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Strategic Choice Between Process and Product Innovation under Different Competitive Regimes

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March 2007

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Abstract

This paper investigates the strategic choice between introducing a process or a product innovation in a duopoly model with vertical differentiation, comparing the outcomes in case of Bertrand and Cournot competition. It is shown that under both competitive regimes three equilibria in innovation adoption may arise: two symmetric equilibria, where firms select the same innovation type, and one asymmetric equilibrium. The competitive regime has an impact on the features of the asymmetric equilibrium, since in case of Bertrand competition, the high (low) quality firm chooses a product (process) innovation, while firms make the opposite choices in case of Cournot competition. The presence of a leapfrogging effect (only in the Cournot case) explains these different outcomes. Last, we find that the Cournot competitors tend to favor the introduction of a new product in comparison with the Bertrand competitors.

JEL classification: D43, L15, O33

Keywords: vertical differentiation, innovation adoption, process and product innovation, competitive regime.

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Strategic Choice Between Process and Product Innovation under Different Competitive Regimes

Abstract

This paper investigates the strategic choice between introducing a process or a product innovation in a duopoly model with vertical differentiation, comparing the outcomes in case of Bertrand and Cournot competition. It is shown that under both competitive regimes three equilibria in innovation adoption may arise: two symmetric equilibria, where firms select the same innovation type, and one asymmetric equilibrium. The competitive regime has an impact on the features of the asymmetric equilibrium, since in case of Bertrand competition, the high (low) quality firm chooses a product (process) innovation, while firms make the opposite choices in case of Cournot competition. The presence of a leapfrogging effect (only in the Cournot case) explains these different outcomes. Last, we find that the Cournot competitors tend to favor the introduction of a new product in comparison with the Bertrand competitors.

1 Introduction

In markets where manufacturing goods are produced managers often face a dilemma: is it better to employ the advances in knowledge and technology to produce a higher quality good or to ensure a higher rate of return by exploiting the benefits of lower unit costs? For example, in the aircraft industry quality is represented by the speed, while a larger size yields lower average operating costs. In 2002 Boeing and Airbus made different choices on these two options. Airbus decided to produce the world's biggest airliner, the A380 (555 seats with the possibility of expanding capacity to 800), Boeing decided instead to go for speed rather than size, trying to develop its Sonic Cruiser (250 seats, which the possibility to fly at 98% of the speed of sound). Championing speed rather than size suggests that Boeing thinks most future growth will come from high quality demand (i.e. fast and frequent point–to–point flights); Airbus, by contrast, still sees a healthy market for a relatively low–cost super–jumbo to connect the world's biggest international airports.[1] The above problem can be classified as the choice between introducing a product or a process innovation. The former consists in the production of new goods, while the latter yields a cost saving benefit in the production of an existing good. This paper tackles this problem and tries to explain what factors might be important in a firm's decision to direct investment (e.g. R&D expenditure) towards the introduction of a product innovation or of a process innovation.

We will show that, under both the competition regimes considered (i.e. Bertrand and Cournot), three types of equilibria concerning the innovation game may arise, two symmetric (both firms introduce either a process innovation or a product innovation) and one asymmetric, where the high quality firm introduces a product innovation and the low quality firm a process innovation under Bertrand competition. On the contrary, in case of Cournot competition, the high quality firm introduces a process innovation while the low quality firm adopts a product innovation. The explanation about the determinant of the prevailing

[1]The Sonic Cruiser project was a failure for Boeing, which decided to produce the 787 Dreamliner, a compromise between speed performances and cost savings.

equilibria is based on the different incentives that the two firms have about adopting a certain type of innovation: for instance, under Bertrand competition the high quality firm has higher incentives to introduce a product innovation than the low quality firm. Quality leader and follower in the ex–ante situation have instead the opposite incentives under Cournot competition: in this case we obtain leapfrogging. Some examples confirm that firms selling goods with different qualities follow different market strategies: high price car manufactures are usually the first to introduce new optional (e.g. CD players, satellite navigators, ABS, etc.), supermarket chains with a good reputation are the first to adopt quality standards, while hard discounts make of price reductions (through costs savings) their mission. Moreover, sometimes we observe a swap in the quality leadership between firms, thanks to the introduction of a new good.

A model where firms strategically choose between either a process or a product innovation can also supply some additional insights about the effects of that decision on the intensity of competition between the two firms. Under both the competitive regimes considered the three above equilibria in innovation adoption have different impacts on the post–innovation prices. For instance, under Bertrand competition the intensity of competition is not relaxed if both firms adopt a process innovation, i.e. they end up with lower prices than the *status quo* levels. On the contrary, price competition becomes less intense if the high quality firm introduces a product innovation and the low quality firm a costs saving innovation and if both firms adopt a new product. Hence, since the adoption of different types of innovation creates an efficiency gap between the two firms, it follows that costs heterogeneity relaxes price competition, and so it can be classified as a supply side effect: *firms strategically choose to have an efficiency gap rather than costs homogeneity because competition becomes less intense.*[2]

Notwithstanding the relevance of this issue there exist almost no attempts to deal with it, since the literature has usually treated the two kinds of innovation

[2]These results are obtained in a duopoly model with vertical differentiation where the market is uncovered, but they also apply to the covered configuration. The literature (see Choi and Shin [1992] and Wauthy [1996]) has shown that the choice of the market configuration (covered or uncovered) is endogenous. A market is covered if all consumers with a positive willingness to pay for the good buy it, while it is uncovered if some consumers do not purchase the good.

separately. Bonanno and Haworth [1998] (henceforth BH) has provided, up to now, the closest contribution to our work. They find that the *type of competitive regime* in which the firms find themselves (Cournot *vs.* Bertrand) may explain why a firm decides to adopt a product innovation and not a process innovation (and vice versa). They study this problem in a vertically differentiated duopoly where only one firm can innovate. BH show that *if the innovator is the high quality firm* a tendency to favor product innovation emerges in case of Bertrand competition and to favor process innovation in presence of Cournot competition.[3] On the other hand, *if the innovator is the low quality firm* and whenever the two regimes lead to different adoptions, the Bertrand competitor chooses to introduce a process innovation, while the Cournot competitor introduces a product innovation.

Few other contributions have weaker links with our work. Rosenkranz [2003] studies, in a Cournot duopoly model with horizontal differentiation, how two competitors will optimally invest into both process and product innovation.[4] She shows that an increase in consumers' reservation price causes firms to increase R&D investments but also to shift them towards product innovation if the relative efficiency of the two types of innovation is kept constant. Lambertini and Orsini [2000] analyze the incentives to introduce a product innovation or a process innovation in a vertical differentiated monopoly (and so there is no strategic interaction).[5] Weiß [2003] presents a duopoly model with horizontal differentiation where firms choose between a process or product innovation. The latter consists in fixing the profit maximizing variety (while the pre–innovation variety is not optimal). In her framework the competitors are engaged in a two–stage competition, i.e. first they select the type of innovation and then they compete

[3] In general if the innovator is the high quality firm one of three things may happen: (1) both the Cournot competitor and the Bertrand competitor choose the process innovation; or (2) both select the product innovation; or (3) they make different choices. Under the latter case the Bertrand (Cournot) competitor choose to introduce a product (process) innovation.

[4] She develops the idea that usually firms have a portfolio of R&D projects, some more targeted at process innovations and some at product innovation, so that the optimal mix between these two types of innovation becomes a key variable in the competitive environment.

[5] They show that the social planner and the monopolist might adopt different type of innovation.

in price. She shows that all feasible moves in innovation adoption may belong to the equilibrium path and that the intensity of competition affects the equilibrium selection (if competition is intense (modest) firms choose product (process) innovation, while if it is intermediate they select asymmetrically).

This paper is an attempt to extend BH's results by considering that, in an oligopolistic environment with vertical differentiation, the choice between a product or a process innovation is taken simultaneously by all the firms in an industry. We shall think of process innovation as a reduction in the firm's production costs, so that it can be defined as *costs saving effect* on firm's efficiency. Product innovation will be interpreted as an improvement in the quality of a firm's product, and we label this as *quality effect*. We will show that BH's results, in a framework where *both* firms innovate, do not take into account for the possibility of leapfrogging. Moreover, we find that the Cournot competitors tend to favor the introduction of a new product w.r.t. the Bertrand competitors.

The paper is organized as follows. Section 2 presents the model, Section 3 analyzes the strategic choice between product and process innovation under Bertrand competition. The investigation is divided in two subsections: the pre–innovation equilibrium, where the two firms' qualities are determined (Section 3.1) and the solution of the innovation game (Section 3.2). Section 4 studies the Cournot case, again splitted in the pre–innovation equilibrium (Section 4.1) and in the innovation equilibrium (Section 4.2). Section 5 presents the main results of the paper, while their proofs are reported in the Appendix.

2 The model

Consider a two–stage duopoly model where firms, given a pre–innovation quality pair $\{\theta_H^0, \theta_L^0\}$ with $\theta_H^0 > \theta_L^0$ ("0" indicates the *status quo*) sell a vertically differentiated good and may be engaged in Bertrand or in Cournot competition. At $t = 1$ firms simultaneously decide whether to adopt a ProCess innovation (PC), or a ProDuct innovation (PD).[6] Hence at time $t = 1$ firm i ($i = H, L$, where L

[6]We rule out the possibility of choosing both types of innovation. Furthermore, notice that the decision not to innovate is not considered since it is easy to show that it is always dominated

stands for "low" quality firm and H for "high" quality firm[7]) chooses I_i, where

$$I_i = \begin{cases} 1 & \text{if firm } i \text{ selects} \quad PC \\ 0 & \text{if firm } i \text{ selects} \quad PD \end{cases}$$

This choice affects firm i's costs function if $I_i = 1$ and instead its market share if $I_i = 0$. We consider that quality is a variable cost (Champseaur and Rochet [1989], Gal–Or [1983] and Mussa and Rosen [1978]) so that firms have the following costs function: $C(y_i, \theta_i) = c\frac{\theta_i^2}{2}y_i$, where y_i is the output of firm i, θ_i its quality and c the constant unit costs of production.[8] A process innovation reduces marginal costs (i.e. it has a costs saving effect) by decreasing c; without loss of generality we assume that under $I_i = 1$ production costs become negligible (i.e. $c = 0$). If instead firm i introduces a new product, it benefits from an increase in its quality from θ_i^0 to $\psi\theta_i^0$ with $\psi > 1$ (see BH p. 502). Hence we label ψ as the quality effect. These two effects are exogenous, since we assume that the innovator has invested in R&D (e.g. it has built a lab and hired a team of scientists) and the corresponding costs are sunk.[9] Note that before choosing which type of innovation to adopt firms have the same costs, and that costs homogeneity is maintained if they make the same type of adoption; instead in case of asymmetric adoptions they have different costs functions. Moreover, it follows from our setup that if at $t = 1$ a firm has selected a process innovation its quality remains fixed at the pre–innovation level, i.e. if $I_i = 1 \rightarrow \theta_i = \theta_i^0$ at $t = 2$. At $t = 2$, after observing the rival's innovation choice, under Bertrand (Cournot) competition firms choose simultaneously the price p_i (quantity y_i).

The market demand is specified as follows: each consumer buys only one unit of the good, and is characterized by the net utility function $U = s\theta - p$, where $s \in [0, 1]$ and p is the price paid for the good. As usual the variable s

by introducing one of the two innovation types.

[7]The choice of being either the high quality firm or the low quality firm (see Herguera and Lutz [1998]) should be studied in a stage before the choice of innovation. We do not solve this stage, but we assign a label to each firm.

[8]Our results are valid also for a costs function where quality is a fixed cost (Bonanno [1986], Motta [1993] and Shaked and Sutton [1982, 1983]), e.g. $C(y_i, \theta_i) = cy_i + \frac{\theta_i^2}{2}$, but are based on a simulation analysis. Results are available upon request.

[9]Without loss of generality we assume that R&D costs are equal to 0.

represents the consumer's willingness to pay (a taste parameter) for the good (Tirole [1988]), and is uniformly distributed over the interval $[0,1]$. From the above and since the consumer with the lowest willingness to pay is located in 0, he/she will never buy the good, unless $p \leq 0$. Hence the market is always "uncovered" and some consumers are always out of the market. The consumer indifferent between buying the low quality good and not buying at all has a utility given by $s\theta_L - p_L = 0$, so that $s = \frac{p_L}{\theta_L}$. The consumer indifferent between buying the low quality good and the high quality good has a taste parameter equal to $s^* = \frac{p_H - p_L}{\theta_H - \theta_L}$. Hence under Bertrand competition the two firms' market shares are

$$y_H = \left[1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right] \tag{1}$$

$$y_L = \left[\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right] \tag{2}$$

while under Cournot competition we have

$$p_H = \theta_H(1 - y_H) - \theta_L y_L \tag{3}$$

$$p_L = \theta_L(1 - y_H - y_L) \tag{4}$$

with $y_H + y_L < 1$. Note that in case of product innovation the innovator receives a "market share premium". For instance, in case of price competition, if the innovator is the high quality firm, its new quality is $\psi\theta_H^0$, and so s^* moves leftwards since $s^{*'} = \frac{p_H - p_L}{\psi\theta_H^0 - \theta_L^0} < s^*$, i.e. $s^{*'} \to s$, thereby increasing its market share. If instead the innovator is the low quality firm $s^{*'} = \frac{p_H - p_L}{\theta_H^0 - \psi\theta_L^0} > s^*$, while $s^{'} = \frac{p_L}{\psi\theta_L^0} < s$, i.e. $s^{'} \to 0$ while $s^{*'} \to 1$ and so $y_L \uparrow$. Moreover, in case of product innovation we will take into account that, if the unique innovator is the low quality firm, a leapfrogging effect may emerge (i.e. firm L may become the high quality firm if $\psi > \frac{\theta_H^0}{\theta_L^0}$).

We look for a subgame perfect equilibrium, i.e. a pair of strategies which forms a Nash equilibrium in each subgame. As usual, we compute the solution by backward induction, starting from the last stage of the game, i.e. the Bertrand (or Cournot) subgame. Firm i's profit in the Bertrand subgame is the following (B stands for Bertrand):

$$\begin{aligned}\pi_i^B(I_i, I_j) = \quad & I_i I_j \left[p_i y_i(\theta_i^0, \theta_i^0)\right] + I_i(1-I_j)\left[p_i y_i(\theta_i^0, \psi\theta_j^0)\right] + \\ & +(1-I_i)I_j\left[\left(p_i - \frac{c{\theta_i^0}^2}{2}\right) y_i(\psi\theta_i^0, \theta_i^0)\right] + \\ & +(1-I_i)(1-I_j)\left[\left(p_i - \frac{c{\theta_i^0}^2}{2}\right) y_i(\psi\theta_i^0, \psi\theta_i^0)\right]\end{aligned} \tag{5}$$

with $i \neq j$ and $i, j = H, L$. Under Cournot competition, the individual profit function is (C is for Cournot):

$$\begin{aligned}\pi_i^C(I_i, I_j) = \quad & I_i I_j \left[p_i(\theta_i^0, \theta_i^0) y_i\right] + I_i(1-I_j)\left[p_i(\theta_i^0, \psi\theta_j^0) y_i\right] + \\ & +(1-I_i)I_j\left[\left(p_i(\psi\theta_i^0, \theta_i^0) - \frac{c{\theta_i^0}^2}{2}\right) y_i\right] + \\ & +(1-I_i)(1-I_j)\left[\left(p_i(\psi\theta_i^0, \psi\theta_i^0) - \frac{c{\theta_i^0}^2}{2}\right) y_i\right]\end{aligned} \tag{6}$$

again with $i \neq j$ and $i, j = H, L$.

3 Innovation adoption under Bertrand competition

In this Section we investigate the strategic choice between product and process innovation if firms compete in prices in the final market. Since the adoption of a certain type of innovation depends upon the *status quo* quality levels (i.e. θ_H^0, θ_L^0), it is necessary to compute the equilibrium before the innovation game.

3.1 The pre–innovation equilibrium

From (1)–(2) we have

$$\pi_H^B(p_h, p_L, \theta_H, \theta_L) = p_H\left(1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right) - \frac{1}{2}c\theta_H^2\left(1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right) \tag{7}$$

and

$$\pi_L^B(p_h, p_L, \theta_H, \theta_L) = p_L\left(\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right) - \frac{1}{2}c\theta_H^2\left(\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right) \tag{8}$$

Firms maximize (7)–(8) by choosing first the quality pair (θ_H^*, θ_L^*) and then the market prices (p_H^*, p_L^*). Starting from the bottom stage we have the following FOCs':

$$\frac{\partial \pi_H^B}{\partial p_H} = 1 - \frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_H}{\theta_H - \theta_L} + \frac{c\theta_H^2}{2(\theta_H - \theta_L)} = 0 \tag{9}$$

$$\frac{\partial \pi_L^B}{\partial p_L} = \frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L} - \left(p_L - \frac{1}{2}c\theta_L^2\right)\left(\frac{1}{\theta_H - \theta_L} + \frac{1}{\theta_L}\right) = 0 \tag{10}$$

Solving the system (9)–(10) gives the pre–innovation equilibrium prices:

$$p_H^* = \frac{\theta_H[4(\theta_H - \theta_L) + c(2\theta_H^2 + \theta_L^2)]}{2(4\theta_H - \theta_L)} \tag{11}$$

$$p_L^* = \frac{\theta_L[2(\theta_H - \theta_L) + c\theta_H(\theta_H + 2\theta_L)]}{2(4\theta_H - \theta_L)} \tag{12}$$

Substituting (11)–(12) in (1)–(2) gives:

$$y_H = \frac{\theta_H[4 - c(2\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \tag{13}$$

$$y_L = \frac{\theta_H[2 - c(\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \tag{14}$$

Replacing (11)–(12) in (7)–(8) we obtain the profits' reduced form at the quality stage, i.e.

$$\pi_H = \frac{\theta_H(\theta_H - \theta_L)[4 - c(2\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \times y_H$$

$$\pi_L = \frac{\theta_L(\theta_H - \theta_L)[2 - c(\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \times y_L$$

By differentiating firm i's profit function w.r.t θ_i we get the following FOCs':

$$\frac{\partial \pi_H^B}{\partial \theta_H} = y_H \frac{4(4\theta_H^2 - 3\theta_H\theta_L + 2\theta_L^2) - c(24\theta_H^3 - 22\theta_H^2\theta_L + 5\theta_H\theta_L^2 + 2\theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{15}$$

$$\frac{\partial \pi_L^B}{\partial \theta_L} = y_L \frac{2\theta_H(4\theta_H^2 - 7\theta_L) + c(4\theta_H^3 - 19\theta_H^2\theta_L + 17\theta_H\theta_L^2 - 2\theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{16}$$

We can now state the equilibrium qualities in the pre–innovation stage and all the corresponding outcomes for both firms.

Proposition 1 *In the equilibrium of the Bertrand pre–innovation game, the quality level of firm H is more then twice the quality level of firm L, since $\theta_H^0 = \frac{0.82}{c}$, $\theta_L^0 = \frac{0.40}{c}$. Moreover, being the quality leader is profitable, since $\pi_H^{B0} = \frac{0.03}{c}$, $\pi_L^{B0} = \frac{0.02}{c}$.*

Proof: See Appendix.

Proposition 1 shows that the high quality firm enjoys a higher profit by selling to the smaller but "richer" market niche ($y_H^0 < y_L^0$ since $y_H^0 = 0.28$, $y_L^0 = 0.34$); to achieve this higher profit level its price is much higher than the low quality one ($p_H^0 = \frac{0.45}{c}$, $p_L^0 = \frac{0.15}{c}$).

3.2 The innovation game

Having solved for the pre–innovation quality levels, we can now compute the subgame perfect equilibrium of the innovation game under price competition.

3.2.1 Subgames

Starting from the last stage of the game, we have to identify the Nash equilibrium in four possible subgames, according to the innovation choices made by the two firms at $t = 1$.

Case a: $I_H = I_L = 1$

Both firms have selected a process innovation at $t = 1$ and so, given that $\theta_H^{11} = \theta_H^0$ and $\theta_L^{11} = \theta_L^0$,[10] the two profit functions, from (5) are:

$$\pi_H^B(I_H = I_L = 1) = \pi_H^{B11} = p_H^{11} \underbrace{\left[1 - \frac{p_H^{11} - p_L^{11}}{\theta_H^0 - \theta_L^0}\right]}_{y_H^{11}} \tag{17}$$

$$\pi_L^B(I_H = I_L = 1) = \pi_L^{B11} = p_L^{11} \underbrace{\left[\frac{p_H^{11} - p_L^{11}}{\theta_H^0 - \theta_L^0} - \frac{p_L^{11}}{\theta_L^0}\right]}_{y_L^{11}} \tag{18}$$

[10] The superscripts indicate the innovation moves at $t = 1$; $\{11\}$ means that $I_H = 1$ and $I_L = 1$.

In this subgame the degree of vertical differentiation is the same of the pre–innovation game, while the two firms have the same unit cost of production in the innovation game. By simultaneously solving $\frac{\partial \pi_H^{B11}}{\partial p_H^{11}} = 0$ and $\frac{\partial \pi_L^{B11}}{\partial p_L^{11}} = 0$, we get the following market shares and profits:

$$y_H^{11} = \frac{2\theta_H^0}{4\theta_H^0 - \theta_L^0}, \qquad y_L^{11} = \frac{\theta_H^0}{4\theta_H^0 - \theta_L^0}$$

$$\pi_H^{B11} = \frac{2\theta_H^0(\theta_H^0 - \theta_L^0)}{4\theta_H^0 - \theta_L^0} \times y_H^{11}, \qquad \pi_L^{B11} = \frac{\theta_L^0(\theta_H^0 - \theta_L^0)}{4\theta_H^0 - \theta_L^0} \times y_L^{11} \tag{19}$$

By substituting for the quality outcomes in the pre–innovation game, i.e. $\theta_H^0 = \frac{0.82}{c}$ and $\theta_L^0 = \frac{0.40}{c}$, we get the two firms' profit functions at $t = 1$ if they both adopt a process innovation, i.e.

$$\pi_H^{B11} = \frac{0.14}{c}, \qquad \pi_L^{B11} = \frac{0.02}{c} \tag{20}$$

Notice that $\pi_i^{B11} \gg \pi_i^0$ (π_L^{B11} is slightly higher than π_L^0) and that firm H enjoys a large profit increase in comparison with its pre-innovation profit.

Case b: $I_H = 0, I_L = 1$

The high quality firm has adopted a product innovation, so that $\theta_H^{01} = \psi\theta_H^0$, while firm L has chosen a process innovation, i.e. $\theta_L^{01} = \theta_L^0$. The two profit functions before choosing prices are:

$$\pi_H^B(I_H = 0, I_L = 1) = \pi_H^{B01} = \left(p_H^{01} - \frac{1}{2}c(\theta_L^0)^2\right) \underbrace{\left(1 - \frac{p_H^{01} - p_L^{01}}{\psi\theta_H^0 - \theta_L^0}\right)}_{y_H^{01}}$$

$$\pi_L^B(I_H = 0, I_L = 1) = \pi_L^{B01} = p_L^{01} \underbrace{\left(\frac{p_H^{01} - p_L^{01}}{\psi\theta_H^0 - \theta_L^0} - \frac{p_L^{01}}{\theta_L^0}\right)}_{y_L^{01}}$$

The degree of vertical differentiation increases in comparison with that arising in the pre–innovation game. However firm L enjoys a cost advantage. By differentiating the two profit function w.r.t. prices and then by solving the two FOCs' we get the following outcomes:[11]

$$y_H^{01} = \frac{\theta_H^0[4\psi(\psi\theta_H^0 - \theta_L^0) - c\theta_H^0(2\psi\theta_H^0 - \theta_L^0)]}{2(4\psi\theta_H^0 - \theta_L^0)(\psi\theta_H^0 - \theta_L^0)}, \quad y_L^{01} = \frac{\psi\theta_H^0[2(\psi\theta_H^0 - \theta_L^0) + c\theta_H^2]}{2(4\psi\theta_H^0 - \theta_L^0)(\psi\theta_H^0 - \theta_L^0)}$$

$$\pi_H^{B01} = \frac{\theta_H^0[4\psi(\psi\theta_H^0 - \theta_L^0) - c\theta_H^0(2\psi\theta_H^0 - \theta_L^0)]}{2(4\psi\theta_H^0 - \theta_L^0)(\psi\theta_H^0 - \theta_L^0)} \times y_H^{01}, \pi_L^{B11} = \frac{\theta_L^0(\theta_H^0 - \theta_L^0)}{4\theta_H^0 - \theta_L^0} \times y_L^{01}$$

By substituting for θ_H^0 and θ_L^0 we obtain the following profit functions:

$$\pi_H^{01} = \frac{1.80(\psi - 0.13)^2(\psi - 0.77)^2}{c(8.81\psi^3 - 6.43\psi^2 + 1.17\psi - 0.06)}, \tag{21}$$

$$\pi_L^{01} = \frac{0.22\psi(\psi - 0.08)^2}{c(8.81\psi^3 - 6.43\psi^2 + 1.17\psi - 0.06)} \tag{22}$$

Case c: $I_H = 1, I_L = 0$

The two firms make different choices also in this case, since firm H goes for a process innovation while firm L chooses a product innovation. This implies that $\theta_H^{10} = \theta_H^0$ and $\theta_L^{10} = \psi\theta_L^0$. Hence for ψ increasing, there is the possibility of leapfrogging: the latter happens when $\psi\theta_L^0 \geq \theta_H^0$, i.e. when $\psi \geq \frac{\theta_H^0}{\theta_L^0} = 2.05$. We have then to investigate two sub–cases: (1) no leapfrogging, i.e. $1 \leq \psi < 2.05$; (2) leapfrogging ($\psi > 2.05$).

Sub–case 1: no leapfrogging

The two profit functions are:

$$\pi_H^B(I_H = 1, I_L = 0) = \pi_H^{B10} = p_H^{10} \underbrace{\left[1 - \frac{p_H^{10} - p_L^{10}}{\theta_H^0 - \psi\theta_L^0}\right]}_{y_H^{10}}$$

$$\pi_L^B(I_H = 1, I_L = 0) = \pi_L^{B10} = \left(p_L^{10} - c\frac{(\theta_L^0)^2}{2}\right) \underbrace{\left(\frac{p_H^{10} - p_L^{10}}{\theta_H^0 - \psi\theta_L^0} - \frac{p_L^{10}}{\psi\theta_L^0}\right)}_{y_L^{10}}$$

[11] Note that $0 < y_H^{01} < 1$, $0 < y_L^{01} < 1$ and $y_H^{01} + y_L^{01} < 1$ for $\psi > 1$, while $p_H^{01} > 0$ and $p_L^{01} > 0$ for $\psi > 1$.

Solving $\frac{\partial \pi_H^{B10}}{\partial p_H^{10}} = 0$ and $\frac{\partial \pi_L^{B10}}{\partial p_L^{10}} = 0$ yields the following market outcomes and profits:

$$y_H^{10} = \frac{\theta_H^0[4(\theta_H^0 - \psi\theta_L^0) + c(\theta_L^0)^2]}{2(4\theta_H^0 - \psi\theta_L^0)(\theta_H^0 - \psi\theta_L^0)}, \;\; y_L^{10} = \frac{\theta_H^0[2\psi(\theta_H^0 - \psi\theta_L^0) - c\theta_L^0(2\theta_H^0 - \psi\theta_L^0)]}{2(4\theta_H^0 - \psi\theta_L^0)(\theta_H^0 - \psi\theta_L^0)}$$

$$\pi_H^{B10} = \frac{\theta_H^0[4(\theta_H^0 - \psi\theta_L^0) + c(\theta_L^0)^2]}{2(4\theta_H^0 - \psi\theta_L^0)} \times y_H^{10}$$

$$\pi_L^{B10} = \frac{\theta_L^0[2\psi(\theta_H^0 - \psi\theta_L^0) - c\theta_L^0(2\theta_H^0 - \psi\theta_L^0)]}{2(4\theta_H^0 - \psi\theta_L^0)} \times y_L^{10}$$

Since $\theta_H^0 = \frac{0.82}{c}$ and $\theta_L^0 = \frac{0.40}{c}$ we obtain:

$$\pi_H^{B10} = \frac{(0.65\psi - 1.41)^2}{c(8.81 - 6.43\psi + 1.17\psi^2 - 0.06\psi^3)} \tag{23}$$

$$\pi_L^{B10} = \frac{0.05(\psi - 0.45)^2(\psi - 1.80)^2}{c(8.81 - 6.43\psi + 1.17\psi^2 - 0.06\psi^3)} \tag{24}$$

At this stage we have to consider an important issue: since in this sub–case we have always a reduction in the degree of vertical differentiation (which is equal to 0 if $\psi = 2.05$), firm L may have not the incentive to choose $I_L = 0$ in response to $I_H = 1$. This is because the reduction in the degree of vertical differentiation may reduce its profit, rather than increasing it. The literature has pointed out (Gabszewicz and Thisse [1979,1980], Shaked and Sutton [1982], BH [1998]) that firm L, under Bertrand competition may refrain to increase its quality even if it can do this at no costs (as in this model). If we evaluate $\frac{\partial \pi_L^{B10}}{\partial \psi} < 0$ we obtain that $\frac{\partial \pi_L^{B10}}{\partial \psi} < 0$ if $1.20 < \psi \leq 1.80$, while the derivative is positive for $1.80 < \psi < 2.05$. However under the latter situation y_L^{10} becomes negative and so this solution is unfeasible. Hence we can say that to respond with $I_L = 0$ to $I_H = 1$ is rational only when $1 \leq \psi \leq 1.20$. If instead $1.20 < \psi < 2.05$ the only chance that firm L will consider as a reply to $I_H = 1$ is $I_L = 1$.

Sub–case 2: leapfrogging

In this situation firm L becomes the quality leader. Moreover, the degree of vertical differentiation increases, in comparison with the pre–innovation game, when $\psi > \left(\frac{\theta_H^0}{\theta_L^0}\right)^2$. Hence the structure of the two firms' demand function changes, as it is shown by the following profit functions:

$$\pi_H^B(I_H = 1, I_L = 0) = \pi_H^{B10} = p_H^{10} \underbrace{\left[\frac{p_L^{10} - p_H^{10}}{\psi\theta_L^0 - \theta_H^0} - \frac{p_H}{\theta_H^0}\right]}_{y_H^{10}}$$

$$\pi_L^B(I_H=1,I_L=0)=\pi_L^{B10}=\left(p_L^{10}-c\frac{(\theta_L^0)^2}{2}\right)\underbrace{\left(1-\frac{p_L^{10}-p_H^{10}}{\psi\theta_L^0-\theta_H^0}\right)}_{y_L^{10}}$$

After solving the Bertrand subgame we get:

$$y_H^{10}=\frac{\psi\theta_L^0[2(\psi\theta_L^0-\theta_H^0)+c(\theta_L^0)^2]}{2(4\psi\theta_L^0-\theta_H^0)(\psi\theta_L^0-\theta_H^0)},\ \ y_L^{10}=\frac{\theta_L^0[4\psi(\psi\theta_L^0-\theta_H^0)-c\theta_L^0(\psi\theta_L^0-\theta_H^0)]}{2(4\psi\theta_L^0-\theta_H^0)(\psi\theta_L^0-\theta_H^0)}$$

$$\pi_H^{B10}=\tfrac{\theta_H^0[2(\psi\theta_L^0-\theta_H^0)+c(\theta_L^0)^2]}{2(4\psi\theta_L^0-\theta_H^0)}\times y_H^{10}$$
$$\pi_L^{B10}=\tfrac{\theta_L^0[4\psi(\psi\theta_L^0-\theta_H^0)-c\theta_L^0(\psi\theta_L^0-\theta_H^0)]}{2(4\psi\theta_L^0-\theta_H^0)}\times y_L^{10}$$

If we substitute for θ_H^0 and θ_L^0 we obtain:

$$\pi_H^{10}=\frac{0.05\psi(\psi-1.86)^2}{c(1.01\psi^3-3.13\psi^2+2.41\psi-0.55)},\tag{25}$$

$$\pi_L^{10}=\frac{0.10(\psi-0.09)^2(\psi-2.16)^2}{c(1.01\psi^3-3.13\psi^2+2.41\psi-0.55)}\tag{26}$$

and so the impact of ψ on π_L^0 is the following:

$$\frac{\partial\pi_L^{10}}{\partial\psi}=\frac{0.10(\psi-0.09)(\psi-0.51)(\psi-1.96)(\psi-2.16)(\psi^2-1.44\psi+1.95)}{c(0.30-2.65\psi+9.25\psi^2-16.19\psi^3+14.67\psi^4-6.34\psi^5+1.03\psi^6)}\tag{27}$$

Computation shows that the denominator is always positive for $\psi\geq 2.06$, while the numerator is negative if $2.06\leq\psi\leq 2.16$. Hence in this case we have that $\frac{\partial\pi_L^{10}}{\partial\psi}<0$ and so the strategy $I_L=0$ as a response to $I_H=1$ is unfeasible. Indeed firm L will introduce a product innovation when this leads to leapfrogging only if $\psi>2.16$. Hence, to sum up, the strategy pair $(I_H=1,I_L=0)$ is feasible only for $1\leq\psi\leq 1.20$ and for $\psi>2.16$. In the first interval no leapfrogging occurs and firms' profits are given by (23)–(24), while in the second one they are (25)–(26).

Case d: $I_H=I_L=0$

We have that $\theta_H^{00}=\psi\theta_H^0$ and $\theta_L^{00}=\psi\theta_L^0$. Hence the two profit functions are:

$$\pi_H^B(I_H=I_L=0)=\pi_H^{B00}=\left(p_H-c\frac{(\theta_H^0)^2}{2}\right)\underbrace{\left(1-\frac{p_H-p_L}{\psi(\theta_H^0-\theta_L^0)}\right)}_{y_H^{00}}$$

Figure 1: Firm H's profits as function of ψ if $I_L = 1$

$$\pi_L^B(I_H = I_L = 0) = \pi_L^{B00} = \left(p_L - c\frac{(\theta_L^0)^2}{2}\right)\underbrace{\left(\frac{p_H - p_L}{\psi(\theta_H^0 - \theta_L^0)} - \frac{p_L}{\psi\theta_L^0}\right)}_{y_L^{00}}$$

Again, the degree of vertical differentiation is unchanged in this subgame. Solving the game at $t = 2$ gives the market outcomes and profits displayed below:

$$y_H^{00} = \frac{\theta_H^0[4\psi - c(2\theta_H^0 + \theta_L^0)]}{2\psi(4\theta_H^0 - \theta_L^0)}, \qquad y_L^{00} = \frac{\theta_H^0[2\psi + c(\theta_H^0 - \theta_L^0)]}{2\psi(4\theta_H^0 - \theta_L^0)}$$

$$\pi_H^{00} = \frac{\theta_H^0(\theta_H^0 - \theta_L^0)[4\psi - c(2\theta_H^0 + \theta_L^0)]}{2(4\theta_H^0 - \theta_L^0)} \times y_H^{00},$$

$$\pi_L^{00} = \frac{\theta_L^0(\theta_H^0 - \theta_L^0)[2\psi + c(\theta_H^0 - \theta_L^0)]}{2(4\theta_H^0 - \theta_L^0)} \times y_L^{00},$$

By applying the quality levels relative to the pre–innovation game we obtain the following profits:

$$\pi_H^{B00} = \frac{0.14(\psi - 0.51)^2}{c\psi} \tag{28}$$

$$\pi_L^{B00} = \frac{(0.13\psi + 0.03)^2}{c\psi} \tag{29}$$

Figure 2: Firm H's profits as function of ψ if $I_L = 0$

3.2.2 Equilibrium

Having identified the reduced form of firm i's profit under each possible innovation moves at $t = 1$, we can now identify the subgame perfect equilibrium. First we have to compute firm H's best reply to $I_L = 1$ and to $I_L = 0$. If firm L selects $I_L = 1$, then firm H's profits, i.e. (20) and (21), are those shown in Figure 1). Hence we can write, by solving the inequality $\pi_H^{B11} \geq \pi_H^{B01}$, the best response function of the high quality firm when the low quality one adopts a process innovation:

$$I_H^B(I_L = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.65 \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

If instead firm L chooses $I_L = 0$, we have that firm H profit is given by, if it adopts a process innovation, (23) if ψ is small and so no leapfrogging occurs, and by (25) if ψ is large (and so we have leapfrogging); if instead firm H chooses a product innovation as well, its profit is given by (28). Its profit functions are shown in Figure 2. Clearly, firm H finds more profitable to select $I_H = 1$ if $1 \leq \psi \leq 1.20$ (π_H^{B10} does not exist for $1.20 < \psi \leq 2.16$), so that its best reply to $I_L = 0$ is the following:

$$I_H^B(I_L = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.20 \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

We move now to analyze firm L's best responses in the innovation game. If firm H adopts a process innovation, the low quality firm has the profit functions displayed in Figure 3. The best reply is clearly the following one (notice that if $\psi \geq 2.33$ the low quality firm adopts a product innovation and becomes the quality leader):

$$I_L^B(I_H = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 2.33 \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

Figure 3: Firm L's profits as function of ψ if $I_H = 1$

Last, we need to study firm L behavior when the high quality firm chooses a product innovation. Figure 4 shows that firm L's profit is decreasing in ψ if it adopts a process innovation (π_L^{B01}) and increasing in ψ if it selects a product innovation as well (but it remains the quality follower). By comparing (24)–(26) and (29) we have that $\pi_L^{B01} \geq \pi_L^{B00}$ when $\psi \leq 1.75$, i.e. that firm L's best reply to $I_H = 0$ is:

$$I_L^B(I_H = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.75 \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

Now we can identify the equilibrium in innovation adoption.

Proposition 2 *In case of Bertrand competition, the innovation game has the following equilibria: (i) both firms adopt a process innovation if quality effect is relatively small compared to cost reduction effect (i.e. if* $1 \leq \psi \leq 1.65$*); (ii) the quality leader introduces a product innovation while the quality follower adopts a process innovation if the quality effect is intermediate (i.e. if* $1.65 < \psi \leq 1.75$*); (iii) both firms adopts a product innovation if the quality effect is large (i.e. if* $\psi > 1.75$*). Leapfrogging never occurs and the degree of vertical differentiation either is unchanged (in the two symmetric equilibria) or increased (in the asymmetric equilibrium).*

Figure 4: Firm L's profits as function of ψ if $I_H = 0$

Proof: See Appendix.

Proposition 2 states that only three equilibria can arise in the innovation game under Bertrand competition: two symmetric equilibria (where both firms adopt either a process innovation or a product innovation) and only one asymmetric equilibrium (where the high quality firm adopts a product innovation and the low quality firm a process innovation). Moreover it highlights that the high quality firm is the first to adopt a product innovation, and that there exists an interval where the low quality firm still finds profitable to benefit from a unit costs reduction and not from a product innovation. Note that when the equilibrium is $(I_H^* = 0, I_L^* = 1)$ the two competitors have costs heterogeneity. By comparing the *status quo* (i.e. the pre–innovation equilibrium) and the market outcomes under each innovation equilibrium we can draw several interesting implications.

First, under the symmetric equilibrium $I_H^* = I_L^* = 1$ we have that: $p_H^{11} < p_H^0$, $p_L^{11} < p_L^0$, $y_H^{11} > y_H^0$, $y_L^{11} < y_L^0$, $\pi_H^{B11} > \pi_H^{B0}$ and $\pi_L^{B11} < \pi_L^{B0}$. Hence the competition in R&D gives rise to a decrease in market prices. The latter implies an increase in the intensity of competition between the two firms (with a constant degree of vertical differentiation). For both firms the cost savings effect has a negative strategic effect: the competitor responds to a reduction in firm i's costs by reducing its own price, thereby increasing the intensity of competition. The high quality firm benefits from this: its market share rises up as well as its profits. On the contrary, the low quality firm suffers of a profit loss compared with the *status quo*: since both goods have the same quality than in the *status*

Figure 5: Firms' profits under the innovation equilibria—Bertrand competition

quo but are sold at lower prices, more consumers buy the high quality good.[12] The price reduction operated by the low quality firm is not enough to attract more consumers towards its good.

Second, under the asymmetric equilibrium $\{I_H^* = 0, I_L^* = 1\}$, we have that p_H^{01} increases with ψ, while p_L^{01} shrinks. Moreover, when this equilibrium prevails, $p_H^{B01} \gg p_H^0$ while $p_L^{B01} \ll p_L^0$. Hence price competition is softer than in the *status quo*. Meanwhile, y_H^{01} (y_L^{01}) increases (decreases) with ψ, and both market share are higher than the corresponding levels in the pre–innovation stage. Furthermore, $\pi_H^{B01} > \pi_H^{B0}$ and $\pi_L^{B01} > \pi_L^{B0}$ (with $\pi_H^{B01} > \pi_L^{B01}$). Hence both firms benefit from the reduction in competition due to asymmetric adoption and costs heterogeneity.

Last, if both firms adopt a product innovation, we get that in the interval $\psi > 1.75087$ both market prices arise with ψ and are higher than the *status quo*, as well as the two firms' market shares. Moreover, this market share premium is profitable for both firms (i.e. $\pi_i^{B00} > \pi_i^{B0}$). Figure 5 shows the profitability of the three equilibria for the two firms.

[12] Since firm H adopts a process innovation under this equilibrium, firm L gets a reduction in its profits compared with the pre–innovation level. However, its profits would be lower if it chooses to remain at the *status quo*: in this case firm L would have the same pre–innovation quality and no cost savings, suffering from an efficiency gap from the high quality firm. If $I_H = 1$ and firm L stays fixed at the *status quo*, its costs are $c\frac{\theta_H^0}{2}y_L$ and the solution at $t = 2$ is $y_H = 0.62, y_L = 0.12$, and, above all, $\pi_L = \frac{0.003}{c} < \pi_L^{B11}$. Hence adopting an innovation is always a *dominating* strategy.

4 Innovation adoption under Cournot competition

In this Section we analyze the innovation game when firms compete *á la* Cournot in the final market. First we need to compute the two pre–innovation quality levels.

4.1 The pre–innovation equilibrium

Under quantity competition the two firms' market demand are (3)–(4), and so the two firms' profits are:

$$\pi_H^C(y_H, y_L, \theta_H, \theta_L) = (\theta_H - \theta_L y_H - \theta_L y_L)y_H - \frac{1}{2}c\theta_H^2 y_H \tag{34}$$

$$\pi_L^C(y_H, y_L, \theta_H, \theta_L) = (\theta_L - \theta_L y_H - \theta_L y_L)y_L - \frac{1}{2}c\theta_L^2 y_L \tag{35}$$

Firms maximize (34)–(35) w.r.t. (θ_H, θ_L) first and then w.r.t. (y_H, y_L). By solving the Cournot subgame we have that:

$$\frac{\partial \pi_H^C}{\partial y_H} = \theta_H - 2\theta_H y_H - \theta_L y_L - \frac{1}{2}c\theta_H^2 = 0 \tag{36}$$

$$\frac{\partial \pi_L^C}{\partial y_L} = \theta_L - 2\theta_L y_L - \theta_L y_H - \frac{1}{2}c\theta_L^2 = 0 \tag{37}$$

Solving the system (36)–(37) gives the pre–innovation market shares:

$$y_H^* = \frac{2(2\theta_H - \theta_L) - c(2\theta_H^2 - \theta_L^2)}{2(4\theta_H - \theta_L)} \tag{38}$$

$$y_L^* = \frac{\theta_H(2 + c(\theta_H - 2\theta_L))}{2(4\theta_H - \theta_L)} \tag{39}$$

After substituting y_H^*, y_L^* in (34)–(35) and then differentiating w.r.t. θ_H, θ_L we get

$$\frac{\partial \pi_H^C}{\partial \theta_H} = y_H \frac{16\theta_H^2 - 4\theta_H\theta_L + 2\theta_L^2 - c(24\theta_H^3 - 10\theta_H^2\theta_L + 4\theta_H\theta_L^2 + \theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{40}$$

$$\frac{\partial \pi_L^C}{\partial \theta_L} = y_L \frac{\theta_H(8\theta_H - 2\theta_L + c(4\theta_H^2 - 23\theta_H\theta_L + 2\theta_L^2))}{2(4\theta_H - \theta_L)^2} = 0 \tag{41}$$

The following Proposition points out the equilibrium in the pre–innovation stage.

Proposition 3 *In the equilibrium of the Cournot pre–innovation game, the quality level of firm H is more then a quarter the quality level of firm L, since $\theta_H^0 = \frac{0.74}{c}$, $\theta_L^0 = \frac{0.59}{c}$. Hence the degree of vertical differentiation is lower than under Bertrand. Moreover, being the quality leader is profitable, since $\pi_H^{C0} = \frac{0.0353}{c}$, $\pi_L^{C0} = \frac{0.0350}{c}$, but less than being in the same position under Bertrand.*

Proof: See Appendix.

Again the high quality firm has a lower but more profitable market share than the low quality firm ($y_H^0 = 0.22$, $y_L^0 = 0.24$), while its price is higher ($p_H^0 = \frac{0.43}{c}$, $p_L^0 = \frac{0.31}{c}$). Moreover, both firms enjoy higher profits than under Bertrand.

4.2 The innovation game

The two firms have to decide simultaneously which type of innovation to adopt when they compete *à la* Cournot and have their qualities set at θ_H^0, θ_L^0. To identify the subgame perfect equilibrium we apply the same procedure shown in Section 3.2; hence a less detailed explanation is provided here.

4.2.1 Subgames

Again there are four possible Cournot subgames, according to the innovation choices made at $t = 1$. In each subgame the firms' profit functions vary according to (6), after substituting for each possible $\{I_H, I_L\}$ pair.

Case a: $I_H = I_L = 1$

In this subgame the two profit functions are:

$$\pi_H^{C11} = \underbrace{\left(\theta_H^0 - \theta_H^0 y_H - \theta_L^0 y_L\right)}_{p_H^{11}} y_H^{11}$$

$$\pi_L^{C11} = \underbrace{\left[\theta_L^0 (1 - y_H - y_L)\right]}_{p_L^{11}} y_L^{11}$$

and, by solving the FOCs' $\frac{\partial \pi_H^C}{\partial y_H}=0, \frac{\partial \pi_L^C}{\partial y_L}=0$, we obtain these market outcomes and profits at $t=1$:

$$y_H^{11}=\frac{2\theta_H^0-\theta_L^0}{4\theta_H^0-\theta_L^0}, y_L^{11}=\frac{\theta_L^0}{4\theta_H^0-\theta_L^0}, \pi_H^{C11}=\frac{\theta_H^0(2\theta_H^0-\theta_L^0)}{4\theta_H^0-\theta_L^0}\times y_H^{11}, \pi_L^{C11}=\frac{\theta_H^0\theta_L^0}{4\theta_H^0-\theta_L^0}\times y_L^{11}$$

After substituting for the pre–innovation quality levels we get:

$$\pi_H^{C11}=\frac{0.10}{c} \qquad \pi_L^{C11}=\frac{0.06}{c} \tag{42}$$

Notice that both firms increase profits in comparison with the pre–innovation stage, as well as their market share. On the contrary, market prices shrink: again the process innovation gives rise to a more intense competition.

Case b: $I_H=0, I_L=1$

In this subgame we have the following profits at $t=2$:

$$\pi_H^{C01}=\underbrace{\left(\psi\theta_H^0-\psi\theta_H^0 y_H^{01}-\theta_L^0 y_L^{01}\right)}_{p_H^{01}} y_H^{01}-\frac{c(\theta_H^0)^2}{2}y_H^{01}$$

$$\pi_L^{C01}=\underbrace{\left[\theta_L^0(1-y_H^{01}-y_L^{01})\right]}_{p_L^{01}} y_L^{01}$$

The market outcomes at $t=2$ are:

$$y_H^{01}=\frac{2\psi\theta_H^0-\theta_L^0-c(\theta_L^0)^2}{4\psi\theta_H^0-\theta_L^0}, y_L^{01}=\frac{\theta_H^0(2\psi+c\theta_H^0}{2(4\psi\theta_H^0-\theta_L^0)},$$

while the corresponding profits at $t=1$ are:

$$\pi_H^{C01}=\frac{\psi\theta_H^0(2\psi\theta_H^0-\theta_L^0-c(\theta_H^0)^2)}{4\psi\theta_H^0-\theta_L^0}\times y_H^{01}$$

$$\pi_L^{C01}=\frac{\theta_H^0\theta_L^0(2\psi+c\theta_H^0)}{2(4\psi\theta_H^0-\theta_L^0)}\times y_L^{01}$$

However the two firms' profits, after substituting for the pre–innovation qualities are:

$$\pi_H^{C01}=\frac{1.61(\psi+1.20(10^{-10}))(\psi-0.77)^2}{c(2.95\psi-0.59)^2}, \pi_L^{C01}=\frac{(0.56\psi+0.21)^2}{c(2.95\psi-0.59)^2} \tag{43}$$

Case c: $I_H = 1, I_L = 0$

Again, under these circumstances, leapfrogging occurs when $\psi\theta_L^0 > \theta_H^0 \rightarrow \psi > \frac{\theta_H^0}{\theta_L^0} = 1.26$. We have to study two subcases.

Sub–case 1: no leapfrogging

The two profit functions at $t = 2$ are:

$$\pi_H^{C10} = \underbrace{\left(\theta_H^0 - \theta_H^0 y_H^{10} - \psi\theta_L^0 y_L^{10}\right)}_{p_H^{10}} y_H^{10}$$

$$\pi_L^{C10} = \left[\underbrace{\psi\theta_L^0(1 - y_H^{10} - y_L^{10})}_{p_L^{10}} - c\frac{(\theta_L^0)^2}{2}\right] y_L^{10}$$

After maximizing each profit function wrt price we get:

$$y_H^{10} = \frac{4\theta_H^0 - 2\psi\theta_L^0 + c(\theta_L^0)^2}{2(4\theta_H^0 - \psi\theta_L^0)}, \quad y_L^{10} = \frac{\theta_H^0(\psi - c\theta_L^0}{\psi(4\theta_H^0 - \psi\theta_L^0)}$$

$$\pi_H^{C10} = \frac{\theta_H^0(4\theta_H^0 - 2\psi\theta_L^0 + c(\theta_L^0)^2)}{2(4\theta_H^0 - \psi\theta_L^0)} \times y_H^{10}, \;\; \pi_L^{10} = \frac{\theta_H^0\theta_L^0(\psi - c\theta_L^0)}{4\theta_H^0 - \psi\theta_L^0} \times y_L^{10}$$

If we substitute for θ_H^0 and θ_L^0 we obtain:

$$\pi_H^{C10} = \frac{0.25(\psi - 2.81)^2}{c(0.59\psi - 2.95)}, \quad \pi_L^{C10} = \frac{(0.56\psi - 0.33)^2}{c\psi(0.59\psi - 2.95)} \tag{44}$$

It is now easy to compute that:

$$\frac{\partial\pi_L^{10}}{\partial\psi} = \frac{0.19(\psi - 0.59)(\psi^2 + 3.28\psi + 2.95)}{c\psi^2(25.73 - 15.31\psi + 3.04\psi^2 - 0.20\psi^3)}$$

The numerator is always positive, while the denominator is greater than 0 if $\psi = 1$ and is increasing in ψ when $\frac{\partial(.)}{\partial\psi} > 0$. The latter is equal to $0.60\psi^2 - 6.08\psi + 15.31$, and it is greater than 0 if $\psi \leq 3.57$ and $\psi \geq 6.57$; hence since $1 \leq \psi < 1.26$ the denominator is always positive too. This implies that, differently from the Bertrand case, the low quality firm always benefits from increasing its quality under no leapfrogging and so the solution $I_H = 1, I_L = 0$ is always feasible in this sub–case.

Sub–case 2: leapfrogging

If $\psi > 1.26$ firm L becomes the high quality firm. Hence the two firms swap their positions in the consumers' ranking regarding vertical differentiation, so that the individual demands are defined as follows: $p_H = \theta_H^0(1 - y_H - y_L), p_L = \psi\theta_L^0 - \psi\theta_L^0 y_L - \theta_H^0 y_H$. The profit functions, since $I_H = 1, I_L = 0$, are:

$$\pi_H^{C10} = \underbrace{\theta_H^0(1 - y_H^{10} - y_L^{10})}_{p_H^{10}} y_H^{10}, \quad \pi_L^{C10} = \left(\underbrace{\psi\theta_L^0 - \psi\theta_L^0 y_L^{10} - \theta_H^0 y_H^{10}}_{p_L^{10}} - c\frac{(\theta_L^0)^2}{2} \right) y_L^{10}$$

Solving $\frac{\partial \pi_H^{C10}}{\partial p_H^{10}} = 0$ and $\frac{\partial \pi_L^{C10}}{\partial p_L^{10}} = 0$ we obtain the two equilibrium prices in this specific context, and so outputs and profits are as follows:

$$y_H^{10} = \frac{\theta_L^0(2\psi + c\theta_L^0)}{2(4\psi\theta_L^0 - \theta_H^0)}, \quad y_L^{10} = \frac{2\psi\theta_L^0 - \theta_H^0 - c(\theta_L^0)^2}{4\psi\theta_L^0 - \theta_H^0}$$

$$\pi_H^{C10} = \frac{\theta_H^0\theta_L^0(2\psi + c\theta_L^0)}{2(4\psi\theta_L^0 - \theta_H^0)} \times y_H^{10}, \quad \pi_L^{C10} = \frac{\psi\theta_L^0(2\psi\theta_L^0 - \theta_H^0 - c(\theta_L^0)^2)}{4\psi\theta_L^0 - \theta_H^0} \times y_L^{10}$$

The profit functions, after substituting for the pre–innovation quality levels are:

$$\pi_H^{C10} = \frac{0.25(\psi + 0.29)^2}{c(2.34\psi - 0.74)^2}, \quad \pi_L^{C10} = \frac{0.80\psi(\psi - 0.92)^2}{c(2.34\psi - 0.74)^2} \tag{45}$$

Again, firm L has always an incentive to adopt a product innovation, since

$$\frac{\partial \pi_L^{C10}}{\partial \psi} = \frac{1.88(\psi - 0.92)(\psi^2 - 0.02\psi + 0.29)}{c(12.85\psi^3 - 12.15\psi^2 + 3.83\psi - 0.40)}$$

and both the numerator and the denominator are positive for $\psi > 1$.

We can now define, by considering (44)–(45) the two profit functions if $I_H = 1, I_L = 0$:

$$\pi_H^{C01} = \begin{cases} \frac{\theta_H^0(4\theta_H^0 - 2\psi\theta_L^0 + c(\theta_L^0)^2)}{2(4\theta_H^0 - \psi\theta_L^0)} \times y_H^{10} = \frac{0.25(\psi - 2.81)^2}{c(0.59\psi - 2.95)} & \text{if } 1 \le \psi < 1.26 \\ \frac{\theta_H^0(4\theta_H^0 - 2\psi\theta_L^0 + c(\theta_L^0)^2)}{2(4\theta_H^0 - \psi\theta_L^0)} \times y_H^{10} = \frac{0.25(\psi + 0.29)^2}{c(2.34\psi - 0.74)^2} & \text{if } \psi > 1.26 \end{cases} \tag{46}$$

$$\pi_L^{C01} = \begin{cases} \pi_L^{10} = \frac{\theta_H^0\theta_L^0(\psi - c\theta_L^0)}{4\theta_H^0 - \psi\theta_L^0} \times y_L^{10} = \frac{(0.56\psi - 0.33)^2}{c\psi(0.59\psi - 2.95)} & \text{if } 1 \le \psi < 1.26 \\ \frac{\psi\theta_L^0(2\psi\theta_L^0 - \theta_H^0 - c(\theta_L^0)^2)}{4\psi\theta_L^0 - \theta_H^0} \times y_L^{10} = \frac{0.80\psi(\psi - 0.92)^2}{c(2.34\psi - 0.74)^2} & \text{if } \psi > 1.26 \end{cases} \tag{47}$$

Case d: $I_H = I_L = 0$

If both firms decide to introduce a product innovation, the profit functions at $t = 2$ are:

$$\pi_H^{C00} = \left\{ \left[\underbrace{\psi(\theta_H^0 - \theta_H^0 y_H^{00} - \theta_L^0 y_L^{00})}_{p_H^{00}} - c\frac{(\theta_H^0)^2}{2} \right] \right\} y_H^{00}$$

$$\pi_L^{C00} = \left\{ \left[\underbrace{\psi\theta_L^0(1 - y_H^{00} - y_L^{00})}_{p_L^{00}} - c\frac{(\theta_L^0)^2}{2} \right] \right\} y_L^{00}$$

Firms maximize the above profits at the $t = 2$ by computing the two FOCs $\frac{\partial \pi_H^{C00}}{\partial p_H^{00}}, \frac{\partial \pi_L^{C00}}{\partial p_L^{00}}$ and solving them for the market prices. Outputs and profits are then the following ones:

$$y_H^{00} = \frac{2\psi(2\theta_H^0 - \theta_L^0) - c[2(\theta_H^0)^2 - (\theta_L^0)^2]}{2\psi(4\theta_H^0 - \theta_L^0)}, \quad y_L^{00} = \frac{\theta_H^0[2\psi + c(\theta_H^0 - 2\theta_L^0)]}{2\psi(4\theta_H^0 - \theta_L^0)}$$

$$\pi_H^{C00} = \frac{\theta_H^0\{2\psi(2\theta_H^0 - \theta_L^0) - c[2(\theta_H^0)^2 - (\theta_L^0)^2]\}}{2\psi(4\theta_H^0 - \theta_L^0)} \times y_H^{00},$$

$$\pi_L^{C00} = \frac{\theta_H^0\theta_L^0[2\psi + c(\theta_H^0 - 2\theta_L^0)]}{2\psi(4\theta_H^0 - \theta_L^0)} \times y_L^{00}$$

Since $\theta_H^0 = \frac{0.74}{c}, \theta_L^0 = \frac{0.59}{c}$, by substituting them in the above profits we obtain:

$$\pi_H^{C00} = \frac{0.32\psi - 0.13}{c\psi}, \quad \pi_L^{00} = \frac{0.06(\psi - 0.22)^2}{c\psi} \tag{48}$$

4.2.2 Equilibrium

We are now in a position to compute the equilibrium at $t = 1$. First we need to identify each firm's best reply by comparing the available profit functions when the rival is adopting a given strategy. We start from firm H. If firm L chooses $I_L = 1$, the profit functions shown in (42) and (43). Hence we need to solve $\pi_H^{C11} \geq \pi_H^{C01}$, which is true for $\psi \leq 1.60$. The behavior of the profit functions

Figure 6: Firm H's profits as function of ψ if $I_L = 0$ under Cournot

π_H^{C11} and π_H^{C01} as function of ψ is similar to that displayed in Figure 1. Hence firm H's best reply when $I_L = 1$ is the following one:

$$I_H^C(I_L = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.60 \\ 0 & \text{otherwise} \end{cases} \tag{49}$$

If instead firm L chooses $I_L = 0$ firm H's profit functions are those shown in (46) and (48). The plot of these functions is reported in Figure 6; it is evident, by comparing it with Figure 2, the difference between the Bertrand and the Cournot case in this situation where leapfrogging may occur. While under Bertrand competition the low quality firm has no incentives to adopt a product innovation when the high quality firm has chosen a process innovation, if firms are engaged in Cournot competition the low quality firm always gains from investing in a product innovation when the high quality rival has adopted a cost reduction. To identify the best reply, we need to solve $\pi_H^{C10} \geq \pi_H^{C00}$ for ψ, which is fulfilled when $\psi \leq 1.66$. Hence firm H's best reply when $I_L = 0$ is:

$$I_H^C(I_L = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.66 \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

The next step is to compute firm L's best reply. If firm H selects $I_H = 1$ the two profit functions that we need to compare are shown in (42) and (47). Figure 7 shows the plot of these functions, which is different from that displayed in Figure 3, for the same reason just explained. Notice that π_L^{C10}, differently from

Figure 7: Firm L's profits as function of ψ if $I_H = 1$ under Cournot

the Bertrand case, is always increasing in ψ. We have to study when $\pi_L^{C11} \geq \pi_L^{C10}$, which is true for $\psi \leq 1.54$. Hence firm L's best reply when $I_H = 1$ is:

$$I_L^C(I_H = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.54 \\ 0 & \text{otherwise} \end{cases} \tag{51}$$

The last best reply regards firm L's behavior when the high quality firm adopts a product innovation. To identify it we need to consider the profit functions shown in (43) and in (48) (the plot of these functions is similar to that shown in Figure 4), and so study when $\pi_L^{C01} \geq \pi_L^{C00}$. The latter is true when $\psi \leq 1.65$. Hence firm L's best reply to $I_H = 0$ is the following one:

$$I_L^C(I_H = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.65 \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

We are now in a position to state the equilibrium of the innovation game under Cournot equilibrium.

Proposition 4 *In case of Cournot competition, the innovation game has the following equilibria: (i) both firms adopt a process innovation if quality effect is very small compared to cost reduction effect (i.e. if* $1 \leq \psi \leq 1.54$*); (ii) both firms introduce a product innovation if the quality effect is sufficiently large (i.e. psi* > 1.66*). (iii) If the quality effect is intermediate (i.e.* $1.54 < \psi \leq 1.66$*) leapfrogging occurs since the pre–innovation high quality firm chooses a process innovation while the pre–innovation low quality firm selects a product innovation and becomes the quality leader.*

Proof: See Appendix.

Proposition 4 points out that also in case of Cournot competition three equilibria may arise in the innovation game, two symmetric equilibria and one asymmetric equilibrium. However the latter is different from that prevailing under Bertrand, since the high quality firm selects a process innovation while the low quality firm chooses a product innovation. Since this happens for a quality effect sufficiently high, it involves leapfrogging. *Hence under Cournot the degree of vertical differentiation may also decrease in comparison with it ex–ante level.*

This difference in the asymmetric equilibrium between the two regimes is due to the firms' incentives to adopt a certain type of innovation *as a best reply to the rival's type of innovation.* Under Cournot, the degree of vertical differentiation is lower than under Bertrand, since firms already enjoy a less intense competition. Consequently, the high quality firm, if the size of the product innovation is not too high, may prefer to adopt a process innovation if it anticipates that the low quality firm goes for a product innovation. The intuition is the following: if firm L chooses a product innovation, the high quality firm is subject to leapfrogging and loses its leadership. However, the quality gap is not too big and, instead, the cost advantage granted by a process innovation is large, so that its best reply is to adopt a process innovation. On the other hand, the low quality firm responds to the adoption of a process innovation by the high quality firm with a product innovation because, thanks to leapfrogging, it becomes the quality leader. Vice versa, if it chooses a process innovation as well, the quality gap is as in the ex–ante equilibrium and so it suffers from being the quality follower, and the benefit given by adopting a process innovation is too small. For this reason, in presence of a product innovation of intermediate size, the high quality firm chooses a process innovation, and the low quality firm a product one.

On the contrary, under Bertrand, this asymmetric solution (i.e. $I_H = 1, I_L = 0$) is unfeasible for a large interval of the quality effect because the low quality has no incentive to introduce a product innovation. Rather, the equilibrium asymmetric solution is the one where the high quality firm introduces a product innovation, while the low quality firm adopts a process innovation. In this case no leapfrogging occurs, and the degree of vertical differentiation increases. Moreover, a process innovation grants a robust cost advantage. For these reasons the quality leader prefers a product innovation (the benefit of a higher quality differential is greater than reducing its production cost) while the quality follower chooses a cost reduction innovation (it benefits both of a reduction in the intensity of competition due to the higher degree of vertical differentiation and of a cost advantage).

Last, by comparing Proposition 2 and Proposition 4, we notice that the interval where both firms adopt a process innovation, is greater under Bertrand than under Cournot. Vice versa, the interval where both firms introduce a product innovation is smaller under Bertrand than under Cournot (i.e. both firms select $I_i^* = 0$ *before* in case of quantity competition than in case of price competition). Hence, we add a new insight to the BH's results: the Cournot competitors tend to favor a product innovation in comparison with the Bertrand competitors, *when they both adopt an innovation* (not only a single innovative firm as in BH). Furthermore, the interval where firms behave asymmetrically is larger under Cournot than under Bertrand: a less intense competition involves less need to imitate the rival's behavior.

To sum up, this contribution points out that firms selling goods with a quality gap have different incentives in adopting a product innovation, since introducing the latter becomes a dominant strategy for the high (low) quality firm before than for the quality follower (leader) under Bertrand (Cournot). It is crucial to shed light some intuitions on this issue. BH, for instance, have provided an answer based upon the competitive regime, *but in a model where there is only one innovator*. We have instead obtained that, *when both firms innovate*, the competitive regime may lead to different outcomes because under Cournot leapfrogging is possible, while it is unfeasible under Bertrand. The leapfrogging effect has been not considered by BH.

5 Conclusions

This paper investigates a duopoly model of vertical differentiation where firms simultaneously select whether to adopt a process innovation or a product innovation. This decision is taken both in case of Bertrand and Cournot competition. The two innovations have different impacts on firm's profitability, identified by a costs saving effect (process innovation) and by a market share premium (product innovation). The analysis has produced the following results: First, under both competitive regimes three equilibria in the innovation game may arise: two symmetric (where both firms choose either a process or a product innovation) and one asymmetric (where the two firms adopt different innovation types). Second, under Bertrand competition the asymmetric equilibrium leads to an increase in the degree of vertical differentiation, since the high quality firm introduces a product innovation and the low quality firm adopts a process innovation. On the contrary, under Cournot competition leapfrogging occurs in case of asymmetric information and this may also lead to a decrease in the degree of vertical differentiation. The strategic positions of the two firms explain these different behaviors (in contrast with Bonanno and Haworth [1998] which consider only one innovator): under Bertrand, leapfrogging never occurs because the low quality firm has never the incentive to reduce the degree of vertical differentiation; under Cournot, the low quality firm prefers to become the quality leader rather than selling to the less rich market niche at a lower production cost. On the other hand, the high quality firm does not suffer too much in case of leapfrogging since the quality gap is small, and gets a robust benefit from reducing its costs. Last, the interval where both firms adopt a product innovation is larger under Cournot than under Bertrand. Hence in a strategic context, the Cournot (Bertrand) competitors display a tendency to favor product (process) innovation.

6 Appendix

Proof of Proposition 1: Since $y_H \gg 0$ and $y_L \gg 0$ and in both FOCs' the denominator is positive, (15)–(16) are simultaneously satisfied when

$$4(4\theta_H^2 - 3\theta_H\theta_L + 2\theta_L^2) - c(24\theta_H^3 - 22\theta_H^2\theta_L + 5\theta_H\theta_L^2 + 2\theta_L^3) = 0$$

and

$$2\theta_H(4\theta_H^2 - 7\theta_L) + c(4\theta_H^3 - 19\theta_H^2\theta_L + 17\theta_H\theta_L^2 - 2\theta_L^3) = 0$$

Solving this system yields three solutions: $\{\theta_H = \frac{2}{3}\frac{1}{c}, \theta_L = \frac{8}{3}\frac{1}{c}\}$, $\{\theta_H = 0, \theta_L = 0\}$ and

$$\left\{\theta_H = -\left(\frac{1}{4}\right)\frac{1011\Omega^3 - 4090\Omega^2 + 4632\Omega - 1408}{c(213\Omega^3 - 922\Omega^2 + 1008\Omega - 256)}, \theta_L = \frac{\Omega}{c}\right\}$$

where

$$\Omega = \text{Root of} \quad (128 - 584\chi + 836\chi^2 - 461\chi^3 + 84\chi^4)$$

The first two solutions are unfeasible (the first one has $\theta_H \ll \theta_L$ which has been ruled out by assumption); the third needs to investigate the root of the polynomial $128 - 584\chi + 836\chi^2 - 461\chi^3 + 84\chi^4$. Solving it w.r.t. χ yields two imaginary roots and two real solutions: $\chi_1 = 0.39872$, $\chi_2 = 2.71773$. Then if $\Omega = \chi_2$ we have $\theta_H = \frac{1.77763}{c}$ and $\theta_L = \frac{2.71773}{c}$, so that $\theta_H \ll \theta_L$, which has been ruled out by assumption. If instead $\Omega = \chi_1$ then $\theta_H^0 = \frac{0.81952}{c}$ and $\theta_L^0 = \frac{0.39852}{c}$. The latter is the solution of the pre–innovation quality game.

$\square$

Proof of Proposition 2: From (30)–(33) we get Table 1, where, by inspection, the three equilibria emerge.

$\square$

ψ range	$I_H(I_L=1)$	$I_H(I_L=0)$	$I_L(I_H=1)$	$I_L(I_H=0)$	Nash Eq.
$1 \leq \psi \leq 1.20$	1	1	1	1	$I_H^* = I_L^* = 1$
$1.20 < \psi \leq 1.65$	1	0	1	1	$I_H^* = I_L^* = 1$
$1.65 < \psi \leq 1.75$	0	0	1	1	$I_H^* = 0, I_L^* = 1$
$1.75 < \psi \leq 2.33$	0	0	1	0	$I_H^* = I_L^* = 0$
$2.33 < \psi$	0	0	0	0	$I_H^* = I_L^* = 0$

Table 1: Best replies and Nash equilibria under Bertrand

Proof of Proposition 3: The two FOCs' (40)–(41) are simultaneously satisfied when

$$16\theta_H^2 - 4\theta_H\theta_L + 2\theta_L^2 - c(24\theta_H^3 - 10\theta_H^2\theta_L + 4\theta_H\theta_L^2 + \theta_L^3) = 0$$

and

$$8\theta_H - 2\theta_L + c(4\theta_H^2 - 23\theta_H\theta_L + 2\theta_L^2) = 0$$

The system has three solutions: $\{\theta_H = \frac{2}{7}\frac{1}{c}, \theta_L = \frac{8}{7}\frac{1}{c}\}$, $\{\theta_H = 0, \theta_L = 0\}$ and

$$\left\{\theta_H = \frac{1}{107}\frac{2993\Upsilon^2 - 995\Upsilon + 72}{c(182\Upsilon^2 - 79\Upsilon + 8)}, \theta_L = \frac{2}{c}\Upsilon\right\}$$

where

$$\Upsilon = \text{Root of} \quad (-16 + 126\xi - 463\xi^2 + 749\xi^3)$$

As in the proof of Proposition 1 the first two solutions are unfeasible while the third needs to investigate the root of the above polynomial. The unique real solution is $\xi = 0.29279$. Then we have $\theta_H^0 = \frac{0.73810}{c}$ and $\theta_L^0 = \frac{0.58558}{c}$.

$\square$

Proof of Proposition 4: From (49)–(52), we get Table 2, and the indicated Nash equilibria.

ψ range	$I_H(I_L=1)$	$I_H(I_L=0)$	$I_L(I_H=1)$	$I_L(I_H=0)$	Nash Eq.
$1 \leq \psi \leq 1.54$	1	1	1	1	$I_H^* = I_L^* = 1$
$1.54 < \psi \leq 1.60$	1	1	0	1	$I_H^* = 1, I_L^* = 0$
$1.60 < \psi \leq 1.65$	0	1	0	1	$(I_H^* = 1, I_L^* = 0)$, $(I_H^* = 0, I_L^* = 1)$
$1.65 < \psi \leq 1.66$	0	1	0	0	$I_H^* = 1, I_L^* = 0$
$1.66 < \psi$	0	0	0	0	$I_H^* = I_L^* = 0$

Table 2: Best replies and Nash equilibria under Cournot

If $1.60 < \psi \leq 1.65$ two equilibria in pure strategies arises. We adopt a risk dominance criterion to select the unique equilibrium à la Harsanyi–Selten [1988], following Cabrales *et al.* [2000]. The strategic context when $1.60 < \psi \leq 1.65$ is summarized in the following matrix:

	Firm L	
Firm H	$I_L = 0$	$I_L = 1$
$I_H = 0$	π_H^{00}, π_L^{00}	π_H^{01}, π_L^{01}
$I_H = 1$	π_H^{10}, π_L^{10}	π_H^{11}, π_L^{11}

Table 3: The subgame if $1.60 < \psi \leq 1.65$

The risk dominance criterion compares the product of gains from correct predictions and the equilibrium with the largest product is the one that dominates. We know from Table 2 that the game shown in Table 3 has two strict asymmetric equilibria in pure strategies: $I_H = 0, I_L = 1$, that we label as $H0L1$ and $I_H = 1, I_L = 0$ (denoted as $H1L0$). To compute the risk dominance equilibrium we define $GH0$ as the gain made by firm H by predicting rightly that firm L will play as in $H0L1$ (and best responding to the prediction) instead of predicting wrongly that firm L will play as in $H1L0$ (and best responding to the prediction). Hence $GH0 = \pi_H^{01} - \pi_H^{11}$. Similarly, let $GH1 = \pi_H^{10} - \pi_H^{00}$, and $GL0 = \pi_L^{01} - \pi_L^{00}$, $GL1 = \pi_L^{10} - \pi_L^{11}$. The product of gains from correct predictions on the equilibrium $H0L1$ is then $GH0 \times GL0$, while on the equilibrium $H1L0$ is $GH1 \times GL1$. Notice that:

$$GH0 = \frac{1.61(\psi - 0.03)(\psi - 0.47)(\psi - 1.60)}{c(8.72\psi^2 - 3.46\psi + 0.34)}$$

$$GH1 = -\frac{0.57(\psi - 0.09)(\psi - 1.66)(\psi^2 - 0.16\psi + 0.12)}{c(5.49\psi^2 - 3.46\psi + 0.54)\psi}$$

$$GL0 = -\frac{0.50(\psi - 0.01)(\psi - 1.65)(\psi^2 + 0.19\psi + 0.08)}{c\psi(8.72\psi^2 - 3.46\psi + 0.34)}$$

$$GL1 = \frac{0.80(\psi - 0.04)(\psi - 0.66)(\psi - 1.54)}{c(5.49\psi^2 - 3.46\psi + 0.54)}$$

so that

$$GH0 \times GL0 = -\frac{0.80(\psi - 0.03)(\psi - 0.47)(\psi - 1.60)(\psi - 0.01)(\psi - 1.65)(\psi^2 + 0.19\psi + 0.08)}{c^2(8.72\psi^2 - 3.46\psi + 0.34)^2\psi} \tag{53}$$

$$GH1 \times GL1 = -\frac{0.46(\psi - 0.09)(\psi - 1.66)(\psi^2 - 0.16\psi + 0.12)(\psi - 0.04)(\psi - 0.66)(\psi - 1.54)}{c^2(5.49\psi^2 - 3.46\psi + 0.54)^2\psi} \tag{54}$$

Solving expression (53) greater or equal to expression (54) for ψ we get it is fulfilled for $\psi \geq 1.67$ i.e. outside the relevant range (multiple equilibria arise for $1.60 < \psi \leq 1.65$). Hence the product of gains from correct predictions on the equilibrium $H1L0$ are always greater than the product of gains from correct predictions on the other equilibrium. This means that the equilibrium $I_H = 1, I_L = 0$ risk dominates the equilibrium $I_H = 0, I_L = 1$, and so it is the unique equilibrium surviving the risk dominance refinement when $1.60 < \psi \leq 1.65$.

$\square$

References

- Bonanno, G., 1986, Vertical Differentiation with Cournot Competition, *Economic Notes*, 15, 68–91.

- —,- Haworth, B., 1998, Intensity of Competition and the Choice between Product and Process Innovation, *International Journal of Industrial Organization*, 16, 495–510.

- Cabrales, A.,- Garcia–Fontes, W. and M. Motta, 2000, Risk Dominance Selects the Leader: An Experimental Analysis, *International Journal of Industrial Organization*, 18(1), 137–162.

- Champseaur, P.,- Rochet, J.C., 1989, Multiproduct Duopolists, *Econometrica*, 57, 533–557.

- Choi, C.J.,- Shin, H.S., 1992, A Comment on a Model of Vertical Product Differentiation, *Journal of Industrial Economics*, 40, 229–232.

- Gabszewicz, J.J.,- Thisse, J.-F., 1979, Price Competition, Quality and Income Disparities, *Journal of Economic Theory*, 20, 340–359.

- —, —, 1980, Entry (and Exit) in a Differentiated Industry, *Journal of Economic Theory*, 22, 327–338.

- Gal–Or, E., 1983, Quality and Quantity Competition, *Bell Journal of Economics*, 14, 590–600.

- Harsanyi, J.,- Selten, R., 1988, *A General Theory of Equilibrium Selection in Games*, The MIT Press, Cambridge, MA.

- Herguera, I.,- Lutz, S., 1998, Oligopoly and quality leapfrogging, *The World Economy*, 21, 75–94.

- Lambertini, L.,- Orsini, R., 2000, Process and product innovation in a vertically differentiated monopoly, *Economics Letters*, 68, 333–37.

- Lutz, S., 1997, Vertical Product Differentiation and Entry Deterrence *Journal-of-Economics-(Zeitschrift-fur-Nationalokonomie)*, 65, 79–102.

- Motta, M., 1993, Endogenous Quality Choice: Price vs Quantity Competition, *Journal of Industrial Economics*, 41, 113–131.

- —, Thisse J.F. and A. Cabrales, 1997, On the Persistence of Leadership or Leapfrogging in International Trade, *International Economic Review*, 38, 809–824.

- Mussa, M.,- Rosen, S., 1978, Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301–317.

- Rosenkranz, S., 2003, Simultaneous Choice of Process and Product Innovation when Consumers have a Preference for Product Variety, *Journal of Economic Behavior and Organization*, 50, 183–201.

- Shaked, A.,- Sutton, J., 1982, Relaxing Price Competition through Product Differentiation, *Review of Economic Studies*, 49, 3–14,

- —, —, 1983, Natural Oligopolies, *Econometrica*, 51, 1469–1483.

- Tirole, J., 1988, *The Theory of Industrial Organization*, Cambridge, Mass., MIT Press.

- Wauthy, X., 1996, Quality Choice in Models of Vertical Differentiation, *Journal of Industrial Economics*, 44, 345–353.

- Weiß, P., 2003, Adoption of Product and Process Innovation in Differentiated Markets: The Impact of Competition, *Review of Industrial Organization*, 23, 301–314.