

# Product Market Evidence on the Employment Effects of the Minimum Wage <br> <br> Daniel Aaronson and Eric French 

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Daniel Aaronson and Eric French*

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#### Abstract

We infer the employment response to a minimum wage change by calibrating a model of employment for the restaurant industry. Whereas perfect competition implies employment falls and prices rise after a minimum wage increase, the monopsony model potentially implies the opposite. We show that estimated price responses are consistent with the competitive model. We place fairly tight bounds on the employment response, with the most plausible parameter values suggesting a 10 percent increase in the minimum wage lowers low skill employment by 2 to 4 percent and total restaurant employment by 1 to 3 percent.


[^0]
## 1 Introduction

Until the early 1990s, the consensus was that an increase in the minimum wage causes a small but statistically and economically significant loss in employment (e.g. Brown, Gilroy, and Kohen (1982)). While it was known that this need not be the case if firms have wagesetting power (Stigler (1946)), the empirical results confirmed the qualitative predictions of standard models of perfect competition, which most researchers suspected were relevant for industries which primarily employed minimum wage workers.

However, Card and Krueger's work in the early 1990s spawned a contentious debate over the magnitude, and perhaps even the sign, of the employment response. In a series of papers, they find no or, in some cases, a small positive employment response to an increase in the minimum wage. ${ }^{1}$ Moreover, they discuss a number of other facts that may be inconsistent with competitive markets but consistent with monopsony power, including a spike in the distribution of wages at the minimum and the prevalence of posted vacancies. Recent theoretical developments have been able to generate monopsony-like employment responses after minimum wage increases by introducing labor market search with frictions (Burdett and Mortensen (1998)), efficiency wages (Rebitzer and Taylor (1995), Manning (1995)), or monopsonistic competition with free entry (Bhaskar and To (1999)).

This work has not gone unchallenged, as exemplified by the discussions in Card and Krueger (2000) and Neumark and Wascher (2000). The latter authors, again in a series of papers (e.g. Neumark and Wascher (1996) for a review), consistently find an effect more in line with the Brown et al. (1982) literature review; a 10 percent increase in the minimum wage leads to roughly a 2 percent decrease in teen employment. ${ }^{2}$ Others, including Deere, Murphy, and Welch (1995), Kim and Taylor (1995), and Burkhauser, Couch, and Wittenburg (2000), find even larger negative effects, on the order of 2 to 6 percent. This confusion is particularly acute since the majority of these papers use the same sources of variation to

[^1]identify the employment elasticity (albeit often from different time periods or geographic areas): either the cross-sectional or time series co-movement of teenage employment and the minimum wage.

Many of these papers claim to be testing the market structure of low wage labor markets without explicitly showing what the competitive and monopsonistic models imply. In this paper, we do just that. Using data from the restaurant industry, we calibrate a model of employment determination to infer employment and price responses to a change in the minimum wage implied by the two market structures.

We initially calculate the responses assuming restaurants are price-takers in both the factor and product markets. ${ }^{3}$ Several pieces of information are supplied to the model, including factor costs, the intensity of usage of low-wage workers, the elasticity of substitution between factors, the elasticity of labor supply, and the elasticity of product demand. The model predicts that a 10 percent increase in the minimum wage reduces low skill restaurant industry employment by roughly 2.5 to 4 percent and total restaurant employment by 1.5 to 3 percent under perfect competition. A second implication of perfect competition is that higher labor costs are pushed onto consumers in the form of higher prices.

Next, we augment the model so employers potentially have monopsony power in the labor market. Under monopsony, employment potentially rises in response to an increase in the minimum wage. An implication is that when employment rises, output also rises, and thus output prices fall. Therefore, the competitive and monopsonistic models have different employment and output price predictions.

We use the models' output price implications to test for the potential importance of monopsony behavior in the labor market. This test relies on research (e.g. Aaronson 2001; Aaronson, French, and MacDonald 2005) that shows that most of the higher labor costs incurred by employers are pushed onto consumers in the form of higher prices, a finding that is in sharp contrast to the prediction of monopsony models. Consequently, we infer that few restaurants will increase employment in response to a minimum wage increase. Using the most plausible range of estimates of the model's key parameters, we find that the employment response to a 10 percent change in the minimum wage is likely between 2 and 4 percent for low skill workers and 1 to 3 percent for total restaurant employment. We argue that the

[^2]estimates are well within the bounds set in the empirical literature. All of these predictions are robust to allowing for monopolistic competition in the product market.

It is important to emphasize that our estimates are for the restaurant industry only. Nevertheless, this industry is a major employer of low-wage labor and therefore a particularly relevant one to study. ${ }^{4}$ However, as a result of different intensity of use of minimum wage labor, substitution possibilities, market structure, or demand for their products, other industries might face different employment responses.

The next section describes the basic framework of our study. In this section, we introduce the competitive and monopsony models and show how to use price pass-through to infer the extent of monopsony power. Many of the results from this section are discussed further in a longer version of this paper, Aaronson and French (2006). Section 3 and the appendix at the end of this paper discuss the main parameters used to calibrate the model. Finally, in sections 4 and 5 , the results are described and some concluding comments are offered.

## 2 The Model

This section outlines the structure of the model and the assumptions used to identify the employment response to a minimum wage change. We begin with the perfect competition case and then introduce monopsony power, offering some intuition for the ambiguous impact it has on employment. Finally, we provide a framework for bounding the importance of monopsony power in the labor market, using primarily the output price response to minimum wage changes.

### 2.1 Model Set Up

Throughout the paper, we assume that a large number, ${ }^{5} N$, of firms with identical production technologies and identical products are perfectly competitive in the product market,

[^3]sell their products at a price $p$, and choose their inputs to maximize profits $\pi$ :
\[

$$
\begin{equation*}
\pi(K, L, H, M)=p Q-r K-w_{L} L-w_{H} H-p_{M} M \tag{1}
\end{equation*}
$$

\]

where $Q=F(K, L, H, M)$ is a constant elasticity of substitution aggregator of low skill labor $L$, high skill labor $H$, capital $K$, and materials $M$, purchased at prices $r, w_{L}{ }^{6} w_{H}$, and $p_{M}$, respectively:

$$
\begin{equation*}
Q=\left(\alpha_{K} K^{\rho}+\alpha_{L} L^{\rho}+\alpha_{H} H^{\rho}+\left(1-\alpha_{K}-\alpha_{L}-\alpha_{H}\right) M^{\rho}\right)^{\frac{1}{\rho}} \tag{2}
\end{equation*}
$$

where $\sigma \equiv \frac{1}{1-\rho}$ is the partial elasticity of substitution between $K, L_{L}, L_{H}$ and $M$ in the production of $Q .{ }^{7}$ The assumption of constant elasticities of substitution between all factors is roughly consistent with the empirical literature described in Hamermesh (1993). The market price is:

$$
\begin{equation*}
p=Z\left(\sum_{n=1}^{N} Q_{n}\right)^{-\frac{1}{n}} \tag{3}
\end{equation*}
$$

where $\sum_{n=1}^{N} Q_{n}$ is market output and $\eta$ is the elasticity of demand for the output good. Although the case we consider here is competition in the product market, augmenting the model for monopolistic competition in the product market complicates the analysis but does not change the results. ${ }^{8}$

### 2.2 Long Run Price and Employment Responses of Competitive Firms

Assume all low skilled workers are paid the minimum wage but high skilled workers are paid above it (section 2.5 relaxes the assumption that all low skilled workers are paid the minimum wage). Also assume that the minimum wage only affects the wage of low skilled workers and that the supply of all factors of production to the firm are perfectly elastic.

[^4]Assuming that all firms can adjust all factors, the price and employment responses to a minimum wage hike, derived in appendix B of Aaronson and French (2006), have simple analytical solutions. When firms are perfectly competitive (or monopolistically competitive as in appendix F of Aaronson and French (2006)) and have a constant returns production function, economic profits must be zero both before and after the wage change. Consequently, all changes in labor costs are passed onto the consumer, i.e.

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=s_{L}, \tag{4}
\end{equation*}
$$

where $s_{L} \equiv \frac{w_{L} L}{w_{L} L+w_{H} H+r K+p_{M} M}$ is low skill labor's share of total costs. ${ }^{9}$
Because prices are higher after the minimum wage hike, the quantity produced declines by $s_{L} \eta$ percent for every one percent increase in the minimum wage.

The elasticity of demand for low skill labor $\lambda \equiv \frac{d \ln L}{d \ln w_{m i n}}$, is:

$$
\begin{equation*}
\lambda=-\left(1-\left(\frac{d \ln p}{d \ln w_{\min }}\right)\right) \sigma-\left(\frac{d \ln p}{d \ln w_{\min }}\right) \eta . \tag{5}
\end{equation*}
$$

$\lambda$ is rising (in absolute value) in the elasticity of substitution between labor and the other factors of production (the "substitution effect") ${ }^{10}$ and the elasticity of demand for the output good (the "scale effect"). The substitution effect measures the change in factor ratios given a wage change, holding output fixed. The scale effect measures the change in output given a wage change, holding factor ratios fixed. Inserting equation (4) into equation (5) yields equation (2.7a') in Hamermesh (1993) and (11.6) in Card and Krueger (1995).

Nevertheless, there are several reasons why one might be wary of the long-run competitive employment response shown in equation (5). It is to two of these concerns that we turn next. In Section 2.3, we discuss the short-run employment response when firms cannot adjust their capital stock in response to a minimum wage change. Section 2.4 introduces monopsony

[^5]power in the labor market.

### 2.3 Short-Run Price and Employment Responses of Competitive Firms

In the short-run, firms might not be able to adjust their capital stock in response to a minimum wage change, perhaps because of high adjustment costs or the irreversibility of these investments. ${ }^{11}$ In order to capture the concept of fixed capital, we assume that firms choose capital in order to maximize profits, keeping the wage of low skill labor constant. But after the wage changes, the firm can adjust all factors except capital. ${ }^{12}$

In appendix C of Aaronson and French (2006), we show that if capital cannot adjust, the price response to a change in the minimum wage is:

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=\frac{s_{L}}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)} \tag{7}
\end{equation*}
$$

Inserting equation (7) into equation (5) gives the short run employment response.
An interesting special case arises when $\sigma$, the partial elasticity of substitution between factor inputs, and $\eta$, the elasticity of demand for the output good, are equal. In this situation, the price response is identical whether capital is fixed (equation (7)) or flexible (equation (4)). Additional labor costs are always and fully pushed onto consumers in the form of higher prices. To see why, note that the elasticity of capital in response to changes in the minimum wage when capital can adjust is:

$$
\begin{equation*}
\frac{d \ln K}{d \ln w_{\min }}=s_{L}(\sigma-\eta) \tag{8}
\end{equation*}
$$

Therefore, if $\sigma=\eta$, we should expect no capital response to changes in the minimum wage,

[^6]even if capital can adjust. Thus the fixity of capital is irrelevant.
A second interesting case is when the production function is Leontief (i.e. $\sigma=0$ ). So long as the firm can cover its variable costs (i.e., $p Q>w_{L} L+w_{H} H+p_{K} K$ ), firms cannot increase profits by reducing output. Therefore, prices and employment are unaffected by changes in the minimum wage. One notable application is putty-clay models (Johansen (1959), Gilchrist and Williams (2000)). Although most empirical studies find evidence of positive substitutability between factors of production (Hamermesh, 1993), putty-clay models develop the idea that there may be substitutability between factors for firms being put into place, but no substitutability for firms already in place. Therefore, substitutability between factors only exist in the long run, as new firms enter the marketplace. In the short run, putty-clay models also predict no employment or price response to minimum wage hikes.

Regardless, the key intuition, formalized in equation (5), is that price and employment responses are linked.

Finally, part of the reduction in low skill labor is potentially offset by increases in high skill labor. The employment response of high skill labor is

$$
\begin{equation*}
\frac{d \ln H}{d \ln w_{\min }}=\left(\frac{d \ln p}{d \ln w_{\min }}\right)(\sigma-\eta) . \tag{9}
\end{equation*}
$$

The first term shows the change in product price (which is also equal to the change in marginal cost). The second term has offsetting substitution and scale effects. After the minimum wage increases, high skill labor becomes relatively cheaper than low skill labor, causing firms to substitute to the former. However, the level of output falls, causing the firm to reduce all factors. If $\sigma=\eta$, this offset is exact and we should expect no high skill labor response to changes in the minimum wage.

### 2.4 The Short Run Employment and Price Responses of Monopsonistic Firms

Next, we derive the employment and price effects of minimum wage changes when firms are monopsonists in the labor market, or more specifically, monopsonistically competitive, as in Dickens et al. (1999) or Manning (2003). The contribution of this model relative to others is that we endogenize prices. The production function and product market are the same as in the previous section. We show that employment may rise in response to a minimum wage
hike, but this implies prices will fall.
Many researchers have argued that fast food restaurants are highly competitive and therefore monopsony power is likely negligible. ${ }^{13}$ However, models such as Burdett and Mortensen (1998) where employee search is costly often imply that employers have some degree of monopsony power. Furthermore, Card and Krueger (1995) document several empirical facts that may be inconsistent with competitive models but are consistent with monopsony models. For example, they argue that a well-documented spike in the wage distribution at the minimum implies that, unless there is a similarly sized and placed spike in the distribution of ability, some workers are not paid their marginal revenue product of labor. We show below that the presence of a spike in the minimum wages is not necessarily evidence of monopsony power. But other facts, such as the existence of posted vacancies, are not consistent with competition if it is costly to post a vacancy. Competitive theory implies that an employer can obtain an unlimited number of workers at the going wage rate, and that any vacancy is immediately filled. Therefore, it seems plausible that restaurants have some amount of market power.

In this model, all low skill workers are identical in their productivity, but because of differences in local labor market conditions, all firms in some markets pay above the minimum wage and all firms in other markets pay the minimum wage. ${ }^{14}$ Each of the $N$ firms within a given local labor market face the labor supply curve ${ }^{15}$ for low skill labor:

$$
\begin{equation*}
L_{n}=\theta^{-\gamma_{1}} w_{L, n}^{\gamma_{1}} w_{L,-n}^{\gamma_{2}-\gamma_{1}} \tag{10}
\end{equation*}
$$

where $w_{L, n}$ is the wage offered by the $n^{t h}$ firm and $w_{L,-n}$ is the average wage offered by all other firms in the market. Therefore, all $N$ firms compete in the same product and labor market. ${ }^{16}$ We assume that $\gamma_{1} \geq \gamma_{2}$, so that the quantity of labor supplied to the firm is increasing in the wage offered by the firm, and is weakly falling in the wage offered by other firms in the market. We also assume that $\gamma_{2} \geq 0$; if all firms in the market increase their wage,

[^7]the total quantity of labor supplied in the market will increase. We assume that $\theta$ potentially varies by labor market, although we do not give $\theta$ a subscript for notational convenience. Therefore, within each local labor market, all firms face the same wage and output price but the (low skill) wage and output price varies across labor markets.

Using equation (10), the inverse labor supply curve for low skill labor is:

$$
\begin{equation*}
w\left(L_{n}\right)=\theta L_{n}^{\frac{1}{\gamma_{1}}} w_{L,-n}^{1-\frac{\gamma_{2}}{\gamma_{1}}} \tag{11}
\end{equation*}
$$

and their offered wage is

$$
\begin{equation*}
w_{L}=\max \left\{w(L), w_{\min }\right\} . \tag{12}
\end{equation*}
$$

Note that in equilibrium all firms within a given market will purchase the same amount of factor inputs and will have the same level of output. Therefore, we drop the $n$ subscripts for notational convenience.

Figure 1 shows the competitive and monopsony solutions to the firm's problem. The competitive solution (if the firm is a price taker in the labor market with an exogenous wage $\left.w^{* *}\right)$ is for the firm to hire $L^{* *}$ workers at a wage $w^{* *}$. However, if the firm has monopsony power, the firm will pay only $w^{*}$ and will hire $L^{*}$ workers.

Whether employment rises, falls or remains constant in response to an increase in the minimum wage is determined by the level of the minimum wage. The simplest case occurs when the minimum wage is not binding $\left(w_{\min }<w^{*}\right)$. In this case, a small change in the minimum wage has no effect on employment. Equilibrium employment and wages are $L^{*}$ and $w^{*}$, respectively.

Now, suppose the minimum wage is set between $w^{*}$ and $w^{* *}$. In this case, employment in a labor market with a minimum wage (e.g. $L_{\text {min }}$ in figure 2) is greater than employment in the absence of the minimum wage $\left(L^{*}\right) .{ }^{17}$ The intuition for this result is that although the

[^8]

Figure 1: ILLustration of monopsony EQuilibrium
minimum wage increases the average cost of labor for the firm, it reduces the marginal cost of labor from $\left(1+\frac{1}{\gamma_{1}}\right) w^{*}$ to $w_{\text {min }}$. Below $L_{m i n}$, the marginal cost of labor is the minimum wage. Whether the firm hires $L_{\text {min }}$ or $L_{\text {min }}-1$ workers, all workers are paid $w_{m i n}$. However, for employment levels above $L_{\text {min }}$, the marginal cost of labor is above the minimum wage; no additional workers will work for $w_{\min }$. The employer must increase the pay of all workers in order to obtain an additional one. Consequently, employment is determined by the intersection of the minimum wage and the inverse labor supply function $w(L)$. Therefore, increases in the minimum wage lead to increases in employment for $w_{\min } \in\left[w^{*}, w^{* *}\right)$. Specifically, the percent change in employment in the market (and thus the percent change in employment for every firm in the market) is

$$
\begin{equation*}
\left.\frac{d \ln L}{d \ln w_{\min }}\right|_{w_{L, n}=w_{L,-n}=w_{\min }}=\gamma_{2} . \tag{13}
\end{equation*}
$$

The important insight is that although $\gamma_{1}$ is the relevant parameter for understanding the
gap between the wage and the marginal revenue product of labor, ${ }^{18}$ it is $\gamma_{2}$ that is important for understanding the employment response to the minimum wage.


Figure 2: Illustration of monopsony equilibrium with minimum wage (Bold line denotes $\ln$ MC(L) Curve)

We now solve out for the price response under monopsony when $w_{\min } \in\left[w^{*}, w^{* *}\right)$. Appendix D in Aaronson and French (2006) shows that, given equation (13) but allowing for prices and other factors of production to adjust in the same way we did when allowing for a competitive labor market, yields

$$
\left.\frac{d \ln p}{d \ln w_{\min }}\right|_{w_{L, n}=w_{L,-n}=w_{\min }}= \begin{cases}-\frac{s_{L} \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma^{\prime}\left(1-s_{L}\right)+s_{L} \eta\right)} & \text { if capital can adjust }  \tag{14}\\ -\frac{s_{L} \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\left(s_{L}+s_{K}\right) \eta\right)} & \text { if capital cannot adjust }\end{cases}
$$

for some $\frac{1}{\gamma(\theta)} \in\left[0, \frac{1}{\gamma_{1}}\right]$, where $\frac{1}{\gamma(\theta)}=\left(1+\frac{1}{\gamma_{1}}\right)^{\left(\frac{\ln L^{* *}-\ln L}{\ln L^{* *}-\ln L^{*}}\right)}-1$ measures the difference between the wage and the marginal revenue product of labor, the vertical distance between $\ln w(L)$ and $\ln \operatorname{MRP}(\mathrm{L})$ in figure 1.

[^9]Finally, if the minimum wage lies above the point of intersection of the inverse labor supply function and the marginal revenue product of labor function $\operatorname{MRP}(\mathrm{L})$, i.e., $w_{\text {min }}>w\left(L^{* *}\right)$, employment falls as the minimum wage rises. The marginal cost of labor function, $\mathrm{MC}(\mathrm{L})$, is always equal to the minimum wage. The firm then sets employment by equating the $\operatorname{MRP}(\mathrm{L})$ curve and the minimum wage, as in Sections 2.2 and 2.3. Because the $\operatorname{MRP}(\mathrm{L})$ curve slopes down, increases in the minimum wage unambiguously lead to a reduction in employment. The magnitude of the disemployment effect was discussed in section 2.2 for the case where capital can adjust and 2.3 for the case where capital cannot adjust.

Note that the price response in equation (14) is unambiguously negative. Also recall that equations (4) and (7) showed that, under competition (or if $w_{\min }>w^{* *}$ ), wage hikes unambiguously increase prices. Therefore, in response to a minimum wage hike, employment and price changes are always negatively related. ${ }^{19}$ Employment falls and prices rise under perfect competition, and employment can rise and price can fall under monopsony. Consequently, price data offers an alternative means of inferring the importance of monopsony power in the labor market.

### 2.5 Using Price Pass-Through to Infer the Extent of Monopsony Power

This section proposes a method to infer the extent to which monopsonistic behavior is important using empirical estimates of price pass-through, $E\left[\frac{d \ln p}{d \ln w_{m i n}}\right]$. Furthermore, we show that once we infer the importance of monopsony behavior, we can infer the employment response to increasing the minimum wage.

We allow for the fact that the minimum wage binds in some labor markets, but not in others, because of variation in labor supply (i.e., $\theta$ ). Given the results above, and assuming that capital cannot adjust, the price response to a minimum wage change is:

$$
\frac{d \ln p}{d \ln w_{\min }}(\theta)= \begin{cases}\frac{s_{L}}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)} & \text { if } w_{\min } \geq w^{* *}  \tag{15}\\ -\frac{s_{L} \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\left(s_{L}+s_{K}\right) \eta\right)} & \text { if } w^{*} \leq w_{\min }<w^{* *} \\ 0 & \text { if } w_{\min }<w^{*} .\end{cases}
$$

Line 1 is derived in equation (7) and line 2 in equation (14). If the minimum wage does not

[^10]bind (line 3), output prices will not respond to a minimum wage increase.
The employment response is:
\[

\lambda(\theta)= $$
\begin{cases}-\sigma-(\eta-\sigma)\left(\frac{s_{L}}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)}\right) & \text { if } w_{\min } \geq w^{* *}  \tag{16}\\ \gamma_{2} & \text { if } w\left(L^{*}\right) \leq w_{\min }<w^{* *} \\ 0 & \text { if } w_{\min }<w^{*} .\end{cases}
$$
\]

Line 1 is derived using equation (5), plus the price response in line 1 of equation (15). Line 2 is derived in equation (13). If the minimum wage is not binding, as in line 3 , there is no employment response.

Define $\operatorname{prob}\left(w_{\min }>w^{*}\right)$ as the share of firms that pay the minimum wage and $V=$ $\operatorname{prob}\left(w_{\min } \geq w^{* *} \mid w_{\min }>w^{*}\right)$ as the share of firms facing a binding minimum wage such that the minimum wage is equal to the $\operatorname{MRP}(L)$. If $V=1$, then all firms affected by the minimum wage behave competitively in the labor market. If $V=0$, then all firms affected behave as monopsonists. If we knew $V$, we could use equation (16) to compute the average employment response. If all firms have the same elasticities and factor shares, then
$E[\lambda]=\int_{\theta} \lambda(\theta) d F(\theta)=-\operatorname{prob}\left(w_{\min }>w^{*}\right) \times\left(V\left(\sigma+(\eta-\sigma) \frac{s_{L}}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)}\right)-(1-V)\left(\gamma_{2}\right)\right)$.

Fortunately, we can calculate $V$ by using the extent of price pass-through. Analogous to equation (17), the average price response is

$$
\begin{align*}
E\left[\frac{d \ln p}{d \ln w_{\min }}\right] & =\int_{\theta} \frac{d \ln p}{d \ln w_{\min }}(\theta) d F(\theta) \\
& =\operatorname{prob}\left(w_{\min }>w^{*}\right) s_{L} \times\left(\left[\frac{V}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)}\right]-\left[\frac{(1-V) \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\left(s_{L}+s_{K}\right) \eta\right)}\right]\right) \tag{18}
\end{align*}
$$

Rearranging equation (18), we can solve explicitly for $V$ :

$$
\begin{equation*}
V=\frac{E\left[\frac{d \ln p}{d \ln w_{\text {min }}}\right] \frac{1}{p r o b\left(w_{\text {min }}>w^{*}\right)}+\frac{s_{L} \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\left(s_{L}+s_{K}\right) \eta\right)}}{\frac{s_{L}}{s_{K}\left(\frac{n}{\sigma}\right)+\left(1-s_{K}\right)}+\frac{s_{L} \gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right)}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\left(s_{L}+s_{K}\right) \eta\right)}} . \tag{19}
\end{equation*}
$$

Values of $E\left[\frac{d \ln p}{d \ln w}\right], s_{K}, \gamma_{1}, \gamma_{2}, \eta, \sigma$ can be computed or taken from the existing literature. We report the values that we use in table 1 of section 3 and the appendix. However, we do not have direct estimates of $\operatorname{prob}\left(w_{\min }>w^{*}\right)$ or $s_{L}$. In section 2.6 we show how to infer these objects using additional information.

In deriving equations (18) and (19), we assumed that $s_{L}$ and $s_{K}$ do not vary across labor markets. However, the theory presented above indicates that the factor mix (and thus $s_{L}$ and $s_{K}$ ) varies across labor markets. In the appendix, we provide empirical evidence, based on the 1997 Economic Census for Accommodations and Food Services and the outgoing rotation files of the CPS, that there is little geographic variation in labor's share. Appendix E in Aaronson and French (2006) also shows that, given the calibrated values described in this papers appendix, geographic differences in low skill labor's share are likely small, varying by less than 15 percent (or 4 percentage points) across labor markets. ${ }^{20}$

We make two final observations about the model. First, all of the competitive and monopsony predictions described in this section are robust to allowing for monopolistic competition in the product market, so long as there is a constant elasticity of demand. This result is established in appendix F of Aaronson and French (2006).

Second, this model generates a spike in the distribution of wages at the minimum, even if monopsony power is nonexistent. ${ }^{21}$ This is the case so long as $\sigma$ and $\eta$ are finite (i.e., labor is not a perfect substitute for materials or capital and there is finite elasticity of demand for the output good). Under these reasonable assumptions, low skill labor will still be used as a factor of production, even when the price of labor rises, and increases in output prices will

[^11]lead to a reduction but not cessation in output. In a geographically segmented labor market, with heterogeneous, but not perfectly substitutable, labor, raising the minimum wage slightly above the competitive wage will not shut down the industry. Even if labor were homogenous, increasing the minimum wage will not shut down an industry. Therefore, higher labor costs caused by an increase in the minimum wage can be (partially) pushed onto consumers.

### 2.6 Aggregation Issues

Ideally, we would like to use parameters to calibrate the model that correspond to restaurants that pay at or near the minimum wage. Unfortunately, we often must rely on estimates in the literature that come from more aggregated sources, particularly the entire restaurant industry. If all restaurants were identical, facing identical labor and product markets, using aggregate data would not be a problem. But clearly this is not the case.

Perhaps the most serious concern is that we do not know either the fraction of restaurants that pay the minimum wage (i.e., $\operatorname{prob}\left(w_{\min }>w^{*}\right)$ ), or low wage labor's share at restaurants that pay the minimum wage, $s_{L}$. However, we do have quite a bit of related information, including the share of restaurant workers who are paid the minimum wage $\left(\operatorname{prob}\left(w_{\min }=w_{i}\right)\right)$, labor's share of total costs in the restaurant industry $\left(s_{L}+s_{H}\right)$, the fraction of workers who are low skill $(\operatorname{prob}(L))$ at any given restaurant (and are thus paid the minimum wage in low wage markets), the average number of workers per restaurant by high wage ( $Z_{-\min }$ ) and low wage ( $Z_{\text {min }}$ ) establishments, and minimum wage labor's share of the aggregate wage bill.

In order to recover $\left.\operatorname{prob}\left(w_{\min }>w^{*}\right)\right)$ and $s_{L}$, we assume that there are only three types of labor markets (i.e., $\theta$ can take on only three different values): those that pay the minimum wage to workers and behave competitively (i.e., markets where the minimum wage intersects the $\operatorname{MRP}(\mathrm{L})$ curve for every firm in the market), those that behave monopsonistically (i.e., markets where the minimum wage intersects the labor supply curve for every firm in the market), and markets where the minimum wage does not bind. In markets where the minimum does not bind, all firms pay their low skill workers wagediff $\times w_{\text {min }}$. Furthermore, high wage workers are paid a constant multiple of the wage of low skill workers within the market. Thus, there is a four point wage distribution, with heterogeneity in two dimensions: high and low skill workers, and high and low wage labor markets. We assume that $\operatorname{prob}(L)$
is the same in high wage and low wage labor markets. ${ }^{22}$
In appendix $G$ of Aaronson and French (2006), we show that

$$
\begin{equation*}
\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)=\frac{\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L)} \frac{Z_{-\min }}{Z_{\min }}}{1-\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L)}+\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L)} \frac{Z_{-\min }}{Z_{\min }}} \tag{20}
\end{equation*}
$$

where the right hand side of equation (20) collapses to $\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{p r o b(L)}$ when $\frac{Z_{-\min }}{Z_{\text {min }}}=1$. Using household data, we can measure $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ fairly accurately.

Because we have information on $s_{L}+s_{H}$, but not $s_{L}$ alone, we must infer the relative shares of each factor. In order to infer $s_{L}$, we match minimum wage labor's share of the total wage bill (i.e., the share of total wages going to minimum wage labor) to the data. In appendix G of Aaronson and French (2006), we show that low skill labor's share is

$$
\begin{align*}
s_{L}= & \left(s_{L}+s_{H}\right) \times(\text { minimum wage labor's share of the total wage bill }) \times \\
& \left(\frac{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)+\left(1-\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)\right) * \text { wagediff } * \frac{Z_{-\min }}{Z_{\min }}}{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)}\right) . \tag{21}
\end{align*}
$$

The right hand side of equation (21) has three parts. The first part is labor's share. The product of the second and third parts are the share of labor costs going to low skill labor.

### 2.7 Total Employment Effects

Lastly, we are not only interested in low skill employment but total employment as well. To calculate the impact on total restaurant industry employment, denoted $E$, we must compute a weighted average of the low and high skill workers' employment responses:

$$
\begin{equation*}
\frac{d \ln E}{d \ln w_{\min }}=\left(\frac{d \ln L}{d \ln w_{\min }}(\operatorname{prob}(L))+\frac{d \ln H}{d \ln w_{\min }}(1-\operatorname{prob}(L))\right) \times \operatorname{prob}(\text { lowwage }) \tag{22}
\end{equation*}
$$

$\operatorname{prob}(l o w w a g e)$ is the share of workers who work for a minimum wage restaurant,

$$
\begin{equation*}
\operatorname{prob}(\text { lowwage })=\frac{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right.}{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)+\left(1-\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right) \frac{Z_{-\min }}{Z_{\min }}\right.\right.} \tag{23}
\end{equation*}
$$

[^12]and is derived in equation (95) of Aaronson and French (2006).

## 3 Parameters and Estimation

Table 1 provides a summary of the parameters that we use to calibrate the model. The right hand column gives the baseline values that we use, along with the range of values considered in the robustness checks to follow. The appendix provides a full accounting of how we chose these values.

## 4 Results

### 4.1 Employment and Price Responses

Table 2 reports our estimates of the employment response to a 1 percent minimum wage hike under various scenarios for the key parameters. In the top panel, the employment responses are based on a price response $\left(E\left[\frac{d \ln p}{d \ln w_{\text {min }}}\right]=0.07\right)$ that is consistent with the aggregate results in Aaronson (2001) and Aaronson et al (2005). The bottom panel is based on a price response that is 1.5 standard errors below this estimate $\left(E\left[\frac{d \ln p}{d \ln w_{m i n}}\right]=0.05\right) .{ }^{23}$ Within each panel, the demand elasticity, $\eta$, varies between 0.5 and 1.0 , and the elasticity of substitution, $\sigma$, between 0.5 and 0.8 . The remaining parameters are set to our best assessment of their value, as described in table 1 and the appendix. All reported employment responses are short-run. ${ }^{24}$

The first column displays the low skill employment response to a 1 percent minimum wage increase under the assumption that all firms are price-takers in the labor market, $\lambda_{\text {comp }} \times$ $\operatorname{prob}\left(w_{\min }>w^{*}\right)$. Employment responses are described in equations (5) and (7) for firms where the minimum wage binds. That is, labor markets are perfectly competitive, but some firms pay above the minimum wage. First, consider the case where $\eta=0.5$ and $\sigma=0.8$. Under these conditions, a 1 percent increase in the minimum wage cuts the low skill food away from home workforce by 0.36 percent. If $\eta=1.0$, the disemployment case is only slightly larger, approximately 0.39 percent. The estimates are a bit more sensitive to $\sigma$.

[^13]When $\sigma=0.5, \lambda_{\text {comp }} \times \operatorname{prob}\left(w_{\min }>w^{*}\right)$ falls to between 0.23 and 0.26 . Reducing the elasticity of substitution between factors eases the disemployment effect because firms have less opportunity to substitute from labor to other factors. Accordingly, if $\sigma$ is as high as 1 , the competitive employment response is over 0.5 . While the variety of values are noteworthy, given that the experiment covers what we consider to be the range of plausible values for $\eta$ and $\sigma$ (and other parameters), it appears to us that the employment response is not especially sensitive to reasonable parameter choices. Therefore, we can place reasonably tight bounds on the low skill employment response under fairly strict assumptions about labor market structure.

Next, we introduce monopsony power in the labor market. Recall that increases in the minimum wage trace out the labor supply curve (with elasticity $\gamma_{2}$ ) for monopsonists. Consequently, if all firms are monopsonists, minimum wage hikes significantly increase employment. ${ }^{25}$ Since firms are adding workers in response to a minimum wage change, output increases and prices fall. The model predicts that the price elasticity under monopsony is roughly -0.07 . By comparison, the price elasticity under competition tends to be around 0.07 .

Column 2 reports the value of $V$, which is, as described in equation (19), identified by the degree of actual pass-through relative to the pure monopsony and perfect competition cases. A number substantially below 1 would imply that monopsony power is important. ${ }^{26}$ For example, if $\eta=0.5, \sigma=0.5$, and $E\left[\frac{d \ln p}{d \ln w_{m i n}}\right]=0.07$, then $V=1.04$, suggesting that there is little monopsony power in the industry. However, as factors become more substitutable, the elasticity of demand becomes less elastic, or price responses are muted, the implied amount of monopsony power increases. For example, if $\sigma=0.8, \eta=0.5$, and $E\left[\frac{d \ln p}{d \ln w_{\text {min }}}\right]=0.05$, $V=0.82$.

Column 3 reports our estimate of the low skill employment response to a 1 percent minimum wage change when the extent of monopsony power is accounted for explicitly. Calculations are based on equation (17). Like our estimates developed in the perfect competition setting, these employment responses tend to cluster, in this case in the -0.2 to - 0.4 range, with the former appearing when $\sigma$ is low, $\eta$ is low, and/or price responses are high. To take one fairly moderate reading of the parameters, say $\sigma=0.65, \eta=0.75$, and $E\left[\frac{d \ln p}{d \ln w_{\text {min }}}\right]=0.06$,

[^14]we find $V=0.99$, the low skill employment response is $-0.3 .{ }^{27}$
Of course, these estimates ignore the impact on high skilled employment $\left(E \frac{\Delta \ln H}{\Delta \ln w_{\text {min }}}\right)$ and therefore potentially give a misleading picture of the total employment effect $\left(E \frac{\Delta \ln E}{\Delta \ln w_{m i n}}\right)$. Column 4 shows that the employment response for high skilled workers is always negligible.

To estimate the impact on total restaurant industry employment, we must compute a weighted average of the low and high skill workers' employment responses, as in equation (22). These results are presented in column 5. The total employment response to a 1 percent increase in the minimum wage is roughly -0.2 , with estimates ranging between -0.1 to -0.3 under reasonable parameter choices. ${ }^{28}$

Not only do we compare our predicted aggregate price increases to estimates in the literature, but we also
compare predictions of price increases in areas where the minimum wage does not bind with areas where the minimum wage does bind. Consider the regression:

$$
\begin{equation*}
\left(\frac{\Delta \ln p}{\Delta \ln w_{\min }}\right)_{i}=\gamma \operatorname{prob}\left(w_{i}=w_{\min }\right)+\operatorname{error}_{i} \tag{24}
\end{equation*}
$$

where $\left(\frac{\Delta \ln p}{\Delta \ln w_{\text {min }}}\right)_{i}$ is the average city level price response and $\operatorname{prob}\left(w_{i}=w_{\text {min }}\right)$ is the probability that an individual in city $i$ is paid the minimum wage before the minimum wage hike. Assuming all labor markets are competitive implies that the regression coefficient $\gamma$ should be 0.15 to 0.22 , depending on the specification. ${ }^{29}$ Furthermore, allowing for the inferred amount of monopsony from Table 2, the regression coefficient $\gamma$ should be between 0.15 and $0.21 .{ }^{30}$

Using a panel of city level data, Aaronson et al. (2005) find that this regression has a

[^15]coefficient of 0.36 with a standard error of 0.24 . Therefore, estimated pass-through is greater than, but is not statistically different from, the predicted competitive response or the implied responses that allow for some monopsony power as in Table 2.

### 4.2 The Importance of Aggregation

How does accounting for heterogeneity affect our results? Table 3 shows estimates of the competitive and monopsony price responses, $V$, and the competitive, monopsony, and expected total employment responses to a 1 percent change in the minimum wage under three scenarios. Model 1 allows for labor market heterogeneity but not worker type heterogeneity, model 2 for worker heterogeneity but not labor market type heterogeneity, and model 3 for heterogeneity in both labor market and worker types. Specifically, model 1 assumes that some stores pay all of their workers the minimum wage and the remainder pay none of their workers the minimum wage. There is only one type of labor, but variation in labor supply (i.e., $\theta$ ) causes the minimum wage to bind in some markets and not others. Model 2 assumes that all restaurants within a market are identical and hire both high and low skill labor. High skill labor is always paid above the minimum wage and low skill labor is always paid the minimum wage. Therefore, each firm hires an identical share of the minimum wage labor force and above minimum wage labor force. Note that wage dispersion arises across stores in model 1 but within stores in model 2 . Model 3 is our preferred model, where the amount of heterogeneity within and across firms is calibrated using values in table 1 . Model 3 was used to generate the predicted price and employment responses in table 2 .

Here, it is clear that competitive and monopsony price responses (but not employment responses) are sensitive to assumptions about labor market and worker type heterogeneity. The competitive price response varies from 0.051 in model 2 to 0.099 in model 1 . Relative to the empirical evidence of a price response of 0.07 , model 1 predicts larger price responses than the evidence and model 2 smaller price responses than the evidence. Consequently, model 1 predicts that some firms must have monopsony power in order to reconcile a large predicted competitive response with a relatively small measured response, but model 2 predicts that over $100 \%$ of all firms are competitive. Therefore, relative to a competitive employment response of $-0.17,{ }^{31}$ model 1 predicts larger employment responses (in absolute magnitude)

[^16]and model 2 predicts smaller employment responses.
When we allow for heterogeneity in both worker types and labor markets (model 3), we find that the predicted price response is very close to the empirical estimates and thus we infer that close to $100 \%$ of all firms are competitive. ${ }^{32}$ As a result, the predicted total employment response is also close to the competitive response. As we argue in the conclusion, the estimates derived from model 3 are within the bounds set by the empirical literature. However, ignoring heterogeneity within or across establishments can badly bias price and employment responses.

To illustrate why aggregation affects the price response, and thus our predicted value of $V$, consider the price response assuming all labor markets are competitive and $\sigma=\eta$ (or alternatively, that all factors can adjust). Recall that competition implies that all labor costs are pushed onto consumers in the form of higher prices. Inserting equation (4) into equation (18) and assuming $V=1$ yields

$$
\begin{equation*}
E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=s_{L} \times \operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right) . \tag{25}
\end{equation*}
$$

Now consider what models 1 and 2 imply for $s_{L}, \operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)$, and thus the predicted price responses. Setting $\frac{Z_{-\min }}{Z_{\min }}=1$, inserting equation (20) into (25) (and noting that there is only one type of labor in model 1 , so $\operatorname{prob}(L)=1$ and $\left.s_{L}=s_{L}+s_{H}\right)$ yields:

$$
\begin{equation*}
E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=\left(s_{L}+s_{H}\right) \times \operatorname{prob}\left(w_{\min }=w_{i}\right) . \tag{26}
\end{equation*}
$$

Assuming model 2, where all firms are identical, pay some of their workers the minimum wage, and thus $\operatorname{prob}(L)=\operatorname{prob}\left(w_{i}=w_{\text {min }}\right)$ and $\operatorname{prob}\left(w_{\text {min }} \geq w\left(L^{*}\right)\right)=1$, equations (25) and (21) can be rewritten as:

$$
\begin{equation*}
E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=\left(s_{L}+s_{H}\right) \times(\text { minimum wage labor's share of the total wage bill }) . \tag{27}
\end{equation*}
$$

Using equations (26) and (27) and the values in table 3, we can better understand the predicted price responses from models 1 and 2 . Since minimum wage workers are paid less than other workers, $\operatorname{prob}\left(w_{\min }=w_{i}\right)>($ minimum wage labor's share of the total wage bill) and thus price pass-through is greater in the first model than the second. The difference arises

[^17]from the relative weight of high and low wage workers in the total wage bill. For example, in the empirically relevant case where restaurant size and labors share of the total wage bill are constant across restaurant types, restaurants that hire minimum wage workers have a smaller payroll and fewer sales. Consequently, for a fixed payroll, this implies more minimum wage restaurants in a labor market and a larger aggregate price response to changes in minimum wage laws.

We now quantify the price predictions of models 1 and 2 . Recall that we estimate that roughly 33 percent of restaurant workers are impacted by the level of the minimum wage although their compensation consists of only 17 percent of the aggregate restaurant wage bill. Consider model 1 first. If all restaurants either pay the minimum wage or above the minimum wage, $33 \%$ of all workers are paid the minimum wage by the $33 \%$ of all restaurants that pay the minimum wage. These 33 percent of all restaurants have pass through of 30 percent (because labor's share is $30 \%$ ) and the rest have pass through of 0 percent, for an aggregate pass-though of $33 \% \times 30 \%=9.9 \%$.

Next, consider model 2. If all restaurants have the same distributions of high wage and minimum wage workers, then minimum wage labor is 17 percent of labor costs at every restaurant. If total labor's share is 30 percent, then we should expect pass-through to be $30 \% \times 17 \%=5.1 \%$ at every restaurant. The large differences between these two calculations makes it clear that properly handling the aggregation issue is critical. Our best assessment, model 3 , lies between these two extremes.

## 5 Conclusion

We use a model of labor demand to calibrate the employment response to a change in the minimum wage for the food away from home industry. If all firms are price-takers in the labor market, the model predicts roughly a 3.5 percent fall in low skill employment in response to a 10 percent increase in the minimum wage. A second implication of the competitive model is that
higher labor costs are pushed onto consumers in the form of higher prices. This price response stands in sharp contrast to the monopsony model, offering a way to identify the extent of monopsony power in the labor market for the restaurant industry. Relying on previous research that shows that most of the higher labor costs incurred by employers are
pushed onto consumers in the form of higher prices, we infer that few restaurants will have positive employment responses in reaction to a minimum wage increase. Consequently, using the most plausible range of estimates for the key parameters in the model, we find that the low skill employment response to a 10 percent change in the minimum wage is likely around 3 percent. The total employment response, including high skilled (non-minimum wage) workers is likely around 2 percent. These estimates are just slightly below the perfect competition prediction.

Are our findings consistent with studies that directly estimate the employment response to a minimum wage change? Again, we want to emphasize that the findings reported here are for the restaurant industry only. As a result of different intensity of use of minimum wage labor, substitution possibilities, market structure, and demand for products, other industries should face distinct employment responses. Consequently, it is difficult to compare our results to those that identify the employment effect off the co-movement of teenage employment and the minimum wage, without knowing the full range of these parameters for the major industry employers of teens.

However, if we were to compare our low skill results to the teen studies, we believe our estimates would be well within the range supported in the literature. ${ }^{33}$ Moreover, among those studies that explicitly look at the restaurant industry, our results are quite consistent with Neumark and Wascher (2000), who find the employment response to be around -0.2 but a bit larger than those reported in Card and Krueger (2000). ${ }^{34}$ An employment response of zero, while clearly theoretically plausible, does not seem consistent with the level of monopsony power inferred from the price responses observed in the food away from home sector. Therefore, our results provide evidence against the hypothesis that monopsony power is important for understanding the observed small employment responses in the literature.

[^18]
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## Appendix A: The Parameters of the Model

This appendix details the parameter values we use in the calibration exercise.

Price Pass-Through, $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]$ Aaronson et al (2005) use store-level data from the food away from home component of the Consumer Price Index (CPI) during 1995 to 1997 to identify the extent of price pass-through. ${ }^{35}$ They find that, in the aggregate, a 10 percent increase in the minimum wage increases prices by roughly 0.7 percent during a four month period around the minimum wage enactment date. These results are comparable to Aaronson (2001), who uses panels of U.S. city and Canadian province CPI data from 1978 to 1995. Card and Krueger (1995) also use CPI indexes for Food Away from Home in 27 large metropolitan areas, finding larger price increases in those cities with higher proportions of low-wage workers. Although their estimates are consistent with full pass-through, their standard errors are extremely large. They cannot reject zero price pass-through in many of their specifications. Moreover, additional evidence from specific state increases in Texas and New Jersey suggests close to no price response. As a result, they conclude that their estimates are "too imprecise to reach a more confident assessment about the effects of the minimum wage on restaurant prices." However, Aaronson (2001) and Aaronson et al (2005) use substantially more data and document large and significant increases in food away from home prices immediately surrounding an increase in the minimum wage. These latter results are consistent with studies of price pass-through resulting from other cost shocks, such as sales taxes levies (e.g. Besley and Rosen 1999) and exchange rate movements (e.g. Yang 1997).

In the calibration exercise, which aggregates all markets and types of restaurants, we use a value for $E\left[\frac{d \ln p}{d \ln w_{m i n}}\right]$ of 0.05 or 0.07 .

Labor's Share, $s_{L}+s_{H} \quad$ There are a number of sources for labor share, all of which tend to report similar numbers for the food away from home industry. First, 10-K company reports

[^19]contain payroll to total expense ratios. Of the 17 restaurant companies that appear in a search of 1995 reports using the SEC's Edgar database, the unconditional
mean and median of this measure of labor share is 30 percent and it ranges from 21 to 41 percent. ${ }^{36}$ These numbers are in-line with a sampling of 1995 corporate income tax forms from the Internal Revenue Service's Statistics on Income Bulletin. Because operating costs are broken down by category, it is possible to estimate labor's share. ${ }^{37}$ According to these tax filings, labor cost as a share of operating costs for eating place partnerships is roughly 33 percent. Consequently, we set $s_{L}+s_{H}$ to 30 percent but test the robustness of the results to values between 25 and 35 percent.

Finally, we are particularly interested in labor share in low wage firms. Depending on the elasticity of substitution between factors, firms in high wage labor markets theoretically could have either a higher or lower labor's share. To gauge this heterogeneity, we use the 1997 Economic Census for Accommodations and Food Services, which reports payroll for full service (FS) and limited service (LS) restaurants. LS includes fast-food stores and any restaurant without sit-down service and where customers pay at the counter prior to receiving their meals. Therefore, they tend to be the primary employer of minimum wage labor. According to this 1997 census, labor share, as a fraction of sales, is slightly higher at FS (31 percent) than LS ( 25 percent) stores. ${ }^{38}$ Therefore, there is little evidence of a significant difference in labor share across establishment type.

Furthermore, there is no evidence of an association between labor share and average city wages. Here, we correlate MSA-level labor share from the 1997 Economic Census for Accommodations and Food Services with MSA-level real wage rates computed from the 1990-1999 outgoing rotation files of the CPS. That correlation is small and statistically insignificant for both limited service $(-0.08)$ and full service $(-0.06)$ establishments. The lack of a geographic association between labor share and wages is particular evident when breaking the cities into quartiles based on their average wage rate during the 1990s. For limited service establishments, labor share is (lowest to highest wage quartile) $0.258,0.262,0.260$, and 0.256 . Among

[^20]full-service establishments, labor share is $0.308,0.312,0.311$, and 0.306 . See appendix E of Aaronson and French (2006) for a further discussion of this point.

Capital and Material's Share, $s_{K}$ and $s_{M}$ Based on the same sample of company financial reports used to compute $s_{L}+s_{H}$, we assume that capital share is 30 percent and materials share is 40 percent. When we allow $s_{L}+s_{H}$ to vary, we also allow either capital or material share to adjust accordingly. The results are not sensitive to modifications of these shares between capital and materials.

The Share of Minimum Wage Workers, $\operatorname{prob}\left(w_{\min }=w_{i}\right)$, and Minimum Wage Labor's Share of the Total Wage Bill The minimum wage should affect prices and employment only if it impacts wages. Therefore, we need the share of workers' wages influenced by minimum wage policy. Since this is not available in company reports, we estimate the share of employees that are paid at or very near the minimum wage from the outgoing rotation files of the Current Population Survey (CPS) for the two years prior to the 1996 legislation. ${ }^{39}$ During this time period, 23 percent of restaurant industry workers were within 10 percent of the minimum wage, and therefore clearly impacted by a characteristically-sized 10 percent increase in the minimum wage.

However, this estimate is insufficient for understanding the fraction of workers affected by the minimum wage change. Workers paid slightly above the minimum wage also tend to receive pay increases in response to minimum wage increases. ${ }^{40}$ To approximate this phenomena, we use the outgoing rotation file wage distribution and assume that one-third of workers within 150 percent of the old minimum are impacted by the new minimum wage change. No one beyond this threshold is assumed to be impacted. These assumptions imply $\operatorname{prob}\left(w_{\min }=w_{i}\right)=0.33$. That is, one-third of restaurant workers are influenced by minimum wage legislation. We test the robustness of the results to values of $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ between 0.25 and 0.40 .

Using the same CPS data and assumptions about spillovers, we also estimate that minimum wage labor constitutes $17 \%$ of the wage bill. Therefore, $33 \%$ of all restaurant workers

[^21]are paid the minimum wage, but they are paid only $17 \%$ of the total value of wage payments in the restaurant industry.

The Wage Distribution Parameters, $\operatorname{prob}(L)$, wagediff We allow for wage heterogeneity in two dimensions. We assume that there are two types of workers (high and low skill) and two types of labor markets (high and low wage). We assume that low skill workers in high wage markets are always paid above the minimum wage (as are all high skill workers). However, all low skill workers in low wage labor markets are paid the minimum wage. Furthermore, we assume that all restaurants have the same skill distribution, so the probability of being low skill, $\operatorname{prob}(L)$, does not vary by labor market. Given this assumption, $\operatorname{prob}(L)=$ the probability that someone in a low wage labor market is paid the minimum wage. This pins down the probability that a worker is low skill.

We define a local labor market as a city. Aaronson et al (2005) use the 1979-1995 CPS Outgoing Rotation Groups to estimate the share of restaurant workers that are paid at or near the minimum wage among cities covered in the Consumer Price Index (CPI) survey. ${ }^{41}$ They find that roughly 70 percent of restaurant workers are paid at or near the minimum wage in the lowest wage cities. Therefore, we assume $\operatorname{prob}(L)=0.70$ but allow $\operatorname{prob}(L)$ to vary between 0.60 and 0.80 .

In order to allow for wage differences across labor markets, we set $\theta$ as a two point distribution. The distribution for $\theta$ shifts wages for both high and low skill workers, which in turn shifts wagediff. We pick wagediff $=1.5$ to match the dispersion in wages across cities. Although the ratio of average wages in high wage cities (i.e., the highest paying 50 percent of cities represented in the CPI) to average wages in low wage cities is roughly 1.15 , different measures of dispersion give larger differences in high versus low wage areas. For example, the ratio of wages in the top $15 \%$ of all cities relative to the bottom $15 \%$ of all cities is 1.5 .

Using this information, we can infer the probability that an individual restaurant pays the minimum wage to its low skill workers using equation (20). Furthermore, we can infer the

[^22]share of all restaurant workers who work for restaurants that pay the minimum wage using equation (23).

Average Restaurant Size in High Versus Low Wage Labor Markets, $\frac{Z_{-\min }}{Z_{\min }}$ We compute the average number of employees per restaurant in high wage $Z_{-\min }$ and low wage $Z_{\text {min }}$ markets using the 1997 Economic Census for Accommodations and Food Services. Our definition of a labor market is an MSA. High and low wage cities are determined by the 19901999 average real wage computed from the Outgoing Rotation Files of the CPS. We combine limited and full service establishments to compute restaurant size from all restaurants. The results are similar if we look at all eating and drinking places.

We find that restaurants in high wage cities employ roughly 10 percent fewer workers than in low wage cities. For example, the ratio of restaurant size in the 15 percent of cities with the highest wages relative to 15 percent of cities with the lowest wages is 0.9 .

Admittedly, there is a great deal of restaurant size heterogeneity within a city. Furthermore, much of this heterogeneity could be because local labor markets are smaller than cities. In order to explore the importance of restaurant size within a city, we measure average restaurant size for full service (which rarely pay the minimum wage) and limited service (which often pay the minimum wage) restaurants separately.

We find, on average, that limited service restaurants employ a little over 20 percent fewer workers than full service restaurants across all cities, 10 percent fewer among low wage cities, and 30 percent fewer among high wage cities. Therefore, using restaurant type, $\frac{Z_{-\min }}{Z_{\min }}$ is, on average 1.2 but varies from 1.1 to 1.3 .

Given that the range of estimates for $\frac{Z_{-\min }}{Z_{\min }}$, we set it at 1 as our benchmark, but allow it to vary between 0.9 and 1.3.

The Elasticity of Demand for Food Away From Home, $\eta$ There is little compelling evidence on $\eta$ for the restaurant industry. Brown (1990) uses 1977 and 1982 cross-sectional Census data to calculate an $\eta$ of 0.2 and 1 for food away from home and fast food, respectively. Piggott's (2003) estimates, which use time series data and a generalized model of food demand that nests the main demand systems currently in use, suggest more elastic demand, probably above 1. Hussain (2004) uses repeated cross section data from the Consumer Expenditure Survey and carefully controls for some of the endogeneity problems associated with demand
estimation. Hussain finds an estimate of 0.2 to 0.3 in the basic specification, although he obtains estimates of over 1 when he addresses the endogeneity of labor supply and durables. In our results, we provide estimates when $\eta$ is between 0.5 and 1.0.

The Elasticity of Substitution Between Labor and Capital, $\sigma$ We could not find estimates of $\sigma$ for restaurants. On the one hand, this is unfortunate since there is little reason to expect that factor substitution is equal across industries. In fact, Hamermesh's (1993) review of industry- and product-specific $\sigma$ s reveals a fairly broad range of estimates. However, as Hamermesh stresses, micro-oriented estimates generally do not alter conclusions reached from studies using aggregated data. Substitution between labor and capital is generally between 0.5 and 1.0 for the vast majority of industries, with a mean estimate of 0.75 from Hamermesh's review of aggregate studies and 0.50 from his review of micro studies. Given that the overwhelming majority of these studies are based on manufacturing sectors, the closest parallel to the eating and drinking industry that we could find was Goodwin and Brester (1995), who analyze the food manufacturing industry and find that $\sigma$ is roughly 0.9 in the 1970s and 0.5 thereafter. ${ }^{42}$ Therefore, we allow $\sigma$ to vary between 0.5 and 0.8 .

The Marshallian labor supply elasticities, $\gamma_{1}$ and $\gamma_{2}$ We set the labor supply elasticity, holding wages at other firms fixed, at $\gamma_{1}=5$ but examined the robustness of the results to values between 2 and 10. This range is based on Card and Krueger's (1995, p. 376) interpretation of $\gamma_{1}$ calibrated from estimates of wage elasticities of the hiring and quit functions. They conclude that while $\gamma_{1}$ could vary between 2 and 10 , the upper range is more theoretically and empirically plausible.

We set $\frac{1}{\gamma(\theta)}=\frac{1}{2 \gamma_{1}}$, an approximation that is exact if $\theta$ has a uniform density. However, we found that all values of $\frac{1}{\gamma(\theta)} \in\left(0, \frac{1}{\gamma_{1}}\right)$ to give very similar results.

Bhaskar and To (1999) and Manning (2003) point out that if the labor market is characterized by monopsonistic competition rather than pure monopsony, then setting $\gamma_{1}=\gamma_{2}$ calibrated using hire and quit rates will lead to erroneous inference. Under monopsonistic competition, increases in a firm's wage reduce the quantity of labor supplied at all other firms. Because an increase in a binding minimum wage increases the wage of all firms, it will

[^23]have a smaller effect on a firm's labor supply than just increasing that firm's wage. Although for most demographic groups, labor supply elasticities are close to 0 , the empirical evidence is that labor supply is somewhat elastic for low wage groups. Therefore, we set $\gamma_{2}=0.5$ but allow it to vary between 0 and 1 .

## Appendix B: Solution to the Model Under Perfect Competition

This appendix shows how to solve for the price and employment responses under perfect competition. We discuss price and employment responses under constant returns production (in our case, when $K$ can adjust) below, diminishing returns (when $K$ cannot adjust) in appendix C , and the monopsony case in appendix D .

The first order conditions for maximization of the profit function in equation (1) shows that the marginal products of capital, labor, and materials (obtained by differentiating equation (1) with respect to $K, L, H$ and $M$ ) are equal to their relative prices, $\frac{r}{p}=\frac{\partial Q}{\partial K}, \frac{w_{L}}{p}=$ $\frac{\partial Q}{\partial L}, \frac{w_{H}}{p}=\frac{\partial Q}{\partial H}, \frac{p_{M}}{p}=\frac{\partial Q}{\partial M}$. Solving for the marginal products and taking logs of the first order conditions yields:

$$
\begin{array}{r}
\sigma(\ln r-\ln p)=\sigma \ln \alpha_{K}+(\ln Q-\ln K), \\
\sigma\left(\ln w_{L}-\ln p\right)=\sigma \ln \alpha_{L}+(\ln Q-\ln L), \\
\sigma\left(\ln w_{H}-\ln p\right)=\sigma \ln \alpha_{H}+(\ln Q-\ln H), \\
\sigma\left(\ln p_{M}-\ln p\right)=\sigma \ln \left(1-\alpha_{K}-\alpha_{L}-\alpha_{H}\right)+(\ln Q-\ln M), \tag{31}
\end{array}
$$

respectively.
In this appendix, we make use of the following definitions: $\frac{d \ln K}{d \ln w_{\min }} \equiv \mu, \frac{d \ln M}{d \ln w_{\min }} \equiv$ $\nu, \frac{d \ln H}{d \ln w_{\min }} \equiv \xi, \frac{d \ln Q}{d \ln w_{\min }} \equiv-\tau$. In addition, we define $\frac{d \ln L}{d \ln w_{\min }} \equiv-\lambda$ (as opposed to $\frac{d \ln L}{d \ln w_{m i n}} \equiv$ $\lambda$ in the text) for notational convenience. Note that ${ }^{43}$

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=\frac{\frac{d \ln Q}{d \ln w_{\min }}}{\frac{d \ln Q}{d \ln p}}=\frac{-\tau}{-\eta} . \tag{32}
\end{equation*}
$$

Differentiating equations (28)-(31) with respect to $\ln w_{\min }$, assuming $\frac{d \ln r}{d \ln w_{m i n}}=0, \frac{d \ln p_{M}}{d \ln w_{m i n}}=$

[^24]0 , and $\frac{d \ln w_{H}}{d \ln w_{m i n}}=0^{44}$ and using (32) yields

$$
\begin{gather*}
\sigma\left(\frac{\tau}{\eta}\right)=\tau+\mu,  \tag{33}\\
\lambda=\sigma\left(1-\frac{\tau}{\eta}\right)+\tau  \tag{34}\\
\sigma\left(\frac{\tau}{\eta}\right)=\tau+\xi  \tag{35}\\
\sigma\left(\frac{\tau}{\eta}\right)=\tau+\nu, \tag{36}
\end{gather*}
$$

respectively. Inspection of equations (33), (35), and (36) shows that $\mu=\nu=\xi$.
In order to obtain the output response to a minimum wage change, differentiate output with respect to the minimum wage:

$$
\begin{equation*}
\frac{d Q}{d w_{\min }}=\frac{\partial Q}{\partial K} \frac{d K}{d w_{\min }}+\frac{\partial Q}{\partial L} \frac{d L}{d w_{\min }}+\frac{\partial Q}{\partial H} \frac{d H}{d w_{\min }}+\frac{\partial Q}{\partial M} \frac{d M}{d w_{\min }} . \tag{37}
\end{equation*}
$$

Multiplying both sides by $\frac{w_{\text {min }}}{Q}$, inserting equations (33)-(36), and noting that $\mu=\nu=\xi$, equation (37) can be rewritten as:

$$
\begin{equation*}
-\tau=\left(1-s_{L}\right) \mu+s_{L}(-\lambda) \tag{38}
\end{equation*}
$$

Combining equations (33), (34) and (38) and solving for the unknowns $\{\mu, \lambda, \tau\}$ yields:

$$
\begin{equation*}
\lambda=\left(1-s_{L}\right) \sigma+s_{L} \eta \tag{39}
\end{equation*}
$$

which is equation (5) of the text, and

$$
\begin{equation*}
\tau=s_{L} \eta \tag{40}
\end{equation*}
$$

Using equation (32), equation (40) can be rewritten as

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=s_{L} \tag{41}
\end{equation*}
$$

which is equation (4) of the text.

[^25]
## Appendix C: Labor Demand when Capital Cannot Adjust

When capital cannot adjust, equations (28) and (33) no longer hold. Instead, there is no capital response: i.e., $\mu=0$. However, equations (34)-(37) still apply. Now, the analog of equation (38) is

$$
\begin{equation*}
-\tau=\left(1-s_{L}-s_{K}\right) \nu+s_{L}(-\lambda) \tag{42}
\end{equation*}
$$

Combing equations (34)-(36) with equation (42) yields

$$
\begin{equation*}
\tau=\frac{s_{L} \sigma}{s_{K}+\left(1-s_{K}\right) \frac{\sigma}{\eta}} . \tag{43}
\end{equation*}
$$

Using equations (32) and (43) we get the price response:

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=\frac{s_{L}}{s_{K}\left(\frac{\eta}{\sigma}\right)+\left(1-s_{K}\right)} . \tag{44}
\end{equation*}
$$

Using equations (44), (43) and (34), we get the low skill employment response:

$$
\begin{equation*}
\lambda=\eta \frac{d \ln p}{d \ln w_{\min }}+\sigma\left(1-\frac{d \ln p}{d \ln w_{\min }}\right) \tag{45}
\end{equation*}
$$

where $\frac{d \ln p}{d \ln w_{\text {min }}}$ is defined in equation (44), and high skill employment response:

$$
\begin{equation*}
\xi=(\sigma-\eta) \frac{d \ln p}{d \ln w_{\min }} \tag{46}
\end{equation*}
$$

## Appendix D: Employment and Price Responses with Heterogeneous Labor Markets

This appendix shows how to solve for the equilibrium price and employment responses when firms have monopsony power. To solve the monopsony model, we note that equations (33), (35), and (36) still hold. However, equation (29) must be replaced by equation (47) where the marginal product of labor is:

$$
\frac{\partial Q}{\partial L}= \begin{cases}w_{\min } & \text { if } w_{\min } \geq w\left(L^{* *}\right)  \tag{47}\\ \left(1+\frac{1}{\gamma(\theta)}\right) w_{\min } & \text { if } w\left(L^{*}\right) \leq w_{\min }<w\left(L^{* *}\right) \\ \left(1+\frac{1}{\gamma}\right) w(L) & \text { if } \quad w_{\min }<w\left(L^{*}\right)\end{cases}
$$

$\frac{1}{\gamma(\theta)}=\left(1+\frac{1}{\gamma_{1}}\right)^{\left(\frac{\ln L^{* *}-\ln L}{\ln L^{* *}-\ln L *}\right)}-1$ measures the difference between the wage and the marginal revenue product of labor, the vertical distance between $\ln w(L)$ and $\ln M R P(L)$ in figure 1 , and always lies between 0 and $\frac{1}{\gamma_{1}}$ when $L$ is between $L^{*}$ and $L^{* *}$. Note that equation (47) has three segments; $\theta$ determines which of these segments is equal to the marginal product of labor. As pointed out in the text, we assume that dispersion in $\theta$ causes dispersion in the distribution of wages. Equation (11) shows that for low values of $\theta$, firms need only pay a low wage in order to obtain a given number of workers. Thus, for a sufficiently low value of $\theta$, the minimum wage will bind and employment is determined by the intersection of the minimum wage and the MRPL curve. As $\theta$ increases, firms must pay higher market wages in order to obtain a given number of workers. Therefore, for a sufficiently high value of $\theta$, the minimum wage will not bind.

When employment is determined by the intersection of the minimum wage and the MRPL curve, employment is determined in the same way as when a firm is in a perfectly competitive market. Therefore, the top line of (47) holds and the employment responses are the same as those in Appendices B and C when capital can and cannot adjust, respectively. However, when employment is determined by the intersection of the minimum wage and inverse labor supply curve, equation (45) must be replaced by

$$
\begin{equation*}
\lambda=-\gamma_{2} . \tag{48}
\end{equation*}
$$

We solve for when capital can adjust and cannot adjust, respectively. Consider the case where capital can adjust first. Using equation (47) in place of (34) and assuming that profits are close to zero, equation (37) can be rewritten as:

$$
\begin{equation*}
-\tau=\left(1-s_{L}\right) \mu+s_{L}\left(1+\frac{1}{\gamma(\theta)}\right)(-\lambda) \tag{49}
\end{equation*}
$$

for $\frac{1}{\gamma_{\theta}} \in\left[0, \frac{1}{\gamma_{1}}\right]$. Combining equations (33), (48), and (49) gives us

$$
\begin{equation*}
\tau=-\frac{\gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right) s_{L} \eta}{\left(\sigma\left(1-s_{L}\right)+\eta s_{L}\right)} . \tag{50}
\end{equation*}
$$

Equation (50) allows us to identify two key objects of interest. First, combining it with
equation (33) yields the capital response (which is also equal to the high skill labor response)

$$
\begin{equation*}
\mu=\left(-\frac{\gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right) s_{L} \eta}{\left(\sigma\left(1-s_{L}\right)+\eta s_{L}\right)}\right)\left(\frac{\sigma}{\eta}-1\right) . \tag{51}
\end{equation*}
$$

Second, using $\frac{d \ln p}{d \ln w_{\text {min }}}=\frac{\tau}{\eta}$, the price response is

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=-\frac{\gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right) s_{L}}{\left(\sigma\left(1-s_{L}\right)+\eta s_{L}\right)} . \tag{52}
\end{equation*}
$$

Next, consider the case where capital cannot adjust. Using equation (37), the output response (i.e., the analog to equation (49)) is

$$
\begin{equation*}
-\tau=\left(1-s_{L}-s_{K}\right) \nu+s_{L}\left(1+\frac{1}{\gamma(\theta)}\right)(-\lambda) \tag{53}
\end{equation*}
$$

Combining equations (35), (48), and (53) gives us

$$
\begin{equation*}
\tau=-\frac{\gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right) s_{L} \eta}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\eta\left(s_{L}+s_{K}\right)\right)} \tag{54}
\end{equation*}
$$

with price response

$$
\begin{equation*}
\frac{d \ln p}{d \ln w_{\min }}=-\frac{\gamma_{2}\left(1+\frac{1}{\gamma(\theta)}\right) s_{L}}{\left(\sigma\left(1-s_{L}-s_{K}\right)+\eta\left(s_{L}+s_{K}\right)\right)} . \tag{55}
\end{equation*}
$$

## Appendix E: The Relationship Between Labor's Share and $\theta$

In deriving equations (18) and (19), we assumed that $s_{L}$ and $s_{K}$ do not vary across labor markets. However, the theory presented above indicates that the factor mix (and thus $s_{L}$ and $s_{K}$ ) can vary across labor markets, for two reasons. First, even when comparing two labor markets where the minimum wage binds (and the minimum wage is the same in both markets), differences in $\theta$ can create differences in low skill labor's share. Second, factor shares vary across markets where $\theta$ is sufficiently low that the minimum wage binds versus labor markets where $\theta$ is sufficiently high that the minimum wage does not bind. We presented evidence on geographic variation in labor share in the papers appendix. This appendix further shows that, given the parameter values discussed in Section 3, differences in factor shares across markets are likely to be small, probably between 0 and 14 percent. Therefore, with both of
these pieces of evidence in hand, it seems reasonable to conclude that factor shares do not vary by labor market.

The goal of this appendix is to show how wages, employment and labor's share vary with $\theta$. If $\theta$ is low, the minimum wage will be equal to $w^{* *}$ in Figure 2, and employment will equal $L^{* *}$. As $\theta$ rises, employment falls as employment is determined by the intersection of the minimum wage and inverse labor supply curves. But, for large values of $\gamma_{1}$ (i.e., if the quantity of labor supplied to the firm is elastic), small changes in $\theta$ affect neither wages nor employment. ${ }^{45}$

Next, consider the case where the minimum wage is between $w^{*}$ and $w^{* *}$, as in Figure 2. Increasing $\theta$ along this range does not affect the wage, because the wage is equal to the minimum. However, employment declines as $\theta$ increases because fewer workers are willing to work at the minimum wage. If $\theta$ is sufficiently high that the minimum wage does not bind, increases in $\theta$ increase wages and reduce employment. As we show below, (low skill) labor's share can either increase or decrease with $\theta$, depending on whether the elasticity of substitution between factors, $\sigma$, is less or greater than 1 . Based on the evidence discussed in section 3, we believe $\sigma$ is less than 1 , and therefore higher $\theta$ (and therefore a higher wage) reduces the amount of labor used but increases labor's share.

The fact that wages are increasing in $\theta$ is obvious upon inspection of equation (11). Our primary interest is in understanding differences in employment caused by differences in wages that are in turn caused by differences in $\theta$. Our analysis of CPS data indicates that high wage cities pay, at most, $50 \%$ more than relatively low wage cities.

The first order conditions (28), (30), and (31) still hold, although (29) becomes

$$
\begin{equation*}
\sigma\left(\ln w_{L}-\ln p+\ln \left(1+\frac{1}{\gamma_{1}}\right)\right)=\sigma \ln \alpha_{L}+(\ln Q-\ln L) . \tag{56}
\end{equation*}
$$

Redefine $\frac{d \ln K}{d \ln \theta} \equiv \mu, \frac{d \ln M}{d \ln \theta} \equiv \nu, \frac{d \ln Q}{d \ln \theta} \equiv-\tau$ in addition to $\frac{d \ln H}{d \ln \theta} \equiv-\xi, \frac{d \ln L}{d \ln \theta} \equiv-\lambda$. Then

[^26]equations (33), and (36) still hold. However, equation (34) becomes
\[

$$
\begin{equation*}
\lambda=\sigma\left(\frac{d \ln w}{d \ln \theta}-\frac{\tau}{\eta}\right)+\tau \tag{57}
\end{equation*}
$$

\]

We assume that increases in $\theta$ increase $w_{L}$ and $w_{H}$ equally $\left(\frac{d \ln w_{L}}{d \ln \theta}=\frac{d \ln w_{H}}{d \ln \theta}\right)$. Because of this, $\lambda=\xi$ and thus low and high skill labor have the same elasticities.

Equation (49) must be rewritten as

$$
\begin{equation*}
-\tau=\left(1-s_{L}-s_{H}\right) \mu-\left(s_{H}+s_{L}\left(1+\frac{1}{\gamma(\theta)}\right)\right) \lambda . \tag{58}
\end{equation*}
$$

Solving for $\lambda$ yields:

$$
\begin{equation*}
\lambda=\sigma \frac{d \ln w_{L}}{d \ln \theta}\left(1+\frac{\left(1-\frac{\sigma}{\eta}\right)\left(s_{H}+s_{L}\left(1+\frac{1}{\gamma(\theta)}\right)\right)}{\frac{\sigma}{\eta}-\frac{s_{L}}{\gamma(\theta)}\left(1-\frac{\sigma}{\eta}\right)}\right) \tag{59}
\end{equation*}
$$

Assuming that $\frac{s_{L}}{\gamma(\theta)}\left(1-\frac{\sigma}{\eta}\right)$ is small relative to $\frac{\sigma 46}{\eta}$, equation (59) can be rewritten as

$$
\begin{equation*}
\lambda=\frac{d \ln w_{L}}{d \ln \theta}(\sigma+s(\eta-\sigma)) . \tag{60}
\end{equation*}
$$

In order to calculate the elasticity of low skill labor's share with respect to the wage, we calculate:

$$
\begin{equation*}
\frac{d \ln s_{L}}{d \ln \theta}=\frac{d \ln L}{d \ln \theta}+\frac{d \ln w_{L}}{d \ln \theta}-\frac{d \ln \left(w_{L} L+w_{H} H+p_{M} M+r K\right)}{d \ln \theta} . \tag{61}
\end{equation*}
$$

Equation (61) can be rewritten as

$$
\begin{equation*}
\frac{d \ln s_{L}}{d \ln \theta}=\left(\frac{d \ln w_{L}}{d \ln \theta}-\lambda\right)-\frac{d\left(w_{L} L+w_{H} H+p_{M} M+r K\right)}{d \theta} \frac{\theta}{\left(w_{L} L+w_{H} H+p_{M} M+r K\right)} . \tag{62}
\end{equation*}
$$

[^27]Taking the derivative, noting that $\mu=\nu, \lambda=\xi$ and some algebra yields

$$
\begin{equation*}
\frac{d \ln s_{L}}{d \ln \theta}=\left(1-s_{L}-s_{H}\right)\left(\frac{d \ln w_{L}}{d \ln \theta}-\lambda-\mu\right) \tag{63}
\end{equation*}
$$

which can be rewritten as

$$
\begin{equation*}
\frac{d \ln s_{L}}{d \ln \theta}=\left(1-s_{L}-s_{H}\right)(1-\sigma) \frac{d \ln w_{L}}{d \ln \theta} . \tag{64}
\end{equation*}
$$

Dividing both the left and right hand sides by $\frac{d \ln w_{L}}{d \ln \theta}$ gives an elasticity of low skill labor's share with respect to wage changes (caused by changes in $\theta$ ) equal to $\left(1-s_{L}-s_{H}\right)(1-\sigma)$. Given that most estimates of $\sigma$ are between 0.5 and $1, s_{L}+s_{H}$ is approximately .3 , and wage differentials between high wage and low wage markets are equal to wagediff $=1.5$, the $\log$ differential in low skill labor's share is between 0 and $(1-.3)(1-.5)(\ln 1.5-\ln 1)=.14 \log$ points, or $14 \%$.

## Appendix F: Labor Demand Under Monopolistic Competition when Capital Can Adjust

In this appendix we augment the model to allow for monopolistic competition in the product market, although we note that the results in this section hold if firms are monopolists in the product market as well. The critical assumption is that there is a constant elasticity of demand. If this is the case, there is a constant mark-up over marginal cost. Consequently, increases in labor cost are pushed onto the consumer, as is the case under perfect competition. In this appendix, we show that equilibrium elasticities are the same as in Appendix B.

We assume that consumers have the utility function

$$
\begin{equation*}
U=U\left(Q_{0}, \tilde{Q}\right) \tag{65}
\end{equation*}
$$

where $Q_{0}$ is the numeraire good, $\tilde{Q} \equiv\left(\sum_{n=1}^{N} Q_{n}^{1-\eta_{Z}}\right)^{\frac{1}{\eta_{Z}-1}}$ and $Q_{n}$ denotes output at the $n$th restaurant. Furthermore, we assume that the aggregator $U(.,$.$) is such that$

$$
\begin{equation*}
\frac{d \ln \tilde{Q}}{d \ln \tilde{p}}=-\eta \tag{66}
\end{equation*}
$$

where $\tilde{p}$ is the price index associated with $\tilde{Q}: \tilde{p} \equiv\left(\sum_{n=1}^{N} p_{n}^{\frac{\eta_{Z}-1}{\eta_{Z}}}\right)^{\frac{\eta_{Z}}{\eta_{Z}-1}}$. Equation (66) is
satisfied if $U(.,$.$) is a CES aggregator and the share of income spent on \tilde{Q}$ is close to 0 . Dixit and Stiglitz (1977) point out that two-stage budgeting techniques can be used to analyze this consumer demand problem. In the second stage the consumer solves:

$$
\begin{equation*}
\max _{\left\{Q_{n}\right\}_{n=1}^{N}}\left(\sum_{n=1}^{N} Q_{n}^{1-\eta_{Z}}\right)^{\frac{1}{1-\eta_{Z}}} \tag{67}
\end{equation*}
$$

subject to

$$
\begin{equation*}
\sum_{n=1}^{N} p_{n} Q_{n}=X \tag{68}
\end{equation*}
$$

where $X$ is total expenditure on $\tilde{Q}$. The consumer's first order condition for utility maximization yields

$$
\begin{equation*}
p_{n}=\zeta^{-1}\left(\frac{Q_{n}}{\tilde{Q}}\right)^{-\eta Z} \tag{69}
\end{equation*}
$$

where $\zeta$ is the Lagrange multiplier on the budget constraint. If $N$ is sufficiently large, then $Q_{n}$ is small relative to $\tilde{Q}$ and thus the firm does not take into account the effect of $Q_{n}$ on $\tilde{Q}$ when assessing the effect of $Q_{n}$ on $p_{n}$.

Therefore, the $n$th firm's problem is to maximize

$$
\begin{equation*}
\pi\left(K_{n}, L_{n}\right)=\Omega Q_{n}^{1-\eta_{Z}}-r K_{n}-w_{\min } L_{n}-w_{H} H_{n}-p_{M} M_{n} \tag{70}
\end{equation*}
$$

where $\Omega=\zeta^{-1} \tilde{Q}^{\eta_{Z}}$. The first order conditions for profit maximization are:

$$
\begin{gather*}
w_{\min }=\Omega\left(1-\eta_{Z}\right) Q_{n}^{-\eta_{Z}} \alpha_{L}\left(\frac{Q_{n}}{L_{n}}\right)^{1-\rho}  \tag{71}\\
r=\Omega\left(1-\eta_{Z}\right) Q_{n}^{-\eta_{Z}} \alpha_{K}\left(\frac{Q_{n}}{K_{n}}\right)^{1-\rho}  \tag{72}\\
w_{H}=\Omega\left(1-\eta_{Z}\right) Q_{n}^{-\eta_{Z}} \alpha_{H}\left(\frac{Q_{n}}{H_{n}}\right)^{1-\rho} \tag{73}
\end{gather*}
$$

$$
\begin{equation*}
p_{M}=\Omega\left(1-\eta_{Z}\right) Q_{n}^{-\eta_{Z}}\left(1-\alpha_{L}-\alpha_{H}-\alpha_{K}\right)\left(\frac{Q_{n}}{K_{n}}\right)^{1-\rho}, \tag{74}
\end{equation*}
$$

Taking logs of both sides of equations (71)-(74) and differentiating with respect to $\ln w_{\text {min }}$ yields

$$
\begin{align*}
& 1=-\frac{d \ln \zeta}{d \ln w_{\text {min }}}+\eta_{Z} \frac{d \ln \tilde{Q}}{d \ln w_{\text {min }}}-\eta_{Z} \frac{d \ln Q_{n}}{d \ln w_{\text {min }}}+\frac{1}{\sigma}\left(\frac{d \ln Q_{n}}{d \ln w_{\text {min }}}-\frac{d \ln L_{n}}{d \ln w_{\text {min }}}\right)  \tag{75}\\
& 0=-\frac{d \ln \zeta}{d \ln w_{\min }}+\eta_{Z} \frac{d \ln \tilde{Q}}{d \ln w_{\min }}-\eta_{Z} \frac{d \ln Q_{n}}{d \ln w_{\min }}+\frac{1}{\sigma}\left(\frac{d \ln Q_{n}}{d \ln w_{\min }}-\frac{d \ln K_{n}}{d \ln w_{\min }}\right) .  \tag{76}\\
& 0=-\frac{d \ln \zeta}{d \ln w_{\text {min }}}+\eta_{Z} \frac{d \ln \tilde{Q}}{d \ln w_{\text {min }}}-\eta_{Z} \frac{d \ln Q_{n}}{d \ln w_{\text {min }}}+\frac{1}{\sigma}\left(\frac{d \ln Q_{n}}{d \ln w_{\text {min }}}-\frac{d \ln H_{n}}{d \ln w_{\text {min }}}\right) .  \tag{77}\\
& 0=-\frac{d \ln \zeta}{d \ln w_{\text {min }}}+\eta_{Z} \frac{d \ln \tilde{Q}}{d \ln w_{\text {min }}}-\eta_{Z} \frac{d \ln Q_{n}}{d \ln w_{\text {min }}}+\frac{1}{\sigma}\left(\frac{d \ln Q_{n}}{d \ln w_{\text {min }}}-\frac{d \ln M_{n}}{d \ln w_{\text {min }}}\right) . \tag{78}
\end{align*}
$$

Observation of equations (76)-(78) shows that $\frac{d \ln K_{n}}{d \ln w_{\text {min }}}=\frac{d \ln H_{n}}{d \ln w_{\text {min }}}=\frac{d \ln M_{n}}{d \ln w_{\text {min }}}$.
Note that in equilibrium all firms produce the same amount and thus prices $p_{n}$ are equal. Using the definition of the aggregate quantity index $\tilde{Q}$ and price index $\tilde{p}$ we obtain

$$
\begin{align*}
& \tilde{Q}=N^{\frac{1}{1-\eta_{Z}}} Q_{n}  \tag{79}\\
& \tilde{p}=N^{\frac{\eta_{Z}}{\eta_{Z}-1}} p_{n} \tag{80}
\end{align*}
$$

Inspection of equation (79) shows that

$$
\begin{equation*}
\frac{d \ln \tilde{Q}}{d \ln w_{\min }}=\frac{d \ln Q_{n}}{d \ln w_{\min }} \tag{81}
\end{equation*}
$$

Moreover, equations (69), (79), and (80) show that

$$
\begin{equation*}
\frac{d \ln \zeta}{d \ln w_{\min }}=-\frac{d \ln \tilde{p}}{d \ln w_{\min }} . \tag{82}
\end{equation*}
$$

Inserting equations (81) and (82) into equations (75) and (76) gives equations (34) and (33).
In order to get the output response to a change in the minimum wage, differentiate output with respect to the minimum wage, as in equation (37). Note, however, that the marginal products of capital and labor, and materials are:

$$
\begin{align*}
& \frac{\partial Q_{n}}{\partial K_{n}}=\frac{r}{p_{n}\left(1-\frac{1}{\eta_{Z}}\right)}  \tag{83}\\
& \frac{\partial Q_{n}}{\partial L_{n}}=\frac{w_{\min }}{p_{n}\left(1-\frac{1}{\eta_{Z}}\right)} .  \tag{84}\\
& \frac{\partial Q_{n}}{\partial H_{n}}=\frac{w_{H}}{p_{n}\left(1-\frac{1}{\eta_{Z}}\right)}  \tag{85}\\
& \frac{\partial Q_{n}}{\partial M_{n}}=\frac{p_{M}}{p_{n}\left(1-\frac{1}{\eta_{Z}}\right)} . \tag{86}
\end{align*}
$$

Using equation (37), multiplying both sides by $\frac{w_{\min }}{Q}$ and using equations (83) and (84) yields

$$
\begin{equation*}
\frac{d \ln Q_{n}}{d \ln w_{\min }}=\frac{1}{\left(1-\frac{1}{\eta_{Z}}\right)}\left[\frac{w_{\min } L_{n}}{p_{n} Q_{n}} \frac{d \ln L_{n}}{d \ln w_{\min }}+\frac{r K_{n}}{p_{n} Q_{n}} \frac{d \ln K_{n}}{d \ln w_{\min }}+\frac{w_{H} H_{n}}{p_{n} Q_{n}} \frac{d \ln H_{n}}{d \ln w_{\min }}+\frac{p_{M} M_{n}}{p_{n} Q_{n}} \frac{d \ln M_{n}}{d \ln w_{\min }}\right] . \tag{87}
\end{equation*}
$$

Note that $p_{n} Q_{n}\left(1-\frac{1}{\eta_{Z}}\right)=w_{\text {min }} L_{n}+r K_{n}+w_{H} H_{n}+p_{M} M_{n}$. Inserting this into equation (87) gives equation (38), where $s_{L}=\frac{w L}{w_{\min } L+r K+w_{H} H+p_{M} M}$. Because all firms are identical, $\frac{d \ln Q_{n}}{d \ln w_{m i n}}=\frac{d \ln Q}{d \ln w_{m i n}}$, where $Q$ is aggregate output of all restaurants. Given that the equations and unknowns are the same as in Appendix B, the resulting elasticities are the same as in Appendix B.

Although all elasticities are the same as before, special care must be made in interpreting labor's share, $s_{L}$. Labor's share is labor's share of the firm's costs, not labor's share of the
firm's revenue. If the firm makes an economic profit, then the two will not be the same.
Lastly, we note that if firms are monopsonists in the labor market, the employment response to minimum wage change is as in equation (48). Therefore, the output response is determined by equations (28), (38) and (48), and thus the price response is the same as in equation (52).

## Appendix G: Aggregation

In this appendix, we discuss some of the aggregation issues that were described briefly in Section 2.6. Specifically, we derive $\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)$ and $s_{L}$ using other parameters.

First, we show how to infer $\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)$. We have information on the share of restaurant workers who are paid the minimum wage, $\operatorname{prob}\left(w_{\min }=w_{i}\right)$, the fraction of workers who are low skill, $\operatorname{prob}(L)$ (recall that this fraction is identical for all restaurants), as well as the average number of workers per restaurant that pay low skill workers at and above the minimum wage, $Z_{\text {min }}$. To see how these objects can identify the probability that a restaurant pays the minimum wage, note:

$$
\begin{equation*}
N \times \operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right) \times \operatorname{prob}(L) \times Z_{\min }=I \times \operatorname{prob}\left(w_{\min }=w_{i}\right) \tag{88}
\end{equation*}
$$

where $N$ is the total number of restaurants and $I$ is the total number of restaurant workers. The left hand side of equation (88) is the number of minimum wage workers employed (the employer side) and the right hand side is equal to the number of individuals who report working in the restaurant industry and also report being paid the minimum wage (i.e., the household side). Rearranging equation (88), we get:

$$
\begin{equation*}
\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)=\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L) Z_{\min }} \times \frac{I}{N} . \tag{89}
\end{equation*}
$$

$\frac{I}{N}$ can be identified using information on the relationship between the total number of workers employed by the restaurant industry, as measured from both the employer and household side:

$$
\begin{equation*}
\operatorname{Nprob}\left(w_{\min } \geq w\left(L^{*}\right)\right) \times Z_{\min }+\operatorname{Nrob}\left(w_{\min }<w\left(L^{*}\right)\right) \times Z_{-\min }=I \tag{90}
\end{equation*}
$$

where $Z_{-\min }$ is the number of employees per restaurant that pay above the minimum wage. Combining equations (89) and (90) yields the relationship between the object of interest
$\operatorname{prob}\left(w_{\min }<w\left(L^{*}\right)\right)$ and the object we obtain from known data sources:

$$
\begin{equation*}
\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)=\frac{\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L)} \frac{Z_{-\min }}{Z_{\min }}}{1-\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{rog}(L)}+\frac{\operatorname{prob}\left(w_{\min }=w_{i}\right)}{\operatorname{prob}(L)} \frac{Z_{-\min }}{Z_{\min }}} \tag{91}
\end{equation*}
$$

which is equation (20) of the text.
Next, we infer $s_{L}$. We do this by matching both the share of restaurant workers paid the minimum wage and also their share of the total wage bill, which we define as:

$$
\begin{equation*}
\frac{E\left[\frac{1}{I} \sum_{i=1}^{I} w_{i} 1\left\{w_{i}=w_{\min }\right\}\right]}{E\left[\frac{1}{I} \sum_{i=1}^{I} w_{i}\right]} \tag{92}
\end{equation*}
$$

where $w_{i}$ is the wage of individual $i$, and $1\left\{w_{i}=w_{\min }\right\}$ is a $0-1$ indicator equal to 1 when $w_{i}=w_{\min }$, and is equal to 0 otherwise. In order to proceed, we note that the numerator of equation (92) is equal to $\left(\operatorname{prob}\left(w_{\min }=w_{i}\right) \times w_{\min }\right)$, where $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ is the probability that an individual is both low skill and works for a low wage firm, i.e., $\operatorname{prob}\left(w_{\min }=w_{i}\right)=$ $\operatorname{prob}(L$, lowwage $)$. Because by assumption the share of all workers that are low skill does not vary by labor market, firms in high wage and low wage markets employ high skill and low skill workers in equal proportions, $\operatorname{prob}\left(w_{\min }=w_{i}\right)=\operatorname{prob}(L) \operatorname{prob}($ lowwage $)$. Similar reasoning can be used to obtain the probability that an individual is low skill and works in a high wage market, that an individual is high skill and works for in a low wage market, and that individual is high skill and works for in a high wage market. Wages for low skill-low wage labor market, low skill-high wage labor market, high skill-low wage labor market, and high skill-high wage labor market workers are: $w_{\min },\left(w_{\min } \times\right.$ wagediff $),\left(\frac{w_{H}}{w_{L}} \times w_{\min }\right)$, and $\left(\frac{w_{H}}{w_{L}} \times\right.$ wagediff $\left.\times w_{\text {min }}\right)$, respectively. Now, equation (92) can be rewritten as:

$$
\frac{\operatorname{prob}(L) \operatorname{prob}(\text { lowwage }) w_{\min }}{(A(\operatorname{prob}(\text { lowwage })+A((1-\operatorname{prob}(\text { lowwage })) \times \text { wagediff }))) w_{\min }}
$$

where

$$
\begin{equation*}
A=\left(\operatorname{prob}(L)+(1-\operatorname{prob}(L)) \frac{w_{H}}{w_{L}}\right) . \tag{94}
\end{equation*}
$$

Using equation (91) and $\operatorname{prob}\left(w_{\min }=w_{i}\right)=\operatorname{prob}(L) \times \operatorname{prob}($ lowwage $)$ it is straightforward to show that:

$$
\begin{equation*}
\operatorname{prob}(\text { lowwage })=\frac{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)}{\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)+\left(1-\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right) \frac{Z_{-\min }}{Z_{\min }}\right.\right.} . \tag{95}
\end{equation*}
$$

Using equations (93) and (95),
minimum wage labor's share of the total wage bill $=$

$$
\frac{\operatorname{prob}(L) \operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)}{\left(A \times\left(\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)+\left(1-\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)\right) * \text { wagediff } * \frac{Z_{-\min }}{Z_{\min }}\right)\right)}
$$

where wagediff is the wage differential between high wage and low wage areas (caused by differences in $\theta$ ), and $\frac{w_{H}}{w_{L}}$ is the ratio of wages of high skilled workers to low skilled workers. If all restaurants paid the minimum wage (i.e., if $\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)=1$ ), then equation (96) would be equal $\frac{s_{L}}{s_{L}+s_{H}}$.

Because $\frac{s_{L}}{s_{L}+s_{H}}=\frac{p r o b(L)}{\operatorname{prob}(L)+(1-\operatorname{rrob}(L)) \frac{w_{H}}{w_{L}}}$, it is straightforward to use equation (96) to derive equation (21).

Table 1: Parameters of the model

| Parameters | Baseline <br> (range of values considered in robustness checks) | Source |
| :---: | :---: | :---: |
| Price pass-through, $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]$ | 0.05 and 0.07 | Aaronson (2001) and Aaronson et al (2005) |
| Labor's share, $s_{L}+s_{H}$ | 0.30 (0.25 to 0.35) | 1997 Economic Census for Accommodations and Food Services; IRS Statistics on Income Bulletin; 10-K reports |
| Capital's share, $s_{k}$ | 0.30 (0.25 to 0.35) | 10-K reports |
| Material's share, $s_{M}$ | 0.40 (0.35 to 0.45) | 10-K reports |
| Share of Minimum wage workers, $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ | 0.33 (0.25 to 0.40) | Estimated from the CPS |
| Minimum wage labor's share of total wage bill | 0.17 | Estimated from the CPS |
| Fraction of workers that are low skill, $\operatorname{prob}(L)$ | 0.70 (0.60 to 0.80) | Aaronson et al (2005) |
| Ratio of wages in high wage and low wage labor markets, wagediff | 1.5 | Estimated from the CPS |
| Ratio of the average restaurant employment in high and low wage markets, $\left.\frac{Z_{- \text {min }}}{Z_{\text {min }}}\right]$ | 1 (0.9 to 1.3) | 1997 Economic Census for Accommodations and Food Services |
| Elasticity of demand for food away from home, $\eta$ | 0.5 and 1.0 | Piggott (2003); Hussain (2004) |
| Elasticity of substitution between labor and capital, $\sigma$ | 0.5 and 0.8 | Hamermesh (1993) |
| Marshallian labor supply elasticity, $\gamma_{1}$ | 5 (2 to 10) | Card and Krueger (1995) |
| Marshallian labor supply elasticity, $\gamma_{2}$ | 0.5 (0 to 1) |  |
| $\frac{1}{\gamma(L)}$ | $\frac{1}{2 \gamma_{1}}\left(0 \text { to } \frac{1}{\gamma_{1}}\right)$ |  |

Notes: See the appendix for a detailed description of how these parameter values were determined.

Table 2: Estimates of $\lambda$ Under Different Assumptions about Labor Market Structure, Price Responses, the Elasticity of Demand for Food Away from Home ( $\eta$ ), and the Elasticity of Substitution between Labor and Capital ( $\sigma$ ).

| Parameter <br> Assumptions | Competitive low skill employment response$\left(\lambda_{\text {comp }} * \operatorname{prob}\left(w_{\min }>w^{*}\right)\right)$ | The extent of monopsony power ${ }^{1}$ (V) | Employment response when the extent of monopsony power is included |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{array}{\|l\|} \hline \text { Low } \\ \text { Skill } \end{array}$ | High Skill | Total |
|  | (1) | (2) | (3) | (4) | (5) |
| Price response $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=0.07$ |  |  |  |  |  |
| $\eta=0.5, \sigma=0.8$ | -0.36 | 0.98 | -0.34 | 0.02 | -0.23 |
| $\eta=1.0, \sigma=0.8$ | -0.39 | 1.10 | -0.45 | -0.01 | -0.32 |
| $\eta=0.5, \sigma=0.5$ | -0.23 | 1.04 | -0.25 | 0.00 | -0.18 |
| $\eta=1.0, \sigma=0.5$ | -0.26 | 1.21 | -0.36 | -0.04 | -0.26 |
| Price response $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=0.05$ |  |  |  |  |  |
| $\eta=0.5, \sigma=0.8$ | -0.36 | 0.82 | -0.25 | 0.02 | -0.17 |
| $\eta=1.0, \sigma=0.8$ | -0.39 | 0.90 | -0.33 | -0.01 | -0.23 |
| $\eta=0.5, \sigma=0.5$ | -0.23 | 0.89 | -0.19 | 0.00 | -0.13 |
| $\eta=1.0, \sigma=0.5$ | -0.26 | 1.00 | -0.26 | -0.03 | -0.19 |

Notes: See text for details. See table 1 and the appendix for other parameter values used.
${ }^{1}$ See equation 19. A number below 1 implies monopsony power.

Table 3: Estimates of Price and Low Skill Employment Responses Under Different Assumptions About Aggregation

| Estimates | Model 1 | Model 2 | Model 3 |
| :--- | :--- | :--- | :--- |
| Price response ( $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]$ ) under Perfect Competition | 0.099 | 0.051 | 0.064 |
| Price response ( $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]$ ) under Monopsony | -0.109 | -0.056 | -0.071 |
| The extent of Monopsony power (V) | 0.86 | 1.18 | 1.04 |
|  |  |  |  |
| Total employment response: | -0.17 | -0.17 | -0.17 |
| Under Perfect Competition | 0.17 | 0.17 | 0.17 |
| Under Monopsony | -0.12 | -0.22 | -0.18 |
| Weighted by $V$ |  |  |  |
|  | No | Yes | Yes |
| Heterogeneity in skill type | Yes | No | Yes |
| Heterogeneity in labor market type |  |  |  |

Note: See text for detail. We use $\eta=0.5, \sigma=0.5, E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=0.07$ and the parameter
values from table 2. Wagediff is set to 1.5 for models 2 and 3 and is set to match minimum wage labor's share of the total wage bill and $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ in model 1 .

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[^0]:    *Authors' affiliation is Federal Reserve Bank of Chicago. We thank Derek Neal for suggesting that we write this paper; Gadi Barlevy, Jeff Campbell, John Kennan, Mike Kouparitsas, Lynn Riggs, Dan Sullivan, Ted To, and Marcello Veracierto for useful comments; and Kate Godwin and Tina Lam for outstanding research assistance. Author correspondence to Daniel Aaronson or Eric French, Federal Reserve Bank of Chicago, 230 S. LaSalle St., Chicago, IL 60604 or email at daaronson@frbchi.org or efrench@frbchi.org.

[^1]:    ${ }^{1}$ See Card and Krueger (1995) for a review. A sampling of other papers that corroborate these findings include Wellington (1991), Machin and Manning (1996, 1997), and Dickens, Machin, and Manning (1999). See Neumark and Wascher (2003) for a long list of international studies, many of which find no employment response.
    ${ }^{2}$ These results are consistent with views reported in a survey of leading labor economists (Fuchs, Krueger, and Poterba 1998). However, a full quarter of respondents believe there is no teenage disemployment effect from a 10 percent increase in the minimum wage, and another quarter judge the response to be 3 percent or higher.

[^2]:    ${ }^{3}$ We assume that the minimum wage affects wages and the price of the output good but does not affect the price of other factor inputs.

[^3]:    ${ }^{4}$ Prominent examples of studies that concentrate on the restaurant industry include Katz and Krueger (1992), Card and Krueger (1995, 2000) and Neumark and Wascher (2000). According to the Current Population Survey's Outgoing Rotation Groups, eating and drinking places (SIC 641) is the largest employer of workers at or near the minimum, accounting for roughly a fifth of such employees in 1994 and 1995. The next largest employer, retail grocery stores, employs less than 7 percent of minimum or near minimum wage workers. Moreover, the intensity of use of minimum wage workers in the eating and drinking industry is amongst the highest of the industrial sectors.
    ${ }^{5}$ We do not allow for exit or entry, although we allow the size of businesses to change in response to wage changes.

[^4]:    ${ }^{6}$ We assume the minimum wage covers all employers in the labor market. However, employees in restaurants with revenues of less than $\$ 500,000$ per year are not covered by federal minimum wage law.
    ${ }^{7}$ The constant returns production function assumption implies the size of each firm is indeterminate. However, assuming infinitesimally decreasing returns to scale and an infinitesimally small fixed cost of running the firm preserves all the results yet implicitly defines a firm's size. Therefore, we consider the firm size problem unimportant.
    ${ }^{8}$ So long as firms face constant elasticity of demand, all of the remaining equations in the paper are unaffected. See appendix F in Aaronson and French (2006).

[^5]:    ${ }^{9}$ Under perfect competition, with no profits, $s_{L}$ is also equal to low skill labor's share of revenue, $s_{L}=\frac{w_{L} L}{p Q}$.
    ${ }^{10}$ Although the empirical evidence suggests that all elasticities of substitution between factors of production are roughly similar in magnitude, one could argue that some substitution elasticities are close to zero whereas others (such as the elasticity of substitution between high and low skill labor) are large. However, note that both equations (4) and (5) can also be derived for the production function

    $$
    \begin{equation*}
    Q=\min \left\{\left(\left(1-\alpha_{L}\right) H^{\rho}+\alpha_{L} L^{\rho}\right)^{\frac{1}{\rho}}, \alpha_{K} K, \alpha_{M} M\right\} \tag{6}
    \end{equation*}
    $$

    where $s_{L}$ is redefined as $s_{L} \equiv \frac{w_{L} L}{w_{L} L+w_{H} H}$. Therefore, by properly adjusting $\rho$ and $s_{L}$, it is possible to get some measure of the robustness of our results to different substitution elasticities between factors.

[^6]:    ${ }^{11}$ Empirical work on the disemployment effect of the minimum wage typically focus on annual changes to employment, comparing levels pre- and post- the new minimum wage. This short-run response likely abstracts from many adjustments to the capital-labor ratio that may arise over time in response to higher wage bills. For example, in the fast food industry over the last decade or so, cash registers have been modified so that the cashier need not know the price of a product, only its appearance. These machines also save time by directly transferring orders from the cash register to the cooks. It is these long-run responses that are likely of greater interest to policymakers. Baker, Benjamin, and Stanger (1999) illustrate the potentially distinct employment effects that arise at different time horizons.
    ${ }^{12}$ This approach can be justified by the following three period model. In period 1 , firms choose $K$, and believe they know period 2 prices with probability 1 . In period 2 , all prices are revealed, and the wage potentially changes. The firm can then pick $L$ and $M$. Although $K$ is set for period 2 , the firm can pick period 3 values of $K$. In period 3, all prices were as anticipated. In this case, period 2 represents the short-run equilibrium, and period 3 represents the long-run equilibrium.

[^7]:    ${ }^{13}$ Although Stigler (1946) was the first to observe the potential importance of monopsony power when analyzing minimum wage policy, he was clearly suspicious that this was a relevant scenario.
    ${ }^{14}$ We assume that the wage ratio between high and low skilled workers is constant across labor markets. See section 2.6 for more details.
    ${ }^{15}$ Manning (2003) derives an identical specification
    starting from a specification analogous to Dixit and Stiglitz (1977).
    ${ }^{16}$ Thus we allow for differences in wages and the importance of monopsony power across labor markets, although we assume that all employers of low skill labor are identical within a labor market. This is an extreme assumption given that only 25 percent of all minimum wage workers are employed in the restaurant industry, and thus restaurants compete with many types of firms in the labor market.

[^8]:    ${ }^{17}$ Note that a change in the minimum wage changes both the labor supply curve faced by the firm (because $w_{L,-n}$ changes) and the $\operatorname{MRP}(\mathrm{L})$ curve faced by the firm (because the output price charged by other firms changes). Thus the figures are not helpful for understanding the quantitative price and employment responses, although they are helpful for understanding the qualitative responses. Furthermore, because the labor supply curve and $\operatorname{MRP}(\mathrm{L})$ curve shift in response to the minimum wage, changing the minimum wage also changes the boundary conditions $w^{*}$ and $w^{* *}$. Below, when we refer to $w^{*}$, we mean the value of it for which $w^{*}=w_{\min }$, and $w^{* *}$ the value for which $w^{* *}=w_{\min }$. Therefore, figure 2 is not technically correct, although it helps clarify the intuition of the model.

[^9]:    ${ }^{18}$ In the absence of a minimum wage, the marginal revenue product of labor is $\left(1+\frac{1}{\gamma_{1}}\right)$ times the wage, and for this reason $\frac{1}{\gamma_{1}}$ is sometimes known as the rate of exploitation.

[^10]:    ${ }^{19}$ Again, the substitutability of high and low skill labor can result in an offsetting employment response of high skill labor.

[^11]:    ${ }^{20}$ Given the parameters listed in Section 3, allowing for labor's share to vary across markets had only a modest effect on our results.
    ${ }^{21}$ Within our model, all low skill workers are paid the same wage, although there is wage dispersion across labor markets. If two workers within the same labor market had different wages before the minimum wage hike, they will have different wages after the minimum wage hike. There is some empirical evidence against this implication of the model. Card and Krueger (1995) show that, within states, minimum wage hikes significantly compress wages for those paid near the minimum wage. Unless labor markets are smaller than states (which may be true for minimum wage workers), this is evidence against the model.

[^12]:    ${ }^{22}$ Assuming that wages for high skill workers is a constant multiple of the wages of low skill workers and that the fraction of low skill workers does not vary across labor markets, if fairly consistent with the data. The coefficient of variation of restaurant worker wages within a city varies very little with the average wage within a city, according to our calculations using CPS data. This is consistent with our joint hypothesis.

[^13]:    ${ }^{23}$ These lower estimates are consistent with some specifications reported in Aaronson (2001).
    ${ }^{24}$ Predicted short-run price responses depend on how much the ratio $\frac{\eta}{\sigma}$ varies from 1 . Given the range of estimates for $\eta$ and $\sigma$ that we consider likely, the difference between the long-run and short-run response is very small.

[^14]:    ${ }^{25}$ For example, when $\gamma_{2}=0.5, \lambda_{\text {monop }} \times \operatorname{prob}\left(w_{\min }>w^{*}\right)=0.24$.
    ${ }^{26} V$ can exceed one if firms pass on more of a price increase than would be expected given perfect competition. Empirically, overshifting of ad valorem taxes has been found by, among others, Besley and Rosen (1999) in the retail apparel industry and Karp and Perloff (1989) in the Japanese television market.

[^15]:    ${ }^{27}$ The only other parameters that have a substantive impact on these magnitudes are labor share $\left(s_{L}+s_{H}\right)$ and the share of workers that are low skill $(\operatorname{prob}(L))$. To get an indication of the sensitivity of the results, we start with the baseline described above (where $\sigma=0.65, \eta=0.75$, and $E\left[\frac{d \ln p}{d \ln w_{\min }}\right]=0.06$. Altering $s_{L}+s_{H}$ between 0.25 and 0.35 stretches the low skill employment response to be between -0.36 to -0.27 . If, instead, we allow $\operatorname{prob}(L)$ to be between 0.6 and 0.8 , the low skill employment response falls in the range of -0.37 to -0.26.
    ${ }^{28}$ Furthermore, the other parameters, as discussed in the previous footnote, have a negligible impact (always less than 0.03) on the total employment response.
    ${ }^{29}$ Predicted values are obtained by setting $V=1$ in equation (18), using the value of $s_{L}$ in equation (21), and the formula for the relationship between $\operatorname{prob}\left(w_{\min }=w_{i}\right)$ and $\operatorname{prob}\left(w_{\min } \geq w\left(L^{*}\right)\right)$ derived in equation (20). The range of values for $\gamma$ are obtained by varying the parameters in the same way as Table 2.
    ${ }^{30}$ To make this calculation, we assume there are multiple labor markets within each city. The minimum wage binds in some markets, but not in others. In all cities there is a probability $V$ that firms in the market have monopsony power, if the minimum wage binds in that market. We then use the inferred value of $V$ from the model, and use equations (18), (21), and (20).

[^16]:    ${ }^{31}$ The employment responses do not vary across models only when $\sigma=\eta$.

[^17]:    ${ }^{32}$ Table 2 shows that $V$ ranges from just below 1 to just above 1 for reasonable perturbations of the parameters.

[^18]:    ${ }^{33}$ To get comparable estimates, the teen studies need to be adjusted for coverage (e.g. table 2 in Brown 1999). Brown's adjustments are based on wage distributions for 16-24 year olds, as calculated in Neumark and Wascher (2002). He argues that the share of workers in this age group impacted by the minimum wage is close to 20 percent, suggesting that the elasticity of demand for teenagers should be factored up $1 / 0.2=5$. However, since this calculation is based partly on 20-24 year olds, a more conservative guess for 16-19 year olds would be 3 .
    ${ }^{34}$ Our results are not exactly comparable to theirs, because our results are for the entire restaurant industry, whereas their results are just for the fast food industry, where a higher share of workers are paid the minimum wage.

[^19]:    ${ }^{35}$ While the time frame is somewhat short, this three-year period contains an unusual amount of minimum wage activity. A bill signed on August 20, 1996 raised the federal minimum from $\$ 4.25$ to $\$ 5.15$ per hour, with the increase phased in gradually. An initial increase to $\$ 4.75$ ( 11.8 percent) occurred on October 1, and the final installment ( 8.4 percent) took effect on September 1, 1997. Moreover, additional variation can be exploited by taking advantage of cross-state differences in market wages, state-imposed minimum wages that exceed federal levels, and differences in establishment type.

[^20]:    ${ }^{36}$ The search uses five keywords: restaurant, steak, seafood, hamburger, and chicken.
    ${ }^{37}$ The IRS claims that labor cost is notoriously difficult to decompose for corporations and therefore we restrict our analysis to partnerships, where there is less concern about reporting.
    ${ }^{38}$ Several 10-K reports of individual restaurant companies show that wages account for 85 percent of compensation. Therefore, labor's share based on compensation is roughly 36 and 29 percent at full and limited service restaurants.

[^21]:    ${ }^{39}$ There are no federal changes and only two state changes during these two years. We exclude the two states, Vermont and Washington, with such activity, as well as all data from June to August of 1995, for which there are no geographic identifiers.
    ${ }^{40}$ See Grossman (1983) and Card and Krueger (1995) for empirical evidence and Card and Kruger (1995) Teulings (2000), and Manning (2003) for potential explanations.

[^22]:    ${ }^{41}$ The 27 CPI cities are New York City, Philadelphia, Boston, Pittsburgh, Buffalo, Chicago, Detroit, St Louis, Cleveland, Minneapolis-St. Paul, Milwaukee, Cincinnati, Kansas City, DC, Dallas, Baltimore, Houston, Atlanta, Miami, Los Angeles, San Francisco, Seattle, San Diego, Portland, Honolulu, Anchorage, and Denver. After 1986, prices for 12 of these cities - Buffalo, Minneapolis-St. Paul, Milwaukee, Cincinnati, Kansas City, Atlanta, San Diego, and Seattle, Portland, Honolulu, Anchorage, and Denver - are reported semiannually. Therefore, they only included pre-1986 observations for those cities.

[^23]:    ${ }^{42}$ Likewise, Hamermesh's review of the substitutability between labor and materials suggests similar sized values. Again, this is consistent with the findings in Goodwin and Brester (1995).

[^24]:    ${ }^{43}$ To derive equation (32), recognize that $\tau$ represents the equilibrium output response to a minimum wage change. Moreover, note that $\frac{d \ln Q}{d \ln p}$ is the ratio of the output response to the price response caused by an increase in the minimum wage. Because the demand curve for $Q$ does not change in response to the minimum wage (but the supply curve for $Q$ does), changes in prices and quantities are caused only by the supply curve for $Q$ moving along the demand curve, which has elasticity $\eta$. Therefore, $\frac{d \ln Q}{d \ln p}=-\eta$.

[^25]:    ${ }^{44}$ Since a small share of workers (and an even smaller share of the wage bill) is affected by the minimum wage, the minimum wage likely has small effects on aggregate income and demand. Therefore, the price of capital, materials, and high skill labor are unlikely to be impacted as well.

[^26]:    ${ }^{45}$ To see this point, note that the vertical distance between the inverse log labor supply curve and the log marginal cost curve is $\ln \left(1+\frac{1}{\gamma_{1}}\right) \approx \frac{1}{\gamma_{1}}$. As a result, even if the MRPL curve of the firm is infinitely elastic, the minimum wage can only increase wages by at most a factor of $\frac{1}{\gamma_{1}}$ and still increase employment. Therefore, the most that labor supply could increase in a market by raising the minimum wage is $\Delta \ln L=\gamma_{2} \Delta \ln w=\frac{\gamma_{2}}{\gamma_{1}}$. To the extent that MRPL curves slope down, employment gains of raising
    the minimum wage will be smaller. Given the parameter values in section 3, the employment gains are only about $10 \%$.

[^27]:    ${ }^{46}$ Note that $\frac{s_{L}}{\gamma(\theta)}$ is between 0 and $\frac{s_{L}}{\gamma_{1}}$. For reasonable values, this term is less than 0.02 . Furthermore, $\sigma$ and $\eta$ are similar in size. Therefore, $\frac{\sigma}{\eta}$ is likely close to $1, \frac{s}{\gamma(\theta)}\left(1-\frac{\sigma}{\eta}\right)$ is likely between -0.01 and 0.01 , and our assumption seems reasonable.

