

UNIVERSITA' CATTOLICA DEL SACRO CUORE

WORKING PAPER

DISCE

Dipartimenti e Istituti di Scienze Economiche

POLITICAL GEOGRAPHY AND INCOME INEQUALITIES

Filippo Gregorini

Quaderno n. 46 / settembre 2007



QUADERNI DELL'ISTITUTO DI
TEORIA ECONOMICA E METODI QUANTITATIVI

POLITICAL GEOGRAPHY AND INCOME INEQUALITIES

Filippo Gregorini

Quaderno n. 46 / settembre 2007



UNIVERSITÀ CATTOLICA DEL SACRO CUORE
MILANO

**ISTITUTO DI TEORIA ECONOMICA E METODI QUANTITATIVI
(ITEMQ)**

Membri

Luciano Boggio
Luigi Filippini
Luigi Lodovico Pasinetti
Paolo Varri (*Direttore*)
Domenico Delli Gatti
Daniela Parisi
Enrico Bellino
Ferdinando Colombo
Gianluca Femminis
Marco Lossani

Comitato di Redazione

Luciano Boggio
Luigi Filippini
Luigi Lodovico Pasinetti
Paolo Varri

*I Quaderni dell'Istituto di Teoria Economica e
Metodi Quantitativi possono essere richiesti a:
The Working Paper series of Istituto di Teoria
Economica e Metodi Quantitativi can be requested at:*

Segreteria ITEMQ
Università Cattolica del S. Cuore
Via Necchi 5 - 20123 Milano
Tel. 02/7234.2918 - Fax 02/7234.2923
E-mail ist.temq@unicatt.it

Finito di stampare nel mese di settembre
presso la Redazione stampati
Università Cattolica del Sacro Cuore

*Il Comitato di Redazione si incarica di ottemperare agli obblighi previsti
dall'art. 1 del DLL 31.8.1945, n. 660 e successive modifiche*

“ESEMPLARE FUORI COMMERCIO PER IL DEPOSITO LEGALE AGLI EFFETTI DELLA LEGGE 15
APRILE 2004, N. 106”

POLITICAL GEOGRAPHY & INCOME INEQUALITIES

Filippo Gregorini¹

September 2007

Abstract: This paper studies the effect of the introduction of income inequalities in a model of geopolitical organization. We assume the existence of two groups of agents with different incomes. We focus on the policy effects of changes in income differential between the groups and changes in the fraction of the population belonging to the two groups. In the optimal solution, if size is endogenous and public good provision exogenous size increases as income inequality increases; if both size and public good provision are endogenously determined size is neutral to changes in income inequalities and public good provision decreases as inequality increases. There are cases where a stable solution does not exist and the possibility of non existence increases together with inequality, if both size and public good provision are endogenously determined.

Key Words: Country Size, Public Good, Income Inequality, Tax Distortion

JEL Code: D6, H4, D3, H2

¹Graduate School in the Economics and Finance of the Public Administration, Università Cattolica del Sacro Cuore, Milan. I am grateful to all my professors at DEFAP, in particular my supervisor Mario Gilli, Federico Etro, Ferdinando Colombo, Gerd Weinrich, Lorenzo Cappellari. Furthermore, I'd like to thank Claudio Lucifora, Michel Le Breton, Claudio Zoli, Federico Perali, Sholmo Weber and the participants to the seminars in Milan, Canazei, Verona, Louvain-la-Neuve and Berlin for their useful comments. A special thanks to Marco Giuliani for his "constructive criticism". E-mail: filippo.gregorini@unicatt.it

1 Introduction

1.1 The context

This paper studies the determinants of the size of nations, an issue that have been already explored under many perspectives: the first works are Friedman [17], who studied how countries were shaped by rulers in order to maximize their joint (potential) tax revenues, and Buchanan and Faith [13], who developed a theory about the internal exit and the links of such phenomenon with the level of public spending. They can be considered pioneers of this discipline, whose diffusion increased together with the number of countries in the nineties, when country borders have been redrawn to an extent that is absolutely exceptional for a peacetime period.

The model by Alesina and Spolaore [3] introduced a trade-off between scale economies and heterogeneity, which endogenously determines the size of nations, in a world with uniformly distributed population and coincidence in geographical and preference dimensions. Their analysis was in search for both optimum² and stability³: stable size is smaller than the optimal one. Furthermore, they assumed that public good is exogenous and independent from size. This assumption has been modified by Etro [16]; in his model public good, size and tax revenue are endogenous and there is a structural relationship between them, through a budget constraint; furthermore, Etro introduced a parameter representing the elasticity of marginal utility from public good: the analysis shows how the comparison between the optimal solution and the stable one depends upon this parameter, similar to a degree of substitutability between public and private consumption.

The effects of globalization have been studied by Alesina and Spolaore [3][7], Alesina, Angeloni and Etro [10], Etro [16], and others; the effects of democratization again by Alesina and Spolaore [3][7] Staal [25] and others; Alesina, Spolaore and Wacziarg [6] have listed benefits and costs of size while Spolaore [24] enumerated the determinants of the size of nations. None of them have studied the effects of variations in income distribution.

In this paper we want to analyze the policy effects of the introduction of income inequalities in the model of Alesina and Spolaore modified à la Etro; in particular, we focus on how income inequality affects size and public good provision (if endogenous). These effects have been already explored in a similar context by Bolton and Roland [12], while Haimanko, Le Breton

²The optimal solution is the first best solution.

³A stable solution is an equilibrium where two conditions hold: (i) status quo is preferred to anarchy and (ii) agent's utility is decreasing in size.

and Weber [18] studied the case where population is not uniformly distributed. Bolton and Roland have analyzed how income differences between regions can influence the break-up or unification of countries; they were not interested in the determination of the size of nations; their model emphasized political conflicts over redistribution policies in jurisdictions where the decision to separate or to unify is taken by majority voting. A trade-off between efficiency gains of unification and costs in terms of loss of control on political decisions was highlighted. On the other hand, Haimanko, Le Breton and Weber focused on the differences between efficiency (optimum in our model) and stability (no groups prone to secession) in a model where population is not uniformly distributed, while incomes are not considered; as Bolton and Roland, they focused on threats of secession within a country. They underlined how efficiency implies stability only if the differences in citizens' preferences, due to the geographical distribution of population, are sufficiently small; on the other hand, if the differences are great, efficient countries are not stable, in particular redistribution schemes are needed to prevent secessions.

In our paper population is continuously and uniformly distributed, average income is the same everywhere and the analysis focus on optimum and stability, as in the model by Alesina and Spolaore [3].

We deal with income inequalities, but how to measure them? We created an index looking at the increase in tax distortion due to the introduction of income differences. This index is similar to Gini Index. Following the definition of Lorenz curves, we can measure income inequality (average tax distortion) as the difference between two areas: (i) the area of a distribution where income is the same for everyone, and (ii) the area of the case we are studying. The more there is difference between (i) and (ii), the more there is income inequality (average tax distortion). For a graphical explanation, see Figure 1.

The results of our search for optimum (social planner solution) depend upon the assumptions on the variables size and public good provision. If only size is endogenous, it increases as income differential between rich and poor increases and has ambiguous effects following variations in the percentage of poor. If both size and public good provision are endogenous, size is neutral to variations in income inequalities, while public good provision depends upon it with a behavior similar to the one found by Lindert [19] in his empirical analysis.

In the search for stability (stable solution), we focus on the case where both size and public good provision are endogenously determined. We find that there is the possibility of non existence of a stable solution, and this pos-

sibility increases together with income inequality (average tax distortion). Furthermore, every stable size is smaller than the optimal one and depends upon income inequalities.

1.2 A methodological note

After this introduction, a methodological note is in order. We are interested in variations in the parameters of the distribution of incomes that are independent of variations in the other parameters.

Consider the case of mean-preserving spreads given a two-spike distribution: if, for example, the percentage of poor agents increases, income differential between rich and poor agents changes as a consequence, in order to maintain the same average income. Following the previous example, we consider the case of an increase in the percentage of poor that does not automatically affect income differential; as a consequence, we consider variations in the parameters of income distribution that are not mean preserving (as average income is a variable rather than a constant in real world).

Obviously, in order to focus on the effects of income inequality on the allocation of resources, our results should be explicitly made independent of average income to be able to distinguish between income effects and distribution effects. In the Appendix we discuss the "mean-preserving case"; policy implications are different with respect to non mean-preserving spreads in the sense that the effect of an increase in the percentage of poor is no more ambiguous.

1.3 Summary of the paper

This paper is organized as follows: Section 2 presents the model and discusses the assumptions; Section 3 derives the Social Planner Solution; Section 4 derives the Stable Solution and Section 5 briefly concludes. At the end of the paper, an Appendix contains the optimal solution in case of mean-preserving spreads, as pointed out before.

2 The model

2.1 General assumptions

World population has mass equal to 1; it is continuously and uniformly distributed on the segment $[0, 1]$. Agents' preferences on public good provision

are single peaked; each country has a capital city in the middle of its uni-dimensional space⁴; public good is provided in the capital; the utility that agents derive from it decreases with the distance from the capital.

World population is divided in two groups, call them "poor" and "rich"; within each group, income is the same for everyone; in particular:

$$y_R > \bar{y} > y_P > 0$$

where: y_R = rich's income, \bar{y} = average income, y_P = poor's income.

For simplicity, from now on we will assume $y_P = y$ and $y_R = ky$ with $k > 1$ as indicator of income differential between rich and poor agents.

The parameter α represent the percentage of poor ($1 - \alpha$ is the percentage of rich): we will assume that $\alpha \in (0.5, 1)$ in order to guarantee the skewness to the right of income distribution within population; as a consequence:

$$\bar{y} > y_m = y$$

where: y_m = median income.

As population is continuously and uniformly distributed, in every point of the segment $[0, 1]$ there is a fraction α of poor and a fraction $1 - \alpha$ of rich agents.

2.2 Utility of agent i

Following Etro [16], utility of agent i in country j has the following functional form:

$$U_{ij} = (\lambda - al_{ij}) H(g_j) + u(c_i)$$

where:

$H(\cdot)$ is utility from public spending g .

$u(\cdot)$ is utility from private consumption c .

$(\lambda - al_{ij})$ concerns heterogeneity of preferences between agents depending on the distance l_{ij} from the capital, where a reflects the costs of heterogeneity and $\lambda > 0$ the maximum utility from public good (we assume $a \geq 4\lambda$ to have at least two countries in the optimal solution).

To obtain closed form solutions, specific assumptions are needed.

First of all, linear utility from consumption: $c_i = y_i - t_i$.

⁴The location of the capital is decided by majority rule and the median voter theorem holds.

Then, isoelastic utility from public spending: $H(g_j) = g_j^{1-\theta} / (1 - \theta)$, where $\theta \in [0, 1]$ represent the elasticity of marginal utility of public expenditure (the lower it is, the more public and private consumption are substitutable).

Furthermore, we assume diminishing marginal returns in the production process of public good, with a distortion of taxes increasing and convex in the taxation level. In this simplified context, marginal returns in the provision of public goods and tax distortions are summarized through a quadratic convex cost function of taxation; as a consequence, t_i becomes $t_i^2/2$.

Following these assumptions, utility of agent i (in country j) is:

$$U_{ij} = \frac{g_j^{1-\theta}}{1-\theta} (\lambda - al_{ij}) + y_{ij} - \frac{t_{ij}^2}{2} \quad (1)$$

where y and t are different for agents belonging to different groups: in particular, utility of agent i (in country j), if i is poor, is:

$$U_{ijP} = \frac{g_j^{1-\theta}}{1-\theta} (\lambda - al_{ij}) + y_j - \frac{t_{jP}^2}{2}$$

if $i \in P \subseteq [0, 1]$

while utility of agent i (in country j), if i is rich, is:

$$U_{ijR} = \frac{g_j^{1-\theta}}{1-\theta} (\lambda - al_{ij}) + ky_j - \frac{t_{jR}^2}{2}$$

if $i \in R \subseteq [0, 1]$

2.3 Taxation scheme and budget constraint

Each agent pays taxes and enjoys benefits from public good in the country where he lives; taxes are assumed to be proportional with respect to income. The case with a progressive taxation scheme is not considered in our model as a progressive scheme cannot be Pareto-superior to a proportional one in terms of social welfare⁵.

⁵Distortion from taxation would be minimized with "lump-sum style" taxes, but we prefer not to consider the case of an anti-progressive taxation scheme; on the other hand, we calculated that a progressive scheme cannot be Pareto-superior to a proportional one as tax distortion increases together with progressivity. Following these considerations, we decided to use a proportional taxation scheme in the model.

$$\tau = \text{tax rate} \quad \tau \in (0, 1)$$

$$t_i = \tau y_i$$

The definition of our budget constraint derives from the assumption of continuous and uniform distribution of the population: public spending (public good provision) is equal to tax revenue multiplied by size of nation, where s_j , in case of uniform distribution, represent not only size of nation but also its population.

The budget constraint for country j (with proportional taxation) is:

$$g_j = s_j \tau [\alpha y + (1 - \alpha) ky] \quad (2)$$

3 Social planner solution

3.1 The optimal solution

Given our assumptions of unidimensional world, single-peaked preferences on public good provision and continuously and uniformly distributed population, median agent is the one at a distance $l_m = s/4$ from the government and median voter theorem holds. We assume the existence of an utilitarian social planner whose target is the maximization of world's welfare. The social planner will have to maximize the utility of the median agent to get the "utilitarian" optimal solution.

First of all, let us consider world's Social Welfare Function:

$$W(g, s, t) = \sum_{j=1}^N \int_{s_j} U_{ij} di$$

Following our method to introduce income inequalities, we can derive the Social Welfare Function of the median agent as the sum of the utilities of a poor and a rich agent weighed for percentage of population belonging to the two groups and income differential between them⁶:

⁶We assume that the Social Planner gives the same weight to poor and rich in order to determine the Social Welfare Function to get the first best solution. We studied our model also in case of different weights given to different income levels; we will compare such results with our benchmark case in note 9 on Proposition 3.

$$\begin{aligned}
W(g, s, t) = & \alpha \left[\frac{g^{1-\theta}}{1-\theta} \left(\lambda - a \frac{s}{4} \right) + y - \frac{t_P^2}{2} \right] + \\
& + (1-\alpha) \left[\frac{g^{1-\theta}}{1-\theta} \left(\lambda - a \frac{s}{4} \right) + ky - \frac{t_R^2}{2} \right] \quad (3)
\end{aligned}$$

under the budget constraint $g = st$

From (2) and (3) we can derive the function that the social planner has to maximize to get the first best (optimal) solution.

If we multiply the components of (3) for percentage of poor and percentage of rich, we obtain:

$$W = \frac{g^{1-\theta}}{1-\theta} \left(\lambda - a \frac{s}{4} \right) + [\alpha y + (1-\alpha) ky] - \left[\alpha \frac{t_P^2}{2} + (1-\alpha) \frac{t_R^2}{2} \right]$$

First of all, note that the second component is exactly average income, given percentage of poor and income differential between rich and poor.

Now, consider the last component; it represent the tax revenue that government can use for public spending under the budget constraint; we can transform it through algebraic manipulations.

First, substitute τy to the amount of taxes paid by poor and τky to the amount of taxes paid by rich (as taxes are assumed to be proportional with respect to income, then tax rate is the same for everyone) and obtain:

$$-\frac{1}{2} \tau^2 \left[\alpha y^2 + (1-\alpha) (ky)^2 \right]$$

Note that tax rate is equal to average tax revenue divided by average income:

$$-\frac{1}{2} t^2 \frac{\alpha y^2 + (1-\alpha) (ky)^2}{\bar{y}^2}$$

Furthermore, given the budget constraint where public good provision is equal to size multiplied by tax revenue, we can easily derive:

$$-\frac{1}{2} \left(\frac{g}{s} \right)^2 \frac{\alpha + (1-\alpha) k^2}{[\alpha + (1-\alpha) k]^2} = -\frac{1}{2} \left(\frac{g}{s} \right)^2 \psi$$

To get the first best solution, where the world's welfare would be maximized, the social planner has to solve:

$$\max_{g \geq 0; s \in [0,1]} W(g, s) = \frac{g^{1-\theta}}{1-\theta} \left(\lambda - a \frac{s}{4} \right) + \bar{y} - \frac{1}{2} \left(\frac{g}{s} \right)^2 \psi(\alpha, k) \quad (4)$$

where:

$$\psi(\alpha, k) := \frac{\alpha + (1 - \alpha) k^2}{[\alpha + (1 - \alpha) k]^2} > 1 \quad (5)$$

is our index of income inequality.

The index derives from the component of the Welfare Function concerning the disutility from taxation; it shows us the variation in the average tax distortion in our model that follows the introduction of income inequalities.

Consider the numerator of (5): it approximates the average tax distortion we have in our model given percentage of poor and income differential between rich and poor.

Consider now the denominator of (5): it approximates the tax distortion we would have in case of uniform income within population, if the income of every agent would equal the average income; in such a case everyone would pay the same amount of taxes, then distortion would be minimized.

We can reasonably consider ψ as an index of income inequality; its behavior with respect to percentage of poor and income differential is similar to the one of Gini Index if we consider variations in the parameters of the income distribution.

Let us differentiate our index of income inequality with respect to the variables it depends upon.

Derivative of ψ with respect to income differential:

$$\frac{\partial \psi}{\partial k} = \frac{2\alpha(1-\alpha)(k-1)}{[\alpha + (1-\alpha)k]^3} > 0$$

Derivative of ψ with respect to percentage of poor:

$$\frac{\partial \psi}{\partial \alpha} = \frac{(1-\alpha)k^3 + (\alpha-2)k^2 + (\alpha+1)k - \alpha}{[\alpha + (1-\alpha)k]^3} \geq 0$$

The sign of this last derivative depends upon the values of α and k , as shown in Figure 2.

As income difference between rich and poor increases, income inequality increases whatever the percentage of poor is.

Different is the case of an increase in the percentage of poor: income inequality increases for values of α "next to 0.5" and decreases as α tends to

1⁻; in such a case there is less income inequality with respect to a situation where population is divided in two groups whose number is similar⁷.

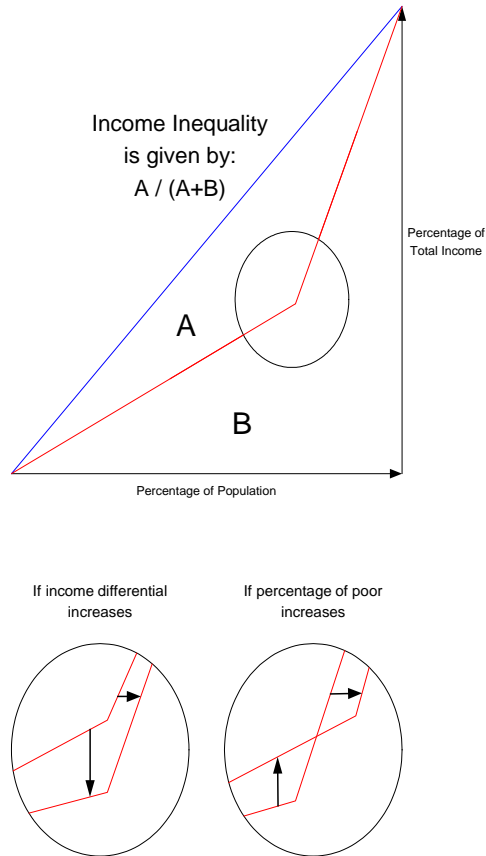


Figure 1: Income Inequality and Lorenz Curves

We will derive now the optimal size of nations and the optimal public good provision through the First Order Conditions of (4).

⁷Consider Figure 1. If k changes, the new Lorenz curve does not intersecate the old one, then Gini Index increases (Rank Dominance). If α changes, the new Lorenz curve intersecates the old one, then Gini Index can either increase or decrease depending on α and k (Generalized Lorenz Dominance).

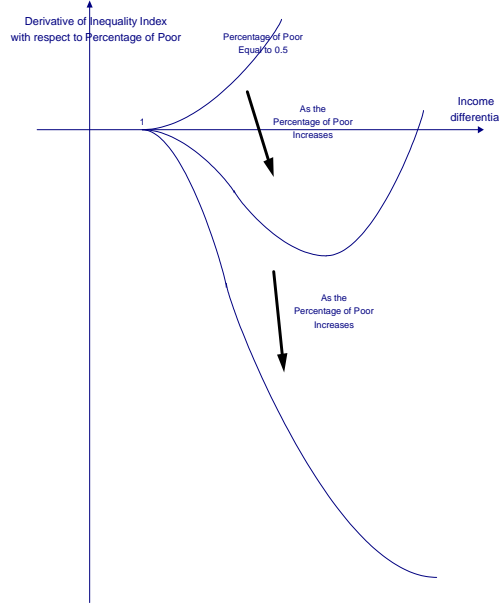


Figure 2: Inequality Index if Percentage of Poor Changes

PROPOSITION 1

If public good provision is exogenous (and size is endogenous), optimal size of nations depends upon income inequality.

Formally:

$$s^{AS*} = g^{\frac{1+\theta}{3}} \left[\frac{4(1-\theta)}{a} \right]^{\frac{1}{3}} \psi(\alpha, k)^{\frac{1}{3}} \quad (6)$$

If inequality increases, then average distortion from taxation increases. If tax rate lowers, the more there is inequality, the more tax distortion lowers as a consequence. Given the budget constraint, and given exogenous public good provision, the social planner increases size to lower tax rate.

Proof. If we derive the Social Welfare Function of the median agent with respect to size, we obtain optimal size of nation as a function of the amount of public good, where size is the only endogenous variable as in Alesina and Spolaore [3]:

$$\frac{\partial W}{\partial s} = -g^{1-\theta} \frac{a}{4(1-\theta)} + \frac{g^2}{s^3} \psi = 0$$

and we obtain:

$$s = g^{\frac{1+\theta}{3}} \left[\frac{4(1-\theta)}{a} \right]^{\frac{1}{3}} \psi(\alpha, k)^{\frac{1}{3}}$$

■

COROLLARY 1.1

If public good provision is exogenous (and size is endogenous), optimal size of nations increases as income differential between rich and poor increases.

Formally:

$$\frac{\partial s(\alpha, k)}{\partial k} > 0$$

Proof. s in (6) increases together with $\psi(\alpha, k)$;
if $\partial\psi/\partial k > 0$, then $\partial s/\partial k > 0$ ■

COROLLARY 1.2

If public good provision is exogenous (and size is endogenous), the range of values of income differential where optimal size of nations is not increasing in the percentage of poor is larger as percentage of poor increases.

Formally:

$$\frac{\partial s(\alpha, k)}{\partial \alpha} \gtrless 0$$

$$\frac{\partial s}{\partial \alpha'} \leq 0 \quad \forall k \in [1, k']$$

$$\frac{\partial s}{\partial \alpha''} \leq 0 \quad \forall k \in [1, k'']$$

With $k' < k''$ as $\alpha' < \alpha''$

Proof. s in (6) increases together with $\psi(\alpha, k)$;
if $\partial\psi/\partial\alpha \gtrless 0$, then $\partial s/\partial\alpha \gtrless 0$ ■

PROPOSITION 2

If both public good provision and size are endogenous and jointly determined, optimal size of nations does not depend upon income inequality.⁸

⁸In our model poor and rich get the same utility from public good provision. We calculated the optimal solution in case of utilities depending on incomes; in such a case optimal size would depend upon income inequality.

Formally:

$$s^{E*} = \frac{4\lambda(1-\theta)}{a(2-\theta)} \quad (7)$$

Proof. Let's consider (6) and the budget constraint (2).

If we substitute the budget constraint in the derivative of the Social Welfare Function of the median agent with respect to size we obtain:

$$t = s^{\frac{2-\theta}{1+\theta}} \left(\frac{a}{4(1-\theta)} \right)^{\frac{1}{1+\theta}} \left(\frac{1}{\psi} \right)^{\frac{1}{1+\theta}} = \Psi(s)$$

If we derive the Social Welfare Function of the median agent with respect to public good provision we obtain:

$$g = s^{\frac{2}{1+\theta}} \left(\lambda - a\frac{s}{4} \right)^{\frac{1}{1+\theta}} \left(\frac{1}{\psi} \right)^{\frac{1}{1+\theta}}$$

If we substitute the budget constraint in it we obtain:

$$t = s^{\frac{1-\theta}{1+\theta}} \left(\lambda - a\frac{s}{4} \right)^{\frac{1}{1+\theta}} \left(\frac{1}{\psi} \right)^{\frac{1}{1+\theta}} = \Phi(s)$$

Now we can equal $\Psi(s)$ and $\Phi(s)$ to derive the optimal size if both size and public good provision are endogenously and jointly determined:

$$\Psi(s) = \Phi(s)$$

and we obtain:

$$s = \frac{4\lambda(1-\theta)}{a(2-\theta)}$$

which is the same result of Etro [16]. ■

PROPOSITION 3

If both public good provision and size are endogenous and jointly determined, optimal amount of public good decreases as income inequality increases⁹.

⁹In our model the social planner gives the same weight to every income class. We calculated the optimal solution in case of different weights to different income classes: the more the weight given to poor is greater than the weight given to rich, the more it would be optimal to raise public expenditure.

Formally:

$$g^{E*} = 2^{\frac{4}{1+\theta}} \left(\frac{\lambda}{2-\theta} \right)^{\frac{3}{1+\theta}} \left(\frac{1-\theta}{a} \right)^{\frac{2}{1+\theta}} \left(\frac{1}{\psi(\alpha, k)} \right)^{\frac{1}{1+\theta}} \quad (8)$$

As we pointed out in the discussion on Proposition 1, if inequality increases, it would be optimal to lower tax. Given the budget constraint, the social planner increases size and/or lowers the provision of public good. If size increases, heterogeneity of preferences on public good provision increases. On the other hand, if public good provision lowers, tax rate and tax distortion lower, and heterogeneity of preferences on public good does not increase. The interpretation of this result is difficult; the literature gives us controversial interpretation, both in theoretical and empirical analysis.

Proof. If we consider (7), which is independent from income differential, we can substitute it in the derivative of the Social Welfare Function of the median agents with respect to public good provision, so we have:

$$g = \left(\frac{4\lambda(1-\theta)}{a(2-\theta)} \right)^{\frac{2}{1+\theta}} \left(\lambda - a \frac{4\lambda(1-\theta)}{a(2-\theta)} \right)^{\frac{1}{1+\theta}} \left(\frac{1}{\psi} \right)^{\frac{1}{1+\theta}}$$

and after algebraic manipulation we obtain:

$$g = 2^{\frac{4}{1+\theta}} \left(\frac{\lambda}{2-\theta} \right)^{\frac{3}{1+\theta}} \left(\frac{1-\theta}{a} \right)^{\frac{2}{1+\theta}} \left(\frac{1}{\psi} \right)^{\frac{1}{1+\theta}}$$

■

COROLLARY 3.1

If both public good provision and size are endogenous and jointly determined, optimal provision of public good decreases as income differential between rich and poor increases.

Formally:

$$\frac{\partial g(\alpha, k)}{\partial k} < 0$$

Proof. g in (8) decreases as $\psi(\alpha, k)$ increases; if $\partial\psi/\partial k > 0$, then $\partial g/\partial k < 0$ ■

COROLLARY 3.2

If both public good provision and size are endogenous and jointly determined, the range of values of income differential where the optimal provision of public good is not increasing in the percentage of poor is larger as percentage of poor increases

Formally:

$$\frac{\partial g(\alpha, k)}{\partial a} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix}$$

$$\frac{\partial g}{\partial \alpha'} \geq 0 \quad \forall k \in [1, k']$$

$$\frac{\partial g}{\partial \alpha''} \geq 0 \quad \forall k \in [1, k'']$$

With $k' < k''$ as $\alpha' < \alpha''$

Proof. g in (8) decreases as $\psi(\alpha, k)$ increases; if $\partial\psi/\partial\alpha \gtrless 0$, then $\partial g/\partial\alpha \gtrless 0$ ■

3.2 Theoretical analysis and empirical analysis

As in Etro [16], optimal size of nations is decreasing in costs of heterogeneity and in non substitutability between public and private goods, while it is increasing in absolute utility from public good.

Our Proposition 3 shows that if inequality increases, it is optimal to lower public expenditure.

This result is in contrast with the one of the model developed by Persson and Tabellini [21][22]; in their model, the tax rate (with a proportional taxation scheme) increases as income polarization increases¹⁰. An important remark need to be made: we focus on optimum, through the vision of a social planner maximizing world's welfare; Persson and Tabellini are looking for the tax rate which maximizes the utility of the median agent (median in terms of income, while we consider the median in terms of geographic location), whose position is the winning one in an election with majority rule. Notwithstanding these different approaches, either in the analysis of Persson and Tabellini an increase in the distortion from taxation (due to an increase in income inequality in our model), leads to a smaller redistribution in equilibrium (a smaller public expenditure).

¹⁰Persson and Tabellini measured income polarization through the median distance from the median; the more median and average income differ, the more income distribution is polarized.

Even the results of the empirical studies on the effects of income inequality on public expenditure are controversial and not easy to interpret. In the econometric analysis by Alesina, Baqir and Easterly [5], income inequality (measured through mean/median income ratio) has negative effect on per capita education spending¹¹, and this empirical result seems similar to the theoretic one of our Proposition 3. More useful to comment our findings is the paper by Lindert [19]¹², whose results on the relationship between income inequality and public expenditure depend upon the way to measure the first variable. Lindert distinguishes between (i) natural logarithm of the ratio between the first and the third income quintiles, named "upper income gap" and (ii) natural logarithm of the ratio between the third and the fifth income quintiles, named "lower income gap"; income inequality is then (i) plus (ii), while income skewness is (i) minus (ii)¹³. Following these definitions of the variables, the econometric analysis by Lindert shows contrasting results. An increase in income skewness leads to more social public expenditure and lowers non-social public expenditure; on the other hand, an increase in income inequality lowers total public expenditure as share of GDP. The anti-spending effect of greater income inequality is in contrast with theories predicting that greater income inequality raises public expenditure, like Persson and Tabellini [21][22], while is coherent with our Proposition 3. Other analysis, like the one of Perotti [20], obtained negative results trying to relate income inequality to government transfers: the problem was the non comparability of different data sets. Finally, we have to note how the variable used in many writing on "inequality and growth", is not income inequality but income skewness, approximated by the ratio of the mean to the median income; Saint-Paul and Verdier [23] agreed that an higher mean/median income ratio raises redistribution.

After this discussion, it is possible to note that in most countries transfers rose more quickly during the 1960s and the 1970s, when income inequality was generally declining; in contrast, during the 1980s and the 1990s, inequality started to increase and government transfers rose less quickly with respect to the previous period. We cannot conclude that our theory is valid following these consideration, and the causality relation between income inequality and public expenditure remains unknown.

¹¹The analysis of Alesina, Baqir and Easterly refers generally to education spending, not to public spending.

¹²This paper is an econometric analysis of the determinants of public spending in 19 OECD countries from 1960 to 1992.

¹³The behavior of Lindert's inequality index with respect to the parameters of the distribution is similar to the one of our inequality index ψ .

Trying to check the correlation between income inequality and public expenditure worldwide, we have analyzed comparative data on population, Gini Index, total public expenditure as share of GDP and priorities (education, health and defense) in public expenditure as share of GDP¹⁴.

Consider again our Proposition 3 (public good provision is endogenous and changes with inequality): if we want to check the negative correlation between income inequality and public expenditure, we need that public expenditure decreases as income inequality, measured through Gini Index, increases. This correlation would be coherent with our analysis. We decided to use the variable on public expenditure on education, health and defense, where direct transfers and subsidies are not (should not be) included. To control the robustness of this correlation in 87 countries, we checked it in different subsamples: (i) OECD countries (ii) OECD countries excluding Mexico (iii) EU countries (iv) countries with at least 5 millions of inhabitants; in all our subsamples a negative correlation between Gini Index and public expenditure on priorities exists.

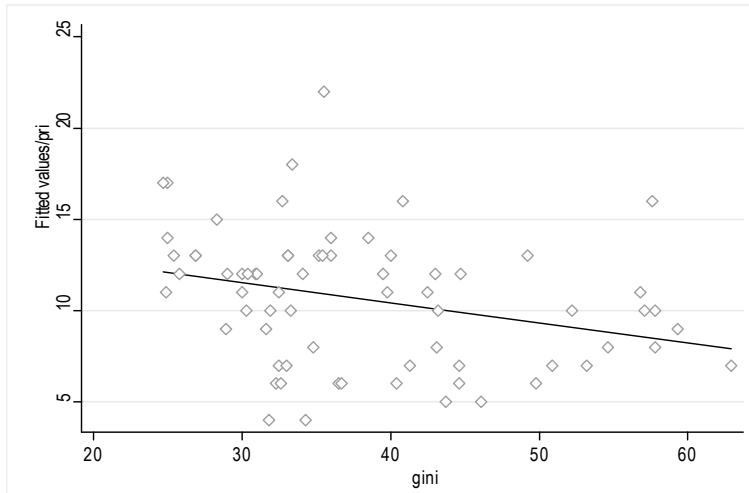


Figure 3: Gini Index & Public Expenditure (>5mlns.)

After our analysis, we are not able to explain the causal effect between

¹⁴The sources of our data set are: Human Development Report of the United Nations (2005) for population; World Bank (2004, 2005) for Gini Index, total public expenditure and public expenditure on health; Unesco (2005) for public expenditure on education and Stockholm International Peace Research Institute (2005) for public expenditure on defense.

income inequality and public expenditure (if exists)¹⁵; on the other hand, we can say that in real world, if income inequality increase, public expenditure as share of GDP is not supposed to increase as a consequence, *ceteris paribus*.

As we pointed out in the introduction, another article dealing with the introduction of income differences in the analysis of models of political geography is the one by Bolton and Roland [12], but the differences between our assumptions and the ones on the basis of their analysis do not allow a direct comparison of the main findings: their paper shows a link between income heterogeneity between regions and incentives to secede; our Proposition 1 (where size is endogenous while public good provision is exogenous) shows how size increases together with income inequality within a country. These results seem contrasting, but they are not: in our model we cannot consider the case of differences in income levels between regions or countries, given our assumption on the distribution of population; on the other hand, Bolton and Roland consider cases where, within a country, a region is poor and another is rich; indeed, after these considerations, the results seem coherent: consider the case of no income differences between regions, with income differences only within regions: if income inequality increases, in Bolton and Roland incentives to secede do not increase and in our paper the optimal size of nation does not decrease.

4 Stable solution

In a country of size s , the favorite public good provision for agent i is:

$$\hat{g}_i = \arg \max \left\{ \frac{g^{1-\theta}}{1-\theta} (\lambda - al_i) + y_i - \frac{1}{2} \left(\frac{g}{s} \right)^2 \right\} = s^{\frac{2}{1+\theta}} (\lambda - al_i)^{\frac{1}{1+\theta}}$$

Given our assumptions, median voter theorem implies that the provision of public good preferred by the median agent is the optimal choice for every country size.

We have to weight the favorite public good provision for poor and rich through income distribution to obtain the provision of public good which is the median agent's preferred choice. The result is given by the derivative of the Social Welfare Function of the median agents with respect to public good provision:

¹⁵ An econometric analysis with control variables and time series data is needed to check the causal effect.

$$\hat{g}_m = \left[\frac{s^2}{\psi(\alpha, k)} \left(\lambda - a \frac{s}{4} \right) \right]^{\frac{1}{1+\theta}} \quad (9)$$

In this model there are income differences, so we have to consider one case for every category of agents; the case of poor agents, and the case of rich ones.

4.1 Stability conditions for poor and rich agents

To obtain the stability conditions for poor and rich agents we have to consider the expected utilities of the poor agent and the rich one living at country borders¹⁶ and then substitute the median agent's preferred public good provision in their expected utilities:

$$V_P(s/2) = \left[\frac{s^2}{\psi(\alpha, k)} \left(\lambda - a \frac{s}{4} \right) \right]^{\frac{1-\theta}{1+\theta}} \left[\frac{\lambda - a \frac{s}{2}}{1-\theta} - \frac{1}{2\phi(\alpha, k)} \left(\lambda - a \frac{s}{4} \right) \right] + y \quad (10)$$

$$V_R(s/2) = \left[\frac{s^2}{\psi(\alpha, k)} \left(\lambda - a \frac{s}{4} \right) \right]^{\frac{1-\theta}{1+\theta}} \left[\frac{\lambda - a \frac{s}{2}}{1-\theta} - \frac{1}{2\beta(\alpha, k)} \left(\lambda - a \frac{s}{4} \right) \right] + ky \quad (11)$$

Note that:

$$\phi = \alpha + (1 - \alpha) k^2 \in (1, +\infty)$$

is average tax distortion in the model, given income distribution. It is increasing in income differential and decreasing in percentage of poor.

On the other hand,

$$\beta = \phi/k^2 \in (0, 1)$$

is average tax distortion in the model divided by income differential squared, given income distribution. It is decreasing both in the income differential and percentage of poor.

¹⁶If stability holds for citizens living at country borders, *a fortiori* it holds for citizens living closer to the government.

To get a stable solution, the two following conditions have to hold both for poor and rich agents:

Condition 1

Status quo has to be preferred to anarchy, so the disutility from taxation cannot be superior to the utility from public good provision.

Formally:

$$V_P(s/2) \geq y \qquad V_R(s/2) \geq ky$$

Condition 2

Utility has to be decreasing in size, so it has to be preferred to join the little country instead of the big one but not vice versa

Formally:

$$\partial V_P(s/2)/\partial s \leq 0 \qquad \partial V_R(s/2)/\partial s \leq 0$$

After some algebra, the sets of stable equilibria satisfying the two stability conditions for poor and rich agents are:

$$I_P(\lambda, \alpha, k, \theta) \qquad I_R(\lambda, \alpha, k, \theta)$$

Every stable size depends upon income distribution and it is not decreasing in ϕ (for poor) and in β (for rich).

Every size satisfying stability conditions for poor is smaller than the optimal, while a stable size satisfying stability conditions for rich can be either smaller or greater than the optimal; it depends upon income inequalities and substitutability between public and private goods¹⁷.

4.2 Existence of a stable solution

Different people living different situations have heterogeneous preferences on the size of nation which satisfies their stability conditions. In particular, a stable solution can exist only if the stable size is smaller than the optimal both for poor and rich agents.

Formally, a stable solution exists if $2\beta + \theta > 1$ and $I_P \cap I_R \neq \emptyset$.

In our model (if $2\beta + \theta > 1$ holds) we have $\min I_P > \min I_R$ and $\max I_P > \max I_R$, then $I_P \cap I_R$ is a non empty set if:

$$\max I_R(\lambda, \alpha, k, \theta) > \min I_P(\lambda, \alpha, k, \theta) \tag{12}$$

¹⁷For rich agents, we have stable size smaller than the optimal if $2\beta + \theta > 1$, stable size greater than the optimal if $4\beta + \theta < 1$, no stable solutions otherwise.

After algebraic manipulation, we can rewrite (12) as:

$$f(\alpha, k, \theta) > 0$$

After tedious algebra, we find the conditions for the existence of stable solutions in the model.

PROPOSITION 4

If $2\beta + \theta > 1$ holds, stable equilibria (with size smaller than the optimal) exist if the value of income differential is between 1 and 1.21 ;if income differential is greater than 1.21 the existence of stable equilibria depend upon the values of α, k, θ .

Formally:

If $2\beta + \theta > 1$ holds, $I_P \cap I_R \neq \emptyset \forall \alpha \in (0.5, 1) \forall \theta \in (0, 1) \Leftrightarrow k \in (1, 1.21]$

4.3 Income inequalities and instability

Optimal size does not depend upon income distribution; stable size depends upon it.

For poor, we have that every stable size is inferior to the optimal solution. Furthermore, we see that for poor agents size increases as income differential increases; in such a case poor could have more pro-capita public good in a greater country because of a multiplicative effect: if income differential increases and size of country doubles, then tax revenue (and the public good provision) is more than doubled; as a consequence, their favorite stable size increases. The effect of an increase of the percentage of poor is opposite: in such a case, poor would have to pay more to get the same public good provision; as a consequence, they would prefer less public good provision and less distance from the government in a smaller country (not to have to share public goods with many other people).

Different is the case of rich agents; if income differential or the percentage of poor increase, the effect for rich is always the same: they have to pay more taxes, and if they pay more taxes they prefer a smaller country to join more benefits from the public goods they have paid for. The difference with the case of poor is clear: if income differential increases, poor pay less taxes, so they prefer greater countries with more public good provision paid by rich.

The most important result of our analysis of stability is that every stable size for poor is smaller than the optimal, while a stable size for rich can be either smaller or greater, and this depends upon income distribution (in particular, it depends upon income differential). As income differential increases, stable size for poor increases (and tends to the optimal solution); on

the other hand, stable size for rich decreases: the more income differential is high the more the preferences of rich and poor are different and the possibility of the non existence of a stable solution increases¹⁸. In extreme cases (where income differential is "high enough" and/or $2\beta + \theta \leq 1$) a stable solution cannot exist, as preferences of rich and poor are irreconcilable¹⁹. What solutions in such a case? Lowers inequality through redistribution? It is not easy to answer this question; from our model, a link between inequality and instability emerges²⁰.

Let's look at the analysis by Haimanko, Le Breton and Weber [18]. They develop a model to study how governments can prevent secession threats through redistribution; in particular, transfers are needed in case of high degree of polarization of individuals' location²¹, given that secessions happen without transfers. In their model geographical and preference dimensions coincide (exactly as in Alesina and Spolaore [3]). Our model is different as we focus on income inequalities within an uniformly distributed population. In spite of this difference, we can consider the degree of population polarization, as measured by them, instead of the degree of income inequality we measure through the index ψ . Following these arguments, a comparison is possible: the main result of Haimanko, Le Breton and Weber is that efficiency, if population polarization is "high enough", does not imply stability without redistribution, and redistribution is needed to prevent secession threats, so the efficient size is greater than the stable one. In our model, with population continuously and uniformly distributed, redistribution is not possible²², so optimal size is greater than the stable one and higher income inequality leads to instability. The results seem similar, even with different assumptions on distribution of population and incomes.

There are different empirical works on the link between income distri-

¹⁸Is it possible to see it through mathematical simulations; in general, the higher k , the smaller the range of α and θ that satisfies $I_P \cap I_R \neq \emptyset$.

¹⁹Remember that any increase in the income differential increases tax distortion (and lowers β as a consequence).

²⁰Note that we have to consider also the effect of the substitutability between public and private goods on existence or non existence of a stable solution. Given that $2\beta + \theta > 1$ is a necessary condition for the existence of a stable solution, it follows that the more public and private goods are substitutable (the lower θ), the more a country is expected to be unstable, given income distribution.

²¹For Haimanko, Le Breton and Weber [18] there is polarization in case of intra-group homogeneity and inter-group heterogeneity (this definition follows Esteban and Ray [15]). Formally, the index of polarization adopted is the median distance to the median (as in Persson and Tabellini [21][22]). Note that in [18] incomes are not considered.

²²Alesina and Spolaore [3] proved that in their model a redistribution scheme cannot be implemented (page 1054-1055).

bution and political instability. The econometric analysis by Alesina and Perotti [2] on 71 countries between 1960 and 1985 shows that political stability is enhanced by the presence of a wealthy middle class: the more the share of total income of the third and fourth quintiles of the population is low, the more a country is expected to be politically instable. Alesina and Perotti focuses on causal relationship, but, as noted by Acemoglu and Robinson [1], in many cases the existing literature on this topic is contradictory and focuses on correlations instead of causal relationship, then it is not useful for scientific purposes.

5 Conclusion

The introduction of income inequalities in the model by Alesina and Spolaore [3] developed by Etro [16] has different policy implications.

If we refer to the optimal solution, our model shows how, given an increase in income inequality, the social planner should lower public expenditure, in order to get the first best solution. Our result is coherent with the results of the econometric analysis by Lindert [19].

If we refer to the stable solution, our model shows how income inequality increases instability, as in the econometric analysis by Alesina and Perotti [2].

From the point of view of a politician, two "worlds" seems to be possible: on one hand, stable countries with less income inequalities and more public expenditure; on the other hand, less stable countries with more income inequalities and less public expenditure. A conflict between these solutions emerges whenever an election take place, even if the distinction between the two positions seems sometimes not so clear as it was in the past²³.

In the very end of the paper, some ideas for further developments within this field of research: to assume that world population and/or incomes are not uniformly distributed; to assume that agents are mobile; to consider the existence of more than one public good in order to distinguish between substitutable and non substitutable goods; to develop an econometric analysis to check the causal effect between income inequality and public expenditure either in recent years²⁴.

²³In some countries (UK for example), the positions of the two main parties seem closer than in the past; in other countries (USA and Italy for example) political polarization seems to be increasing.

²⁴Details on our calculations on: introduction of a progressive taxation scheme, introduction of differences in agents utility from public good provision and comparisons between inequality indices (ψ , Gini and Lindert's Index) are available on request.

Appendix

Social Planner Solution with mean-preserving spreads

As we pointed out in the introduction, the question of mean-preserving spreads versus non mean-preserving spreads is important in order to identify clearly income effects. Does something changes if we consider mean-preserving spreads?

Let's consider a distribution with fixed average income \bar{y} . A percentage $\alpha \in (0.5, 1)$ of the population transfer a fraction of its (uniformly distributed) income to the remaining part of the population.

In particular the income of the agents would be:

$$y_P = \bar{y}(1 - \gamma)$$

if they belong to group α , and

$$y_R = \bar{y} \left(1 + \frac{\alpha\gamma}{1 - \alpha} \right)$$

if they belong to group $1 - \alpha$.

Given this way to model mean-preserving spreads, (4) becomes:

$$\max_{g \geq 0; s \in [0,1]} W(g, s) = \frac{g^{1-\theta}}{1-\theta} \left(\lambda - a \frac{s}{4} \right) + \bar{y} - \frac{1}{2} \left(\frac{g}{s} \right)^2 \varphi(\alpha, \gamma)$$

where:

$$\varphi(\alpha, \gamma) := 1 + \frac{\alpha\gamma^2}{1 - \alpha}$$

is our mean-preserving inequality index.

In such a case, changes in the parameters α and γ won't affect average income.

Let us differentiate our index of income inequality with respect to the variables it depends upon.

$$\frac{\partial \varphi}{\partial \gamma} = \frac{2\alpha\gamma}{1 - \alpha} > 0$$

$$\frac{\partial \varphi}{\partial \alpha} = \frac{\gamma^2}{(1 - \alpha)^2} > 0$$

If we consider mean-preserving spreads, inequality increases together with the percentage of poor and the amount of the regressive transfer.

Let us consider now the similarities and the differences in the results for the optimal solution of our model when both size and public good provision are endogenously determined, as in Proposition 3 (pages 13-15).

If we focus on γ , its behavior is the same of k in our model with non mean-preserving spreads: both γ and k measure the income differential between rich and poor in our two-spike distribution. As a consequence, it emerges how there are no differences between mean preserving and non mean-preserving spreads: income differential between rich and poor raises inequality and lowers public expenditure in the optimal solution.

If we focus on α , we see that the ambiguity of the derivative in our original model depends upon income effects; in case of mean preserving spreads it is always optimal to lower public expenditure following an increase in the percentage of poor.

References

- [1] Acemoglu, D., Robinson J. A. (2006), "Economic Origins of Dictatorship and Democracy", Cambridge University Press
- [2] Alesina, A., Perotti, R. (1996), "Income Distribution, Political Instability, and Investment", *European Economic Review*, vol.40, 1203-1228
- [3] Alesina, A., Spolaore, E. (1997), "On the Number and the Size of Nations", *Quarterly Journal of Economics*, vol.CXIII November, 1027-1056
- [4] Alesina, A., Wacziarg, R. (1998), "Openness, Country Size and Government", *Journal of Public Economics*, vol.69, 305-321
- [5] Alesina, A., Baqir, R., Easterly, W. (1999), "Public Goods and Ethnic divisions" , *Quarterly Journal of Economics*, vol.CXIV November, 1243-1284
- [6] Alesina, A., Spolaore, E., Wacziarg, R. (2000), "Economic Integration and Political Disintegration", *American Economic Review*, vol.90 no.5 December, 1276-1296
- [7] Alesina, A., Spolaore, E. (2003), "The Size of Nations", Massachusetts Institute of Technology Press
- [8] Alesina, A., Spolaore, E., Wacziarg, R. (2004), "Trade, Growth and the Size of Countries", working paper, Harvard University, Brown University and Stanford University
- [9] Alesina, A., Spolaore, E. (2005), "War, Peace, and the Size of Countries", *Journal of Public Economics*, vol.89, 1333-1357
- [10] Alesina, A., Angeloni, I., Etro, F. (2005), "International Unions", *American Economic Review*, vol.95 no.3 June, 602-615
- [11] Bolton, P., Roland, G., Spolaore, E. (1996), "Economic Theories of the Break-Up and Integration of Nations", *European Economic Review*, vol.40, 697-705
- [12] Bolton, P., Roland, G. (1997), "The Breakup of Nations: a Political Economy Analysis", *Quarterly Journal of Economics*, vol.CXIII November, 1057-1090
- [13] Buchanan, J. M., Faith, R. L. (1987), "Secession and the Limit of Taxation: Toward a Theory of Internal Exit", *American Economic Review*, vol.77 no.5 December, 1023-1031

- [14] Cowell, F.A. (2000), "Measurement of Inequality", Chapter 2 of "Handbook of Income Distribution Volume 1", Elsevier North Holland
- [15] Esteban, J. M., Ray, D. (1994) "On the Measurement of Polarization", *Econometrica*, vol.62, 819-851
- [16] Etro, F. (2006), "Political Geography", *Public Choice*, vol.127 June, 329-351
- [17] Friedman D. (1977), "A Theory of the Size and Shape of Nations", *Journal of Political Economy*, vol.85 February, 59-77
- [18] Haimanko, O., Le Breton, M., Weber, S. (2005) "Transfers in a Polarized Country: Bridging the Gap Between Efficiency and Stability", *Journal of Public Economics*, vol.89, 1277-1303
- [19] Lindert, P. (1996), "What Limits Social Spending?", *Explorations in Economic History*, vol.31, 1-36
- [20] Perotti, R. (1996), "Growth, Income Distribution and Democracy: What the Data Say", *Journal of Economic Growth* vol.1, 149-188
- [21] Persson, T., Tabellini, G. (1994), "Is Inequality Harmful for Growth?", *American Economic Review*, vol.84, 600-621
- [22] Persson, T., Tabellini, G. (2000), "Political Economics: Explaining Economic Policy", Massachusetts Institute of Technology Press
- [23] Saint-Paul, G., Verdier, T. (1993), "Education, Democracy and Growth", *Journal of Development Economics*, vol.42, 399-407
- [24] Spolaore, E. (2005), "National Borders and the Size of Nations", working paper, Tufts University
- [25] Staal, K. (2004), "Country Size and Public Good Provision", working paper, Tinbergen Institute, Erasmus University, Rotterdam
- [26] Wittman, D. (2000), "The Wealth and Size of Nations", *Journal of Conflict Resolution*, vol.44 no.6, 868-884
- [27] Zoli, C. (2007), "An Introduction to Inequality, Welfare & Poverty Measurement", Doctorate Winter School on Inequality and Risk, Università di Verona