrow and Cecilia Elena Rouse</i>	WP-05-02
Derivatives and Systemic Risk: Netting, Collateral, and Closeout <i>Robert R. Bliss and George G. Kaufman</i>	WP-05-03
Risk Overhang and Loan Portfolio Decisions <i>Robert DeYoung, Anne Gron and Andrew Winton</i>	WP-05-04
Characterizations in a random record model with a non-identically distributed initial record <i>Gadi Barlevy and H. N. Nagaraja</i>	WP-05-05
Price discovery in a market under stress: the U.S. Treasury market in fall 1998 <i>Craig H. Furfine and Eli M. Remolona</i>	WP-05-06
Politics and Efficiency of Separating Capital and Ordinary Government Budgets <i>Marco Bassetto with Thomas J. Sargent</i>	WP-05-07
Rigid Prices: Evidence from U.S. Scanner Data <i>Jeffrey R. Campbell and Benjamin Eden</i>	WP-05-08
Entrepreneurship, Frictions, and Wealth <i>Marco Cagetti and Mariacristina De Nardi</i>	WP-05-09
Wealth inequality: data and models <i>Marco Cagetti and Mariacristina De Nardi</i>	WP-05-10
What Determines Bilateral Trade Flows? <i>Marianne Baxter and Michael A. Kouparitsas</i>	WP-05-11
Intergenerational Economic Mobility in the U.S., 1940 to 2000 <i>Daniel Aaronson and Bhashkar Mazumder</i>	WP-05-12
Differential Mortality, Uncertain Medical Expenses, and the Saving of Elderly Singles <i>Mariacristina De Nardi, Eric French, and John Bailey Jones</i>	WP-05-13
Fixed Term Employment Contracts in an Equilibrium Search Model <i>Fernando Alvarez and Marcelo Veracierto</i>	WP-05-14
Causality, Causality, Causality: The View of Education Inputs and Outputs from Economics <i>Lisa Barrow and Cecilia Elena Rouse</i>	WP-05-15

Working Paper Series *(continued)*

Competition in Large Markets <i>Jeffrey R. Campbell</i>	WP-05-16
Why Do Firms Go Public? Evidence from the Banking Industry <i>Richard J. Rosen, Scott B. Smart and Chad J. Zutter</i>	WP-05-17
Clustering of Auto Supplier Plants in the U.S.: GMM Spatial Logit for Large Samples <i>Thomas Klier and Daniel P. McMillen</i>	WP-05-18
Why are Immigrants' Incarceration Rates So Low? Evidence on Selective Immigration, Deterrence, and Deportation <i>Kristin F. Butcher and Anne Morrison Piehl</i>	WP-05-19
Constructing the Chicago Fed Income Based Economic Index – Consumer Price Index: Inflation Experiences by Demographic Group: 1983-2005 <i>Leslie McGranahan and Anna Paulson</i>	WP-05-20
Universal Access, Cost Recovery, and Payment Services <i>Sujit Chakravorti, Jeffery W. Gunther, and Robert R. Moore</i>	WP-05-21
Supplier Switching and Outsourcing <i>Yukako Ono and Victor Stango</i>	WP-05-22
Do Enclaves Matter in Immigrants' Self-Employment Decision? <i>Maude Toussaint-Comeau</i>	WP-05-23
The Changing Pattern of Wage Growth for Low Skilled Workers <i>Eric French, Bhashkar Mazumder and Christopher Taber</i>	WP-05-24
U.S. Corporate and Bank Insolvency Regimes: An Economic Comparison and Evaluation <i>Robert R. Bliss and George G. Kaufman</i>	WP-06-01
Redistribution, Taxes, and the Median Voter <i>Marco Bassetto and Jess Benhabib</i>	WP-06-02
Identification of Search Models with Initial Condition Problems <i>Gadi Barlevy and H. N. Nagaraja</i>	WP-06-03
Tax Riots <i>Marco Bassetto and Christopher Phelan</i>	WP-06-04
The Tradeoff between Mortgage Prepayments and Tax-Deferred Retirement Savings <i>Gene Amromin, Jennifer Huang, and Clemens Sialm</i>	WP-06-05
Why are safeguards needed in a trade agreement? <i>Meredith A. Crowley</i>	WP-06-06

Working Paper Series *(continued)*

Taxation, Entrepreneurship, and Wealth <i>Marco Cagetti and Mariacristina De Nardi</i>	WP-06-07
A New Social Compact: How University Engagement Can Fuel Innovation <i>Laura Melle, Larry Isaak, and Richard Mattoon</i>	WP-06-08
Mergers and Risk <i>Craig H. Furfine and Richard J. Rosen</i>	WP-06-09
Two Flaws in Business Cycle Accounting <i>Lawrence J. Christiano and Joshua M. Davis</i>	WP-06-10
Do Consumers Choose the Right Credit Contracts? <i>Sumit Agarwal, Souphala Chomsisengphet, Chunlin Liu, and Nicholas S. Souleles</i>	WP-06-11
Chronicles of a Deflation Unforetold <i>François R. Velde</i>	WP-06-12
Female Offenders Use of Social Welfare Programs Before and After Jail and Prison: Does Prison Cause Welfare Dependency? <i>Kristin F. Butcher and Robert J. LaLonde</i>	WP-06-13
Eat or Be Eaten: A Theory of Mergers and Firm Size <i>Gary Gorton, Matthias Kahl, and Richard Rosen</i>	WP-06-14
Do Bonds Span Volatility Risk in the U.S. Treasury Market? A Specification Test for Affine Term Structure Models <i>Torben G. Andersen and Luca Benzoni</i>	WP-06-15
Transforming Payment Choices by Doubling Fees on the Illinois Tollway <i>Gene Amromin, Carrie Jankowski, and Richard D. Porter</i>	WP-06-16
How Did the 2003 Dividend Tax Cut Affect Stock Prices? <i>Gene Amromin, Paul Harrison, and Steven Sharpe</i>	WP-06-17
Will Writing and Bequest Motives: Early 20th Century Irish Evidence <i>Leslie McGranahan</i>	WP-06-18
How Professional Forecasters View Shocks to GDP <i>Spencer D. Krane</i>	WP-06-19
Evolving Agglomeration in the U.S. auto supplier industry <i>Thomas Klier and Daniel P. McMillen</i>	WP-06-20
Mortality, Mass-Layoffs, and Career Outcomes: An Analysis using Administrative Data <i>Daniel Sullivan and Till von Wachter</i>	WP-06-21

Working Paper Series *(continued)*

The Agreement on Subsidies and Countervailing Measures: Tying One's Hand through the WTO. <i>Meredith A. Crowley</i>	WP-06-22
How Did Schooling Laws Improve Long-Term Health and Lower Mortality? <i>Bhashkar Mazumder</i>	WP-06-23
Manufacturing Plants' Use of Temporary Workers: An Analysis Using Census Micro Data <i>Yukako Ono and Daniel Sullivan</i>	WP-06-24
What Can We Learn about Financial Access from U.S. Immigrants? <i>Una Okonkwo Osili and Anna Paulson</i>	WP-06-25
Bank Imputed Interest Rates: Unbiased Estimates of Offered Rates? <i>Evren Ors and Tara Rice</i>	WP-06-26
Welfare Implications of the Transition to High Household Debt <i>Jeffrey R. Campbell and Zvi Hercowitz</i>	WP-06-27
Last-In First-Out Oligopoly Dynamics <i>Jaap H. Abbring and Jeffrey R. Campbell</i>	WP-06-28
Oligopoly Dynamics with Barriers to Entry <i>Jaap H. Abbring and Jeffrey R. Campbell</i>	WP-06-29
Risk Taking and the Quality of Informal Insurance: Gambling and Remittances in Thailand <i>Douglas L. Miller and Anna L. Paulson</i>	WP-07-01
Fast Micro and Slow Macro: Can Aggregation Explain the Persistence of Inflation? <i>Filippo Altissimo, Benoît Mojon, and Paolo Zaffaroni</i>	WP-07-02
Assessing a Decade of Interstate Bank Branching <i>Christian Johnson and Tara Rice</i>	WP-07-03
Debit Card and Cash Usage: A Cross-Country Analysis <i>Gene Amromin and Sujit Chakravorti</i>	WP-07-04
The Age of Reason: Financial Decisions Over the Lifecycle <i>Sumit Agarwal, John C. Driscoll, Xavier Gabaix, and David Laibson</i>	WP-07-05
Information Acquisition in Financial Markets: a Correction <i>Gadi Barlevy and Pietro Veronesi</i>	WP-07-06