COWLES FOUNDATION DISCUSSION PAPER NO. 48

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On Incentives to Forecasters and to Decision Makers Under Uncertainty

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January 10, 1958

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* Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-358(01), NR 047-006 with the Office of Naval Research.

On Incentives to Forecasters and to Decision Makers Under Uncertainty.

1.

In any organization that is not a "team", i.e., where the members' own interests are not automatically identical with those of the organization, the problem arises of how to provide incentives that make it the own interest of the individuals concerned to act in accordance with the interests of the organization. More specifically, it is necessary to provide incentives for skill, which attract to, and retain within, the organization individuals with appropriate qualifications; and it is necessary to provide incentives for effort, which induce the individuals already attached to the organization to make good use of their skills and to undertake the efforts required by the interests of the organization.

The members' skills and efforts can be judged and their remuneration can be decided by the organization on the basis of three main types of criteria:

1. on the basis of their observed behaviour (as reported by supervisors) as to industry, discipline, skill, etc.;
2. on the basis of external evidence concerning their qualifications and their attitudes towards work, e.g., school certificates, character references, etc.
3. on the basis of the observed results of their activities (from piece-work payment for physical work up to special bonuses for successful executive decisions).

Criteria 1 and 2 can usually fairly well recognize and discourage (or eliminate) gross inefficiency, i.e., grossly insufficient effort and /or skill, but are less

effective in distinguishing and encouraging high levels of skill and effort -- or of originality and initiative. Criterion 3 (based on the results achieved), on the other hand, is very sensitive to such qualities, but has the disadvantage of often making remuneration to a considerable extent subject to chance, as the actual results of a person's activities may depend not only on his skill and effort but also on random influences not under his control. This is particularly true in the case of individuals who specialize in making decisions under uncertainty (e.g., higher executives) or in making predictions (e.g., statisticians).

This fact would not be a disadvantage if the individuals concerned had positive or at least neutral attitudes towards risk, i.e., had cardinal utility functions convex or at least linear in terms of money (or in terms of whatever rewards they receive). But in actual fact it seems to be more realistic to assume that most individuals dislike risk and display utility functions concave at least at their normal earned incomes - even when they are quite agreeable to risk-taking in other connections, e.g., in gambling. Hence, the larger the chance fluctuations to which a given individual's income is subject the larger the average income (expected income) that he must receive in order to attain any specified level of utility - including the minimum level of utility needed to keep him attached to the organization.

Thus the more dependent an individual's income is made on the actual results of his activities the greater will be his incentive for good performance, but the greater will be also the expected costs of this arrangement to the organization.

2.

I shall make the following assumptions. Each member (or employee) of the organization has a utility function convex in money. More specifically, a given employee's expected utility, $Eu = U$, which he tries to maximize, has the following form

(1) $U = \varphi\,(W, \sigma_w) - e$ with $\frac{\delta \varphi}{\delta W} > 0$ $\frac{\delta \varphi}{\delta \sigma_w} < 0$

where $W = E\,w$ is the expected value of his money income; $\sigma_w = \mathrm{Var}\,(w)$ is the variance of his income; and e is the disutility to him of the effort needed to earn the income w.

I also assume that, in order to retain this individual's services, the organization has to keep him at least at a certain specified utility level $\bar{U}$ so that his expected utility

(2) $U \geq \bar{U}$

Moreover, I assume that it is possible to distinguish each particular employee's personal contribution to the organization's total gross profit. The employee's income w will be made an increasing function of the value v of his contribution to profit, i.e.,

(3) $w = \psi(v)$ with $\frac{d\,w}{d\,v} > 0$

ψ will be called the <u>incentive function</u> of this employee. In particular, I shall consider the case where ψ has the simple linear form of

(3)' $w = a\,v + b$ with $a > 0$

The net profit of the organization on this employee's activities will be

(4) $x = v - w$

The organization itself is likely to be much less reluctant to bear the risks connected with a given employee's activities than this employee himself. First of all, for the organization as a whole the risks associated with different employees will tend to cancel out to some extent. Moreover, the organization will have much larger reserves against temporary losses, and can much more easily afford to take a longer view, than its individual employees. Actually I shall assume for the sake of simplicity that the organization takes a fully neutral attitude towards risk and tries to maximize simply its expected net profit

$$X = E_x = E\,v - E\,w = V - W \,. \tag{5}$$

But our conclusions would be much the same if we made the more general assumption that the organization has a negative attitude towards risk, only less so than its individual employees.

Obviously, the incentive effect will be larger the more dependent the employee's income on his net contribution v, i.e., the larger the value of the coefficient a in (3)'.

But the larger the value of a the larger the variance σ_w of the employee's income as by (3)'

$$\sigma_w = a^2 \,.\, \mathrm{Var}\,(v) = a^2 \,.\, \sigma_v$$

Consequently, in view of (1) and (2), the larger the value of a the larger must be the employee's expected income W in order to keep him at the desired level, $\bar{U}$ of utility.

Hence, in deciding on the form of the employee's incentive function, the organization has to weigh the advantage of a larger incentive effect, against the disadvantage of having to pay the employee a higher salary on the average, if the employee's salary is made strongly dependent on his contribution to profit.

For instance, suppose that the employee has a choice among actions $1,2, \ldots, k$, which would yield for the organization the expected gross profits $V_1, V_2, \ldots, V_k$, with $V_1 > V_2 > \ldots > V_k$. Suppose also that the more profitable actions also require larger effort so that $e_1 > e_2 > \ldots > e_k$.

Let us first assume that the variance of the profit, i.e., σ_v, would be the same whichever action is taken. Then the employee's choice among these alternative actions will hinge on how strongly his pay depends on the profit achieved. The variance σ_v, the disutilities $e_1, \ldots, e_k$ and the utility function φ of (1) together will determine certain constants $a_1, a_2, \ldots, a_{k-1}$ such that if in (3)' we choose the coefficient $a \geqq a_1$ then the individual will perform action 1; if we choose $a_1 > a \geqq a_2$ then he will perform action 2 etc. while if we choose $a < a_{k-1}$ then he will perform action k. In view of requirement (2), the larger we make the coefficient a the higher expected income has to be secured for our individual [by appropriate choice of the constant b in (3)']. Suppose that in case we choose $a = a_1$ then our individual must obtain the expected income $W = W_1$; if we choose $a = a_2$ then he must obtain $W = W_2$ etc. Then, with the choice of $a = a_1$ the organization's expected net profit will be $x_1 = V_1 - W_1$; with the choice of $a = a_2$ the expected net profit will be $x_2 = V_1 - W_2$ etc. Hence it will be profitable for the organization to choose $a = a_1$ only if $x_1 = \max_i X_i$, i.e., if $V_1 - V_2 \geqq W_1 - W_2$ as well as $V_1 - V_3 \geqq W_1 - W_3$ etc.

On the contrary, if $V_1 - V_2 < W_1 - W_2$, it will be perferable for the organization to choose only $a = a_2$ and let the employee perform the less profitable action 2 instead of the more profitable action 1, as it would be too expensive to induce him to perform the latter. On the other hand, if $V_1 - V_2 \geqq W_1 - W_2$ it will be in the interest of the organization to choose $a = a_1$, i.e., to choose a no higher than is just necessary for inducing the employee to perform action 1. For, if $a = a_1$ is chosen the employee will have to receive the expected income $W = W_1$ while if $a > a_1$ were chosen he would have to receive an even higher expected income $W > W_1$.

For similar reasons it will always be in the interest of the organization to find ways of reducing the variance of the employee's income if this can be done without reducing the incentive effect. For instance, at least in the case of employees not connected with marketing, it will be preferable to make their incomes dependent on their physical outputs rather than on the market values of their outputs because this arrangement will eliminate the effects of short-run price changes on their incomes. (In the case of members of the sales department, of course, the position may be different as making their incomes dependent on the prices achieved may have an important incentive function.)

One way of reducing the variance of the employee's income, without changing its incentive effect, will be to replace the linear incentive function defined in (3)' by an incentive function of a more general form. More particularly, the variance of the employee's income will decrease if the marginal incentive $d w / d v$ is decreased for very high and very low values of v, but is increased for intermediate values of v. There will not be any change in the total incentive effect if these adjustments in the derivative $d w / d v$ properly offset each other.* __

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* It may be particularly important to reduce the derivative $d w / d v$, possibly to nil, for very low values of v. That is, it may be necessary to quarantee a certain minimum income to the employee irrespective of his performance (though of course it will be understood that he is liable to dismissal if his performance too often fails to make the mark). For, without a guaranteed minimum income, the employee will feel he has to take a very high risk and will require an unduly high average (or expected) income in compensation.

If we now drop the assumption that the variance σ_v of the gross profit is the same whichever action is taken, our conclusions will naturally depend on whether this variance tends to increase or decrease if actions more profitable to the organization are chosen. If the more profitable actions tend to decrease this variance, this will give an additional incentive for the employee to select such profitable actions since by this means he can reduce the variance σ_w of his own income, also: therefore it will become less expensive for the organization to induce him to perform these actions. The opposite will be true if σ_v tends to increase with actions that yield higher expected profits.

3.

People's negative attitudes towards risk are the main reason why the entrepreneurial unction tends to be concentrated in the hands of the organization itself. If people did not shun risk-taking, each member of the organization could be paid simply the value of his own marginal contribution to profit less a certain constant depending on his bargaining position vis-à-vis the organization. This would mean that each individual would have to bear the whole risk associated with his own activities: that is, each individual would perform full entrepreneurial functions. Hence the organization would be in a position to give the fullest possible scope to the individual's initiatives because the losses that might arise from their mistakes would be fully borne by these individuals themselves.

But, apart from the fact that it may be physically impossible to recover major losses from individual employees, this arrangement is rendered impracticable by the average employee's aversion to risk. Rather than paying its employees huge risk premiums as would be necessary if these employees had to bear themselves the risks of the organization's operations, the organization will be better off if it itself underwrites the main part

of these risks - even if this means reducing the incentives to the employees for good performance and means narrowing the scope that can be given to their initiative.

4.

Even assuming that all individuals employed by the organization have negative attitudes to risk, some of them no doubt will have less negative attitudes to risk, and will require smaller risk premiums for bearing risks, than others. This will be due partly to differences in the shapes of the individuals' cardinal utility functions, and partly to the fact that in the same situations some individuals will face (or will think they face) a smaller risk of failure than do others.

Thus it will be in the interest of the organization to behave like a discriminating monopsonist, and to make use of these differences in different individuals' supply prices for risk bearing.

Full exploitation of these individual differences would require finding out the exact values of each individual's supply prices for bearing various risks, and to strike a separate bargain with each individual on the basis of this information.

But a fairly effective and much easier way of discrimination would be to offer each individual a free choice between two (or more) alternative payment schemes: one would make the individual's salary highly dependent on the results of his activities without, however, offering a high risk premium (a high expected income) to compensate him for this risk-bearing; the other would make his salary independent of (or only slightly dependent on) his results. Individuals less averse to risk-taking, or individuals very confident in their ability to achieve good results, will choose the first payment scheme even though no high risk premium is offered; other individuals will choose the second. In spite of the fact that no high risk premium would be offered, the organization would not incur the risk of depressing the utility levels of some individuals below the admissible minimum by asking them to bear heavy risks without adequate compensation, as every individual would be free to choose the second payment scheme, which involves negligible risk.

This arrangement would also have the additional advantage for the organization that it would give some guidance in deciding how much confidence the organization should place in the judgement of different individuals (in particular when different individuals advocate different policies): other things being equal, it will be reasonable to place more confidence in the recommendations of those individuals who have enough confidence themselves in their own judgement concerning the issue in question as to choose the first payment scheme, i.e., as to back their judgement by a bet.

To sum up, an arrangement where each member of the organization would perform full entrepreneurial functions would have considerable advantages because it would furnish strong incentives for good performance and would enable the organization to give full scope to individual initiative. But such an arrangement is rendered impracticable by most people's aversion to risk-taking. However, a good deal of the advantages associated with this scheme could be achieved if all individuals were given a free choice of how much risk-bearing and entrepreneurial responsibility they wished to undertake.

5.

I now propose to apply our conclusions to the problem of making incentive payments to forecasters (statisticians), who have to predict certain future events. I shall consider only the case where the forecaster has to specify the precise value that a certain random variable is likely to take in the future, and shall not consider the case where he has to specify only the probabilities of alternative possible values.*

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* Concerning this latter case, see I.J. Good, "Rational Decisions," Jour. Roy. Stat. Soc., Ser. B, 14 (1952), pp. 107-114; and J. McCarthy, "Measures of the Value of Information," Proc. Nat. Acad. of Sc., 42 (1956), pp. 654-655.

The forecaster will have to be paid a fixed salary w_o less a deduction ω depending on the size (and possibly also on the direction) of the error ϵ in his prediction. Thus the incentive function will have the form

$$w = w_o - \omega(\epsilon) .$$

Obviously the form of ω will depend on how the organization's profit is affected by the size of the error ϵ . Other things being equal, it will be desirable to make the marginal incentive (or rather marginal penalty) $d\omega/d\epsilon$ larger for those ranges of ϵ where the marginal loss $-dv/d\epsilon$ itself suffered by the organization is larger. (This fact may make it desirable e.g., to set up different penalties for overestimating and for underestimating the predicted quantity.) Moreover, it will always be undesirable to make the marginal penalty $d\omega/d\epsilon$ larger in absolute value than the marginal loss $dv/d\epsilon$, because a marginal penalty equal to the marginal loss will always fully compensate the organization and further increase in the marginal penalty would only unnecessarily increase the variance of the employee's income.

It is perhaps less obvious but follows from our previous argument that it will be desirable to make the marginal penalty relatively small (or even zero) both for very small and for very large errors. For, small errors can never be avoided however large the forecaster's skill and effort: hence there is no point in giving strong incentives for avoiding them. On the other hand, very heavy penalties for occasional large errors unduly increase the variance of the employee's income and make it necessary to pay very high salary to him as a risk premium.

Moreover, in the case of forecasters it will also be true that the organization can greatly strengthen the incentives for good performance without having to pay large risk premiums if all forecasters are given a free choice, in connection with each prediction, between a payment scheme involving heavy penalties for prediction error

and one involving less heavy penalties.*

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* Of course, if on grounds of convenience the forecaster's salary is made equal to a constant less a penalty for errors, rather than making it directly a function of the profit achieved, then it will be always necessary, in the case of increasing this penalty, to compensate the forecaster by increasing the constant term of his income. But it will be still true that a smaller increase in this constant will suffice if the forecaster has a free choice between a payment scheme with, and one without, heavy penalties -- than if he is brought under the scheme with heavy penalties without any choice.

Moreover, the organization can use the fact that a forecaster chooses the payment scheme with heavy penalties, as an indication that the forecaster himself has a strong confidence in his prediction. If different employees of the organization make opposite predictions then the organization can give greater credit to those employees' predictions who have a better record of successful predictions -- as well as to the predictions of those who are prepared to incur heavier penalties in the case of a failure.