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## *Biased Contests*

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JEL Classification numbers: C72, D72, D74.
Keywords: Endogenous Contests, Contest Success Function, Information provision.

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# Biased Contests*

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**Abstract**

We examine the effects of providing more accurate information to a political decision-maker who is lobbied by competing interests. Conventional wisdom holds that such a bias in the direction of the correct decision improves the efficiency of government. We provide a formal definition of bias which is derived from the same fundamentals that give rise to a contest model of lobbying. Efficiency of government is measured by both the probability of taking the correct decision and the amount of social waste associated to lobbying activities. We present a benchmark model in which increasing the bias always improves the efficiency of government under both criteria. However, this result is fragile in the sense that reasonable alternative assumptions in the micro-foundations lead to slightly different models in which – due to different strategic effects of bias – under either criterion there is no guarantee that more accurate information improves government.

Keywords: Endogenous Contests, Contest Success Function, Information Provision.

JEL Classification: C72 (Noncooperative Games), D72 (Economic Models of Political Processes: Rent-Seeking, Elections), D74 (Conflict; Conflict Resolution; Alliances).

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# 1 Introduction

Situations in which political decision-makers have to take decisions under uncertainty abound. In many cases there exists a possibility to invest in additional information (e.g. research, policy evaluations or hearings) in order to obtain more accurate information. Such an investment biases the decision in the direction of the correct decision. Conventional wisdom holds that such a bias improves the efficiency of government. This paper analyzes whether this is true when the decision-maker is lobbied by competing interests.

A prominent way to capture lobbying by competing interests is through a contest model. Lobbies compete by making irreversible outlays, while the success of lobbying is non-deterministic and governed by a so-called contest success function. In this paper we focus on two competing lobbies $A$ and $B$ who make outlays $e_A$ and $e_B$, respectively. A contest success function associates, to each vector of efforts $(e_A, e_B)$, a lottery specifying for each agent a probability of getting the object. That is, $\Psi(e_A, e_B)$ is such that, for each $i \in \{A, B\}$, $\Psi_i(e_A, e_B) \geq 0$, and $\Psi_A(e_A, e_B) + \Psi_B(e_A, e_B) = 1$. It is assumed that the decision-maker commits to determine the winner through the contest success function $\Psi$. The standard starting point for the strategic analysis of contest games is then the following specification. Given a contest success function $\Psi$ and the vector of efforts $(e_A, e_B)$, the lobbies' expected utility from participating in the contest is given by

$$E\Pi_i(e_A, e_B) = \Psi_i(e_A, e_B) V_i - e_i, \text{ for } i \in \{A, B\}. \tag{1}$$

In his 1980 paper Gordon Tullock proposed the by now 'classical' rent-seeking game. He specified a proportional form which prescribes in its simplest version (under constant returns to scale) that the probability of winning of the interest groups, given a vector of lobbying efforts $(e_A, e_B)$, is given by

$$\Psi_i(e_A, e_B) = \frac{e_i}{e_A + e_B} \text{ for } i \in \{A, B\}. \tag{2}$$

Moreover, in the seminal paper in which Gordon Tullock introduced these contest games in general and a more general functional form of the contest success function in equation (2), he also makes the following assertion (p. 109 and p. 111):

> "...One way to lower the social costs is to introduce bias into the selection process. Note that we normally refer to bias as a bad thing, but one could be biased in the direction of the correct decision. ...we would like to have court proceedings biased in such a way that whoever is on the right side need not to make very large investments in order to win, and if this is true, the people on the wrong side will not make very large investments either, because they do not pay."
> "...this kind of bias ...would have very large payoffs, not only in reducing rent-seeking activity but also in increasing efficiency of government in general."

Note that this quote not only provides a statement of the conventional wisdom. It also provides a very convincing intuition of *how bias might improve the efficiency of government.* In



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what follows we will use this quote as a guide to our analysis in several respects. On one hand, it will prove helpful for the understanding of the underlying forces in biased contest games to see whether Tullock's intuition applies to these games. On the other hand, the quote suggests two efficiency criteria:

- Social waste: it is desirable that bias reduces overall rent-seeking activity.
- Taking the correct decision: it is desirable that bias improves government in general.

Note that while with the second criterion points at a very important property of contests, the rent-seeking literature has so far only concentrated on the first efficiency criterion. Taking into account the second criterion requires to formalize the idea of a 'correct' decision in a contest game. We do so by deriving both the notion of bias and the crucial element in the specification of a contest game, the contest success function, from the same primitives based on the classical jury setting (see for instance Austen-Smith and Banks (1996) or Feddersen and Pesendorfer (1998 and 1999)). This way we derive the reduced form of a contest given by equation (1) from the actions of rational decision-makers.

We start our analysis of the effects of bias in a game that is very close to the classical Tullock rent-seeking game. We call this game 'benchmark' because, broadly speaking, it shows that Tullock's conjectures are logically consistent in the sense that we can formalize them by a reasonable game. A crucial feature of the benchmark is that both lobbies are always active. Despite the fact that bias never completely deters lobbying activities, the strategic effects of the contest conform to Tullock's reasoning. This implies that irrespective of the efficiency criteria used increasing the accuracy of the politician's information is always efficiency-enhancing: *Tullock's statement is fully supported.*

We make then successively two reasonable modifications in the micro-foundations in order to see how robust this result is. Both games share the a priory desirable feature that in equilibrium additional information might deter (at least) one interest group from lobbying. This highlights that it is important whether there can exist situations in which only one interest group might be deterred from lobbying, because in such situations additional information might increase competition. Moreover, it turns out to be important whether the equilibrium win probabilities of the lobbies are related to the likelihood that the lobby supports the correct policy. The robustness section implies that *the efficiency properties of biased contests must be qualified.*

This paper bridges two strands of literature. First, there is a literature on contest and rent-seeking games (see e.g. the surveys by Nitzan (1994) and Konrad (2004)).[1] While one of the

[1] Some papers in this literature derive, as we do, contest models from micro-foundations (eg. Lazear and Rosen (1981), the working paper version of Hillman and Riley (1989), Fullerton and McAfee (1999), Baye and Hoppe (2003) or Corchón and Dahm (2006)). However, these models differ from ours in a technical sense (e.g the elements of the primitives are different and/or the underlying uncertainty has a different structure). Moreover, they differ in the interpretations that are determined through the choice of the technical assumptions. There is also a relationship to probabilistic voting models (Coughlin (1992)). Skaperdas (1996) and Clark and Riis (1998) motivate contest models through an axiomatic characterization. Amegashie (2006) proposes a contest model on tractability grounds.



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main themes of this literature is the question how to reduce the social waste associated with rent-seeking activities, it abstracts – contrary to the present paper – from the informational aspects of decision-making. Consequently, in this literature efficiency of government is only measured by social waste. Second, there is a literature on informational lobbying (see e.g. the survey by Austen-Smith (1997)) and there are recent attempts by Bennedsen and Feldmann (2006) as well as Dahm and Porteiro (2006a and b) to analyze the incentives of lobbies to either provide a political decision-maker with policy relevant information or to exert political pressure (for instance in the form of bribes or campaign contributions). By contrast, the present paper abstracts from the source of additional information and focusses entirely on the strategic consequences that more accurate information has on the lobbies' competition in pressuring a political decision-maker. Frequently this literature does not analyze the efficiency of government. When it does it measures it only by the probability of an erroneous decision (see Austen-Smith and Wright (1992) and Dahm and Porteiro (2006b)) and not through the other criterion.

This paper is organized as follows. The next section presents the basic model. In this section we also derive our notion of bias and how we measure efficient government. Section 3 analyzes the effect of bias in the benchmark game which essentially confirms Tullock's conjecture. Section 4 analyzes the robustness of these findings and Section 5 offers some concluding remarks.

## 2 The Model

Consider the following simplified version of the classical jury setting. A political decision-maker is pivotal in a political decision among two policies $A$ and $B$. There are two states of the world $a$ and $b$ and we assume that both states are equally likely. We denote by $\omega$ the true state of the world and by $D$ the decision taken. In a world without lobbying the payoffs of the politician depend on the match between the state of the world and on the policy chosen as represented by the following table:[2]

| $\omega \backslash D$ | $A$ | $B$ |
|---|---|---|
| $a$ | $R$ | $0$ |
| $b$ | $0$ | $1-R$ |

with $R \in [0, 1]$.

### 2.1 Bias in the Direction of the Correct Decision

In order to capture the best case for investment in information we assume that the politician has access to a costless test that reveals with probability $q \in \left(\frac{1}{2}, 1\right]$ the true state of the world. By abuse of notation we denote the result of the test by $t \in \{a, b\}$. Given our assumption that initially both states are equally likely, this implies that the probabilities of the states conditional on a signal are $q$ and $1-q$, respectively:

$$\Pr(\omega = a \mid t = a) = q \text{ and } \Pr(\omega = b \mid t = a) = 1 - q,$$

[2] In Feddersen and Pesendorfer's language $1-R$ reflects the threshold of reasonable doubt. A politician who believes that the likelihood of state $a$ is larger than $1-R$ will prefer decision $A$.



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and similarly if $t = b$. It is easy to check that both signals are equally likely. This setting implies realistically that if new evidence indicating state $b$ is revealed, the benefit $R$ from a correct match $(a, A)$ must be relatively high to induce that the politician chooses $A$.

## 2.2 Lobbying: Micro-Founded Contests

There are two interest groups and by abuse of notation we denote each lobby in the same way as his preferred policy. Since the final decision is denoted by $D$ we write, say, $D = A$ if lobby $A$ succeeds. The stakes of each lobby in the decision are given by $V_A$ and $V_B$, respectively.[3] W.l.o.g. we assume that $V_A \geq V_B$. Both lobbies are risk neutral, and exert simultaneously effort in order to influence the political decision. We specify later how effort influences the politician. The effort levels are irreversible and denoted by $e_A$ and $e_B$.

The informational assumptions are as follows. The fact that both states are initially equally likely, and the 'quality' $q$ of the test, are common knowledge. Moreover, the lobbies observe the result of the test when the politician obtains additional information. Thus, the probability that the state of the world is $a$, denoted by $p_a \in \{\frac{1}{2}, q, 1-q\}$, is common knowledge ($p_b$ is similarly defined). If there is no additional information this probability is the initial prior $p_a = \frac{1}{2}$ and after investment in information it equals the posterior belief $p_a \in \{q, 1-q\}$. However, there is asymmetric information about the payoffs of the politician: The decision-maker knows his type $R$ but the lobbies know only that $R$ is uniformly distributed on the line segment $[0, 1]$.[4]

The timing of the game is sequential. In the first stage lobbies exert effort simultaneously. Given this effort, the politician makes in the second stage his policy choice. The alternative chosen is the one that gives the highest payoffs to the politician. Thus, this model has a structure similar to the all-pay auction (see e. g. in Baye *et al.* (1993 and 1996)). The only difference is informational. Lobbies do not know the politician's type.

The fact that lobbies do not know the politicians type when they decide on how much effort to exert implies that, although a lobby's effort may be higher as the one of his opponent, the lobby may be unsuccessful. Therefore, from the lobby's view point the award of the prize is non-deterministic. We will see that this gives rise to a contest which is governed by a contest success function in the sense of equation (1). We derive thus non-deterministic contest games as the consequence of the actions of *rational* decision-makers.

[3] One interpretation is that, say, lobby $A$ values decision $A$ by $V_A$ and decision $B$ with zero. But the lobbies valuation of the other lobby's preferred policy could also be positive. In this case $V_A$ measures how much lobby $A$ prefers policy $A$ over policy $B$ (and analogously for $V_B$).

[4] This is, of course, an assumption we make for simplicity. Generalizations of the distribution function will affect the precise functional form of the lobbies' win probabilities (see the contest success functions derived below). For instance, if $R$ is distributed according to a symmetric density function, then it is straightforward to see that the win probabilities will be monotonically increasing transformations of our specifications, since then for a given $R$ it holds that $1 - F(R) = F(1 - R)$. However, for this and further generalizations it will still be true that efforts affect which type of the politician will be a 'threshold'-type in the sense that given a vector of efforts all higher types prefer policy $A$, while all lower types prefer policy $B$.



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## 2.3 Efficient Government

In the contest literature the predominant efficiency criterion is the question how much the contest deters overall lobbying activities as measured by social waste. Hence, as usual, our first efficiency criterion is social waste measured by total rent-seeking outlays in equilibrium.[5] We wish to analyze the effect of more accurate information and need, thus, to compute social waste in a situation after investment in information in which $p_a \in \{q, 1-q\}$. We have

$$\begin{aligned} SW(e^*|t) &= \Pr(t=a)(e_A^*(p_a=q)+e_B^*(p_a=q))+\Pr(t=b)(e_A^*(p_b=q)+e_B^*(p_b=q)) \\ &= \frac{1}{2}\Big(e_A^*(p_a=q)+e_B^*(p_a=q)+e_A^*(p_b=q)+e_B^*(p_b=q)\Big). \end{aligned} \tag{3}$$

We wish, thus, to establish whether this function is monotonically decreasing (strictly) in the 'quality' $q$ of the additional information. Of course, absent lobbying investment in information is never undesirable because by assumption there is no lobbying effort (or social waste) independently of the information the decision-maker possesses.

In the literatures on the Condorcet Jury Theorem (Feddersen and Pesendorfer (1998)) and informational lobbying (Austen-Smith and Wright (1992)) the natural criterion to judge the efficiency of government is the probability of an erroneous decision. Consequently, this probability will be our second efficiency criterion. Since in equilibrium the contest success function measures the probability of each decision, we measure it by

$$\Pr(Err) = \Pr(\omega=a)\,\Psi_B(e^*) + \Pr(\omega=b)\,\Psi_A(e^*),$$

the ex ante probability that the 'wrong' policy is chosen. Taking into account additional information, we have

$$\begin{aligned} \Pr(Err|t) &= \Pr(t=a)\Big(\Pr(\omega=a|t=a)\,\Psi_B^{p_a=q}(e^*)+\Pr(\omega=b|t=a)\,\Psi_A^{p_a=q}(e^*)\Big) \\ &\quad +\Pr(t=b)\Big(\Pr(\omega=a|t=b)\,\Psi_B^{p_b=q}(e^*)+\Pr(\omega=b|t=b)\,\Psi_A^{p_b=q}(e^*)\Big) \\ &= \frac{1}{2}\Big(q\Psi_B^{p_a=q}(e^*)+(1-q)\Psi_A^{p_a=q}(e^*)+(1-q)\Psi_B^{p_b=q}(e^*)+q\Psi_A^{p_b=q}(e^*)\Big) \\ &= \frac{1}{2}\left[1-(2q-1)\left(\Psi_A^{p_a=q}(e^*)-\Psi_A^{p_b=q}(e^*)\right)\right]. \end{aligned} \tag{4}$$

Again, we wish to establish whether this function is monotonically decreasing (strictly) in the 'quality' $q$ of the additional information.

Notice also that – absent lobbying – this is true. Assuming that $R$ is uniformly distributed we can calculate the probability of an error ex ante the realization of $R$ as

$$\Pr(Err \mid t, NoL) = 2q(1-q) > 0,$$

which is monotonically decreasing (strictly) in $q \in [1/2, 1]$.[6] Thus, absent lobbying under both criteria there is no reason not to invest in costless information.

[5] The underlying assumption necessary to consider the lobbying efforts as real resources wasted is that they are expenditures on socially unproductive activities.

[6] Assuming that $R$ is known, additional information makes a difference when the politician is moderate enough



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## 3 A Benchmark Game

Consider the following contest success function.

**Definition 3.1** *In the benchmark the contest success function is given by*

$$\Psi_A(e_A, e_B) = \frac{p_a e_A}{p_a e_A + p_b e_B} \text{ and } \Psi_B(e_A, e_B) = 1 - \Psi_A(e_A, e_B).$$

Note that in the initial situation when both states are equally likely $\Psi$ simplifies to the 'classical' Tullock contest (with constant returns to scale) which was also proposed in the seminal article that contained the introductory quote to the present paper.[7] The 'informational advantage' of each policy, represented by the probability that the policy is correct, and lobbing effort are combined multiplicatively. Since efforts are multiplied by probabilities that may be close to zero or one, this formulation captures intuitively Tullock's intuition that "whoever is on the right side need not to make very large investments in order to win, and if this is true, the people on the wrong side will not make very large investments either, because they do not pay." We provide first a micro-foundation for this contest success function.

**Assumption 3.1** *Suppose that the politician's payoffs from lobbying efforts are given as represented by the following table:*

| $\omega \backslash D$ | $A$ | $B$ |
|---|---|---|
| $a$ | $e_A R$ | $0$ |
| $b$ | $0$ | $e_B(1-R)$ |

.

Assumption 3.1 captures a simple game in which lobbying increases multiplicatively the politician's advantage from the lobby's favorite policy independently of the state of the world.[8] However, since mismatches between state of the world and policy are normalized to zero, lobbying

relative to the informativeness of the test ($1-q \leq R \leq q$). In this range better information induces better decisions since here $\Pr(Err \mid t, NoL) = 1 - q$. Outside this range the politician is 'extreme' in the sense that he prefers a policy so much that he ignores contrary evidence through the test. However, as the 'quality' $q$ of the test improves some 'extreme' politicians also adjust their behavior to the additional information. Thus, also from this perspective better information induces better decisions.

[7] Notice that in this and in Tullock's original formulation – but not in Definitions 4.1 and 4.2 below – the contest success function is not well defined when $e_A = e_B = 0$. For the moment we postulate, as e.g. in Baye *et al.* (1994), that a fair lottery takes place when efforts are zero. However, in Subsection 4.1 we specify a function that is very similar to Definition 3.1 and that takes this criticism explicitly into account. See also the discussion of this discontinuity in Corchón (2000). Note also that, since the prize never remains with the decision-maker, this contest success function is different from the one proposed in Amegashie (2006). This function is similar but also different from the one in Leininger (1993), since there the sum of coefficients of effort is strictly larger than one.

[8] Notice that when $e_A = 0$ or $e_B = 0$ successful matches between states and policies yield zero payoffs for the politician. This is slightly inconsistent with the table in Section 2. We choose to start with the present game for simplicity of the equilibrium analysis. But in Subsection 4.1 we present a game yielding very similar predictions that is fully consistent in this sense. In order to derive the next Lemma we assume that the politician randomizes equally between both lobbies when $e_A = e_B = 0$.



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efforts only make a difference when a policy is matched with the corresponding state of the world. Comparing the expected payoffs of the politician of both policies we have that

$$E\Pi(D=A) \geq E\Pi(D=B) \quad \Leftrightarrow \quad p_a e_A R \geq (1-p_a) e_B (1-R)$$
$$\Leftrightarrow \quad R \geq \frac{(1-p_a)e_B}{p_a e_A + (1-p_a)e_B} \equiv \bar{R}.$$

An effort vector $(e_A, e_B)$ leads to a winning probability for lobby $A$ of $\Psi_A(e_A, e_B) = 1 - F(\bar{R})$. This implies Definition 3.1 and we have the following result.

**Lemma 3.1** *Assumption 3.1 yields the benchmark contest. That is, lobbies choose effort as to maximize equation (1), where $\Psi_i(e_A, e_B)$ is specified in Definition 3.1.*

We determine now the equilibria of this game. But before doing this we define a measure for the asymmetry of the game

$$\hat{e} \equiv \frac{p_a p_b V_A V_B}{(p_a V_A + p_b V_B)^2}.$$

The parameter $\hat{e} \in [0, 1/4]$ reaches a maximum of $1/4$ if $p_a V_A = p_b V_B$. This situation can be considered the 'symmetric case' since a lobby can be 'strong' because he is likely to be right or because he values the policy much more than the other lobby. In situations of asymmetry (that is, $p_a V_A \neq p_b V_B$), the value of $\hat{e}$ decreases in the asymmetry of the contest.[9] In the case of equal valuations $\hat{e}$ is equal to $q(1-q)$.

Since the objective functions of the lobbies (as specified in Lemma 3.1) are concave, the first order conditions characterize a maximizer of expected utility for each lobby. The first order conditions are

$$\frac{p_a p_b V_A e_B}{(p_a e_A^* + p e_B)^2} \equiv 1 \text{ and } \frac{p_a p_b V_B e_A}{(p_a e_A + p e_B^*)^2} \equiv 1.$$

This implies that $e_A^* V_B = e_B^* V_A$ must hold and allows to establish the following result.

**Proposition 3.1** *For any $V_A \geq V_B$ and $p_a$, there exists a unique pure strategy Nash equilibrium to the benchmark game. In this equilibrium the optimal effort levels are given by $e_A^* = \hat{e} V_A$ and $e_B^* = \hat{e} V_B$. Equilibrium win probabilities are therefore*

$$\Psi_A(e^*) = \frac{p_a V_A}{p_a V_A + p_b V_B} \text{ and } \Psi_B(e^*) = \frac{p_b V_B}{p_a V_A + p_b V_B}.$$

Notice that in the initial situation when both states are equally likely $\hat{e} = V_A V_B / (V_A + V_B)^2$. This implies that the equilibrium effort levels and win probabilities coincide with the ones in the 'classical' Tullock rent-seeking game. Note also that additional information will affect the asymmetry of the contest measured by $\hat{e}$. Once the decision-maker has obtained additional information, depending on the test result, we have either $p_a = q$ or $p_a = 1 - q$ and we can define $\hat{e}(t = a)$ and $\hat{e}(t = b)$ accordingly. In other words, both the optimal effort level of the lobby

[9] To see this note that we can rewrite $\hat{e} = a/(a+1)^2$, where $a = p_a V_A / (p_b V_B)$. This function reaches a unique global maximizer at $a = 1$. Notice also that $\hat{e}$ is homogenous of degree zero in the valuations.



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on the 'right' side (who got additional support through the test) and optimal effort level of the lobby on the 'wrong' side (whose position was damaged by the test) depend in the same way on the asymmetry. As proposed by Tullock (in the introductory quote) in response to additional information both lobbies adjust their equilibrium effort in the same way. But does this mean that both lobbies will reduce their equilibrium effort? Not always, when the informational gain is not very important (for values of $q$ close to 1/2), if $t = a$ both lobbies reduce their effort, but if $t = b$ both interest groups increase their lobbying outlays. Additional information of low quality has the potential to increase competition when the information acquired goes against the position of the 'stronger' lobby, the one with the higher valuation. Therefore, although this contest behaves in many respects as proposed by Tullock, there is in principle room for situations in which bias decreases the efficiency of government (as measured e.g. by social waste). However, we turn now to a careful analysis of these effects on the efficiency of government and show that such a non-monotonicity cannot occur in this model.

We start by analyzing the effect of additional information on the probability of an erroneous decision. Given the equilibrium win probabilities we can rewrite equation (4) as

$$\begin{aligned}\Pr(Err|t) &= \frac{1}{2}\Big(q\frac{(1-q)V_B}{qV_A+(1-q)V_B} + (1-q)\frac{qV_A}{qV_A+(1-q)V_B} \\ &\quad +(1-q)\frac{qV_B}{(1-q)V_A+qV_B} + q\frac{(1-q)V_A}{(1-q)V_A+qV_B}\Big) \\ &= \frac{q(1-q)(V_A+V_B)^2}{2}\Big(\frac{1}{qV_A+(1-q)V_B} + \frac{1}{(1-q)V_A+qV_B}\Big) \\ &= \frac{(V_A+V_B)^2}{2}\frac{1}{(V_A^2+V_B^2)+\frac{1-2q+2q^2}{q(1-q)}V_AV_B} \equiv \Gamma(q).\end{aligned}$$

Since $\frac{1-2q+2q^2}{q(1-q)}$ is strictly increasing in $q$, the function $\Gamma(q)$ is strictly decreasing in $q$.

Consider now the amount of social waste generated through the contest. Given the equilibrium efforts we can rewrite equation (3) as

$$\begin{aligned}SW(e^*|t) &= \frac{V_A+V_B}{2}\Big(\hat{e}(t=a)+\hat{e}(t=b)\Big) \\ &= \frac{(V_A+V_B)^2V_AV_B}{2}\frac{1}{(V_A^2+V_B^2)+\frac{1-2q+2q^2}{q(1-q)}V_AV_B} = V_AV_B\Gamma(q).\end{aligned}$$

Again, additional information is beneficial and the effect is monotonic. We summarize this in the following result.

**Proposition 3.2** *For any $V_A \geq V_B$ and $q$, in the benchmark game additional information always (strictly) improves the efficiency of government independently of whether efficiency is measured by the probability of taking an erroneous decision or by the social waste generated through the contest. Moreover, the beneficial effect of bias is monotonically increasing (strictly) in the 'quality' $q$ of the additional information.*



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Although bias affects the asymmetry of the contest and the asymmetry induces both lobbies either to increase or to decrease their efforts, from an ex ante point of view (before the realization of the test) additional information is always beneficial. The next section studies how general this result is.

## 4 Robustness

It is straightforward to modify Assumption 3.1 in order to generalize the game of the previous section to a family that depends on a positive scalar $\alpha$ analogously to the 'classical' Tullock rent-seeking game. This yields $\Psi_A(e_A, e_B) = p_a e_A^\alpha / (p_a e_A^\alpha + p_b e_B^\alpha)$. The parameter $\alpha$ specifies how deterministic the contest is and consequently includes (for $\alpha = 0$) a lottery independent of the lobbies' efforts that only depends on the likelihood of the states of the world and (for $\alpha \to \infty$) a 'biased' all-pay auction. As in other contests (for the 'classical' Tullock game see Pérez-Castrillo and Verdier (1992) and for the ratio-form contest see Alcalde and Dahm (2006)), it is straightforward to check that if $\alpha$ is low enough a pure strategy Nash equilibrium exists. In this generalized benchmark contest $\hat{e}$ must be generalized to $\hat{e}(\alpha) \equiv \alpha p_a p_b V_A^\alpha V_B^\alpha / (p_a V_A^\alpha + p_b V_B^\alpha)^2$. As a result equilibrium efforts are affected ($e_A^* = \hat{e}(\alpha) V_A$ and $e_B^* = \hat{e}(\alpha) V_B$) but equilibrium win probabilities are not. Replicating the previous line of reasoning allows to derive an analogous result to Proposition 3.2 for the generalized benchmark contest.

Despite this robustness to modifications of the contest that maintain the basic functional form of the contest success function we show now that once this basic functional form is modified the contest behaves differently. The following subsection presents a minor modification while Subsection 4.2 specifies a larger departure from the benchmark. Both modifications can be traced back to reasonable changes in the micro-foundations.

### 4.1 The Multiplicative Tullock Rent-Seeking Game

Suppose we desire to work with a contest success function which is well defined when no lobby exerts effort. It is straightforward to modify Assumption 3.1 in order to generate the following function which represents a minor change to the benchmark.[10]

**Definition 4.1** *In the multiplicative game the contest success function is given by*

$$\Psi_A(e_A, e_B) = \frac{p_a(1 + e_A)}{p_a(1 + e_A) + p_b(1 + e_B)} \text{ and } \Psi_B(e_A, e_B) = 1 - \Psi_A(e_A, e_B).$$

The equilibria of this contest are very related to the ones of the benchmark game. However, there is an important difference: in an interior equilibrium a constant is subtracted from the equilibrium effort of the benchmark (case 1 in the next Proposition). Consequently, an interior

[10] A payoff table in which the payoffs of the four combinations $(a, A)$, $(a, B)$, $(b, A)$, and $(b, B)$ are $R(1 + e_A)$, 0, 0, and $(1 - R)(1 + e_B)$, respectively, yields Definition 4.1. Note that, contrary to Assumption 3.1, this is fully consistent with the table in Section 2.



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equilibrium does not always exist. More precisely, it will cease to exist when additional information becomes very precise. Since it is still true that competition is higher when the weaker lobby obtains support for his position through the test result, there is a region in which after $t = a$ only lobby $A$ is active while after $t = b$ both groups lobby (case 2). The last possibility is that lobby $B$ is never active while lobby $A$ is only active when the test is not too precise (case 3). In this region at least lobby $B$ is not active because the informational advantage of one of the lobbies is too high. Contrary to the benchmark, this makes it possible that bias in the decision-making process deters lobbying efforts completely. More formally, we have the following result which is proved in Appendix A.[11]

**Proposition 4.1** *For any $V_A \geq V_B$ and $p_a$, there exists a unique pure strategy Nash equilibrium to the multiplicative contest. This equilibrium is as follows:*

1. [Both lobbies active] *If $\hat{e}(t = a)V_B \geq 1$, then for all $t \in \{a, b\}$*

$$\begin{array}{ll} e_A^* = \hat{e}V_A - 1 & e_B^* = \hat{e}V_B - 1 \\ \Psi_A(e^*) = \frac{p_a V_A}{p_a V_A + p_b V_B} & \Psi_B(e^*) = \frac{p_b V_B}{p_a V_A + p_b V_B}. \end{array}$$

2. *If $\hat{e}(t = b)V_B \geq 1 \geq \hat{e}(t = a)V_B$, then:*

   *(a)* [Only lobby A active] *if $t = a$, then*

$$\begin{array}{ll} e_A^* = \frac{\sqrt{q(1-q)V_A} - 1}{q} & e_B^* = 0 \\ \Psi_A(e^*) = 1 - \sqrt{\frac{1-q}{qV_A}} & \Psi_B(e^*) = \sqrt{\frac{1-q}{qV_A}}. \end{array}$$

   *(b)* [Both lobbies active] *if $t = b$, then*

$$\begin{array}{ll} e_A^* = \hat{e}V_A - 1 & e_B^* = \hat{e}V_B - 1 \\ \Psi_A(e^*) = \frac{(1-q)V_A}{(1-q)V_A + qV_B} & \Psi_B(e^*) = \frac{qV_B}{(1-q)V_A + qV_B}. \end{array}$$

3. *If $1 > \hat{e}(t = b)V_B$, then for all $t \in \{a, b\}$:*

   *(a)* [Only lobby A active] *if $q(1-q)V_A \geq 1$, then*

$$\begin{array}{ll} e_A^* = \frac{\sqrt{p_a p_b V_A} - 1}{p_a} & e_B^* = 0 \\ \Psi_A(e^*) = 1 - \sqrt{\frac{p_b}{p_a V_A}} & \Psi_B(e^*) = \sqrt{\frac{p_b}{p_a V_A}}. \end{array}$$

   *(b)* [No lobby active] *if $q(1-q)V_A \leq 1$, then*

$$\begin{array}{ll} e_A^* = 0 & e_B^* = 0 \\ \Psi_A(e^*) = p_a & \Psi_B(e^*) = p_b. \end{array}$$

[11] The difference-form contests analyzed in the literature display the striking feature that in many situations in a pure strategy equilibrium there is only one player active. Notice that the multiplicative contest (and the additive game analyzed below) behave very similarly in the sense that whenever the informational advantage of one player is high enough in any pure strategy Nash equilibrium there is at most the other player active.



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This Proposition implies that the comparative statics of the multiplicative game behave in the same way as the ones of the benchmark as long as in equilibrium both lobbies are active. However, if the 'quality' of information is high enough this will no longer be true and requires a careful analysis. We analyze next the effect of bias on the probability of an erroneous decision. We have the following result which is again proved in Appendix A.

**Proposition 4.2** *For any $V_A \geq V_B$ and $q$, in the multiplicative contest additional information always (strictly) reduces the probability of taking an erroneous decision. Moreover, the beneficial effect of bias is monotonically increasing (strictly) in the 'quality' $q$ of the additional information.*

The intuition for this result is that in equilibrium a lobby's win probability is still related to the likelihood with which the group defends the 'right' policy. Additional information assures thus that more often an error in the political decision can be avoided. However, with respect to social waste the efficiency properties of this model are different.

Since in this game sufficiently accurate additional information deters lobbying efforts completely, there are always values for $q$ in which social waste in the multiplicative rent-seeking game is lower than in the benchmark. However, in this game social waste might be increased through bias as the next example shows dramatically.

**Example 1** *Suppose $V_A = 100$ and $V_B = 100/9$. These parameter values imply that there is no value for $q$ such that both lobbies are active independently of the test result (case 1 in Proposition 4.1). However, for almost all possible values of $q$, that is, $q \in [0.5, 0.987\,8]$, additional information raises competition when it supports lobby B's position. As a result lobby A is always active and lobby B only when $t = b$ (case 2). There is a very small interval in which independently of the test result only lobby A is active, more precisely, for $q \in [0.9878, 0.98990]$ (case 3a) and for the remaining values of $q$ both lobbies are deterred by the bias (case 3b). Although this increased competition for intermediate values of $q$ through the test result does not reverse the beneficial effect of bias on the error probability it does reverse the effect on social waste (see Figure 1)*

To understand the driving force of this example it is useful to remember the discussion after Proposition 4.1. We mentioned there the potential for a non-monotonicity of total effort due to the possibility that $\hat{e}(t = b)$ increases for some values of $q$. As it turned out this non-monotonicity cannot occur in the benchmark game. The multiplicative contest is different because of the possibility that lobby B is not active in equilibrium. The example is chosen such that $\hat{e}(t = b)$ increases very strongly on a long interval. This implies that total effort increases strongly when $t = b$ is revealed. When $t = a$ is revealed, only lobby A is active and his effort decreases according to a different functional form. With the parameters chosen the increase after $t = b$ is much stronger than the reduction after $t = a$, generating the dramatic increase in social waste depicted in Figure 1.

To summarize, a slight modification of the fundamentals of our model yields instead of the benchmark game the multiplicative contest. While in the former game both lobbies are always



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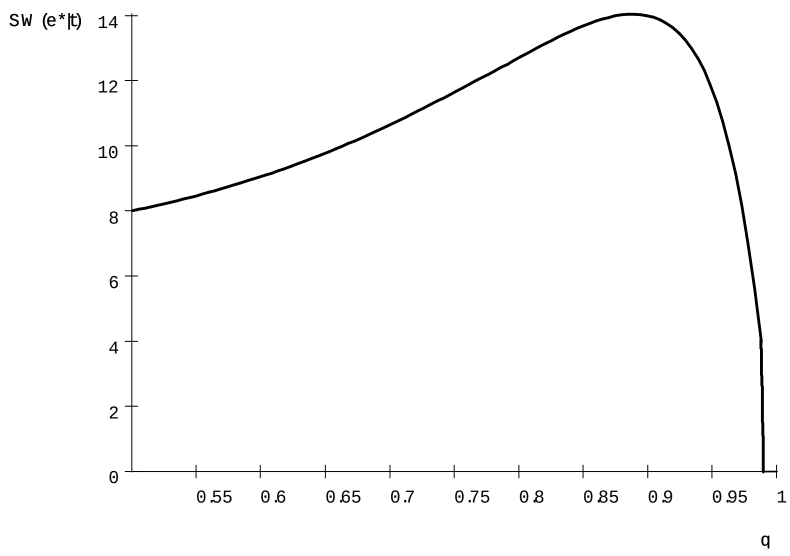


Figure 1: Social Waste in Example 1

active, in the latter (at least) the 'weaker' lobby might be deterred from lobbying. This has consequences for the effect of bias on the efficiency of government:

- Providing additional information has the potential to increase competition. The 'weaker' lobby might obtain additional support and fight harder. The 'stronger' lobby on the 'wrong' side might accept the fight instead of giving in. Tullock's reasoning – which worked in the benchmark – does not need to apply and as a result social waste might be increased in response to bias.

- However, there is still a relationship between the lobbies' equilibrium win probabilities and the likelihood that they defend the 'right' policy. This assures that bias in the political decision reduces the probability of an error.

- Moreover, it is still true that sufficiently accurate information is desirable under both efficiency criteria. In fact, it is easy to show that as $q \to 1$ both $\Pr(Err|t)$ and $SW(e^*|t)$ converge to zero.

## 4.2 The Additive Tullock Rent-Seeking Game

Note that in order to provide a micro-foundation for the multiplicative game a table in which the payoffs of the four combinations $(a, A)$, $(a, B)$, $(b, A)$, and $(b, B)$ are $R(1 + e_A)$, 0, 0, and $(1 - R)(1 + e_B)$, respectively, might be used. Although in this formulation effort multiplies the payoffs of the politician, in some instances we can also think of it as being additive. More precisely, this is the case when state and policy are matched. However, in the cases in which state of the world and policy are not matched, effort does not affect the politician's payoffs. There



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might be situations in which this is not a desirable assumption. This consideration motivates the following contest success function.

**Definition 4.2** *In the additive game the contest success function is given by*

$$\Psi_A(e_A, e_B) = \frac{e_A + p_a}{e_A + e_B + 1} \text{ and } \Psi_B(e_A, e_B) = 1 - \Psi_A(e_A, e_B).$$

Again, we can recover the standard Tullock contest with constant returns to scale; defining $e'_A = e_A + p_a$ and $e'_B = e_B + p_b$. The 'informational advantage' of each policy and the lobbing effort are combined additively. We provide first a micro-foundation for this contest.

**Assumption 4.1** *Suppose that the politician's payoffs from lobbying efforts are given as specified in the following table:*

| $\omega \backslash D$ | $A$ | $B$ |
|---|---|---|
| $a$ | $R + e_A R$ | $e_B(1-R)$ |
| $b$ | $e_A R$ | $1 - R + e_B(1-R)$ |

Under Assumption 4.1 lobbying increases the payoffs independently of the state of the world. The effectiveness of lobbying efforts depends on the type $R$ of the politician. Reasoning as before allows to derive the following result.

**Lemma 4.1** *Assumption 4.1 yields the additive contest. That is, lobbies choose effort as to maximize equation (1), where $\Psi_i(e_A, e_B)$ is specified in Definition 4.2.*

We determine next the equilibria of this game. Remember that in Tullock's original game (under constant returns to scale) the optimal effort levels are given by

$$\tilde{e}_A = \left(\frac{V_A}{V_A + V_B}\right)^2 V_B \text{ and } \tilde{e}_B = \left(\frac{V_B}{V_A + V_B}\right)^2 V_A.$$

Note that $\tilde{e}_A \geq \tilde{e}_B$ holds.

The equilibria of this contest follow the pattern of those of the multiplicative game, because there are three possible configurations: either both lobbies are active, only one interest group lobbies or none enters the contest. There are, however, important differences arising from the fact that, in the additive game the lobbies' effort and their informational advantage are perfect substitutes. As a result, when both lobbies are active (case 1 in the next Proposition), their equilibrium effort is the one in Tullock's original game ($\tilde{e}_i$) minus the support received through information. When this difference is not positive at least one lobby is deterred from the contest (case 2), because his informational advantage is higher than what the lobby is willing to bid given his valuation and the other lobby's effort. In Appendix A we proof the following result.

**Proposition 4.3** *For any $V_A \geq V_B$ and $p_a$, there exists a unique pure strategy Nash equilibrium to the additive game. This equilibrium is as follows:*



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1. [Both lobbies active] *If $\tilde{e}_i \geq p_i$ for both $i \in \{A, B\}$, then for both $i \in \{A, B\}$*

$$e_i^* = \tilde{e}_i - p_i \quad \textit{and} \quad \Psi_i(e^*) = \tfrac{V_i}{V_i + V_j}.$$

2. *If there exists $i \in \{A, B\}$ such that $\tilde{e}_i < p_i$, then:*

   *(a)* [One lobby active] *if there exists $j \in \{A, B\}$ such that (for $k \neq j$) $p_k V_j \geq 1$*

$$\begin{array}{lcl} e_j^* = \sqrt{p_k V_j} - 1 & \textit{and} & e_k^* = 0 \\ \Psi_j(e^*) = 1 - \sqrt{\frac{p_k}{V_j}} & \textit{and} & \Psi_k(e^*) = \sqrt{\frac{p_k}{V_j}}. \end{array}$$

   *(b)* [No lobby active] *if such a $j$ does not exist, then for both $i \in \{A, B\}$*

$$e_i^* = 0 \quad \textit{and} \quad \Psi_i(e^*) = p_i.$$

It is worth to point out that in this game there are strategic effects which are in line with Tullock's intuition. *Any* lobby might abstain because his informational advantage is high enough. Notice that this is not true in the multiplicative game where (given that one interest group is active) only the 'weaker' lobby $B$ might be deterred from lobbying.

We turn now to an analysis of the efficiency of government. Suppose, first, that valuations are high enough so that after any test result both lobbies are active. This requires, on one hand, that valuations are high enough and, on the other, that they are symmetric enough as formalized in the following condition.

**Assumption 4.2** *The lobbies' valuations are such that*

$$V_A > 4 \textit{ and } V_B \in \left[ \frac{V_A\left(1 + \sqrt{V_A}\right)}{V_A - 1}, V_A \right].$$

This assumption implies that valuations can not be too asymmetric because the lower bound for $V_B$ is increasing in $V_A$. Under Assumption 4.2 the expression for the probability of an error in the political decision in equation (4) simplifies to

$$\Pr\left(Err|t\right) = \frac{1}{2}\Big(q\frac{V_A}{V_A + V_B} + (1-q)\frac{V_B}{V_A + V_B} + (1-q)\frac{V_B}{V_A + V_B} + q\frac{V_A}{V_A + V_B}\Big) = \frac{1}{2}.$$

Consider now the amount of social waste generated through the contest. Given the equilibrium efforts we can rewrite equation (3) as

$$SW(e^*|t) = \frac{1}{2}\Big(\tilde{e}_A - q + \tilde{e}_B - (1-q) + \tilde{e}_A - (1-q) + \tilde{e}_B - q\Big) = \frac{V_A V_B}{V_A + V_B} - 1.$$

In both cases additional information has no effect on the efficiency of government. Each lobby simply reacts to additional information by maintaining the sum of effort and informational advantage constant. This implies that a lobby's equilibrium probability of obtaining the prize (and, hence, the probability that the political decision taken is erroneous) remains unaltered. Moreover, since information supporting one side always damages the opposite side, the effects on the equilibrium efforts cancel out and total outlays (i.e., social waste) are also constant. We summarize this in the following result.

**Proposition 4.4** *Suppose that the lobbies' valuations are high and symmetric enough such that Assumption 4.2 holds. For any q, in the additive contest additional information does not affect the efficiency of government independently of whether efficiency is measured by the probability of taking an erroneous decision or by the social waste generated through the contest. However, individual lobbying behavior is adjusted according to the bias.*

One implication of this result is that once we move away from our ideal scenario for information acquisition to a world in which additional information is costly, investment in information is suboptimal for any arbitrarily small cost.

When Assumption 4.2 does not hold it can be shown that an analogous result to Proposition 4.2 holds.[12] However, we show now that the strategic effects of the additive rent-seeking game induce worse efficiency properties than the ones of the games previously studied. Consider the following example.

**Example 2** *Let $V_A = V_B = 2$. This implies that $\tilde{e}_A = \tilde{e}_B = 1/2$ and therefore in the initial situation no lobbying takes place. After biasing the contest, however, the situation of one lobby is necessarily worse than before inducing this lobby to expend resources in the political process. Depending on the result of the test this lobby is sometimes lobby A (when $t = b$) and sometimes lobby B (when $t = a$). Since both valuations are equal to two, in both cases the active lobby exerts effort of $\sqrt{2q} - 1 > 1$ and obtains a win probability of $1 - \sqrt{\frac{q}{2}}$. This implies that social waste is given by $SW(e^*|t) = \sqrt{2q} - 1$, which is a strictly increasing function of the 'quality' q of the additional information. Moreover, the probability of an error can be simplified to*

$$\Pr(Err) = q - (2q-1)\sqrt{\frac{q}{2}},$$

*which is a strictly decreasing function in q. However, it does not converge to zero as $q \to 1$, since*

$$\lim_{q\to 1}\left(q - (2q-1)\sqrt{\frac{q}{2}}\right) = 1 - \frac{1}{2}\sqrt{2} = 0.292\,89.$$

To summarize, a further slight modification of the fundamentals of the model leads from the multiplicative to the additive game. The strategic effects in the two games are very different because a lobby's informational advantage and effort are now perfect substitutes. As a result the capacity of information to increase efficiency is much more limited:

- As in the multiplicative game, bias has the potential to increase competition. Contrary to the multiplicative game, Tullock's intuition applies only partially because the lobby on the 'wrong' side might fight harder in order to prevent to lose too much ground. Increases in social waste are not confined only to relative small values of $q$ and social waste might be monotonically increasing for any 'quality' of additional information. The more precise additional information is, the more detrimental it might be under this efficiency criterion.

[12] Details are available on request.



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- Again contrary to the multiplicative game increasing the precision of additional information does not imply that one lobby is eventually deterred. This might happen but it does not need to. As a result, even acquiring ex-ante perfect information is not a sufficient condition to ensure a fully accurate decision. In fact, for the parameter values of Example 2, even if the system has perfect information ($q \to 1$) the lobbies' influence implies that one third of the times the decision taken is not the correct one.

## 5 Concluding Remarks

This paper has investigated the conventional wisdom that in situations in which a political decision-maker is lobbied by competing interests the efficiency of government can be increased by providing more accurate information to the decision-maker. We have derived this bias in the direction of the correct decision from fundamentals of the model that also give rise to the lobbying game as a contest or rent-seeking game. Our results suggest that whether the conventional wisdom is true or not is very sensible to the fundamentals of the model. Slight changes in the micro-foundations generate very different strategic effects and there is no guarantee that investment in additional information will improve government.

Our analysis has highlighted the importance of the possibility that in biased contests one lobby does not enter the contest. Since these situations go hand in hand with situations in which both lobbies abstain from the contest this possibility seems desirable on first sight. However, it opens the door to situations in which additional information increases competition. As a result social waste might be increased. Concerning the second efficiency criterion that we analyze the effects of bias seem more efficiency enhancing. Our analysis suggests that additional information increases the frequency with which the right policy is chosen if there is a relationship between a lobby's equilibrium win probability and the likelihood that the lobby favors the correct policy. In many situations this will be the case. However, we have also shown that sometimes such a relationship does not exist and consequently information provision is undesirable whenever there is an arbitrary small cost of information. Moreover, there are situations in which even very precise information cannot prevent wrong decisions to be taken frequently.

We have presented our analysis in a framework that represents only slight departures from Tullock's classical rent-seeking game as this is the most prominent contest in the literature. But it is important to point out that our approach can also be used to derive very different games and that there is no reason to believe that in these games the strategic effects will be more in line with the ones in the benchmark game. Another prominent class of contest games are difference-form contests (see e.g. Che and Gale (2000)). Note that in some circumstances it might be reasonable to assume that lobbying efforts increase the payoffs of the politician independently of the politician's type $R$. It is straightforward to see that a payoff table in which (for any positive scalar $s$) the payoffs of the four combinations $(a, A)$, $(a, B)$, $(b, A)$, and $(b, B)$ are $R + se_A$, $se_B$, $se_A$, and $1 - R + se_B$, respectively, yields a biased version of Che and Gale's difference-form contest. Analyzing this contests it turns out that in pure strategy



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equilibria only lobby A is active and exerts exactly the amount of effort necessary to outweigh the 'informational' advantage of the other lobby. Thus, the 'stronger' lobby wins the contest for sure and the efficiency of the contest – as measured by both criteria – does not depend on the information of the decision maker. In this sense the qualitative properties of the difference-form contest when lobby $B$ is deterred from lobbying are similar to the ones of our additive game when both lobbies are active.

Our main conclusions are also robust under an alternative efficiency criterion. Note that in the fundamentals of our models (e.g. Assumption 3.1) the effort of the lobby that does not get his favored policy is wasted in a different sense from the standard notion of social waste. It is wasted in that it does not yield a benefit to the politician. While we could have specified a more complicated payoff table that, given a policy choice, depends positively on both lobbies' efforts, this suggests that alternative definitions of waste in the political process might be reasonable.[13] One such notion might, hence, be the fraction of effort which is 'lost in the process': $SW' = \Psi_A(e_A^*, e_B^*)e_B^* + \Psi_B(e_A^*, e_B^*)e_A^*$. Given that this formulation is a part of the standard notion of social waste it does not reverse our conclusions. Indeed, in Example 2 we obtain $SW' = \sqrt{2/q}SW$, which is still strictly increasing in the quality of additional information.

The preceding implies that our approach to derive contest games from micro-foundations shifts the crucial element in the specification of the contest from the contest success function to an assumption relating states of the world, policy choice and the lobby's effort. This assumption is not only crucial for the strategic and normative properties of the contest but also for determining which normative criteria are the 'right' ones. Our work points thus at the importance of investigating these relationships in future research.

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[13] For example, all the results of our benchmark game remain true if we consider that, given a positive parameter $k$, the payoffs of the four combinations $(a, A)$, $(a, B)$, $(b, A)$, and $(b, B)$ are $(1+k)e_AR+ke_B(1-R)$, $ke_AR+ke_B(1-R)$, $ke_AR + ke_B(1 - R)$, and $ke_AR + (1 + k)e_B(1 - R)$, respectively. For the other games similar specifications are possible. We stick to our formulations for clarity of the exposition.



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# A Appendix

## A.1 Proof of Proposition 4.1

The proof builds on the following lemma.

**Lemma A.1** *For any $V_A \geq V_B$ and $p_a$, there exists a unique pure strategy Nash equilibrium to the multiplicative game. This equilibrium is as follows:*

1. *If $\hat{e}V_B \geq 1$, then $e_A^* = \hat{e}V_A - 1$ and $e_B^* = \hat{e}V_B - 1$.*
2. *If $\hat{e}V_B < 1$, then:*
   (a) *$e_A^* = \frac{\sqrt{p_a p_b V_A} - 1}{p_a}$ and $e_B^* = 0$, if $p_a p_b V_A \geq 1$; and*
   (b) *$e_A^* = 0$ and $e_B^* = 0$, otherwise.*

**Proof of Lemma A.1:** We show first that the strategy profile specified in the statement constitutes an equilibrium. By symmetry consider lobby $A$'s objective function (1), with $\Psi_i(e_A, e_B)$ defined as in Definition 4.1, is (for any $p_a \in (0,1)$) a strictly concave function of $e_A$ with derivative

$$\frac{\partial E\Pi_A(e_A, e_B)}{\partial e_A} = \frac{p_a p_b (1 + e_B)}{(p_a(1 + e_A) + p_b(1 + e_B))^2} V_A - 1. \tag{5}$$



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The first order conditions for a maximizer of expected utility imply $e^* = \hat{e}V_A - 1$ and $e^* = \hat{e}V_B - 1$. Notice also that if $e_B = 0$, then the first order condition of lobby $A$ implies that $e_A^* = \frac{\sqrt{p_a p_b V_A} - 1}{p_a}$. In addition, we have that

$$\left.\frac{\partial E\Pi_B(e_A = \frac{\sqrt{p_a p_b V_A}-1}{p_a}, e_B)}{\partial e_B}\right|_{e_B=0} \leq 0 \Leftrightarrow \hat{e}V_B \leq 1.$$

This proves that when $\hat{e}V_B \geq 1$, then $e^* = \hat{e}V_A - 1$ and $e^* = \hat{e}V_B - 1$ is the unique equilibrium. Suppose that $\hat{e}V_B < 1$. The preceding implies that $e_B^* = 0$ and $e_A^* = \max\{\frac{\sqrt{p_a p_b V_A}-1}{p_a}, 0\}$ is an equilibrium. Concerning uniqueness, it only remains to prove that $e_A^* = \frac{\sqrt{p_a p_b V_A}-1}{p_a}$ and $e_B^* = 0$ imply that $e_A^* = 0$ and $e_B^* = \frac{\sqrt{p_a p_b V_B}-1}{p_b} > 0$ is not an equilibrium. We proceed by contradiction and suppose both were an equilibrium. We have that $\hat{e}V_A \leq 1 < p_a p_b V_B$ must hold. This implies $(V_A)^2 < (p_a V_A + p_b V_B)^2$ or $V_A < V_B$, a contradiction which proves the Lemma.

In order to prove Proposition 4.1, note that $\hat{e}(t = b)V_B \geq 1$ implies that $p_a p_b V_A = q(1 - q)V_A \geq 1$, since because of $V_A \geq V_B$

$$\frac{q(1-q)V_A\,(V_B)^2}{((1-q)V_A + qV_B)^2} \leq q(1-q)V_A \Leftrightarrow V_B \leq (1-q)V_A + qV_B$$

holds. This fact and Lemma A.1 imply Proposition 4.1 *Q.E.D.*

## A.2 Proof of Proposition 4.2

It is straightforward to derive the functional form of the probability of an error in the political decision $\Pr(Err)$ from equation (4) for each of the cases in Proposition 4.1 and to check that it is continuous. The probability of an error in the political decision $\Pr(Err)$ in equation (4) is monotonically decreasing (strictly) in $q$ if

- $\frac{\partial \Psi_A^{p_a=q}(e^*(q),q)}{\partial q} > \frac{\partial \Psi_A^{p_b=q}(e^*(q),q)}{\partial q}$, and
- $\Psi_A^{p_a=q}(e^*) > \Psi_A^{p_b=q}(e^*)$.

We proceed by analyzing each of the of the cases in Proposition 4.1:

1. If $\hat{e}(t = a)V_B \geq 1$, then $\Psi_A^{p_a=q}(e^*) = \frac{qV_A}{qV_A + (1-q)V_B}$ and $\Psi_A^{p_b=q}(e^*) = \frac{(1-q)V_A}{(1-q)V_A + qV_B}$. We have that $\frac{\partial \Psi_A^{p_a=q}(e^*(q),q)}{\partial q} > 0$ and $\frac{\partial \Psi_A^{p_b=q}(e^*(q),q)}{\partial q} < 0$. Since for $q = 1/2$ we have that $\Psi_A^{p_a=q}(e^*) = \Psi_A^{p_b=q}(e^*)$ both conditions are fulfilled.

2. Suppose $\hat{e}(t = b)V_B \geq 1 \geq \hat{e}(t = a)V_B$. If $t = a$, then $\Psi_A^{p_a=q}(e^*) = 1 - \sqrt{\frac{1-q}{qV_A}}$, while for $t = b$, $\Psi_A^{p_b=q}(e^*) = \frac{(1-q)V_A}{(1-q)V_A + qV_B}$. We have that $\frac{\partial \Psi_A^{p_a=q}(e^*(q),q)}{\partial q} > 0$ and $\frac{\partial \Psi_A^{p_b=q}(e^*(q),q)}{\partial q} < 0$. Moreover,

$$\Psi_A^{p_a=q}(e^*) > \Psi_A^{p_b=q}(e^*) \Leftrightarrow \hat{e}(t = b)V_B > \left(\frac{1-q}{q}\right)^2,$$

which is true.


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3. Suppose $1 > \hat{e}(t=b)V_B$ and $q(1-q)V_A \geq 1$. In this case it is true that $\Psi_A^{p_a=q}(e^*) = 1 - \sqrt{\frac{1-q}{qV_A}}$ and $\Psi_A^{p_b=q}(e^*) = 1 - \sqrt{\frac{q}{(1-q)V_A}}$. This implies that $\frac{\partial \Psi_A^{p_a=q}(e^*(q),q)}{\partial q} > 0$ and $\frac{\partial \Psi_A^{p_b=q}(e^*(q),q)}{\partial q} < 0$. Moreover,
$$\Psi_A^{p_a=q}(e^*) > \Psi_A^{p_b=q}(e^*) \Leftrightarrow q^2 V_A > (1-q)^2 V_A,$$
which is true.

4. The case $1 > \hat{e}(t=b)V_B$ and $q(1-q)V_A < 1$ represents a world without lobbying where we already know that the result holds. *Q.E.D.*

## A.3 Proof of Proposition 4.3

We show first that the strategy profile specified in the statement constitutes an equilibrium. Consider $i,j \in \{A,B\}$ and $j \neq i$. For any $e_j$, equation (1), with $\Psi_i(e_A, e_B)$ defined as in Definition 4.2, is a strictly concave function of $e_i$ with derivative
$$\frac{\partial E\Pi_i(e_A,e_B)}{\partial e_i} = \frac{e_j + p_j}{(e_A+e_B+1)^2} V_i - 1. \tag{6}$$
The first order conditions for a maximizer of expected utility are therefore
$$e_j^* = \frac{(e_A^* + e_B^* + 1)^2}{V_i} - p_j \text{ for } i,j \in \{A,B\}. \tag{7}$$
Adding the previous expression for both lobbies yields $e_A^* + e_B^* + 1 = \frac{V_A V_B}{V_A + V_B}$. Substituting in the previous line gives as a unique solution $e_i^* = \tilde{e}_i - p_i$ for both $i \in \{A,B\}$. Notice also that if $e_j = 0$, then the first order condition of lobby $i$ implies that $e_i^* = \sqrt{p_j V_i} - 1$. In addition, we have that
$$\left.\frac{\partial E\Pi_j(e_i = \sqrt{p_j V_i} - 1, e_j)}{\partial e_j}\right|_{e_j=0} = \frac{\sqrt{p_j V_i} + p_j}{p_j V_i} V_j - 1 \leq 0 \Leftrightarrow \tilde{e}_j \leq p_j.$$

This proves that when $\tilde{e}_i \geq p_i$ for both $i \in \{A,B\}$, then $e_i^* = \tilde{e}_i - p_i \geq 0$ for both $i \in \{A,B\}$ is the unique equilibrium. Suppose that there exists $i \in \{A,B\}$ such that $\tilde{e}_i < p_i$. The preceding implies that $e_i^* = 0$ and $e_j^* = \max\{\sqrt{p_i V_j} - 1, 0\}$ is an equilibrium.

Concerning uniqueness, it only remains to prove that $e_i^* = 0$ and $e_j^* = \sqrt{p_i V_j} - 1 > 0$ imply that $e_i^* = \sqrt{p_j V_i} - 1 > 0$ and $e_j^* = 0$ is not an equilibrium. We proceed by contradiction. Suppose both were an equilibrium. We have that, on one hand, $\tilde{e}_i \leq p_i$ must hold for both $i \in \{A,B\}$, while on the other hand, $p_a V_B > 1$ and $p_b V_A > 1$ must be true. Adding the first two inequalities yields
$$\tilde{e}_A + \tilde{e}_B \leq 1 \Leftrightarrow V_A V_B \leq V_A + V_B.$$
Adding the second two inequalities after rewriting them as $p_a > \frac{1}{V_B}$ and $p_b > \frac{1}{V_A}$ we obtain
$$\frac{1}{V_A} + \frac{1}{V_B} < 1 \Leftrightarrow V_A + V_B < V_A V_B,$$
the desired contradiction. *Q.E.D.*


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