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Optimal federal taxes with public inputs

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JEL Classification numbers: H2, H4, H7.
Keywords: vertical externalities, public input, federal taxes.

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OPTIMAL FEDERAL TAXES WITH PUBLIC INPUTS

by

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Abstract: This paper deals with the solution to vertical expenditure externalities in a federation with two levels of government sharing taxes. Under these circumstances, the Nash equilibrium does not satisfy the condition for production efficiency in the provision of public inputs. This vertical expenditure externality is removed when the federal government, behaving as Stackelberg leader, chooses the optimal tax rate on labor income. The sign of this tax rate depends on the elasticity of marginal productivity of the public input with respect to employment. Moreover, the previous result concerning both vertical (tax and expenditure) externalities are independent each other is confirmed here.

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1 Introduction

The study of interactions between different levels of goverment has been traditionally focussed on vertical tax externalities. As is well-known, they arise when taxes of one level of government affect the tax revenues of another level of government. The origin of it lies in the federal version of the problem of commons, which usually leads to over-explotation of tax bases and to excessively high taxes. A number of papers has dealt with the efficiency implications coming from this issue, highlighting its consequences on fiscal gap (Boadway and Keen, 1996; Kotsogiannis and Martínez, 2007), the differences between Leviathan or benevolent governments (Flowers, 1988; Johnson, 1991), the existence of heterogeneous agents and redistribution (Boadway et al., 1998) or the qualifications which have to be done when fiscal competition at horizontal level is considered as well (Keen and Kotsogiannis, 2002).

By contrast, vertical expenditure externalities have been less studied. These situations appear when the expenditure decided by one level of government affect others levels of governments' revenues. The case of productivity-enhancing public spending (say, public infrastructure) is a good example of this. The highway built by a state government has a clear impact on productivity of production factors and, consequently, on tax revenues levied by federal and local authorities. While in the case of vertical tax externalities the usual result is overtaxation (compared to the second-best outcome achieved in a unitary country), things are not so clear when vertical expenditure externalities are involved. Indeed, public inputs have a positive effect on labor productivity and hence on income taxes, but the profit tax base may either increase or decrease, affecting ambiguously federal and local tax revenues based on rents.

Dahlby (1996) finds that the federal government can solve vertical expenditures externalities providing a matching grant to states equal to the additional federal revenue that is generated from one dollar spent by states on productivity-enhancing expenditures. Dahlby and Wilson (2003) show how the government providing the public input may over-estimate or under-estimate the marginal benefit, leading to an inefficient provision. To undo this externality and replicate the second-best outcome, they propose to offer the state governments a matching grant which is defined on the basis of the federal tax rates on labor income and profits.

This paper extends the contribution of Dahlby and Wilson (2003) in two directions. First, the second-best outcome is replicated here with the federal government behaving as a Stackelberg leader; it implies a more elaborate framework than that of Dahlby and Wilson (2003), with the federal government moving first and taking account the states' reaction functions with respect to the federal policy variables. Second, their finding (in their Proposition 3) that the vertical tax and expenditure externalities are independent each other is confirmed here but using an alternative approach. Instead of discussing in terms of the bias in the provision of public inputs and its relationship with the tax externality as they do, I use separately two conditions neutralizing both vertical externalities by replicating the second-best outcome.

Therefore, this paper shows how the federal government can replicate the second-best unitary outcome in the provision of public inputs. The highest level of government sets an optimal tax rate on labor income, whose sign depends on the elasticity of marginal productivity of public input to employment: if this is elastic, the federal government should tax the labor income; if inelastic, the federal government ought to subsidize this tax base. Additionally, the paper finds that both vertical externalities have to be faced by the federal government independently each other.

After this Introduction the paper sets up the general framework of the model. Section 3 obtains the optimal solutions for the provision of public inputs and public goods in a unitary country, serving us as a benchmark which can be compared with the fiscal decisions taken by states in a federal country. Section 4 describes how the federal government can replicate the unitary outcome, and finally section 5 concludes.

2 The theoretical framework

The model I use is simple and well-known from Boadway and Keen (1996), in which two levels of government occupy the same tax bases on labor. However, by contrast to Boadway and Keen (1996), two new relevant features are included here. First, instead of using specific taxes on labor, I consider ad valorem taxes on labor income. In such a case, the vertical tax externality can be positive and overlapping tax bases may result in inefficiently low tax rates. Second, this paper deals with public inputs, affecting productivity of labor, which means scope for vertical expenditure externalities. In a sense, the theoretical framework I present here is a mix between Boadway and Keen (1996) and Dahlby and Wilson (2003).

Particularly, I characterize a model where different levels of government provide two types of public expenditure. The federal country consists of k (symmetric) states, populated by nk identical, but immobile, households. Each representative household has the following utility function:

$$u(x,l) + B(G), \tag{1}$$

where x is a private good (and numeraire), l is labor, and G is a federal public good. The sub-utility $u(x,l)$ has the usual properties (quasi-concave, increasing in x and decreasing in l) and $B(G)$ is increasing and concave.

The representative consumer maximizes (1) subject to the constraint $x = (1-\tau)wl$, where τ is the tax rate on labor income and w denotes the gross wage rate. It yields the labor supply $l(\bar{w})$, where $\bar{w} = (1-\tau)w$ is net wage. It is assumed that $l'(\bar{w}) > 0$.[1] Indirect utility is then given by $v(\bar{w}) = u(\bar{w}l(\bar{w}), l(\bar{w}))$, with $v' = u_x l$ by Roy's identity.

Each state is endowed with the same amount of some fixed factor. Output in each state is produced by applying the services of labor and public input to the fixed factor according to the technology $f(nl, g)$, with g being the state public input. This production function has the usual properties $f_L, f_g, f_{Lg} > 0$, $f_{Lg} = f_{gL}$ and $f_{LL}, f_{gg} < 0$, with $L = nl$. Output can be costlessly used for x, g and G. The private sector maximizes profits, given by $\pi = f(L, g) - wL$, and thus chooses labor demand that satisfies

$$f_L(L, g) = w \tag{2}$$

On this basis, and given $l(\bar{w})$, the equilibrium gross wage rate $w((1-\tau), n, g)$ can be obtained. For later use, it can be proved that $w_\tau > 0$ and $w_g > 0$. The positive sign of w_τ comes from differentiating totally (2):

$$-f_{LL}nl'w + f_{LL}nl'(1-\tau)w_\tau - w_\tau = 0. \tag{3}$$

Solving for w_τ gives

$$w_\tau = \frac{-f_{LL}nl'w}{1 - f_{LL}nl'(1-\tau)}, \tag{4}$$

which is positive given the properties of the production function. The positive sign of w_g is obtained through a similar way. Differentiating totally (2) we have:

$$f_{Lg} + f_{LL}nl'(1-\tau)w_g - w_g = 0,$$

which solving for w_g gives:

$$w_g = \frac{f_{Lg}}{1 - f_{LL}nl'(1-\tau)}, \tag{5}$$

that is positive on the basis of the properties of the production function.

Comparative statics on profits (rents) π gives:

$$\pi_\tau = -nlw_\tau < 0. \tag{6}$$

$$\pi_g = f_g - nl\left[f_{Lg} + (1-\tau)\, f_{LL}nl'w_g\right] \lesseqgtr 0. \tag{7}$$

The underlying intuition behind the ambiguity in the sign of (7) is clear. Additional units of public input increase production, and consequently π, but the positive effect of g on wages through improving labor productivity may decline the rents[2].

[1] A subscript denotes the derivative of a function of several variables whereas a prime denotes the derivative of a function of one variable.

[2] It can be proved that π_g is always positive with a Cobb-Douglas technology. In general, only for values of the elasticity of the marginal product of the public input with respect to labor input higher than 1, a negative sign for π_g is to be expected.

3 Governments and vertical externalities

As is usual in fiscal federalism literature, the analysis now aims at obtaining the equilibrium solution which would be achieved by a unitary country. This will serve as a benchmark for efficiency comparisons when fiscal policies are chosen by different levels of government. Equilibrium in a unitary country involves maximization of $v(\bar{w}) + B(G)$, choosing τ, G, g, subject to the consolidated budget constraint $G + kg = nk\tau w((1-\tau), n, g)l(\bar{w}) + \pi((1-\tau), n, g)$.

From the first order conditions of this optimization problem, it is straightforward to show that the optimality rules for the provision of the national public good G and the state public input g are given, respectively, by:

$$\frac{nkB'(G)}{u_x} = \frac{1}{1 - \frac{\tau w l'}{l}} \tag{8}$$

$$f_g = 1 \tag{9}$$

Equation (8) simply states that at the unitary optimum the tax rate τ is set such that the sum of the marginal rates of substitution between the federal and the private good x must be equal to the marginal cost of public funds (MCPF), given by $1/(1-(\tau w l'/l))$. Equation (9) is the condition for production efficiency in the public sector, familiar from Diamond and Mirrlees (1971). In essence, it means that, at optimum, the marginal productivity of the public input is just equal to its marginal cost of production, which is 1 in the present model; and that occurs in spite of using distortionary taxation.

I turn now to the characterization of the equilibrium when different levels of government are involved in deciding on fiscal policy. Under the new federal structure for the country, each state government provides the local public input g, which is financed by taxing, at the rate t, labor income wl. The federal government provides the national public good G, financed by taxing labor income at the rate T. Consolidated taxation is denoted by $\tau \equiv t + T$. Profits π are taxable by the federal government, at a fixed rate θ, and by the state governments at the rate of $(1-\theta)$. Denoting by S the vertical transfer, the state public input is then given by

$$g(t, T, \tau, S, n, \theta) = ntw((1-\tau), n, g)l[(1-\tau)w((1-\tau), n, g))] + (1-\theta)\pi((1-\tau), n, g) + S, \tag{10}$$

and the federal public good by

$$G(t, T, \tau, S, n, \theta) = nkTw((1-\tau), n, g)l[(1-\tau)w((1-\tau), n, g))] + \theta\pi((1-\tau), n, g) - kS, \tag{11}$$

with

$$
\begin{aligned}
G_T &= nkwl + nkTw_\tau l + nkTwl'((1-\tau)w_\tau - w) + k\theta\pi_\tau, &(12)\\
G_t &= nkTw_\tau l + nkTwl'((1-\tau)w_\tau - w) + k\theta\pi_\tau, &(13)\\
G_g &= nkTw_g(l + wl'(1-\tau)) + k\theta\pi_g &(14)\\
G_S &= -k < 0. &(15)
\end{aligned}
$$

Notice, from (12) and (13), that $G_T = nkwl + G_t$. Equations (13) and (14) are central to the present analysis. They show the effects of states' fiscal decisions on the federal budget constraint, i. e. they are measures of the two vertical externalities existing in the model. The signs of both of them are indeterminate here because ad valorem taxation and the provision of public inputs at state level may increase or decrease the federal revenues (Dahlby and Wilson, 2003).

Nevertheless, the best way of showing how the equilibrium in a federal country moves away from the second-best solution is to discuss the case in which state governments behave as Nash players. In such a situation, each state government ignores the impact of its fiscal decisions on federal revenues and, consequently, vertical externalities are expected to appear.

The typical state chooses (t, g) to maximize $v(\bar{w}) + B(G)$ subject to (10), taking as given n, θ and the decision variables of the federal government (T, S, G). The necessary conditions of this are given by

$$[(1-T)nl + (1-\tau)(T-\theta)n^2 l f_{LL} l'] w_g + (1-\theta)(f_g - nlw_g) - 1 = 0 \equiv \Omega \qquad (16)$$

$$g - ntwl - (1-\theta)\pi - S = 0 \equiv \Psi, \qquad (17)$$

where (16) comes from manipulations on the first order conditions for t and g, and (17) from the state's budget constraint (10). Obviously, the condition for production efficiency in the provision of the public input does not hold at this scenario, and this is an indication of the existence of a vertical expenditure externality. Anyway, consistently with intuition, the condition $f_g = 1$ is trivially achieved when $T = \theta = 0$ is imposed in (16), that is, when only one level of government is considered.

4 Equilibrium with the federal government behaving as Stackelberg

Consider now the federal government is assumed to be able to act as first mover (or Stackelberg leader), deciding on its fiscal variables and anticipating the effect of them on states' behavior. Before focussing on the problem of federal government, a characterization of the response of states to federal policy variables is needed. With this aim, I take as basis the expressions (16) and (17). Both of them implictly define the states' reaction function

$$
\begin{aligned}
t &= \xi(T, \theta, S, n) && (18)\\
g &= \gamma(T, \theta, S, n). && (19)
\end{aligned}
$$

Differentiating totally (16) and (17) we obtain a two-equation system which provides information on comparative statics of states' reaction function:

$$g_T = \Lambda(\Psi_t \Omega_T - \Omega_t \Psi_T) \tag{20}$$

$$g_S = \Lambda(\Psi_t \Omega_S - \Omega_t \Psi_S) \tag{21}$$

$$t_T = \Lambda(-\Psi_g \Omega_T + \Omega_g \Psi_T) \tag{22}$$

$$t_S = \Lambda(-\Psi_g \Omega_S + \Omega_g \Psi_S), \tag{23}$$

where $\Lambda = -\frac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t}$, and Ω_t and Ω_g are the second order conditions of the state government optimization problem.

The federal government chooses (T, S), and residually G, to maximize $v(\bar{w}) + B(G)$ subject to (11) and the states' reaction functions (18) and (19). First order conditions for T and S are respectively:

$$\Gamma V' + (G_T + G_t t_T + G_g g_T)B' = 0 \tag{24}$$

$$\Phi V' + (G_S + G_t t_S + G_g g_S)B' = 0, \tag{25}$$

where $\Gamma = ((1-\tau)w_\tau - w)(1 + t_T) + (1-\tau)w_g g_T$ and $\Phi = ((1-\tau)w_\tau - w)t_S + (1-\tau)w_g g_S$. Combining both expressions

$$\Gamma \frac{G_S}{G_T} - \Phi + \Gamma \frac{G_t t_S + G_g g_S}{G_T} - \Phi \frac{G_t t_T + G_g g_T}{G_T} = 0, \tag{26}$$

and taking account w_g, w_τ, (20)-(23), and that $\Omega_S = 0$, we have

$$n l f_{Lg} \Psi_g + \frac{(1-\tau) n l f_{Lg}^2 \Psi_t}{w} = 0, \tag{27}$$

where the conditions $G_t = G_g = 0$ have been imposed. Solving (27) for $f_g = 1$, the optimal federal tax rate is obtained:

$$T^* = \theta \frac{E_g - 1}{E_g}, \tag{28}$$

where E_g is the elasticity of marginal productivity of g with respect to employment. The sign of T^* depends on the value of E_g; particularly, $T^* \gtreqless 0$ when $E_g \gtreqless 1$. The more intense the complement relationship between labor and public inputs, the higher the optimal federal tax rate. While Dahlby and Wilson (2003) suggest a matching grant for correcting both vertical externalities, this paper provides an alternative policy tool based on the asymmetry of government behaviors at state and federal levels.

Additionally, conditions $G_t = G_g = 0$ appear as sufficient to achieve the unitary outcome[3]. Note that Dahlby and Wilson (2003) and Kotsogiannis and Martínez (2007) show that both conditions characterize the second-best solution with *ad valorem* taxation for the provision of public inputs and public goods, respectively. In this sense, I have confirmed that both vertical externalities are independent each other as long as the federal government neutralizes the effect of sharing taxation and the impact of state public input on its revenues dealing with both externalities separately. The rule for replicating the second-best solution does not consist of taking account a combination of G_t and G_g (which should be zero), but the federal government cancels out the tax externality $G_t = 0$ and the expenditure externality $G_g = 0$ without considering reciprocal links between both of them. And this is done with the federal tax rate conveniently chosen. To summarize:

Proposition 1 *The federal government facing vertical tax and expenditure externalities replicates the unitary solution with the optimal federal tax rate* $T^* = \theta \frac{E_g - 1}{E_g}$. *More specifically,*

(a) if the marginal productivity of g to employment is elastic, then the federal government taxes labor income.

(b) if the marginal productivity of g to employment is inelastic, then the federal government subsidizes labor income, and

(c) both vertical externalities are independent each other.

5 Concluding remarks

The study of vertical expenditure externalities has not been so intense as the case of vertical tax externalities. However, the real federations show a number of cases in which the public spending decided by one level of government affects revenues of other levels of government. Public investment constitutes a good example of this, with extensions beyond national federations, namely, the structural funds financed by the European Union and impacting on federal, regional and local budgets.

[3]Assuming a Cobb-Douglas utility function, it can be numerically proved that the conditions $G_t = G_g = 0$ become necessary and sufficient to replicate the unitary second-best.

This paper has dealt with a model in which vertical expenditures externalities appear in a federal country with taxes on rents and *ad valorem* taxation on labor income. State governments provide a public input affecting federal government's revenues, which in turn are used to finance a national public good. It is clear that the unitary government would achieve the second-best outcome in the provision of both public expenditures, that in the case of public inputs means holding the production efficiency condition. But when different levels of government are involved in policy decisions, vertical externalities damage efficiency.

A way of correcting them is to assume that the federal government behaves as Stackelberg leader, that is, moving first by deciding its fiscal policy and taking account the reaction of states to changes in federal variables. The result is the optimal federal tax rate, which crucially depends on the elasticity of marginal productivity of public input to employment. The higher this elasticity, the more likely to have a positive federal tax rate. Moreover, this paper confirms a previous result on the absence of links between both vertical externalities.

Some policy implications can be derived from this. First, information appears as a relevant point for designing optimal federal policies. This information does not only consists of the behavior of subnational governments when face changes in federal policy variables, but also of the economic features and impact of expenditures decided by state governments. Note that the optimal federal tax rate found in this paper is closely related to the way through which the state public input enters the production function. Second, the power of federal government to replicate the second-best outcome dramatically depends on its ability to become a Stackelberg leader. This contrasts with some real cases of federations in which the federal government behaves as a Nash player, in line with the behavior assumed in this paper for states. In a sense, one can guess that a strong federal government is a necessary condition for avoiding inefficient decisions on public spending.

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Appendices

Appendix A

Derivation of equations (8) and (9) in text.
In this Appendix I derive the optimal rules for the provision of the public good G and the public input g in a unitary country.

The maximization problem for the unitary government is to maximize $v(\bar{w}) + B(G)$, choosing τ, G and g, and subject to the consolidated budget constraint as given by

$$G + kg = nk\tau w((1-\tau), n, g)l(\bar{w}) + \pi\left((1-\tau), n, g\right). \tag{A.1}$$

Denoting by μ the Lagrange multiplier associated with the budget constraint, necessary conditions of this maximization problem are given by

$$(\tau) \quad : \qquad v'((1-\tau)w_\tau - w) + \mu A = 0, \tag{A.2}$$
$$(G) \quad : \qquad B'(G) - \mu = 0, \tag{A.3}$$
$$(g) \quad : \qquad v'((1-\tau)w_\tau - w) + \mu B = 0, \tag{A.4}$$

where

$$A \equiv nkwl + \tau nkw_\tau l + \tau nkwl'((1-\tau)w_\tau - w) + k\pi_\tau, \tag{A.5}$$

and

$$B \equiv -k + nkl\tau w_g + \tau nkwl'(1-\tau)w_g + k\pi_g. \tag{A.6}$$

Note also that π_τ can be re-written as

$$\pi_\tau = \frac{f_{LL}n^2 l' wl}{1 - f_{LL}nl(1-\tau)} < 0. \tag{A.7}$$

Notice now that we also have (4) and, following from the firm's first order condition $f_L(L, g) = w$, that

$$l'(w) = 1/(nf_{LL}(L, g)). \tag{A.8}$$

Substituting (A.8) into (4), and that and (A.7) into (A.5) and simplifying, one arrives at

$$A \equiv kn((1-\tau)w_\tau - w)(\tau wl' - l). \tag{A.9}$$

Making use now of the fact that $v' = u_x l$ and (A.9), straightforward manipulation of the first order conditions (A.2)-(A.3) gives the second-best tax rule in (8).

For the case of the optimal rule of the public input (9), manipulation of the first order conditions (A.2)-(A.4) gives:

$$\frac{nv'(1-\tau)w_g}{u_x} = \frac{1 - nl\tau w_g - \tau nwl'(1-\tau)w_g - \pi_g}{1 - \frac{\tau wl'}{l}}, \tag{A.10}$$

where the expresions (A.7) and (4) have been used again. Substituting (5) and (7) into (A.10), the production efficiency condition (9) is obtained.

□

Appendix B

Derivation of equation (16) in text.
In this Appendix I derive one of the equations defining the optimal rules for the provision of public input g by a state government in a federal country.

The maximization problem for the state government is to maximize $v(\bar{w}) + B(G)$, choosing t and g, and subject to the state budget constraint (10). Denoting by μ the Lagrange multiplier associated with the budget constraint, the first order conditions for t and g are given by

$$(t) \quad : \quad v'((1-\tau)w_\tau - w) - \mu C = 0, \tag{B.1}$$
$$(g) \quad : \quad v'((1-\tau)w_g) + \mu D = 0, \tag{B.2}$$

where $C = nwl + ntw_\tau l + ntwl'((1-\tau)w_\tau - w) + (1-\theta)\pi_\tau$ and $D = -ntw_g l - ntwl'(1-\tau)w_g - (1-\theta)\pi_g + 1$. Since $v' = u_x l$, manipulations with (B.2) and (B.1) give

$$\frac{v'((1-\tau)w_g)}{u_x} = \frac{l((1-\tau)w_\tau - w)D}{C}.$$

Dividing the top and the bottom of the RHS of above expression by $l((1-\tau)w_\tau - w)$ and making use of (A.7), (4), (7) and (5), the expression (16) is obtained.

□

Appendix C

Derivation of expressions (20), (21), (22) and (23) in text.
Differentiating totally (16) and (17), I have:

$$\Omega_g dg + \Omega_t dt + \Omega_T dT + \Omega_S dS = 0 \tag{C.1}$$
$$\Psi_g dg + \Psi_t dt + \Psi_T dT + \Psi_S dS = 0. \tag{C.2}$$

This two-equation system can be re-written using a matricial form,

$$\begin{pmatrix} \Omega_g & \Omega_t \\ \Psi_g & \Psi_t \end{pmatrix} \begin{pmatrix} dg \\ dt \end{pmatrix} = - \begin{pmatrix} \Omega_T & \Omega_S \\ \Psi_T & \Psi_S \end{pmatrix} \begin{pmatrix} dT \\ dS \end{pmatrix},$$

which solving for dg and dt gives

$$\begin{pmatrix} dg \\ dt \end{pmatrix} = - \begin{pmatrix} \Omega_g & \Omega_t \\ \Psi_g & \Psi_t \end{pmatrix}^{-1} \begin{pmatrix} \Omega_T & \Omega_S \\ \Psi_T & \Psi_S \end{pmatrix} \begin{pmatrix} dT \\ dS \end{pmatrix}. \quad \text{(C.3)}$$

Matricial manipulations on (C.3) show that

$$g_T = -\tfrac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t} (\Psi_t \Omega_T - \Omega_t \Psi_T) \quad \text{(C.4)}$$

$$g_S = -\tfrac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t} (\Psi_t \Omega_S - \Omega_t \Psi_S) \quad \text{(C.5)}$$

$$t_T = -\tfrac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t} (-\Psi_g \Omega_T - \Omega_g \Psi_T) \quad \text{(C.6)}$$

$$t_S = -\tfrac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t} (-\Psi_g \Omega_S - \Omega_g \Psi_S) \quad \text{(C.7)}$$

□

Appendix D

Derivation of expression (28) in the text

Setting $G_t = G_g = 0$ in (26), the following is obtained:

$$\Gamma \frac{G_S}{G_T} - \Phi = 0. \quad \text{(D.1)}$$

Given $G_T = nkwl + G_t$ and the expressions of w_τ and w_g, (D.1) can be rewritten as

$$\frac{1}{1-(1-\tau)nl' f_{LL}} \left[-\frac{1}{nwl} (-w(1+t_T) + (1-\tau) f_{Lg} g_T) + w t_S - (1-\tau) f_{Lg} g_S \right] = 0.$$

It is clear that the first term of this expression is non-zero, then the expression inside brackets must be zero. Upon using the comparative statics of the states' reaction functions (20), (21), (22) and (23), some manipulations give

$$\Lambda \left(nl f_{Lg} \Psi_g + \frac{nl f_{Lg} \Psi_t (1-\tau) f_{Lg}}{w} \right) = 0, \quad \text{(D.2)}$$

where $\Lambda = -\frac{1}{\Omega_g \Psi_t - \Psi_g \Omega_t}$ and the facts that $\Omega_S = 0$, $\Omega_t = \Omega_T + nl f_{Lg}$ and $\Psi_T = \Psi_t + nwl$ have been used. Given that $\Lambda \neq 0$, solving the expression inside parentheses in (D.2) for $f_g = 1$, one obtains

$$T^* = \theta \frac{E_g - 1}{E_g},$$

where $E_g = \frac{nl f_{Lg}}{f_g}$.

□