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Fernando Alvarez and Marcelo Veracierto

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Fernando Alvarez
University of Chicago and NBER

Marcelo Veracierto
Federal Reserve Bank of Chicago

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Abstract

Fixed term employment contracts have been introduced in number of European countries as a way to provide flexibility to economies with high employment protection levels. We introduce these contracts into the equilibrium search model in Alvarez and Veracierto (1999), a version of the Lucas and Prescott island model, adapted to have undirected search and variable labor force participation. We model a contract of length J as a tax on separations of workers with tenure higher than J. We show a version of the welfare theorems, and characterize the efficient allocations. This requires solving a control problem, whose solution is characterized by two dimensional inaction sets. For $J = 1$ these contracts are equivalent to the case of firing taxes, and for large J they are equivalent to the laissez-faire case. In a calibrated version of the model, we evaluate to what extent contract lengths similar to those observed in Europe, close the gap between these two extremes.

1 Introduction

In this paper we construct a general equilibrium search model to analyze the effects of fixed term employment contracts (or, for short, temporary contracts). This type of contracts were introduced in economies with high employment protection levels in Europe and Latin America as a way of giving firms some flexibility in the process of hiring and firing workers. Fixed term contracts stipulate a period of time, typically between one and three years, during which workers can be dismissed at very low or zero separation costs. If workers are retained beyond this period, standard separation costs apply.

Since the introduction of fixed term contracts during the eighties, the fraction of workers hired under this modality has expanded steadily in Europe to reach more than 13 percent in 2000. There are large cross-country differences behind this number, however, due to differences in the scope and duration of the fixed term contracts allowed for. For instance, some countries restrict these contracts to certain occupations and type of workers while others given them broad applicability. In this paper we focus on the case of Spain because in 1984 it substantially liberalized the applicability of temporary contracts at a time when the country had one of the highest employment protection levels in Europe (see, Cabrales and Hopenhayn 1997, and Heckman and Pages-Serra 2000). From 1984 to 1991 the fraction of workers with fixed term contracts in Spain went from 11 to more than 30 percent and almost all the hiring in the economy became under this form (see Hopenhayn and Garcia-Fontes, 1996). These reforms were partly undone during the nineties, when the maximum length of the fixed term contracts was reduced from 3 years to one year and the severance payments for ordinary indefinite-length contracts were substantially reduced. However, even after this partial reversal, the fraction of workers under fixed term contracts stabilized at about 33 percent.

Figure 1, which is taken from Hopenhayn and Cabrales (1997), displays estimates for the one-quarter transition probabilities from employment to unemployment during the six years before and after the 1984 reform, as a function of the length of the employment spells. The firing rates increased significantly after the reform and a spike formed at an employment duration of 3 years which, not surprisingly, corresponds to the maximum fixed term contract length allowed by the reform. Thus, the introduction of the fixed term contracts appears to have significant effects on worker reallocation. In fact, there is considerable agreement in the literature that the main effects of introducing fixed term contracts are a substantial increase in the flows from unemployment to employment (i.e. a decrease in the average duration of unemployment) and a significant increase in

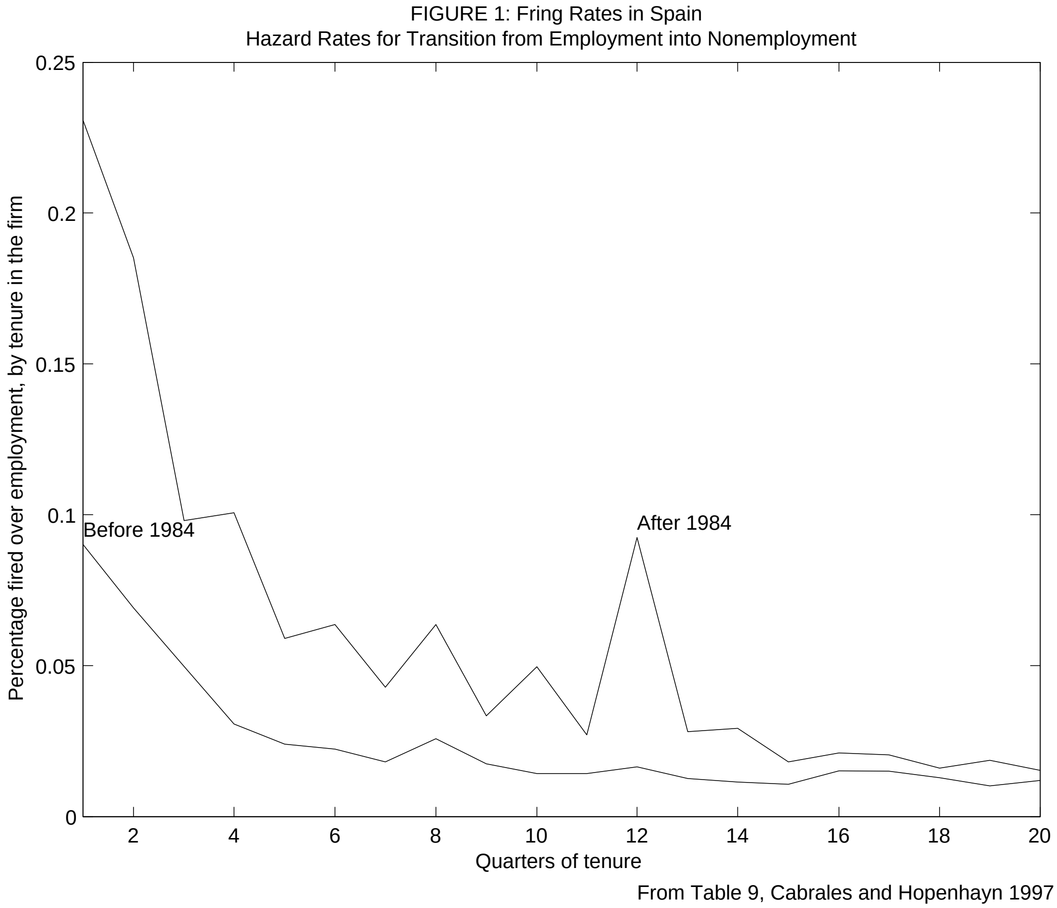

Figure 1:

the flows from employment to unemployment (i.e. an increase in the firing rate) as can be seen, for example, in the literature survey by Dolado et al (2001). The net effect of these two opposing forces on the unemployment rate is not as clear, but the evidence seems to indicate a small increase.

In order to analyze fixed term employment contracts we introduce them into the equilibrium search model of Alvarez and Veracierto (1999), which is a version of the Lucas and Prescott island search model with undirected search and variable labor force participation. Similarly to Lucas and Prescott (1974), production takes place in a large number of locations (or islands) that use identical neoclassical decreasing returns to scale technologies. There are many firms in each island, all of them subject to the same island-specific productivity shock. Changes in the island-specific productivity shocks give raise to changes in labor demand across locations. Moving a worker across locations is costly: It requires one period during which the agent does not enjoy leisure nor works. In addition, agents that search arrive randomly to all the islands in the economy (i.e. search is undirected). When a worker leaves an island, he can choose either to work at home (stay out of the labor force) or to search (become unemployed). Within each island, firms and workers participate in competitive labor markets. We assume that agents have access to perfect insurance markets, so that firms maximize expected discounted profits and households maximize expected discounted wages.

The employment protection system that we analyze is characterized by two parameters: the firing tax τ and the length of the fixed term contracts J. In particular, firms must pay a firing tax τ per unit reduction in the employment of permanent workers (those that have a tenure level equal to or greater than J) but are exempt from paying firing taxes on temporary workers (those with a tenure level less than J). Because firing taxes are tenure dependent, the state of an island is not only described by the idiosyncratic productivity level but by the distribution of workers across tenure levels. Since workers are differentiated by their tenure levels, they participate in different labor markets and receive different wages. Given that the firms and workers problems are dynamic, they must take into account the equilibrium law of motion of wages across tenure levels. The presence of the tenure dependent firing cost implies that firms must solve a modified sS optimization problem. In turn, workers at each tenure level must solve a search problem, deciding whether to stay in the island where they are currently located or to become non-employed. A stationary equilibrium for this economy requires solving the process for the island-level equilibrium wages, so that the demand for labor equals its supply at each tenure level and island-wide state. The economy-wide equilibrium is described by an invariant distribution across islands states. This economy-wide distribution is needed to describe the benefit of search and the aggregate demand for labor.

If the separation cost are considered a technological feature of the environment, a version of the first and second welfare theorems hold for our recursive competitive equilibrium (RCE). If instead, the separation cost are taxes rebated lump-sum, the most interesting case to consider, the welfare theorems do not apply but we can still use a modified version of the planning problem to characterize a RCE. In particular, we can break the economy-wide planning problem into a series of island-wide planning problems, one for each island. Each of these island-wide social planners solves a similar problem: To maximize the expected discounted value of output by deciding how many workers to keep and how many to take out of the island. In this problem the island-wide planner takes as given the constant flow U of searches that arrive to the island. This flow is independent of the characteristics of the island because of the assumption of undirected search. The island's planner also takes as given the shadow value of returning a worker to non-employment. This shadow value is tenure dependent, to take into account the separation taxes τ. While the state of this problem is the distribution of workers by tenure levels, a J dimensional object, we show how to reduce it to a two dimensional object: the number of temporary and permanent workers. We also show that the solution to this control problem is characterized by two-dimensional sets of inaction, one set for each value of the idiosyncratic productivity shock. Given the solution to the island-wide planning problem, the economy-wide equilibrium is obtained by solving for two unknowns: the equilibrium shadow value of workers and the equilibrium number of searchers U.

We take the case with no fixed term contracts and large firing costs as our benchmark and calibrate it to reproduce a stylized version of the Spanish economy before the 1984 reform. We use this calibrated version to evaluate to what extent fixed term contracts of different lengths add flexibility to the labor market. For large values of J, fixed term contracts are equivalent to the laissez-faire case, since for large J most workers will be temporary, and hence if they were dismissed they will be zero separation costs. Thus, we phrase the question of the added flexibility by computing how much of the gap between the firing tax and the laissez-faire cases is closed when fixed term contracts of empirically reasonable length are introduced. We find that even when the firing tax τ is small, introducing temporary contracts of a short length J sharply increases the average firing rate and decreases the average duration of unemployment. Nevertheless, for firing taxes of about a year of average wages (the value that we argue corresponds to Spain during the eighties) unemployment rate, productivity and welfare change smoothly with J. For instance, the unemployment rate is 2.4 percent points higher in laissez-faire than in the benchmark case of firing taxes (and no temporary contracts). With temporary contracts of three years duration, the length of contracts after the 1984 reform in Spain, we find that the unemployment rate is 1.25 percentage points higher than in the benchmark case. We also find that the welfare cost of firing taxes is about 2.5 percentage points in the benchmark case (in perpetual consumption equivalent units), while the welfare cost of temporary contracts of three years of length is about 1 percent. Thus temporary contracts of 3 years provide substantial flexibility, closing more than half of the gap between the benchmark and laissez-faire cases.

Several papers have analyzed the effect of temporary contracts, including a theoretical analysis of them, such as Blanchard and Landier (2001) and Nagypal (2002). The models that are more similar in spirit to our paper, however, are Bentolila and Saint Paul (1992), Hopenhayn and Cabrales (1993), Aguiregabiria and Alonso-Borrego (2004) and Alonso-Borrego et al (2005), since they all study labor demand models with dynamic adjustment costs. One difference with the models in

Bentolila and Saint Paul (1992), Hopenhayn and Cabrales (1993), Aguiregabiria and Alonso-Borrego (2004) is that these papers consider partial equilibrium models (with exogenous wages) and do not consider unemployment. The paper that is closest to ours is Alonso-Borrego et al (2005) since it performs a general equilibrium analysis in a model with search frictions. However, there are important differences. First, agents are subject to exogenous borrowing limits. Second, employment contracts are constrained to have a constant wage rate as long as the employment relation lasts. Third, workers under temporary contracts are assumed to be less productive than under ordinary contracts, regardless of their actual or expected tenure. Fourth, fixed term contracts con only last one model period. Some of these assumptions, such as lack of insurance, are meant to provide realism. However, they substantially complicate the interpretation of the results. For example, it is unclear to what extent the results depend on the rigid wage contracts.[1] We think that by performing the analysis in an economy with efficient contracts, this paper not only provides easily interpretable results but provides a useful benchmark for evaluating deviations from the complete contracts case. Other assumptions, such as one period contracts, are introduced for tractability. However, the restriction to one period temporary contracts may be important, given that the actual length goes up to 3 years. As a consequence, we think that the two papers should be considered complementary.

The paper is organized as follows. Section 2 describes the economy. Section 3 defines efficient allocations. Section 4 characterizes efficient stationary allocations. Section 5 defines and characterizes a stationary recursive competitive equilibrium. Section 6 gives a more realistic, although more complicated, definition of a competitive equilibrium and establishes that it is equivalent to the more tractable specification of Section 5. Finally, Section 7 performs the computational experiments. Seven appendices provide all the proofs and supporting material to the paper.

2 Description of the Economy

Production takes place in a continuum (measure one) of different locations, or "islands". In each island consumption goods are produced according to $F(E, z)$, a neoclassical production function, where E is employment and z a productivity shock that takes values in the set Z. The process

[1] The analysis in Alvarez-Veracierto (2002), on which Alonso and Borrego's paper is based, indicates that the rigid wage contracts probably play a critical role in the results.

for z is Markov with transition function $Q(z_{t+1}|z_t)$, and realizations are i.i.d. across islands. We let $f(E,z) \equiv \partial F(E,z)/\partial E$ and assume that f is continuous and strictly decreasing in E, strictly increasing in z, and that

$$\lim_{E\to 0} f(E,\underline{z}) = \infty$$

where $\underline{z} \equiv \min\{z : z \in Z\}$.

There is a continuum of agents with mass equal to N. Agents participate in one of the following three activities: to work in an island, to perform home production (or, equivalently, to enjoy leisure), or to search. Non-employed agents, which we sometimes refer to as "agents being at a central location", either work at home (enjoy leisure) or search. If they work at home during the current period, they start the following period as non-employed. If a non-employed agent searches in the current period, she does not produce during the current period but arrives randomly to an island at the beginning of the next period. We assume that search is undirected, so the probability of arriving to an island of any given type is given by the fraction of islands of that type in the economy. An agent that is located at an island at the beginning of the period can decide whether to stay in the island or to become non-employed. If she stays, she works and starts the following period in the same location.

We let L_t the number of agents engaged in home production at time t, and U_t the fraction engaged in search at time t. The period utility function for the household consuming c units of consumption goods and L units of leisure are:

$$u(c,L) = \frac{c^{1-\gamma}-1}{1-\gamma} + \omega L..$$

As it is well known, the linearity of leisure in household preferences can represent an economy with indivisible labor and employment lotteries, as in Rogerson (1988). To simplify the description of the planner's problem we will focus in the case where consumption and leisure are perfect substitutes, which is obtained setting $\gamma = 0$. In this case we consider home production as an alternative activity that produces ω consumption goods per period, and let the household's utility function simply be

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t.$$

As we explain in Section 7, this assumption is without loss of generality, in the sense that there is a simple mapping between stationary allocations with different values of γ.

Up to here the environment is a modification of the equilibrium search model of Lucas and Prescott (1974) that introduces household production and undirected search, as in Alvarez and

Veracierto (1999). We now introduce a tenure-dependent separation cost. In this section we introduce this separation cost as a technological feature of the environment. In Section 5 we show how to use the efficient allocation of this economy to construct an equilibrium where the separation cost is a tax levied to firms and rebated to households in a lump-sum way.

The tenure-dependent separation cost works as follows: if an agent has worked for J or more periods in a location, then at the time that she returns to the central location τ consumption goods are lost from production in the island. If she returns to the central location after less than J periods, no separation cost is incurred. In Section 5 and 6 we present equilibrium concepts that show that this tenure-dependent separation cost at the island's level captures the temporary employment contracts used in the real world.

3 Efficient Allocations: Formal Definition

Since the separation cost depends on tenure levels, an allocation must include the distribution of workers by tenure in each island. We refer to workers with tenure $j = 1, ..., J-1$ in a location as temporary workers, and to those with tenure $j \geq J$ as permanent workers. Thus the state of a location is given by its productivity shock z and by a J dimensional vector T indicating the number of workers with different tenures. In the sequential notation locations are indexed with their state at time $t = 0$, denoted by $X = T_0$. We use $z^t = (z_t, z_{t-1}, ..., z_0)$ for the history of shocks of length t, and index each location at time t by (z^t, X), its history of shocks and its initial state. The initial state of the economy is described by a distribution of locations across pairs (z_0, X) and by U_{-1}, the number of agents that searched during $t = -1$. We let $\eta(X|z_0)$ be the fraction of locations with state X conditional on z_0, and $q_0(z)$ the initial distribution of z_0. We assume that q_0 equals the unique invariant distribution associated with the transition Q. We denote by $q_t(z^t)$ the fraction of islands with history z^t, which by the Law of Large Numbers satisfies,

$$q_{t+1}\left(z^t, z_{t+1}\right) = Q\left(z_{t+1}|z_t\right) q_t\left(z^t\right).$$

We indicate employment of agents with tenure j at a location $(z^t,\ X)$ by $E_{jt}(z^t, X)$, for $j = 0, ..., J,\ \ z^t \in Z^t$ and $t \geq 0$. Likewise, we denote by $S_{jt}(z^t, X)$ the separations, i.e. the number of agents with tenure j that return to the central location.

Formally we say that $\{E_{jt}, S_{jt}, U_t, H_t\}$, given η and U_{-1}, is a feasible allocation if the following conditions hold: i) the island's law of motion

$$E_{j,t}\left(z^t, X\right) = E_{j-1,t-1}\left(z^{t-1}, X\right) - S_{j,t}\left(z^t, X\right),\ j = 1, 2, ..., J-1,$$

$$E_{J,t}\left(z^t, X\right) = E_{J-1,t-1}\left(z^{t-1}, X\right) + E_{J,t-1}\left(z^{t-1}, X\right) - S_{J,t}\left(z^t, X\right),$$

$$E_{0,t}\left(z^t\right) = U_{t-1} - S_{0,t}\left(z^t, X\right),$$

$S_{j,t}(z^t, X) \geq 0$ for $t \geq 0,\ z^t \in Z^t, X \in \text{supp}(\eta)$, ii) the feasibility constraint for the labor market

$$U_t + \sum_{z^t}\sum_{X}\sum_{j=0}^{J} E_{j,t}\left(z^t, X\right) q_t\left(z^t\right)\eta\left(X|z_0\right) + L_t = N$$

$U_t, H_t \geq 0$ for all $t = 0, 1, ...$ and iii) the initial conditions given by

$$E_{j-1,-1} = X_j \text{ for } j = 1, 2, ..., J-1,$$

$$E_{J-1,-1} + E_{J,-1} = X_J,$$

where $E_{0,-1} = U_{-1}$, are given.

The first constraint states that the number of employed workers of tenure $j \leq J-1$ is given by the number of workers of tenure $j-1$ that were employed in the island during the previous period, minus the number of these workers that are taken out of the island during the current period. The second constraint is analogous to the first constraint for workers of tenure J or higher. It differs from the first one because we don't keep track of workers of tenure $j \geq J$ separately (they are all lumped together into tenure J). The third constraint says that the employment of tenure zero workers is given by those that just arrived to the island, minus the number of them that are taken out of the island. The fourth constraint states that sum of total unemployment, total employment and agents out of the labor force equals the population N. The fifth equation defines $E_{j-1,-1}$ in terms of the initial conditions X_j.

Hereon we define $T_{j,t}(z^t, X)$ as the number of workers of tenure j available at the beginning of the period t in an island of type (z^t, X), so that

$$T_{j,t}\left(z^t, X\right) = E_{j-1,t-1}\left(z^{t-1}, X\right)\ j = 1, 2, ..., J-1,$$

$$T_{J,t}\left(z^t, X\right) = E_{J-1,t-1}\left(z^{t-1}, X\right) + E_{J,t-1}\left(z^{t-1}, X\right),$$

$$T_{0,t}\left(z^t\right) = U_{t-1}$$

Hence condition i) in the definition of feasibility is equivalent to

$$E_{j,t}\left(z^t, X\right) \leq T_{j,t}\left(z^t\right) \text{ for all } j$$

With these objects at hand we can define a planning problem whose solutions characterize the set of efficient allocations. We say that $\{E_{jt},\ S_{jt},\ T_{j,t},\ U_t,\ L_t\}$ is an efficient allocation if it maximizes

$$\sum_t \beta^t \sum_{z^t} \sum_X F\left(\sum_{j=0}^J E_{j,t}\left(z^t, X\right), z_t\right) q_t\left(z^t\right) \eta\left(X|z_0\right)$$
$$+\sum_t \beta^t \omega L_t - \tau \sum_t \beta^t \sum_{z^t} \sum_X S_{J,t}\left(z^t, X\right) q_t\left(z^t\right) \eta\left(X|z_0\right)$$

for all feasible allocations given the initial conditions η and U_{-1}. A feasible allocation $\{E_{jt}, S_{jt}, T_{j,t}, U_t, L_t\}$ given the initial conditions η, U_{-1} is stationary if $U_t,\ L_t$ and the cross sectional distribution η_t are constant, where η_t is given by

$$\eta_{t+1}\left(A|z'\right) = \sum_{z^t \in Z^t} \sum_X I_A\left(z^t, X\right) \eta_0\left(X|z_0\right) q_t\left(z^t\right) Q\left(z'|z_t\right)$$

and where I_A is an indicator defined as

$$I_A\left(z^t, X\right) = \left\{\begin{array}{c} 1, \text{ if } \left[T_{1,t}\left(z^t, X\right), ..., T_{J,t}\left(z^t, X\right)\right] \in A \\ 0, \text{ otherwise} \end{array}\right\}$$

for all $z^t \in Z^t$, $X \in \text{supp}(\eta)$, and Borel measurable $A \subset R^J_+$. Finally, we say that $\{L, U, \eta\}$ is a stationary efficient allocation if there is some efficient allocation $\left\{\hat{E}_{jt}, \hat{S}_{jt}, \hat{T}_{j,t}, \hat{U}_t, \hat{L}_t\right\}$ with initial condition $\hat{U}_{-1},\ \hat{\eta}$ which is stationary and for which

$$\hat{U}_{-1} = \hat{U}_t = U, \quad \hat{L}_t = L, \text{ and } \hat{\eta}_t = \eta$$

for all $t \geq 0$.

4 Characterization of efficient stationary allocations.

We refer to efficient allocations being interior, as those in which are agents engaged in all three activities: search, home production, and work. Our characterization of interior efficient stationary allocations consists on the solution of two equations in two unknowns: (U, θ), where U is the unemployment and θ is the shadow value of being non-employed. One equation states that the shadow value of search equals the expected value of arriving next period to an island randomly, according to the invariant distribution. The second equation ensures that agents are indifferent between doing search or home production. The first equation is quite complex, it involves solving a dynamic programing problem and using the invariant distribution generated by its optimal policies. We refer to this dynamic programing problem as the island planning problem.

The state of this problem is given by (T, z), where T is a vector describing the number of workers across tenure levels $j = 1, 2, ..., J$ at the beginning of the period, and where z is the current productivity shock. The island planner receives U workers with tenure $j = 0$ every period. The planner decides how many workers to employ at each tenure level, and returns workers to the central location at a shadow value given by θ. The planner incurs a cost τ per worker with tenure J that is returned to the central location. Formally,

$$\begin{aligned} & V(T, z; U, \theta) \\ = \; & \max_{\{E_j\}} \left\{ F\left(\sum_{j=0}^{J} E_j, z\right) + \theta\left([U - E_0] + \sum_{j=1}^{J} [T_j - E_j]\right) - \tau [T_J - E_J] \right. \\ & \left. + \beta \sum_{z'} V(E_0, E_1, ..., E_{J-2}, E_{J-1} + E_J, z'; U, \theta) Q(z'|z) \right\} \end{aligned}$$

subject to $0 \leq E_j \leq T_j$ for $j = 1, .., J$ and $0 \leq E_0 \leq U$. We let $G(T, z; U, \theta)$ be the optimal employment decision and $T' = A(T, z)$ the implied transition function with $T'_{j+1} = G_j(T, z)$ for $j = 0, ..., J-2$ and $T'_J = G_J(T, z) + G_{J-1}(T, z)$.

It is intuitive to see that if U is the economy-wide efficient unemployment level, and θ is the economy-wide shadow value of non-employment, the employment decisions of the island planners' problem recovers the economy-wide efficient employment decisions. To see why, notice that each island faces the same value for U, since search is undirected, and the same value of θ, since workers are identical once they leave the island and arrive to the central location.

As stated above, the shadow value of non-employment equals the discounted expected value of arriving at an island with zero tenure under the invariant distribution. To find the shadow value of workers with tenure zero at each island we define the problem of an island's planner that faces a flow of unemployed workers $\hat{U}$ for one period, and then it reverts to the constant flow U thereafter:

$$\begin{aligned} & \hat{V}\left(T, z; \hat{U}, \theta\right) \qquad (1) \\ = & \max_{E_j} \left\{ F\left(\sum_{j=0}^{J} E_j, z\right) + \theta\left(\left[\hat{U} - E_0\right] + \sum_{j=1}^{J} [T_j - E_j]\right) - \tau\left[T_J - E_J\right] \right. \\ & \left. + \beta \sum_{z'} V\left(E_0, E_1, ..., E_{J-2}, E_{J-1} + E_J, z'; U, \theta\right) Q\left(z'|z\right) \right\} \end{aligned}$$

subject to $0 \leq E_j \leq T_j$ for $j = 1, .., J$ and $E_0 \leq \hat{U}$. Using this problem we define the value of an extra zero tenure worker in a location with (T, z) as:

$$\lambda\left(T, z; U, \theta\right) = \frac{\partial \hat{V}\left(T, z; \hat{U}, \theta\right)}{\partial \hat{U}} |_{\hat{U}=U} \ . \qquad (2)$$

where $\partial \hat{V}/\partial \hat{U}$ is a subgradient of $\hat{V}$ in the case it which is not differentiable. The next theorem gives a characterization of the stationary efficient allocations.

Theorem 1 . *Let (U, θ) be an arbitrary pair. Let $V\left(\cdot; U, \theta\right)$ be the solution of the island planning problem, and let $G\left(\cdot; U, \theta\right)$ $\lambda\left(\cdot; U, \theta\right)$ be the their associated optimal policies and shadow value for zero tenure workers. Suppose that:*

i) $\mu\left(\cdot; U, \theta\right)$ is a stationary distribution for the process (T, z) with transition functions given by $Q\left(z'|z\right)$ for z' and by $A\left(T, z\right)$ for T'.

ii) the value of search σ is given by

$$\sigma = \beta \int \lambda\left(T, z\ ; U, \theta\right) \mu\left(dT \times dz\ ; U, \theta\right)$$

iii)the number of agents engaged in home production N satisfy

$$L = N - U - \int \left[\sum_{j=0}^{J} G_j\left(T, z\ ; U, \theta\right)\right] \mu\left(dT \times dz\ ; U, \theta\right) \geq 0$$

iv)the labor force participation decisions are optimal, in the sense that

$$\begin{aligned} \theta &= \max\left\{\sigma,\ \omega + \beta\theta\right\}, \\ 0 &= L\ \left[\theta - \omega + \beta\theta\right]. \end{aligned}$$

Finally, define $\eta\left(T, z\right) = \mu\left(T|z\right)$, as the distribution of T conditional on z. Then $\{L, U, \eta\}$ is an efficient stationary allocation.

Conditions (i) and (ii) have been explained above. Condition (iii) defines the number of agents doing home production as total population minus the sum of unemployment and employment, and states that home production must be nonnegative. The first equation in condition (iv) states that the value of non-employment must be the best of two alternatives: the value of search, which is σ, and the value of doing home production during the current period and being non-employed the following period, which is $\omega + \beta\theta$. The second equation in condition (iv) is a complementary slackness condition for home production.

Theorem 1 implies that characterizing efficient stationary allocations is reduced to solving two equations in two unknowns, and checking that an inequality is satisfied. Given an arbitrary pair (U, θ), the functions $V(\cdot, U, \theta)$, $G(\cdot, U, \theta)$, $\lambda(\cdot, U, \theta)$, and the distribution $\mu(\cdot, U, \theta)$ can be found using standard recursive techniques. Defining $\sigma(U, \theta)$ and $L(U, \theta)$ as the left hand sides of conditions (ii) and (iii), respectively, the two equations that U and θ must satisfy are:

$$\begin{aligned} \theta &= \max\left\{\sigma\left(U, \theta\right), \omega + \beta\theta\right\} \\ 0 &= L\left(U, \theta\right)\left[\theta - \omega - \beta\theta\right]. \end{aligned}$$

and the inequality that must be satisfied is that$L\left(U, \theta\right) \geq 0$. A consequence of this simple characterization is that Theorem 1 can be used for constructing a computational algorithm and for establishing the existence and uniqueness of a stationary efficient allocation.

The island planning problem is at the center of this characterization, so the next section turns to its analysis.

4.1 Island's planner problem

We start by analyzing the derivatives of V, which can be shown to be differentiable. The standard proof by Benveniste and Scheikman does not apply because the optimal choice for E is not interior. In Appendix B we construct an alternative proof and find expressions for the derivatives of V. Intuitively, the marginal value of an extra worker of tenure j is the sum of two terms. The first term is the sum of the expected discounted marginal productivity during those periods in which no worker of the same cohort has ever been sent back to the central location. The second term is the expected discounted net shadow value the first time that a worker of the same has been sent back. Formally, for $T_j > 0$, $\partial V\left(T, z\right)/\partial T_j = V_j^*\left(T, z\right)$, where V_j^* is define as follows. Denote the current date by 0 and define the stopping time n_j as the first date s at which the number of workers with current tenure j is reduced. We let $E_{i,s}^{*}$ be the optimal employment level s periods from now of workers with tenure level i, and we let $T_{i,s}$ be the begining-of-period number of workers s periods from now with tenure level i, so that

$$n_j = \text{ first date } s \text{ at which } E_{\min\{J,\ j+s\},s}^{*} < T_{\min\{J,\ j+s\},s}$$

Now we are ready to define $V_j^{*}(T,z)$ as:

$$\begin{aligned} V_j^{*}(T,z) &= \sum_{s=0}^{\infty} \beta^s E_0 \left[f\left(\sum_{i=0}^{J} E_{i,s}^{*}, z_s \right) |n_j > s \right] + \\ & \quad E_0 \left[\beta^{n_j}\ \theta \right] - E_0 \left[\beta^{n_j}\ \tau\ |n_j \geq J \right] \end{aligned} \tag{3}$$

This implies that if some workers of tenure j are sent back, i.e. if $E_j = G_j(T,z) < T_j$, then the marginal value of all workers of this tenure level is $V_j^{*}(T,z) = \theta$ for $j \leq J-1$ and is equal to $\theta - \tau$ for $j = J$.

In Appendix A we show the following three properties of the solution to this problem.

First, it is immediate to show that if $T_j > 0$, then $\partial V(T,z)/\partial T_j \geq \theta$ for $j \leq J$ and $\geq \theta - \tau$ for $j = J$, since the planner has the option of sending the workers back to the central location.

Second, it is easy to see that if a permanent worker is fired, i.e. if $E_J = G_J(T,z) < T_J$, then all the temporary workers must have been fired as well, i.e. $E_j = G_j(T,z) = 0$ for all $j = 0, ..., J-1$. A policy with this property saves on the separation cost τ, which are only paid by permanent workers.

The third important property is that the first workers to be fired are the temporary workers with the longest tenure. The intuition for this property is that while all workers are perfect substitutes in production, these workers are the closest to becoming subject to the separation cost τ, and thus this policy saves on potential separation costs. In an economy where all islands planner have followed this policy in the past, and a constant flow U of tenure $j = 0$ workers has arrived every period, the states T in the ergodic set take a particular form. Formally, the ergodic set is a subset of $\mathcal{E}$, which is given by

$$\mathcal{E} = \left\{ T \in [0,U]^{J-1} \times R_+ : T = (U, ..., U, T_j, 0, ..., 0, T_J),\ \text{for some } j:\ 1 \leq j \leq J-1 \right\}$$

This property allow us to reduce the dimensionality of the endogenous state of the island planning problem from J to 2. Hence we analyzed a simplified island planning problem, to which we turn next.

4.2 Simplified Island's planner problem

States for the island planning problem T that belong to $\mathcal{E}$ can be described by two numbers: t, the total number of temporary workers (workers with tenure less or equal to J), and p, the number permanent workers (workers with tenure greater than J). We use this feature to consider the island planning problem with a simplified state (t, p, z). In this simplified problem, the choices are employment of temporary workers, e_t and employment of permanent workers e_p. The law of motion for the endogenous state is:

$$t' = U + e_t - \max\{e_t - (J-1)U,\ 0\} \text{ and } p' = e_p + \max\{e_t - (J-1)U,\ 0\} \tag{4}$$

The number of temporary workers next period, t', equals those that are employed this period, e_t, plus those that arrive next period, U, net of those that will become permanent, $\max\{e_t - (J-1)U,\ 0\}$. Likewise, the number of next period permanent workers, p', equals those that are employed this period, e_p plus those temporary workers that will become permanent. The planner's value function v : $[U, J \cdot U] \times R_+ \times Z$ satisfies

$$\begin{aligned} v(t,p,z) = \max_{e_t, e_p, t', p'} \quad & \{F(e_t + e_p, z) + \theta[t - e_t] + (\theta - \tau)[p - e_p] \\ & + \beta \int v(t',\ p', z') Q(z, dz') \Big\} \end{aligned}$$

subject to

$$0 \le e_t \le t, \quad 0 \le e_p \le p,$$

and the law of motion (4).

Formally, v is related to V for states $T \in \mathcal{E}$ is as follows:

$$v(T_1 + T_2 + ... + T_{J-1}, T_J, z) = V(T_1, T_2, ..., T_{J-1}, T_J, z).$$

Since v and V are closely related, and V is concave, then v is concave in (t, p), even though the graph of the feasible set for this problem is not convex. From the definition of v and the properties of V we have that v is differentiable with respect to t for all $t > 0$ which are not integer multiples of U, and differentiable with respect to p for all $p > 0$. Thus, for all (t, p, z) with $p > 0$

$$\frac{\partial v(t,p,z)}{\partial p} = \frac{\partial V(T,z)}{\partial T_J}$$

and for all t that can be written as $t = (j-1)U + T_j$ with $T_j \in (0, U)$,

$$\frac{\partial v(t,p,z)}{\partial t} = \frac{\partial V(T,z)}{\partial T_j}$$

At the points t given by $t = j \times U$ for some $j = 1, ..., J-2$, the right derivative of v with respect to t is $\partial V / \partial T_j$, and its left derivative is $\partial V / \partial T_{j+1}$.

The main result of this section is the characterization of the optimal policies. The optimal policy is characterized by a two-dimensional set of inaction $I(z)$. For each z, the optimal policy $(e_t(t,p,z), e_p(t,p,z))$ is to stay in the set of inaction $I(z)$ and otherwise to go to its boundary, as explained below. The boundary of the set of inaction is described by two continuous functions, $\hat{p}$ and $\hat{t}$ defined in $\hat{p} : Z \to R_+$ and $\hat{t} : R_+ \times Z \to [0,\ J \cdot U]$. The function $\hat{t}$ is decreasing in p and hits zero at a value of $p \le \hat{p}(z)$. The function $\hat{t}$ is the boundary of the set of inaction for the values t that are strictly positive. Formally, these functions define the set of inaction $I(z)$ as follows:

Definition 2 *For each $z \in Z$,*

$$I(z) = \left\{ (t,p) \in [0, J \cdot U] \times R_+ : p \le \hat{p}(z),\ \textit{and}\ t \le \hat{t}(p,z)\ \right\} \tag{5}$$

The optimal policy is as follows: if $p \le \hat{p}(z)$ and the state is outside the set of inaction $I(z)$, temporary workers are fired until the boundary of $I(z)$ is hit, with no change in permanent workers. If $p > \hat{p}(z)$, all temporary workers are fired, and permanent workers are fired to hit $\hat{p}(z)$. Formally,

$$\begin{aligned} e_t(t,p,z) &= \min\left\{t,\ \hat{t}(p,z)\right\}, \\ e_p(t,p,z) &= \min\left\{p,\ \hat{p}(z)\right\} \end{aligned}$$

Figure 2 illustrates a typical shape of the Inaction set for a given z and the nature of the optimal policy.

The threshold $\hat{p}(z)$ solves

$$\theta - \tau = f(\hat{p}(z), z) + \beta \int \frac{\partial v}{\partial p}(U, \hat{p}(z))\, Q(z, dz')$$

so that $\hat{p}$ is lowest value of the permanent workers for which the marginal value of an extra permanent worker is $\theta - \tau$, and hence if the island planner were to have one extra one, she will be returned to the central location.

Given (p, z), the function $\hat{t}(p,z)$ is defined as the lowest value of t for which the marginal value of an extra temporary worker is θ, so that if the island planner were to have an extra temporary worker it will return her to the central location .The function $\hat{t}(p,z)$ solves

$$\theta = f\left(\hat{t}(p,z) + p, z\right) + \beta \int \frac{\partial v}{\partial t}\left(\hat{t}(p,z) + U,\ p\right) Q(z, dz')$$

Figure 2
Optimal Decision Rule for Employment

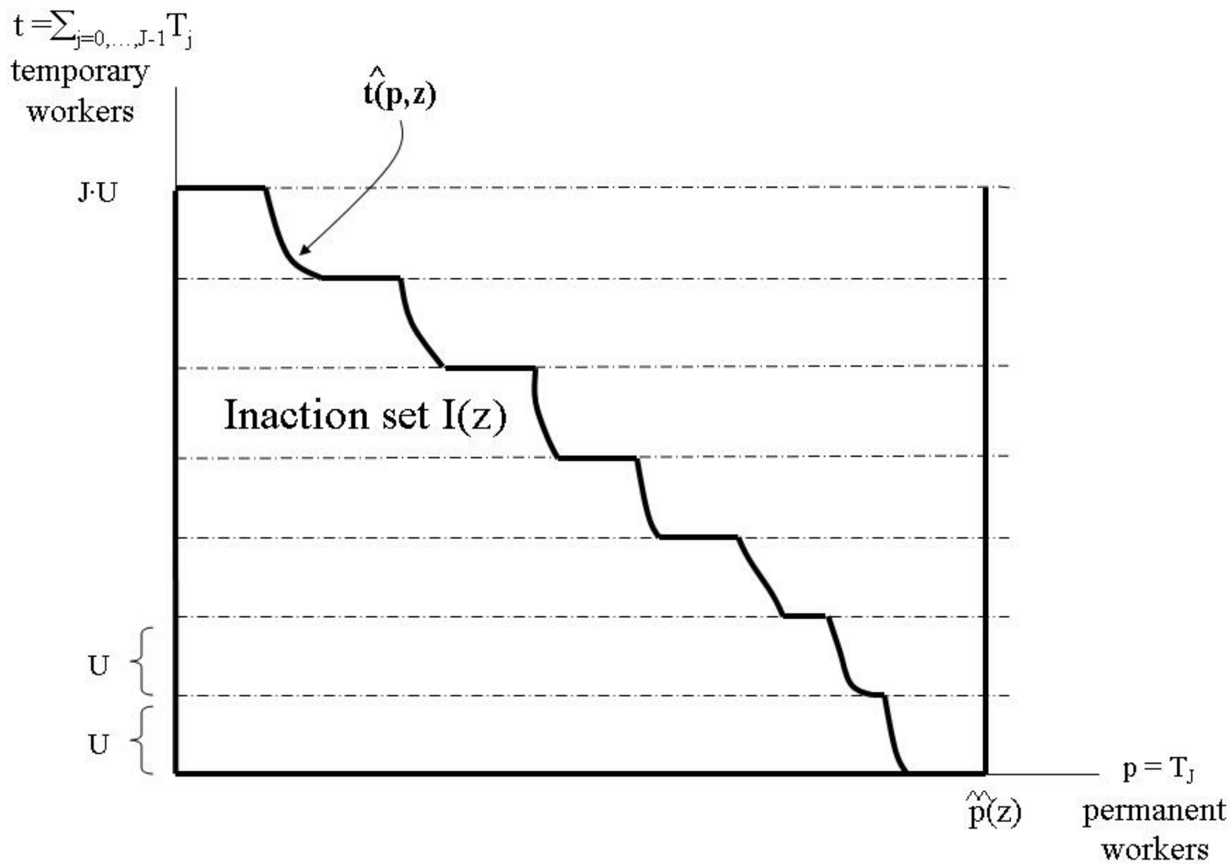

Figure 2:

for $\hat{t}(p,z) \leq (J-1)U$ and

$$\theta = f\left(\hat{t}(p,z) + p, z\right) + \beta \int \frac{\partial v}{\partial p}\left(JU,\ p + \hat{t}(p,z) - (J-1)U\ \right) Q(z, dz')$$

for $\hat{t}(p,z) \in ((J-1)U, JU]$. To simplify the exposition we have written the expressions assuming that v is differentiable. If v is evaluated at integers multiples of U, so that v is not differentiable, these expressions have to be written in terms of the subgradients of v.

The intuition for why the frontier of the set of inaction, given by $\hat{t}$, is decreasing in p, is that temporary and permanent workers are perfect substitutes in production. Indeed, it can be shown that $\hat{t}$ is strictly decreasing for values of p such that $\hat{t}(p,z)$ is not an integer multiple of U. At the points on which $\hat{t}$ is an integer multiple of U, this function can be flat: on these point the function v may not be differentiable, as explained above. While all these properties are quite intuitive, the proofs are involved because of the non-differentiability of v, Appendix B contains a formal treatment of these results.

5 Stationary Recursive Competitive Equilibrium

In this section we describe a convenient recursive competitive equilibrium (RCE) for this economy. Without loss of generality, we consider the case where $\gamma = 0$, so that leisure and consumption goods are perfect substitutes. We also treat the separation cost τ as being a technological feature of the environment. In this version of the economy the 1st and 2nd welfare theorems hold, so stationary equilibria can be found by computing the efficient stationary equilibrium described in Section 4. At the end of the section we explain how to map the equilibrium allocations obtained in this case into equilibrium allocations for any $\gamma > 0$. We also describe the mapping to the case where the separation costs are firing taxes rebated to households as lump-sum transfers and where, consequently, the welfare theorems do not hold.

In a RCE firms and workers participate in competitive labor markets in each island. Wages are indexed by j, the workers's tenure in the island, and by (T, z), the island-wide state. As in the previous sections, a permanent worker is defined as having tenure $j \geq J$ in the island. Whenever a firm decreases its employment of permanent workers, it must pay a separation cost τ per unit. Notice that it is the tenure in the island, as opposed to tenure in the firm, what determines if a worker separation is subject to the separation cost τ. This unrealistic assumption affords tractability by allowing a descentralization with spot labor markets. The reason is that, since the separation costs are at the island level, workers are not tied to the firms that hire them. Next section will remove this unrealistic specification by introducing long term contractual arrangements.

Current wages across tenure levels are given by

$$w(T,z) = (w_0(T,z), w_1(T,z), ..., w_{J-1}(T,z), w_J(T,z)),$$

a function of the island-wide state (T,z). The law of motion for wages can then be obtained from the island-wide equilibrium employment rule and the associated law of motion for the island-wide state. The equilibrium employment rule is denoted by

$$G(T,z) \equiv (G_0(T,z), G_1(T,z), ..., G_{J-1}(T,z), G_J(T,z)).$$

The law of motion for the endogenous state $T' = A(T,z)$ is then given by

$$A(T,z) = (G_0(T,z), G_1(T,z), ..., G_{J-2}(T,z), G_{J-1}(T,z) + G_J(T,z)).$$

The problem for a worker with tenure j in an island of state (T,z) is to decide whether to become non-employed or to stay and work. Becoming non-employed entails a value given by θ. By staying, the worker receives a wage rate w_j during the current period and gains tenure $\min\{j+1, J\}$ for the following period. We denote the value function for a tenure j worker in a (T,z) island as $W_j(T,z)$. This value function must solve

$$W_j(T,z) = \max\left\{\theta,\ w_j(T,z) + \beta \int W_{\min\{j+1,J\}}(A(T,z), z')\, Q(z, dz')\right\}$$

for all (T,z) and $j = 0, ..., J$.

The value function $B(p;T,z)$ of a firm that employed p permanent workers during the previous period in an island with state (T,z) solves:

$$\begin{aligned} & B(p;T,z) \\ = \ & \max_{\{g_j \geq 0\}_{j=0}^{J}} \left\{ F\left(\sum_{j=0}^{J} g_j,\ z\right) - \sum_{i=0}^{J} w_j(T,z)\, g_j - \tau \max\{p - g_J, 0\} \right. \\ & \left. + \beta \sum_{z'} B(g_J + g_{J-1};\ A(T,z), z')\, Q(z|z') \right\} \end{aligned}$$

The optimal decision rule is denoted by

$$g_j = m_j(p;T,z),$$

for $0 \leq j \leq J$, describing the optimal employment level at each tenure j. For future reference, notice that $B(p;T,z)$ is decreasing in p, since having employed more permanent workers in the previous period makes the firm subject to higher potential separation costs. Thus, provided that B is differentiable, $-\tau \leq \partial B/\partial p \leq 0$, and $\partial B/\partial p = -\tau$ if some permanent workers are fired, i.e. if $g_J = m_J(p;T,z) < p$.

A recursive stationary competitive equilibrium (RCE) is given by numbers $\{\theta, U, \sigma\}$ and functions $\{w, G, B, m, W\}$ that satisfy the following conditions:

i).Given wages $w(\cdot)$, employment $G(\cdot)$, and the law of motion $A(\cdot)$, the representative firm is representative:

$$m_j(T_J;T,z) = G_j(T,z),$$

for all (T,z) and all $0 \leq j \leq J$; and

ii). Given wages $w(\cdot)$, employment $G(\cdot)$ and law of motion $A(\cdot)$ the decision of the representative worker is representative:

$$\begin{aligned} W_j(T,z) &> \theta \Rightarrow G_j(T,z) = T_j, \quad \text{for } j > 0 \text{ and} \\ W_0(T,z) &> \theta \Rightarrow G_0(T,z) = U. \end{aligned}$$

And if $G_j(T,z) > 0$, then

$$W_j(T,z) = w_j(T,z) + \beta \int W_{\min\{J,j+1\}}(A(T,z), z') Q(z, dz') \ .$$

iii) The law of motion A defines an invariant distribution μ across states (T,z) as follows

$$\mu(D, z') = \sum_{z \in Z} \left[\int_{\{T,z:\ A(T,z) \in D\}} \mu(dT \times z) \right] Q(z'|z)$$

iv) Feasibility in the labor market is satisfied:

$$N - U - \int G(T,z)\,\mu(dT \times dz) \geq 0, \ \ U \geq 0,$$

v) The value of search σ and the value of becoming non-employed θ satisfy

$$\sigma = \beta \int W_0(T,z)\,\mu(dT \times dz), \quad \theta = \max\{\omega + \beta\theta,\ \sigma\}$$

vi) The labor force participation decision is optimal:

$$
\begin{aligned}
0 &= \left[N - U - \int G(T,z)\,\mu(dT \times dz)\right]\,[\theta - \omega - \beta\theta] \\
0 &= U\,[\theta - \sigma]\ .
\end{aligned}
$$

The next theorem establishes the 1st and 2nd welfare theorem for this economy and provides a partial characterization of the RCE.

Theorem 3 *Welfare Theorems and equilibrium characterization:*
i) Let $\{U, \theta, w, G, B, m, W, \mu\}$ be an recursive stationary equilibrium, then there is an island planner value function V, for which $\{V, G, U, \theta, \mu\}$ is an stationary efficient allocation.
ii) Conversely, let $\{V, G, U, \theta, \mu\}$ be a stationary efficient allocation, then there are wages and value functions $\{w, B, m, W\}$ for which $\{U, \theta, w, E, B, m, W, \mu\}$ is a recursive stationary equilibrium.
iii) the functions B, W and V related as in i) and ii) satisfy

$$
\begin{aligned}
W_j(T,z) &= \partial V(T,z)/\partial T_j \text{ for } j = 0, ..., J-1 \qquad (6)\\
\partial B(T_J, T, z)/\partial p + W_J(T,z) &= \partial V(T,z)/\partial T_J
\end{aligned}
$$

The reasons for the equivalence shown in i) and ii) are the same as in the Prescott and Mehra (1980) result about equivalence between recursive competitive equilibrium and efficient allocations. Our set up does not directly maps into theirs, so in Appendix C we offer a constructive proof of i) and ii).

Condition iii) are obtained by comparing the first order conditions from the planning problem with the optimality conditions for the workers and firms in the recursive competitive equilibrium. These conditions give some intuition on how the prices decentralize the efficient allocation. Recall that $\partial V/\partial T_j$ is the shadow value of a tenure j worker in the island planning problem. Condition iii) says that the shadow value of an extra temporary worker for the planner is the same as the equilibrium value function W_j. Instead the shadow value of a permanent worker for the planer, $\partial V/\partial T_j$, is lower than the equilibrium value function for a worker W_J. This difference is exactly the shadow value of an extra permanent worker for the firm, $\partial B/\partial p$, which, due to the separation cost, is a number between $-\tau$ and 0.

The next proposition gives a partial characterization of equilibrium wages.

Proposition 4 *Let $\{U, \theta, w, G, B, m, W, \mu\}$ be an recursive stationary equilibrium. Without loss of generality, the equilibrium wage w can be chosen to satisfy*

a) for all $j = 0, 1, ..., J-2$

$$w_j(T, z) = f\left(\sum_{i=0}^{J} G_i(T, z), z\right),$$

b) for all $j = 0, 1, ..., J-2$

$$\begin{aligned} w_j(T, z) - \beta\tau &\leq w_{J-1}(T, z) \leq w_j(T, z) \\ w_j(T, z) - \beta\tau &\leq w_J(T, z) \leq w_j(T, z) + \tau \\ w_{J-1}(T, z) &\leq w_J(T, z) \leq w_{J-1}(T, z) + \tau \end{aligned}$$

and if $E_J(T, z) < T_J$:

$$w_{J-1}(T, z) \leq w_j(T, z) < w_J(T, z),$$

c) and the equilibrium value function W for workers can be chosen so that they satisfy:

$$\begin{aligned} W_0(T, z) &\geq W_1(T, z) \geq \cdots \geq W_{J-1}(T, z) \\ W_J(T, z) &\geq W_{J-1}(T, z) \ . \end{aligned}$$

This proposition says that there are three equilibrium levels of wages in a given location: one level for temporary workers with tenures $j = 0, ..., J-2$, a second level for workers that are about to become permanent, i.e. those with tenure $J-1$, and a third level of wages for permanent workers, i.e. those with tenure J or higher.

Temporary workers with tenures $j = 0$ to $j = J-2$ are hired in spot markets and paid their marginal productivity. Wages of workers with tenure $J-1$, i.e. those that would become permanent if they were to work during the current period, are (weakly) smaller than their marginal productivity. This gives the right incentive to workers and firms. They give the incentive to workers to leave the location as their tenure gets closer to $J-1$, as condition c) makes precise. Firms do not hire them spite of the low wages because if they do so, the firms will be subject to separation cost in the future. Wages of permanent workers are (weakly) higher than those with tenure $J-1$. This also gives the right incentives to workers and firms. They induce workers with tenures J and higher to stay in the location, as condition c) explicitly shows. This is consistent with the firms decision of firing permanent workers last in order to avoid the separation tax τ.

The proof of Proposition 4 follows, essentially, from the analysis of the first order conditions of the firm problem. Appendix C contains a joint proof of Theorem 3 and Proposition 4.

Stationary Equilibrium for $\gamma > 0$ and separation taxes

Here we describe how to use the stationary allocation obtained in the case where $\gamma = 0$ and the separation cost is a technological feature of the environment, to find the equilibrium for the case where $\gamma > 0$ and the separation cost τ is a tax levied to firms and rebated lump sum to households.

First we describe how the equilibrium conditions for the households change when $\gamma > 0$. We assume that there are perfect insurance markets, so that all households consume the same amount, equal to the aggregate consumption level, which we denote as c. The household first order condition for an interior equilibrium (one with a strictly positive amount of time dedicated to leisure and a strictly positive amount of search) equates the marginal rate of substitution with the flow value of search:

$$\frac{\omega}{u'(c)} = (1-\beta)\,\sigma. \tag{7}$$

In such interior equilibrium the value of search equates the value of non-employment, so that $\sigma = \theta$.

Second, we describe how the equilibrium changes when the separation cost τ is a tax, rebated lump sum to households, as opposed to a technological cost. In this case aggregate consumption in a stationary equilibrium is given by

$$c = \int F\left(\sum_{j=0}^{J} G_j(T,p),\ z\right)\mu(dT \times dz) \tag{8}$$

Given these changes, the allocation corresponding to an interior stationary equilibrium can be described by $\{V, G, U, \theta, \mu\}$, where V is the value function and G the optimal policy for the island planning problem for (U, θ), and where μ is the invariant distribution for $\{(T, z)\}$ generated by (G, Q) such that:

a) the value of search is generated by $\hat{V}, \mu$

$$\sigma = \beta \int \left[\frac{\partial \hat{V}\left(T, z; \hat{U}, \theta\right)}{\partial \hat{U}}\Big|_{\hat{U}=U}\right] \mu(dT \times dz\ ; U, \theta)$$

where $\hat{V}$ is defined in terms of V as in (1),

b) the marginal condition (7) holds where aggregate consumption is given by (8).

Alternatively we could have defined an equilibrium for $\gamma > 0$ with separation taxes in terms of the firms and workers problem, as we have done for the RCE. We chose to define it in terms of the stationary allocations to simplify the notation. Using the arguments in Theorem 3, it is easy to show that the two definitions would have been equivalent.

Finding a Stationary Equilibrium with $\gamma > 0$ and separation taxes

Now we describe how to obtain the allocations corresponding to an equilibrium with $\gamma > 0$ and separation taxes using the stationary efficient allocations for $\gamma = 0$. Start an efficient stationary equilibrium described by $\{V, G, U, \theta, \mu\}$ and with aggregate consumption $c(U, \theta)$ given by the right hand side of (8). Let (U', θ') satisfy

$$U' = \phi\, U \quad \text{and} \quad \theta' = \phi^{\alpha-1}\, \theta \tag{9}$$

where the scalar ϕ solves:

$$\frac{\omega}{\left[c(U,\theta)\ \phi^{\alpha}\right]^{-\gamma}} = (1-\beta)\ \theta' \ . \tag{10}$$

We claim that such (U', θ') and its associated island planning problem value function of optimal decision rules $\{V', G'\}$ and invariant distribution μ', describe the allocations for an equilibrium with $\gamma > 0$ and separation taxes.

The key to this result is the following homogeneity property of the stationary efficient allocations.

Homogeneity Property. Let the pair (U, θ) index an island planning problem with value function $V(\cdot; U, \theta)$ and optimal policies $G(\cdot; U, \theta)$. Let $\phi > 0$ be a positive factor and define the pair (U', θ') as

$$\begin{aligned} U' &= \phi\, U \\ \theta' &= \phi^{\alpha-1}\, \theta. \end{aligned}$$

Then, in the case of Cobb-Douglas production function $F(E, z) = z\ E^{\alpha}$, one can easily verify that the value function is homogeneous of degree α in the sense that

$$V(\phi T, z; U', \theta') = V(T, z; U, \theta)\ \phi^{\alpha}$$

and that the policies are homogenous of degree one in the sense that

$$G(\phi T, z\ ; U', \theta') = G(T, z; U, \theta)\ \phi.$$

Using this homogeneity property and the value of ϕ given in (10), it is immediate to verify that one obtains an equilibrium for γ with separation taxes.

6 Interpretation of separation cost as temporary contracts

In the previous section the separation cost was modeled as a *tax* on employment reductions of workers with tenure $j \geq J$ at the *island level*. This allowed for a very simple competitive structure with spot labor markets. However, in reality, temporary contracts specify *severance payments* as a function of the workers' tenure at the *firm level*. Modeling severance payments as separation taxes in the context of competitive equilibria is standard in the literature, (see for instance Bentolila and Bertola, 1990, and Hopenhayn and Rogerson, 1993). However modelling the tenure level at the island level (as opposed to the firm level) is specific to this paper. In this section we introduce an alternative and more realistic definition of a competitive equilibrium that specifies the tenure of workers at the firm level. This specification ties workers with firms, and hence requires long term contracting to achieve efficiency. In fact, we will argue that the competitive equilibrium with long term contrats and tenure at the firm level supports the same equilibrium allocation as the RCE of the previous section. This is an important result: There is no loss of realism in specifying that the tenure relevant for temporary contracts is at the islands level instead of the firm level.

To obtain this equivalence result certain restrictions on the type of temporary contracts allowed are needed. However, this is not a weakness of the model. On the contrary, these restrictions resemble those observed in actual countries. Temporary contracts have often been introduced with the purpose of reducing unemployment by encouraging hiring, yet retaining employment protection in the form of firing costs. Thus the implementation of temporary contracts have typically included restrictions such as eligibility clauses. Indeed the Spanish reform of 1984, which broadened the scope of fixed term contracts, specified that workers must be registered as unemployed to be eligible to be hired under a temporary employment contract (see the Appendix in Cabrales and Hopenhayn, 1997).[2] In Portugal temporary contracts can only be used by new firms, or by firms hiring the long term unemployed or first-time job seekers (see Table 1 in Dolado et. al, 2001).

To incorporate this type of elegibility restrictions we assume not only that the separation taxes

[2] Cabrales and Hopenhayn (1997) describes the elegibility requirements of "Fixed term employment promotion contracts" (which includes: "general fixed-term employment promotion contracts", "contracts, work practice and formation", "social collaboration contracts"), "indefinite length employment promotion contracts" (which includes "indefinite length contracts for the older-than-45", "indefinite length contracts for women in underepresented occupations", "indefinite length contracts for younger than 25 or between 25 and 29"). Only "indefinite length contracts for the handicapped" are exempt from the requirement of being previously a registered unemployed.

are assessed based on the tenure of the workers at the firm's level (as opposed to the island level), but that only workers that searched during the previous period (i.e. that were unemployed) are eligible to be hired under temporary contracts. If a firm hires a worker that was employed somewhere else in the island during the previous period, the worker becomes subject to regular firing taxes immediately.[3] In this scenario, the market structure would have to be changed to accommodate for the fact that workers would try to exploit the bargaining power that they would gain by staying in a same firm. To avoid this, we assume that firms and workers participate in island-wide competitive markets for binding, long-term, state-contingent, wage contracts at the time of hiring. Below we offer an informal description of the equilibrium using long term contracts and Appendix F provides a more formal treatment.

6.1 Binding contracts and tenure at the firm level (an informal description)

In this decentralization, firms and workers trade state contingent contracts in competitive labor markets, specifying the periods of time that the worker will supply labor to the firm as a function of the sequence of productivity shocks z^t. Since employment must be continuous over time, each contingent contracts is effectively reduced to a stopping time specifying the time of separation. These stopping times are perfectly enforceable. When the realized sequence of productivity shocks triggers a separation, the worker can choose to offer a new stopping time to the market or to leave the island and receive the outside value θ. Each stopping time has its own price, which is taken as given by firms and workers.

There are two type of workers in the island: "incumbent" workers and "new arrivals". An "incumbent" is a worker that has been previously employed by some firm in the island. A "new arrival" is a worker that has just arrived to the island for the first time. The stopping times sold

[3] While the restrictions that we impose in the model captures the specific elegibility clauses in Portugal and Spain, it is clear that in general actual regulations must somehow preclude the possibility of firms completely avoiding the firing penalties by reshuffling workers. To be concrete, think of the following extreme case: a firm that divides itself into two units and every period it fires all the workers from each unit and hires them in the other. In this way, the tenure of its workforce is always zero and, hence, the separation tax does not apply. The assumption used in Section 5 that firing taxes are assessed based on worker's tenure at the island level can also be thought as a one type of restriction that precludes firms from following this type of scheme.

by the different type of workers differ in terms of the separation costs involved. In particular, the stopping times sold by "new arrivals" are subject to the separation cost τ only if the separation occurs after J periods (the length of the trial periods in the fixed term contracts). On the contrary, the stopping times sold by "incumbents" are always subject to the separation cost τ. Since the stopping times sold by the different types of workers are different commodities they have, in general, different prices. Intuitively, a stopping time sold by an "incumbent" worker will have a lower price than the same stopping time sold by a "new arrival" to compensate firms for the potentially higher separation costs.

Taking prices as given, firms decide how many stopping times of each type to purchase from the different type of workers. Their objective is to maximize the expected present value of their profits, net of separation costs.

Despite the unusual commodities traded and the indivisibility in the supply of contracts, the competitive equilibrium considered is standard and, hence, the welfare theorems hold. The equilibrium allocation can then be characterized as the solution to a social planner's problem. In this problem, the planner chooses stopping times for "incumbents" and "new arrivals" taking into account that the separation cost τ applies to "incumbent" workers in every separation, but that it applies to "new arrivals" only in separations that take place after J periods of employment.

A brief analysis of the planner's problem will help understand the equivalence between this type of equilibrium and the one considered in the main text of the paper. Clearly, the social planner will never want to separate a "newly arrived" worker and rehire him as a an "incumbent" before the trial period for the fixed term contracts is over. The reason is that being rehired as "incumbent" makes the worker liable to separation costs, while maintaining his "newly arrived" status saves on separation costs during the trial period. Also, the social planner will never want to separate a "newly arrived" worker after the trial period is over and rehire him under an "incumbent" contract because this entails incurring the separation cost τ without any benefit. As a consequence, the planner will choose the stopping times for "newly arrived" workers in such a way that they separate only to leave the island (and receive the value θ). This means that the social planner will never use "incumbent" workers.

Being left with only "newly arrived" workers, the planner's problem is formally identical to the Island's Planner problem described in Section 4. This has an important implication: The allocation obtained in the competitive equilibrium with long term contracts and tenure at the firm level described in this section is identical to the one obtained in the competitive equilibrium with spot labor contracts and tenure at the island level that was described in Section 5. Moreover, the price of a stopping time sold by a "new arrival" (in the equilibrium with binding contracts and tenure at the firm level) must be equal to the expected discounted value of the spot wages (in the equilibrium with spot labor contracts and tenure at the island level of Section 5) obtained by a worker that arrives to the island for the first time and follows an employment plan described by that stopping time.

7 Computational Experiments

In this section we calibrate our economy to evaluate the long-run consequences of introducing temporary employment contracts. Temporary contracts have been introduced in a number of countries with high employment protection policies as a way of providing firms some flexibility in the process of hiring and firing workers. The reform of the Spanish labor market during the mid-eighties is perhaps the most extreme case, given the scope of the temporary contracts introduced (see Cabrales and Hopenhayn, 1997, or Alonso-Borrego et all, 2004). To assess the extent by which temporary contracts add flexibility to the labor market we calibrate our economy to one with high separation taxes and no temporary contracts, such as the Spanish economy previous to the 1984 reform. This benchmark case, which we refer to as the “firing-tax case", is obtained by setting $J = 1$ and $\tau > 0$.[4] Using the parameter values calibrated in the firing-tax case, we compute competitive equilibria under temporary employment contracts of different lengths, i.e. with different values for J, and evaluate their effects.

For comparison purposes we also compute the equilibrium allocation under zero separation taxes, which we refer to as the “laissez-faire" case. This is an interesting case to consider because, as we argue below, the equilibrium allocations with temporary employment contracts of long duration coincide with the equilibrium allocation for the laissez-faire case. Based on this property, we compare how much of the gap between the firing-tax and laissez-faire cases is closed by introducing temporary contracts of different lenght J.

Before describing our calibration, we state two properties that will be useful for interpreting the results.

[4] Consider an equilibrium with $J = 1$ and $\tau > 0$. Since $J = 1$, the dismissal of anyone that has worked, even for one period, triggers a separation tax τ. Thus in this case there are no temporary workers.

The laissez-faire case.

In the laissez-faire case, which is obtained by setting $\tau = 0$, the value of J as well as the tenure levels of the different workers are immaterial since temporary and permanent workers become perfect substitutes. This means that while total employment is uniquely determined, the hiring and firing rates across the different tenure levels are undetermined. Despite of this, we choose to focus on the employment adjustments obtained as the limit when $\tau \to 0$ (or equivalently, when τ is arbitrarily small). This is useful because it helps emphasize the type of adjustments that temporary contracts lead to, even in the case where they are totally unimportant. The employment adjustments in the laissaiz-faire case are characterized by the functions $\hat{t}$ and $\hat{p}$ obtained as the limit when $\tau \to 0$. The limit functions $\hat{t}$ and $\hat{p}$ have the following three properties for each value of z: 1) $\hat{t}\left(\hat{p}\left(z\right) - JU, z\right) = JU$, 2) $\hat{t}$ has slope -1 with respect to p, and 3) $\hat{t}\left(\hat{p}\left(z\right), z\right) = \hat{p}\left(z\right)$.

Temporary contracts as $J \to \infty$.

To simplify the argument, assume that z is bounded. Let (U^*, θ^*) be the equilibrium values corresponding to the laissez-faire case, i.e. to $\tau = 0$, and let p^* be an upper bound on the size of firms under the invariant distribution for this case. For instance we take

$$p^* = \max_{z \in Z} \hat{p}_{LF}\left(z\right)$$

where $\hat{p}_{LF}\left(z\right)$ is the employment threshold for permanent workers in the island planning problem for (U^*, θ^*) and $\tau = 0$ (the laissez-faire case).

Now consider the length J^* given by the smallest integer such that $J^*U > p^*$. We claim that regardless of the value of τ, if $J \geq J^*$ the pair (U^*, θ^*) and the associated island planning problem value function, optimal policies and invariant distribution, constitute a stationary efficient allocation. The idea is quite simple: with such a large J, firms can replicate completely the employment decisions under the laissez-faire case using only temporary workers, and hence the value of the separation tax τ becomes immaterial.

Calibration

We calibrate the model to an economy with high employment protection and no temporary contracts that resembles the Spanish economy previous to the 1984 reform. In terms of policy parameters, we set τ equal to one year of average wages and $J = 1$. Our choice of τ reflects the expected discounted cost (at the time that a worker is hired) of dismising a woker, which is the measure proposed by Heckman and Pages-Serra (2000). In Appendix G we compute this measure for the policies in place in Spain before 1984.

We use a value $\alpha = 0.64$ for the share parameter in the production function, which roughly corresponds to the labor share. This choice implicitely assumes that that all other factors, such as capital, are fixed across locations. We use a quarterly time period, so we choose $\beta = 0.96$ to generate an annual interest rate of 4 percent.

For z we use a discrete Markov chain approximation to the following AR(1) process:

$$\log z' = \rho \log z + \sigma \ \varepsilon,$$

where ε is a standard normal. We choose the values of ρ and σ so that the unemployment rate is just above 6.75 % and the duration of unemployment is just above 1 year. The exact values that we use are $\rho = 0.955$ and $\sigma^2 = 0.075$, which correspond to a discrete approximation that uses six truncated values for z so that the absolute value of ε never exceeds two standard deviations. The quarterly firing rate (total separations divided by employment) is 1.77% (Garcia-Fontes and Hopenhayn, 1996, estimate a firing rate of 1.84% per quarter for the years 1978-1984). Our choices are meant to capture the situation in Spain before the 1984 reform. The reason why we chose a lower unemployment rate and a lower duration of unemployment than those observed in Spain is that we are abstracting from the unemployment insurance system. In Alvarez and Veracierto (1999) we analyzed the effects of introducing unemployment insurance benefits into the model with firing taxes, finding that they increase the unemployment rate and the average duration of unemployment quite significantly.[5] Given these results, we believe that, in the context of this model, it is reasonable to calibrate to the values for unemployment rate and average duration of unemployment described above.

We report the equilibrium for different values of γ. For each value of γ we use a different value of the parameter ω so that labor force participation is always 65% in the benchmark case, and hence the employment rate is 60.6%. [6]The rest of the parameters are the same for each pair (γ, ω).

We have currently calibrated to a quarterly period to conserve on grid points in our numerical implementation of the simplified island planning problem. In this case the search technology is

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[5]See the analysis of section "UI benefits, firing subsidies, firing taxes and severance payments" which considers the net effects of unemployment insurance benefits and firing taxes, and the results in Table 5 of Alvarez and Veracierto (1999).

[6]The different combinations of (γ, ω) are: $(0,\ 1.3047)$, $(1/2,\ 1.0739)$, $(1,\ 0.883)$ and $(8,\ 0.058)$. With $\gamma = 0$, there are no income effects, since preferences are linear. With $\gamma = 1$, income and substitution effects of a permanent increase in wages cancel. With $\gamma = 8$, the income effects is much higher, so that the uncompensated labor supply elasticity is lower, similar to the ones estimated by Nickel (19??).

such that workers get at most one offer per quarter. We view our current calibration as tentative since we have not yet explored what are the empirically reasonable values for the number of offers per-period.[7]

Experiments

We compute the stationary equilibria, which we refer to as "the general equilibrium case" for different values of J, the length of temporary contracts. We compare the effect of varying J against the benchmark case of firing taxes and against the laissez-faire case. Recall that for J large enough, the equilibrium allocation with temporary contracts coincides with the laissez-faire one. Thus, this comparison allows us to see how much flexibility is added by increasing the length of temporary contracts.

We also compute the allocations that correspond to the laissez-faire case for different values of J. As explained above, the value of J is immaterial for the allocation, but we concentrate on the employment dynamics that correspond to a very small value of τ or, formally, to the limit when $\tau \to 0$.

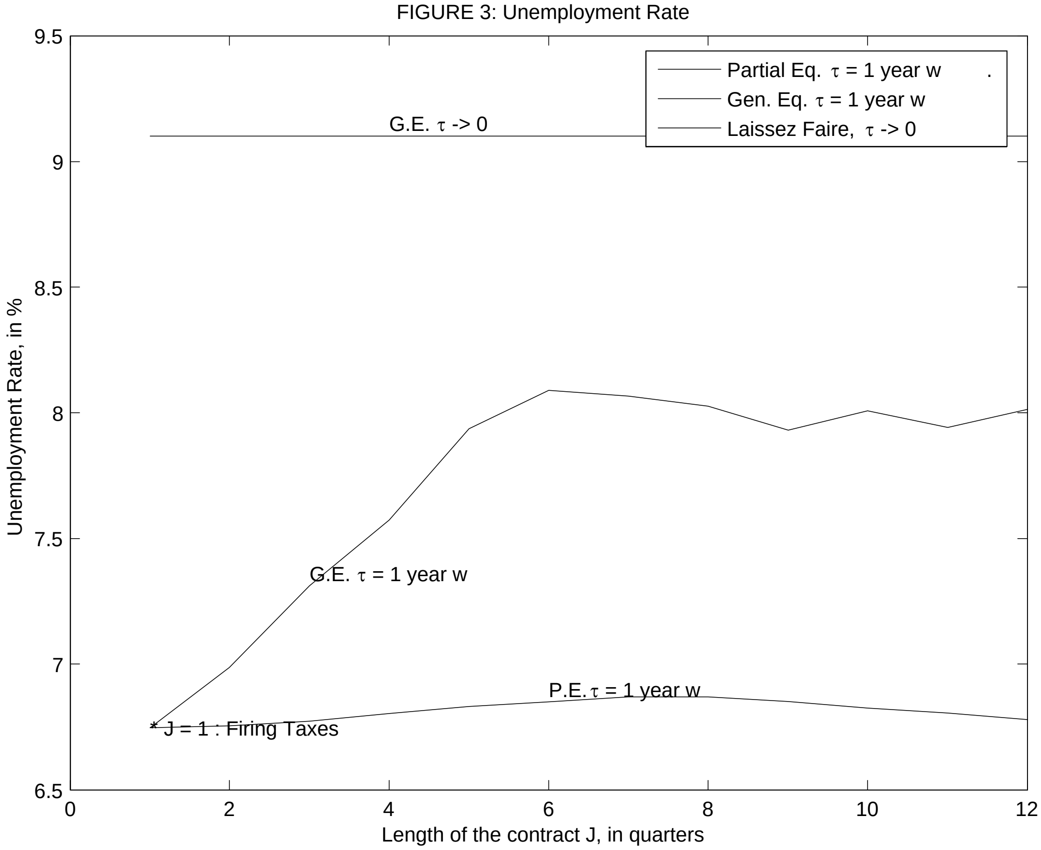
Finally, for comparisons purposes, we compute statistics for what we refer to as the "partial equilibrium" case. For each J, this corresponds to the equilibrium for an industry that takes as given the value of search θ and the number of new arrivals U. This equilibrium is constructed by solving the island planning problem keeping fixed the values θ and U that correspond to the benchmark case, i.e. the equilibrium with firing taxes. Comparing the statistics for the partial equilibrium case with the general equilibrium case gives the effects of the endogenous changes in θ and U as the length of the temporary contracts changes.

Given the homogeneity property described in section 5, a number of statistics are independent of the intertemporal substitution parameter $1/\gamma$. In particular, those that refer to magnitudes relative to unemployment, employment or the labor force, such as the unemployment rate, the average duration of unemployment and firing rates, are the same in all cases. We start by describing the effects of temporary contracts on this set of common statistics. Without loss of generality we set $\gamma = 0$. This is the simplest case to understand because consumption and leisure are perfect substitutes, and thus the equilibrium value of θ is $\omega/\left(1-\beta\right)$, a parameter independent of the policies.

[7] An alternative specification for the search technology that allows for more flexibility in terms of the number of offers per-period is to assume that if U workers search per period, only $p\,U$ arrive to the islands.

Figure 3 shows the effects of the different policies on the unemployment rate. In the context of our model we define the unemployment rate as $ur = U/(U+E)$. In this and all subsequent figures, the effects are depicted as a function of the length of the temporary contracts J. The equilibrium for $J = 1$ corresponds to the benchmark case with firing taxes and no temporary contracts. The unemployment rate in the laissez-faire case is almost 2.5 percent higher than in the benchmark case. This is a feature common to many other search models: firing taxes deter firing and hiring, but the largest effect is on the firing margin. The intuition for this difference is that the effect of the firing taxes on hiring is mitigated by time discounting. In the partial equilibrium case the unemployment rate does not change much with the length of the temporary contracts J, so the general equilibrium effects are important to understand the effect on the unemployment rate.

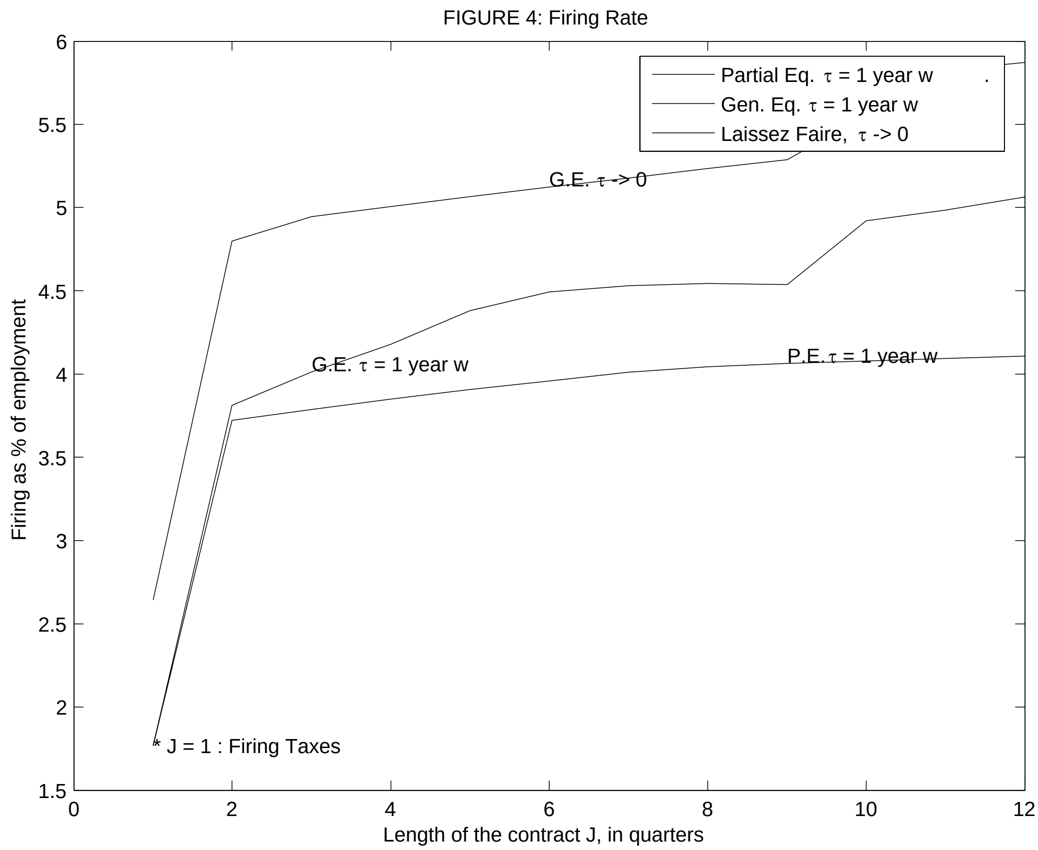
FIGURE 3: Unemployment Rate



In the general equilibrium case the unemployment rate increases with the length of the temporary contracts J. With temporary contracts of 3 years ($J = 12$), the unemployment rate is 1.3 percent points higher than with firing taxes, about half way in closing the gap between the benchmark case and laissez-faire case. In the data the pattern between the level of unemployment and the presence of temporary contracts is not clear. Dolado et all (2001) survey the literature and conclude that the Spanish evidence support that the effects of temporary contracts is a "neutral of slightly positive effect on unemployment". To better understand the effect of temporary contracts in unemployment in the model (Figure 3) it is helpful to decompose the changes in the unemployment rate ur into changes in the firing rate (Figure 4) and changes in the average duration of unemployment (Figure 5).

In Figure 4 we plot the value for the firing rate fr, defined as total firing over total employment. Recall that for the laissez-faire and the partial equilibrium cases, the values for U and θ are constant across all J. As it should be expected, the firing rates for laissez-faire are higher than the ones for the partial equilibrium case for all values of J. Notice that the firing rates in these two cases are increasing in J, with a large jump at $J = 2$. To understand this pattern we concentrate on the laissez-faire case where the employment in each island stays constant. Recall that we compute employment by tenure in the laissez-faire case as the limit for an equilibrium with $\tau \to 0$. The increase in the firing rate helps to avoid the (arbitrarily small) separation tax. The firing rate jumps between $J = 1$ and $J = 2$ because with $J = 2$ the temporary workers with longest tenure are fired and replaced by newly arrived workers. This reshuffling cannot be done with $J = 1$. The smooth increase in the firing rate with J is due to the fact that with higher J firms can accumulate a larger proportion of their workforce as temporary workers. With this larger proportion, if they need to decrease total employment they can do so at the same time that they hire newly arrived workers.[8] Notice that the pattern of firing rates as a function of J for the partial equilibrium case, where the separation cost are substantial (one year of average wages), is the same as in the laissez faire case, with essentially zero firing taxes.

[8] To understand this it is helpful to consider the case of an island where an increase in $J = J'$ to $J = J'+1$ triggers an increase in firing. Suppose that the island suffers a negative shock in z and that for J' the number of temporary workers in the island is just enough to make the adjustment in total employment purely firing temporary workers, without resorting to fire any permanent worker. If we now consider the case of $J = J' + 1$, then there will be more temporary workers than needed to make the adjustment in total employment. In this case the island can fire temporary workers in excess of what is needed to make the adjsutment in total employment, and hire some newly arrived workers.



The value for the firing rate for the general equilibrium case lies in between the value for the partial equilibrium case and the one for the laissez-faire case, and it gets closer to the one for the laissez-faire case as J increases. To understand why the value for the general equilibrium case lies between the other two cases, notice that the equilibrium value of U is higher in the general equilibrium case than in the partial equilibrium case, and that U increases with J. The value of U is larger than in the partial equilibrium because as J increases there are less impediments to labor mobility. With fewer impediments, the shadow value of a worker in the production sector increases, which induces a larger fraction of the population to search. Since in general equilibrium firms receive a higher flow of newly arrived workers (i.e. a higher U), they can engage more in the replacement of temporary workers of high tenure by newly arrived workers to save on separation costs.

The quarterly firing rate for the general equilibrium case goes from 1.77% for $J = 1$ to 5.1% for $J = 12$, which are roughly similar to the ones for Spain before and after 1984: Garcia-Fontes and Hopenhayn (1996) estimate quarterly firing rates of 1.84% during the six years prior to the extension of temporary contracts, and of 4.8% for the six years after. The model overestimate these effects a bit, since comparing the effect in the model for $J = 1$ with $J = 12$ does not corresponds exactly to Spain before and after 1984, since before 1984 some temporary contracts were allowed, as we explain below.

Figure 5 shows the average duration of unemployment d, defined as $d = (1/fr)\ ur\ /\ (1 - ur)$. The three cases display similar values. There is a large drop in the average duration between the benchmark case and $J = 2$. This is the result of the increase in hiring of newly arrived workers, as explained in the case of Figure 4. Since d is similar for the three cases, the effects on unemployment are accounted for the behavior of firing rates discussed above. Notice that, as opposed to the jumps at $J = 2$ for the firing rate and average duration of unemployment, the increase in the unemployment rate for the general equilibrium is smooth (compare Figure 3 with Figures 4 and 5). This is because for $J = 2$, the sharp decrease in the average duration of unemployment coincides with a sharp increase in the firing rate.

FIGURE 5: Average Duration of Unemployment

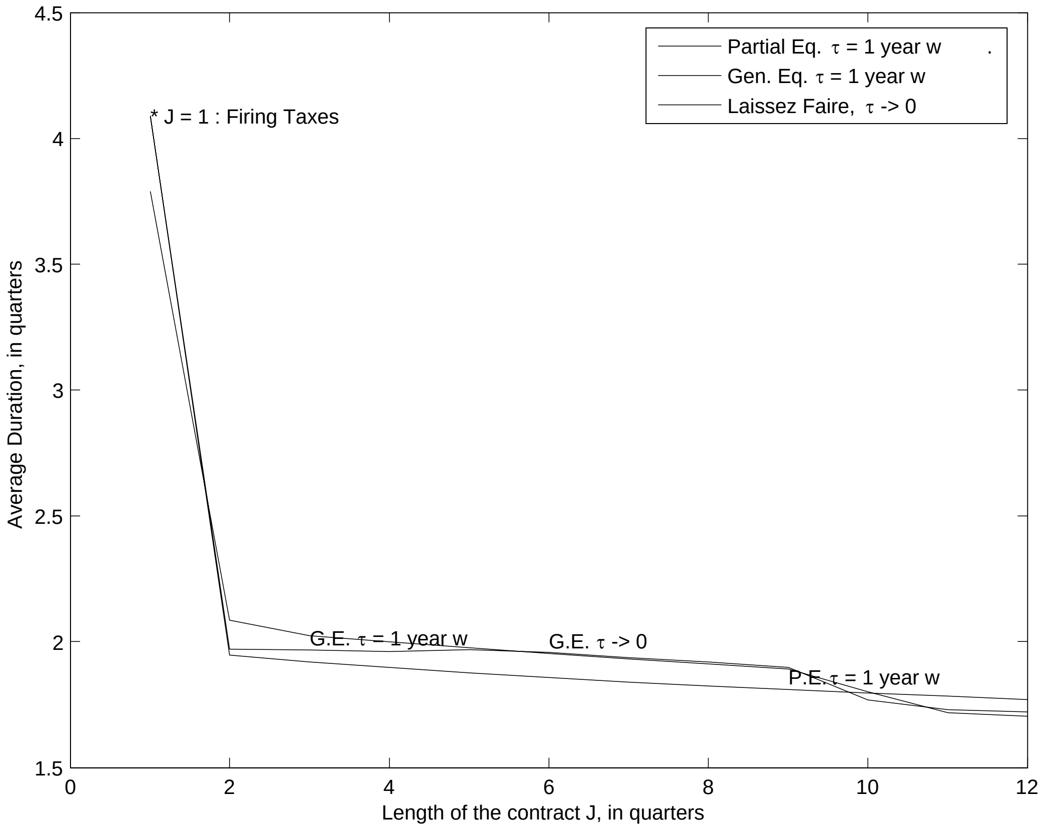


Figure 6 displays the fraction of permanent workers in total employment for the general equilibrium and the laissez-faire cases. The fraction of permanent workers is higher for the general equilibrium case than for the laissez-faire case, since in the general equilibrium case firms retain more permanent workers to avoid the high separation cost. Nevertheless, the fraction of permanent workers is very similar in the two cases. Notice also that as J increases, the fraction of permanent workers decreases steadily. For $J = 12$, which corresponds to temporary contracts of 3 years, 33 percent of workers are in temporary contracts. In Europe in the nineties, the fraction of workers with temporary contracts has been increasing steadily over time to about 12 percent, reaching its highest value for Spain. In Spain this fraction went from 11 percent before 1984 to an average of 33 percent during the nineties.

Figure 7 displays the firing rates by tenure of employment for temporary contracts of length $J = 8$ in the general equilibrium and laissez-faire cases. As in Figure 6, the values are very similar for both cases. The firing rates are initially decreasing in tenure, due to a compositional effect. For $j = J - 1$, Figure 7 shows a spike in firing, due to the high firing rate of the temporary workers with the highest tenure. The firing rate for permanent workers is the smallest of all. This pattern is similar to the one estimated in the data by Cabrales and Hopenhayn in Spain after the generalization of this contracts, which we have reproduced above.[9]

FIGURE 6: Share of Permanent Workers in Total Employment for differentg contract length J

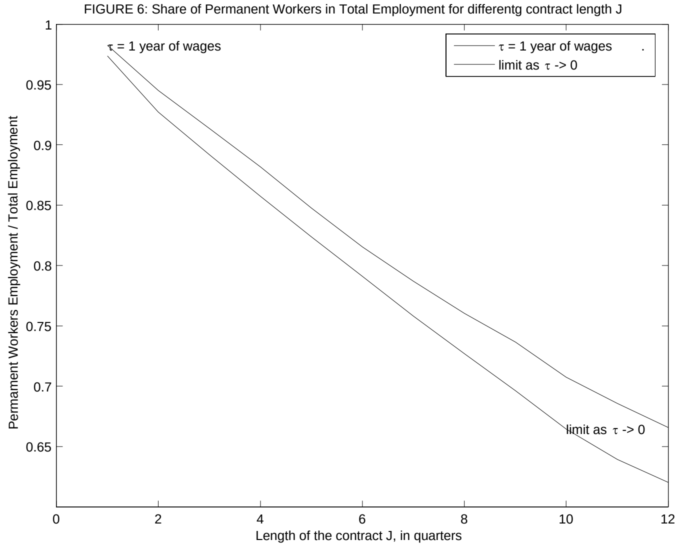
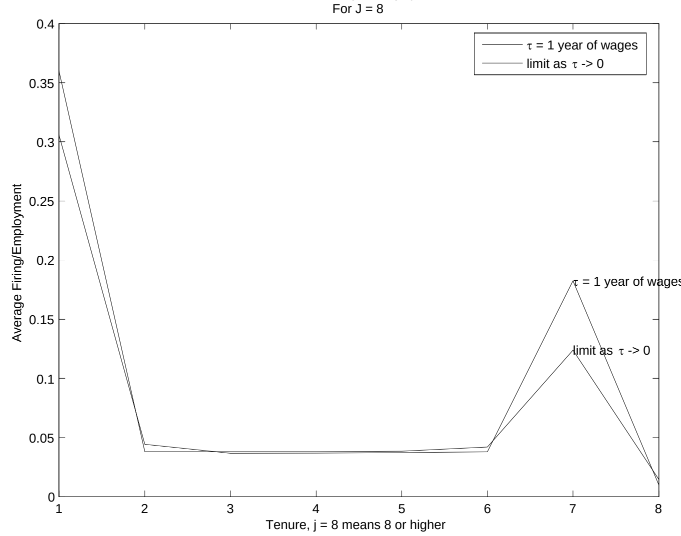


FIGURE 7: Average Firing by Tenure
For J = 8



Notice that the patterns displayed in Figures 5, 6 and 7 for the average duration of unemployment, share of permanent workers in total employment, and the firing rate by tenure are similar to the ones found in Spain after the mid-eighties and have typically being interpreted as evidence that temporary contracts play an important role. However, in our model similar patterns are obtained for τ equal to one year of average wages as well as for τ arbitrarily small, which shows that in on its itself, large changes in turnover do not necesarilly entails large changes in welfare relevant variables, such as employment, unemployment, aggregate consumption and productivity. We obtain this result under the extreme assumption that workers with different tenure are perfect substitutes.

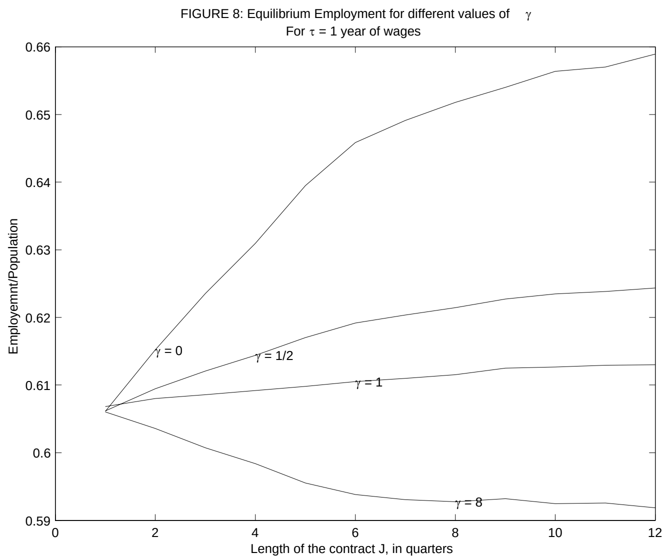
[9] This pattern is present in the calibrated model for $J \leq 9$. For higher values it is much noisy.

Under a different specification, such as on the job learning, this result will not be obtained. In particular, if the effect of on the job learning is large enough, small separation cost may have very small effect on turnover rates. Nevertheless, we interpret the spike in figure 1 for tenure of about 3 years, as evidence that the effects of separation taxes are not completely outweigh by the learning. [10]We leave the examination of a model that incorporate both features for future work.

Figure 8 shows the behavior of employment for the general equilibrium case for different values of γ. As J increases there are both income and substitution effects. The income effect is due to the fact that as J increases firms have more flexibility and thus working in the market is more attractive, i.e. the equilibrium value of θ increases. The income effect is due to the fact that the economy is more productive. For low values of γ, the substitution effect dominates and thus aggregate employment increases with J. For low values of γ the income effect dominates and thus aggregate employment decreases with J.

Figure 9 displays output and employment for the general equilibrium case for $\gamma = 1$ and compares its value with the ones in the laissez-faire case. Notice that while output seems to converge monotonically to the laissez-faire case, employment does not seem to have converged to the laissez-faire case for $J = 12$. Indeed, the value of employment for the general equilibrium case seems to overshoot the laissez faire value. This is a sign that even for $J = 12$, i.e. temporary contracts of length 3 years, the allocation is in some dimensions far away from converging to the laissez-faire case. This can also be seen in Figure 6, that shows the for $J = 12$ the fraction of permanent workers for $J = 12$ is still about 65 percent (recall that for $\tau > 0$, the allocation converges to laissez-faire when the fraction of permanent workers goes to zero).

[10]With on the job learning and firing cost, if the effect of learning is strong enough, it will not be optimal for the firm to fire first the temporary workers with higher tenure. In this case, the spike at the end of the fixed term contracts shown for Spain in Figure 1 will not obtain.



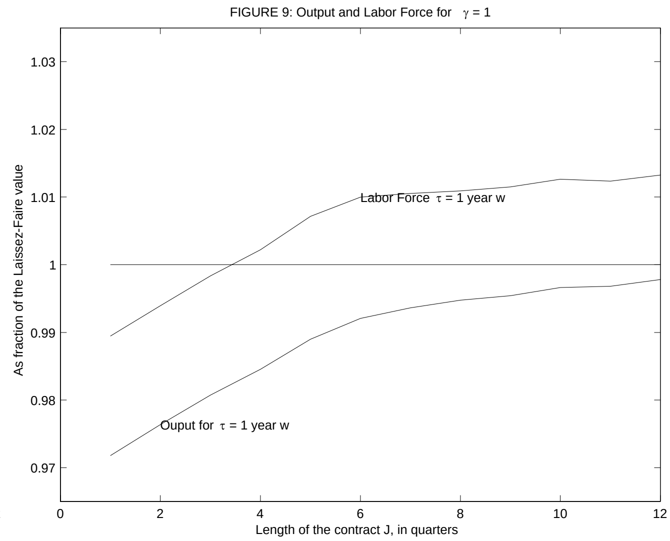
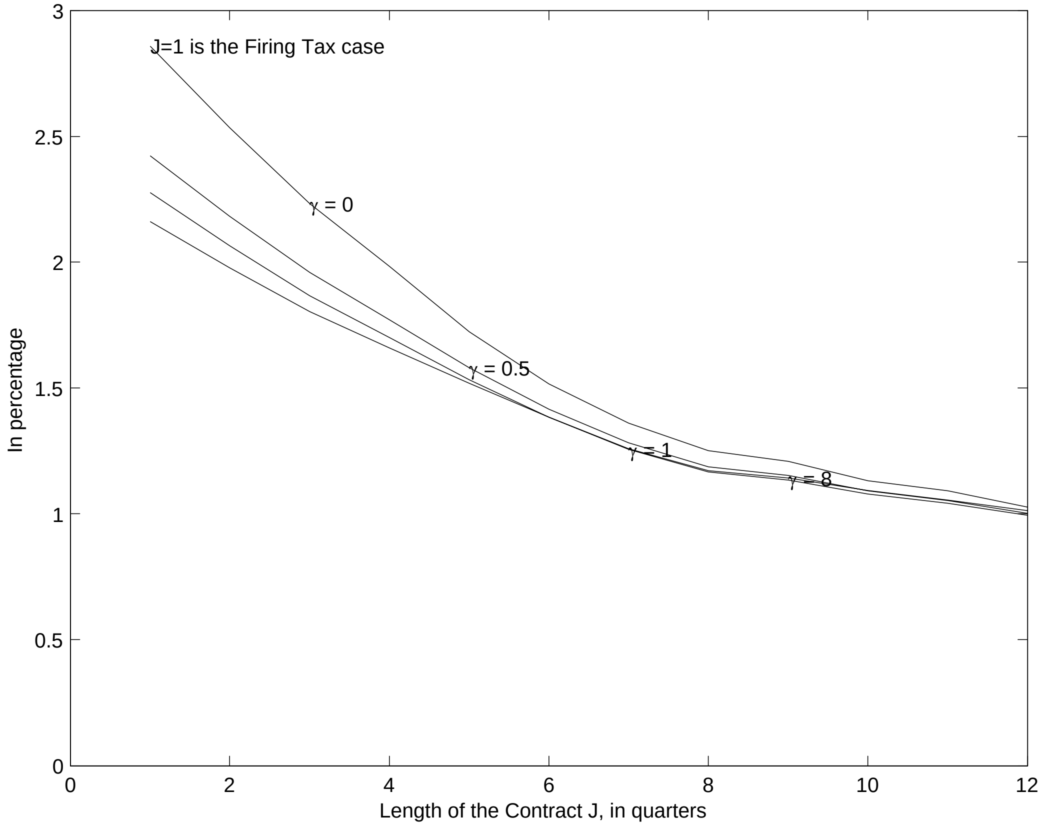


Figure 10 displays the welfare cost of temporary contracts of different lengths. This figure plots the extra perpetual consumption flow needed to make the representative household indifferent between living in the economy with temporary contracts of length J and living in the laissez-faire economy. This calculation compares the stationary equilibrium of the two economies, and hence does not take into account the transition after a change in policy. For the same J, the welfare cost are higher for smaller γ, since in this case there is more substitution between consumption and leisure. For $J = 1$, Figure 10 shows the welfare cost of firing taxes, which are about 2.5 percent. This number is similar to the one found by Hopenhayn and Rogerson (1993) and by Veracierto (2001). As J increases the welfare cost decreases: it goes from about 2.5 percent for length of a quarter and decreases smoothly with J until a value of 1 percent for contract length of 3 years, or $J = 12$. Thus, even if some of the characteristic of the allocation (such as employment in figure 9) do not converge monotonically to their laissez faire value as J increases, the welfare cost, which in a sense takes all the relevant features into consideration, does converges monotonically.

Figure 10: Welfare Cost of Contracts
Consumption equivalent relative to Laissez-Faire



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Technical appendix to:

Fixed Term Employment Contracts

in an Equilibrium Search Model

Fernando Alvarez
University of Chicago and NBER

Marcelo Veracierto
Federal Reserve Bank of Chicago

This document contains 7 appendices:
Appendix A: Analysis of the Island Planning Problem.
Appendix B: Analysis of the Simplified Island Planning Problem.
Appendix C: Proofs.
Appendix D: Definition of Auxiliary Competitive Equilibrium ("ACE").
Appendix E: Lagrangian for the Recursive Island Planning Problem.
Appendix F: Binding contracts and tenure at the firm level (a formal description)
Appendix G: Calibration of τ.

Appendix A: Analysis of the Island Planning Problem

The next set of results establish that the fixed point $V = H[V]$, the fixed point of the corresponding Bellman equation, is differentiable and that its derivatives are indeed given by V_j^*, define in equation (3). The results in the next three lemmas and two propositions are analogous to standard manipulations of first order conditions, except for the fact that V may not be differentiable.

Consider the problem of the planner of an island that receives U workers per period and that starts with workers $(T_1, T_2, ..., T_{J-1}, T_J)$ where T_i is the number of workers with tenure $i = 1, 2, ..., J$. Define E as the set of possible workers tenure profiles, $E = [0, U]^{J-1} \times R_+$. The planners value function $V : E \times Z$ solves

$$\begin{aligned} & H[V](T_1, T_2, ..., T_{J-1}, T_J, z) \quad (11) \\ = & \max_{\{E_i\}_{i=0}^J} \left\{ F\left(\sum_{i=0}^J E_i, z\right) + \sum_{i=0}^{J-1} \theta [T_i - E_i] + (\theta - \tau)[T_J - E_J] \right. \\ & \left. + \beta \int V(E_0, E_1, ..., E_{J-2}, E_{J-1} + E_J, z') Q(z, dz') \right\} \end{aligned}$$

subject to

$$\begin{aligned} 0 & \leq E_0 \leq U, \\ 0 & \leq E_i \leq T_i \text{ for } i = 1, 2, ..., J. \end{aligned}$$

The fixed point of H gives the stationary version of island planning problem defined in 37.

Proposition 5 *H maps concave functions into concave ones.*

The proof of this Proposition is standard, so we omit it..

We use the following notation for subgradients. Let $G : X \to R$ a concave function. We use $\partial G(x)$ to denote its subgradient at x (if it is clear the value of x from the context we simply use ∂G). In our case $X \subset R^n$, we use $\partial G_{x_i}(x)$ for $i = 1, 2, ..., n$ (and ∂G_{x_i} when it is clear) to denote the projection of $\partial G(x)$ into the subspace of the $x_i's$. Abusing notation, we use $G_{x_i}(x)$ (and G_i when it is clear) to denote a generic element of $\partial G_{x_i}(x)$, so that $G_{x_i}(x) \in \partial G_{x_i}(x)$.

The next proposition gives a useful result, ordering the subgradients of V

Proposition 6 *Consider a function V satisfying*

$$\begin{aligned} V_{T_1} & \geq V_{T_2} \geq \cdots \geq V_{T_{J-1}} \geq V_{T_J}, \quad (12) \\ V_{T_1} & \leq V_{T_J} + \tau \quad (13) \end{aligned}$$

for all z and $T > 0$, where

$$\left(V_{T_1}, V_{T_2}, \ldots, V_{T_{J-1}}, V_{T_J}\right) \in \partial V(T, z).$$

Then,

$$\begin{aligned} H[V]_{T_1} & \geq H[V]_{T_2} \geq \cdots \geq H[V]_{T_{J-1}} \geq H[V]_{T_J}, \quad (14) \\ H[V]_{T_1} & \leq H[V]_{T_J} + \tau \quad (15) \end{aligned}$$

for all z and $T > 0$, where

$$\left(H[V]_{T_1}, H[V]_{T_2}, \ldots, H[V]_{T_{J-1}}, H[V]_{T_J}\right) \in \partial H[V](T, z).$$

Intuitively it follows from the assumption that workers are perfect substitutes and from the fact that $\tau > 0$.

The following proposition and corollaries are important to characterize the solution of the problem and to reduce its dimensionality.

Proposition 7 *Let V satisfy (12). Then the policies for $H[V]$ satisfy the following. Let $E = (E_0, E_1, ..., E_{J-1}, E_J) \in [0, U]^J \times R_+$ be feasible given T. Consider an alternative $\tilde{E} = \left(\tilde{E}_0, \tilde{E}_1, ..., \tilde{E}_{J-1}, \tilde{E}_J\right)$ such that: i) it is feasible for T, ii)*

$$\sum_{j=0}^{J-1} E_j = \sum_{j=0}^{J-1} \tilde{E}_j \text{ and } E_J = \tilde{E}_J,$$

and iii) there is a j' such that $\tilde{E}_j \geq E_j$ for all $j \leq j' \leq J-1$ and that $\tilde{E}_j = 0$for all j, $j' < j \leq J-1$. Then $\tilde{E}$ is weakly preferred to E.

Proof. Replacing any policy by one with these properties can not decrease output but can decrease the separation cost τ. ■

Corollary 8 *The optimal policy can be chosen with the following property:*
(*) If $E_j < T_j$ for some j, $1 \leq j \leq J-1$, then $E_{j'} = 0$ for all $j' : j < j' \leq J-1$.

Corollary 9 *If $T \in \mathcal{E}$ and T' is given by the optimal policy*

$$T' = (T'_1, T'_2, ..., T'_J) = (E_0, E_1, ..., E_{J-2}, E_{J-1} + E_J)$$

then $T' \in \mathcal{E}$.

The next set of results establish that the fixed point $V = H[V]$ is differentiable and that its derivatives are indeed given by V_j^*. The results in the next three lemmas and two propositions are analogous to standard manipulations of first order conditions, except for the fact that V may not be differentiable.

Let define the function $\hat{R}(E, z)$, as follows: $\hat{R} : R_+^{J+1} \times Z \to R$

$$\begin{aligned} \hat{R}(E, z) &= F\left(\sum_{i=0}^{J} E_i, z\right) - \theta \sum_{i=0}^{J-1} E_i - (\theta - \tau) E_J \\ &+ \beta \int V(E_0, E_1, \ldots, E_{J-2}, E_{J-1} + E_J, z') Q(z, dz'). \end{aligned}$$

The first lemma shows a standard saddle-type result for the problem defining $H[V]$.

Lemma 10 *Let V be concave. Fix T, z and let*

$$\begin{aligned} H[V](T, z) &= \max_E \left\{ \hat{R}(E, z) + \hat{\theta} T : 0 \leq E \leq T \right\}, \qquad (16) \\ E(T, z) &= \arg\max_E \left\{ \hat{R}(E, z) : 0 \leq E \leq T \right\}. \end{aligned}$$

Then

$$\hat{\theta} + \lambda^* = (H[V]_0, H[V]_1, ..., H[V]_J) \in \partial H[V](T, z)$$

if and only if λ^* is a Lagrange multiplier, i.e.

$$\begin{aligned} \hat{R}(E^*, z) + \lambda(T - E^*) &\geq \hat{R}(E^*, z) + \lambda^*(T - E^*) \qquad (17) \\ &\geq \hat{R}(E, z) + \lambda^*(T - E) \end{aligned}$$

for all non-negative E, λ, where $\hat{\theta} = (\theta, ..., \theta, \theta - \tau)$, $E^* = E(T, z)$ and $U = T_0$.

Notice that since $\hat{R}$ is concave and the restrictions are linear, $E(T, z)$ solves problem (16) if and only if there (E^*, λ^*) is a saddle as in equation (17) -see, for example, "Analytical Method in Economics", Takayama, Theorem 2.9-.

The next lemma shows the Kuhn-Tucker conditions for this problem.

Lemma 11 *Let V be concave. A necessary and sufficient condition for $E^* = \{E_i^*\}_{i=0}^J$ solves*

$$E^* \in \arg\max_E \hat{R}(E, z) \;\; s.t. \;\; 0 \leq E \leq T$$

given T, z is that there exists a $\left\{\hat{R}_i\right\}_{i=0}^J \in \partial \hat{R}(E^*, z)$ such that (E^*, λ^*) is a saddle where,

$$\lambda_i^* = \hat{R}_i^*. \qquad (18)$$

Given our previous results we can now write the analogous to the Euler equations.

Proposition 12 *Let V be concave. Fix T, z. Then, $0 \leq E^* \leq T$ is an optimal choice given T, z if and only if for all $\{H[V]_i(T,z)\}_{i=0}^J \in \partial H[V](T,z)$ there is a $\left\{\hat{R}_i\right\}_{i=0}^J \in \partial \hat{R}(E^*, z)$ such that*

$$\begin{aligned}
H[V]_i(T,z) &= \hat{R}_i(E^*,z) + \theta \text{ for } i = 0, \ldots, J-1 \\
\hat{R}_i(E^*,z) &\geq f\left(\sum_{i=0}^J E_i^*, z\right) - \theta \\
&\quad + \beta \int V_{i+1}\left(E_0^*, \ldots, E_{J-2}^*, E_{J-1}^* + E_J^*, z'\right) Q(z, dz') \\
\text{with} &= \text{ if } E_i^* > 0 \\
\hat{R}_i(E^*,z) &\geq 0, \\
0 &= \left(H[V]_i(T,z) - \theta\right)(T_i - E_i^*), \text{ and} \\
H[V]_J(T,z) &= \hat{R}_J(E^*,z) + \theta - \tau \ , \\
0 &= \left(H[V]_J(T,z) - (\theta - \tau)\right)(T_J - E_J^*), \\
\hat{R}_J(E,z) &\geq f\left(\sum_{i=0}^J E_i^*, z\right) - (\theta - \tau) \\
&\quad + \beta \int V_J\left(E_0^*, \ldots, E_{J-2}^*, E_{J-1}^* + E_J^*, z'\right) Q(z, dz') \\
\text{with} &= \text{ if } E_J^* > 0 \\
\hat{R}_J(E^*,z) &\geq 0
\end{aligned}$$

where we let $U = T_0$.

The next lemma shows that employment is bounded below, and hence marginal productivity is bounded above.

Lemma 13 *There is an $e > 0$ such that for all T, z*

$$\sum_{i=0}^J E_i(T,z) \geq e > 0.$$

By this lemma, the solution for V_j^* is well defined because, since $f\left(\sum_{i=0}^J E_{i,s}^*, z_s\right)$ are uniformly bounded.

Proposition 14 *Let V be the fixed point of H. Assume that $U > 0$. Then V is differentiable with respect to T_i when $T_i > 0$.*

Appendix B: Analysis of the Simplified Island Planning Problem

The planner's value function $v : [0, J \cdot U] \times R_+ \times Z$ has to satisfy the functional equation h:

$$\begin{aligned} & h[v](t,p,z) \\ = \; & \max_{e_t,e_p} \{F(e_t+e_p,z) + \theta[t-e_t] + (\theta-\tau)[p-e_p] \\ & +\beta \int v(t',p',z') Q(z,dz') \Big\} \end{aligned} \tag{19}$$

subject to

$$\begin{aligned} 0 &\leq e_t \leq t, \\ 0 &\leq e_p \leq p, \end{aligned}$$

and where the law of motion is given by

$$\begin{aligned} t' &= \min\{U+e_t, JU\} \\ p' &= e_p + \max\{U+e_t-JU, 0\} \end{aligned}$$

Proposition 15 *Consider V and v such that*

$$v(T_1+T_2+...+T_{J-1}, T_J, z) = V(T_1, T_2, ..., T_{J-1}, T_J, z) \tag{20}$$

for all $(T_1, T_2, ..., T_{J-1}, T_J) \in \mathcal{E}$. Then

$$h[v](T_1+T_2+...+T_{J-1}, T_J, z) = H[V](T_1, T_2, ..., T_{J-1}, T_J, z) \tag{21}$$

for all $(T_1, T_2, ..., T_{J-1}, T_J) \in \mathcal{E}$.

Proof. By Proposition 7 and its corollaries, $h[v] = H[V]$ in $\mathcal{E}$. ■

Lemma 16 *Assume that V satisfies (12). Consider T and $\hat{T}$ and V such that*

$$T_1+T_2+...+T_{J-1} = \hat{T}_1+\hat{T}_2+...+\hat{T}_{J-1} \text{ and } T_J = \hat{T}_J. \tag{22}$$

for any $\hat{T} \in \mathcal{E}$ and $T \in E$ then

$$H[V](T,z) \leq H[V]\left(\hat{T}, z\right).$$

Proof. If follows directly from the definition of $\mathcal{E}$ and the assumed property (12). ■

Proposition 17 *Let v be the function corresponding to V as in (20) defined for $T \in \mathcal{E}$. Assume that $V(\cdot, z)$ is concave, and that V satisfies (12). Then $h[v](\cdot, z)$ is concave in t, p.*

Remark 18 *The previous proposition is not obvious since the feasible set of the problem defined by the right hand side of $h[v]$ is not convex.*

We now introduced the R, which is the objective function being maximized in $h[v]$. The "derivatives' of R are used to define the functions $\hat{t}$ and $\hat{p}$.

Definition 19 *Given v, define $R(e_t, e_p, z)$ as*

$$R(e_t,e_p,z) = F(e_t+e_p,z) - \theta e_t - (\theta-\tau) e_p$$

$$+\beta \int v(U+\min\{e_t,(J-1)U\}, e_t+e_p-\min\{e_t,(J-1)U\}, z') Q(z,dz')$$

Consider an island planner with no temporary workers ($t = 0$) and a given z. The quantity $\hat{p}(z)$ is the number of permanent workers that leaves the island's planner indifferent between firing "one" permanent worker and keeping all $\hat{p}(z)$ of them.

Definition 20 *Let R be defined as in (19). For each z define $\hat{p}(z)$, such that*

$$0 \in \partial R_{e_p}(0, \hat{p}(z), z) \ .$$

Consider an island planner with $0 < p < \hat{p}(z)$, so it does it not want to fire any permanent worker for that z. The quantity $\hat{t}(p, z)$ is the number of temporary workers that leaves the island's planner indifferent between firing "one" transitory worker and keeping all $\hat{t}(p, z)$ of them. Formally:

Definition 21 *Let R be defined as in (19). For each p, z define $\hat{t}(p, z)$ as follows:*
(i) if $R_{et} > 0$ for all $R_{et} \in \partial R_{et}(U \cdot J, p, z)$, then $\hat{t}(p, z) = J \cdot U$,
(ii) if $R_{et} < 0$ for all $R_{et} \in \partial R_{et}(0, p, z)$, then $\hat{t}(p, z) = 0$,
(iii) otherwise $\hat{t}(p, z)$ solves $0 \in \partial R_{e_t}\left(\hat{t}(p, z), p, z\right)$.

The remaining of this section shows that $\hat{p}, \hat{t}$ exists, that they are unique, and that $\hat{t}$ is decreasing in p. The proofs are complicated by the fact that R is not differentiable.

Proposition 22 *Let v be functions corresponding to V as in (20), assume that V is concave and satisfies (12). The function $R(\cdot, z)$ is strictly concave.*

Define $M : [0, U \cdot J] \to R_+$ as

$$M(e_t) \equiv \min\{e_t, (J-1)U\}$$

notice that

$$\begin{aligned} & e_p + \max\{e_t - (J-1)U, 0\} \\ = \quad & e_p + e_t - \min\{e_t, (J-1)U\} \\ = \quad & e_p + e_t - M(e_t) . \end{aligned}$$

Remark 23 *It is standard to show that $h[v]$ is increasing in t, p and z if v has that properties.*

Remark 24 *Assume that V satisfies (12) and (13). Let v be defined as in (20). Denote by $\partial h[v]$ the subgradient of $h[v](t, p, z)$ when v is considered as a function of t and p. A corollary of Proposition (15) and Proposition (6) is that*

$$h[v]_p \le h[v]_t \le h[v]_p + \tau,$$

for all $\left(h[v]_t, h[v]_p\right) \in \partial h[v](t, p, z)$.

Proposition 25 *Fix t, p, z. Assume that v satisfies (12), (13), and is concave. Define v as in (20). Let $\left(h[v]_t, h[v]_p\right) \in \partial h[v](t, p, z)$. Then $h[v]_p \ge \theta - \tau$. Moreover, there exists a $\bar{p}(z)$ such that for all $p \ge \bar{p}(z)$ and t: $h[v]_p = \theta - \tau$ for any $h[v]_p \in \partial h[v]_p(t, p, z)$.*

Given v define

$$b(e_t, e_p, z) \equiv \int v(U + M(e_t), e_t + e_p - M(e_t), z') Q(z, dz')$$

as a function of e_t and e_p and z. Let ∂B be its subgradient with respect to (e_t, e_p).

Lemma 26 *Assume that v is concave and that it satisfies*

$$v_p \leq v_t \leq v_p + \tau,$$

for all t, p, z. Define v as in (20). Fix any z, e_t, e_p. Let $\left(b_{e_t}, b_{e_p}\right) \in \partial b\left(e_t, e_p, z\right)$. Then

$$b_{e_p} \leq b_{e_t} \leq b_{e_p} + \tau.$$

Let $\partial R\left(e_t, e_p, z\right)$ be the subgradient of R when considered as a function of $\left(e_t, e_p\right)$.

Lemma 27 *Assume that v is concave and that is satisfies*

$$v_p \leq v_t \leq v_p + \tau,$$

for all t, p, z. Fix any z, e_t, e_t. For all $\left(R_{e_p}, R_{e_t}\right) \in \partial R\left(e_t, e_p, z\right)$

$$R_{e_p} \geq R_{e_t} + \tau\left(1 - \beta\right).$$

Corollary 28 *Let e_p, e_t be the optimal choice of employment for Problem (19). If $e_p < p$ and $t > 0$, then $e_t = 0$. If this were not true, i.e. if $e_p < p$ and $e_t > 0$, then $R_{e_p} = R_{e_t} = 0$, which contradicts Lemma 27.*

Lemma 29 *Let v be functions corresponding to V as in (20), assume that V is concave and satisfies (12). Let R be defined as in (19).*
For each z there is a unique $\hat{p}$ satisfying (20). Moreover, $0 < \hat{p}\left(z\right) < \bar{p}\left(z\right) < +\infty$.

Using the concavity of R and strict concavity of F we define $\hat{t}$ as follows.

Lemma 30 *Let v be functions corresponding to V as in (20), assume that V is concave and satisfies (12). Let R be defined as in (19).*
Then for each $\left(p, z\right)$, $0 < p < \hat{p}\left(z\right)$, there exists a unique $\hat{t}$ that satisfies (21).

Proof. The existence and uniqueness of $\hat{t}$ in follows from the strict concavity of R. ■

Proposition 31 *Assume that V is concave and that satisfies (13) and (12). Let v be given by V as in (20). Assume, without loss of generality that v is concave in $\left(t, p\right)$. Then,*
i) The optimal decision rules of $h\left[v\right]$ are described by the set of Inaction for R as

$$\begin{aligned} e_t\left(t, p, z\right) &= \min\left\{t, \hat{t}\left(p, z\right)\right\}, \\ e_p\left(t, p, z\right) &= \min\left\{p, \hat{p}\left(z\right)\right\} \end{aligned}$$

for all t, p, z.
ii) $H\left[V\right]$ is concave, satisfies (13) and (12).
iii) $h\left[v\right]$ and $H\left[V\right]$ satisfy (20) and $h\left[v\right]$ is concave.

Proof. It follows from the definition of $\hat{t}$, $\hat{p}$ and various of the previous results. ■

Lemma 32 *Let V be concave, and satisfy (12) and (13). Let v be defined as in (20). Let $\hat{t}$, $\hat{p}$ and I be defined as in (29), (**??**), (5). Then, the subgradients of $h\left[v\right]$ are as follows:*
If $t \neq iU$ for $i = 1, 2, .., J-1$, then $h\left[v\right]\left(t, p, z\right)$ is differentiable with respect to t.
If $\left(t, p\right) \in Int\left(I\left(z\right)\right)$:

$$h\left[v\right]_t\left(t, p, z\right) = f\left(t + p, z\right) + \beta\int b_{e_t}\left(t, p, z'\right) Q\left(z, dz'\right) > \theta,$$

If $\left(t, p\right) \in Int\left(I\left(z\right)^C\right)$:

$$h\left[v\right]_t\left(t, p, z\right) = \theta > f\left(t + p, z\right) + \beta\int b_{e_t}\left(t, p, z'\right) Q\left(z, dz'\right),$$

If $\left(t, p\right)$: $t = \hat{t}\left(p, z\right) < JU$:

$$\left[\underline{h}\left[v\right]_t\left(t, p, z\right), \bar{h}\left[v\right]_t\left(t, p, z\right)\right] = \left[\theta,\ f\left(t + p, z\right) + \beta\bar{b}_{et}\left(t, p, z\right)\right]$$

Definition 33 *We say that* $\partial v_t\,(t,p,z)$ *is decreasing in* p *if it satisfies the following property. If* $p < p'$*, then define* $\underline{v_t}', \bar{v}_t', \underline{v_t}$ *and* $\bar{v}_t$ *satisfying*

$$\left[\underline{v_t}', \bar{v}_t'\right] = \partial v_t\,(t,p',z)\,,$$

and

$$\left[\underline{v_t}, \bar{v}_t\right] = \partial v_t\,(t,p,z)\,.$$

Then

$$\underline{v}_t' \leq \underline{v}_t \text{ and } \bar{v}_t' \leq \bar{v}_t$$

Notice that if v is differentiable at (t,p,z) this property simply says that $\partial v\,(t,p,z)\,/\partial t$ is decreasing in p.

Lemma 34 *. Let* V *be concave, and satisfy (12), and (13). Let* v *be defined as in (20). Assume that the subgradient of* v_t *is decreasing in* p*, i.e. it satisfies the condition 33. Let* $\hat{t}\,(p,z)$ *be defined as in (**??**) for the optimal rule that attains the right hand side of* $h\,[v]$ *. Then, the subgradient of* $h\,[v]_t$ *is decreasing in* p *too, i.e. it satisfies the condition 33 and* $\hat{t}\,(p,z)$ *is weakly decreasing in* p*.*

Finally

Proposition 35 *Let* v *be the fixed point of* h*. Let* $\hat{t}$ *be defined as in definition* **??***. Then* $\hat{t}\,(p,z)$ *is decreasing in* p*. Moreover, if* $\hat{t}$ *is not a multiple of* U*, then* $\hat{t}$ *is strictly decreasing in* t*.*

Appendix C: Proofs

Proof of Theorem 1. To show this proposition we characterize the competitive equilibrium of a particular decentralization of the economy. Since the 1st welfare theorem holds, characterizing this equilibrium gives us a characterization of the efficient allocations. We call this equilibrium "auxiliary competitive equilibrium" or "ACE" for short. See Appendix D below for a definition of the ACE. The characterization of a stationary ACE coincides with conditions i) to vi) of Theorem 1.

We start by providing some of the necessary conditions that an ACE must satisfy.

Lemma 36 *Let $\{\theta_t,\ \lambda_t(z^t,X),\ E_{j,t}(z^t,X),\ T_{j,t}(z^t,X),\ S_{j,t}(z^t,X),\ U_t,\ L_t;\ all\ t,z^t,j,X\}$ be an AC equilibrium. Then, there is sequence $\{\sigma_t\}$ where σ_t is the value of search at t, for which:*
i) without loss of generality, $\theta_t(z^t,X)=\theta_t$,
ii)

$$\begin{aligned}
\sigma_t &= \beta\sum_X\sum_{z^{t+1}}\lambda_{t+1}\left(z^{t+1},X\right)\eta\left(X|z_0\right)q_t\left(z^t\right)\\
\theta_t &= \max\left\{\omega+\beta\theta_{t+1},\sigma_t\right\}\\
0 &= L_t\left[\theta_t-\omega-\beta\theta_{t+1}\right]
\end{aligned}$$

and
iii) for all z^t,X,

$$\begin{aligned}
\theta_t &\le \lambda_t\left(z^t,X\right),\\
0 &= \left[\lambda_t\left(z^t,X\right)-\theta_t\right]\left[T_{0,t}\left(z^t,X\right)-E_{0,t}\left(z^t,X\right)\right].
\end{aligned}$$

The proof of this Lemma follows directly from the linearity in the problem of firms of type II.

This Lemma shows, among other things, that the value to a firm of type I of reallocation (firing) a worker does not depend on the characteristic of the island, so that θ_t does not depend on (z^t,X) and that the value of search σ_t is related to the value of "selling" (assigning) a worker to the different islands randomly, i.e. in proportion to the number of island of each type.

We will show that the ACE allocation can be obtained by solving a particular dynamic programing problem given two numbers (θ,U) and by checking two appropriate equilibrium conditions. We develop this characterization in a sequence of results.

The solution of the dynamic programing problem will give the equilibrium quantities chosen by firms of type I and the equilibrium prices $\lambda_t(z^t,X)$. This problem has the interpretation of the maximization problem solved for a coalition of firms of type I that are endowed with a flow $\mathbf{U}=\{U_t\}_{t=0}^{\infty}$ of newly arrived workers. We refer to this problem as the "island planner problem", i.e. the problem of a planner in charge of the island employment decision by tenure. The planner chooses how many workers of each tenure to employ and how many to send back, obtaining θ_t for each of them, net of the cost τ.

Definition 37 *Let $V_t: R_+^J\times Z\times R_+^\infty\to R$*

$$\begin{aligned}
&V_t\left(T_1,...T_J;z_t,\mathbf{U}\right)\\
=\ &\max_{E_j,j=0,...J}\Bigg\{F\left(\sum_{j=0}^J E_j,z_t\right)\\
&+\sum_{j=0}^J\left[T_j-E_j\right]\theta_t-\tau\left[T_J-E_J\right]\\
&\beta\sum_{z_{t+1}\in Z}V_{t+1}\left(E_0,...,E_{J-1}+E_J;z_{t+1},\mathbf{U}\right)\Bigg\}Q\left(z_{t+1}|z_t\right)\Bigg\}
\end{aligned}$$

subject to

$$\begin{aligned}
T_0 &= U_t\\
E_j &\le T_j\ \ j=0,1,...,J.
\end{aligned}$$

where $\mathbf{U}=\{U_t;\ all\ t\ge 0\}\in R_+^\infty$.

The next Lemma links the island planning problem with the equilibrium quantities chosen by type I firms and the prices $\{\lambda_t\}$.

Lemma 38 *Let* $\left\{\theta_t^*,\ \lambda_t^*\left(z^t,X\right),\ E_{j,t}^*\left(z^t,X\right),\ T_{j,t}^*\left(z^t,X\right),\ S_{j,t}^*\left(z^t,X\right),\ U_t^*,\ L_t^*;\ all\ t, z^t, j, X\right\}$ *be an auxiliary competitive equilibrium given initial conditions* $U_{-1}^*, \eta^*\left(X|z_0\right)$. *Define* $\hat{V}_t$ *for* $\{U_t^*, \theta_t^*\}$ *and let* $\hat{E}_{j,t}\left(T,z\right)$ *be its optimal policy. Then,* $\left\{E_{j,t}^*\left(z^t,X\right)\right\}$ *solves* V_t *for all the initial conditions* X*, i.e.*

$$\hat{E}_{j,t}\left(T_t^*\left(z^t,X\right),z_t\right)=E_{j,t}^*\left(z^t,X\right)\ for\ all\ t,z^t,X$$

and

$$\lambda_t^*\left(z^t,X\right)=\partial V_t\left(T_t^*\left(z^t,X\right),z_t;\mathbf{U}^*\right)\ for\ all\ t,z^t,X$$

where $\partial V_{_t}\left(T,z_t;\mathbf{U}^*\right)$ *is and element of the subgradient of* $V_{_t}\left(T,z_t;\left\{U_0^*,...,U_{t-1}^*,\cdot,U_{t+1}^*,...\right\}\right)$ *with respect to* U_t^*.

The proof of this Lemma follows from comparing the island planning problem with the problem of firms of type I in a competitive equilibrium, and from the definition of a subgradient.

The next Lemma gives the characterization of ACE.

Lemma 39 . *Let some arbitrary initial distribution* $\eta^*\left(X|z_0\right)$ *be given. Let also some arbitrary sequence* $\{U_t^*,\theta_t^*: all\ t\}$ *be given. Let* $\hat{E}_{j,t}\left(T,z\right)$ *be the optimal policy of island planning problem (37) defined for* $\{U_t^*,\theta_t^*\}$. *Define* $\left\{E_{j,t}^*\right\}$ *as*

$$E_{j,t}^*\left(z^t,X\right)=\hat{E}_j\left(T_t^*\left(z^t,X\right),z_t\right)$$

where $T_{t,j}^*$ *has been generated by* $\left\{\hat{E}\right\}$ *and the initial condition* X*, i.e.*

$$\begin{aligned}
T_{0,j}^* &= X_j\\
T_{t,j}^*\left(z^t,X\right) &= \hat{E}_{j-1}\left(T_{t-1}^*\left(z^{t-1},X\right),z_{t-1}\right)\ for\ j=1,...,J-1\\
T_{t,J}^*\left(z^t,X\right) &= \hat{E}_{J-1}\left(T_{t-1}^*\left(z^{t-1},X\right),z_{t-1}\right)+\hat{E}_J\left(T_{t-1}^*\left(z^{t-1},X\right),z_{t-1}\right)
\end{aligned}$$

and let $\partial V_{_t}\left(T,z_t;U^*\right)$ *be an element of the subgradient of* $V_{_t}\left(T,z_t;\left\{U_0^*,...,U_{t-1}^*,\cdot,U_{t+1}^*,...\right\}\right)$ *with respect to* U_t^*.
i) Define $\left\{\lambda_{t+1}^*\right\}$ *as*

$$\lambda_t^*\left(z^t,X\right)=\partial V_{_t}\left(T_t^*\left(z^t,X\right),z_t;U^*\right).$$

ii)Define, the value of search $\{\sigma_t\}$ *as*

$$\sigma_t=\beta\sum_{z^{t+1}}\sum_X\lambda_{t+1}^*\left(z^{t+1},X\right)q_{t+1}\left(z^{t+1}\right)\eta^*\left(X|z_0\right)\ for\ all\ t.$$

iii)Define $\{L_t^*\}$ *as*

$$L_t^*=N-U_t^*-\sum_{z^t}\sum_X E_{j,t}^*\left(z^t,X\right)q_t\left(z^t\right)\eta^*\left(X|z_0\right)\geq 0\ for\ all\ t.$$

iv) Suppose that the following optimal labor force participation conditions are satisfied

$$\theta_t^*=\max\left\{\sigma_t,\omega+\beta\theta_{t+1}^*\right\},$$

$$L_t^*\ \left[\theta_t-\omega+\beta\theta_{t+1}\right]=0$$

for all t. *Then* $\left\{\theta_t^*,\ \lambda_t^*\left(z^t,X\right),\ E_{j,t}^*\left(z^t,X\right),\ T_{j,t}^*\left(z^t,X\right),\ S_{j,t}^*\left(z^t,X\right),\ U_t^*,\ L_t^*;\ all\ t,z^t,j,X\right\}$ *is a auxiliary competitive equilibrium given the initial conditions* U_{-1}^* *and* η^*.

The proof of this Lemma follows by construction and by the definition of competitive equilibrium and the properties of Problem (37).

Since the first welfare theorem hold for this economy, the characterization of the allocation for an ACE in the previous Lemma applies to the efficient allocations.

Now we define stationary ACE in terms of the objects used our previous characterization of the ACE.

Definition 40 *We say that the auxiliary competitive equilibrium* $\{\theta_t, \lambda_t, L_t, U_t, E_{jt}, S_{jt}\}$ *for initial measure* η *is a stationary equilibrium if there are constants,* θ, U, L, *and functions,* $E_j^* : R_+^{J+1} \times Z \to R,\ j = 0, ..., J,\ \lambda^* : R_+^{J+1} \times Z \to R$, *for which*

$$\begin{aligned}
\theta_t &= \theta,\ \text{all } t \\
U_t &= U,\ \text{all } t \\
L_t &= N,\ \text{all } t \\
E_{i,t}\left(z^t, X\right) &= E_j^*\left(T_t\left(z^t, X\right), z_t\right),\ \text{all } t, z^t \\
\lambda_t\left(z^t, X\right) &= \lambda^*\left(T_t\left(z^t, X\right), z_t\right),\ \text{all } t, z^t
\end{aligned}$$

and where defining $T_j' : R_+^{J+1} \times Z \to R$ *as*

$$\begin{aligned}
T_0'(T, z) &= U, \\
T_j'(T, z) &= E_{j-1}^*(T, z) \text{ for } j = 1, ..., J-1, \\
T_J'(T, z) &= E_J^*(T, z) + E_{J-1}^*(T, z)
\end{aligned}$$

and letting μ *be an invariant distribution of the joint process* (T, z), *with transition given by* (T', Q), *we have*

$$\eta(T|z)\,\zeta(z) = \mu(T, z)$$

where $\zeta(z)$ *is the invariant distribution of* z.

Finally, since a stationary ACE is a particular type of ACE, then by the previous application of the 1st welfare theorem, the stationary version of conditions i) to iv) in Lemma 39 characterizes a stationary efficient allocation. Since the stationary version of conditions i) to iv) in Lemma 39 coincide with conditions i) to iv) of this Theorem, we have finished its the proof.

■

Proof. of Proposition 6 We first show that (15). Consider two states $T > 0$ and $T' > 0$, where T' is obtained from T by increasing the number of workers with tenure J by δ and by decreasing decreasing the number of workers with tenure 1 by δ:

$$\begin{aligned}
T_j' &= T_j \text{ for } j = 2, ..., J-1 \\
T_1' &= T_1 - \delta \text{ and } T_J' = T_J + \delta
\end{aligned}$$

It suffices to show that there is a feasible policy for T' that produces a reduction in total payoff at most by τ and thus

$$H[V(T', z)] - H[V(T, z)] \geq -\tau.$$

To establish this consider two cases, depending on whether in the original \plan more than δ workers with tenure 1 were fired or not. Let δ be a positive number smaller than $T_1/2$. In the case where more than δ workers with tenure 1 were fired, then reduce the firing of workers with tenure 1 by δ and increase the firing of workers with tenure J by δ. Then there is a reduction in current payoff of τ, and no change in the future state. In the second case, let

$$\begin{aligned}
& \frac{1}{\delta}\left(H[V(T', z)] - H[V(T, z)]\right) \\
\geq\ & \frac{1}{\delta}\beta E\left[V\left(\tilde{E}_0, ..., \tilde{E}_{J-2}, \tilde{E}_{J-1} + \tilde{E}_J, z'\right)|z\right] \\
& -\frac{1}{\delta}\beta E\left[V\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right)|z\right]
\end{aligned}$$

where

$$\begin{aligned}
\tilde{E}_j &= E_j \text{ for } j = 2, ..., J-1 \\
\tilde{E}_1 &= E_1 - \delta \\
\tilde{E}_J &= E_J + \delta
\end{aligned}$$

which is feasible given the stated assumptions. Thus using the properties of directional derivatives and subgradients of concave functions

$$\begin{aligned}
& \lim_{\delta \to 0} \frac{1}{\delta} V\left(\tilde{E}_0, ..., \tilde{E}_{J-2}, \tilde{E}_{J-1} + \tilde{E}_J, z'\right) - V\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right) \\
= & \min_{(V_1,...,V_J) \in \partial V} \left\{ (V_J - V_1)\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right) \right\} \\
= & -\tau + \min_{(V_1,...,V_J) \in \partial V} \left\{ (V_J - V_1)\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right) + \tau \right\} \\
= & -\tau + \min_{(V_1,...,V_J) \in \partial V} \left\{ \lim_{\varepsilon \downarrow 0} (V_J - V_1)\left(E_0 + \varepsilon, ..., E_{J-2} + \varepsilon, E_{J-1} + E_J + \varepsilon, z'\right) + \tau \right\} \\
= & -\tau + \lim_{\varepsilon \downarrow 0} \min_{(V_1,...,V_J) \in \partial V} \left\{ (V_J - V_1)\left(E_0 + \varepsilon, ..., E_{J-2} + \varepsilon, E_{J-1} + E_J + \varepsilon, z'\right) + \tau \right\} \\
\geq & -\tau
\end{aligned}$$

where we use theorem 24.4, page 233, of Rockafellar (1997) which shows that the graph of ∂f is closed for a concave function on $R^{n \cdot}$, the hypothesis that (13) holds for all subgradients with $T > 0$, and where we denote

$$\begin{aligned}
& (V_J - V_1)\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right) \\
\equiv \; & V_J\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right) + \tau - V_1\left(E_0, ..., E_{J-2}, E_{J-1} + E_J, z'\right).
\end{aligned}$$

Finally since for all subgradients:

$$H[V]_J(T, z) - H[V]_1(T, z) \geq \lim_{\delta \to 0} \frac{1}{\delta}\left(H\left[V\left(T', z\right)\right] - H\left[V\left(T, z\right)\right]\right)$$

then

$$H[V]_J(T, z) - H[V]_1(T, z) \geq -\tau.$$

The argument to show that (14) follows from a similar argument, where we let

$$T'_j = T_j + \delta \text{ and } T'_{j+1} = T_{j+1} - \delta$$

for $j = 1, ..., J-1$. ■

Proof of Lemma 10. Let λ^* be a Lagrange multiplier, then $\lambda^*(T - E^*) = 0$. Consider T', and $E' = E(T', z)$, then

$$\begin{aligned}
& H[V](T, z) - \hat{\theta} T \\
= \; & \hat{R}(E(T), z) \\
\geq \; & \hat{R}(E(T'), z) + \lambda^*(E(T) - E(T')) \\
\geq \; & \hat{R}(E(T'), z) + \lambda^*(T - T') \\
= \; & H[V](T', z) - \hat{\theta} T' + \lambda^*(T - T')
\end{aligned}$$

thus $\hat{\theta} + \lambda^*$ is a subgradient of $H[V]$. Let $\hat{\theta} + \lambda^*$ be a subgradient of $H[V](T, z)$. Since workers can always be sent back and get $\hat{\theta}$, then $\lambda^* \geq 0$. Also,

$$H[V](T, z) = H[V](E^*, z) + \hat{\theta}[T - E^*]$$

for $E^* = E(T, z)$. Then, by definition of subgradient

$$\hat{\theta}[T - E^*] = H[V](T, z) - H[V](E^*, z) \geq \left(\hat{\theta} + \lambda^*\right)(T - E^*)$$

or

$$0 = \hat{R}(T, z) - \hat{R}(E^*, z) \geq \lambda^*(T - E^*)$$

but $E^* \leq T$ so $\lambda^*(T - E^*) = 0$. This equality, together with the definition of a subgradient imply that λ^* is a Lagrange multiplier. ■

Proof of 11. Let (E^*,λ^*) be a saddle satisfying (18). Then, by theorem 2.9 in Takayama E^* is optimal. Let E^* be optimal. Then, by theorem 2.9 in Takayama there are $\lambda^* \geq 0$ such that (E^*,λ^*) is a saddle. It rests to show that $\lambda_i^* = \hat{R}_i^*$ for some subgradient. From the definition of a saddle,

$$\hat{R}\left(E^*,z\right)+\lambda^*\left(T-E^*\right)\geq \hat{R}\left(E,z\right)+\lambda^*\left(T-E\right)$$

or

$$\hat{R}\left(E^*,z\right)\geq \hat{R}\left(E,z\right)+\lambda^*\left(E^*-E\right).$$

which is the definition of a subgradient. ■

Proof of Proposition 12. Let E^* be optimal. Take any $\left\{H\left[V\right]_i\left(T,z\right)\right\}_{i=0}^J \in \partial H\left[V\right]\left(T,z\right).$ By lemma 10 λ^* is a Lagrange multiplier, where

$$\left\{H\left[V\right]_i\left(T,z\right)\right\}_{i=0}^J = \lambda^* + \hat{\theta}.$$

By lemma 18, $\lambda_i^* = \hat{R}_i^*$ for some subgradient. Then $\hat{R}_i \geq 0$, and

$$0=\hat{R}_i^*\left(T_i-E_i^*\right)=\left(H\left[V\right]_i\left(T,z\right)-\theta\right)\left(T_i-E_i^*\right).$$

Let $\left\{H\left[V\right]_i\left(T,z\right)\right\}_{i=0}^J \in \partial H\left[V\right]\left(T,z\right)$, and let R_i^* be a subgradient of $\hat{R}^*$ evaluated at some $0\leq E^*\leq T$ such that the above conditions are satisfied. Define λ^* as

$$\lambda^* = \left\{H\left[V\right]_i\left(T,z\right)\right\}_{i=0}^J - \hat{\theta} = \left\{R_i^*\right\}_{i=0}^J$$

where the last equality follow by the assumed properties. We will show that (E^*,λ^*) is a saddle. From the above conditions,

$$0=\lambda_i^*\left(T_i-E_i^*\right)$$

Hence,

$$\hat{R}\left(E^*,z\right)+\lambda\left(T-E^*\right)\geq \hat{R}\left(E^*,z\right)+\lambda^*\left(T-E^*\right), \text{ for every } \lambda\geq 0$$

Since, by the above conditions, λ^* is a subgradient of $\hat{R}^*$ evaluated at $0\leq E^*\leq T$, it follows that

$$\hat{R}\left(E,z\right)\leq \hat{R}\left(E^*,z\right)+\lambda^*\left(E-E^*\right), \text{ for every } E$$

Hence,

$$\hat{R}\left(E^*,z\right)+\lambda^*\left(T-E^*\right)\geq \hat{R}\left(E,z\right)+\lambda^*\left(T-E\right), \text{ for every } E$$

so that E^* is optimal.

If $E_i^*>0$,

$$\begin{aligned}\hat{R}_i\left(E^*,z\right) &= f\left(\sum_{i=0}^J E_i^*,z\right)-\theta \\ &+\beta\int V_{i+1}\left(E_0^*,\ldots,E_{J-2}^*,E_{J-1}^*+E_J^*,z'\right)Q\left(z,dz'\right)\end{aligned}$$

follows since $\partial\left(g+h\right)\left(x\right)=\partial g\left(x\right)+\partial h\left(x\right)$, see Rockafeller, Thm 23.8 and since F is differentiable with derivative f. When $E_i^*=0$, the subgradient of F are any numbers greater than f, and hence the previous expression hold with inequality. ■

Proof of Lemma 13. By contradiction , for all $e>0$, there is a T,z such that

$$\sum_{i=0}^J E_i\left(T,z\right)\leq e,$$

Take $e<U$ and such that

$$f\left(e,\underline{z}\right)>\theta.$$

where $\underline{z}=min\left\{z:z\in Z\right\}.$ Since $T_0=U>0,\ E_0\left(T,z\right)<T_0$.From 12

$$0=\left[H\left[V\right]_0\left(T,z\right)-\theta\right]\left[T_0-E_0\right]$$

thus

$$H[V]_0(T,z) = \theta$$

but

$$H[V]_0(T,z) = \hat{R}_0(E^*,z) + \theta$$

so $\hat{R}_0(E^*,z) = 0$. Since

$$\begin{aligned} 0 &= \hat{R}_0(E^*,z) \geq f\left(\sum_{i=0}^{J} E_i^*, z\right) - \theta \\ &\quad + \beta \int V_1\left(E_0^*, \ldots, E_{J-2}^*, E_{J-1}^* + E_J^*, z'\right) Q(z, dz') \\ &> \beta \int V_1\left(E_0^*, \ldots, E_{J-2}^*, E_{J-1}^* + E_J^*, z'\right) Q(z, dz') \\ &\geq 0. \end{aligned}$$

■

Proof of Proposition 14. Let T, z be such that $T_i > 0$. Assume that $\left\{E_{j,s}^*\right\}$ is optimal. Take a subgradient $V_i(T,z) = H[V]_i(T,z)$.

First consider the case where $E_{i,s}^* = 0$, then

$$[H[V]_i(T,z) - \theta_i(s)]\left[T_{i,s} - E_{i,s}^*\right] = 0$$

thus, its unique solution is $H[V]_i(T,z) = \theta_i(s)$, provided that $T_{i,s} > 0$. Thus, as an special case, if $E_{i,0}^* = 0$, then $\theta_i(0)$ is the derivative of V.

Now consider the case where $E_{i,s}^* > 0$. We use the formulae in Proposition 12 and replace its value repeatedly, solving it forward until $\hat{\tau}_j = s$, the first time that for this cohort employment is smaller than the number of workers present at the location. Since $E_{i,s}^* > 0$ at each iteration

$$\begin{aligned} H[V]_i(T,z) &= f\left(\sum_{i=0}^{J} E_i^*, z\right) \\ &\quad + \beta \int V_{i+1}\left(E_0^*, \ldots, E_{J-2}^*, E_{J-1}^* + E_J^*, z'\right) Q(z, dz'). \end{aligned}$$

Notice that in this case, we argue above that $V_i = \theta_i(s)$. Thus, we find that unique solution of $H[V]_i(T,z)$ is $V_i^*(T,z)$. Hence the subgradient is unique, and thus $V(T,z)$ is differentiable. ■

Proof. of Proposition 17. Take (t_1, p_1) and (t_2, p_2) and consider $(t_\lambda, p_\lambda) = (\lambda t_1 + (1-\lambda) t_2,\ \lambda p_1 + (1-\lambda) p_2)$. Let the unique corresponding elements in $\mathcal{E}$ for (t_1, p_1) and (t_2, p_2) be T_1 and T_2. Consider $T_\lambda = \lambda T_1 + (1-\lambda) T_2$, which is not necessarily on $\mathcal{E}$. Let $\hat{T}_\lambda$ be the unique element in $\mathcal{E}$ that corresponds to T_λ. Note that (t_λ, p_λ) satisfies

$$t_\lambda = \sum_{j=1}^{J-1} \hat{T}_\lambda \text{ and } p_\lambda = \hat{T}_J.$$

Then,

$$\begin{aligned} & \lambda h[v](t_1, p_1, z) + (1-\lambda) h[v](t_2, p_2, z) \\ = \ & \lambda H[V](T_1, z) + (1-\lambda) H[V](T_2, z) \\ \leq \ & H[V](T_\lambda, z) \\ \leq \ & H[V]\left(\hat{T}_\lambda, z\right) \\ = \ & h[v](t_\lambda, p_\lambda, z), \end{aligned}$$

where the first equality follows from Proposition 15, the first inequality follows from concavity of V and Proposition 5 , the second inequality follows from Lemma 16, and the last equality follows from Proposition 15. ■

Proof. of Proposition 22. First define

$$\begin{aligned}\hat{R}(E,z) &= F\left(\sum_{j=0}^{J} E_j, z\right) - \theta\sum_{j=0}^{J-1} E_j - (\theta-\tau) E_J \\ &+\beta\int V\left(U, E_0, \ldots, E_{J-2}, E_{J-1}+E_J, z'\right) Q\left(z, dz'\right).\end{aligned}$$

Since V and F are concave, then $\hat{R}$ is concave. Now take $\left(e_t^i, e_p^i\right)$ for $i = 1,2$ and consider $\left(e_t^\lambda, e_p^\lambda\right) = \left(\lambda e_t^1 + (1-\lambda) e_t^2, \lambda e_p^1 + (1-\lambda\right.$ Let the unique corresponding elements to $\left(e_t^i, e_p^i\right)$in $[0,U]^J \times R_+$ that satisfies property (*) be denoted by $\tilde{E}^i$ for $i = 1,2$. Define $E^\lambda = \lambda\tilde{E}^1 + (1-\lambda)\tilde{E}^2$. Note that

$$\sum_{j=0}^{J-1} E_j^\lambda = e_t^\lambda \text{ and } E_J = e_p^\lambda.$$

Define $\tilde{E}^\lambda$ as the unique element in $[0,U]^J \times R_+$ that satisfies property (*) and such that

$$\sum_{j=0}^{J-1} E_j^\lambda = \sum_{j=0}^{J-1} \tilde{E}_j^\lambda \text{ and } E_J = \tilde{E}_J.$$

Then

$$\begin{aligned} & \lambda R\left(e_p^1, e_t^1, z\right) + (1-\lambda) R\left(e_p^2, e_t^2, z\right) \\ = \ & \lambda\hat{R}\left(\tilde{E}^1, z\right) + (1-\lambda)\hat{R}\left(\tilde{E}^2, z\right) \\ \leq \ & \hat{R}\left(E^\lambda, z\right) \\ \leq \ & \hat{R}\left(\tilde{E}^\lambda, z\right) \\ = \ & R\left(e_p^\lambda, e_t^\lambda, z\right),\end{aligned}$$

where the first equality follows construction of $\tilde{E}^i$ and since by assumption v and V satisfies (20), the first inequality follows from the concavity of $\hat{R}$, the second inequality follows by assumption (12) and Proposition 7 and its corollaries, and the last equality follows from the same argument than in Proposition 15. ■

Proof. of Proposition 25 Define the operator $\bar{h}$ as

$$\begin{aligned}\bar{h}[v](t,p,z) &= \max_{0\leq e_t, 0\leq e_p}\left\{F\left(e_t+e_p, z\right) + \theta\left[t-e_t\right] + (\theta-\tau)\left[p-e_p\right] + \right. \\ &\left. +\beta\int v\left(U+M\left(e_t\right), e_p+e_t-M\left(e_t\right), z'\right) Q\left(z, dz'\right)\right\}\end{aligned}$$

Comparing this problem with (**??**) the constraints $e_t \leq t$ and $e_p \leq p$ were removed, hence

$$h[v](t,p,z) \leq \bar{h}[v](t,p,z).$$

The optimal policies e_t, e_p do not depend on t and p, thus the function $\bar{h}[v]$ is linear with derivatives

$$\begin{aligned}\bar{h}[v]_p(t,p,z) &= \theta-\tau \\ \bar{h}[v]_t(t,p,z) &= \theta\end{aligned}$$

for all t,p,z. By concavity of $h[v]$,

$$\begin{aligned} h[v](t,0,z) &\leq h[v](t,p,z) + h[v]_p(0-p) \text{ or} \\ h[v](t,p,z) &\geq h[v]_p p + h[v](t,0,z)\end{aligned}$$

where $\left(h[v]_t, h[v]_p\right) \in \partial h[v](t,p,z)$. Rearranging and using the linearity of $\bar{h}[v]$:

$$h[v]_p p + h[v](t,0,z) \leq h[v](t,p,z) \leq \bar{h}[v](t,p,z) = \bar{h}[v](t,0,z) + [\theta-\tau] p$$

for all p. Thus by monotonicity of $h[v]$ and $\bar{h}[v]$ on t :

$$h[v]_p(t,p,z)\,p + h[v](0,0,z) \le \bar{h}[v]((J-1)U,0,z) + [\theta-\tau]\,p$$

$$\sup_{t\in[0,U(J-1)]} h[v]_p(t,p,z)\,p + h[v](0,0,z) \le \bar{h}[v]((J-1)U,0,z) + [\theta-\tau]\,p$$

Hence

$$\begin{aligned} &\lim_{p\to\infty}\inf \frac{h[v](0,0,z)-\bar{h}[v](U(J-1),0,z)}{p} \\ =\ & 0 \le \lim\inf_{p\to\infty}\left([\theta-\tau]-\sup_{t\in[0,U(J-1)]} h[v]_p(p,t,z)\right) \end{aligned}$$

or

$$\lim_{p\to\infty}\sup\left[\sup_{t\in[0,U(J-1)]} h[v]_p(p,t,z)\right] \le \theta-\tau.$$

On the other hand, for the original problem (**??**) for (p_0,t,z). A feasible policy for $p \ge p_0$ is to set $e_p^0 = e_p(p_0,t,z)$, in which case each additional unit of p yields $\theta-\tau$. Hence the right derivative of $h[v](t,p_0,z)$ is greater or equal than $\theta-\tau$. Since $h[v]$ is concave, then $h[v]_p(t,p_0,z) \ge \theta-\tau$ for all (t,p_0,z).

Combining the two inequalities, for large enough p, $h[v]_p(p,t,z) = \theta-\tau$ for all t. ■

Proof. of Lemma 26. Consider two cases. First $e_t < (J-1)U$. In this case $M(e_t) = e_t$, which implies that

$$b(e_t,e_p,z) = \int v(U+e_t,e_p,z')\,Q(z,dz'),$$

thus

$$\begin{aligned} b_{e_t} &= \int v_t dQ, \\ b_{e_p} &= \int v_p dQ \end{aligned}$$

where

$$(v_t,v_p) \in \partial v(U+e_t,e_p,z')$$

for the corresponding elements. Second, if $e_t > (J-1)U$,

$$b(e_t,e_p,z) = \int v(JU,e_p+e_t-(J-1)U,z')\,Q(z,dz'),$$

thus

$$\begin{aligned} b_{e_t} &= \int v_p dQ, \\ b_{e_p} &= \int v_p dQ. \end{aligned}$$

Since, by assumption,

$$v_p \le v_t \le v_p + \tau,$$

we have shown the required result, except for the case where $e_t = (J-1)U$. This case follows by continuity, since the graph of the subgradient of a concave function is closed (Rockafellar, 1997, Theorem 24.4, page 233). ■

Proof. of Lemma 27. By the definition of R :

$$\begin{aligned} R_{e_p} &= f(e_t,e_p,z)-(\theta-\tau)+\beta b_{e_p}, \\ R_{e_t} &= f(e_t,e_p,z)-\theta+\beta b_{e_t} \end{aligned}$$

where

$$\left(b_{e_t},b_{e_p}\right) \in \partial b(e_t,e_p,z).$$

Then

$$R_{e_p} - R_{e_t} = \tau + \beta \left[b_{e_p} - b_{e_t} \right] \geq \tau \left(1 - \beta \right)$$

where the inequality follows from the previous lemma. ■

Proof. of Lemma 29. The existence of $\hat{p}$ follows by the concavity of R with respect to p, the Inada conditions on F and from Proposition 25, which shows that $v_p = \theta - \tau$ for large p. The uniqueness of p follows by the strict concavity of F. That $\hat{p} < \bar{p}$ follows from concavity of R with respect of e_p and Lemma 27. ■

Proof. of Lemma 32. The first statement follows by considering the case where $T \in \mathcal{E}$ so that there is a $i \in \{1, 2, ..., J-1\}$ and T_i such that

$$T_1, ..., T_J = (U, .., U, T_i, 0, ..., 0, T_J)$$

for $T_i \in (0, U)$

$$V\left(U, ..., U, T_i, ..., 0, T_J\right) = v\left(\left(i-1\right)U + T_i, T_J\right) \text{ for all } T_i \in (0, U)$$

Thus

$$V_i\left(U, ..., U, T_i, ..., 0, T_J\right) = v_t\left(\left(i-1\right)U + T_i, T_J\right) \text{ for } i = 1, 2, ..., J-1.$$

The second and third claims follows from the form of the optimal decision rules, i.e. the definition of the range of inaction and the strict concavity of R. The third follows since, for $t \geq \hat{t}(p, z)$ it is feasible to fire any extra temporary workers, so that we know the right derivative of $h[v]$ with respect to t. ■

Proof. of Lemma 34. First we establish that $\hat{t}(p, z)$ is decreasing in p. Then we use this result, to show that $h[v]_t$ is decreasing in p.

By definition of $\hat{t}$,

$$0 \in R_{et}\left(\hat{t}(p, z), p, z\right)$$

for the case when $0 < \hat{t} < JU$. The main idea is to show that $R_{et}(t, p, z)$ is decreasing in p, and then use that, by concavity, $R_{et}(t, p, z)$ is decreasing in t.

The subgradient R_{et} is given by

$$R_{e_t}(t, p, z) = f(t + p, z) - \theta + \beta b_{et}(t, p, z)$$

where $b(t, p, z)$ is given by

$$b(t, p, z) = \int v\left(U + \min\{t, U(J-1)\}, t + p - \min\{t, U(J-1)\}, z'\right) Q(z, dz')$$

We can then write b by cases as

$$\begin{aligned} b(t, p, z) &= \int v\left(U + t, p, z'\right) Q(z, dz') \text{ if } t \leq U(J-1) \\ b(t, p, z) &= \int v\left(UJ, t + p - U(J-1), z'\right) Q(z, dz') \text{ for } t \geq U(J-1) \end{aligned}$$

and hence its subgradients are

$$\begin{aligned} b_{et}(t, p, z) &= \int v_t\left(U + t, p, z'\right) Q(z, dz') \text{ if } t < U(J-1) \\ b_{et}(t, p, z) &= \left[\int \underline{v}_p\left(UJ, p, z'\right) Q(z, dz'), \int \bar{v}_t\left(UJ, p, z'\right) Q(z, dz') \right] \text{ if } t = U(J-1) \\ b_{et}(t, p, z) &= \int v_p\left(UJ, t + p - U(J-1), z'\right) Q(z, dz') \text{ for } t > U(J-1) \end{aligned}$$

Now we are ready to show that $R_{et}(t, p, z)$ is strictly decreasing in p. Consider first the case where $t < U(J-1)$. In this case it follows from the hypothesis that v_t is decreasing in p and the strict concavity of f. Consider the case where $t > (J-1)U$. In this case it follows from the concavity of v, so that v_p is decreasing, and the strict concavity of f. Finally, for the case where $t = (J-1)U$, we combine the previous two arguments for the right and left derivatives.

Having established that $R_{et}(t, p, z)$ is strictly decreasing in p, then it follows that $\hat{t}$ is decreasing in p since R_{et} is decreasing in t by concavity of R.

The cases where $\hat{t} = UJ$ or $\hat{t} = 0$ are similar.

Now we turn to show that $h[v]_t$ is decreasing in p. We consider three cases. First, let $(t,p) \in Int(I(z))$. In this case,

$$\begin{aligned} h[v]_t(t,p,z) &= f(t+p,z) + \beta b_{et}(t,p,z) \\ &= R_{et}(t,p,z) + \theta \end{aligned}$$

and thus $h[v]_t$ is decreasing in p since, as shown above, $R_{et}(t,p,z)$ is decreasing in p. In the case where $(t,p) \in Int\left(I(z)^C\right)$, then $h[v]_t = \theta$, and hence $h[v]$ is differentiable, and its derivative constant, so that it is weakly decreasing in p. Finally, consider the case where (t,p,z) is such that $t = \hat{t}(p,z)$. As shown above $\hat{t}$ is weakly decreasing in p, thus for $p' > p,\ t \geq \hat{t}(p',z)$. Also, the derivative subgradient of $h[v]_t$ are

$$\left[\underline{h}[v]_t(t,p,z), \bar{h}[v]_t(t,p,z)\right] = \left[\theta,\ f(t+p,z) + \beta\bar{b}_{et}(t,p,z)\right]$$

If $\hat{t}(p',z) = \hat{t}(p,z)$, then, using the expression for the left derivative of $h[v]_t$, it follows since f is concave and since, as shown above, b_{et} is decreasing in p. If $\hat{t}(p',z) < \hat{t}(p,z)$, then, it must be that the point $\left(\hat{t}(p,z), p', z\right)$ is in the interior of the complement of the range of inaction, and thus $h[v]_t\left(\hat{t}(p,z), p', z\right) = \theta$. Thus $h[v]_t$ has decreased in this case too, since the subgradient has collapsed to its right derivative. ■

Proof. of Proposition 35. That $\hat{t}$ is decreasing in t follows using Lemma 34. Notice that starting with $V^0 = 0$ and $v^0 = 0$ satisfies all the hypothesis of this lemma. Since all these properties are preserved in the limit, they hold for the fixed point. That see that $\hat{t}$ is strictly consider first the case where $t < U(J-1)$. In this case if follows by using that in a neighborhood of those points $v(t+U,p,z')$ is differentiable with respect to t -see Proposition **??**-, that it satisfies

$$\theta = f\left(\hat{t}(p,z) + p, z\right) + \beta \int v_t\left(\hat{t}(p,z) + U, p, z'\right) Q(z,dz'),$$

that $v_t(t+U,p,z')$ is decreasing in p, and that f is strictly decreasing. A similar argument holds when $t > U(J-1)$, where

$$\theta = f\left(\hat{t}(p,z) + p, z\right) + \beta \int v_p\left(JU, p + \hat{t}(p,z) - JU, z'\right) Q(z,dz').$$

■

Proof of Theorem 3 and Proposition 4.

To prove the theorem it is convenient to define a sequential economy that corresponds to the island planning problem taking as given U, θ.. This economy has a firm whose problem corresponds to that one of the firm with value function B in the RCE and a family whose problem has solution that gives the workers value function W in the RCE.

I) We define this economy in a standard Arrow-Debreu sequential way. This definition allows to use the 1st and 2nd welfare theorem to link the allocation that solves the island sequence planning problem with an allocation that solves the firms and workers problem in the island economy as well as to link it with the equilibrium wages w.

The commodities for the sequential island economy with initial state X, z_0 is given by processes for employment by tenure E and consumption C

$$(E,C) = \left\{C_t\left(z^t\right), E_{jt}\left(z^t\right) : j = 0,...,J,\ z^t \in Z\right\}.$$

We use g_{jt} to denote the labor choice of the firms in a sequential island problem. We use the h_t and s_t for hiring and firing of permanent workers. The net output of firms is to produce the following date t history z^t amount of consumption good

$$F\left(\sum_{j=0}^{J} g_{jt}\left(z^t\right), z_t\right) - \tau\ s_t\left(z^t\right) \tag{23}$$

The choices of g for the firms are subject to the restrictions that $g_{j,-1}(z_{-1}) = X_j$ for $j = 0,...,J$, the law of motion of the permanent workers

$$g_{Jt}\left(z^t\right) = g_{Jt-1}\left(z^{t-1}\right) + g_{J-1t-1}\left(z^{t-1}\right) - s_t\left(z^t\right) + h_t\left(z^t\right) \tag{24}$$

and the non-negativity of hiring and firing

$$s_t\left(z^t\right) \geq 0 \text{ , } h_t\left(z^t\right) \geq 0, g_{jt}\left(z^t\right) \geq 0$$

for all $j = 0, ..., J,\ z^t, t \geq 0$.

We use e to denote the labor choice and c for the consumption choice of the household in the sequential island problem. This household "owns" as an endowment a stream of U unemployed workers per period, that arrive to the island every period. The household is risk neutral in terms of consumption $c_t\left(z^t\right)$. Its decision is to assign a worker to work on the island or to permanently work outside the island, which gives value θ per worker, in units of the final good. The utility function of the household is

$$\sum_{t=0}\sum_{z^t}\beta^t Q\left(z^t|z_0\right) \times$$

$$\times\left[c_t\left(z^t\right)+\theta\left(\sum_{j=0}^{J-1}\left[e_{j-1t-1}\left(z^{t-1}\right)-e_{jt}\left(z^t\right)\right]+\left[e_{Jt-1}\left(z^{t-1}\right)+e_{J-1t-1}\left(z^{t-1}\right)-e_{Jt}\left(z^t\right)\right]\right)\right]$$

The household is subject to the following restrictions:

$$e_{j,-1}\left(z_{-1}\right) = X_j \text{ for } j = 0, ..., J$$

and for all t, z^t non-negative e_{jt} subject to:

$$\begin{aligned} e_{0,t}\left(z^t\right) &\leq U, \\ e_{jt}\left(z^t\right) &\leq e_{j-1t-1}\left(z^{t-1}\right) \text{ for } j = 1, 2, ..., J-1 \\ e_{Jt}\left(z^t\right) &\leq e_{Jt-1}\left(z^{t-1}\right)+e_{J-1t-1}\left(z^{t-1}\right) \end{aligned} \tag{25}$$

Market clearing for the sequential economy is given by

$$\begin{aligned} e_{jt}\left(z^t\right) &= g_{jt}\left(z^t\right) \\ c_t\left(z^t\right) &= F\left(\sum_{i=0}^{J} g_{it}\left(z^t\right),\ z_t\right)-\tau s_t\left(z^t\right) \end{aligned}$$

for all $j = 0, ..., J$, and all $t,\ z^t$. Prices in this sequence island economy are given by intertemporal consumption prices, $P_t\left(X, z^t\right)$ and wages by tenure $w_{jt}\left(X, z^t\right)$ in terms of date t history z^t consumption goods. Given the household preferences for consumption we impose

$$P_t\left(X, z^t\right) = \beta^t Q\left(z^t|z_0\right)$$

With these prices the problem for the firm is to maximize profits, i.e.

$$\begin{aligned} & B_0\left(x_J, X, z_0\right) \\ = & \max_{\{g\}}\sum_{t=0}\beta^t\sum_{z^t}\left[F\left(\sum g_{jt}\left(z^t\right), z_t\right)-\sum_{j=0}^{J} g_{jt}\left(z^t\right) w_{jt}\left(X, z^t\right)-\tau s_t\left(z^t\right)\right] Q\left(z^t|z_0\right) \end{aligned}$$

subject to

$$g_{J-1} = x_J$$

and the law of motion for s, h and g. Let $\beta^t\xi_t\left(z^t\right) Q\left(z^t|z_0\right)$ be the multiplier of the restriction (24). The first order conditions for the firm's problem are:

$$f\left(\sum g_{jt}\left(z^t\right), z_t\right)-w_{jt}\left(X, z^t\right) \leq 0$$

for $j = 0, ..., J-2$ with equality if $g_{jt}\left(z^t\right) > 0$. For $j = J-1$

$$f\left(\sum g_{jt}\left(z^t\right), z_t\right)-w_{J-1t}\left(X, z^t\right)+\beta\sum_{z_{t+1}}\hat{\xi}_{t+1}\left(z^t, z_{t+1}\right) Q\left(z_{t+1}|z_t\right) \leq 0$$

with equality if $g_{J-1t}(z^t) > 0$. For $j = J$

$$\hat{\xi}_t(z^t) = f\left(\sum_{j=0}^{J} g_{jt}(z^t), z_t\right) - w_{Jt}(X, z^t) + \beta \sum_{z_{t+1}} \hat{\xi}_{t+1}(z^t, z_{t+1}) Q(z_{t+1}|z_t) \tag{26}$$

if $g_{Jt}(z^t) > 0$. The first order conditions for $h_t(z^t)$ is

$$\hat{\xi}_t(z^t) \leq 0$$

with equality if $h_t(z^t) > 0$. The first order conditions for $s_t(z^t)$ is

$$-\tau - \hat{\xi}_t(z^t) \leq 0$$

with equality if $s_t(z^t) > 0$.The last three inequalities imply (26). The slackness condition for (24) gives:

$$g_{J,t-1}(z^{t-1}) + g_{J-1,t-t}(z^{t-1}) > g_{J,t}(z^t) \text{ then } \hat{\xi}_t(z^t) = -\tau$$

$$g_{J,t-1}(z^{t-1}) + g_{J-1,t-t}(z^{t-1}) < g_{J,t}(z^t) \text{ then } \hat{\xi}_t(z^t) = 0.$$

Now we turn to the household problem in a sequential island economy. Letting $\beta^t Q(z^t|z_0)\hat{\nu}_{j,t}(z^t)$ be the Lagrange multiplier for (25) the first order conditions of the household problem are equivalent to

$$W_{jt}(X, z^t) = \max\left\{\theta,\ w_{jt}(X, z^t) + \beta \sum_{z_{t+1}} W_{j+1t+1}(X, z^t, z_{t+1}) Q(z_{t+1}|z_t)\right\},$$

for $j = 0, ..., J-1$ and

$$W_{Jt}(X, z^t) = \max\left\{\theta,\ w_{Jt}(X, z^t) + \beta \sum_{z_{t+1}} W_{Jt+1}(X, z^t, z_{t+1}) Q(z_{t+1}|z_t)\right\}$$

where

$$W_{jt}(X, z^t) = \hat{v}_{jt}(X, z^t) + \theta$$

with slackness

$$e_{jt}(z^t) < e_{j-1t-1}(z^{t-1}) \text{ then } W_{jt}(X, z^t) = \theta,$$

and $e_{jt}(z^t) > 0$,

$$W_{jt}(X, z^t) = w_{jt}(X, z^t) + \beta \sum_{z_{t+1}} W_{j+1t+1}(X, z^t, z_{t+1}) Q(z_{t+1}|z_t)$$

for $j = 0, ..., J-1$ and analogously for $j = J$.

To see why this is the case, write the Lagrangian of the household problem as

$$\sum_{t=0} \beta^t \sum_{z^t} Q(z^t|z_0) \times$$

$$\left\{\sum_{j=0}^{J} e_{jt}(z^t) w_{jt}(X, z^t) + \right.$$
$$\theta\left(\sum_{j=0}^{J-1}\left[e_{j-1t-1}(z^{t-1}) - e_{jt}(z^t)\right] + \left[e_{J\ t-1}(z^{t-1}) + e_{J-1t-1}(z^{t-1}) - e_{Jt}(z^t)\right]\right)$$
$$+\hat{\nu}_{0t}(z^t)\left[U - e_{0t}(z^t)\right] +$$
$$\left. + \sum_{j=1}^{J-1} \hat{\nu}_{jt}(z^t)\left[e_{j-1t-1}(z^{t-1}) - e_{jt}(z^t)\right] + \hat{\nu}_{jt}(z^t)\left[e_{J-1t-1}(z^{t-1}) + e_{Jt-1}(z^{t-1}) - e_{Jt}(z^t)\right]\right\}$$

The first order conditions of this problem are as follows. For $e_{jt}\left(z^{t}\right)$:

$$w_{jt}\left(X,z^{t}\right)-\left(\theta+\hat{\nu}_{jt}\left(z^{t}\right)\right)+\beta\sum_{z_{t+1}}\left(\theta+\hat{v}_{j+1\ t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\leq 0$$

with $=$ if $e_{jt}\left(z^{t}\right)>0$ for $j=0,1,...,J-1$ and ,

$$w_{Jt}\left(X,z^{t}\right)-\left(\theta+\hat{\nu}_{Jt}\left(z^{t}\right)\right)+\beta\sum_{z_{t+1}}\left(\theta+\hat{v}_{J\ t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\leq 0$$

with $=$ if $e_{Jt}\left(z^{t}\right)>0$.

The slackness conditions are: if $e_{jt}\left(z^{t}\right)<e_{j-1t-1}\left(z^{t-1}\right)$ then $\hat{\nu}_{jt}\left(z^{t}\right)=0$ for $j=0,...,J-1$ and if $e_{Jt}\left(z^{t}\right)<e_{J-1t-1}\left(z^{t-1}\right)+e_{J-1t-1}\left(z^{t-1}\right)$ then $\hat{\nu}_{Jt}\left(z^{t}\right)=0$.

To compare a competitive equilibrium with the planning problems it helps define a sequential island planning problem. In this problem the planner maximizes the expected discounted value of net output (23) subject to the feasibility constraints (24) and (25). This is the sequential version of the recursive island planning problem. Let $V_{0}\left(X,z_{0}\right)$ be the value attained by this planning problem.

Let $\beta^{t}\xi_{t}\left(z^{t}\right)Q\left(z^{t}|z_{0}\right)$ be the multiplier of the constraint (24) and $\beta^{t}\nu_{jt}\left(z^{t}\right)Q\left(z^{t}|z_{0}\right)$ the multiplier of the constraints (25). The first order conditions of the island sequential planning problem are equivalent to:

$$\theta+\nu_{jt}\left(z^{t}\right)=\max\left\{\theta,\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{j+1\ t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\right\}$$

with $\nu_{jt}\left(z^{t}\right)=0$ if $E_{jt}\left(z^{t}\right)<E_{j-1t-1}\left(z^{t-1}\right)$ and

$$\theta+\nu_{jt}\left(z^{t}\right)=\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{j+1\ t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)$$

if $E_{jt}\left(z^{t}\right)>0$ for $j=0,..,J-2$. For $j=J-1$ we have

$$\begin{aligned}&\theta+\nu_{J-1t}\left(z^{t}\right)\\=&\max\left\{\theta,\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t},z_{t+1}\right)+\xi_{t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\right\}\end{aligned}$$

with $v_{J-1t}\left(z^{t}\right)=0$ if $E_{J-1t}\left(z^{t}\right)<E_{J-1t-1}\left(z^{t-1}\right)$ and

$$\begin{aligned}&\theta+\nu_{J-1t}\left(z^{t}\right)\\=&\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t},z_{t+1}\right)+\xi_{t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\end{aligned}$$

if $E_{J-1t}\left(z^{t}\right)>0$. For $j=J$ we have

$$\begin{aligned}&\theta+\nu_{Jt}\left(z^{t}\right)+\xi_{t}\left(z^{t}\right)\\=&\max\left\{\theta,\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t},z_{t+1}\right)+\xi_{t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\right\}\end{aligned}$$

with $v_{Jt}\left(z^{t}\right)=0$ if $E_{Jt}\left(z^{t}\right)<E_{Jt-1}\left(z^{t-1}\right)+E_{J-1t-1}\left(z^{t-1}\right)$ and

$$\begin{aligned}&\theta+\nu_{Jt}\left(z^{t}\right)+\xi_{t}\left(z^{t}\right)\\=&\ f\left(\sum E_{jt}\left(z^{t}\right),z_{t}\right)+\beta\sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t},z_{t+1}\right)+\xi_{t+1}\left(z^{t},z_{t+1}\right)\right)Q\left(z_{t+1}|z_{t}\right)\end{aligned}$$

if $E_{Jt}\left(z^{t}\right)>0$.

To see why this is the case, write the Lagrangian for the planning problem is:

$$V_{0}\left(X,z_{0}\right)=\max_{\{E\}}\sum_{t=0}\beta^{t}\sum_{z^{t}}Q\left(z^{t}|z_{0}\right)\times$$

$$
\begin{aligned}
&\Big\{F\left(\sum E_{jt}\left(z^{t}\right), z_{t}\right)-\tau S_{t}\left(z^{t}\right) \\
&+\theta\left(\sum_{j=0}^{J-1}\left[E_{j-1 t-1}\left(z^{t-1}\right)-E_{jt}\left(z^{t}\right)\right]+\left[E_{J\ t-1}\left(z^{t-1}\right)+E_{J-1 t-1}\left(z^{t-1}\right)-E_{Jt}\left(z^{t}\right)\right]\right) \\
&+\nu_{0t}\left(z^{t}\right)\left[U-E_{0t}\left(z^{t}\right)\right]+\sum_{j=1}^{J-1} \nu_{jt}\left(z^{t}\right)\left[E_{j-1}\left(z^{t-1}\right)-E_{jt}\left(z^{t}\right)\right] \\
&+\nu_{Jt}\left(z^{t}\right)\left[E_{J-1 t-1}\left(z^{t-1}\right)+E_{Jt-1}\left(z^{t-1}\right)-E_{Jt}\left(z^{t}\right)\right] \\
&\xi_{t}\left(z^{t}\right)\left[-E_{Jt}\left(z^{t}\right)+E_{Jt-1}\left(z^{t-1}\right)+E_{J-1 t-1}\left(z^{t-1}\right)-S_{t}\left(z^{t}\right)+H_{t}\left(z^{t}\right)\right]\Big\}
\end{aligned}
$$

The f.o.c. are:

$$
f\left(\sum E_{jt}\left(z^{t}\right), z_{t}\right)-\theta-\nu_{jt}\left(z^{t}\right)+\beta \sum_{z_{t+1}}\left(\theta+v_{j+1\ t+1}\left(z^{t}, z_{t+1}\right)\right) Q\left(z_{t+1} | z_{t}\right) \leq 0
$$

with equality if $E_{jt}\left(z^{t}\right)>0$.

$$
\begin{gathered}
f\left(\sum E_{jt}\left(z^{t}\right), z_{t}\right)-\theta-\nu_{J-1t}\left(z^{t}\right)+ \\
\beta \sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t}, z_{t+1}\right)\right) Q\left(z_{t+1} | z_{t}\right)+\beta \sum_{z_{t+1}} \xi_{t+1}\left(z^{t}, z_{t+1}\right) Q\left(z_{t+1} | z_{t}\right) \leq 0
\end{gathered}
$$

with equality if $E_{J-1t}\left(z^{t}\right)>0$

$$
\begin{gathered}
f\left(\sum E_{jt}\left(z^{t}\right), z_{t}\right)-\theta-\xi_{t}\left(z^{t}\right)-\nu_{Jt}\left(z^{t}\right)+ \\
\beta \sum_{z_{t+1}}\left(\theta+v_{J\ t+1}\left(z^{t}, z_{t+1}\right)\right) Q\left(z_{t+1} | z_{t}\right)+\beta \sum_{z_{t+1}} \xi_{t+1}\left(z^{t}, z_{t+1}\right) Q\left(z_{t+1} | z_{t}\right) \leq 0
\end{gathered}
$$

with equality if $E_{Jt}\left(z^{t}\right)>0$.

The first order condition for $H_{t}\left(z^{t}\right)$ is

$$
\xi_{t}\left(z^{t}\right) \leq 0
$$

with equality if $H_{t}\left(z^{t}\right)=0$. The first order condition for $S_{t}\left(z^{t}\right)$ is

$$
-\tau-\xi_{t}\left(z^{t}\right) \leq 0
$$

with equality if $S_{t}\left(z^{t}\right)>0$.

(II) We now show i), the 1st welfare theorem, and iii). We start with an island RCE $\{w, W, B, G\}$. Pick an arbitrary state $(T, z)=\left(X, z_{0}\right)$ in the support of μ. We construct the sequential CE with $\left(X, z_{0}\right)$ as initial condition as follows. Let wages be:

$$
w_{jt}\left(X, z^{t}\right)=w_{j}\left(D^{t-1}\left(X, z^{t-1}\right), z_{t}\right).
$$

and let multipliers and employment be

$$
\begin{aligned}
\theta+\hat{v}_{jt}\left(X, z^{t}\right) & = W_{j}\left(D^{t-1}\left(X, z^{t-1}\right), z_{t}\right) \\
e_{jt}\left(X, z^{t}\right) & = G_{j}\left(D^{t-1}\left(X, z^{t-1}\right), z_{t}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
D^{t}\left(X, z^{t}\right) &= G\left(T, z_{t}\right) \quad \text{for} \\
T &= \left(U, D_{0}^{t-1}\left(X, z^{t-1}\right), \ldots, D_{J-2}^{t-1}\left(X, z^{t-1}\right), D_{J-1}^{t-1}\left(X, z^{t-1}\right)+D_{J}^{t-1}\left(X, z^{t-1}\right)\right) \text{ and} \\
D^{-1}\left(X, z_{0}\right) &= X
\end{aligned}
\tag{27}
$$

It is immediate to verify that $\{e, \hat{\nu}\}$ solves the f.o.c. of the household problem in a sequential island equilibrium, and hence it solves the household problem. For future reference we define

$$
W_{j0}\left(X, z_{0}\right)=\hat{\nu}_{j0}\left(z_{0}, X\right)+\theta.
$$

Define the Lagrange multiplier and employment for the firms problem as:

$$\begin{aligned} \hat{\xi}_t\left(X, z^t\right) &= B_p\left(D_J^{t-1}\left(X, z^{t-1}\right), D^{t-1}\left(X, z^{t-1}\right), z_t\right) \\ g_{jt}\left(X, z^t\right) &= G_j\left(D^{t-1}\left(X, z^{t-1}\right), z_t\right) \end{aligned}$$

It is immediate to verify that $\left\{g, \hat{\xi}\right\}$ solves the firms order conditions of the firm's problem in an island sequential CE, and hence it solves the firm's problem. Let B_0 be the value of the firm in the sequential island CE. For future reference, from the envelope theorem, we have

$$\partial B_0\left(x_J, X, z_0\right) / \partial x_J = \hat{\xi}_0\left(X, z_0\right)$$

evaluated at $x_J = X_J$.

By the 1st welfare thm. applied to the sequential island economy, $\{e\} = \{g\}$ is a P.O. allocation, and hence solves the sequential island planning problem. By inspection, the Lagrange multipliers $\{\xi_t, \nu_{jt}\}$ that satisfy the first order conditions of the sequential planning problem are identical to the Lagrange multipliers for the firm's problem $\left\{\hat{\xi}_t\right\}$ and to the Lagrange multipliers $\{\hat{\nu}_{jt}\}$ of the households problem in the sequential CE. From these first order conditions:

$$W_{j0}^*\left(X, z_0\right) = \hat{\nu}_{j0}\left(z_0, X\right) + \theta = \nu_{j0}\left(z_0, X\right) + \theta = \partial V_0\left(X, z_0\right) / \partial X_j$$

for $j = 0, ..., J-1$ and

$$\begin{aligned} W_{J0}\left(X, z_0\right) + \partial B_0\left(x_J, X, z_0\right) / \partial x_J &= \hat{\nu}_{J0}\left(X, z_0\right) + \theta + \hat{\xi}_0\left(X, z_0\right) \\ &= \nu_{J0}\left(X, z_0\right) + \theta + \xi_0\left(X, z_0\right) = \partial V_0\left(X, z_0\right) / \partial X_J \end{aligned}$$

evaluated at $x_J = X_J$. The allocation described by G is, by hypothesis, recursive, so it solves the recursive island planning problem with initial condition X, z_0. Repeating this argument for each initial condition (X, z_0) we show that

$$\begin{aligned} V_0\left(T, z\right) &= V\left(T, z\right), \\ B_0\left(T_J, T, z\right) &= B\left(T_J, T, z\right), \\ W_{j0}\left(T, z\right) &= W_j\left(T, z\right) \end{aligned}$$

for all $(T, z.)$ Hence we have shown the first welfare theorem for the recursive representation of the island problem, and that (6), condition iii) of the theorem, holds.

(III). We now show ii), the 2nd welfare theorem, condition iii) of Theorem 3 and condition (b) of Proposition 4. We start with a solution of the recursive planning problem, and with $\nu\left(T, z\right)$ and $\xi\left(T, z\right)$ which, by the envelope theorem satisfy

$$\frac{\partial V\left(T, z\right)}{\partial T_j} = \theta + \nu_j\left(T, z\right).$$

for $j = 0, ..., J-1$ and

$$\frac{\partial V\left(T, z\right)}{\partial T_J} = \theta + \nu_J\left(T, z\right) + \xi\left(T, z\right)$$

If if were the case that there are more than one pair ν_J, ξ for a given T, z, utilize a selection that only depends on (T, z). >From the principle of optimality, the solution of the recursive island problem V is the same as the value function for the sequential island problem V_0, so that $V\left(T, z\right) = V_0\left(T, z\right)$.

Choose any initial state X, z_0 to be used as initial condition to the sequential island problem. Define

$$\begin{aligned} \nu_{jt}\left(X, z^t\right) &= \nu_j\left(D^{t-1}\left(X, z^{t-1}\right), z_t\right) \\ \xi_t\left(X, z^t\right) &= \xi\left(D^{t-1}\left(X, z^{t-1}\right), z_t\right) \\ E_{jt}\left(X, z^t\right) &= G\left(D^{t-1}\left(X, z^{t-1}\right), z_t\right) \end{aligned}$$

where D^{t-1} is defined as in (27) using the optimal decision rule from the recursive planning problem. By comparing the first order conditions of the recursive island planing problem with the first order conditions of the sequential island planning problem, it can be seen that $\{E_t, \nu_{jt}, \xi_t\}$ so defined solve the f.o.c. of the sequence island's planning problem. Next define wages as follows:

$$w_{jt}\left(X, z^t\right) = f\left(\sum_{j=0}^{J} E_{jt}\left(z^t, X\right), z_t\right) \tag{28}$$

for $j = 0, 1, 2, ..., J-2$, for $j = J-1$ let

$$w_{J-1t}\left(X, z^t\right) = f\left(\sum_{j=0}^{J} E_{jt}\left(X, z^t\right), z_t\right) + \beta \sum_{z^{t+1}} \xi_{t+1}\left(X, z^t, z_{t+1}\right) Q\left(z_{t+1}|z_t\right) \tag{29}$$

Finally, for $j = J$

$$w_{Jt}\left(X, z^t\right) = f\left(\sum_{j=0}^{J} E_{jt}\left(X, z^t\right), z_t\right) - \xi_t\left(X, z^t\right) + \beta \sum_{z^{t+1}} \xi_{t+1}\left(X, z^t, z_{t+1}\right) Q\left(z_{t+1}|z_t\right) \tag{30}$$

The function $B_0\left(x_J, X, z_0\right)$ is defined as the solution of the firm problem for wages w_{jt} in the sequential island equilibrium.Given wages w_{jt}, the functions W_{jt} are defined as:

$$W_{jt}\left(X, z^t\right) = \nu_{jt}\left(X, z^t\right) + \theta \tag{31}$$

for $j = 0, ..., J$.

Define the candidate multipliers for the sequential firm problem as $\hat{\xi}_t = \xi_t$. Given wages w_{jt}, and multipliers $\hat{\xi}_t$, it is easy to verify that the allocation $g_{jt} = E_{jt}$, and its implied $\{s_t, h_t\}$ solve the first order conditions of the firms in the island sequential economy. To verify this, one uses the first order conditions for the island planner problem in the island sequential economy. >From the envelope condition it is immediate that

$$\partial B_0\left(x_J, X, z_0\right) / \partial x_J = \hat{\xi}_0\left(X, z_0\right)$$

where $x_J = X_J$.

Define the candidate multipliers for the sequential household problem $\hat{v}_{jt} = v_{jt}$.Given wages w_{jt} and multipliers $\hat{v}_{jt}$ it is easy to verify that the allocation $e_{jt} = E_{jt}$ solve the first order conditions of the household problem in the island sequential economy. To verify this, one uses the first order conditions for the island planner problem in the island sequential economy.

Thus we have established that the sequential allocation constructed out of the solution of the recursive island planning problem from an initial state X, z_0 can be decentralized as a sequential island competitive equilibrium. Finally, we define the elements of the recursive competitive island equilibrium as follows:

$$\begin{aligned} w_j\left(X, z_0\right) &= w_{j0}\left(X, z_0\right), \\ W_j\left(X, z_0\right) &= W_{j0}\left(X, z_0\right), \\ B\left(X_J, X, z_0\right) &= B_0\left(X_J, X, z_0\right) \end{aligned}$$

By repeating this construction for all (X, z_0) in the support of μ, we construct the functions w, W and B. These functions constitute a RCE since they are constructed from the sequential island competitive equilibrium.

>From the previous arguments we have:

$$\begin{aligned} \frac{\partial V\left(T, z\right)}{\partial T_j} &= \theta + \nu_j\left(T, z\right) = \theta + v_{j0}\left(z, T\right) \\ &= \theta + \hat{\nu}_{j0}\left(z, T\right) = W_{j0}\left(T, z\right). \end{aligned} \tag{32}$$

for $j = 0, ..., J-1$ and

$$\begin{aligned} \frac{\partial V\left(T, z\right)}{\partial T_J} &= \theta + \nu_J\left(T, z\right) + \xi\left(T, z\right) = \theta + \nu_{J0}\left(, z\right) + \xi_0\left(z, T\right) \\ &= \theta + \hat{\nu}_{J0}\left(z, T\right) + \hat{\xi}_0\left(z, T\right) = W_{J0}\left(T, z\right) + \frac{\partial}{\partial x_J} B_0\left(T_J, T, z\right), \end{aligned} \tag{33}$$

and thus condition iii) is satisfied.

(IV) We establish conditions (b) of Proposition 4. Since in (II) and (III) we have shown the 1st and 2nd welfare theorems, we can, without loss of generality, start with an efficient allocation and examine the equilibrium wages w that we constructed in (III) in equations (28), (29) and (30). The multiplier $\xi \in [-\tau, 0]$ and $\xi_t(X, z^t) = -\tau$ if $S_t(z^t) > 0$, i.e. if permanent workers are being fired. Thus, the inequalities in (b) follows from these definitions and the properties of ξ.

(V). We establish condition c) of Proposition 4. Since in (II) and (III) we have shown the 1st and 2nd welfare theorems, we can, without loss of generality, start with an efficient allocation and examine the equilibrium value function for workers W that we constructed in (III) in equation (31). Using equations (32), (33) we have that

$$\begin{aligned} \frac{\partial V(T,z)}{\partial T_j} &= W_j(T,z), \text{ for } j = 0, ..., J-1 \\ \frac{\partial V(T,z)}{\partial T_J} &= W_J(T,z) + \xi(T,z) \ . \end{aligned}$$

In Proposition (6) we have shown that

$$V_{T_1} \geq V_{T_2} \geq \cdots \geq V_{T_{J-1}},$$

Thus

$$W_1 \geq W_2 \geq \cdots \geq W_{J-1}.$$

Finally since W are part of an equilibrium, they satisfy

$$\begin{aligned} W_{J-1}(T,z) &= w_{J-1}(T,z) + \beta E[W_J(A(T,z), z') | z] \\ W_J(T,z) &= w_J(T,z) + \beta E[W_J(A(T,z), z') | z] \end{aligned}$$

and since we have already established (c), $w_J \geq w_{J-1}$, and thus we have $W_{J-1} \leq W_J$. This finishes the proof of IV).
■

Appendix D: Definition of Auxiliary Competitive Equilibrium ("ACE").

This appendix defines the competitive equilibrium (ACE) used in the proof of Theorem 1. There are two types of firms, type I and II, and families. The are as many markets to "buy" and "sell" workers as islands of type z^t, X.

Preferences of the family.

The families own all firms of both type and consume final consumption goods. They are risk neutral, and discount at rate β.

$$\sum_t \beta^t \sum_{z^t} C_t\left(z^t\right) q_t\left(z^t\right)$$

Notice that firms do not "own" they all labor. The "labor" is allocated initially to the two types of firms.

To simplify the notation we anticipate that, given the risk neutrality of households, the price for final goods sold at date t, state z^t is $\beta^t q_t\left(z^t\right)$.

Problem of Firms type I.

There is a continuum of firms of type I in each island of type X by "buying" workers from a central location at price $\lambda_t\left(z^t, Z\right)$. They start at period $t=0$ with a profile of workers given by their type X. Workers that are "bought" in this period are given tenure $j=0$. The operate the technology F. They can sale workers to the central location they obtained a price $\theta_t\left(z^t, X\right)$. If they "sale" workers with tenure J or higher in the island, they lose τ per worker.

The sequence problem for the firms in the islands who “buys” workers at price λ_t and sell them at price θ_t. He also pays the separation cost τ.

For each $(X|z_0)$ they maximize:

$$\sum_{t=0} \beta^t \sum_{z^t}\left\{F\left(\sum_{j=1}^{J} E_{j,t}\left(z^t, X\right), z_t\right)-T_{0,t}\left(z^t, X\right) \lambda_t\left(z^t, X\right)\right\} q_t\left(z^t\right) \tag{34}$$

$$+\sum_{t=0} \beta^t \sum_{z^t}\left\{\sum_{j=0}^{J}\left[T_{j,t}\left(z^t, X\right)-E_{j,t}\left(z^t, X\right)\right] \theta_t\left(z^t, X\right)-\left[T_{J,t}\left(z^t, X\right)-E_{J,t}\left(z^t, X\right)\right] \tau\right\} q_t\left(z^t\right)$$

by choice of $\left\{E_{jt}, T_{jt}\right\}_{t \geq 0}$ subject to to the technological constraints in hiring and firing:

$$E_{j,t}\left(z^t, X\right) \leq T_{j,t}\left(z^t, X\right) \text{ for } j=0,1,..,J$$

$$T_{j,t}\left(z^t, X\right)=E_{j-1,t-1}\left(z^{t-1}, X\right) \text{ for } j=1,2,...,J-1$$

$$T_{J,t}\left(z^t, X\right)=E_{J-1,t-1}\left(z^{t-1}, X\right)+E_{J,t-1}\left(z^{t-1}, X\right)$$

given initial conditions

$$T_{j,0}\left(z^0, X\right)=X_j \text{ for } j=1,2,...,J.$$

Problem for firms type II.

The sequence problem for the firms that produce home goods and reallocation of workers. They sell workers to each islands, subject to the undirected search technology, and buy them back workers from islands. The firms also operate the home production technology. "Purchases" are denoted by $S_{jt}\left(z^t, X\right)$ with price $\theta_t\left(z^t, X\right)$ and "sales" are denoted by $Y_t\left(z^t, X\right)$ at the price $\lambda_t\left(z^t, X\right)$.

Firms type II maximize:

$$\sum_t \beta^t \omega L_t+\sum_t \beta^t \sum_{z^t} \sum_X Y_t\left(z^t, X\right) \lambda_t\left(z^t, X\right) \eta\left(X|z_0\right) q_t\left(z^t\right) \tag{35}$$

$$-\sum_t \beta^t \sum_{z^t} \sum_X\left[\sum_{j=0}^{J} S_{j,t}\left(z^t, X\right)\right] \theta_t\left(z^t, X\right) \eta\left(X|z_0\right) q_t\left(z^t\right)$$

by choice of $\left\{Y_t, S_{jt}, U_t\right\}_{t \geq 0}$ subject to the undirected search technology, so that they cannot sell different quantities to different islands, which is written as

$$U_{t-1}=Y_t\left(z^t, X\right) \text{ for all } t, z^t, X.$$

and the flow constraint stating that workers "bought" can be allocated to either increase the stock producing at home or to search:

$$U_t+L_t-L_{t-1} \leq \sum_x \sum_{z^t} \sum_{j=0}^{J} S_{jt}\left(z^t, X\right) \eta\left(X|z_0\right) q_t\left(z^t\right) \text{ for all } t$$

and where U_{-1} and L_{-1} are given.

Market clearing:

For final goods:

$$\begin{aligned} & \tau \sum_x \sum_{z^t} \left[T_{J,t}\left(z^t, X\right) - E_{J,t}\left(z^t, X\right)\right] q_t\left(z^t\right) \eta\left(X|z_0\right) + C_t \\ = \quad & L_t \omega + \sum_x \sum_{z^t} F\left(\sum_{j=1}^{J} E_{j,t}\left(z^t, X\right), z_t\right) q_t\left(z^t, X\right) \eta\left(X|z_0\right) \end{aligned}$$

for the market of new (tenure $j = 0$) workers:

$$U_{t-1} = T_{0,t}\left(z^t, X\right) \text{ for all } t, z^t, X$$

for the market of incumbent (tenure $j > 0$) workers:

$$S_{j,t}\left(z^t, X\right) = E_{j,t}\left(z^t, X\right) - T_{j,t}\left(z^t, X\right) \text{ for all } j, t, z^t, X.$$

Appendix E: Lagrangian for the Recursive Island Planning Problem.

It is helpful to rewrite the Recursive Island Planning Problem using the Lagrange ν and ξ for the constraints:

$$V(T,z) = \max_{g\geq 0, s\geq 0, h\geq 0} \min_{\nu\geq 0, \xi\geq 0} \{ F\left(\sum g_j, z\right) - \tau s + \beta \sum_{z'} V(U, g_0, g_1, ..., g_{J-2}, g_{J-1} + g_J) Q(z'|z)$$

$$+\theta\left(\sum_{j=0}^{J-1} [T_j - g_j] + [T_J - g_J]\right)$$

$$+\nu_0 [U - g_0] + \sum_{j=1}^{J-1} \nu_j [T_j - g_j] + \nu_J [T_J - g_J]$$

$$\xi [-g_J + T_J - s + h] \}$$

It is immediate to obtain the following envelope conditions:

$$\frac{\partial V(T,z)}{\partial T_J} = \theta + \nu_J + \xi$$

for $j = 0, ..., J-1$

$$\frac{\partial V(T,z)}{\partial T_j} = \theta + \nu_j.$$

Appendix F: Binding contracts and tenure at the firm level (a formal description)

There are competitive markets in the island. At each date t and event z^t the set of commodities traded is $S\left(z^t\right)$. A commodity $s \in S(z^t)$ is a stopping time indicating the time at which a worker will be dismissed under each possible continuation sequence $z_{t+1}^{\infty} = \{z_{t+1}, z_{t+2}, ...\}$ following the history z^t. Formally, $S(z^t)$ is the set of all functions

$$s\left(z^t; z_{t+1}^{\infty}\right) : Z^{\infty} \rightarrow \{t+1, t+2, ..., \infty\}$$

satisfying that

$$\begin{aligned} s\left(z^t; z_{t+1}^{\infty}\right) &= T \Rightarrow s\left(z^t; \hat{z}_{t+1}^{\infty}\right) = T, \\ \text{for all } \hat{z}_{t+1}^{\infty} \text{ such that } &: \{z_{t+1}, z_{t+2}, ..., z_T\} = \{\hat{z}_{t+1}, \hat{z}_{t+2}, ..., \hat{z}_T\}. \end{aligned}$$

When a worker arrives for the first time to the island at date and event z^t, he is a "newly arrived worker" and can supply only one stopping time in the set $S\left(z^t\right)$. The worker cannot supply a new stopping time before the previous stopping time is actually executed, i.e. before the worker is separated from his previous employer. The first time that the worker separates he becomes an "incumbent worker" for the rest of his stay in the island. An incumbent worker at date and event z^t can also supply any one stopping time in the set $S(z^t)$ as long as he has no outstanding stopping time from a previous sale. "Newly arrived workers" and "incumbent workers" sell different commodities, though. The stopping time sold by an "incumbent worker" at date and event z^t entails a cost τ at date $s\left(z^t; z_{t+1}^{\infty}\right)$, for every possible realization z_{t+1}^{∞}. On the contrary, the stopping time sold by a "newly arrived worker" at date and event z^t entails a cost τ at date $s\left(z^t; z_{t+1}^{\infty}\right)$, only if the realization z_{t+1}^{∞} is such that $s\left(z^t; z_{t+1}^{\infty}\right) \geq t + J$.

Each stopping time, being a different commodity, has a price associated with it. We express the price of the stopping times traded at time and event z^t in terms of the final consumption good at that time and event, and denote them for each $s \in S\left(z^t\right)$ by $P^A\left(z^t, s\right)$ and $P^I\left(z^t, s\right)$ for the "newly arrived" and "incumbent" stopping times, respectively. Workers and firms take the prices $P^A\left(z^t, s\right)$ and $P^I\left(z^t, s\right)$ for all $t \geq 0,\ z^t \in Z^t$, and $s \in S\left(z^t\right)$ as given.

The problem of an "incumbent" worker at time and event z^t, if she has no outstanding stopping times at the time, is the following:

$$I\left(z^t\right) = \max\left\{\theta,\ \max_{s \in S(z^t)} \left\{P^I\left(z^t, s\right) + E\left[\beta^{s-t}\ I\left(z^s\right)\right]\right\}\right\} \tag{36}$$

where the expectation is taken with respect to all possible realizations $z_{t+1}^{\infty} = \{z_{t+1}, z_{t+2}, ...\}$, conditional on z^t. This equation states that an incumbent worker can choose to leave the island, obtaining θ, or sell the stopping time $s \in S\left(z^t\right)$ that provides the highest value. A stopping time $s \in S\left(z^t\right)$ provides $P^I\left(z^t, s\right)$ units of the consumption good during the current period and the value $I\left(z^s\right)$ of being an incumbent worker at the (random) stopping time s. Observe that, since the worker maximizes the present expected value of his earnings, equation (36) implicitly assumes linear preferences.[11]

The problem of "a newly arrived worker" at time t state z^t is given by

$$A\left(z^t\right) = \max\left\{\theta,\ \max_{s \in S(z^t)} \left\{P^A\left(z^t, s\right) + E\left[\beta^{s-t}\ I\left(z^s\right)\right]\right\}\right\}.$$

This problem is analogous to the "incumbent" worker problem, except that the "newly arrived worker faces a different price for the stopping time that she sells and becomes an "incumbent" worker at the end of the stopping time (i.e. she changes its type).

We let $N^A\left(z^t, s\right)$ be the quantity of newly arrived workers hired with contract $s \in S\left(z^t\right)$ at time and event z^t. Likewise, we let $N^I\left(z^t, s\right)$ be the quantities of incumbent workers hired with contract $s \in S\left(z^t\right)$ at time and event z^t. The firm chooses $N^A\left(z^t, s\right)$ and $N^I\left(z^t, s\right)$ for every z^t and $s \in S\left(z^t\right)$ to maximize expected discounted profits, taking as given the prices $P^A\left(z^t, s\right)$ and $P^I\left(z^t, s\right)$ and the fact that the stopping times of the different types of workers entail potentially different separation costs at termination. Without loss of generality, we assume that the firm never employed any workers previous to $t = 0$. This will has no consequence in the analysis given our focus on steady state equilibria.

The problem of the representative firm is the following:

$$\max_{N^A, N^I} \sum_{t=0} \sum_{z^t \in Z^t} \beta^t \left[F(n_t\left(z^t\right), z_t) - \sum_{s \in S(z^t)} \left(P^A\left(z^t, s\right) N^A\left(z^t, s\right) + P^I\left(z^t, s\right) N^I\left(z^t, s\right)\right) - T_t\left(z^t\right)\right] \mu_t\left(z^t\right)$$

[11] The linear preferences assumption in this "island-economy" is justified by the existence of perfect insurance markets in the original economy.

subject to:

$$n_t\left(z^t\right)=\sum_{i=0}^{t}\left\{\sum_{s\in S\left(z_0^i\right):\ s\left[z_0^i;\left(z_{i+1}^t,z_{t+1}^\infty\right)\right]\ >\ t,\text{ for every } z_{t+1}^\infty}\left[N^A\left(z^i,s\right)+N^I\left(z^i,s\right)\right]\right\} \tag{37}$$

$$T_t\left(z^t\right)=\tau\sum_{i=0}^{t-1}\left\{\sum_{s\in S\left(z_0^i\right):\ s\left[z_0^i;\left(z_{i+1}^t,z_{t+1}^\infty\right)\right]\ =\ t,\text{ for every } z_{t+1}^\infty}N^I\left(z^i,s\right)\right\}+\tau\sum_{i=0}^{t-J}\left\{\sum_{s\in S\left(z_0^i\right):\ s\left[z_0^i;\left(z_{i+1}^t,z_{t+1}^\infty\right)\right]\ =\ t,\text{ for every } z_{t+1}^\infty}N^A\left(z^i,s\right)\right\} \tag{38}$$

where z_j^i in equations (37) and (38) denotes the partial history $\{z_j, z_{j+1}, ..., z_{i-1}, z_i\}$ embodied in z^t. The firm maximizes the expected discounted value of profits, which are given by output minus the purchase of the stopping times supplied both by "new arrival" and "incumbent" workers, minus separation costs. The employment of the firm at time and event z^t, is given by equation (37). This equation says that total employment is the sum of all the workers, both "new arrivals" and "incumbents", that were hired between periods zero and t and that have been never fired along the history z^t. Equation (38) describes the separation costs at time and event z^t as the sum of two terms. The first term is the sum of all "incumbent" workers that have been hired between periods 0 and $t-1$, which have been contracted to separate at date t if event z^t took place. The second term is the sum of all "newly arrived" workers that have been hired between periods 0 and $t-J$, which have been contracted to separate at date t if event z^t took place. Observe that those "newly arrived" workers that have been hired between periods $t-J+1$ and $t-1$ and separate at date t and event z^t are not included in equation (38) because they separate during the trial period stipulated by the fixed term contracts and, thus, are not subject to separation costs.

The market clearing conditions are as follows. If $N^A\left(z^t,s\right)>0$ at some time and event z^t and some $s\in S(z^t)$, then

$$A\left(z^t\right)=P^A\left(z^t,s\right)+E\left[\beta^{s-t}\ I\left(z^s\right)\right]$$

Also,

$$\sum_{s\in S(z^t)}N^A\left(z^t,s\right)<U\ \Rightarrow A\left(z^t\right)=\theta.$$

The conditions for "incumbent" workers are similar. If $N^I\left(z^t,s\right)>0$ at some time and event z^t and some $s\in S(z^t)$, then

$$I\left(z^t\right)=P^I\left(z^t,s\right)+E\left[\beta^{s-t}\ I\left(z^s\right)\right]$$

Also,

$$\sum_{s\in S(z^t)}N^I\left(z^t,s\right)<X^I\left(z^t\right)\Rightarrow I\left(z^t\right)=\theta.$$

where $X^I\left(z^t\right)$ is the number of "incumbent" workers available for hire at the beginning of time and event z^t, which is given as follows:

$$X^I\left(z^t\right)=\sum_{i=0}^{t-1}\left\{\sum_{s\in S\left(z_0^i\right):\ s\left[z_0^i;\left(z_{i+1}^t,z_{t+1}^\infty\right)\right]\ =\ t,\text{ for every } z_{t+1}^\infty}\left[N^I\left(z^i,s\right)+N^A\left(z^i,s\right)\right]\right\} \tag{39}$$

Finally, the hiring of each type of workers cannot exceed the amount initially available:

$$\sum_{s\in S(z^t)}N^A\left(z^t,s\right)\leq U \tag{40}$$

$$\sum_{s\in S(z^t)}N^I\left(z^t,s\right)\leq X^I\left(z^t\right) \tag{41}$$

Observe that the supply of stopping time is indivisible: Workers can supply only one stopping time $s\in S(z^t)$, and only in the case that the worker has no previous stopping time outstanding. However, the linear preferences assumed, together with the convex production possibility set of the firm, guarantee that the welfare theorems hold. The competitive allocation is then obtained as the solution to the social planner's problem, which is to maximize

$$\sum_{t=0}\sum_{z^t\in Z^t}\beta^t\left[F(n_t\left(z^t\right),z_t)+\theta\left(U-\sum_{s\in S(z^t)}N^A\left(z^t,s\right)\right)+\theta\left(X^I\left(z^t\right)-\sum_{s\in S(z^t)}N^I\left(z^t,s\right)\right)-T_t\left(z^t\right)\right]\mu_t\left(z^t\right)$$

subject to equations (37), (38), (39), (40) and (41).

A few remarks are in order. Clearly, the social planner will never want to separate a "newly arrived" worker and rehire him as a an "incumbent" before the trial period for the fixed term contracts is over. The reason is that being rehired as "incumbent" makes the worker liable to separation costs, while maintaining his "newly arrived" status saves on separation costs during the trial period. Also, the social planner will never want to separate a "newly arrived" worker after the trial period is over and rehire him under an "incumbent" contract because this entails incurring the separation cost τ without any benefit. As a consequence, the planner will choose the stopping times for "newly arrived" workers in such a way that they separate only when they are to leave the island (and receive the value θ). This means that $N^I(z^t, s) = 0$ for every z^t and every $s \in S(z^t)$.

Being left with only "newly arrived" workers, the planner's problem is formally identical to the Island's Planner problem described in Section ??.[12] This has an important implication: The competitive equilibrium with long term contracts and tenure at the level of the firm described in this Appendix is equivalent to the competitive equilibrium with spot labor contracts and tenure at the level of the island that was described in the main text of the paper. Moreover, for every z^t and $s \in S(z^t)$ such that $N^A(z^t, s) > 0$ the price $P^A(z^t, s)$ must be equal to the expected discounted value of the spot wages obtained (in the equilibrium with spot labor contracts and tenure at the island level) by a worker that arrives to the island at time and event z^t, and follows an employment plan described by the stopping time s.

[12] In particular, it is identical to the problem of an Island's Planner endowed with no worker of positive tenure at $t = 0$.

Appendix G: Calibration of τ.

Heckman and Pages-Serra (2000) propose to summarize employment protection policies into a single statistic. The measure they use is the expected discounted cost at the time that a worker is hired of dismissing that worker in the future as a summary. Their index I is given by

$$I = \sum_{t=1}^{T} \beta^t \delta^{t-1} (1-\delta) \left\{ b_t + a S_t^j + (1-a) S_t^u \right\}$$

where T is the maximum tenure consider in the index, β a time discount factor, δ is the survival rate (prob. of remaining employed next period if employed during the current period), b_t is wage earning during the advance notice period for a worker of tenure t, S_t^j is the severance payment to a worker of tenure t if the dismissal is classified as "justified" (i.e. "fair" or "objective") and S_t^u is the severance payment to a worker of tenure t if the dismissal is "not justified".

Heckman and Pages-Serra use a year as a time period, and the following values: $\beta = 0.92$ (an 8 percent interest rate), $\delta = 0.88$ (a turnover rate of 12 percent, based on data for the US), a value of T of 20 years, and for Spain they advocate to use $a = 0.2$ for the period before 1997, based instead on the information on Bertola, Boeri and Cazes (2000), "Employment protection, the case of Industrialized countries: the case for new indicators", International Labor Review, 139(1):2000. Heckman and Pages-Serra compute their Job security index for Spain for the late 90s. Since we calibrate our model to the period before the broadening applicability of temporary contracts, we recompute their index for the policies in place before the 1984 reform. We use the following values:

- b_t : one month of wages for tenure 1 and 2 and 3 months for higher tenure (from Chapter 2 of OECD Labor Outlook, 1999, Table 2.2)

- a : 0.2 (since their argument applies prior to 1984)

- S_t^j : 2/3 months per year up to a maximum of 12 months (from Chapter 2 of OECD Labor Outlook, 1999, page 96)

- S_t^u : 1 1/2 months per year up to a maximum of 42 months (from Chapter 2 of OECD Labor Outlook, 1999, page 101).

We consider two cases. Case a: with these choices for $b_t, a,$ S_t^u and S_t^j, and using the values for β and δ used by Heckman and Pages-Serra, we obtain that I prior to 1984 equals to 0.42 as a fraction of annual average wages. Case 2, if instead we use $\beta = 0.96$, which is the value we use in our paper, and $\delta = 0.93$, which is closer to the one for Spain prior to 1984 according to Hopenhayn and Cabrales, we obtain a value of I prior to 1984 of 0.56 as a fraction of annual wages.

Finally, since in our benchmark case the firing taxes do not depend on the tenure of the workers, we select the value of τ that so that the value of the index above will give the value we calibrate for Spain prior to the reform. These value solves the equation:

$$I = \sum_{t=1}^{T} \beta^t \delta^{t-1} (1-\delta) \ \tau = \tau \ (1-\delta) \beta \frac{1-(\beta\delta)^T}{1-\beta\delta}$$

or

$$\tau = I \frac{1-\beta\delta}{(1-\delta) \ \beta \ \left(1-(\beta\delta)^T \right)}$$

The value of τ that corresponds to the first case is 0.74, and to the second case is 0.98 of annul wages. We think that for our purposes the choices of the second case better reflect the situation prior to 1984 and hence calibrate the model to τ equivalent to one year of average wages.

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