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Fixed-Prize Tournaments versus First-Price Auctions in Innovation Contests

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Fixed-Prize Tournaments versus First-Price Auctions in Innovation Contests*

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Abstract

This paper analyzes a procurement setting with two identical firms and stochastic innovations. In contrast to the previous literature, I show that a procurer who cannot charge entry fees may prefer a fixed-prize tournament to a first-price auction since holding an auction may leave higher rents to firms when the innovation technology is subject to large random factors.

Keywords: innovation contest, auction, tournament, quality

JEL classification: D44, H57, L15

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1 Introduction

A buyer who wishes to procure an innovative good or service usually cares about the quality of the innovation, which is affected by suppliers' investments in R&D. Since investments and quality are often non-contractible, procurers frequently hold contests among potential suppliers to induce investments in R&D.[1] Two popular contest mechanisms are fixed-prize tournaments and first-price auctions. In the tournament, the best innovator receives an ex ante fixed prize. In the auction, the buyer procures the innovation from the firm that offers the most favorable combination of quality and price. Both mechanisms prevent opportunistic behavior ex post: The procurer cannot lower payments to firms by understating quality, and firms do not benefit from overstating their costs.

Che and Gale [2003] show that, under a deterministic innovation technology, a first-price auction is optimal within the broad range of contest mechanisms in which only the winner receives a prize. In particular, the auction always outperforms a fixed-prize tournament. Fullerton et al. [2002] compare first-price auctions and fixed prizes in a stochastic environment. They also find that the auction generally leads to lower costs for the buyer.

In contrast, however, we often observe fixed-prize tournaments in R&D settings. A prominent historic example is the 1829 contest where Liverpool and Manchester Railway announced a prize of £500 for the best performing engine for the first passenger line between two British cities (Fullerton and McAfee [1999]). Currently, the Defense Advanced Research Projects Agency (DARPA) of the U.S. Department of Defense sponsors the "Grand Challenge 2005" to promote R&D in autonomous ground vehicle technology. A prize of $2 millon will be awarded to the team whose vehicle completes

[1] Seminal papers on innovation contests include, e.g., Taylor [1995], Fullerton et al. [1999], Fullerton and McAfee [1999].

a certain route within a specified time (DARPA [2005]). In the private sector, the InnoCentive company provides an online forum where "seeker companies" post R&D challenges in chemistry or biology for which scientists then submit solutions. The best solution is rewarded by a prespecified prize (InnoCentive [2005]).[2]

The purpose of this paper is to provide a possible answer to the question of when a buyer may prefer a fixed-prize tournament to a first-price auction. To do so, I consider a two-firm setting where firms have the same stochastic innovation technology and are liquidity constrained, so that the buyer cannot charge entry fees. Before a firm bids a price, it observes not only the quality of its own innovation but also the one of the other firm.[3] This assumption is a significant departure from Che and Gale [2003] and Fullerton et al. [2002]. It is not essential for the results, but greatly simplifies the analysis. In section 5, I briefly discuss the case in which the quality of a firms' innovation is observed only by this firm and the buyer.

In the auction, if firms' innovations differ significantly, the high-quality firm can demand a much higher price than the low-quality firm. Therefore, assuming that the buyer cannot charge entry fees, firms may earn higher rents under an auction than under a prespecified fixed prize. As a result, the buyer prefers a fixed-prize tournament to an auction when the innovation technology is subject to large random factors, so that firms are likely to realize quite different innovations.

"Randomness" is measured in terms of conditional stochastic dominance (CSD).[4] Formally, the buyer preannounces a fixed prize if the cumulative distribution function (cdf) of the quality difference is dominated by an exponential distribution. Another,

[2] For further examples of fixed-prize tournaments, see, e.g., Windham [1999], Che and Gale [2003], or Maurer and Scotchmer [2004].

[3] For example, this is the case if submitting innovations involves testing prototypes (e.g., of a military plane), and employees of both firms are present when prototypes are tested.

[4] The concept of CSD is common in the literature on first-price auctions. See, e.g., Lebrun [1998], Maskin and Riley [2000], or Arozamena and Cantillon [2004].

stronger, sufficient condition for the optimality of a fixed prize is the log-convexity of the cdf. In contrast, if the cdf is log-concave, an auction without a minimum price is optimal.

The latter point is useful to understand why my findings do not contradict Fullerton et al. [2002], who also analyze a stochastic environment in the absence of entry fees but find that the auction generally dominates. However, they consider a special class of log-concave cdfs. The main difference between my paper and Che and Gale [2003] is that, in their framework, identical firms never earn rents because their deterministic innovation technology implies a mixed-strategy equilibrium and thus complete rent dissipation.

The remainder of the paper is organized as follows. In section 2, the model is introduced. Section 3 analyzes the bidding and investment stage. In section 4, the buyer's optimization problem is solved. The last section concludes.

2 The model

A buyer holds a contest to procure an innovation from one of two ex ante identical firms. All parties are risk-neutral and the buyer cannot charge entry fees. Firm i, $i \in \{1, 2\}$, receives an innovation of quality

$$q_i = x_i + \mu_i \tag{1}$$

if it invests $c(x_i) + \bar{c}$, where $x_i, \bar{c} \geq 0$, and μ_i is a random variable. I call x_i firm i's investment strategy. I assume that $c(x_i)$ is strictly increasing, strictly convex, and twice differentiable for all $x_i > 0$. Furthermore, $c(0) = 0$, $\lim_{x_i \to +0} c'(x_i) = 0$, and

$\inf_{x_i \geq 0} c''(x_i) > 0$.[5] If a firm decides to participate in the contest, it must at least invest $\bar{c}$ to be able to submit an innovation. Thus, $\bar{c}$ captures all fixed and opportunity costs from contest participation.

The random variables μ_1 and μ_2 are identically and independently distributed. Additionally,

$$E[\max\{\mu_1, \mu_2\}] - 2\bar{c} \geq \bar{u}, \tag{2}$$

where $\bar{u} \in \mathbb{R}$ denotes the buyer's utility if she does not procure the innovation. This assumption guarantees that the buyer ex ante benefits from holding the contest.[6]

Since the difference between the qualities of firms' innovations will be crucial, I define the random variable $\eta := \mu_2 - \mu_1$ with cdf $G(\eta)$ and density $g(\eta)$. Because μ_1 and μ_2 are identically distributed, $g(\eta)$ is symmetric around zero. Let S denote the support of g. I assume that $G(\eta)$ is continuous on $\mathbb{R}$ and both $G(\eta)$ and $g(\eta)$ are differentiable on the interior of S.[7] Furthermore, let $g(\eta)$ have a local maximum at $\eta = 0$.

A firm's investment strategy and investment costs are non-observable. Both qualities q_1, q_2 are observed by the buyer and the firms. However, qualities are non-verifiable. Third parties can only verify payments to firms, whether a firm submitted an innovation, and from which firm the buyer procured the innovation.

Timing is as follows. In the first stage, the buyer specifies a fixed payment $f \geq 0$ to each firm submitting an innovation. Additionally, she commits to procuring one of the submitted innovations, where the winning firm receives at least a minimum price of $\underline{p} \geq 0$ and at most a maximum price of $\overline{p} \geq \underline{p}$. In the second stage, firms choose

[5] As will be shown in section 3, the last assumption is required to guarantee the existence of a pure strategy equilibrium in the investment stage.

[6] The buyer can always implement $x_i = 0$ if she commits to paying $\bar{c}$ to each firm that submits an innovation (assuming that the submission of an innovation is verifiable).

[7] For many distributions of μ_i (e.g., the uniform distribution), g is not differentiable at zero, which does, however, not affect the results.

their investment strategies x_1, x_2. Afterwards, random variables μ_1, μ_2 are realized, firms submit their innovations, and qualities q_1, q_2 are observed.[8] In the last stage, the bidding takes place. Firms announce prices p_1 and p_2, respectively, such that $\underline{p} \leq p_1, p_2 \leq \overline{p}$. Firm i wins if it offers the higher surplus to the buyer, i.e., if $q_i - p_i > q_j - p_j$. In this case, firm i receives $p_i + f$, and firm j receives f. If surpluses are identical, the winner is chosen by flipping a fair coin.

Observe that the mechanism nests both fixed-prize tournaments and first-price auctions. Specifically, if the buyer chooses $\underline{p} = \overline{p}$, the mechanism amounts to a fixed-prize tournament where the prize is awarded to the firm with the higher quality. In contrast, with $\underline{p} = 0$ and $\overline{p} = \infty$, the mechanism is a first-price auction without a minimum or maximum allowable price.

3 Firms' decisions

The game is solved by backwards induction. In the last stage, when bidding occurs, all parties involved know q_1 and q_2. Suppose that $q_i > q_j$. Then, firm j bids $p_j = \underline{p}$, and firm i wins the bidding by setting

$$p_i = \begin{cases} \underline{p} + (q_i - q_j) & \text{if } q_i - q_j \leq \Delta \\ \underline{p} + \Delta & \text{if } q_i - q_j > \Delta \end{cases}, \tag{3}$$

where $\Delta := \overline{p} - \underline{p}$, and $q_i - q_j$ reflects the increase in the buyer's surplus if she procures the innovation from the high-quality firm.

Thus, the firm with the higher quality wins the bidding. It receives the minimum price $\underline{p}$ plus a quality premium which is bounded above by Δ and equals $|q_1 - q_2|$ if

[8] Alternatively, all parties involved could observe non-verifiable quality signals $s_i = q_i + \epsilon_i$, where ϵ_i is some noise occurring when quality is measured.

$|q_1 - q_2| \leq \Delta$. Given investment strategies x_1 and x_2, we have that $q_1 - q_2 = x_1 - x_2 - \eta$. Consequently, the payment that firm 1 receives in the last stage of the game in addition to f is:

$$\begin{cases} 0 & \text{if} \quad x_1 - x_2 < \eta \\ \frac{1}{2}\underline{p} & \text{if} \quad x_1 - x_2 = \eta \\ \underline{p} + x_1 - x_2 - \eta & \text{if} \quad x_1 - x_2 - \Delta \leq \eta < x_1 - x_2 \\ \underline{p} + \Delta & \text{if} \quad \eta < x_1 - x_2 - \Delta \end{cases} \tag{4}$$

Analogously, firm 2 obtains:

$$\begin{cases} 0 & \text{if} \quad \eta < x_1 - x_2 \\ \frac{1}{2}\underline{p} & \text{if} \quad x_1 - x_2 = \eta \\ \underline{p} + x_2 - x_1 + \eta & \text{if} \quad x_1 - x_2 < \eta \leq x_1 - x_2 + \Delta \\ \underline{p} + \Delta & \text{if} \quad x_1 - x_2 + \Delta < \eta \end{cases} \tag{5}$$

In the investment stage, each firm chooses its investment strategy to maximize its expected payment in the auction net of investment costs. Given that firm 2 adopts investment strategy x_2, firm 1 chooses x_1 to maximize

$$\int_{x_1 - x_2 - \Delta}^{x_1 - x_2} (\underline{p} + x_1 - x_2 - \eta) g(\eta) d\eta + \int_{-\infty}^{x_1 - x_2 - \Delta} (\underline{p} + \Delta) g(\eta) d\eta - c(x_1). \tag{6}$$

Given x_1, firm 2 maximizes

$$\int_{x_1 - x_2}^{x_1 - x_2 + \Delta} (\underline{p} + x_2 - x_1 + \eta) g(\eta) d\eta + \int_{x_1 - x_2 + \Delta}^{\infty} (\underline{p} + \Delta) g(\eta) d\eta - c(x_2). \tag{7}$$

The first-order conditions for a pure-strategy Nash-equilibrium (x_1^N, x_2^N) are

$$\underline{p}g(x_1^N - x_2^N) + [G(x_1^N - x_2^N) - G(x_1^N - x_2^N - \Delta)] = c'(x_1^N), \tag{8}$$

$$\underline{p}g(x_1^N - x_2^N) - [G(x_1^N - x_2^N) - G(x_1^N - x_2^N + \Delta)] = c'(x_2^N). \tag{9}$$

For the purpose of this paper and given that firms are identical, I only consider symmetric pure-strategy Nash-equilibria $x_1^N = x_2^N =: x$. Symmetry of $g(\eta)$ implies that $G(-\Delta) = 1 - G(\Delta)$ and $G(0) = \frac{1}{2}$, so that the first-order conditions simplify to

$$\underline{p}g(0) + \left[G(\Delta) - \frac{1}{2}\right] = c'(x). \tag{10}$$

Starting from identical investment strategies, the left-hand side of (10) gives a firm's marginal benefit from increasing investments. The term $\underline{p}g(0)$ indicates the higher probability of winning the bidding and obtaining at least $\underline{p}$. The term in square brackets represents the increase in the expected quality premium that the winning firm receives in addition to $\underline{p}$.

Naturally, high values of $\underline{p}$ and Δ provide strong investment incentives. Furthermore, investments increase in $g(0)$ and, given Δ, in $G(\Delta)$. A large $g(0)$ indicates that the probability of winning responds strongly to changes in investments. A large $G(\Delta)$ reflects that the quality premium is likely to equal the difference between qualities (instead of Δ). Both implies that the outcome of the mechanism is relatively sensitive to changes in investments.

Firms' objective functions (6) and (7) are not necessarily concave, so that we need further assumptions to ensure that x as given by (10) is indeed a pure-strategy equilibrium. A sufficient condition for the strict concavity of (6) in x_1 (given an arbitrary x_2)

is that[9]

$$\sup_{\eta\in \text{int}S}\{\underline{p}g'(\eta)+g(\eta)\}<\inf_x c''(x), \tag{11}$$

i.e., random influences are sufficiently significant (g is sufficiently "flat").

Because $g(\eta)$ is symmetric around zero, this condition also guarantees strict concavity of (7) in x_2 (given an arbitrary x_1). Since condition (11) should be satisfied for all $\underline{p}$ that the buyer might choose, we need to specify an upper bound on $\underline{p}$. It can be shown that the buyer never wants to implement an x that is larger than the socially optimal investment level (given that two firms invest), denoted x^*,

$$x^* := \text{argmax}_x\ x+E[\max\{\mu_1,\mu_2\}]-2(c(x)+\bar{c}). \tag{12}$$

Since $c'(x^*)=1/2$, we obtain from (10) that $\underline{p}\leq 1/(2g(0))$. Therefore, I henceforth assume that

$$\sup_{\eta\in \text{int}S}\left\{\frac{g'(\eta)}{2g(0)}+g(\eta)\right\}<\inf_x c''(x). \tag{13}$$

That is, I restrict attention to the class of problems for which the exogenously given functions $g(\eta)$ and $c(x)$ are such that a firm's objective function is concave for all $\underline{p}$ and Δ that the buyer might choose.[10]

[9]int$S$ denotes the interior of S.

[10]Usually, this is the case if var(η) $=$ 2var(μ_i) is large enough. For example, if μ_i is uniformly distributed on $[0,u]$, we have

$$g(\eta)=\begin{cases}\frac{1}{u}+\frac{\eta}{u^2} & \text{if} \quad -u\leq\eta\leq 0\\ \frac{1}{u}-\frac{\eta}{u^2} & \text{if} \quad 0<\eta\leq u\end{cases},$$

so that (13) is equivalent to $3/(2u)<D$.

4 The buyer's problem

I now analyze the first stage in which the buyer specifies $f, \underline{p}$, and Δ. By (10), each firm chooses the efficient investment strategy x^* if the buyer holds a first-price auction without minimum and maximum price ($\underline{p} = 0$, $\overline{p} = \infty$).[11] However, as firms may earn rents, this choice of $\underline{p}$ and $\overline{p}$ will in general not be optimal from the buyer's point of view.

Instead of maximizing the buyer's expected surplus, I consider the problem of minimizing her expected costs for implementing a given investment strategy x. This greatly simplifies the analysis. Moreover, for the purpose of this paper, since the sufficient conditions for the optimality of a fixed-prize tournament will not depend on x, it is also not necessary to determine the surplus-maximizing investment strategy.

In the last stage, assuming identical investments, the price that the buyer has to pay is

$$\begin{cases} \underline{p} + |\eta| & \text{if } |\eta| \leq \Delta \\ \underline{p} + \Delta & \text{otherwise} \end{cases}. \tag{14}$$

Let $\bar{G}$ denote the cdf of $|\eta|$ and $\bar{g}$ the corresponding density function.[12] Then, the expected price before observing qualities is

$$P(\underline{p}, \Delta) := \underline{p} + \int_0^{\Delta} y\bar{g}(y)dy + (1 - \bar{G}(\Delta))\Delta. \tag{15}$$

[11] This mechanism maximizes the buyer's expected surplus in Che and Gale [2003] when firms are identical.

[12] It is easily verified that $\bar{G}(y) = 2G(y) - 1$ for all $y \geq 0$. Also note that $|\eta| = \mu_{(2)} - \mu_{(1)}$, where $\mu_{(2)}, \mu_{(1)}$ denote the first- and second-order statistic of the the sample μ_1, μ_2.

This leads to the following optimization problem for the buyer:

$$C(x) := \min_{f,\underline{p},\Delta} \; P(\underline{p},\Delta) + 2f \tag{16}$$

$$\text{s.t.} \quad \underline{p}g(0) + \left[G(\Delta) - \frac{1}{2}\right] - c'(x) \;=\; 0 \tag{17}$$

$$f + \frac{1}{2}P(\underline{p},\Delta) - c(x) \;\geq\; \bar{c} \tag{18}$$

$$f, \underline{p}, \Delta \;\geq\; 0 \tag{19}$$

Equation (17) is the incentive compatibility constraint, inequality (18) is a firm's participation constraint, and (19) are the non-negativity constraints. By eliminating $\underline{p}$ using (17) and observing that the buyer will choose the smallest non-negative f that satisfies (18), the problem becomes

$$C(x) = \min_{\Delta} \; \left[\max\left\{2(c(x)+\bar{c}), \hat{P}(\Delta;x)\right\}\right] \quad \text{s.t. } 0 \leq \Delta \leq \bar{G}^{-1}\left(2c'(x)\right), \tag{20}$$

where

$$\hat{P}(\Delta;x) := \frac{1}{\bar{g}(0)}\left[2c'(x) - \bar{G}(\Delta)\right] + \int_0^\Delta y\bar{g}(y)dy + (1-\bar{G}(\Delta))\Delta. \tag{21}$$

$\hat{P}(\Delta;x)$ is the expected price that the buyer has to pay depending on her choice of Δ and given x. The buyer minimizes her expected costs by choosing Δ to minimize $\hat{P}(\Delta;x)$. Let $\Delta^*(x)$ denote an optimal choice of Δ given x. Then, two cases can be distinguished. If $2(c(x)+\bar{c}) > \hat{P}(\Delta^*;x)$, the expected price in the auction is smaller than firms' investment and opportunity costs. Thus, f must be positive to make firms participate in the contest. The buyer chooses f such that firms are just compensated for their costs, i.e., firms' participation constraints are binding.[13] On the other hand, if

[13]Actually, in this case, the buyer is indifferent between all Δ for which $2(c(x)+\bar{c}) > \hat{P}(\Delta;x)$, i.e.,

$2(c(x)+\bar{c}) \leq \hat{P}(\Delta^*;x)$, fixed payments are zero and firms earn (ex ante) rents.

Implementing a fixed-prize tournament ($\Delta^*(x) = 0$) is optimal if $\hat{P}(\Delta;x)$ increases in Δ, i.e.,

$$\frac{\partial \hat{P}(\Delta;x)}{\partial \Delta} = -\frac{\bar{g}(\Delta)}{\bar{g}(0)} + [1-\bar{G}(\Delta)] \geq 0 \quad \text{for all} \quad \Delta \geq 0. \tag{22}$$

Increasing Δ while holding x constant has two effects. First, the minimum price $\underline{p}$ decreases (given by $-\frac{\bar{g}(\Delta)}{\bar{g}(0)}$). Second, the expected quality premium that the winning firm receives in addition to $\underline{p}$ increases (given by $[1-\bar{G}(\Delta)]$). The second effect always dominates the first one if

$$\lambda_{\bar{G}}(\Delta) := \frac{\bar{g}(\Delta)}{1-\bar{G}(\Delta)} \leq \bar{g}(0) \quad \text{for all} \;\; \Delta \geq 0. \tag{23}$$

Consequently, $\Delta^*(x) = 0$ *for all* x if the hazard rate of $\bar{G}$, $\lambda_{\bar{G}}(\Delta)$, is sufficiently small. If, on the other hand, $\hat{P}(\Delta;x)$ strictly decreases in Δ over some interval $(0,a]$, $a > 0$, the buyer should not use a fixed-prize scheme. The optimal minimum and maximum price then depends on $\bar{G}$ and x.[14] However, if $\hat{P}(\Delta;x)$ always decreases in Δ, i.e.,

$$\lambda_{\bar{G}}(\Delta) \geq \bar{g}(0) \quad \text{for all} \;\; \Delta \geq 0, \tag{24}$$

the buyer minimizes expected procurement costs by setting $\underline{p} = 0$ and $\Delta = \bar{G}^{-1}(2c'(x))$. That is, the buyer uses an auction without a minimum price and sets the maximum price so that firms invest x.

Applying the concept of conditional stochastic dominance (CSD), we can now distinguish distributions of $|\eta|$ with respect to the optimality of a fixed-prize tournament

minimizing $\hat{P}(\Delta;x)$ is sufficient but not necessary for minimizing procurement costs.

[14] In particular, since $\hat{P}(\Delta;x)$ is in general not convex in Δ, the first-order condition for minimizing $\hat{P}(\Delta;x)$ is not sufficient to characterize $\Delta^*(x)$.

or an auction with $\underline{p} = 0$.

Definition 1 *Consider two cdfs $\bar{G}_1$ and $\bar{G}_2$. $\bar{G}_1$ dominates $\bar{G}_2$ in terms of conditional stochastic dominance, denoted $\bar{G}_1 \succeq \bar{G}_2$, if*

$$\lambda_{\bar{G}_1}(y) \geq \lambda_{\bar{G}_2}(y)$$

for all y for which $\lambda_{\bar{G}_1}(y)$ and $\lambda_{\bar{G}_2}(y)$ are well defined.

This means that, conditional on any maximum quality premium Δ, this maximum quality premium is more likely to be paid under $\bar{G}_2$ than under $\bar{G}_1$. It also implies that $\bar{G}_1(y) \geq \bar{G}_2(y)$ for all y, i.e., $\bar{G}_1$ first-order stochastically dominates $\bar{G}_2$.[15]

Inequality (23) is binding at $\Delta = 0$ for every distribution of $|\eta|$. Furthermore, if $|\eta|$ is exponentially distributed, we have $\lambda_{\bar{G}}(\Delta) = \bar{g}(0)$ for all $\Delta \geq 0$ so that (23) is always binding. Therefore, we obtain the following proposition.

Proposition 1 *Let $H(y) := 1 - \exp[-\bar{g}(0)y]$.*

(i) If $H \succeq \bar{G}$, then the buyer uses a fixed-prize tournament.

(ii) If $\bar{G} \succeq H$, then the buyer uses an auction with $\underline{p} = 0$.

Thus, if it is possible to rank $\bar{G}$ in terms of CSD relatively to an exponential distribution with the same marginal probability of having the higher quality under identical investments ($H'(0) = \bar{G}'(0)$), the optimal procurement mechanism is either a fixed-prize tournament or an auction with minimum price zero. This holds independently of the investment strategy x the buyer wants to implement. In the special case of $\bar{G} = H$, all combinations of $\underline{p}$ and Δ that implement x lead to the same expected costs for the buyer.

[15]See, e.g., Krishna [2002], p. 260.

The intuition of proposition 1 is as follows. If $H \succeq \bar{G}$, the realization of $|\eta|$ is likely to be relatively large, i.e., the values of the innovations for the buyer will probably differ greatly. As a result, in an auction, the high-quality firm can demand a large quality premium, i.e., the buyer is likely to pay $\underline{p} + \Delta$. Setting incentives through an auction is then too expensive from the buyer's point of view since it leaves higher rents to firms than a fixed prize.

The concept of CSD allows to capture the intuition for the superiority of a fixed-prize tournament under certain probability distributions. However, we do not know yet which of the common distributions (e.g, normal or uniform distribution) favor an auction or a fixed-prize scheme. To answer this question, note that the optimality condition for a tournament, inequality (23), holds if $\lambda_{\bar{G}}(y)$ is monotone decreasing. By contrast, a monotone increasing $\lambda_{\bar{G}}(y)$ (i.e., inequality (24) holds) favors an auction with minimum price zero. Monotonicity of $\lambda_{\bar{G}}$ turns out to be equivalent to the log-concavity or log-convexity of G on $S \cap \mathbb{R}_{-}$.

Definition 2 *A function $F : \mathbb{R} \to (0, \infty)$ is log-concave on the interval $(a, b) \subseteq \mathbb{R}$ if the function $\ln F$ is concave on (a, b) and log-convex if $\ln F$ is convex on (a, b).*

Thus, G is log-convex (log-concave) on a certain interval if and only if $g(y)/G(y)$ is increasing (decreasing) on this interval. For $y \geq 0$, we have that

$$\lambda_{\bar{G}}(y) = \frac{\bar{g}(y)}{1 - \bar{G}(y)} = \frac{g(y)}{1 - G(y)} = \frac{g(-y)}{G(-y)}, \tag{25}$$

which implies that $\lambda_{\bar{G}}$ is monotone decreasing (increasing) if and only if G is log-convex (log-concave) on $S \cap \mathbb{R}_{-}$. This yields the following proposition.

Proposition 2 *(i) If G is log-convex on $S \cap \mathbb{R}_{-}$, then the buyer uses a fixed-prize tournament.*

(ii) If G is log-concave on $S \cap \mathbb{R}_-$, then the buyer uses an auction with $\underline{p} = 0$.

As is well known from the literature, most "named" distributions are log-concave (see, e.g., An [1998] or Bagnoli and Bergstrom [2005]). However, cdfs that exhibit log-convexity are easy to construct. For example,

$$G(y) := \begin{cases} \frac{1}{2(1-y)} & y \leq 0 \\ 1 - \frac{1}{2(1+y)} & y > 0 \end{cases} \tag{26}$$

is log-convex on $\mathbb{R}_-$ so that, under this distribution function, a fixed-prize tournament dominates an auction.[16]

Log-convexity of G, or, equivalently, a decreasing $\lambda_{\bar{G}}$ means that the instantaneous probability that a certain quality difference $|\eta|$ is realized, given that the quality difference is at least $|\eta|$, decreases in $|\eta|$. This is a strong requirement. However, by (23), it is sufficient (and necessary) for a tournament to be superior that $\lambda_{\bar{G}}(y)$ decreases for small y, while it may increase for relatively large y as long as it does not exceed $\lambda_{\bar{G}}(0)$. Intuitively, $\lambda_{\bar{G}}(y)$ decreases for small y if it becomes more likely that firms' innovations differ relatively strongly, given that they have not found very similar innovations. This may be the case if the innovation technology is subject to large random factors.

5 Discussion and conclusion

This paper shows that a fixed-prize tournament can dominate a first-price auction in procurement settings. The key point leading to this result is that, under stochastic innovations and in the absence of entry fees, holding an auction may leave higher rents to firms than announcing a fixed prize. The technical results on CSD and log-convexity

[16]It is easily verified that, for this distribution function, condition (13) holds if $1.5 < \inf_x c''(x)$. In general, log-concavity or log-convexity of the distribution function does not contradict (13).

versus log-concavity, however, are presumably particular to the way the stochastic innovation technology is modelled, namely, that an increase in a firm's investment shifts its cdf of quality to the right.

For simplicity, I considered a two-firm setting. The extension to the case of n identical firms is straightforward. With n firms, the quality premium in the auction is the difference between the two highest order statistics, $\mu_{(n)} - \mu_{(n-1)}$, of the sample $\mu_1, \ldots, \mu_n$. Then, the hazard rate of the cdf of $\mu_{(n)} - \mu_{(n-1)}$ is crucial for making the right choice between auction and tournament.[17]

Furthermore, I assumed that all parties involved observe qualities before the bidding process. If instead only the buyer and firm i observes q_i, firm i bids $q_i - E[q_j|q_j < q_i]$ in a first-price auction without any minimum or maximum price (compare Fullerton et al. [2002]). It can be shown that this leads to the same investments and expected costs for the buyer as in the case analyzed above. Although optimal bidding strategies change, parties' expected payoffs in the stages *before* qualities are observed remain the same. With a fixed prize, it does not matter whether a firm can observe the quality that the other contestant can supply. Therefore, the buyer still prefers a fixed prize if the expected price in the auction, $\mu_{(2)} - \mu_{(1)}$, is so high that firms earn large rents.

It is often argued that holding an auction (without a minimum or maximum price) has a substantial advantage over announcing a fixed-prize tournament since the latter requires more knowledge on the side of the buyer. To calculate an appropriate fixed prize, the buyer has to know firms' costs and innovation technologies. In this paper, if the buyer conducts an auction without restricting the set of allowable prices, firms even choose efficient investments. However, this is often not optimal from the buyer's point of view. Then, in the auction, the buyer also needs detailed knowledge on invest-

[17]The decision on the optimal number of contest participants is non-trivial. Holding investments constant, more participants lead to a higher expected quality, while the effect on the buyer's cost function depends heavily on the underlying probability distribution.

ment costs and innovation technologies to calculate the appropriate maximum and/or minimum price. Therefore, as soon as the buyer wishes to direct firms' investment behavior, the informational advantage of the auction disappears.

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