SFB 649 Discussion Paper 2005-013

Nonparametric Productivity Analysis

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

SFB 649 ECONOMIC RISK BERLIN

12 Nonparametric Productivity Analysis

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How can we measure and compare the relative performance of production units? If input and output variables are one dimensional, then the simplest way is to compute efficiency by calculating and comparing the ratio of output and input for each production unit. This idea is inappropriate though, when multiple inputs or multiple outputs are observed. Consider a bank, for example, with three branches A, B, and C. The branches take the number of staff as the input, and measures outputs such as the number of transactions on personal and business accounts. Assume that the following statistics are observed:

- Branch A: 60000 personal transactions, 50000 business transactions, 25 people on staff,
- Branch B: 50000 personal transactions, 25000 business transactions, 15 people on staff,
- Branch C: 45000 personal transactions, 15000 business transactions, 10 people on staff.

We observe that Branch C performed best in terms of personal transactions per staff, whereas Branch A has the highest ratio of business transactions per staff. By contrast Branch B performed better than Branch A in terms of personal transactions per staff, and better than Branch C in terms of business transactions per staff. How can we compare these business units in a fair way? Moreover, can we possibly create a virtual branch that reflects the input/output mechanism and thus creates a scale for the real branches?

Productivity analysis provides a systematic approach to these problems. We review the basic concepts of productivity analysis and two popular methods

DEA and FDH, which are given in Sections 12.1 and 12.2, respectively. Sections 12.3 and 12.4 contain illustrative examples with real data.

12.1 The Basic Concepts

The activity of production units such as banks, universities, governments, administrations, and hospitals may be described and formalized by the production set:

$$\Psi = \{(x, y) \in \mathbb{R}^p_+ \times \mathbb{R}^q_+ \mid x \text{ can produce } y\}.$$

where x is a vector of inputs and y is a vector of outputs. This set is usually assumed to be *free disposable*, i.e. if for given $(x, y) \in \Psi$ all (x', y') with $x' \geq x$ and $y' \leq y$ belong to Ψ, where the inequalities between vectors are understood componentwise. When y is one-dimensional, Ψ can be characterized by a function g called the *frontier function* or the production function:

$$\Psi = \{(x, y) \in \mathbb{R}^p_+ \times \mathbb{R}_+ \mid y \leq g(x)\}.$$

Under free disposability condition the frontier function g is monotone nondecreasing in x. See Figure 12.1 for an illustration of the production set and the frontier function in the case of $p = q = 1$. The black curve represents the frontier function, and the production set is the region below the curve. Suppose the point A represent the input and output pair of a production unit. The performance of the unit can be evaluated by referring to the points B and C on the frontier. One sees that with less input x one could have produced the same output y (point B). One also sees that with the input of A one could have produced C. In the following we describe a systematic way to measure the efficiency of any production unit compared to the peers of the production set in a multi-dimensional setup.

The production set Ψ can be described by its sections. The input (requirement) set $X(y)$ is defined by:

$$X(y) = \{x \in \mathbb{R}^p_+ \mid (x, y) \in \Psi\},$$

which is the set of all input vectors $x \in \mathbb{R}^p_+$ that yield at least the output vector y. See Figure 12.2 for a graphical illustration for the case of $p = 2$. The region over the smooth curve represents $X(y)$ for a given level y. On the other hand, the output (correspondence) set $Y(x)$ is defined by:

$$Y(x) = \{y \in \mathbb{R}^q_+ \mid (x, y) \in \Psi\},$$

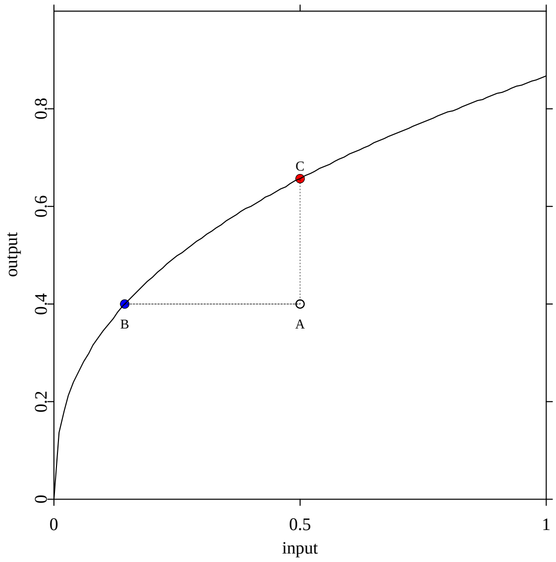

Figure 12.1: The production set and the frontier function, $p = q = 1$.

the set of all output vectors $y \in \mathbb{R}_+^q$ that is obtainable from the input vector x. Figure 12.3 illustrates $Y(x)$ for the case of $q = 2$. The region below the smooth curve is $Y(x)$ for a given input level x.

In productivity analysis one is interested in the input and output isoquants or efficient boundaries, denoted by $\partial X(y)$ and $\partial Y(x)$ respectively. They consist of the attainable boundary in a radial sense:

$$\partial X(y) = \begin{cases} \{x \mid x \in X(y),\ \theta x \notin X(y), 0 < \theta < 1\} & \text{if } y \neq 0 \\ \{0\} & \text{if } y = 0 \end{cases}$$

and

$$\partial Y(x) = \begin{cases} \{y \mid y \in Y(x),\ \lambda y \notin X(y), \lambda > 1\} & \text{if } Y(x) \neq \{0\} \\ \{0\} & \text{if } y = 0. \end{cases}$$

Given a production set Ψ with the scalar output y, the production function g can also be defined for $x \in \mathbb{R}_+^p$:

$$g(x) = \sup\{y \mid (x, y) \in \Psi\}.$$

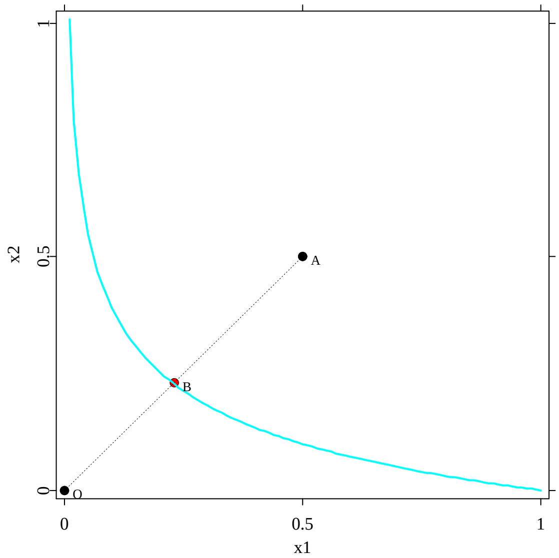

Figure 12.2: Input requirement set, $p = 2$.

It may be defined via the input set and the output set as well:

$$g(x) = \sup\{y \mid x \in X(y)\} = \sup\{y \mid y \in Y(x)\}.$$

For a given input-output point (x_0, y_0), its input efficiency is defined as

$$\theta^{\mathrm{IN}}(x_0, y_0) = \inf\{\theta \mid \theta x_0 \in X(y_0)\}.$$

The efficient level of input corresponding to the output level y_0 is then given by

$$x^{\partial}(y_0) = \theta^{\mathrm{IN}}(x_0, y_0) x_0. \tag{12.1}$$

Note that $x^{\partial}(y_0)$ is the intersection of $\partial X(y_0)$ and the ray θx_0, $\theta > 0$, see Figure 12.2. Suppose that the point A in Figure 12.2 represent the input used by a production unit. The point B is its efficient input level and the input efficient score of the unit is given by OB/OA. The output efficiency score $\theta^{\mathrm{OUT}}(x_0, y_0)$ can be defined similarly:

$$\theta^{\mathrm{OUT}}(x_0, y_0) = \sup\{\theta \mid \theta y_0 \in Y(x_0)\}. \tag{12.2}$$

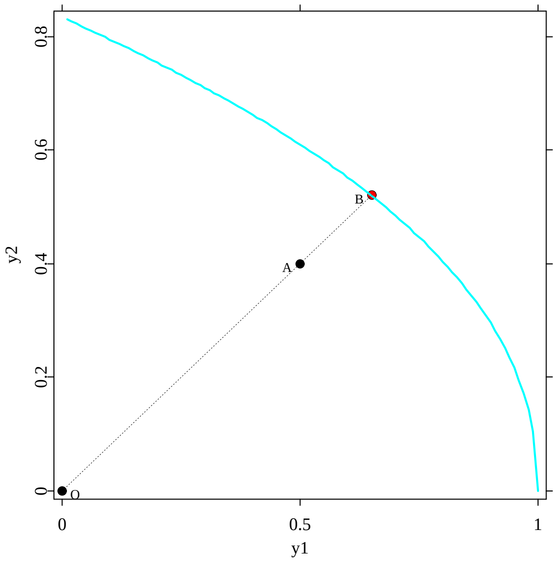

Figure 12.3: Output corresponding set, $q = 2$.

The efficient level of output corresponding to the input level x_0 is given by

$$y^{\partial}(x_0) = \theta^{\mathrm{OUT}}(x_0, y_0) y_0.$$

In Figure 12.3, let the point A be the output produced by a unit. Then the point B is the efficient output level and the output efficient score of the unit is given by OB/OA. Note that, by definition,

$$\begin{aligned} \theta^{\mathrm{IN}}(x_0, y_0) &= \inf\{\theta \mid (\theta x_0, y_0) \in \Psi\}, \\ \theta^{\mathrm{OUT}}(x_0, y_0) &= \sup\{\theta \mid (x_0, \theta y_0) \in \Psi\}. \end{aligned} \tag{12.3}$$

Returns to scale is a characteristic of the surface of the production set. The production set exhibits constant returns to scale (CRS) if, for $\alpha \geq 0$ and $P \in \Psi$, $\alpha P \in \Psi$; it exhibits non-increasing returns to scale (NIRS) if, for $0 \leq \alpha \leq 1$ and $P \in \Psi$, $\alpha P \in \Psi$; it exhibits non-decreasing returns to scale (NDRS) if, for $\alpha \geq 1$ and $P \in \Psi$, $\alpha P \in \Psi$. In particular, a convex production set exhibits non-increasing returns to scale. Note, however, that the converse is not true.

For more details on the theory and method for productivity analysis, see Shephard (1970), Färe, Grosskopf, and Lovell (1985), and Färe, Grosskopf, and Lovell (1994).

12.2 Nonparametric Hull Methods

The production set Ψ and the production function g is usually unknown, but a sample of production units or decision making units (DMU's) is available instead:

$$\mathcal{X} = \{(x_i, y_i), i = 1, \ldots, n\}.$$

The aim of productivity analysis is to estimate Ψ or g from the data $\mathcal{X}$. Here we consider only the deterministic frontier model, i.e. no noise in the observations and hence $\mathcal{X} \subset \Psi$ with probability 1. For example, when $q = 1$ the structure of $\mathcal{X}$ can be expressed as:

$$y_i = g(x_i) - u_i, \ i = 1, \ldots, n$$

or

$$y_i = g(x_i)v_i, \ i = 1, \ldots, n$$

where g is the frontier function, and $u_i \geq 0$ and $v_i \leq 1$ are the random terms for inefficiency of the observed pair (x_i, y_i) for $i = 1, \ldots, n$.

The most popular nonparametric method is Data Envelopment Analysis (DEA), which assumes that the production set is convex and free disposable. This model is an extension of Farrel (1957)'s idea and was popularized by Charnes, Cooper, and Rhodes (1978). Deprins, Simar, and Tulkens (1984), assuming only free disposability on the production set, proposed a more flexible model, say, Free Disposal Hull (FDH) model. Statistical properties of these hull methods have been studied in the literature. Park (2001), Simar and Wilson (2000) provide reviews on the statistical inference of existing nonparametric frontier models. For the nonparametric frontier models in the presence of noise, so called nonparametric stochastic frontier models, we refer to Simar (2003), Kumbhakar, Park, Simar and Tsionas (2004) and references therein.

12.2.1 Data Envelopment Analysis

The Data Envelopment Analysis (DEA) of the observed sample $\mathcal{X}$ is defined as the smallest free disposable and convex set containing $\mathcal{X}$:

$$\begin{aligned}\widehat{\Psi}_{\mathrm{DEA}} &= \{(x,y) \in \mathbb{R}^p_+ \times \mathbb{R}^q_+ \,|\, x \geq \sum_{i=1}^n \gamma_i x_i,\ y \leq \sum_{i=1}^n \gamma_i y_i, \\ &\quad \text{for some } (\gamma_1, \ldots, \gamma_n) \text{ such that} \\ &\quad \sum_{i=1}^n \gamma_i = 1, \gamma_i \geq 0\ \forall i = 1, \ldots, n\}.\end{aligned}$$

The DEA efficiency scores for a given input-output level (x_0, y_0) are obtained via (12.3):

$$\begin{aligned}\widehat{\theta}^{\mathrm{IN}}(x_0, y_0) &= \min\{\theta > 0 \,|\, (\theta x_0, y_0) \in \widehat{\Psi}_{\mathrm{DEA}}\}, \\ \widehat{\theta}^{\mathrm{OUT}}(x_0, y_0) &= \max\{\theta > 0 \,|\, (x_0, \theta y_0) \in \widehat{\Psi}_{\mathrm{DEA}}\}.\end{aligned}$$

The DEA efficient levels for a given level (x_0, y_0) are given by (12.1) and (12.2) as:

$$\widehat{x^\partial}(y_0) = \widehat{\theta}^{\mathrm{IN}}(x_0, y_0) x_0; \quad \widehat{y^\partial}(x_0) = \widehat{\theta}^{\mathrm{OUT}}(x_0, y_0) y_0.$$

Figure 12.4 depicts 50 simulated production units and the frontier built by DEA efficient input levels. The simulated model is as follows:

$$x_i \sim \mathrm{Uniform}[0,1],\ y_i = g(x_i)e^{-z_i},\ g(x) = 1 + \sqrt{x},\ z_i \sim \mathrm{Exp}(3),$$

for $i = 1, \ldots, 50$, where $\mathrm{Exp}(\nu)$ denotes the exponential distribution with mean $1/\nu$. Note that $\mathrm{E}[-z_i] = 0.75$. The scenario with an exponential distribution for the logarithm of inefficiency term and 0.75 as an average of inefficiency are reasonable in the productivity analysis literature (Gijbels, Mammen, Park, and Simar, 1999).

The DEA estimate is always downward biased in the sense that $\widehat{\Psi}_{\mathrm{DEA}} \subset \Psi$. So the asymptotic analysis quantifying the discrepancy between the true frontier and the DEA estimate would be appreciated. The consistency and the convergence rate of DEA efficiency scores with multidimensional inputs and outputs were established analytically by Kneip, Park, and Simar (1998). For $p = 1$ and $q = 1$, Gijbels, Mammen, Park, and Simar (1999) obtained its limit distribution depending on the curvature of the frontier and the density at the boundary. Jeong and Park (2004) and Kneip, Simar, and Wilson (2003) extended this result to higher dimensions.

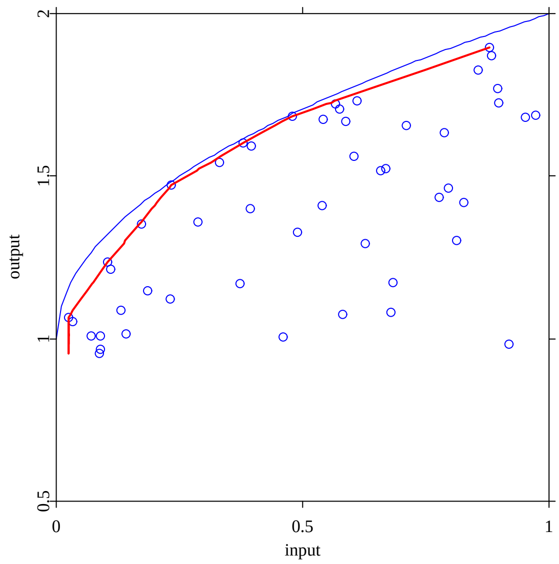

Figure 12.4: 50 simulated production units (circles), the frontier of the DEA estimate (solid line), and the true frontier function $g(x) = 1 + \sqrt{x}$ (dotted line).
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12.2.2 Free Disposal Hull

The Free Disposal Hull (FDH) of the observed sample $\mathcal{X}$ is defined as the smallest free disposable set containing $\mathcal{X}$:

$$\widehat{\Psi}_{\mathrm{FDH}} = \{(x, y) \in \mathbb{R}_+^p \times \mathbb{R}_+^q \,|\, x \geq x_i,\ y \leq y_i,\ i = 1, \ldots, n\}.$$

We can obtain the FDH estimates of efficiency scores for a given input-output level (x_0, y_0) by substituting $\widehat{\Psi}_{\mathrm{DEA}}$ with $\widehat{\Psi}_{\mathrm{FDH}}$ in the definition of DEA efficiency scores. Note that, unlike DEA estimates, their closed forms can be derived by a straightforward calculation:

$$\widehat{\theta}^{\mathrm{IN}}(x_0, y_0) = \min_{i|y \le y_i} \max_{1 \le j \le p} x_i^j \Big/ x_0^j,$$
$$\widehat{\theta}^{\mathrm{OUT}}(x_0, y_0) = \max_{i|x \ge x_i} \min_{1 \le k \le q} y_i^k \Big/ y_0^k,$$

where v^j is the jth component of a vector v. The efficient levels for a given level (x_0, y_0) are obtained by the same way as those for DEA. See Figure 12.5 for an illustration by a simulated example:

$$x_i \sim \mathrm{Uniform}[1,2], y_i = g(x_i)e^{-z_i}, g(x) = 3(x-1.5)^3+0.25x+1.125, z_i \sim \mathrm{Exp}(3),$$

for $i = 1, \ldots, 50$. Park, Simar, and Weiner (1999) showed that the limit distribution of the FDH estimator in a multivariate setup is a Weibull distribution depending on the slope of the frontier and the density at the boundary.

12.3 DEA in Practice: Insurance Agencies

In order to illustrate a practical application of DEA we consider an example from the empirical study of Scheel (1999). This concrete data analysis is about the efficiency of 63 agencies of a German insurance company, see Table 12.1. The input $X \in \mathbb{R}_+^4$ and output $Y \in \mathbb{R}_+^2$ variables were as follows:

X_1 : Number of clients of Type A,

X_2 : Number of clients of Type B,

X_3 : Number of clients of Type C,

X_4 : Potential new premiums in EURO,

Y_1 : Number of new contracts,

Y_2 : Sum of new premiums in EURO.

Clients of an insurance company are those who are currently served by the agencies of the company. They are classified into several types which reflect, for example, the insurance coverage. Agencies should sell to the clients as many contracts with as many premiums as possible. Hence the number of clients (X_1, X_2, X_3) are included as input variables, and the number of new contracts (Y_1)

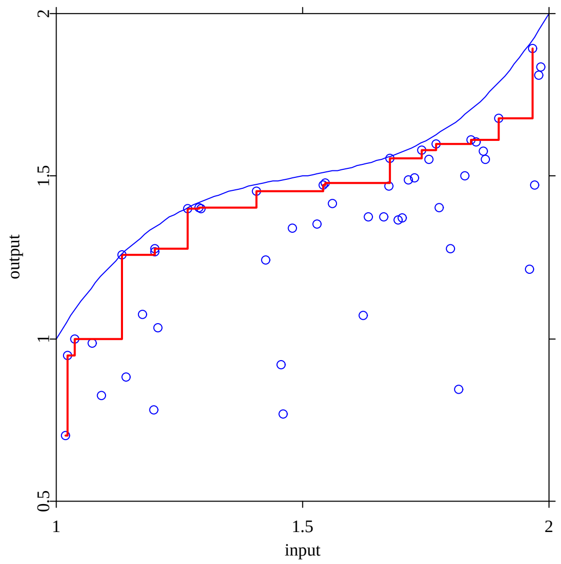

Figure 12.5: 50 simulated production units (circles) the frontier of the FDH estimate (solid line), and the true frontier function $g(x) = 3(x - 1.5)^3 + 0.25x + 1.125$ (dotted line). STFnpa02.xpl

and the sum of new premiums (Y_2) are included as output variables. The potential new premiums (X_4) is included as input variables, since it depends on the clients' current coverage.

Summary statistics for this data are given in Table 12.2. The DEA efficiency scores and the DEA efficient levels of inputs for the agencies are given in Tables 12.3 and 12.4, respectively. The input efficient score for each agency provides a gauge for evaluating its activity, and the efficient level of inputs can be interpreted as a 'goal' input. For example, agency 1 should have been able to yield its activity outputs ($Y_1 = 7$, $Y_2 = 1754$) with only 38% of its inputs, i.e., $X_1 = 53$, $X_2 = 93$, $X_3 = 4$, and $X_4 = 108960$. By contrast, agency 63, whose efficiency score is equal to 1, turned out to have used its resources 100% efficiently.

Table 12.1: Activities of 63 agencies of a German insurance company

	inputs				outputs	
Agency	X_1	X_2	X_3	X_4	Y_1	Y_2
1	138	242	10	283816.7	7	1754
2	166	124	5	156727.5	8	2413
3	152	84	3	111128.9	15	2531
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
62	83	109	2	139831.4	11	4439
63	108	257	0	299905.3	45	30545

Table 12.2: Summary statistics for 63 agencies of a German insurance company

	Minimum	Maximum	Mean	Median	Std.Error
X1	42	572	225.54	197	131.73
X2	55	481	184.44	141	110.28
X3	0	140	19.762	10	26.012
X4	73756	693820	258670	206170	160150
Y1	2	70	22.762	16	16.608
Y2	696	33075	7886.7	6038	7208

12.4 FDH in Practice: Manufacturing Industry

In order to illustrate how FDH works, the Manufacturing Industry Productivity Database from the National Bureau of Economic Research (NBER), USA is considered. This database is downloadable from the website of NBER [http://www.nber.org]. It contains annual industry-level data on output, employment, payroll, and other input costs, investment, capital stocks, and various industry-specific price indices from 1958 on hundreds of manufacturing industries (indexed by 4 digits numbers) in the United States. We selected data from the year 1996 (458 industries) with the following 4 input variables, $p = 4$, and 1 output variable, $q = 1$ (summary statistics are given in Table 12.5):

Table 12.3: DEA efficiency score of the 63 agencies

Agency	Efficiency score
1	0.38392
2	0.49063
3	0.86449
.	.
.	.
.	.
62	0.79892
63	1

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Table 12.4: DEA efficiency level of the 63 agencies

	Efficient level of inputs			
Agency	X_1	X_2	X_3	X_4
1	52.981	92.909	3.8392	108960
2	81.444	60.838	2.4531	76895
3	131.4	72.617	2.5935	96070
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
62	66.311	87.083	1.5978	111710
63	108	257	0	299910

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X_1 : Total employment,

X_2 : Total cost of material,

X_3 : Cost of electricity and fuel,

X_4 : Total real capital stock,

Y : Total value added.

Table 12.5: Summary statistics for Manufacturing Industry Productivity Database (NBER, USA)

	Minimum	Maximum	Mean	Median	Std.Error
X1	0.8	500.5	37.833	21	54.929
X2	18.5	145130	4313	1957.2	10771
X3	0.5	3807.8	139.96	49.7	362
X4	15.8	64590	2962.8	1234.7	6271.1
Y	34.1	56311	3820.2	1858.5	6392

Table 12.6 summarizes the result of the analysis of US manufacturing industries in 1996. The industry indexed by 2015 was efficient in both input and output orientation. This means that it is one of the vertices of the free disposal hull generated by the 458 observations. On the other hand, the industry 2298 performed fairly well in terms of input efficiency (0.96) but somewhat badly (0.47) in terms of output efficiency. We can obtain the efficient level of inputs (or outputs) by multiplying (or dividing) the efficiency score to each corresponding observation. For example, consider the industry 2013, which used inputs $X_1 = 88.1$, $X_2 = 14925$, $X_3 = 250$, and $X_4 = 4365.1$ to yield the output $Y = 5954.2$. Since its FDH input efficiency score was 0.64, this industry should have used the inputs $X_1 = 56.667$, $X_2 = 9600$, $X_3 = 160.8$, and $X_4 = 2807.7$ to produce the observed output $Y = 5954.2$. On the other hand, taking into account that the FDH output efficiency score was 0.70, this industry should have increased its output upto $Y = 4183.1$ with the observed level of inputs.

Table 12.6: FDH efficiency scores of 458 US industries in 1996

		Efficiency scores	
	Industry	input	output
1	2011	0.88724	0.94203
2	2013	0.79505	0.80701
3	2015	0.66933	0.62707
4	2021	1	1
.	.	.	.
.	.	.	.
.	.	.	.
75	2298	0.80078	0.7439
.	.	.	.
.	.	.	.
.	.	.	.
458	3999	0.50809	0.47585

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".