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Via Necchi 5 - 20123 Milano
Tel. 02/7234.2918 - Fax 02/7234.2923
E-mail ist.temq@unicatt.it

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Spillovers, disclosure lags, and incentives to innovate: Do oligopolies over-invest in R&D?

Gianluca Femminis[a,+] Gianmaria Martini [b]
[a] Università Cattolica di Milano,
[b] Università di Bergamo

Abstract

We develop a dynamic duopoly, in which firms have to take into account a technological externality, that reduces over time their innovation costs, and an inter-firm spillover, that lowers only the second comer's R&D cost. This spillover exerts its effect after a disclosure lag. We identify three possible equilibria, which are classified, according to the timing of R&D investments, as early, intermediate, and late. The intermediate equilibrium is subgame perfect for a wide parameters range. When the innovation size is large, it implies underinvestment. Hence, even in presence of a moderate degree of inter-firms spillover, the competitive equilibrium calls for public policies aimed at increasing the research activity. When we focus on minor innovations – the case in which, according to the earlier literature, the market equilibrium under-invests – our results imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before.

Keywords: R&D, knowledge spillover, dynamic oligopoly
JEL classification: L13, L41, O33.

[+]Correspondence to: G. Femminis, ITEMQ, Università Cattolica di Milano, Largo Gemelli 1, I-20123 Milano. email: gianluca.femminis@unicatt.it
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1 Introduction

Understanding firms' decision to innovate is of fundamental importance to design policies aimed at maximizing welfare. The firms' choices are driven by their incentives; hence the market structure in which firms operate plays a crucial role in determining the pace of technical progress. This provides a strong motivation for the analysis of oligopolies, which are the most widespread market configuration.

In our study, we analyze a duopoly in which – as in many recent contributions (e.g. Stenbacka and Tombak (1994), Hoppe (2000), and Schmidt-Dengler (2006)) – the R&D cost shrinks over time thanks to general advances in knowledge and technology. In addition to this standard technological externality, firms take into account a spillover that lowers the second comer's innovation cost,[1] exerting its effect after a time period which we label "disclosure lag". Whenever the follower wants to exploit the spillover, he grants to the leader a competitive advantage period (at least) equal to the disclosure lag. Of course, the first innovator is aware of this fact, accordingly the behavior of the interacting firm is significantly influenced by the presence of the spillover and of the disclosure lag.

What we find is that in our framework three types of equilibria arise, while the existing contributions, following Fudenberg and Tirole (1985), detect two possible market equilibria: an early and a late one.

This literature, which starts with Reinganum (1981), and is excellently surveyed by Hoppe (2002), identifies two driving forces characterizing the equilibria: the length of the follower's strategic delay, and the intensity of the competitive pressure. In the early equilibrium, the second innovator delays his decision to invest for a relatively long period. This choice is driven by the desire to grasp the benefit of technical progress, that reduces the innovation cost as time goes by. The follower's optimal choice implies a long competitive advantage period for the innovator leader, which favors the latter's payoff at the expenses of the former's one. Hence, to avoid being preempted, the first mover invests "very soon", and the R&D investment is socially excessive. The preemption possibility also implies rent equalization. In contrast, a late equilibrium arises once technical progress has substantially reduced the innovation costs, so that an innovation leader cannot emerge, because the rival would immediately copy her decision. In this case, any innovator – anticipating that there will be no leadership – waits until her

[1] The importance of spillovers for R&D is underscored in De Bondt (1996), who provides many reference of earlier contributions, which, however, adopt static even if multi-stage frameworks.

choice maximizes the joint discounted stream of net profits. The collusive flavour of this equilibrium is apparent: accordingly, Fudenberg and Tirole's analysis implies that this type of market equilibrium underinvests.

A distinctive feature of the new type of equilibrium we identify, is that the follower invests exactly at the end of the disclosure lag, i.e. as soon as he can exploit the spillover. We label "intermediate" this third type of equilibrium, since the decisions to innovate occur at dates positioned between the early, and the late ones. The intermediate equilibrium takes place after the early one, in fact the shorter competitive advantage period it implies, is an optimal choice for the follower only when the R&D costs have become sufficiently low. The intermediate equilibrium anticipates the early one, since the foresaking of the spillover benefits becomes optimal only when the R&D costs are very low.

The intermediate equilibrium is particularly relevant because it is subgame perfect for a large range of the parameters set.

To understand this point, consider first the case of an innovation of limited size. In this situation, when the spillover is (relatively) high, the follower grasps (relatively) large benefits from investing at the end of the disclosure lag, so that he finds optimal to select this strategy for a long time interval. This makes the leader unwilling to wait until the late equilibrium prevails, which gives rise to the intermediate equilibrium. Instead, the late equilibrium is subgame perfect when the spillover is very low, because in this case the "immediate reply" strategy for the follower becomes optimal at earlier dates.[2]

When the innovation size is large, an early equilibrium may emerge, because a major innovation, bringing a large cost advantage to the leader, enhances her incentive to be first. However, due to the reduction in innovation costs, the higher the spillover, the sooner the second comer optimally invests in reply to an early leader's investment. This reduces the leaders' efficiency advantage period, leading to the dominance of the intermediate equilibrium. Moreover, a (relatively) high spillover increases the second comer's payoff in the intermediate time interval, and this softens the leader's preemption incentive to invest. This milder competition implies higher payoffs for both firms in the intermediate equilibrium.

When the innovation size is large, the intermediate equilibrium implies that the duopolistic market equilibrium involves underinvestment. An underinvesting equilibrium in presence of a major innovation is a result that contrasts not only with the literature following Fudenberg

[2] As in the previous literature, a small cost reduction, implying a weak incentive to innovate first, does not give rise to an equilibrium with preemption.

and Tirole, but also with the previous contributions inspired by Loury (1979), and by Lee and Wilde (1980).[3] The relevant implication is that, according to our model, the competitive equilibrium calls for public policies aimed at increasing the research activity. Notice that the natural indicators of an highly competitive environment, namely an equilibrium with R&D diffusion and rent equalization, do not imply that the R&D investment is excessive from the social planner's perspective.

With minor innovations – the case in which, according to the earlier literature, the market equilibrium underinvests – the prevalence of the intermediate equilibrium imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, despite the presence of an inter-firm spillover. Notice that the equilibrium we describe is more realistic than the late one, which is characterized by simultaneous adoptions, a phenomenon seldom observed in the real world.

These results, being driven by the assumption of an inter-firm spillover coupled with the one of a disclosure lag, differ from the ones already obtained in the literature. In fact, Riordan (1992) focuses on the early equilibrium, and analyses the impact of price and entry regulations on the timing of adoption. Because these regulatory schemes tend to reduce the first innovator's rents, they are likely to delay the early adoption, which can be socially beneficial.

Stenbacka and Tombak (1994) analyze the role of experience, which implies that the probability of successful implementation of an innovation is an increasing function of the time distance from the investment date. As for welfare, they show that a collusive adoption timing may improve welfare when compared with the market equilibrium. This happens when the pace of technical progress is fairly high: when this is the case, a collusive adoption is beneficial because the industry can fully take advantage of the reduction of innovation cost. In contrast, a competitive market equilibrium, being driven by the incentives to obtain a strategic advantage, induces a premature adoption.

In Hoppe (2000), firms are uncertain about the profitability of the in-

[3] Loury, and Lee and Wilde assume that a new technique becomes suddenly available, and immediately triggers the industry's investment in R&D. The competitive pressure induced by the market structure implies that the equilibrium involves an R&D investment that is higher than the social optimum. This result can partially be ascribed to the tournament nature of these models. In a non-tournament model, Beath *et al* (1989) underscore the role of the competitive threat as a major determinant of R&D expenditure. Because the larger is the competitive threat the more resources firms invest in R&D, overinvestment is more likely the larger is the size of the innovation. Delbono and Denicolò (1991), again in a non-tournament framework, find that the equilibrium R&D effort can be lower than the social optimum if the marginal efficiency of the R&D expenditure is low (hence each firm invests less and gets a small R&D output).

novation. Her framework differs from the one by Fudenberg and Tirole, thanks to the presence of technological uncertainty, which induces an asymmetry between the leader and the follower. The latter observes the leader's outcome, and hence becomes aware about the actual profitability characterizing the new technique. This informational spillover may bring about a second-mover advantage. Moreover, an high probability of failure induces a late simultaneous adoption because it curtails the first mover expected payoff. When the late equilibrium is subgame perfect, Hoppe finds that an earlier simultaneous adoption would be welfare increasing, while the result are less definite when the early equilibrium prevails.

Weeds (2002) presents a tournament version of Fudenberg and Tirole (1985), in which profits evolve stochastically. She suggests that the early (late) equilibrium over(under)-invests; however the late equilibrium is closer to the social optimum than the early one.[4]

The paper proceeds in the standard way. In Section 2 we present our model. In Section 5 we discuss the equilibrium concept adopted in the analysis and we compute the different market equilibria, in which firms compete both in the innovation and in the product stages. Then, subgame perfectness is invoked as a selection device among market equilibria. In Section 6 we spell out the welfare implications of our analysis. Concluding comments in Section 7 end the paper.

2 The model

We consider an industry composed of two firms, i and j, which – in each (infinitesimally short) time period – are involved in a two–stage interaction: first they decide whether to innovate or not, then they compete *à la* Cournot in the final market.

Market demand is linear and equal to: $P = a - Q$, where P is the market clearing price and $Q = q_i + q_j$ is the total quantity supplied. Each firm has a unit cost of production c. Notice that, at the beginning, the two firms are symmetric.

The investment in R&D immediately yields a cost-reducing process

[4] The presence of an inter-firm spillover assimilates our model to the frameworks proposed by Katz and Shapiro (1987), Dutta *et al.* (1995), and Hoppe and Lehmann-Gruber (2005) among others. Katz and Shapiro introduce an extreme form of technological spillover, assuming that, in a duopoly, the follower can adopt at no cost the new technology as soon as the leader has invested. This hypothesis induces the possibility of a second mover advantage. A similar approach is followed in Dasgupta (1988). Dutta *et al.* demonstrate that the second mover advantage may prevail as subgame perfect equilibrium output in product innovation games. Hoppe and Lehmann-Gruber generalize the previous results by analyzing the issue of multiple peaks in the leader's payoff function.

innovation, which shrinks the unit production cost by an amount x, with $x < c$. Hence firm h's post–innovation production cost is $C(q_h) = (c - x)q_h$, $h = i, j$.

Each firm's payoff will depend not only on its adoption date but also on its rival's one. If both firms have not invested up to period t, their individual profits in the Cournot subgame at t are those of the pre–innovation stage, i.e.

$$\pi_0 = \frac{A^2}{9}, \tag{1}$$

where $A = a - c$. The subscript indicates the number of firms which have already introduced the innovation. The instantaneous welfare (computed *à la Marshall*) is then equal to:

$$W_0 = \frac{4A^2}{9}. \tag{2}$$

If instead only one firm, say firm 1, invests in R&D at t, it benefits of an efficiency advantage, and obtains a higher market share. The market price at t decreases in comparison with the pre-innovation level, while the individual profits become:

$$\pi_1^L = \frac{(A + 2x)^2}{9}; \pi_1^F = \frac{(A - x)^2}{9}, \tag{3}$$

where L and F stands for 'leader' and 'follower'. Notice that $\pi_1^L > \pi_1^F$, $\pi_1^L > \pi_0$ and $\pi_1^F < \pi_0$. Because the quantity produced by the firm that has not innovated is $(A - x)/3$, to preserve the duopolistic structure characterizing our market we need to assume that $A > x$. This hypothesis implies that, in a Cournot environment, the cost-reducing innovation is non−drastic. In case of asymmetric behavior at t, welfare is:

$$W_1 = \frac{8A(A + x) + 11x^2}{18}, \tag{4}$$

with $W_1 > W_0$.

Finally, we need to compute the outcomes when both firms have innovated at t. In this case, being more efficient, they both produce more than in the *status quo*; therefore, the market price is lower. Individual profits at t are:

$$\pi_2 = \frac{(A + x)^2}{9}. \tag{5}$$

Obviously, $\pi_1^L > \pi_2$; notice that the difference between π_1^L and π_2 is increasing in x: when only one firm enjoys a cost advantage, she obtains a larger market share while benefiting from an higher price to cost margin. When both firms have innovated, the social welfare is:

$$W_2 = \frac{4(A+x)^2}{9}, \tag{6}$$

with $W_2 > W_1$, since $A > x$.

When firms simultaneously invest in R&D, individual profits rise from (1) to (5) and welfare jumps from (2) to (6). Alternatively, firms may behave asymmetrically, so that there are both an innovation leader and a follower. Under these circumstances individual profits first change from (1) to (3) (and welfare from (2) to (4)), and then, when the follower invests in R&D, profit change from (3) to (5) (and welfare from (4) to (6)).

Time is continuous and firms' horizon is infinite. Firms discount future profits at the common rate r.

In our set-up, the research project has a fixed size, as in Fudenberg and Tirole (1985), Hoppe (2000), and many others. If the first firm investing in R&D sinks the cost as soon as the innovation becomes technically feasible, i.e. at time 0, it pays γ. Such a cost then decreases at the constant rate $\rho > 0$, thanks to the advances in pure research, and to the availability of new results obtained in related fields. Of course, this form of technical progress is exogenous to any single firm. This picture is captured by the following R&D cost function

$$C_L(t_L) = \gamma e^{-\rho t_L}, \text{ for } \; t_L \in [0, \infty), \tag{7}$$

where t_L is the calendar time when the innovation leader introduces the technical improvement.

In his classic study, Mansfield (1985) reports that in 59% of cases the innovator's rivals needs more than twelve months to obtain the relevant information. More recently, Cohen *et al.* (2002) compute that the average adoption lag for unpatented process innovation in Japan, and in the US, is, respectively, 2.03 and 3.37 years. Accordingly, we introduce in the innovation follower R&D cost function, an element representing the delay needed to grasp the benefit stemming from the leader's innovative activity. More precisely, we assume an exogenously determined disclosure lag, Δ. An obvious but important consequence of our assumption is that – whenever the follower wants to exploit the spillover – the introduction of an innovation grants to the leader a competitive advantage period (at least) equal to Δ years.[5]

[5]Miyagiwa and Ohno (2002) adopt the same assumption in a R&D model built in the spirit of Lee and Wilde (1980).

As for the innovation follower, the R&D cost evolution is described by:

$$C_F(t_F) = \begin{cases} \gamma e^{-\rho t_F} & \text{for } \ t_F \in [t_L, t_L + \Delta) \\ (1-\theta)\gamma e^{-\rho t_F} & \text{for } \ t_F \in [t_L + \Delta, \infty) \end{cases}, \tag{8}$$

where t_F is the adoption time for the innovation follower. $\theta \in [0, \bar{\theta}]$ is the inter-firm spillover parameter; $\bar{\theta}$ shall be assumed as being strictly lower than unity. In fact, for realistic values of Δ, if θ were close to unity, the follower – bearing almost no innovation cost – would always invest at the end of the disclosure lag. Hence, this particular case would deliver results close to the ones in Katz and Shapiro (1987). Whenever $\theta > 0$, the innovation is only partially appropriable: the second comer enjoys a reduction in R&D costs by imitating his competitor at $t_F \geq t_L + \Delta$.

The way we introduce the spillover and the disclosure lag into the model is extremely simple: it would have been preferable to consider a stochastic disclosure lag, with a probability of information diffusion depending upon the time elapsed from the introduction of the innovation, and on the follower's imitation effort. The latter should influence also the spillover size.[6] However, even the simplest stochastic formulation – namely the one involving a constant probability of information diffusion coupled with a fixed spillover size – precludes the attainment of explicit results.[7] Hence, our formulation has been chosen as the optimal compromise between analytical tractability and "realism".

We denote by $V_L(t_L, t_F)$ the stream of future profits, discounted back to time 0, obtained by the firm investing at t_L while her rival sinks the innovation cost at $t_F \geq t_L$. Hence, we have that:

$$V_L(t_L, t_F) = \frac{\pi_0}{r} + \frac{\pi_1^L - \pi_0}{r} e^{-rt_L} + \frac{\pi_2 - \pi_1^L}{r} e^{-rt_F} - C_L(t_L) e^{-rt_L},$$

and therefore, from (1), (3), and (5):

$$V_L(t_L, t_F) = \frac{A^2}{9r} + \frac{4(A+x)x}{9r} e^{-rt_L} - \frac{(2A+3x)x}{9r} e^{-rt_F} - C_L(t_L) e^{-rt_L}, . \tag{9}$$

[6] To endogenize θ we could have followed Jin and Troege (2006), which suggest that firms can raise it, paying a convex imitation cost. Nevertheless, we preferred not to pursue this development of the model, because our framework is already fairly complex. For the same reason we do not endogenize the lenght of the disclosure lag.

[7] A constant probability of information disclosure does not represent an improvement upon our formulation, since the sparse empirical evidence available suggests that the probability of successful imitation increases over time.

The second addendum represents the first innovator stand-alone incentive, while the third one is the profit reduction imposed to the innovation leader by the follower's decision to adopt.

The second innovator's payoff is:

$$V_F(t_L, t_F) = \frac{\pi_0}{r} + \frac{\pi_1^F - \pi_0}{r} e^{-rt_L} + \frac{\pi_2 - \pi_1^F}{r} e^{-rt_F} - C_F(t_F) e^{-rt_F},$$

and hence, from (1), (3), and (5) we have that:

$$V_F(t_L, t_F) = \frac{A^2}{9r} - \frac{(2A - x)x}{9r} e^{-rt_L} + \frac{4Ax}{9r} e^{-rt_F} - C_F(t_F) e^{-rt_F}. \quad (10)$$

Here, the incentive to innovate is summarized by the third addendum, while the profit externality imposed by the leader to the follower is captured by the second addendum.

Before describing the firms' value functions, we introduce some technical assumptions, restricting the admissible values for θ, Δ and γ.

Assumption 1 : $1 - \frac{A}{A+x} \left[\frac{r(2A+3x)+\rho(6A+3x)}{r(2A-x)+\rho(6A+3x)} \right]^{\rho/r} < \bar{\theta} < 1.$

As we shall discuss, if the maximum spillover $\bar{\theta}$ were close to zero, the results delivered by our model would be similar to those obtained by Fudenberg and Tirole (1985). Accordingly, by requiring that the maximum spillover is high, we make the discussion more interesting. Actually, Assumption 1 allows the spillover levels to be sufficiently high that all the cases considered in what follows are relevant.

Assumption 2 : $\Delta \leq \bar{\Delta} = \frac{1}{r} \ln \left(1 + \frac{r}{\rho} \frac{2A+3x}{6A+3x} \right).$

The purpose of Assumption 2 is to limit the number of cases that we need to consider. Notice that Assumption 1 guarantees that $\bar{\Delta} > 0$; when we present some numerical exercises, we will verify that Assumption 2 does not restrict Δ to values too short to be sensible.

Assumption 3 : $\gamma \geq \bar{\gamma} = \frac{4Ae^{\rho\bar{\Delta}}}{9(r+\rho)(1-\bar{\theta})}.$

This hypothesis implies that, when the innovation becomes feasible, its lump-sum cost is sufficiently high that the second comer does not wish to innovate before the disclosure lag is complete. Accordingly, there is a time span in which the spillover is not foresaken by the follower, which provides a role for the spillover itself.

Notice that $\bar{\gamma} > 0$, by Assumption 1.

3 The follower's investment problem

Since the follower optimally reacts to the leader's decisions, it is natural to analyze first his behavior.[8]

When the leader has invested in the early stages of the game, the follower prefers to delay his adoption more than Δ years. In fact, opting for a delay longer than Δ, the follower not only nets the benefits from imitation, but he can also grasp relevant additional gains from pure research, which is still producing results that are quantitatively important to reduce the R&D cost. Maximizing (10) with respect to t_F, we obtain the follower's optimal choice, which is to invests at

$$T_F^* = -\frac{1}{\rho} \ln \left(\frac{4A}{9\gamma(r+\rho)(1-\theta)} \right). \tag{11}$$

This solution applies when the leader sinks the costs at $t_L \leq T_F^* - \Delta$.[9]

The comparative statics on T_F^* gives sensible results. In particular, the higher the inter-firms spillover, the sooner the second comer invests: a high θ reduces – *ceteris paribus* – the follower's costs and therefore anticipates his investment date.[10]

When the leader does not invest before $T_F^* - \Delta$, the follower does not sink the fixed cost at T_F^*: at that time, the disclosure lag has not elapsed yet, which increases the follower's cost, and prevents this choice from being optimal.

If $t_L > T_F^* - \Delta$, the follower's choice is among to wait exactly Δ periods before investing (to grasp the inter-firm spillover), to wait less than Δ, and to copy immediately.

We now discuss separately the case of high, and of low spillovers. We define

$$\theta'(\Delta) = 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta},$$

and we notice that $\theta'(0) = 0$, that $\partial\theta'(\Delta)/\partial\Delta > 0$, and that $\partial^2\theta'(\Delta)/(\partial\Delta)^2 > 0$.

[8] For ease of exposition, we refer to the follower as if it were headed by a male CEO.

[9] Assumptions 2 and 3 guarantee that $T_F^* - \Delta \geq 0$ for any $\Delta \in [0, \bar{\Delta}]$, $\theta \in [0, \bar{\theta}]$.

[10] An higher $\pi_2 - \pi_1^L$ increases the follower's incentive to innovate, and hence anticipates his decision; an increase in γ or in r delays his investment decision, because the innovation is more costly, or the future profits are more heavily discounted. The technical progress parameter ρ plays a twofold role: on the one hand, its increase implies that, at any date, the innovation costs are lower, which calls for an earlier investment; on the other hand, a faster reduction in innovation costs may induce a firm to wait because it knows that the cost will quickly become smaller. With a low spillover, the first direct effect prevails over the second indirect one; in contrast, when θ is high, the impact of an increase in ρ on T_F^* may well be positive.

Suppose first that $\theta > \theta'(\Delta)$, i.e. that the spillover is high. In this case, the choice of waiting less than Δ is never optimal. In fact, when the spillover is sizable, and the innovation cost is still high enough, waiting Δ years implies an R&D cost saving that is high enough to compensate for an efficiency disadvantage period equal to the disclosure lag. Hence, there is a time interval in which the follower's optimal choice is to wait Δ periods before innovating. We define $\bar{T}$ as the first date such that the second firm payoff gained by the "immediately following" strategy, becomes as high as the payoff granted by the decision of waiting Δ periods before investing in R&D. Solving the equation $V_F(t_L, t_L) = V_F(t_L, t_L + \Delta)$, where the follower's value function is given by (10), it is immediate to obtain:

$$\bar{T} = -\frac{1}{\rho} \ln \left[\frac{4A}{9\gamma r} \frac{1 - e^{-r\Delta}}{1 - (1 - \theta) e^{-(r+\rho)\Delta}} \right]. \tag{12}$$

Notice that an increase in the spillover parameter raises $\bar{T}$. In fact, a more relevant benefit from imitation postpones the undertaking of a line of action that prescribes the forsaking of the benefit itself.[11]

Finally, if the innovation leader decides to invest "late" (i.e. when $t_L \in [\bar{T}, \infty)$) the R&D cost is so low that it is optimal for the second firm to immediately enter upon his rival's investment, without exploiting the inter-firm spillover.

The above arguments are formally presented in:

Proposition 1 *When $\theta \in [\theta'(\Delta), \bar{\theta}]$, the follower's optimal strategy is to invest at*
(a) T_F^* if $t_L \in [0, T_F^* - \Delta]$
(b) $t_L + \Delta$ if $t_L \in (T_F^* - \Delta, \bar{T}]$
(c) t_L if $t_L \in (\bar{T}, \infty)$.

Proof: See the Appendix.

When the spillover is low ($\theta < \theta'(\Delta)$), the above analysis must be partly modified for $t_L > T_F^* - \Delta$. In this case, waiting Δ periods is less rewarding for the follower, and it gets less and less rewarding as time goes by, due to the shrinking in the R&D cost. Accordingly, the follower's choice of waiting less than Δ becomes optimal for some $t_L > T_F^* - \Delta$.

Defining

$$T_F' = -\frac{1}{\rho} \ln \left(\frac{4A}{9\gamma(r + \rho)} \right) \tag{13}$$

[11] Apart from the effect of θ, the comparative static for $\bar{T}$ is quite similar to the one for T_F^*.

as the follower's optimal investment date in absence of spillover, his optimal choice is summarized by:

Proposition 2 *When* $\theta \in [0, \theta'(\Delta))$*, the follower's optimal strategy is to invest at*
(a) T_F^* *if* $t_L \in [0, T_F^* - \Delta]$;
(b) $t_L + \Delta$ *if* $t_L \in (T_F^* - \Delta, \breve{T}_L]$ *where* $\breve{T}_L \in (T_F^* - \Delta, T_F']$ *is the time instant such that* $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$;
(c) T_F' *if* $t_L \in (\breve{T}_L, T_F']$;
(d) t_L *if* $t_L \in (T_F', \infty)$.

Proof: See the Appendix.

Notice that, $\lim_{\theta \to 0} T_F^* = T_F'$; moreover Proposition 2 has the interesting

Corollary 3 $\lim_{\theta \to 0} \breve{T}_L = T_F^* - \Delta$

Proof: See the Appendix.

Accordingly, when $\theta = 0$, the time interval sub (b) in Proposition 2 collapses to Ø, and the follower's optimal strategy is to invest at $T_F'(= T_F^*)$ if $t_L \in [0, T_F']$, and to immediately follow the leader's investment for $t_L \in (T_F', \infty)$. This result comes as no surprise: when there is no spillover, the disclosure lag cannot have any effect on the follower's optimal decision. Hence, what we find is the follower's optimal strategy identified by Fudenberg and Tirole (1985).

4 The leader's investment decision and the value functions behavior

We now solve the leader's optimal decision problem, determining her payoff.[12]

In a model without spillovers, if the leader opts for an early adoption, she is aware that her competitor will postpone his investment for quite a long time. This allows for an inverted-U leader's payoff function. This shape is determined by two opposing forces. An increase in the leader's adoption time induces a reduction in her innovation cost, which increases her value function, but implies also a shortening in her efficiency advantage period, which reduces her payoff. When t_L is relatively low, the

[12]Notice that – for ease of exposition – we refer to the leader as if it were run by a female CEO.

former effect dominates the latter because the cost reduction induced by the technological externality is quantitatively relevant. From the value function (9), it is clear that the leader's optimal entry date is obtained balancing the later attainment of the profits yielding the stand-alone incentive, with the decrease in the innovation cost. Hence, the leader's optimal investment date is independent from the follower's decision, and therefore from the spillover. Maximizing (9) given t_F, one immediately obtains that

$$T_L^* = -\frac{1}{\rho} \ln \left(\frac{4(A+x)}{9\gamma(r+\rho)} \right), \tag{14}$$

which applies also for $\theta > 0$.

However, the presence of a spillover and of a disclosure lag brings about several effects. Because θ reduces T_F^*, it shrinks the leader's cost advantage period, reducing the leader's value function.[13] Moreover, a longer Δ reduces the time interval during which the leader knows that the follower is playing T_F^* as his optimal reply. This happens because T_F^* has been computed on the ground of the attainment of the spillover, and hence on the completion of the disclosure lag. The joint effect of the presence of Δ and θ (that reduces T_F^*) may imply that $T_L^* > T_F^* - \Delta$, so that the leader's value function is increasing in $[0, T_F^* - \Delta]$. In Figure 1, the dashed line represents the leader's value function, which – for $t_L \in [0, T_F^* - \Delta]$ – has the usual inverted-U shape, while Figure 2 considers cases in which T_L^* is larger than $T_F^* - \Delta$ (and henceforth it is not shown).

[Figure 1 about here]

[Figure 2 about here]

For $t_L \in (T_F^* - \Delta, \bar{T}]-$ the leader's value function tends again to assume an inverted-U shape. This behavior can be easily understood with reference to the case $\theta > \theta'(\Delta)$, in which the leader is aware that the follower grants her an efficiency period of lenght equal to the disclosure lag, and hence constant (refer to Proposition 1). Therefore, if t_L is close to $T_F^* - \Delta$, the reduction in the fixed cost due to the technological externality outweights the effects of the postponement of the post-innovation higher profits (that – in current value – do not change over time). As t_L gets larger, the second negative effect prevails over the former positive one.[14]

[13] Exploiting equations (9) and (11), it is immediate to notice that $\frac{\partial V_L(t_L, T_F^*)}{\partial \theta} < 0$.

[14] When θ gets smaller, $\bar{T}$ is reduced (as implied by Eq. (12)). Hence, the negative effect may not have the time to become strong enough to induce the inverted-U shape for the innovation leader value function.

We now characterize in more details the maximum value functions under different assumptions concerning θ and Δ, focusing first on $t_L \in (0, \bar{T}]$, and then on $t_L \in (\bar{T}, \infty)$.

Accordingly, we first analyze, for $t_L \in (0, \bar{T}]$, the case in which θ is small with respect to Δ, a concept that we make precise by defining

$$\theta''(\Delta) = 1 - \frac{4Are^{(r+\rho)\Delta}}{(r+\rho)(6A+3x)(e^{r\Delta}-1)+4Ar},$$

and assuming that $\theta \leq \theta''(\Delta)$. Figure 3 is helpful to locate the portion of space we are considering.[15]

[Figure 3 about here]

In this case, due to the low spillover, the first innovator payoff in the early interval is higher than the follower's one for some t_1, as depicted in Figure 1, Panels (A_1) and (A_2). Moreover, a relatively long disclosure lag calls for an early end of the interval $[0, T_F^* - \Delta]$, which implies that the innovation leader value function is higher than the follower's one at the end of this interval (i.e. for $\theta \leq \theta''(\Delta)$, we have that $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$).

The fact that the disclosure lag is relatively long has another important implication: in the intermediate interval, i.e. for $t_L \in (T_F^* - \Delta, \bar{T}]$, the value function for the leader is higher than the one for the follower, who must let the leader enjoy a long cost advantage period.

These behaviors are formally stated in:

Proposition 4 *(a) When $\theta \in [0, \theta''(\Delta)]$, $V_L(t_L, T_F^*) \geq V_F(t_L, T_F^*)$ for some $t_L \in [0, T_F^* - \Delta]$;*
(b) when $\theta \in (\theta'(\Delta), \theta''(\Delta)]$, $V_L(t_L, t_L + \Delta) > V_F(t_L, t_L + \Delta)$ for $t_L \in (T_F^* - \Delta, \bar{T}]$;
(c) when $\theta \in [0, \theta'(\Delta)]$, $V_L(t_L, t_L + \Delta) > V_F(t_L, t_L + \Delta)$ for $t_L \in (T_F^* - \Delta, \breve{T}_L]$, and $V_L(t_L, T_F') \geq V_F(t_L, T_F')$, for $t_L \in (\breve{T}_L, T_F']$, with the equality applying at $t_L = T_F'$.

Proof: See the Appendix.

We now consider – again for $t_L \in (0, \bar{T}]$ – the effects of a relatively large spillover.

In this case, the fact that $\theta > \theta''(\Delta)$ implies that $V_L(T_F^* - \Delta, T_F^*) \leq V_F(T_F^* - \Delta, T_F^*)$ (refer to Figure 2). As it will become clear in Section 5,

[15] Notice that $\theta''(0) = 0$, and that, for $\Delta < \bar{\Delta}$, $\partial\theta''(\Delta)/\partial\Delta > 0$, while $\theta''(\Delta) > \theta'(\Delta)$.

it is important to verify whether the leader's maximum value function is increasing in $t_L \in (0, T_F^* - \Delta]$. Simple calculations show that $T_L^* \geq T_F^* - \Delta$ if $\theta \geq \theta^*(\Delta)$, where

$$\theta^*(\Delta) = 1 - \frac{A}{A+x} e^{\rho\Delta}.$$

It is obvious that $\partial\theta^*(\Delta)/\partial\Delta < 0$, and that $\theta^*(0) > 0$. In Figure 3, $\theta^*(\Delta)$ is the downward sloping bold curve, which is portrayed only for values such that $\theta^*(\Delta) > \theta''(\Delta)$.

We now restrict our attention to the sub-case $\theta > \max\{\theta^*(\Delta), \theta''(\Delta)\}$, so that the innovation leader's value function is increasing in the interval $[0, T_F^* - \Delta]$, and $V_L(t_L, T_F^*)$ and $V_F(t_L, T_F^*)$ can be drawn for $t_L \in [0, T_F^* - \Delta]$ as in Figure 2.

In comparison to the previous case (i.e. $\theta \leq \theta''(\Delta)$), the higher spillover reduces the leader's payoff in the early interval. This happens because an increase in θ anticipates the follower's optimal reply date, and therefore reduces the leader's efficiency advantage period.[16]

The relatively high spillover bears important consequences also for the intermediate interval. Delaying for Δ periods his entry, the follower obtains large benefits in terms of fixed costs, which moves upward his value function. The increase in the inter-firm spillover parameter involves also a second effect, namely the rise in $\bar{T}$. As already underscored, a larger benefit from imitation postpones the undertaking of a line of action that implies the renounce to the benefit itself. These two effects imply that – in the early stages of the interval $(T_F^* - \Delta, \bar{T}]$ – the innovation follower enjoys a payoff larger than the first mover's one, while in later stages the innovation leader grasps higher payoffs. We denote by T^{ip}, the unique solution for the equation $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$. (The superscript stands for 'intersection point').

An increase in θ benefits the follower, and therefore augments $V_F(t_L, t_L + \Delta)$, postponing T^{ip}. In Figure 2, Panel (B$_1$), we plot the value functions for a relatively low spillover, so that T^{ip} is lower than $\hat{T}_L$, the date at which $V_L(t_L, t_L + \Delta)$ is maximum. When instead the spillover is substantial, $V_L(t_L, t_L + \Delta)$ reaches $\hat{T}_L$ at a date earlier than T^{ip}, as in Figure 2, Panel (B$_2$). As we shall argue in the next section, this may lead to second-mover advantage games.

Having definined

$$\theta'''(\Delta) = 1 - \frac{[(6A+3x)\rho + 4Ar](e^{-r\Delta} - 1) + (2A+x)r}{r[4(A+x) - (2A+3x)e^{-r\Delta}]} e^{(r+\rho)\Delta},$$

[16] Formally, we have that $\partial V_L(t_L, T_F^*)/\partial T_F^* > 0$.

that can be traced in Figure 3, we formally present the above arguments in:[17]

Proposition 5 *(a) When* $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, *the maximum for leader's value function in the interval* $[0, \bar{T}]$ *lies in the sub-interval* $(T_F^* - \Delta, \bar{T}]$;
(b) when $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$, *at* T^{ip} *the leader's value function has not reached its maximum;*
(c) when $\theta \in (\theta'''(\Delta), \bar{\theta}]$, *at* T^{ip} *the leader's value function is non-increasing.*

Proof: See the Appendix.

To complete the discussion of the parameter space, we need to consider the small area $\theta \in [\theta''(\Delta), \theta^*(\Delta))$ (refer to Figure 3). There, the spillover is sufficiently low to allow $V_L(t_L, T_F^*) \geq V_F(t_L, T_F^*)$ for some $t_L \in [0, T_F^* - \Delta]$. On the other hand, θ is high enough to induce a second mover advantage for some $t_L \in (T_F^* - \Delta, \bar{T}]$. In this case, the firms' value function are depicted in Figure 4.

[Figure 4 about here]

Eventually, we consider the time interval $t_L \in (\bar{T}, \infty)$.

In this case, the R&D cost is so low that it is optimal for the second firm to immediately enter upon his rival's investment, without exploiting the inter-firm spillover. Accordingly, the first firm is aware that—as soon as she innovates—the second firm will immediately follow her decision, and invest. Hence, each firm takes her decision anticipating such a follower's behavior. This leads to an equilibrium where the two firms maximize their joint payoff: knowing that it will be immediately followed, each firm delays its innovation until its discounted sum of profits reaches its maximum. In this context, where firms remain symmetric, the maximization of a single firm's payoff coincides with their joint maximization.

When the leader decides to invest "late" she knows that – as soon as she innovates – the rival firm immediately sinks the innovation cost. Accordingly, the payoff for both the first firm is:

$$V_S(t_S, t_S) = \frac{A^2}{9r} + \frac{(2A + x)x}{9r} e^{-rt_S} - \gamma x e^{-(r+\rho)t_S}, \tag{15}$$

where S stands for 'symmetric'.

[17] It is possible to show that $\theta'''(0) = 0$, and that $\partial\theta'''(\Delta)/\partial\Delta\big|_{\Delta=0} > 0$.

Maximization of (15) with respect to t_S under the constraint $t_S \geq \bar{T}$ yields that the first firm optimal timing is:

$$T^{le} = \begin{cases} \bar{T} & \text{if } \theta > 1 - e^{(r+\rho)\Delta}\left[1-(r+\rho)\frac{1-e^{-r\Delta}}{r}\frac{4A}{2A+x}\right] \\ -\frac{1}{\rho}\ln\left(\frac{2A+x}{\gamma(r+\rho)}\right) & \text{if } \theta \leq 1 - e^{(r+\rho)\Delta}\left[1-(r+\rho)\frac{1-e^{-r\Delta}}{r}\frac{4A}{2A+x}\right] \end{cases},$$

where the superscript stands for 'late'.

$V_S(t_S, t_S)$ tends to display an inverted-U shape. When t_S is close to $\bar{T}$, the reduction over time in the fixed cost for the innovation prevails over the reduction in the value function due to the delay in the attainment of the post-innovation higher profits. However, when $\bar{T}$ is high, which is the case if θ is large, the second effect prevails for $t_S \in (\bar{T}, \infty)$, and the value function is decreasing. In Figures 1, 2 and 4 we have depicted the case in which $V_S(t_S, t_S)$ takes its standard inverted-U shape.

5 The market equilibrium

In this Section we discuss the equilibrium in the non-cooperative R&D game. Subgame perfectness is the natural criterion to apply in these contexts. As in many dynamic games, we restrict our attention to pure strategies. Accordingly, before applying subgame perfectness, we need to introduce the assumptions that allow us to disregard mixed strategies.

As already mentioned, in our set-up, only one research project is available to the firms: hence, the choice to innovate at time t is an irreversible stopping decision. Therefore, our model belongs to the class of symmetric timing games, which can be divided into two sub-classes, depending upon which firm (the one that moves first or the one that moves second) obtains the higher payoff.

We can make this point more precise, by assuming for the moment that we have exogenously assigned the task of moving first to one of the two firms. In this case, there is a first mover advantage if the firm that must move first obtains the higher payoff. If, instead, the first mover obtains the lower payoff, there is a second mover advantage. Obviously the first mover is assumed to behave optimally, choosing the innovation time that maximizes its payoff, given the second mover optimal choice.

To deal with first mover advantage games, we drop the hypothesis of exogenously assigned roles and we follow Hoppe and Lehman-Gruber (2005) assuming that:

Assumption 4 : if the two firms – at t – are indifferent between the roles of the first or of the second mover, then each firm aims at becoming the leader. Each firm is randomly selected with probability 1/2 as holding the right of moving first at t, while the other firm may postpone its adoption.[18] If the leader is indifferent between adopting at time t or later, then it chooses t.

Assumption 4 is used to rule out, as it happens in most of the literature, the possibility of coordination failures as an equilibrium outcome. In other words, firms do not choose to move at the same instant of time if they know that they would regret this choice afterwards.[19]

In dealing with second mover advantage games, we assume that the equilibrium is driven by expectations, and we make the following

Assumption 5 : Whenever the innovation leader payoff is lower than the second comer's, one firm – randomly chosen with probability 1/2 – believes that the other one never enters first.

The above hypothesis (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out the mixed-strategies equilibria, often referred to as a war of attrition (Fudenberg and Tirole (1991)). However−if we reject Assumption 5 −our firms would start to randomize at $\hat{T}_L$, obtaining, in every instant of time an expected payoff equal to the leader's one. Hence, the rejection of Assumption 5 leads – in the second mover advantage cases – to the attainment of equilibria implying later adoption dates but the same expected payoff for the leader than the one we study. In what follows, it will become apparent that removing Assumption 5 is harmless for our results.

Notice that Fudenberg and Tirole (1985) argue that the most reasonable outcome is the equilibrium that Pareto-dominates the others. In our case, Pareto ranking implies that all firms prefer the pure strategy equilibrium involving an advantage for the follower.

Subgame perfectness requires that the equilibrium must survive all the possible off-equilibrium deviations. Accordingly we need to compare the leader's payoff at any candidate equilibrium with her payoff at any instant earlier than the one that is part of the proposed equilibrium. If we can find an instant at which the leader's payoff is higher than the discounted value of her payoff at the candidate equilibrium, the leader prefers to invest at that date rather than to wait for the proposed equilibrium, which therefore is not subgame perfect. When the leader's payoff is higher than the follower's one, we also need to take into account the possibility of preemption by the follower. This follows from the fact that the roles of the leader and of the follower are not pre-assigned: if

[18] Adoption by one firm may result in an instantaneous follow-on adoption by the other firm, i.e. the two firms adopt 'consecutively but at the same instant of time', obtaining the same payoff.

[19] From a technical standpoint – as Hoppe and Lehman-Gruber (2005) remark – an equilibrium involving coordination failures cannot be obtained in the case of a continuous-time game without a grid, in which equilibria are defined to be the limits of discrete-time mixed strategies (Fudenberg and Tirole (1985), and (1991)).

the follower's payoff is lower than the leader's one, the former has an incentive to preempt the latter, becoming the leader.

The logic to obtain the unique subgame perfect equilibrium in first-mover advantage games can be described by exploiting Panel (A_1) in Figure 1. When both firms invest simultaneously at T^{le}, they obtain $V_S(T^{le}, T^{le})$. However, the leader would like to adopt first at T_L^*, the date at which her discounted payoff is at its maximum. But the roles of innovation leader and follower are not pre-assigned. Hence, when the second firm knows that the other will adopt at time T_L^*, it is in his interest to preempt at time $T_L^* - dt$. By backward induction, we conclude that the equilibrium strategy for the first innovator is to invest as soon as the leader's payoff is equal to the follower's one (i.e. at $\underline{T}_L$). (Assumption 4 grants us that each firm has a 50% chance of being the first innovator, and that only one firm invests at $\underline{T}_L$.) Notice that the preemption argument spelled out above yields equal payoffs to the two firms in the subgame perfect equilibrium. Hence, in this case the equilibrium involves rent dissipation.

To conclude that the SPNE dictates to the leader to invest at $\underline{T}_L$, and to the follower at T_F^* in the cases portrayed in Figure 1, it is not needed that $V_L(T_L^*, T_F^*) > V_S(T^{le}, T^{le})$. Actually, it is sufficient to find a $V_L(t_L, t_L + \Delta) > V_S(T^{le}, T^{le})$. When this is the case, it is in the first mover's interest to deviate from $\{T^{le}, T^{le}\}$, and backward induction leads to the rent dissipation equilibrium $\{\underline{T}_L, T_F^*\}$. (Refer to Figure 1, Panel (A_2)).

As an example of second-mover advantage game consider Panel (B_2) in Figure 2. Investing simultaneously at T^{le}, both firms obtain $V_S(T^{le}, T^{le})$. However, $V_L(\hat{T}_L, \hat{T}_L + \Delta) > V_S(T^{le}, T^{le})$. Hence, by Assumption 5 , the firm that believes that the other one never enters first chooses $t_L = \hat{T}_L$; the other firm has no incentive to preempt its rival before date t_L.

Having clarified the equilibrium concept, we may now exploit the results obtained in the previous Sections to qualify the SPNE.

When $\theta \in [\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, Propositions 1 and 5 imply that the SPNE is either T^{le}, or it is in the intermediate interval $(T_F^* - \Delta, \bar{T}]$. The equilibrium is at T^{le} (i.e. it is "late") if $V_S(T^{le}, T^{le}) > V_L(\hat{T}_L, \hat{T}_L + \Delta)$. If, on the contrary, $V_L(\hat{T}_L, \hat{T}_L + \Delta) \geq V_S(T^{le}, T^{le})$, it is "intermediate" (i.e. the entry dates belong to the interval $(T_F^* - \Delta, \bar{T}]$). Moreover, from Proposition 5, Part (b), if $\theta \in [\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$, we have that $\hat{T}_L \geq T^{ip}$ so that in the SPNE the first firm invests at T^{ip}, and the second at $T^{ip} + \Delta$. Notice that this equilibrium is of the first mover advantage type, and implies preemption (refer to Figure 2, Panel (B_1)). When $\theta \in [\max\{\theta^*(\Delta), \theta'''(\Delta)\}, \bar{\theta}]$, Proposition 5, Part (c), implies that $\hat{T}_L < T^{ip}$, and the subgame perfect equilibrium is of the second-mover advantage type (refer to Panel (B_2) in Figure 2). As intuition suggests, if θ gets higher for a given disclosure lag, the candidate equilibrium shifts from the first-mover advantage type to the second-mover one: the higher is the spillover, and hence the lower is the fixed cost for the follower, the more likely is that his payoff is higher than the leader's one. The difference between our second-mover's advantage equilibria and Hoppe's (2000) one lies in the fact that – in our model – the information spillover takes time to materialize; accordingly the entry is sequential. Of course, the presence of a disclosure lag limits the area in which the second-mover advantage prevails. When Δ shrinks to zero, our analysis converges to Hoppe's one.[20]

If $\theta \in [0, \theta''(\Delta)]$, the SPNE is either T^{le}, or it prescribes to the leader the adoption in $[0, T_F^* - \Delta]$ (and hence it is "early"). The equilibrium is late (at T^{le}) when $V_S(T^{le}, T^{le})$ is larger than the leader's maximum deviation payoff in $[0, \bar{T}]$. When this is not the case, Propositions 1, 2, and 4 guarantee that the SPNE is the preemptive equilibrium in which the leader adopts at $\max\{0, \underline{T}_L\}$, and the follower adopts at T_F^*. Both Panels in Figure 1, provide examples of this equilibrium.

Finally, if $\theta \in [\theta''(\Delta), \theta^*(\Delta))$, we have three candidate equilibria. Not surprisingly, there is – as usual – the simultaneous entry date that maximizes the firms' joint payoff. Moreover, the spillover is sufficiently low to allow for a candidate equilibrium in $[0, T_F^* - \Delta]$, where the advantage for the innovation leader is still high. Nonetheless, θ is high enough to induce – for some $t_L \in (T_F^* - \Delta, \bar{T}]$ – an higher payoff for the follower. Accordingly, we have a candidate SPNE also in $(T_F^* - \Delta, \bar{T}]$ (refer to Figure 4). As before, the candidate equilibrium in the intermediate interval $(T_F^* - \Delta, \bar{T}]$ is of the first-mover advantage type if $\hat{T}_L \geq T^{ip}$, while it involves a second-mover advantage when $\hat{T}_L < T^{ip}$.

5.1 Numerical results

The determination of the SPNE as a function of the parameters cannot be performed analytically, due to the high degree of non linearity in our model. Hence, we now present some numerical results.[21]

In our simulations, we normalize to unity the market dimension parameter A, and we fix the discount rate r to 0.03, which is consistent

[20] From the technical standpoint, consider $\hat{T}_L$ as defined in the Proof for Proposition 5, and notice that the limit for $\Delta \to 0$ of $\hat{T}_L$ converges to $\hat{\tau}_M$ in Hoppe (2000), p. 322.

[21] Our routine has been written in Matlab, and it is based on a discretization of the space $[\theta \text{ x } \Delta]$, for $\theta \in [10^{(-10)}, 0.8]$ and $\Delta \in [10^{(-10)}, 4]$. We have used 300.000 gridpoints, however our results do not relevantly change for any number of evaluation points larger than 15.000. This routine is available upon request from the authors.

with computing calendar time in years. The parameter γ does not play any substantial role (provided that $\gamma \geq \bar{\gamma}$): the effect of an higher γ (i.e. of a less efficient R&D) is to postpone all of the equilibria, without changing their relative convenience. Hence, we choose $\gamma = 50$ with no loss of generality. As for ρ, we study industry-specific rates of reduction in innovation costs. Referring to Cummins and Violante (2002), and to the related literature, one can find estimates of the technical change in sector specific capital goods. The sector in which productivity (of capital goods) has grown at the fastest pace is – not surprisingly – "computers and office equipment", where productivity grew by more than 20% a year in US, for the entire post-war period. Apart from this outlier, the greatest technical change occurred in communications equipment (9% a year), aircraft (8%), and instruments (6%). We then have a 5% change for the "service industry machinery". The productivity growth in all the other sectors is between 0.1% and 3.8% a year. Because a non-negligible share of the productivity increase is retained by the producer, we simulate the model for $\rho \in \{0.01; 0.04; 0.07\}$.

The first value characterizes technologically mature sectors, which still benefits from some technical progress in the sectors producing their machinery. We label this sectors as Industry I. In industry II, $\rho = 0.04$, which is the case of a fairly dynamic sector. Finally, Industry III is a frontier sector, where $\rho = 0.07$.

As mentioned in Section 2, to preserve the duopolistic structure of our market, we consider only non-drastic innovation. Hence, the size of the R&D output, x, is lower than A ($x < 1$). We investigate two types of innovative output: a minor innovation where $x = 0.05A(= 0.05)$ and a major innovation where $x = 0.5A(= 0.5)$.[22]

Because the lower is x, the lower is also $\bar{\Delta}$, we compute $\lim_{x\to 0} \bar{\Delta}$ for $\rho \in \{0.01; 0.04; 0.07\}$, and we find that it is equal to {23.105, 7.438, 4.451}. Hence, the restriction implied by Assumption 2 is realistic in most contexts.

Figure 5 portrays the equilibria arising in the case of a minor innovation. Panel (a) highlights that in Industry I a low spillover implies, for a given Δ, a late equilibrium, while as the spillover increases the intermediate equilibrium prevails. For instance, when $\Delta = 2.5$, (refer to Table 1) the late equilibrium prevails when $\theta \leq 0.058$, while if $\theta > 0.058$ we have the intermediate equilibrium.

[Figure 5 about here]

The intuition for this result is the following: as underscored by Fu-

[22] We have verified that reasonable perturbations in r, ρ, and x do not significantly affect our results.

denberg and Tirole (1985), the smaller the cost reduction, the weaker is the incentive to innovate first.[23] Hence, a small x means that the highest deviation payoff for an early innovator is low, so that the early equilibrium never prevails over the late one. Moreover, a low spillover gives rise to a late equilibrium because it shrinks the intermediate region, since the second firm has a weak incentive to wait Δ to enjoy a modest R&D cost-reducing spillover (refer to the definition for $\bar{T}$ and to Figure 2). Hence, the late equilibrium prevails over any possible deviation occurring in the intermediate period.

Industry	**Innovation**			
	minor		**major**	
I	$\theta \leq 0.058$	late	$\theta \leq 0.103$	early
	$\theta > 0.058$	intermediate	$\theta > 0.103$	intermediate
II	$\theta \leq 0.061$	late	$\theta \leq 0.111$	early
	$0.061 < \theta \leq 0.069$	early		
	$\theta > 0.069$	intermediate	$\theta > 0.111$	intermediate
III	$\theta \leq 0.061$	late	$\theta \leq 0.136$	early
	$0.061 < \theta \leq 0.082$	early		
	$\theta > 0.082$	intermediate	$\theta > 0.136$	intermediate

Table 1: R&D equilibria ($\Delta = 2.5$)

When θ grows, the intermediate region enlarges, leading to a situation in which the first firm's deviation payoff becomes greater than her late equilibrium payoff. This leads to the prevalence of the intermediate equilibrium.

Panel (b) in Figure 5 shows the equilibria arising in Industry II. Again, for a given Δ, if the spillover is very low, the equilibrium in the R&D stage is the late one, for the reasons explained before. However, as θ increases (but it is still lower than $\theta''(\Delta)$), the early equilibrium prevails. This happens in the small area contained between the two curves exiting from the origin in Figure 5 (refer also to Table 1). To understand this result, bear in mind that an increase in ρ raises the payoffs in the intermediate region, because the R&D costs are lower.[24] The increase in the deviation payoff in the intermediate region destroys the late equilibrium, and moves the equilibrium to the early stage, as shown in Figure 1, Panel (A_1) (refer also to the discussion in Section 5).

[23] This happens because the single innovator profit function, π_1^l, is more convex in x than π_2 (Refer to Eqs. (3) and (5)).

[24] The effect of ρ on the late equilibrium payoff is of course similar, but it is less significant since at that time the R&D cost are already very low.

Finally, a further increases in θ (above $\theta''(\Delta)$), reducing the first innovator's payoff in the early stage, makes the intermediate equilibrium dominant, as shown in Figure 2.

Figure 5, panel (c) shows the equilibrium selection in Industry III ($\rho = 0.07$): we have the same pattern observed for Industry II, with the only difference being that the θ threshold that discriminates the intermediate equilibrium from the early one is higher. This happens because the payoffs are higher in the early region than in the intermediate one. In fact, the former payoffs benefit more from a rapid technical progress.

The case of a major innovation is portrayed in Figure 6, in which $x = 0.50A (= 0.50)$. Here, the late equilibrium never prevails: a high x favors the selection of the early equilibrium, as shown in Fudenberg and Tirole (1985). However, in our framework, an early equilibrium arises only for moderate values of the spillover parameter. In fact, when θ increases so that the intermediate equilibrium exists, the latter prevails for two reasons. First, a high θ negatively influences the first innovator payoffs in the early interval, because it anticipates the follower's investment date (equation (11)). Second, in the intermediate interval, as the spillover increases, the second comer's payoff gets larger, softening the incentive to invest for the leader. This milder competition implies higher payoffs for both firms, inducing the selection of the intermediate equilibrium.

[Figure 6 about here]

In sum, our analysis of the equilibrium selection process suggests that the intermediate equilibrium is the subgame perfect one in large portions of the parameter space.

This may help to explain the results in Schmidt-Dengler (2006). He estimates the determinants of the adoption of equipment for magnetic resolution images, which allows him to disentangle the preemption from the stand alone profit-maximizing effect. He finds that preemption accounts only for a relatively small share of the acceleration of investment timing that caracterizes the duopolistic market solution when compared to the collusive scenario. This is what our model prescribes for $\theta > \theta''(\Delta)$.

6 Welfare analysis

In order to assess the welfare properties of our equilibria, we now design and solve the benevolent planner problem, and then we compare the welfare maximizing investment levels with those realized by the market equilibria. Hence, we determine under which parameter configurations the market implies an excessive investment.

In dealing with the planner problem, we need to introduce some hypotheses.

First, we adopt a second best perspective, assuming that neither the number of firms acting in the market nor the way they compete in the second stage quantity game lies within the regulatory power of the benevolent planner. Therefore, what this non-omnipotent planner can choose, is the timing of innovation.[25] Hence, its decisions will be based on the instantaneous welfare levels, given by Eqs. (2), (4) and (6), that are attained by the Cournot decentralized solution.

Second, the spillover obtained by firms engaging in a joint R&D project at the dates prescribed by the planner, is the same that is grasped by the second entrant when he waits Δ. While this assumption is debatable, it allows us to analyze the classic problem of the internalization of the inter-firm spillover.[26]

Therefore, the social planner maximizes – with respect to the adoption dates t_L and t_F – the following welfare function

$$\begin{aligned} W(t_L, t_F) = & \frac{W_0}{r} + \frac{W_1 - W_0}{r} e^{-rt_L} + \frac{W_2 - W_1}{r} e^{-rt_F} \\ & -\gamma e^{-(r+\rho)t_L} - (1-\theta)\gamma e^{-(r+\rho)t_F}, \end{aligned} \tag{16}$$

where $t_L \leq t_F$ is a natural constraint.

The maximization of (16) yields

$$T_L^{SP} = -\frac{1}{\rho} \ln \left(\frac{(8A + 11x)x}{18\gamma(r+\rho)} \right), \quad T_F^{SP} = -\frac{1}{\rho} \ln \left(\frac{(8A - 3x)x}{18\gamma(r+\rho)(1-\theta)} \right),$$

if $\theta \leq \frac{14x}{8A+11x}$, and

$$T_L^{SP} = T_F^{SP} = T^{SP} = -\frac{1}{\rho} \ln \left(\frac{(8A + 4x)x}{9\gamma(r+\rho)(2-\theta)} \right),$$

[25] This approach is standard in the literature: see Stenbacka and Tombak (1994), Hoppe (2000), and Weeds (2002). The first best equilibrium for an omnipotent planner implies the presence of only one firm: whenever there are non-decreasing returns in the innovation size or probability, it is optimal to have only one firm to innovate and cover the entire market at the marginal (post-innovation) cost.

[26] Our approach implies that the joint R&D activity grants a faster information flow, but not a cost advantage, when compared to a decentralized solution. The spillover parameter is actually unlikely to be significantly increased by an R&D agreement when the innovation costs incorporate large expenditures for the training of the employees required by the new production process, for some new machineries (or for adaptation of the existing plant), and so on (see De Bondt (1996)).

However, in the literature various alternative hypothesis have been discussed (see e.g. Poyago-Theotoky (1999), Hinloopen (2003), Leahy and Neary (2007)). The assumption of an increase in θ will be briefly discussed later.

if not. The superscript SP stands for "social planner".

To verify whether the decentralized solution induces overinvestment in comparison with the centralized one, we weight the discounted (to 0) innovation costs implied by the subgame perfect market solution against those obtained by the social planner. When the market innovation costs are higher (lower) than the planner solution ones, there is overinvestment (underivestment). The difference in the firms' timings between the centralized and the decentralized solutions adds to the inefficiency due to the use of a non-optimal amount of resources.

Because the market game often does not have a closed form solution, to appreciate the differences in the discounted innovation costs, we need to rely on numerical simulations, which allow to obtain the following results:

i) Whenever the early equilibrium prevails, the market solution implies an excessive use of resources (i.e. overinvestment).

ii) Symmetrically, when the late equilibrium is subgame perfect, the decentralized solution involves a too low level of investment.

iii) When the intermediate equilibrium dominates, it implies underinvestment, but for a small parameters sub-set, in which the size of the innovation is small, and the speed of the exogenous technical progress is high.

While the first two results are intuitive, the third deserves more attention.

To understand why an overinvesting intermediate equilibrium is possible only if the innovation size is small, consider the Eqs. (1-6), which show that both the instantaneous social welfare, and the firms profits increase more than proportionally with the size of the innovation. Because the social welfare is larger than the firms profit, also the wedge between the social and the private incentives to innovate increases with x, which acts against the possibility of overinvestment with a large innovation.

An increase in ρ reduces both the social planner's optimal adoption date(s) and the intermediate equilibrium ones. In the market game a steeper cost reduction profile, benefits, *ceteris paribus*, the leader, who pays the full cost, more than the follower. This provides an incentive for his preemptive behavior, which leads to overinvestment for low values of the spillover.[27]

[27] When the spillover is high, in fact, the internalization of the spillover has strong positive effects on welfare.

Accordingly, the portion of the parameter space with overinvestment is the wider, the larger is ρ, and the lower is x. However, even in this case, the overinvestment area is very small: e. g. for $\rho = 0.09$, $x = 0.05$, and $\Delta = 2$, the intermediate equilibrium implies overivestment for $\theta \in [0.080, 0.115]$.[28]

Hence, not only the intermediate equilibrium prevails for most of the parameter configurations (as shown in Sub-section 5.1), but it also implies that the duopolistic market equilibrium involves underinvestment. This applies even when the innovation size is large, and hence the incentives to hasten innovation are remarkable. Therefore, the market equilibrium calls for public policies aimed at increasing the research activity even in this case, unless the inter-firm spillover is very low. Notice that the natural indicators of a highly competitive environment, namely a diffusion equilibrium and rent equalization, do not necessarily imply that the R&D investment is excessive from the social planner's perspective.

When we focus on minor innovations – the case in which the market equilibrium underinvests, according to the earlier literature – our results imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, because the underinvesting intermediate equilibrium is closer to the social optimum than the late equilibrium.[29]

7 Conclusions

In our duopoly game, firms, in addition to a technological externality, takes into account a spillover that lowers the second comer's innovation cost. This spillover exerts its effect after a "disclosure lag". In this setting, a new equilibrium arises, in which the R&D investment takes place at intermediate dates in comparison with those already identified in the literature.

Preemption, R&D diffusion, and the possibility of rent equalization characterize the intermediate equilibrium, which is competitive, although in a mild form. The intermediate equilibrium is subgame perfect for a large range of the parameters set; moreover, it is socially inefficient,

[28] Notice that Assumption 5 applies only if $\theta > \theta'''(\Delta)$, i.e. when the intermediate equilibrium already implies underinvestment. Hence, it is not crucial for these results.

[29] Suppose that a joint R&D activity guarantees not only a faster but also an easier, and hence less costly, information flow. In this case the spillover parameter in Eq. (16) should be higher than in the market game, and the social planner should dictate earlier investment date(s). Under this alternative assumption, result ii) is unaffected, result iii) is strenghtened, because it applies for even larger parameters set, while result i) weakens. In fact, it is possible – for a sizeable (and somehow irrealistic) increase in θ – that the second best optimal timing anticipates the early equilibrium ones.

implying a low level of investment in R&D.

This happens even in presence of major innovations, despite the large incentive to invest in R&D provided by this type of innovation. This result has important implications for innovation policy. For example, research joint ventures should be assessed in more favorable terms than those implied by the literature following d'Aspremont and Jacquemin (1988), and Kamien, Muller, and Zhang (1992). In fact, while a RJV may underinvest in comparison to an highly competitive equilibrium, it is likely to improve social welfare over a 'mildly competitive', underinvesting, market outcome. Furthermore, our paper suggests that R&D subsidies should be set in place in a range of market configurations wider than that has been previously proposed. Finally, our analysis provides an argument against the use of entry regulations (or price caps), which are sometimes used to slow technology adoption, e.g. in telecommunication industries. We leave the analysis of these policy instruments for further research.

When the innovation size is small, the prevalence of the intermediate equilibrium implies that R&D enhancing policies must be less intense than devised in the earlier literature. Actually, policies designed without taking into account the inter-firm spillover can be largely oversized, even when the spillover is quantitatively modest. Notice also that the intermediate equilibrium calls for moderate policies, which may prove easy to implement from a political economy perspective.

Our setting can be extended in various directions, which however, would require an heavy use of numerical techniques. For example, it would be interesting to consider a stochastic inter-firm spillover, in which the probability of information diffusion depends upon the time elapsed from the introduction of the innovation, and on the follower's imitation effort. Also, we would like to consider the possibility that the leader actively (and hence costly) attempts to prevent information leakages, thereby increasing the disclosure lag. Whenever the combined effects of the firms' efforts lenghten this lag, they reduce the follower's equilibrium payoff, and hence, also the leader's one. Therefore, they tend to reduce the intermediate equilibrium dominance area. However, the numerical analysis developed in Sub-section 5.1 suggests that the effect of the disclosure lag on the dominance areas are weak. Hence, our main result should not be undermined by the adoption of a richer framework.

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A Appendix A: Proofs

Proof of Proposition 1

As a preliminary, notice that Assumptions 2 and 3 guarantees that the interval $[0, T_F^* - \Delta]$ is non empty for $\theta \in [0, \bar{\theta}], \Delta \in [0, \bar{\Delta}]$. Notice, moreover, that $\bar{T} > T_F^* - \Delta$ for $\theta \geq 0$.

Proof of part (a). The payoff at time 0 for the second firm, when it invests at t_F, is given by (10).

Suppose that the innovation leader has sunk the innovation cost at time $t_L \in [0, T_F^* - \Delta]$, and that the second comer decides to wait more than Δ, to grasp the inter-firms spillover. In this case, according to Eq. (8) the innovation cost is $C_F(t_F) = (1-\theta)\gamma e^{-\rho t_F}$, and a few straightforward calculations show that T_F^*, as given by (11), maximizes $V_F(t_L, t_F)$.

Alternatively, the second comer could decide not to wait for Δ periods, and in this case he should invest at (13). This second alternative requires that $T_F' \in [t_L, t_L + \Delta)$. Had the latter restriction not been satisfied, the innovation follower would have benefited from the spillover. Since $T_F' > T_F^*$, whenever $t_L \in [0, T_F^* - \Delta]$ the innovation follower grasps the imitation benefits and invests at T_F^*.

Because of this, his payoff can be written as:

$$V_F(t_L, T_F^*) = \frac{A^2}{9r} + \left(\frac{2A - x}{9r}\right) e^{-rt_L} + \frac{4A\rho}{9r(r+\rho)} \left[\frac{4A}{9\gamma(r+\rho)(1-\theta)}\right]^{\frac{r}{\rho}},$$

which implies: $\frac{\partial V_F(t_L, T_F^*)}{\partial t_L} > 0$, and $\frac{\partial^2 V_F(t_L, T_F^*)}{(\partial t_L)^2} < 0$ in the whole interval $[0, T_F^* - \Delta]$. Also notice that $\frac{\partial V_F(t_L, T_F^*)}{\partial \theta} > 0$ for every $t_L \in [0, T_F^* - \Delta]$. (This explains the behavior of $V_F(t_L, T_F^*)$ for $t_L \in [0, T_F^* - \Delta]$ in Figures 1, 2, and 4)

Proof of part (b). When $t_L > T_F^* - \Delta$, the innovation follower will never wait more than Δ, simply because $t_L > T_F^* - \Delta$. Hence, his available strategies are:

(1) wait exactly Δ periods to grasp the benefit of the spillover,

(2) invest immediately after the innovation leader, and

(3) wait for a time span shorter than Δ (to exploit the exogenous technological externality), and then invest (therefore, without exploiting the inter-firm spillover).

First we compare what the innovation follower obtains by waiting Δ periods (strategy 1) with what he gets by investing immediately after the innovation leader (strategy 2). Hence, we determine when $V_F(t_L, t_L + \Delta) \geq V_F(t_L, t_L)$. This inequality immediately boils down to:

$$\frac{4A}{9r} e^{-rt_L} - (1-\theta)\gamma e^{-(r+\rho)(t_L+\Delta)} \geq \frac{4A}{r} e^{-r(t_L+\Delta)} - \gamma e^{-(r+\rho)t_L},$$

which, in its turn, is satisfied when: $t_L \leq \bar{T}$. Hence, the innovation follower never chooses to immediately follow the leader for any $t_L \in (T_F^* - \Delta, \bar{T}]$.

Next, we compare strategy 1 with strategy 3; this comparison will be carried out for $t_L \in [T_F^* - \Delta, T_F' - \Delta]$ first, then for $t_L \in (T_F' - \Delta, T_F']$, and finally for $t_L \in (T_F', \bar{T}]$.

As a preliminary, notice that the inequality $\bar{T} \geq T_F'$ is satisfied for $\theta \geq \theta'(\Delta)$.

Suppose now that the leader invests at $t_L \in [T_F^* - \Delta, T_F' - \Delta]$. In this interval, the payoff function for a follower who does not exploit the inter-firm spillover is always increasing. In fact, this function is concave with a global maximum at $t_F = T_F' \ \forall \ t_L$. Hence, it is optimal for the follower to invest with a delay not lesser than Δ, which implies that the spillover is actually exploited.

When $t_L \in (T_F' - \Delta, T_F']$, the optimal strategy for the innovation follower must be determined by comparing what it gets by delaying its investment for Δ periods, with what can be obtained by investing at T_F'. Hence, we need to determine when $V_F(t_L, t_L + \Delta) - V_F(t_L, T_F') \geq 0$. This inequality immediately boils down to:

$$\frac{4A}{9r}\left[e^{-r(t_L+\Delta)} - e^{-rT_F'}\right] - \gamma\left[(1-\theta)e^{-(r+\rho)(t_L+\Delta)} - e^{-(r+\rho)T_F'}\right] \geq 0. \tag{A.1}$$

It is easy to show that the left hand side of (A.1) is non-increasing in t_L in the whole interval $(T_F' - \Delta, T_F']$. Evaluate equation (A.1) at $t_L = T_F'$, and–exploiting equation (13)–substitute out T_F' when convenient, to obtain:

$$e^{-rT_F'}\frac{4A}{9r}\left[e^{-r\Delta} - 1 - r(1-\theta)\frac{e^{-(r+\rho)\Delta}}{r+\rho} + \frac{r}{r+\rho}\right] \geq 0,$$

which is fulfilled when $\theta \geq \theta'(\Delta)$. Hence, under this restriction, the follower's strategy of waiting Δ periods is chosen for any $t_L \in (T_F' - \Delta, T_F']$.

Finally, strategy 3 can never be optimal for $t_L \in (T_F', \bar{T}]$ simply because the payoff function for a follower who does not exploit the spillover is decreasing in $t_F \in (t_L, \bar{T}]$ and thus there is no point in waiting when the leader has already invested; recall moreover that the immediate investment strategy has already been proven to be dominated by a time Δ delay.

Proof of Part (c).

The Proof of Part (b) implies that the innovation follower will never wait Δ, for any $t_L \geq \bar{T}$. Hence, if $t_L \in (\bar{T}, \infty)$, his available strategies are:

(1) invest immediately after the innovation leader, and

(2) wait for a time span shorter than Δ (to exploit the exogenous technological externality), and then invest without exploiting the inter-firm spillover.

The Proof of Part (b) implies that – when the innovation follower decides to wait for a time span shorter than Δ – he invests at T_F' for any $t_L \in (T_F' - \Delta, T_F']$. In fact, the payoff function for the follower, $V_F(t_L, t_F)$ has a maximum at T_F'. We have already noticed that, for $\theta > \theta'(\Delta)$, $\bar{T} \geq T_F'$. Hence, under this parameter restriction, the second innovator invests immediately after the innovation leader. In fact, it is never in the follower's interest to wait Δ periods, because $t_L > \bar{T}$, while $V_F(t_L, t_F)$ is decreasing in t_F in the whole interval $t_L \in [\bar{T}, \infty)$. Hence, the follower has no point in waiting.

This completes the Proof. ■

Proof of Proposition 2

The Proof for Part (a) follows the corresponding one for Proposition 1.

Proof for Parts (b) and (c). From the Proof for Proposition 1, we already know that when $\theta \in [0, \theta'(\Delta))$, then $\bar{T} < T_F'$. Notice, also, that it is possible to prove that $T_F' - \Delta < \bar{T}$.

In the time interval $t_L \in [T_F^* - \Delta, T_F' - \Delta]$ the optimal strategy is again to wait Δ and exploit the inter-firm spillover, because the follower's payoff function $V_F(t_L, t_F)$ is increasing in $t_F \in [t_L, T_F' - \Delta]$.

When $t_L \in (T_F' - \Delta, \bar{T}]$, the optimal strategy for the innovation follower must be determined by comparing what he gets by delaying his investment for Δ periods with what can be obtained by investing at T_F'. Unfortunately, it is not possible to characterize analytically the sub-intervals in which the two alternative strategies prevail. Let us denote by $\breve{T}_L$ the instant when $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$. $\breve{T}_L \in (T_F' - \Delta, \bar{T}]$ because: $V_F(t_L, t_L + \Delta) - V_F(t_L, T_F')$ is non-increasing in t_L; $\lim_{\epsilon \to 0} [V_F(T_F' - \Delta + \epsilon, T_F' + \epsilon) - V_F(T_F' - \Delta + \epsilon, T_F')] > 0$ and $V_F(\bar{T}, \bar{T} + \Delta) - V_F(\bar{T}, T_F') < 0$, in fact, by definition, $V_F(\bar{T}, \bar{T} + \Delta) = V_F(\bar{T}, \bar{T})$, and $V_F(\bar{T}, \bar{T}) < V_F(\bar{T}, T_F')$. Hence, for $t_L \in (T_F' - \Delta, \breve{T}_L]$ strategy (1) is optimal, while for $t_L \in (\breve{T}_L, T_F']$ the innovation follower decides to innovate at T_F' (strategy 3).

Notice that Assumption 1 guarantees that $V_F(t_L, t_L + \Delta)$ has a maximum in $(T_F^* - \Delta, T_F']$.

Proof for Part (d). Because $\bar{T} < T_F'$, when $t_L \in (T_F', \infty)$, the innovation follower invests immediately after the innovation leader because its payoff function is decreasing in t_F. ■

Proof of Corollary 3

$\breve{T}_L$ has been defined as the time instant such that $V_F(t_L, t_L + \Delta) = V_F(t_L, T_F')$. Substitute $\breve{T}_L$, and T_F' in the value function (10), and consider the cost function (8) with $\theta = 0$, to conclude that $\breve{T}_L + \Delta = T_F'$. Because in this case $T_F^* = T_F'$, the Proof is completed. ■

Proof of Proposition 4

Proof for Part (a). For $t_l \in [0, T_F^* - \Delta]$, the innovation leader payoff is given by (9) in which the innovation costs are provided by (7) and $t_F = T_F^*$.

Exploiting equation (11), we obtain:

$$V_L(t_L, T_F^*) = \frac{A^2}{9r} + \left[\frac{4(A+x)x}{9r} - \gamma x e^{-\rho t_L}\right] e^{-rt_L} - \frac{(2A+3x)x}{9r}\left(\frac{4A}{9\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}.$$

A few calculations show that the restriction $\theta \leq \theta''(\Delta)$, implies: $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$.

Notice also that $\frac{\partial V_L(t_L, T_F^*)}{\partial t_L} \gtrless 0$ when $t_L \lesseqgtr T_L^*$ (with T_L^* given by (14)).

Proof for Part (b). When $\theta \in [\theta'(\Delta),\ \theta''(\Delta))$, the first innovator is aware of the fact that – if she invest later than $T_F^* - \Delta$– her competitor will invest with a delay of Δ periods (Proposition (1), Part (b)). In this case one can show that the unique solution for the equation $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$, $T^{ip} = -\frac{1}{\rho}\ln\left(\frac{(6A+3x)(1-e^{-r\Delta})}{9\gamma r[1-(1-\theta)e^{-(r+\rho)\Delta}]}\right)$, lies outside the interval $[T_F^* - \Delta, \bar{T}]$ (namely $T^{ip} < T_F^* - \Delta$). Hence, the follower's payoff is lower than the first innovator's one for $t_L \in (T_F^* - \Delta, \bar{T}]$.

Proof for Part (c). If $t_L \in (T_F^* - \Delta, \breve{T}_L]$, because $T^{ip} < T_F^* - \Delta$, it is obvious that $V_L(t_L, t_L + \Delta) > V_F(t_L, t_L + \Delta)$ for any $t_L \in [T_F^* - \Delta, \bar{T}]$ and hence, a fortiori, for any $t_L \in [T_F^* - \Delta, \breve{T}_L]$.

When $t_L \in (\breve{T}_L, T_F']$, the follower innovates at time T_F' (Proposition 2, Part (c)). In this case, we have again that $V_L(t_L, T_F') \geq V_F(t_L, T_F')$ for $t_L \in [T_F' - \Delta, T_F']$ (the equality applies at $t_L = T_F'$). To show this, consider first that $V_L(T_F', T_F') = V_F(T_F', T_F')$, and that $\partial[V_L(t_L, T_F') - V_F(t_L, T_F')]/\partial t_L < 0$. Then, notice that $V_L(T_F^* - \Delta, T_F^*) \geq V_F(T_F^* - \Delta, T_F^*)$, which implies $V_L(T_F^* - \Delta, T_F') \geq V_F(T_F^* - \Delta, T_F')$, and therefore $\frac{A^2}{9r} + \frac{4(A+x)x}{9r}e^{-r(T_F^* - \Delta)} - \frac{(2A+3x)x}{9r}e^{-rT_F'} - \gamma e^{-(r+\rho)(T_F^* - \Delta)} \geq \frac{A^2}{9r} - \frac{(2A-x)x}{9r}e^{-r(T_F^* - \Delta)} + \frac{4Ax}{9r}e^{-rT_F'} - \gamma e^{-(r+\rho)T_F'}$. Because the two last functions intersect only twice (and they are identical at T_F', where the weak inequality above reduces to $V_L(T_F', T_F') = V_F(T_F', T_F')$), we have that, in the whole interval $[T_F^* - \Delta, T_F']$, and hence, a fortiori, in the interval $(\breve{T}_L, T_F']$, $\frac{A^2}{9r} + \frac{(2A+x)x}{9r}e^{-rt_L} - \frac{(2A+3x)x}{9r}e^{-rT_F'} - \gamma e^{-(r+\rho)t_L} \geq \frac{A^2}{9r} - \frac{(2A-x)x}{r}e^{-rt_L} + \frac{4Ax}{9r}e^{-rT_F'} - \gamma e^{-(r+\rho)T_F'}$.

Accordingly, the follower's payoff is lower than or equal to the first innovator's one for $t_L \in (T_F^* - \Delta, T_F']$.■

Proof of Proposition 5

As a preliminary, notice that $\theta'''(\Delta) \geq \theta''(\Delta)$, $\Delta \in [0, \bar{\Delta}]$.

Proof of Part (a). Recall that, for $t_L \in [0, T_F^* - \Delta]$, the innovation follower invests at T_F^* (Proposition 1), and the leader is aware of this behavior. Notice that, if $\theta \geq \theta^*(\Delta)$, we have that $T_L^* \geq T_F^* - \Delta$, because the latter inequality requires $(A+x) \leq \frac{A}{(1-\theta)}e^{\rho\Delta}$, and hence $\theta \geq 1 - \frac{A}{A+x}e^{\rho\Delta}$. Because $\theta \geq \theta''(\Delta)$, we have that $V_L(T_F^* - \Delta, T_F^*) \leq V_F(T_F^* - \Delta, T_F^*)$. Hence, for $t_L \in [0, T_F^* - \Delta]$, $V_L(t_L, T_F^*)$ is increasing, moreover, when it reaches its highest value, namely $V_L(T_F^* - \Delta, T_F^*)$, the first firm payoff is still lower than the second one.

When $t_L \in (T_F^* - \Delta, \bar{T}]$, the follower innovates with a delay of Δ. Accordingly, the first innovator's payoff for $t_L \in [T_F^* - \Delta, \bar{T}]$ is:

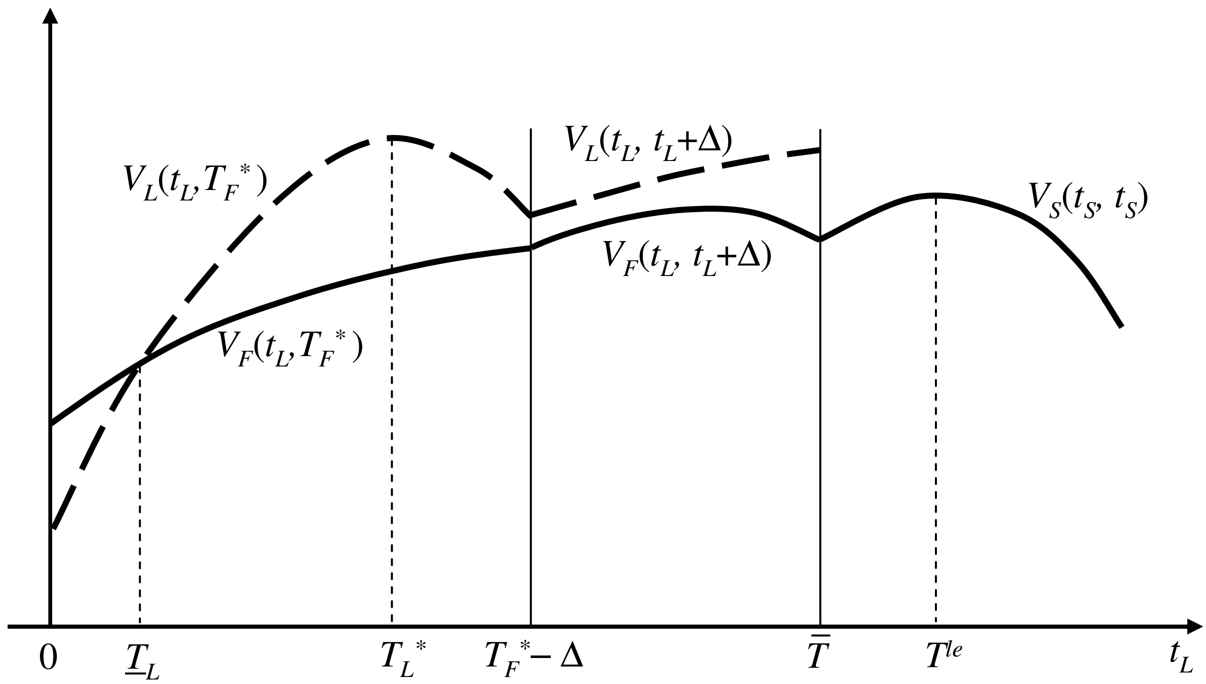
$$V_L(t_L, t_L + \Delta) = \frac{A^2}{9r} + \left\{ \left[\frac{4(A+x)x}{9r} - \gamma e^{-\rho t_L} \right] - \frac{(2A+3x)x}{9r} e^{-r\Delta} \right\} e^{-rt_L}.$$

Notice that $V_L(t_L, t_L + \Delta)$ reaches its maximum at

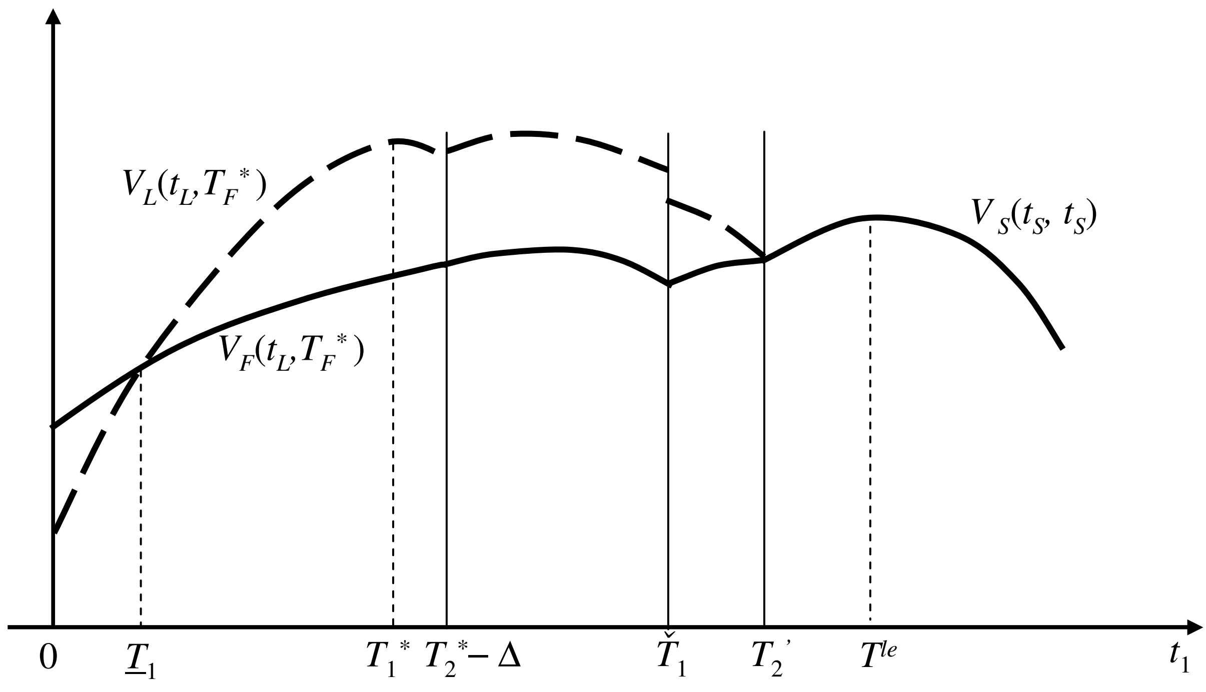
$$t_L = \hat{T}_L = -\frac{1}{\rho} \ln \left[\frac{4(A+x) - (2A+3x)e^{-r\Delta}}{9\gamma(r+\rho)} \right];$$

a few calculations now allow to show that $\hat{T}_L > T_F^* - \Delta$. Hence, the maximum for the first firm value function in the interval $[0, \bar{T}]$ lies in the sub-interval $(T_F^* - \Delta, \bar{T}]$.

Proof of Parts (b) and (c). For $\theta \geq \max\{\theta^*(\Delta), \theta''(\Delta)\}$, notice that T^{ip}, the unique solution for the equation $V_L(t_L, t_L + \Delta) = V_F(t_L, t_L + \Delta)$, belongs to the interval $[T_F^* - \Delta, \bar{T}]$. Then compute that $T^{ip} < \hat{T}_L$, when $\theta < \theta'''(\Delta)$. ■

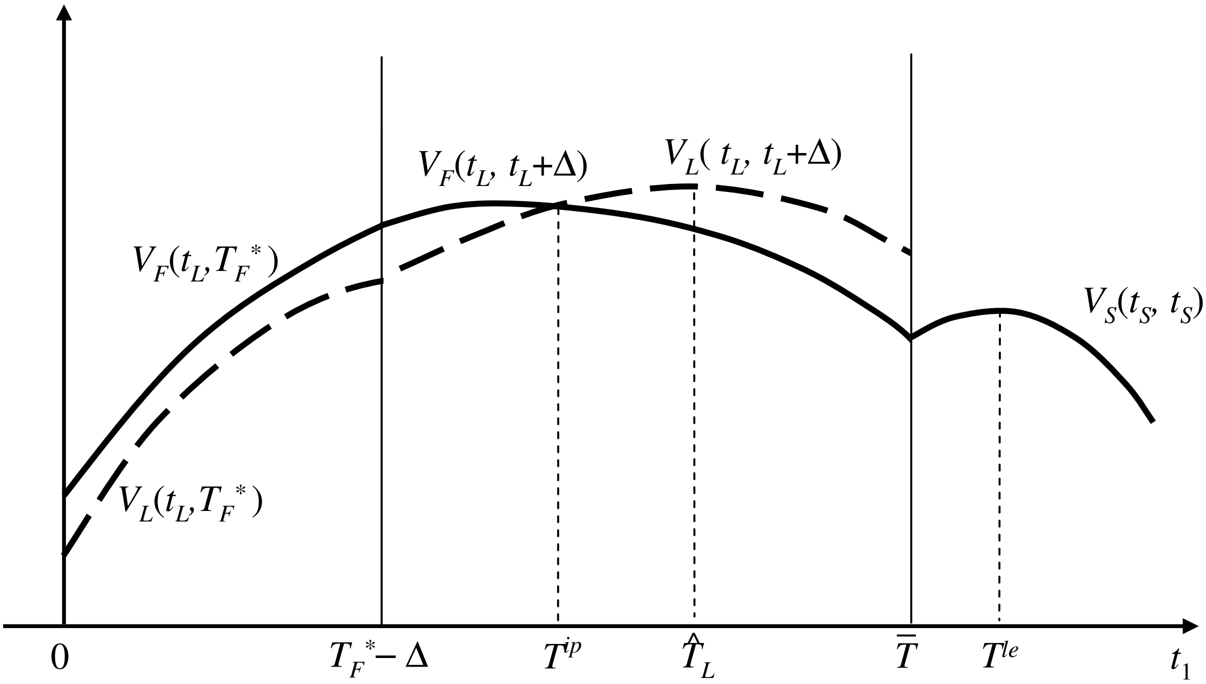


Panel (A_1) : $\theta \in (\theta'(\Delta), \theta''(\Delta)]$

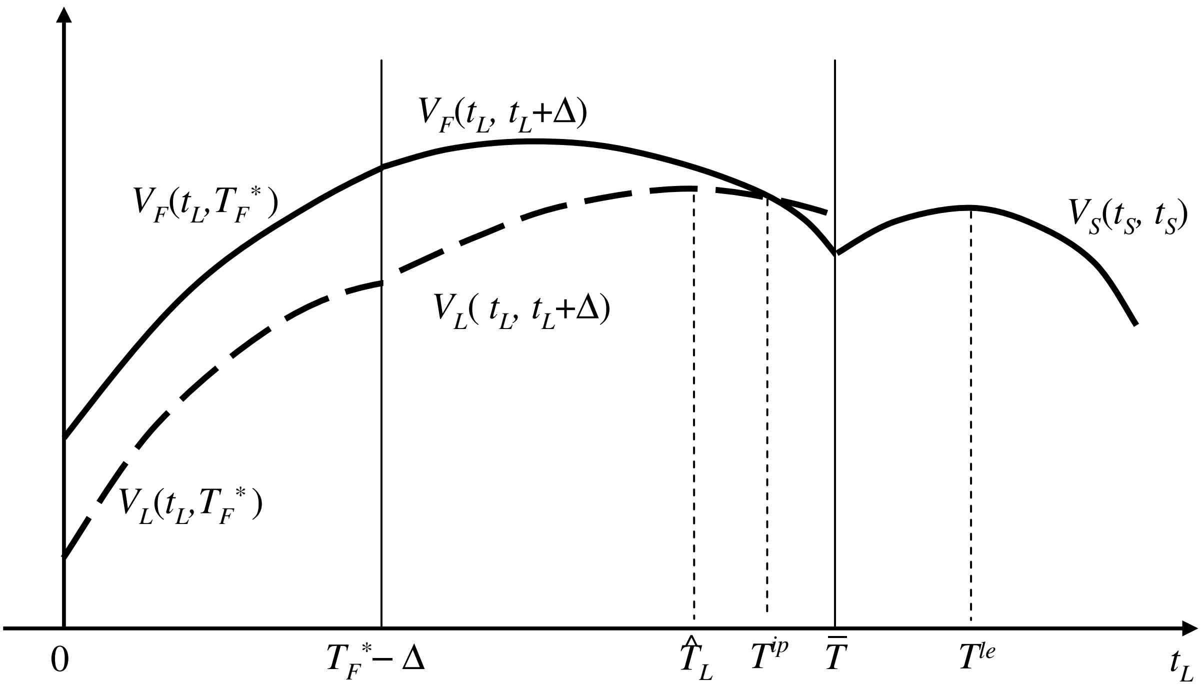


Panel (A_2) : $\theta \in (0, \theta'(\Delta)]$

Figure 1: Alternative behaviors of the firms' discounted payoffs for $\theta \in [0, \theta''(\Delta)]$.



Panel (B_1) : $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \theta'''(\Delta)]$



Panel (B_2) : $\theta \in (\theta'''(\Delta), \bar{\theta}]$

Figure 2: Alternative behaviors of the firms' discounted payoffs for $\theta \in (\max\{\theta^*(\Delta), \theta''(\Delta)\}, \bar{\theta}]$

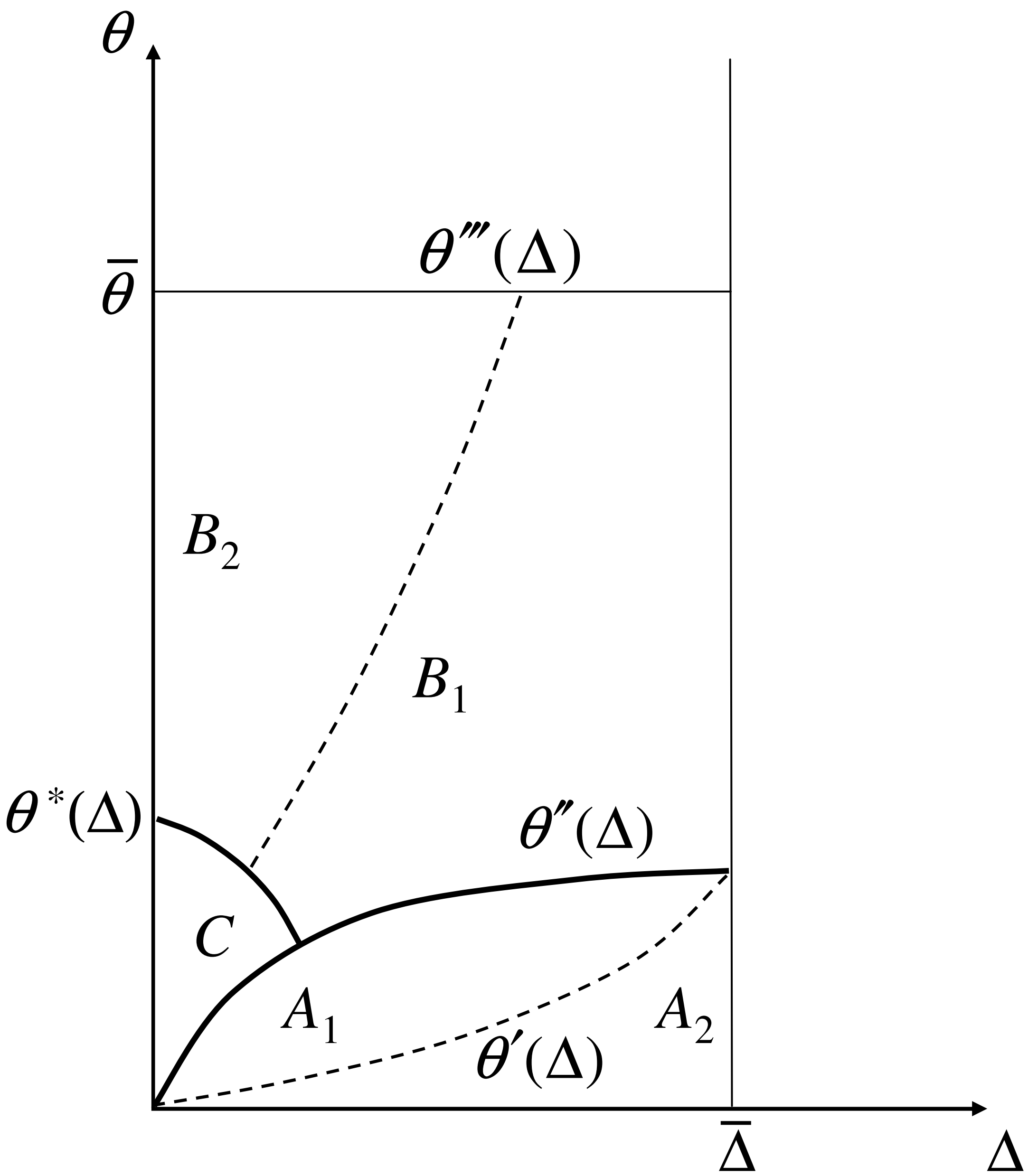


Figure 3. Parameter sets leading to alternative behaviors of the payoffs functions in the early and in the intermediate intervals.

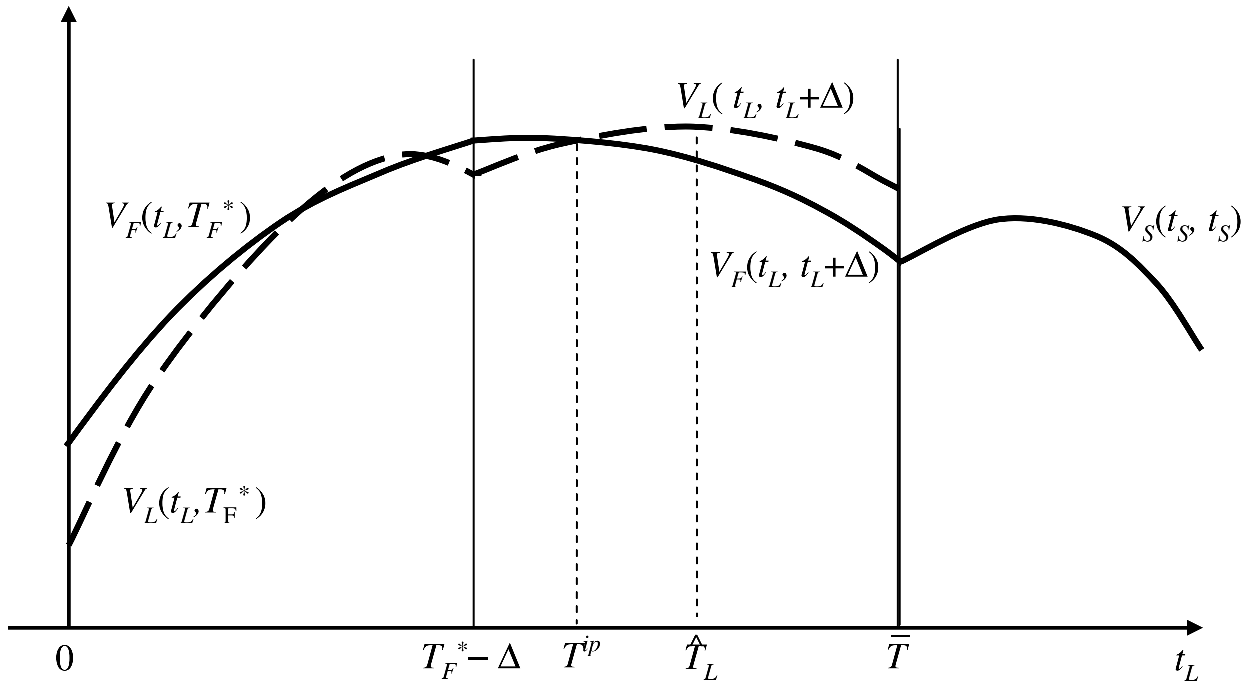
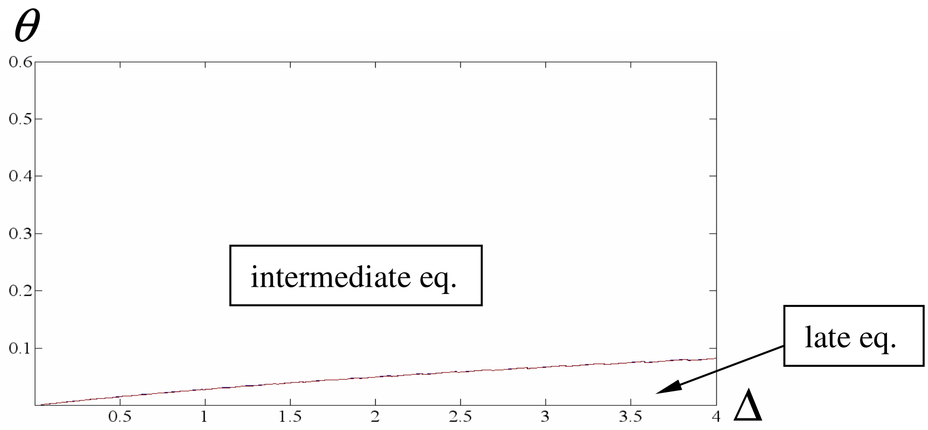
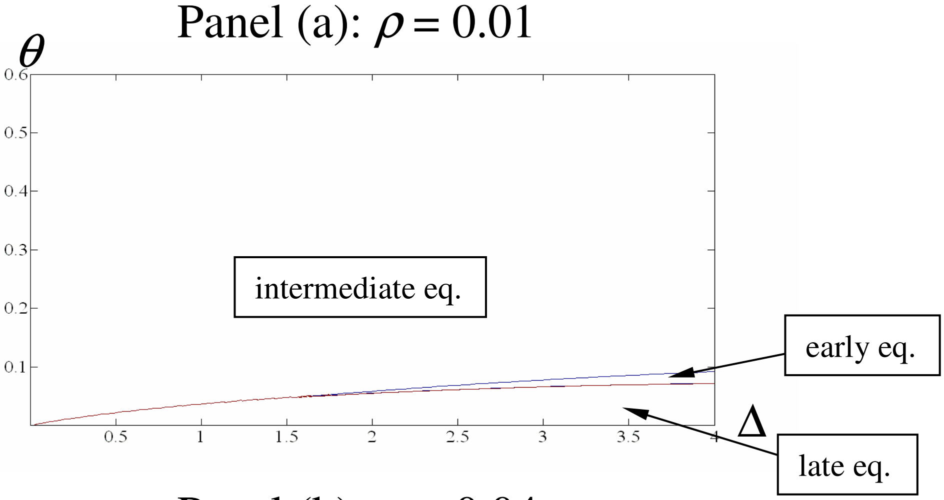


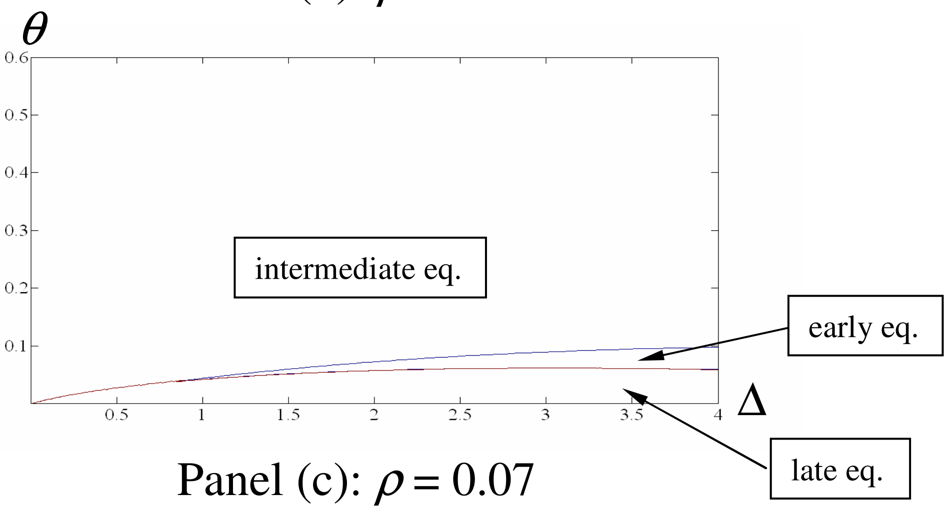
Figure 4: Behaviors of the firms' discounted payoffs for $\theta \in (\theta''(\Delta), \theta^*(\Delta)]$.



Panel (a): $\rho = 0.01$

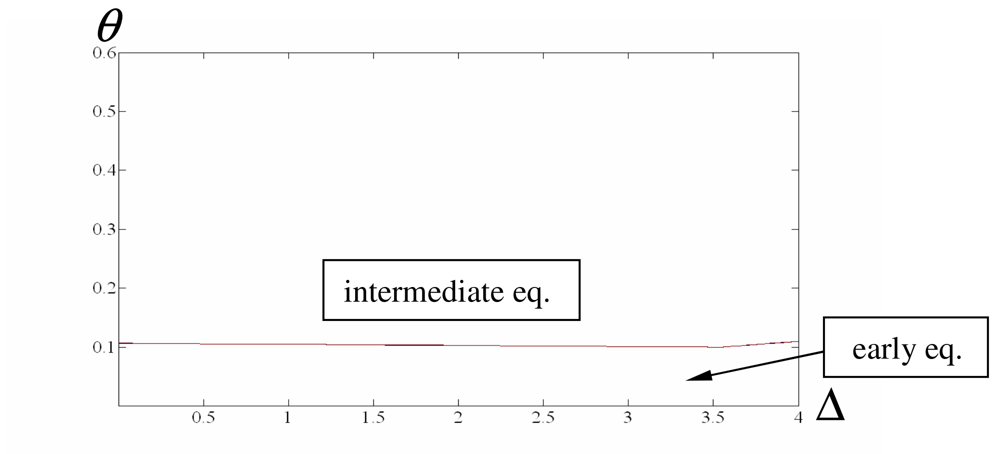


Panel (b): $\rho = 0.04$

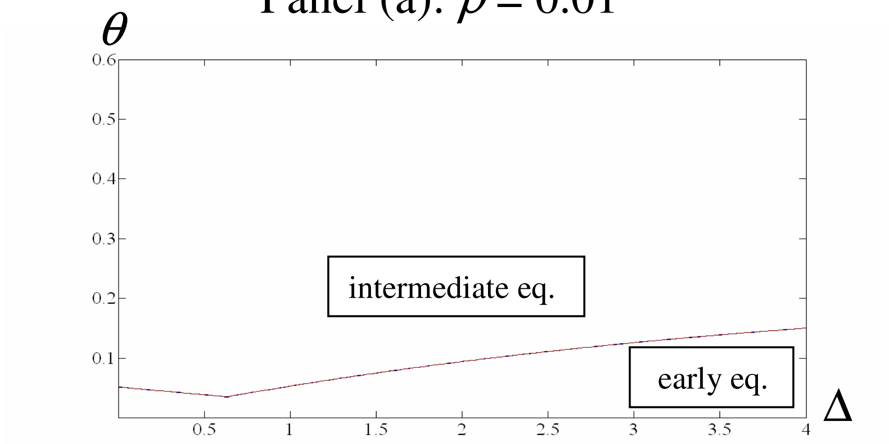


Panel (c): $\rho = 0.07$

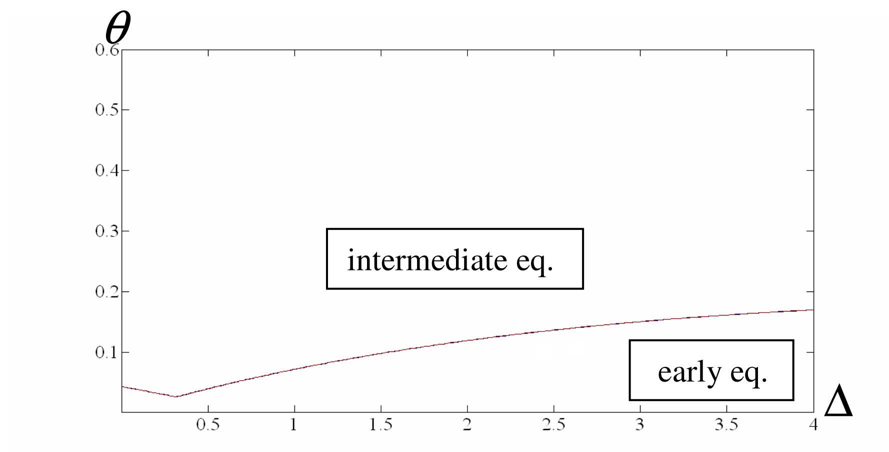
Figure 5 Equilibrium selection – minor innovation ($x = 0.05$)



Panel (a): $\rho = 0.01$



Panel (b): $\rho = 0.04$



Panel (c): $\rho = 0.07$

Figure 6 Equilibrium selection – major innovation ($x = 0.50$)