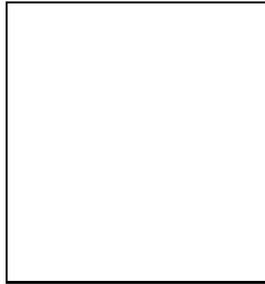


MODULAR HYBRID INFLATION WITHOUT FLAT DIRECTIONS AND THE CURVATON

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I study two-stage hybrid inflation driven by moduli fields, corresponding to flat directions of supersymmetry, lifted by supergravity corrections. The first stage corresponds to a period of either fast-roll or 'locked' inflation, induced by an oscillating inflaton. This is followed by a second stage of fast-roll inflation. Enough total e-foldings to encompass the cosmological scales are achieved. Structure in the Universe is generated due to a curvaton field.

1 Introduction

The latest CMB observations suggest that structure formation in the Universe is due to the existence of a *superhorizon* spectrum of curvature perturbations. This strongly implies that, during the early stages of its evolution, the Universe underwent a period of cosmic inflation.

According to the inflationary paradigm, inflation is realized through the domination of the Universe by the potential density of a light scalar field, which is slowly rolling down its almost flat potential. One of the reasons for using a flat potential is that one requires inflation to last long enough for the cosmological scales to exit the horizon during the period of accelerated expansion, so as to solve the horizon and flatness problems. Thus, inflation seems to require the presence of a suitable flat direction in field space. Unfortunately, this is hard to attain in supergravity because Kähler corrections generically lift the flatness of the scalar potential¹.

Still, there have been attempts to overcome this so-called η -problem of inflation. A first step toward inflation without a flat direction is fast-roll inflation², which, however, may last only for a limited number of e-foldings and, hence, it is probably not capable to explain the observations. Recently, another mechanism for inflation without a flat direction was suggested³. Rapid oscillations in a hybrid-type potential keep the field 'locked' on top of a saddle point and prevent it from rolling toward the minima. Unfortunately, oscillatory inflation is also too brief.

In this paper⁴ we point out that in a hybrid-type non-flat potential one can have two consecutive stages of inflation. Depending on the curvature of the potential, the first stage is a period of either fast-roll or oscillatory ‘locked’ inflation. This is followed by a second period of tachyonic fast-roll inflation. In total, inflation may last long enough to solve the horizon and flatness problems, without imposing stringent bounds on the curvature of the potential. Since our inflaton is not a light field it cannot be responsible for the observed spectrum of curvature perturbations. We, therefore, consider that these perturbations are due to a curvaton field⁵.

2 Fast–Roll versus Locked Inflation

Consider two moduli fields, which parameterize flat directions of supersymmetry (whose flatness is lifted by supergravity corrections) with a hybrid type of potential of the form

$$V(\Phi, \phi) = \frac{1}{2} m_\Phi^2 \Phi^2 + \frac{1}{2} \lambda \Phi^2 \phi^2 + \frac{1}{4} \alpha (\phi^2 - M^2)^2 \quad , \quad (1)$$

where Φ and ϕ above are taken to be real scalar fields with $m_\Phi \sim M_S^2/m_P \sim m_{3/2}$, $M \sim m_P$, $\alpha \sim (M_S/m_P)^4$, $\lambda \sim 1$ and $M_S \sim \sqrt{m_{3/2} m_P} \sim 10^{11} \text{ GeV}$ corresponding to gravity mediated supersymmetry breaking, where $m_{3/2} \sim \text{TeV}$. The tachyonic mass of ϕ is $m_\phi \sim \sqrt{\alpha} M \sim m_{3/2}$.

The above potential has global minima at $(\Phi, \phi) = (0, \pm M)$ and an unstable saddle point at $(\Phi, \phi) = (0, 0)$ similarly to hybrid inflation⁶. However, in contrast to regular hybrid inflation, for $|\Phi|, |\phi| \leq m_P$, the potential does not satisfy the slow-roll requirements.

Now, since the effective mass-squared of ϕ is $(m_\phi^{\text{eff}})^2 = \lambda \Phi^2 - \alpha M^2$, ϕ is driven to zero if $\Phi > \Phi_c \equiv \sqrt{\alpha/\lambda} M \sim m_{3/2}$. Suppose, that originally the system lies in the regime, where, $m_{3/2} < \Phi \leq m_P$ and $\phi \simeq 0$. With such initial conditions the effective potential for Φ becomes: $V(\Phi, \phi = 0) = \frac{1}{2} m_\Phi^2 \Phi^2 + M_S^4$. Since $\Phi < m_P$, when ϕ remains at the origin, the scalar potential is dominated by a false vacuum density $V_{\text{inf}} \simeq M_S^4$, resulting in a period of inflation. During this period, we have $V_{\text{inf}} \sim (m_P H_{\text{inf}})^2$, which means that $H_{\text{inf}} \sim m_{3/2}$. This is why there is no slow roll; because all the masses are of the order of the Hubble parameter, as expected by the action of supergravity corrections¹. And yet, there is inflation as long as ϕ remains at the origin.

During this period the Klein-Gordon equation for Φ is: $\ddot{\Phi} + 3H_{\text{inf}}\dot{\Phi} + m_\Phi^2 \Phi = 0$, which has a solution of the form $\Phi \propto e^{\omega t}$, where $\omega = -\frac{3}{2}H_{\text{inf}} \left[1 \pm \sqrt{1 - \frac{4}{9}(m_\Phi/H_{\text{inf}})^2} \right]$. Therefore, the evolution of Φ depends on whether m_Φ is larger or not from $\frac{3}{2}H_{\text{inf}}$.

2.1 Fast–Roll Inflation ($m_\Phi \leq \frac{3}{2}H_{\text{inf}}$)

In this case, there are two exponential solutions to the Klein-Gordon equation, both decreasing with time. The solution with the positive sign decreases faster and rapidly disappears. Thus,

$$\Phi = \Phi_0 \exp(-F_\Phi \Delta N), \quad \text{with} \quad F_\Phi \equiv \frac{3}{2} \left[1 - \sqrt{1 - \frac{4}{9}(m_\Phi/H_{\text{inf}})^2} \right] \quad , \quad (2)$$

where, $\Delta N \equiv H_{\text{inf}} \Delta t$. Therefore, the total number of e-foldings of fast-roll inflation is

$$N_{\text{FR}} = -\frac{1}{F_\Phi} \ln(\Phi_{\text{end}}/\Phi_0) \simeq \frac{1}{F_\Phi} \ln(m_P/m_{3/2}) \quad , \quad (3)$$

where $\Phi_0 \sim m_P$ and $\Phi_{\text{end}} = \Phi_c \sim m_{3/2}$. The larger m_Φ is the smaller is the number N_{FR} of the total e-foldings of fast-roll inflation. However, this number cannot become arbitrarily small because, if m_Φ is bigger than $\frac{3}{2}H_{\text{inf}}$, then the dynamics of Φ becomes distinctly different.

2.2 Locked Inflation ($m_\Phi > \frac{3}{2}H_{\text{inf}}$)

In this case the Klein-Gordon equation for Φ is solved by an equation of the form:

$$\Phi = \Phi_0 e^{-\frac{3}{2}\Delta N} \cos(\omega_\Phi \Delta t), \quad \text{with} \quad \omega_\Phi = H_{\text{inf}} \sqrt{(m_\Phi/H_{\text{inf}})^2 - \frac{9}{4}} \approx m_\Phi \quad . \quad (4)$$

The field is oscillating instead of rolling toward the true minimum of the potential because, provided the frequency of the oscillations is large enough, the time $(\Delta t)_s$ that the system spends on top of the saddle point is too small to allow its escape from the oscillatory trajectory. Indeed, $(\Delta t)_s \sim \Phi_c/m_\Phi \bar{\Phi} \sim \bar{\Phi}^{-1}$, where $\bar{\Phi}$ is the amplitude of the oscillations. Originally $\bar{\Phi} \sim m_P$ but the expansion of the Universe dilutes the energy of the oscillations and $\bar{\Phi}$ decreases, which means that $(\Delta t)_s$ grows. However, until $(\Delta t)_s$ becomes large enough to be comparable to m_ϕ^{-1} , ϕ has no time to roll away from the saddle. Hence, the oscillations continue until $\bar{\Phi}_{\text{end}} \sim m_{3/2} \sim \bar{\Phi}_c$, at which point ϕ departs from the origin and rolls down toward its VEV.

During the oscillations the density of the oscillating Φ is $\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}m_\Phi^2\Phi^2 \simeq \frac{1}{2}m_\Phi^2\bar{\Phi}^2$. Hence, for $\bar{\Phi} < m_P$, the overall density is dominated by V_{inf} , which remains constant as long as ϕ remains locked at the origin. Consequently, the Universe undergoes inflation when $\bar{\Phi}$ lies in the range $\bar{\Phi} \in (m_{3/2}, m_P)$. Therefore, the total number of e-foldings of locked inflation is

$$N_{\text{lock}} = \frac{2}{3} \ln(m_P/m_{3/2}) \simeq 24 \quad . \quad (5)$$

Hence, $N_{\text{FR}} > N_{\text{lock}}$, i.e. N_{lock} is the minimum number of e-foldings that the Universe inflates while ϕ remains at the origin. Thus, regardless of the curvature along the Φ -direction, a minimum number of e-foldings is guaranteed. However, locked inflation alone cannot provide the necessary number of e-foldings corresponding to the cosmological scales. Fortunately, there is a subsequent period of inflation, driven by the scalar field ϕ after it departs from the origin.

3 Tachyonic Fast-Roll Inflation

The potential for ϕ is: $V(\phi) = V_{\text{inf}} - \frac{1}{2}|(m_\phi^{\text{eff}})^2|\phi^2 + \frac{1}{4}\alpha\phi^4$. Since the roll of ϕ begins after $\bar{\Phi} < \Phi_c$, we have $|(m_\phi^{\text{eff}})^2| \sim m_\phi^2$. The Klein-Gordon satisfied by ϕ is: $\ddot{\phi} + 3H_{\text{inf}}\dot{\phi} - m_\phi^2\phi = 0$, which admits solutions of the form $\phi \propto e^{\omega_\phi t}$ with $\omega_\phi = -\frac{3}{2}H_{\text{inf}} \left[1 \pm \sqrt{1 + \frac{4}{9}(m_\phi/H_{\text{inf}})^2} \right]$. The positive sign solution corresponds to the decreasing mode which rapidly disappears. Hence,

$$\phi = \phi_0 \exp(F_\phi \Delta N), \quad \text{with} \quad F_\phi \equiv \frac{3}{2} \left[\sqrt{1 + \frac{4}{9}(m_\phi/H_{\text{inf}})^2} - 1 \right] \quad . \quad (6)$$

From the above, we see that the total number of e-foldings of tachyonic fast-roll inflation is

$$N_\phi = \frac{1}{F_\phi} \ln(\phi_{\text{end}}/\phi_0) \simeq \frac{1}{F_\phi} \ln(M/m_\phi) \quad , \quad (7)$$

where the final value of ϕ is its VEV: $M \sim m_P$, while the initial value of ϕ is determined by the tachyonic fluctuations which send it off the top of the potential, and is given by $\phi_0 = m_\phi/2\pi$.

From Eqs. (3), (5) and (7) we see that the total number of inflationary e-foldings is

$$N_{\text{tot}} = N_\Phi + N_\phi \simeq \left(\frac{1}{F_\Phi} + \frac{1}{F_\phi} \right) \ln(m_P/m_{3/2}) \quad , \quad (8)$$

where $F_\Phi \geq \frac{3}{2}$ and $N_\Phi \equiv \max\{N_{\text{FR}}, N_{\text{lock}}\}$ corresponds to the first stage of inflation. It is N_{tot} that has to be compared to the necessary e-foldings for the cosmological scales.

4 The necessary e-foldings

The inflationary period has to be sufficiently long to encompass the cosmological scales. This results in the following lower bound on the total number of e-foldings of inflation⁸:

$$N_C = 72 - \ln \left(m_P / V_{\text{inf}}^{1/4} \right) - \frac{1}{3} \ln \left(V_{\text{inf}}^{1/4} / T_{\text{reh}} \right) \quad , \quad (9)$$

where $T_{\text{reh}} \sim \sqrt{\Gamma m_P}$, is the reheat temperature, $\Gamma \simeq g^2 m_\phi$ is the decay rate for the inflaton field ϕ (corresponding to the last stage of inflation) and g is the coupling of ϕ to the decay products. If the coupling of the field to other particles is extremely weak then ϕ decays gravitationally, in which case $\Gamma \sim m_\phi^3 / m_P^2$. Thus, the effective range for g is: $m_{3/2} / m_P \leq g \leq 1$.

Using that $V_{\text{inf}} \sim M_S^4$, we find: $N_C = 54 - \frac{1}{3} \ln g$. Demanding that $N_{\text{tot}} > N_C$ provides an upper bound on m_ϕ , which is more stringent the smaller N_Φ is. Hence, the tightest bound corresponds to $N_\Phi = N_{\text{lock}}$. After a little algebra we obtain the bound

$$m_\phi / H_{\text{inf}} < \frac{3}{2} \left\{ \left[\left(\frac{108 + \ln \sqrt{g}}{\ln(m_P / m_{3/2})} - \frac{7}{4} \right)^{-1} + 1 \right]^2 - 1 \right\}^{1/2} . \quad (10)$$

Considering the range of g it is easy to check that the above bound interpolates between 2 and 3. Thus, it is possible to satisfy the cosmological observations with $m_\Phi \sim m_\phi \sim m_{3/2} \sim H_{\text{inf}}$. Hence, *the combination of locked and fast-roll inflation is capable of providing enough e-foldings to encompass the cosmological scales without the use of any flat direction*. If the mass of Φ is below $\frac{3}{2} H_{\text{inf}}$ then N_Φ is given by $N_{\text{FR}} > N_{\text{lock}}$. In this case the bound on m_ϕ is further relaxed (N_ϕ does not need to be as large). Hence, *regardless of m_Φ , the required e-foldings corresponding to the cosmological scales, can be obtained only with a mild upper bound on m_ϕ* .

5 Discussion and conclusions

Using natural values for the parameters and a generic, hybrid-type potential we showed that moduli fields, corresponding to flat directions of supersymmetry, whose flatness is lifted by supergravity corrections, can naturally generate enough e-foldings of inflation to solve the horizon and the flatness problems with only a mild upper bound on the tachyonic mass of the inflaton and without employing slow roll at all. That way inflation can escape from the famous η -problem.

Structure formation, in our model, is due to the existence of a curvaton field, which is not linked to the moduli inflatons. The curvaton *must* be a flat direction. Being unrelated to inflationary dynamics, the curvaton can be protected by a symmetry (other than supersymmetry), which may even be exact during inflation (e.g. a global U(1) for a PNGB curvaton⁹).

The tachyonic fluctuations at the phase transition which terminates the first stage of inflation can generate primordial black holes with mass comparable to the mass of the horizon volume at the time of their creation¹⁰. Hence, the earlier they form the smaller they are and the sooner they evaporate. Therefore, to protect nucleosynthesis, V_{inf} has to be bounded from below as¹¹ $V_{\text{inf}}^{1/4} \gtrsim 10^{11} \text{ GeV}$, which our model marginally satisfies. Problems may also arise from the possible formation of topological defects at the phase transition, depending on their stability. Nevertheless, both black holes and defects can be avoided provided that $\phi_0 > m_\phi$.

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