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Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach

Olivier Jeanne and Anton Korinek

Abstract

This paper analyzes prudential controls on capital flows to emerging markets from the perspective of a Pigouvian tax that addresses externalities associated with the deleveraging cycle. It presents a model in which restricting capital inflows during boom times reduces the potential outflows during busts. This mitigates the feedback effects of deleveraging episodes, when tightening financial constraints on borrowers and collapsing prices for collateral assets have mutually reinforcing effects. In our model, capital controls reduce macroeconomic volatility and increase standard measures of consumer welfare.

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Olivier Jeanne has been a senior fellow at the Peterson Institute for International Economics since 2008. He is a professor of economics at the Johns Hopkins University and has taught at UC Berkeley (1997) and at Princeton University (2005–06). From 1998 to 2008 he held various positions in the Research Department of the International Monetary Fund. He is affiliated with the National Bureau of Economic Research and is a research fellow at the Center for Economic Policy Research (London). **Anton Korinek** has been an assistant professor of economics at the University of Maryland since 2007. He received his PHD from Columbia University and his current research focuses on the ongoing financial crisis with particular emphasis on financial accelerator effects, systemic risk, policy measures to address the crisis, and regulatory measures to reduce the vulnerability of the economy to future crises.

Peterson Institute for International Economics

1750 Massachusetts Avenue, NW Washington, DC 20036-1903
Tel: (202) 328-9000 Fax: (202) 659-3225 www.piie.com

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A number of emerging market economies have recently imposed or considered imposing controls on capital inflows in the face of fierce capital flow bonanzas.[1] For example, Brazil imposed a 2 percent levy on on foreign investments in Brazilian stocks and fixed-income securities on Oct. 24, 2009 after experiencing a 36 percent appreciation of its currency earlier during the year, and Taiwan followed suit with a similar measure in November.[2] However, while policymakers around the world are clearly concerned about the effects of volatility in capital flows, the theoretic welfare case for such intervention has been less clear. The existing literature has studied how capital flow volatility can trigger feedback cycles that work through the depreciation of the real exchange rate. See e.g. Javier Bianchi (2010) and Anton Korinek (2009, 2010). This paper contributes to the debate by presenting a model based on a more general mechanism that involves asset price deflation.

1 Model

We describe a small open economy in a one-good world with three time periods $t = 0, 1, 2$. The economy is populated by a continuum of atomistic identical consumers, with a mass normalized to one. The consumer issues debt in period 0 and repays it in

[1] See *Financial Times*, "Worried nations try to cool hot money," November 19, 2009.

[2] Capital controls had also been imposed by Chile over the period of 1991-98, amid mixed reviews. See e.g. Francisco Gallego et al. (2002) for a discussion.

periods 1 and 2. In period 1, his ability to roll over debt may be affected by a collateral constraint. Period 2 represents the long term. Optimism about the future may lead to a large volume of debt inflows in period 0, making the economy vulnerable to a sudden stop/credit crunch in period 1.

The utility of the representative consumer is given by

$$u(c_0) + u(c_1) + c_2.$$

The riskless world interest rate is normalized to zero. Thus the first-best level of consumption is the same in periods 0 and 1 and is given by c^* satisfying $u'(c^*) = 1$.

Domestic income involves two components, an endowment e that is obtained in period 1 and is not pledgeable to foreign creditors, and the return y on an asset that materializes in period 2 and can be pledged as collateral on loans from foreign investors. (We assume that the asset is not acquired by foreign investors because residents have a strong comparative disadvantage in managing it). Each domestic consumer owns one unit of the asset, and the price of the asset at time t is denoted by p_t. For simplicity, we assume that the asset return y and the endowments are deterministic, except for e, which is revealed in period 1. Because of a credit constraint, low realizations of e may trigger countercyclical capital outflows or "sudden stops".[3]

Under these assumptions the budget constraints of a domestic consumer are given by

$$\begin{cases} c_0 = d_1, \\ c_1 + d_1 = e + d_2, \\ c_2 + d_2 = y, \end{cases}$$

where d_t is the debt to be repaid at the beginning of period t. The interest rate is equal to zero because there is no default in equilibrium. Each consumers faces a collateral

[3]We could also assume that y is stochastic, leading to a model in which booms and busts in capital flows are driven by the price of domestic assets (see Olivier Jeanne and Anton Korinek, 2010).

constraint of the form

$$d_2 \leq \theta_1 p_1, \tag{1}$$

where θ_1 is the quantity of domestic collateral held by the consumer at the beginning of period 1. Domestic consumers can buy or sell the asset in a perfectly competitive domestic market but in a symmetric equilibrium we must have $\theta_1 = 1$. The micro-foundation for constraint (1) is that a consumer could walk away from his debt, following which foreign creditors could seize his asset and sell it to other consumers in the domestic market.[4]

2 Laissez-faire equilibrium

We solve for the equilibrium going backwards.[5] Decentralized agents first solve for the period-1 equilibrium, taking initial liquid net worth $m_1 = e - d_1$ as given:

$$V_{lf}(m_1) = \max_{d_2} \{u(c_1) + c_2\} \quad \text{s.t.} \quad d_2 \leq p_1.$$

In equilibrium, the period-1 price of the asset is equal to its expected return times the marginal utility of period-2 consumption (1) divided by the marginal utility of period-1 consumption,

$$p_1 = \frac{y}{u'(c_1)}. \tag{2}$$

The first-order condition to the period 1 maximization problem is

$$u'(c_1) = 1 + \lambda_{lf},$$

where λ_{lf} is the shadow cost of constraint (1). If the equilibrium is unconstrained, then $c_1 = c^*$ and $p_1 = y$. The equilibrium is indeed unconstrained if and only if the value

[4] As we show in Jeanne and Korinek (2010), the constraint can also involve the end-of-period collateral and be written $d_2 \leq \phi\theta_2 p_1$ with $\phi < 1$. The only thing that matters is that the collateral constraint depend on the current price of the asset, p_1.

[5] While the main steps of the derivation are reported below, some details have been omitted and can be found in a technical appendix available at request to the authors.

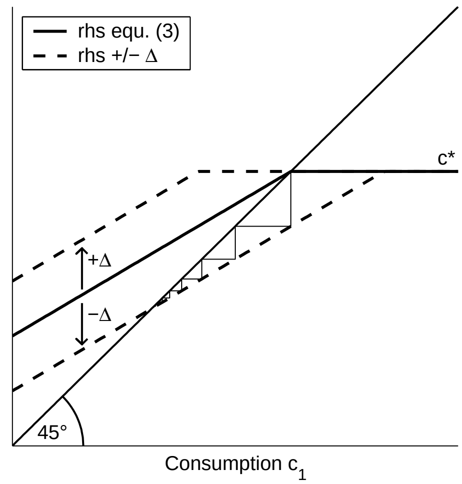

Figure 1: Dynamics of Financial Amplification

of collateral is sufficiently high to cover $d_2 = c^* - e + d_1$, that is, if net worth is higher than a threshold

$$m_1 \geq m^* \equiv c^* - y.$$

If this condition is violated, the equilibrium is constrained and is characterized by

$$c_1 = m_1 + \frac{y}{u'(c_1)}. \tag{3}$$

Both sides of equation (3) are increasing with c_1. When the constrained value of c_1 reaches c^*, the equilibrium is unconstrained. In figure 1 we illustrate the resulting equilibrium. Since both lines are upward-sloping in the constrained region, small shocks to liquid net worth can lead to large movements in consumption and asset prices. The zigzag line in the figure illustrates how the economy reacts to a small change in the endowment e by $-\Delta$, as indicated by the downward shift in the dashed line: For the original level of consumption, the borrowing constraint would be violated, hence consumption has to decline. But this reduces the asset price $p_1 = y/u'(c_1)$ and therefore tightens the borrowing constraint, leading to a downward spiral of declining consumption and dropping asset prices. This is the the general mechanism behind standard models of financial acceleration or debt deflation. In the unconstrained region, by contrast, a change in endowment by $+\Delta$ (as illustrated by the upper dashed line) does not affect consumption c_1.

We restrict our attention to the case where equation (3) is satisfied by at most one c_1 because the derivative of the r.h.s. with respect to c_1 is strictly smaller than 1,

$$y\frac{d(1/u'(c))}{dc} < 1 \quad \forall y, \forall c \leq c^*. \tag{4}$$

If this condition is not satisfied there might be multiple equilibria, in which a fall in the price of the domestic asset is self-fulfilling because it depresses domestic consumption.[6] Equation (3) has a solution c_1 if and only if the debt coming to maturity can be repaid with the available liquid net worth ($m_1 > 0$), and this solution is unique. In reduced form, we can write the period-1 level of consumption and the price of the asset as increasing functions of net worth, $c(m_1)$ and $p(m_1)$.

In the unconstrained regime, capital inflows are decreasing in e as a higher endowment shock reduces the need of consumers to borrow abroad. Conversely, if the economy is credit-constrained (in the "sudden stop regime"), capital flows become procyclical, i.e., a lower endowment shock e leads to a lower value of the collateral asset, reduced borrowing from abroad.

In period 0, decentralized agents solve the maximization problem $\max u(c_0) + E_0 V_{lf}(m_1)$. Using $V'_{lf}(m_1) = u'(c_1)$, this yields the first-order condition

$$u'(c_0) = E_0\left[u'(c_1)\right]. \tag{5}$$

The left-hand-side and right-hand-side of this equation are respectively decreasing and increasing in d_1. The equation uniquely determines the equilibrium level of d_1 under laissez-faire.

3 Social planner equilibrium

We compare the laissez-faire equilibrium with the allocations chosen by a constrained social planner who internalizes the asset pricing equation in the economy (2) and

[6] In the following we abstract from multiplicity for the sake of simplicity.

realizes that changes in aggregate consumption entail changes in asset prices, which in turn affect the borrowing constraint. In period 1, the social planner chooses the same allocation as under laissez-faire. The social planner sets d_1 in period 0 to maximize expected welfare $u(c_0) + E_0 V_{sp}(e - d_1)$ where the planner's measure of period-1 welfare is given by

$$V_{sp}(m_1) = \max_{d_2} \left\{ u(c_1) + c_2 + \lambda_{sp} \left[p(m_1) - d_2 \right] \right\},$$

where $p(m_1) = y/u'(c_1)$, and λ_{sp} denotes the shadow price on the credit constraint for the social planner. The first-order condition with respect to d_2 remains $u'(c_1) = 1 + \lambda_{sp}$. The difference with laissez-faire is that the social planner internalizes the endogeneity of the price to the aggregate level of liquid wealth, m_1, which decentralized agents take as given. By implication the social planner recognizes that the marginal value of liquid wealth in period 1 is

$$V'_{sp}(m_1) = u'(c_1) + \lambda_{sp} \cdot p'(m_1).$$

In the constrained regime, the social marginal value of liquid wealth is larger than its private marginal value because it includes the impact of aggregate wealth on the price of collateral.

The planner's first-order condition with respect to first-period debt d_1 is therefore

$$u'(c_0) = E_0 \left[u'(c_1) + \lambda_{sp} \cdot p'(m_1) \right]. \tag{6}$$

Whenever there are states in which the borrowing constraint is binding in period 1, both the shadow price λ_{sp} and the derivative $p'(m_1)$ are positive, and hence the social planner makes the economy consume less and issue less debt in period 0 than under laissez-faire (compare with (5)). This can be interpreted as a macro- (or systemic) precautionary motive: the social planner internalizes the impact of aggregate debt on the probability and severity of a sudden stop.

The optimal level of debt could be implemented in a decentralized way by a tax τ on debt inflows that is rebated in lump sum fashion. The first-order condition on d_1

under such a tax $u'(c_0) = (1+\tau)E_0(u'(c_1))$ implies that the optimal tax is given by

$$\tau = \frac{E_0\left[\lambda_{sp} \cdot p'\left(m_1\right)\right]}{E_0\left[u'\left(c_1\right)\right]}.$$

4 Illustration

We assume that utility is logarithmic ($u(c) = \log c$) and that e is uniformly distributed over the interval $[\bar{e} - \varepsilon, \bar{e} + \varepsilon]$. The logarithmic utility implies $c^* = 1$. As shown in the technical appendix, under those assumptions the model can be solved almost entirely in closed form. We assume $m^* = 0.2$ and $\bar{e} = 1.3$.

Figure 2 shows how the probability of a sudden stop (under laissez-faire and with the social planner) and the optimal tax τ vary with the maximum size of the endowment shock ε. For $\varepsilon < \bar{e} - m^* - 1 = 0.1$, the economy is never constrained under laissez-faire so that the optimal tax is equal to zero. If $\varepsilon > 0.1$, the probability of a sudden stop is positive and increasing in the downside risk—it reaches almost 20 percent for $\varepsilon = 0.3$ under laissez-faire. Meanwhile the expected consumption gap $(c^* - c_1)/c^*$ conditional on a sudden stop increases from zero to about 28 percent (not shown on the figure).

The figure illustrates the extent to which the social planner insures the economy against the risk of a sudden stop. For $\varepsilon \simeq 0.13$, the probability of sudden stop is reduced from 10 percent under laissez-faire to 6.8 percent by the social planner with a rather moderate tax of $\tau \simeq 1.3$ percent.[7] The optimal tax increases more than proportionately with the probability of a sudden stop because large sudden stops are costly in terms of domestic welfare. If $\varepsilon = 0.3$, the social planner imposes a hefty tax of 11.4 percent on debt inflows so as to reduce the probability of a sudden stop from 19 to 12 percent.

[7] The social planner reduces not only the probability but also the average size of the sudden stops. The expected consumption gap conditional on a sudden stop is lowered from 6.8 percent to 4.6 percent by the tax.

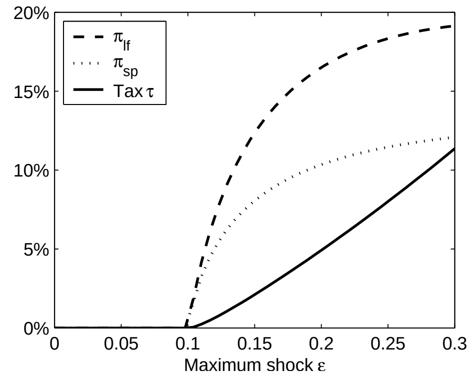

Figure 2: Probability of Sudden Stop and Optimal Tax

5 Discussion

Contingent Liabilities If other forms of liability are available, the amplification dynamics in the economy are mitigated, and so are the resulting externalities. However, in practice risk markets are often incomplete due to problems such as asymmetric information, and international debt flows are pervasive. Even if decentralized agents have access to ex ante complete insurance markets, there may be reasons why they choose to expose themselves to binding constraints and trigger inefficient financial accelerator dynamics in some states of nature. This is the case for instance if lenders are risk-averse, as discussed in more detail in Korinek (2009, 2010).

Investment If we introduce risky investment decisions into the model, we find similar distortions. Decentralized agents undervalue the social costs of losses in low output states, and therefore expose themselves excessively to risky projects that fail when aggregate output is low. By the same token, they undervalue insurance and invest insufficiently in counter-cyclical projects that would yield positive payoffs in states with low aggregate endowment shocks.

Bailouts Our analysis above assumed that the only intervention available to a social planner was the imposition of ex-ante taxes on borrowing. In the real world, another common policy instrument is bailouts that aim to loosen binding constraints by directly transferring resources to constrained agents. In our setup above, a one dollar transfer to constrained agents would relax constraints and trigger positive amplification effects that lead to a total increase in consumption by $1 + p'(m_1) = 1 + \frac{y}{m^*} = \frac{1}{m^*}$ in the log-utility example.

However, there are two important limitations to bailouts: First, a self-financing bailout, i.e. a bailout that does not involve a permanent resource transfer from outside the economy, is only possible if the planner has either accumulated resources *ex ante* or has a superior capacity *ex post* to collect repayments after the bailout.[8] Secondly, to the extent that bailouts are anticipated, they create significant moral hazard concerns, i.e., they induce decentralized agents to increase their borrowing *ex ante*, making it more likely that constraints will be binding and crises will occur.

6 Conclusion

This paper presents a simple model of collateralized international borrowing, in which the value of collateral assets endogenously depends on the state of the economy. When financial constraints are binding in such a setup, financial amplification effects (sudden stops) arise as declining collateral values, tightening financial constraints and falling consumption mutually reinforce each other.

Such amplification effects are not internalized by individual borrowers and constitute a negative externality that provides a natural rationale for the Pigouvian taxation of international borrowing. In a sample calibration we found the optimal Pigouvian tax on foreign debt to be 1.3 percent per dollar borrowed for an economy that experiences sudden stops with 10 percent probability. A fuller characterization of the externalities stemming from financial amplification effects in an infinite-horizon DSGE framework as well as the resulting optimal Pigouvian taxes are presented in Jeanne and Korinek (2010).

[8] In other words, the bailout loan will only be repaid if lending by the planner is not subject to constraint (1).

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Technical Appendix

A1. Derivation of first-order conditions

The budget constraints of a domestic consumer are given by

$$\begin{cases} c_0 = d_1 + (1-\theta_1)p_0, \\ c_1 + d_1 = e + d_2 + (\theta_1 - \theta_2)p_1, \\ c_2 + d_2 = \theta_2 y, \end{cases}$$

where θ_t is the quantity of the domestic collateral asset held by the consumer at the beginning of period t and d_t is the debt to be repaid at the beginning of period t. The budget constraints in the text assume $\theta_t = 1$, which is true in equilibrium.[9] However to derive the asset pricing equation (2) we need to take into account the fact that θ_t could be different from 1 at the individual level.

Utility is maximized under the collateral constraint (1). In period 1, thus, the representative consumer solves the problem,

$$\max_{d_2,\theta_2} u\left(e + d_2 + (\theta_1 - \theta_2)p_1 - d_1\right) + \theta_2 y - d_2 + \lambda_{lf}(\theta_1 p_1 - d_2),$$

where λ_{lf} is the shadow cost of the constraint. The first-order condition for θ_2 is $u'(c_1)p_1 = y$, which gives equation (2). The first-order condition for d_2 is $u'(c_1) = 1 + \lambda_{lf}$.

A2. Numerical illustration: the log-utility-uniform-distribution case

If $u(c) = \log c$, then $c^* = 1$. Condition (4), which ensures equilibrium uniqueness, is satisfied iff $y < 1$.

We assume e is uniformly distributed in $[\bar{e} - \varepsilon, \bar{e} + \varepsilon]$. Equation (3) has a solution $c_1 \geq 0$ if and only if $m_1 = e - d_1 \geq 0$. Since this inequality must be satisfied for any realization of e we must have $d_1 < \min e = \bar{e} - \varepsilon$. This is true provided that

$$\varepsilon < \bar{e} - d_1.$$

The equilibrium level of consumption is the min of $c^* = 1$ and the constrained level of consumption given by equation (3), $c_1 = m_1 + yc_1$. This implies

$$c_1 = \min\left(1, \frac{m_1}{m^*}\right),$$

where $m^* \equiv 1 - y > 0$.

[9] In our model consumers are in fact indifferent between holding the collateral asset or bonds between periods 1 and 2 so long as they are unconstrained in period 1. As a result, their portfolio composition may be indeterminate. For simplicity and without loss of generality, we limit our focus on the symmetric equilibrium where the asset and bond holdings of all agents are identical.

Equation (2) implies that the price of collateral is given by

$$p_1 = yc_1 = y \min\left(1, \frac{m_1}{m^*}\right).$$

The calibration will be characterized in terms of three parameters: m^*, $\bar{e}$ and ε. First, let us derive a condition that is necessary and sufficient for the economy to be constrained with some probability. If the economy is never constrained then $c_1 = 1$ and condition (5) implies $c_0 = 1$ so that $d_1 = c_0 = 1$. The economy is then indeed unconstrained in period 1 iff $d_2 = c^* + d_1 - e = 2 - e$ is smaller than y for all possible realizations of e, that is if

$$\bar{e} > 1 + m^* + \varepsilon.$$

Conversely, if $\bar{e} < 1 + m^* + \varepsilon$ there is a nonzero probability that the credit constraint binds in period 1. The constraint is binding in period 1 if and only if $m_1 < m^*$, that is

$$e < m^* + d_1.$$

We will consider calibrations such that the economy would not be constrained in the absence of uncertainty but may be constrained for sufficiently large negative shocks, that is

$$1 + m^* < \bar{e} < 1 + m^* + \varepsilon.$$

Let us assume $\varepsilon > \bar{e} - m^* - 1$, so that the economy is constrained in period 1 with a nonzero probability. With a uniform distribution for e, the equilibrium condition (5) can be written

$$\begin{aligned}
\frac{1}{d_1} &= E_0[u'(c_1)], \\
&= \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon}^{m^*+d_1} \frac{m^*}{e-d_1} de + \frac{1}{2\varepsilon}\int_{m^*+d_1}^{\bar{e}+\varepsilon} de, \\
&= \frac{1}{2\varepsilon}\left[m^* \log\left(\frac{m^*}{\bar{e}-\varepsilon-d_1}\right) + \bar{e} + \varepsilon - m^* - d_1\right]. \qquad (7)
\end{aligned}$$

This equation determines the level of debt under laissez-faire, d_1^{lf}. One can show that this equation determines a unique level of d_1 through the following steps. First, note that $E_0[u'(c_1)] > 1$ (since $c_1 < 1$ in the constrained state and $c_1 = 1$ in the unconstrained state). This implies that the rhs is above the lhs for $d_1 \geq 1$, and that d_1^{lf} must be strictly lower than 1. Second, note that $d_1^{lf} > \bar{e} - m^* - \varepsilon$: otherwise the economy is never credit constrained and the first-order condition above cannot be satisfied (the lhs is above the rhs, so that the consumer would increase its debt until the constraint becomes binding with some probability). Third, one can show that the rhs is strictly increasing with d_1 in the interval $[\bar{e} - m^* - \varepsilon, 1]$ by taking the first derivative

$$\frac{\partial \text{rhs}}{\partial d_1} = \frac{1}{2\varepsilon}\left[\frac{m^*}{\bar{e}-\varepsilon-d_1} - 1\right].$$

Thus it follows that the rhs and lhs are equal for one unique d_1^{lf}, which satisfies

$$\bar{e} - m^* - \varepsilon < d_1^{lf} < 1.$$

The fixed-point equation for d_1 can be solved numerically to find the laissez-faire equilibrium level d_1^{lf} given the exogenous parameters m^*, $\bar{e}$ and ε.

To derive the equations with a social planner note that

$$\lambda_{sp} = \frac{1}{c_1} - 1 = \left(\frac{m^*}{m_1} - 1\right)^+,$$

and that

$$p'(m_1) = \begin{cases} 1/m^* - 1 & \text{if } m_1 < m^*, \\ 0 & \text{if } m_1 \geq m^*. \end{cases}$$

It follows that the social planner first-order condition (equation (6)) can be written

$$\begin{aligned} \frac{1}{d_1} &= E_0[u'(c_1) + \lambda_{sp} p'(m_1)], \\ &= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{m^*+d_1} \left(\frac{1}{e - d_1} + 1 - \frac{1}{m^*}\right) de + \frac{1}{2\varepsilon} \int_{m^*+d_1}^{\bar{e}+\varepsilon} de, \\ &= 1 + \frac{1}{2\varepsilon} \left[\log\left(\frac{m^*}{\bar{e} - \varepsilon - d_1}\right) - 1 - \frac{d_1 - \bar{e} + \varepsilon}{m^*}\right], \end{aligned} \tag{8}$$

which again determines one unique level of d_1, which we denote by d_1^{sp}. One can show that d_1^{sp} satisfies

$$\bar{e} - m^* - \varepsilon < d_1^{sp} < d_1^{lf} < 1.$$

The optimal tax rate on debt inflows satisfies

$$1 + \tau = \frac{E_0[u'(c_1) + \lambda_{sp} p'(m_1)]}{E_0[u'(c_1)]} = \frac{1}{d_1^{sp} E_0[u'(c_1)]}.$$

Then using the fixed-point equation for d_1^{sp} one can compute

$$\begin{aligned} E_0[u'(c_1)] &= \frac{1}{2\varepsilon} \left[m^* \log\left(\frac{m^*}{\bar{e} - \varepsilon - d_1^{sp}}\right) + \bar{e} + \varepsilon - m^* - d_1^{sp}\right], \\ &= m^* \left(\frac{1}{d_1^{sp}} - 1\right) + 1, \end{aligned}$$

so that

$$1 + \tau = \frac{1}{m^* + (1 - m^*) d_1^{sp}}. \tag{9}$$

One would like to calibrate the model so as to obtain "reasonable values" for the probability and size of a sudden stop. We now explain how to derive the underlying parameters m^*, $\bar{e}$ and ε from assumptions about the levels of the probability of a sudden stop and of the expected consumption gap $c^* - c_1$ conditional on a sudden stop. The probability of sudden stop is given by

$$\begin{aligned} \pi &= \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon}^{m^*+d_1} de, \\ &= \frac{1}{2} - \frac{\bar{e}-m^*-d_1}{2\varepsilon}. \end{aligned} \tag{10}$$

Note that since $d_1 < 1$, we have $d_1 + m^* < \bar{e}$ so that the probability of a sudden stop must be lower that $1/2$.

The minimum level of c_1 is given by

$$\begin{aligned} \min c_1 &= \frac{\min e - d_1}{m^*} = \frac{\bar{e}-\varepsilon-d_1}{m^*} \\ &= 1 - 2\frac{\varepsilon\pi}{m^*}, \end{aligned}$$

which implies, if we denote by Δc the expected consumption gap $c^* - c_1$ conditional on a sudden stop,

$$\Delta c = \frac{\varepsilon\pi}{m^*}. \tag{11}$$

Thus from π and Δc we can derive the ratio m^*/ε,

$$\frac{m^*}{\varepsilon} = \frac{\pi}{\Delta c},$$

and a relationship between $\bar{e}$, d_1 and ε,

$$\frac{\bar{e}-d_1}{\varepsilon} = 1 + \pi\left(\frac{1}{\Delta c} - 2\right).$$

We can then compute the level of debt under laissez-faire,

$$\frac{1}{d_1^{lf}} = \frac{m^*}{2\varepsilon}\log\left(\frac{m^*/\varepsilon}{(\bar{e}-d_1)/\varepsilon - 1}\right) + \frac{1}{2} + \frac{\bar{e}-d_1}{2\varepsilon} - \frac{m^*}{2\varepsilon},$$

which, substituting out the terms in $(\bar{e}-d_1)/\varepsilon$ and m^*/ε, gives

$$d_1^{lf} = \left[1 - \pi - \frac{\pi}{2\Delta c}\log(1-2\Delta c)\right]^{-1}.$$

Given a value for $\bar{e}$, the values of ε and m^* can be derived using

$$\varepsilon = \frac{\bar{e}-d_1^{lf}}{1+\pi\left(1/\Delta c - 2\right)},$$

$$m^* = \frac{\pi}{\Delta c}\varepsilon.$$

The condition $1 + m^* < \bar{e}$ is satisfied iff

$$\bar{e} > 1 + \frac{\pi}{1 - 2\pi} \frac{1 - d_1^{lf}}{\Delta c}.$$

One can choose $\bar{e}$ arbitrarily subject to this condition.

Section 4 of the paper presents the following numerical illustration: $\overline{e} = 1.3$ and $y = 0.8$ (or $m^* = 0.2$). Thus the constraint is binding with nonzero probability if $\varepsilon > \overline{e} - m^* - 1 = 0.1$. Figure 2 was constructed as follows. For ε between 0 and 0.3 (we use an evenly spaced grid with 50 points) we compute:

• the levels of d_1^{lf} and d_1^{sp} by solving for the fixed-point equations (7) and (8);

• the probability of sudden stop under laissez-faire and the social planner, π^{lf} and π^{sp}, using equation (10);

• the expected consumption gaps under laissez-faire and the social planner using equation (11);

• the optimal tax on capital inflows τ using equation (9).

Figure 2 shows the variation with ε of π^{lf}, π^{sp} and τ.