Production extended research joint ventures and welfare*

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Abstract

A wider RJV extension hastens process innovations at the cost of increasing collusion in the final market. In a Cournot model, an extended RJV is welfare enhancing only when the Antitrust Authority is strong, so that the increase in distortion is limited, and when the size of the technical improvement is large, so that the introduction of the innovation is more valuable.

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1 Introduction

In 1984 the US Congress approved the National Cooperative Research Act (NCRA), which grants antitrust immunity to firms involved in RJVs focused on the research stage only. The Act induced a relevant increase of RJV agreements during the 90s, and also a pressure on the legislator to extend the RJV breadth. Firms claimed that RJV members cannot make the most of the results obtained in research if they cannot cooperate also in later stages. An extended cooperation yields savings in the fixed costs involved by the upgrade of the existing capital stock, by its enlargement, by the fine-tuning of the new product lines, etc. When the outcome of R&D is a new product, the cost reduction entailed by an enhanced RJV concerns also design, marketing, and others. In 1993 the Congress approved the National Cooperative Research and Production Act (NCRPA), that extended the antitrust immunity to RJVs dealing up to the production stage.[1] Such a wider RJV breadth has generated, mostly among practitioners but also within the profession, a widespread concern of an increase in the product market degree of collusion, which may reduce welfare. Recently, Goeree and Helland (2008), and Seldeslachts *et al.* (2008) provide some indirect evidence that RJVs may facilitate collusion.

In this note, we assume that – in the pre-innovation market equilibrium – noncooperative collusion is not viable, because the probability of detecting a defection is too low. When firms form a RJV under a NCRA-type legislation (that we label type-I RJV), each firm, at the time of discovery, must sustain a fixed cost that represents all the expenditures pertaining to the development stage. Because type-I RJVs are focused only on research, we assume that the probability of detecting a defection from collusion is not increased. On the contrary, if firms are allowed to cooperate up to the production stage – and hence they are involved in a type-II RJV – the enhanced level of cooperation leads to the development of information channels that increase the probability

[1] The NCRPA covers cooperative efforts in R&D, production, application for patents, granting licenses for the venture's results, and the general management of the proprietary interests of the venture. The following activities are instead excluded: exchange of information about prices, market shares and profits, agreements to restrict sales, or to fix prices, etc. For details, one can consult the NCRP Act at http://www.usdoj.gov/atr/public/guidelines/guidelin.htm.

of detecting a defection. Such an increase is high enough that firms may now tacitly collude (as in many papers based on Friedman (1971)).

In our model, a type-II RJV hastens innovation not only because it implies savings in fixed costs – which is socially valuable in itself – but also due to the perspective of higher profits. This comes to the cost of an increased distortion in the final product market. When a process innovation is introduced in a Cournot market, an NCRPA-type legislation is welfare enhancing only when the Antitrust Authority is strong, so that the increase in distortion is limited, and when the size of the technical improvement is large.

The literature on RJV and collusion is relatively sparse, and leaves the issue of the desirability of a wider RJV breadth unexplored.

In fact, Martin (1995) develops a model in which tacit collusion is important. However, he assumes collusion in the pre-innovation stage, and shows that RJVs strengthen the possibility of a tacit agreement. This happens because a defection breaks up the RJV, making the retaliation harsher. In a context of horizontal product differentiation, Lambertini *et al.* (2002) prove that RJVs – resulting in the lack of differentiation – destabilize collusion. This happens because, with identical products, a small deviation grants, albeit temporarily, the whole market. When the decisions to join a cartel and a RJV are simultaneous, Catilina and Feinberg (2006) show that a RJV may provide the additional synergy that makes the cartel stable. Cooper and Ross (2008) highlight that a RJV in one market may provide a credible punishment for firms colluding in another market, thereby facilitating collusion there. Miyagiwa (2008) suggests that RJVs favour collusion, because innovation sharing eliminates inter-firm asymmetries, and increase industry profits, which contributes to the stability of collusion.

2 The Model

2.1 A simple stochastic innovation set-up

We consider an industry composed of n symmetric firms.When the possibility to develop a new technology opens up, every firm finds profitable to form the RJV. Accordingly, they establish a joint research lab, allowing them to obtain the technical improvement with a probability intensity h, which is assumed to be linear in the overall cost that the firms agree to sustain. We introduce the linearity assumption because it favours the welfare dominance of type-II RJVs. In fact, the stronger incentive to innovate provided by a type-II RJVs yields an higher increase in the hazard rate under constant returns, than under decreasing ones.[2]

When the RJV is of type-I, each firm i must sink, at the discovery time, a fixed cost F, then it obtains the non-collusive profit, Π_n^i, that is higher than the initial one, Π_0^i. Notice that the subscript indicates the number of firms which have introduced the new technology. If RJVs are of type-II, each member sustains only a share ν of the fixed cost F, then it grasps $\Pi_n^{i,C} (> \Pi_n^i)$, which is the profit obtained under implicit collusion.

Firms discount future profits at the common rate r.

Under a type-I RJV, the value for firm i is:

$$V^{I,i} = \int_0^\infty \left[h^I \left(\frac{\Pi_n^i}{r} - F \right) + \Pi_0^i \right] e^{-(r+h^I)t} = \frac{h^I \left(\Pi_n^i - rF \right) + r\Pi_0^i}{r(r+h^I)},$$

and the RJV problem is:

$$\max_{\{z^{I,i}\}} \sum_{i=1}^n (V^{I,i} - z^{I,i}), \quad s.t.\ h^I = \delta \left(\sum_{i=1}^n z^{I,i} \right),$$

where $z^{I,i}$ is firm i's contribution to the joint lab, and δ is a research productivity parameter. It is trivial to obtain

$$h^I = [n\delta(\Pi_n - rF - \Pi_0)]^{1/2} - r, \tag{1}$$

which leads to introduce the following natural assumption, guaranteeing that – for a type-I RJV – it is profitable to invest in research:

A1: $\Pi_n^i - rF - \Pi_0^i > r^2/(n\delta)$.

The comparative statics for Equation (1) delivers sensible results: the discovery hazard rate depends positively on the profit incentive $\Pi_n^i - \Pi_0^i$, on

[2]Notice that, with constant returns, formulating the problem as in Loury (1979), or as in Lee and Wilde (1980) yields the same results.

the number of firms benefiting from the discovery, and on the productivity of the research lab; it depends negatively on the size of the fixed cost, and on the interest rate.

If the RJV is of type-II, the value for firm i is:

$$V^{II,i} = \int_0^\infty \left[h^{II} \left(\frac{\Pi_n^{i,C}}{r} - \nu F \right) + \Pi_0^i \right] e^{-(r+h^{II})t} = \frac{h^{II} \left(\Pi_n^{i,C} - rvF \right) + r\Pi_0^i}{r(r+h^{II})},$$

and the RJV problem is:

$$\max_{\{z^{II,i}\}} \sum_{i=1}^n (V^{II,i} - z^{II,i}), \qquad s.t.\ h^{II} = \delta \left(\sum_{i=1}^n z^{II,i} \right),$$

that is solved by:

$$h^{II} = \left[n\delta(\Pi_n^{i,C} - r\nu F - \Pi_0^i) \right]^{1/2} - r. \tag{2}$$

Notice that $h^{II} > h^I$, due to the increase in the post-innovation profits, and to the reduction in the fixed cost born by each firm.

2.2 Profits and Welfare with process innovation

We consider an industry producing an homogeneous good. Market demand is linear and equal to $P = a - Q$, where P is the market clearing price and $Q = \sum_{i=1}^n q_i$ is the total quantity supplied. Each firm incurs the same unit cost of production c.[3] When the R&D investment is successful, the ensuing process innovation shrinks the unit production cost by an amount x, with $x < c$. Hence firm i's post–innovation production cost is $C(q^i) = (c - x)q^i$.

Before introducing the process innovation, firms compete *à la* Cournot, obtaining the instantaneous profits:

$$\Pi_0^i = \frac{A^2}{(n+1)^2}, \ \forall i,$$

[3] We use this framework, since it is standard in the RJV literature from the seminal papers by d'Aspremont and Jacquemin (1988) and by Kamien *et al.* (1992). Moreover, in horizontal differentiation models following d'Aspremont *et al.* (1979), either there is full marker coverage – and hence collusion for given locations does not alter social surplus – or the firms act as local monopolists.

where $A = a - c$ is a market dimension measure. The instantaneous welfare (computed *à la Marshall* as the sum of consumers' and producers' surpluses) is then equal to:

$$W_0 = \frac{n(n+2)}{(n+1)^2}\frac{A^2}{2}.$$

Once firms have obtained the cost-reducing innovation by means of a type-I RJV, they produce more than in the *status quo*, selling at a lower market price. Individual profits are:

$$\Pi_n^i = \frac{(A+x)^2}{(n+1)^2}, \ \forall i.$$

Obviously, $\Pi_n^i > \Pi_0^i$.

When the new technique has been introduced, the social welfare is:

$$W_n = \frac{n(n+2)}{(n+1)^2}\frac{(A+x)^2}{2}.$$

with $W_n > W_0$, since $x > 0$.

If firms are allowed to enter in a type–II agreement, they may collude on the product market once the process innovation has been obtained. Rather than assuming that each firm obtains $1/n^{th}$ of the monopoly profit, we consider a situation in which each firm i solves the problem:

$$\max_{q_i} \Pi_n^i + \alpha \sum_{j \neq i} \Pi_n^j, \tag{3}$$

where $\alpha \in [0,1]$ is a parameter capturing the reduction in the horizontal externality among firms, which is associated to collusion.[4] When $\alpha = 0$, firms compete *à la Cournot*, and the horizontal externality is maximum; the case $\alpha = 1$ corresponds to monopoly, and the externality is fully internalized. We interpret α as reflecting the effectiveness of the Antitrust Authority (henceforth Antitrust).[5] When the latter is strong, to avoid triggering its reaction,

[4] Notice that α can also be interpreted as the constant conjectural variation parameter. (See Martin, 2001, Chapter 3).

[5] If the Antitrust could freely decide upon its effectiveness, it would induce a value for α implying the welfare-maximizing RJV breadth.

firms solve Problem (3) agreeing on a low α. When the Antitrust is powerless, α is set close to unity.

Standard calculations show that:

$$\Pi_n^{i,C}(\alpha) = \frac{[\alpha(n-1)+1](A+x)^2}{[n+1+\alpha(n-1)]^2},\ \forall i,$$

while the instantaneous social welfare is:

$$W_n(\alpha) = \frac{n[2\alpha(n-1)+n+2]}{[n+1+\alpha(n-1)]^2}\frac{(A+x)^2}{2}.$$

Under the two alternative RJV regimes, the intertemporal social welfare is:

$$SW^I = \int_0^\infty \left[h^I\left(\frac{W_n}{r} - nF\right) + W_0\right] e^{-(r+h^I)t} - \frac{h^I}{\delta};$$
$$SW^{II}(\alpha) = \int_0^\infty \left[h^{II}\left(\frac{W_n(\alpha)}{r} - n\nu F\right) + W_0\right] e^{-(r+h^{II})t} - \frac{h^{II}}{\delta},$$

where $h^I (= n\delta z^{I,i})$ and $h^{II} (= n\delta z^{II,i})$ are given by (1) and (2), respectively.

2.3 The result

We now prove:

Proposition 1 *When Assumption 1 is fulfilled, there exist at most one α, which is $\hat{\alpha}$, such that, for $\alpha \in [0, \hat{\alpha}]$, $SW^{II}(\alpha) \geq SW^I$.*

Proof. Refer to the Appendix ■

When $SW^{II}(1) > SW^I$, the NCRPA is always preferable; if instead $SW^{II}(1) \leq SW^I$ there is a unique α guaranteeing that – for lower market distortions – a type-II RJV improves welfare upon a type-I agreement.

However, the Antitrust effectiveness depends upon her budget constraint, and upon the degree of RJV private information (Besanko and Spulber (1989)). These features limit the Antitrust ability to manipulate α. Such extensions to our model, make the analysis more complex. Accordingly they are left for future research.

It is interesting to compute $\hat{\alpha}$, to gauge which is the maximum degree of final market collusion allowing for a positive welfare effect of a wider RJV breadth.

Because we compute calendar time in years, we set the interest rate to 0.03. We consider the size of innovation x as a proportion of the market dimension parameter A, so that – without loss of generality – we can set the latter equal to 1. The innovation sizes we consider are $x \in \{0.1;\ 0.3;\ 0.5;\ 0.7\}$. For each x, the R&D efficiency parameter δ is calibrated with reference to a "baseline" case, in which the type–I RJV success rate, h^I, is equal to 0.20. In each baseline case, $n = 6$, and the R&D cost $z^{I,i}(= h^I/(n\delta))$ is equal to the fixed cost F. The fact that h^I it is not increasing in x, but rather is constant, favours the welfare dominance of the type–II RJV for high x levels. In fact, the higher is the success probability, the less welfare increasing is the hastening of the innovation process induced by a type-II RJV. The choice $n = 6$ favours the type–II RJV for high n: an increase in the number of firms reduces h^I because Π_n shrinks,[6] and the "hastening effect" of type–II RJVs becomes socially more valuable.

In Figure 1, $\hat{\alpha}$ is portrayed as a function of $z^{I,i}/F$, the continuos line represents $\hat{\alpha}$ for $x = 0.1$; the dashed line is drawn for $x = 0.3$; the dotted line for 0.5, and the dashed-dotted for 0.7.

[Figure 1 about here]

The three Panels in Figure 1 depicts $\hat{\alpha}$ for $n = \{2,\ 6,\ 12\}$[7] if $\nu = 0.5$. Clearly, these scenarios favours the type–II RJV, since the savings in fixed costs they entail are relevant. In particular, in case of a duopoly, any cost duplication is avoided.

The fact that $\hat{\alpha}$ is decreasing in $z^{I,i}/F$ is not surprising: when the weight of F is low compared to the pure R&D cost, a type–II RJV becomes less

[6] This effect is weak for low n because $\partial h^I/\partial n < 0$, and $\partial h^{II}/(\partial n)^2 < 0$ (refer to Eq. (1)).

[7] Seldeslachts et al. (2008) report that the 43% of US RJVs is composed of less than four firms, the 36% is made up of four to nine enterprises (with an average of 5.75), while the remaining 21% is composed of ten or more firms.

appealing since the savings in fixed costs it entails are relatively smaller. An increase in x shifts upward the threshold $\hat{\alpha}$, because a larger innovation size makes the introduction of the innovation more valuable from the perspective of the intertemporal welfare. Accordingly, a type–II RJV – hastening the innovative process – becomes more appealing.

The values for $\hat{\alpha}$ reported in Figure 1 are low. Hence, our calculations imply that a wider RJV breadth is welfare enhancing only if: *i*) the action of the Antitrust is effective in limiting the degree of collusion, *ii*) the saving in fixed costs it allows is significant, and *iii*) the size of the technical improvement is large.

Accordingly, a higher RJV breadth, as granted under the NCRPA, is unlikely to improve social welfare.

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4 Appendix

Proof for Proposition 1.

We first show that $SW^{II}(0) > SW^I$, that assures us that the savings in the fixed cost guaranteed by a type-II RJV improve welfare, had this joint venture implied no increase in the market distortion.

In fact, $SW^{II}(0) = SW^I$when $v = 1$; notice moreover that

$$\frac{\partial SW^{II}(\alpha)}{\partial v} = \left[\frac{W_n(\alpha) - W_0 - rn\nu F}{(r + h^{II})^2} - \frac{1}{\delta}\right]\frac{\partial h^{II}}{\partial v} - \frac{h^{II}nF}{r + h^{II}}.$$

Taking advantage of Eq. (2), to substitute $(r + h^{II})$ out of the addendum in the big square brackets in the expression above, we obtain

$$\frac{\partial SW^{II}(\alpha)}{\partial v} = \left[\frac{W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i)}{\delta n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)}\right]\frac{\partial h^{II}}{\partial v} - \frac{h^{II}nF}{r + h^{II}}.$$

From Eq. (2) we immediately see that $\partial h^{II}/\partial v < 0$; Notice that the numerator of the expression in the big square brackets, when evaluated at $\alpha = 0$, is positive. (This can be easily obtained exploiting the expressions in Sub-section (2.2)). The denominator in the big square brackets in the expression above is positive by Assumption 1. Accordingly, we have shown that

$$\left.\frac{\partial SW^{II}(\alpha)}{\partial v}\right|_{\alpha=0} < 0,$$

and hence that $SW^{II}(0) > SW^I$, for $v \in [0, 1)$.

We now show that the decrease in $SW^{II}(\alpha)$, caused by an increase in α, implies the existence of at most $\alpha \in [0, 1]$, such that $SW^{II}(\hat{\alpha}) = SW^I$, so that, for $\alpha \in [0, \hat{\alpha}]$, $SW^{II}(\alpha) \geq SW^I$.

Since

$$SW^{II}(\alpha) = \frac{h^{II}[W_n(\alpha) - rn\nu F] + rW_0}{r(r + h^{II})} - \frac{h^{II}}{\delta},$$

we immediately obtain that

$$\frac{\partial SW^{II}(\alpha)}{\partial \alpha} = \left[\frac{W_n(\alpha) - rn\nu F - W_0}{(r + h^{II})^2} - \frac{1}{\delta}\right]\frac{\partial h^{II}}{\partial \alpha} + \frac{h^{II}}{r\,(r + h^{II})}\frac{\partial W_n(\alpha)}{\partial \alpha}. \quad \text{(A1)}$$

From Eq. (2), it is easy to derive:

$$\frac{\partial h^{II}}{\partial \alpha} = \frac{1}{2}\frac{n\delta}{r + h^{II}}\frac{\partial \Pi_n^{i,C}(\alpha)}{\partial \alpha},$$

which, using $\Pi_n^C(\alpha)$, becomes:

$$\frac{\partial h^{II}}{\partial \alpha} = \frac{n\delta(n-1)^2(1-\alpha)(A+x)^2}{2\left(r+h^{II}\right)[n+1+\alpha(n-1)]^3}. \tag{A2}$$

Notice that $\partial h^{II}/\partial \alpha > 0$, for $\alpha \in [0,1)$: an increase in profits hastens innovation.

As for $\partial W_n(\alpha)/\partial \alpha$, simple calculations yield:

$$\frac{\partial W_n(\alpha)}{\partial \alpha} = -\frac{n(n-1)[1+\alpha(n-1)]}{[n+1+\alpha(n-1)]^3}(A+x)^2. \tag{A3}$$

Obviously, $\partial W_n(\alpha)/\partial \alpha < 0$: an increase in α, and therefore in the market distortion, reduces the instantaneous welfare.

Using Eqs. (A3), and (A2), in (A1) gives:

$$\frac{\partial SW^{II}(\alpha)}{\partial \alpha} = \frac{n(n-1)(A+x)^2}{2\left(r+h^{II}\right)[n+1+\alpha(n-1)]^3} \left\{ \left[\frac{W_n(\alpha) - W_0 - rn\nu F}{\left(r+h^{II}\right)^2} - \frac{1}{\delta} \right] \delta(n-1)(1-\alpha) - \frac{2h^{II}}{r}[1+\alpha(n-1)] \right\}.$$

Exploiting Eq. (2), substitute $(r+h^{II})$ out of the addendum in the big square brackets in the expression above to obtain

$$\frac{\partial SW^{II}(\alpha)}{\partial \alpha} = \frac{n(n-1)(A+x)^2}{2\left(r+h^{II}\right)[n+1+\alpha(n-1)]^3} \left\{ \left[\frac{W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i)}{n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)} \right] (n-1)(1-\alpha) - \frac{2h^{II}}{r}[1+\alpha(n-1)] \right\}. \tag{A4}$$

When $W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i) \leq 0$, the derivative (A4) is negative; hence, there is at most one $\alpha \in [0,1]$ such that for $\alpha \in [0,\hat{\alpha}]$, $SW^{II}(\alpha) \geq SW^I$.

When instead $W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i) > 0$, which happens if $\alpha < \frac{(n+1)x}{(n-1)A}$, it is convenient to define:

$$G(\alpha) = \frac{n(n-1)(A+x)^2}{2\left(r+h^{II}\right)[n+1+\alpha(n-1)]^3},$$

and

$$F(\alpha) = \left\{ \left[\frac{W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i)}{n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)} \right] (n-1)(1-\alpha) - \frac{2h^{II}}{r}[1+\alpha(n-1)] \right\}.$$

It is immediate to notice that $G(\alpha) > 0$. As for $F(\alpha)$, notice that: $F(0) \lesseqgtr 0$, and that $F(1) < 0$, while

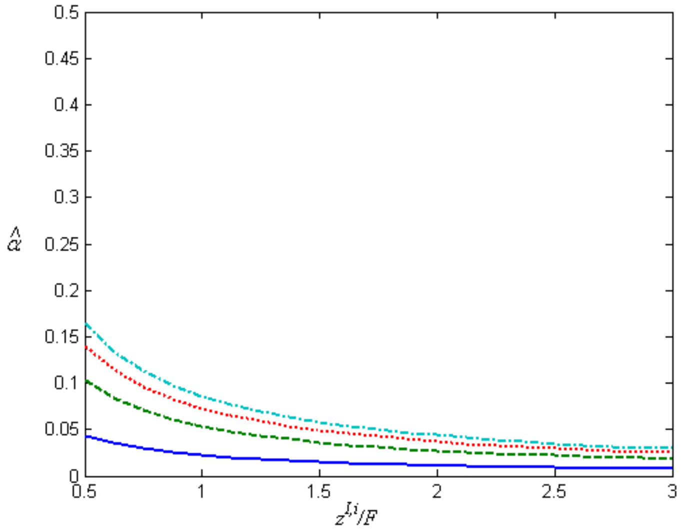
$$
\begin{aligned}
\frac{\partial F(\alpha)}{\partial \alpha} = & \frac{(n-1)(1-\alpha)}{n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)} \left[\frac{\partial W_n(\alpha)}{\partial \alpha} - n \frac{\partial \Pi_n^{i,C}}{\partial \alpha} \right] + \\
& - \left[\frac{W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i)}{n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)^2} \right] (n-1)(1-\alpha) \frac{\partial \Pi_n^{i,C}}{\partial \alpha} + \\
& - \left[\frac{W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i)}{n(\Pi_n^{i,C} - r\nu F - \Pi_0^i)} \right] (n-1) - \frac{2h^{II}}{r}(n-1) + \\
& - \frac{2}{r}[1 + \alpha(n-1)] \frac{\partial h^{II}}{\partial \alpha}.
\end{aligned}
$$

Because $\partial W_n(\alpha)/\partial\alpha < 0$, (Eq. (A2)), and $\partial \Pi_n^{i,C}/\partial\alpha > 0$, the first addendum is negative. Because we are considering the case $W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i) > 0$, the fact that $\partial \Pi_n^C/\partial\alpha > 0$ guarantees that also the second addendum is negative. It is obvious that also the third and the fourth addenda of the expression above are negative, while the fifth is so because $\partial h^{II}/\partial\alpha > 0$ (Eq. (A3)).

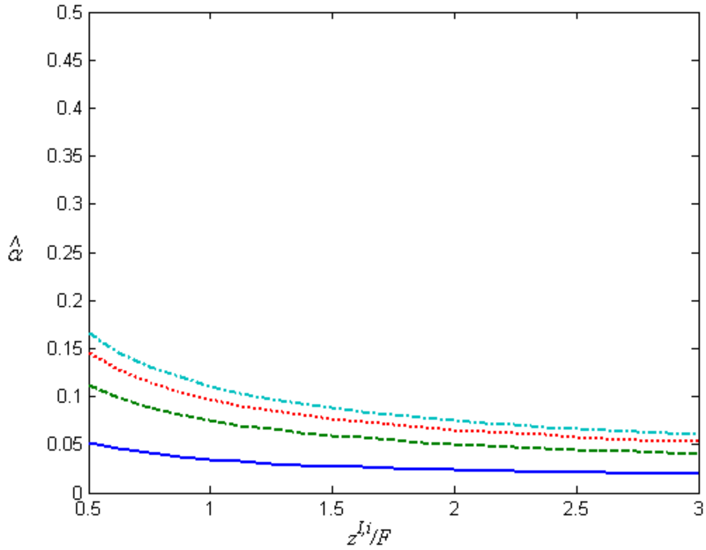
Accordingly, for $W_n(\alpha) - n\Pi_n^{i,C} - (W_0 - n\Pi_0^i) > 0$ we have that $\partial F(\alpha)/\partial\alpha < 0$, which guarantees that either $\partial SW^{II}(\alpha)/\partial\alpha < 0$ for $\alpha \in [0,1]$ (when $F(0) < 0$), or $\partial SW^{II}(\alpha)/\partial\alpha = 0$ for a unique $\alpha \in [0,1]$ (when $F(0) > 0$).

In both cases there is at most one $\alpha \in [0,1]$ such that for $\alpha \in [0,\hat{\alpha}]$, $SW^{II}(\alpha) \geq SW^I$.■

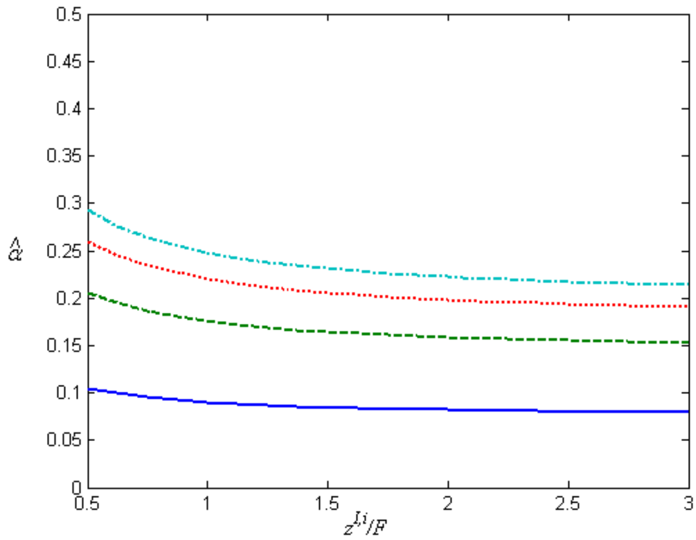
Figure 1



Panel (a): n = 2



Panel (b): n = 6



Panel (c): n = 12