DOLLAR DENOMINATED DEBT AND OPTIMAL SECURITY DESIGN

By

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Dollar Denominated Debt and Optimal Security Design*

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Abstract

During a crisis, developing countries regret having issued dollar denominated debt because they have to pay more when they have less. Ex ante, however, they may be worse off issuing local currency debt because the equilibrium interest rate might rise, making it more expensive for them to borrow. Many authors have assumed that lenders and borrowers have contrary goals, and that local currency (peso) debt is better for the borrower (Bolivia), and dollar debt is better for the lender (America).

We show that if each country is represented by a single consumer with quadratic utilities, in perfect competition, then both will agree ex ante on whether dollar debt or peso debt is better. (In fact all assets can be Pareto ranked). But we show that it might well be dollar debt that Pareto dominates. In particular, if the lender is sufficiently risk averse and the borrower sufficiently impatient, and the lender's endowment is sufficiently riskless, then dollar debt Pareto dominates peso debt. However, if there are persistent gains to risk sharing between the countries, then peso debt Pareto dominates dollar debt.

In the special case where utilities are linear in the first period and quadratic in the second period, we can completely characterize the Pareto ranking of any asset by a formula depending only on marginal utilities at autarky.

In the more general case where utilities are linear in the first period and have positive third derivative in the second period, we show that when persistent gains to risk sharing hold, America must gain from Peso debt but Bolivia might lose. Thus the presumption that peso debt is more favorable to Bolivia than to America is false.

Our framework of optimal security design can be used to demonstrate one rationale for credit controls. If the purchasing power of a dollar overseas varies with the quantity of debt issued, then both America and Bolivia can gain from capital controls, because a tax that reduces the quantity of Bolivian debt might make the real dollar payoffs in Bolivia more 'peso-like', and therefore under persistent gains to risk pooling, better for America and Bolivia.

Keywords: Dollar debt, Currency, Indexed bonds, Security design, Capital controls

JEL Classification: D61, F31, F34, G10, G12

1 Introduction

Experience suggests that a country in economic crisis usually finds that its currency is steeply depreciating. If the country has borrowed heavily in foreign currency ("dollar" denominated debt), then its plight is made all the worse by the need to pay more just when it has less. Immediately following the Latin American crisis of 1984, Eskridge (1985) called attention to the dangers of dollar denominated debt, and again after the Asian crisis of 1998, Frankel (2000) and Krugman (1999) listed dollar denominated debt as one of the many factors contributing to the financial panic. Clearly the crisis would have been eased had the same borrowing, at the same rates, been taken out in local currency. But of course had the debt been "peso" denominated (or indexed to domestic output) in the first place, the interest rate would have been different and the level of debt would have changed.

In this paper we explore the ex ante consequences of dollar versus local currency denominated debt in a perfectly competitive economy. We consider security design in a two agent economy (A = America, B = Bolivia = Borrower), with one asset and one international good. We apply the analysis to the international debt problem by assuming that the purchasing power of each currency in each state is a function of the economic fundamentals; the better a country's fundamentals, the higher its currency's value in terms of the international good. An alternative interpretation is that we compare a debt-contract indexed to Bolivia's output to a debt contract indexed to America's output. Peso denominated debt reduces Bolivia's burden when Bolivia's income is lower. Compared to peso debt, dollar denominated debt makes Bolivia's repayment burden harder when Bolivia's income is relatively worse than American income. Nonetheless, the proposition that peso debt is ex ante Pareto better, or at least ex ante better for Bolivia, turns out to be valid only under additional conditions.

When America is risk neutral and large, and Bolivian and American endowments are inversely monotonic, we show that peso debt is always better for Bolivia than dollar debt. Bohn (1990) had previously demonstrated the same conclusion under the additional hypotheses that American endowments are riskless, and Bolivian utility is mean-variance. Even with the heroic assumption of American risk-neutrality, restrictions must be placed on endowments to guarantee that peso debt is better than dollar debt for Bolivia.

Risk neutrality is clearly an unacceptable hypothesis. Without it the question arises: is there some restriction on endowments (or utilities) that makes peso debt better than dollar debt for Bolivia? What about for America? If American and Bolivian endowments are inversely monotonic, and American endowments are relatively large, in that aggregate endowments are still inversely monotonic with Bolivian endowments, we say that there are endowment generated persistent gains to risk pooling. Under this hypothesis at least one country must benefit by a switch from dollar debt to peso debt, no matter what the utilities.

The conventional view has been that peso debt might be better for Bolivia even when it is worse for America. We show, however, that if both America and Bolivia have quadratic preferences, then peso debt is better for Bolivia if and only if it is better for America. Indeed we show that with quadratic utilities and two agents, all debt contracts can be Pareto-ranked. Thus with quadratic utilities and endowment generated persistent gains to risk pooling, both countries prefer peso debt to dollar debt.

When utilities are linear-quadratic, we give an explicit numerical score for each asset, equal to the difference in marginal utility to A and B of the asset payoffs, evaluated at autarky. Assets with higher score give equilibria that Pareto dominate equilibria with lower scoring assets. These theorems permit us to describe general conditions under which peso debt Pareto dominates dollar debt, but also conditions under which dollar debt Pareto dominates peso debt.

We show that if utilities are linear-quadratic, then dollar debt will Pareto dominate peso debt, provided that Bolivia is sufficiently impatient, America is sufficiently risk-averse and American endowments are sufficiently close to riskless. (These hypotheses clearly violate endowment generated persistent gains to risk pooling, since the sum of the endowments is no longer inversely monotonic with Bolivian endowments).

When endowment generated persistent gains to risk pooling holds but utilities are not quadratic, which country benefits from peso debt? Surprisingly, that country is America! We show that with linear-concave utilities, if $u_h''' \geq 0$ for $h = A, B$, America must always be better off with peso debt than dollar debt, and we give an example to show that Bolivia may be worse off.

Finally, we use our framework of optimal security design to demonstrate one welfare rationale for credit controls. Suppose that the real purchasing power of the dollar in Bolivia changes with the quantity of debt issued by Bolivia (say increasing in the quantity of debt when Bolivian output gets lower). Taxes on foreign loans could take advantage of the externality; less loans means the dollar debt in real terms would become more "peso-like". With quadratic utilities (and persistent gains to risk pooling) this would help both Bolivia and America.

There is a large formal literature on the welfare gains from international risk sharing (see e.g., Athanasoulis-Shiller (1995), van Wincoop (1999) and the references therein). This literature, however, treats a completely different problem - the question there is how much welfare improves with the introduction of new assets. In the context of dollar denominated debt, the important question is how welfare changes as the payoffs of existing assets change. From a theoretical viewpoint the analysis in this paper is closest to Demange and Laroque (1995). They characterize Pareto-optimal asset structures in a linear Gaussian (mean-variance) setup. By contrast, in our quadratic (mean-variance) setup, we Pareto-rank inefficient asset structures.

We have assumed a competitive framework throughout, ignoring default and moral hazard both on the part of the borrower and the central bank. Atkenson (1991) argues that with moral hazard on the part of the borrowers, dollar denominated debt might be optimal.

There is also a large literature discussing the implementation of debt relief programs for the so-called 'Highly Indebted Poor Countries' (HIPC) (see e.g., Berlage et al (2000)). This paper presents circumstances under which it would be better for the lenders as well as the HIP countries to restructure their dollar denominated debt as local currency debt.

Our model takes price levels and exchange rates as exogenous. In a companion paper (1999) we introduce multiple currencies, multiple goods, and multiple agents per country and we derive these exchange rates endogenously from preferences and bank policy.

2 The Real Economy

2.1 The Underlying Economy

Consider a model of an exchange economy with two periods and a single perishable good. There are S possible states $s = 1, ..., S$ in the second period, and the first period state is denoted by $s = 0$. There are two agents, a lender, A (as in America), and a borrower, B (as in Bolivia), with initial endowments $e^h \equiv (e_0^h, \tilde{e}^h) \in \mathbb{R}_{++}^{S+1} = \mathbb{R}_{++} \times \mathbb{R}_{++}^S$ and time-separable, von-Neumann-Morgenstern utility functions

$$U^h(x) = v^h(x_0) + \sum_{s=1}^{S} \gamma_s u^h(x_s)$$

with strictly increasing, concave and C^2 functions v^h, u^h for $h = A, B$. We assume the state probabilities γ_s are identical across agents. The underlying economy is then described by $E = ((U^h, e^h)_{h=A,B})$. If all the v^h and u^h are quadratic, we call it a quadratic economy; if $v^h(x_0) = x_0$ (constant marginal utility) and the u^h quadratic, we call the economy linear-quadratic; and if $v^h(x_0) = x_0$ and the u^h are strictly concave, we call the economy linear-concave.

2.2 Arrow–Debreu Equilibrium

We define an Arrow-Debreu equilibrium as a vector of prices $p \in \mathbb{R}_+^{S+1}$ and consumptions $x^h \equiv (x_0^h, \tilde{x}^h) \in \mathbb{R}_+^{S+1} = \mathbb{R}_+ \times \mathbb{R}_+^S$, $(p, (x^h)_{h=A,B})$, such that

$$x_s^A + x_s^B = e_s \equiv e_s^A + e_s^B \text{ for } s = 0, 1, ..., S$$

and such that for $h = A, B$,

$$x^h \in \arg\max_{x \in \mathbb{R}_+^{S+1}} U^h(x) \text{ s.t. } p \cdot (x - e^h) \leq 0$$

In an Arrow-Debreu equilibrium the borrower who needs money at time 0 can arrange to pay back precisely in those states that are easiest for him to pay, or of most interest to the lender, or both. When markets are incomplete, the borrower has no choice but to pay back in the proportions specified by the available assets.

2.3 Incomplete Markets Economy

Suppose now that agents can only trade via a single asset, paying $d_s \in \mathbb{R}$ units of the good in states $s = 1, ..., S$. Denote agent h's holding of the asset by $\theta^h \in \mathbb{R}$, and consumption by $x^h \in \mathbb{R}_+^{S+1}$. The incomplete markets economy is therefore described by $(E, d) = ((U^h, e^h)_{h=A,B}, d)$. In what follows we shall interpret different kinds of debt by different vectors d. Dollar denominated debt will be taken to be $d^A = \frac{\tilde{e}^A}{\|\tilde{e}^A\|_2}$; peso debt will be understood as $d^B = \frac{\tilde{e}^B}{\|\tilde{e}^B\|_2}$ and real debt will be represented by $\tilde{d} = (1, \ldots, 1)$, where $\|x\|_n = (\sum_{s=1}^S \gamma_s x_s^n)^{1/n}$ for $n \geq 1$.

2.4 Incomplete Markets Equilibrium

We define an incomplete financial market equilibrium (GEI equilibrium) for (E, d) as a vector $(\pi, (x^h, \theta^h)_{h=A,B})$ of the asset price, consumption and portfolio choices for each agent such that

$$\theta^A + \theta^B = 0$$

and for $h = A, B$,

$$\begin{aligned}(x^h, \theta^h) \in \arg\max_{(x,\theta)\in\mathbb{R}_+^{S+1}\times\mathbb{R}} \quad & U^h(x) \text{ s.t.} \\ & x_0 = e_0^h - \pi\theta \\ & x_s = e_s^h + \theta d_s \text{ for } s = 1, ..., S.\end{aligned}$$

Given an Arrow-Debreu equilibrium $(\bar{p}, (\bar{x}^h)_{h=A,B})$ we define A-D dividends $\bar{d} \in \mathbb{R}^S$ by

$$\bar{d}_s = \bar{x}_s^A - e_s^A \text{ for } s = 1, ..., S.$$

It is easy to verify that with these dividends $\bar{d}$ for the single asset, the economy $(E, \bar{d})$ has a GEI-equilibrium $(\pi, (x^h, \theta^h)_{h=A,B})$ with $(x^h = \bar{x}^h)_{h=A,B}$ as the equilibrium allocation and with $\theta^A = -\theta^B = 1$.

If there is a dividend d^0 such that the economy (E, d^0) has a GEI equilibrium $(\pi, (x^h, \theta^h)_{h=A,B})$ with $x^h = e^h$ and $\theta^h = 0$, then d^0 is called autarkic.

2.5 Comparative Statics and Welfare

Observe that as long as $d \neq 0$, the scale of d does not matter. Doubling d and replacing each θ^h with $\frac{1}{2}\theta^h$ and π with 2π gives the same real equilibrium. Thus nothing is lost if we restrict d to the sphere $\mathcal{S}^{S-1} = \{d \in \mathbb{R}^S : \sum_{s=1}^S \gamma_s d_s^2 = 1\}$.

Define the Welfare correspondence $\mathcal{W}_E : \mathcal{S}^{S-1} \to \mathbb{R}^2$ by

$$\mathcal{W}_E(d) = \{(U^A(x^A), U^B(x^B)) : \exists(\pi, \theta) \text{ such that } (\pi, (x^A, \theta^A), (x^B, \theta^B)) \text{ is a GEI equilibrium}\}.$$

Define the interior equilibrium correspondence $\mathcal{W}_E^0(d) = \{(U^A(x^A), U^B(x^B)) : x^A, x^B \gg 0, \exists(\pi, \theta)$ such that $(\pi, (x^A, \theta^A), (x^B, \theta^B))$ is a GEI equilibrium$\}$.

Ultimately we wish to describe the whole correspondence $\mathcal{W}_E$ and compare equilibrium welfare for different specifications of the asset payoff, d. One might imagine that we would get something like the graph in Figure 1.

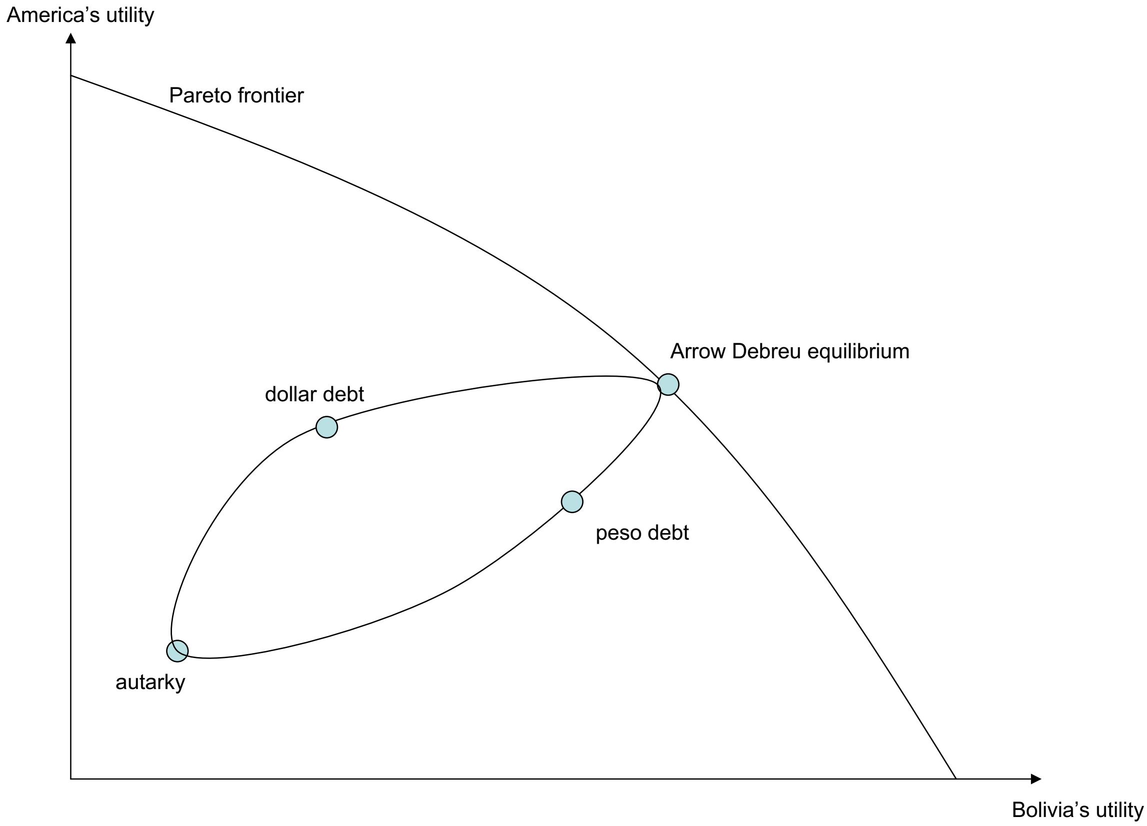

FIGURE 1: A welfare correspondence ?

In the picture, peso debt is better for the borrower and dollar debt is better for the lender. We shall show that in fact (1) in some prominent cases, dollar debt equilibria and peso debt equilibria are Pareto comparable, (2) when they are Pareto comparable peso debt may be Pareto worse than dollar debt and (3) when they are not Pareto comparable, it is usually America who prefers peso debt.

We show that in the quadratic case, peso debt is Pareto better than dollar debt if there are persistent gains to risk pooling between A and B.

3 Dollar Denominated Debt vs. Local Currency Debt

Dollar denominated debt may be dangerous for a developing country because in those states when it can least afford to pay, it may be called upon to pay the most, in terms of its own currency or goods. The idea is that when an economy is performing well, its currency is typically strong. Thus to keep the promise of delivering one U.S. dollar, or its equivalent in pesos, more pesos will have to be found in those states when the developing country has less production. Of course whether more pesos translates into more goods depends on the central bank and the connection between inflation and output.

Our formal model so far has no place for money, or for multiple currencies, or for central banks. In our companion paper (1999) we add a full-fledged monetary sector and multiple currencies to the underlying world economy, and derive the rates of inflation, and the exchange rates endogenously. Here we prefer to keep a simpler reduced form model by taking the connection between price levels and exchange rates on the one hand, and economic fundamentals on the other, as exogenous.

We do so for three reasons. First, we do not wish to be bogged down by the added complexity of a model with multiple currencies, multiple goods, and so on. The connection between asset design and welfare is complicated enough. Second, there is no universally accepted model of money. Our study of dollar denominated debt should be independent of the monetary transmission mechanism between economic activity and price levels. Third, we wish for a theory that does not involve the complications of central bank policy.

3.1 Price Levels Assumption

We have assumed that the real commodity payoff of dollar debt is $d^A = \frac{\tilde{e}^A}{\|\tilde{e}^A\|_2}$, and of peso debt is $d^B = \frac{\tilde{e}^B}{\|\tilde{e}^B\|_2}$. This can be rationalized by assuming that the price level of each currency is inversely proportional to its country's endowment. One can in turn justify such an assumption with a crude quantity theory of money: if the whole endowment of a country is sold against the stock of the local currency (and if the supply of money is held constant by the passive central bank), then indeed price levels will be inversely proportional to endowments.

From the price levels assumption and no-arbitrage, we can deduce that the exchange rate in each state s for the local currency against the dollar is proportional to e_s^B/e_s^A. The currency of each country is then worth the most precisely when its endowment is relatively highest, which accords with intuition.

3.1.1 Indexed Bonds vs. Real Bonds

Our results can also be interpreted without the price level assumption. We are comparing debt indexed to Bolivia's output to debt indexed to America's output. Even without any assumptions on the relation between output and exchange rates, this is an interesting exercise. In many cases it will be impossible for developing countries to issue debt in their local currency. The country's central banks can influence the exchange rates by inflating the money supply and can therefore reduce the real value of local currency debt whenever the debt levels are high. Because of this moral hazard problem Americans may be unwilling to buy Peso-denominated bonds. If the bonds are indexed to actual output, however, this problem disappears and such securities could be actively traded in world markets. Thus in one interpretation of the dividends d^A and d^B, we assume the central banks are completely passive, holding money supply fixed. In another interpretation of the same dividends we assume the central banks are so active that investors choose to index the debt to real output.

In addition to comparing debt indexed to Bolivia's output to debt indexed to America's output, we also compare it to the riskless bond, $\tilde{d} = \tilde{1}$, i.e., to a security that pays a fixed quantity of the international good in each state.

4 Small Borrower and Risk Neutral Lender

In the classic partial equilibrium model of international finance, the borrower is small and the lender is risk neutral. Even in this case peso debt is not necessarily better than dollar debt. Ideally Bolivia would like to pay exclusively when it has the most, not proportionally to how much it has. The claim that dollar debt is worse for Bolivia than peso debt implicitly relies on the assumption that dollar debt is even further from the ideal than peso debt.

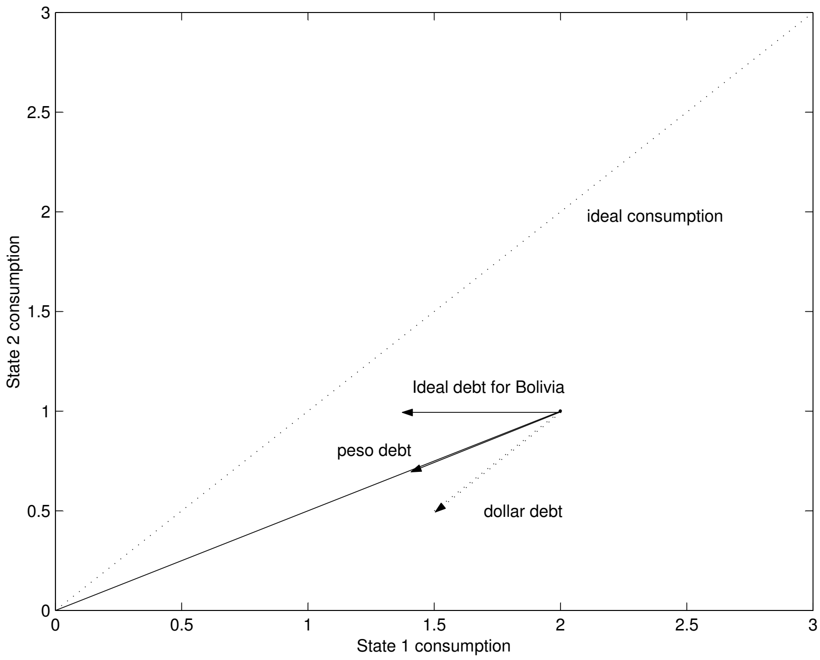

Figure 2: Different debt instruments

Bohn (1990) took it for granted that dollar debt could be taken to be equivalent to riskless debt. As can be seen in Figure 2, that is indeed further from ideal than peso debt. In reality, dollar debt is far from riskless, and so we consider more general dollar payoffs. In Theorem 1 dollar debt is only assumed to be inversely monotonic with Bolivian endowments. Riskless dollar debt is a special case. Bohn also restricted attention to mean-variance utilities for Bolivia, with a large risk-neutral lender. We show more generally that when America is large and risk neutral, and Bolivia has any risk-averse von Neumann-Morgenstern utility, peso debt is ex ante better for Bolivia than dollar debt, provided that dollar debt is inversely monotonic with Bolivian endowments. This partial equilibrium intuition is probably responsible for the presumption that peso debt is better for Bolivia than dollar debt.

Theorem 1 *Suppose America is risk neutral and large, so that $\pi(d) = \delta \sum_{s=1}^{S} \gamma_s d_s$ is the price of the asset in any equilibrium with dividends d, where $\delta > 0$. Suppose that dollar debt is inversely monotonic with Bolivian endowments, $\tilde{e}_s^B > \tilde{e}_t^B \Rightarrow d_s^A \leq d_t^A$ for all $s, t \in S$. Suppose Bolivia has von Neumann-Morgenstern utility $U^B(x) = v^B(x_0) + \sum_{s=1}^{S} \gamma_s u^B(x_s)$ where u^B is strictly concave. If Bolivia is the borrower, then it is better off with peso debt than dollar debt (and better off with peso debt than riskless debt).*

Proof. Suppose in the dollar equilibrium that Bolivia sells θ units of the dollar asset, obtaining final consumption $\hat{x}^B$ in period 1. In the peso economy, Bolivia could have sold the same value of peso debt, guaranteeing it the same consumption in period 0, and consumption $\tilde{x}^B$ in period 1. Then $\hat{x}^B - \tilde{x}^B = (\tilde{e}^B - \lambda\tilde{e}^A) - (\tilde{e}^B - \mu\tilde{e}^B) = \mu\tilde{e}^B - \lambda\tilde{e}^A$, which is strictly co-monotonic with $\hat{x}^B$ and $\tilde{x}^B$. But from the small country pricing hypothesis and the fact that the same value of debt was sold, we know that $E_\gamma(\hat{x}^B - \tilde{x}^B) = 0$. Take any vector y on the line from $\tilde{x}^B$ to $\hat{x}^B$. Then from strict concavity, the vector $u_B'(y)$ is strictly inversely monotonic with $\hat{x}^B - \tilde{x}^B$. But any two vectors that are strictly inversely monotonic must have negative covariance. Hence

$$\sum_{s=1}^{S} \gamma_s u_B'(y_s)(\hat{x}_s^B - \tilde{x}_s^B) < 0.$$

Integrating over all y along the line shows that utility is lower at $\hat{x}^B$ than at $\tilde{x}^B$. But this proves that Bolivia will always do better with Peso debt than dollar debt. ∎

Risk neutrality is an unacceptable hypothesis. American utility will also be affected by the choice of debt. The interest rate on the debt will vary with the quantity of debt issued. America might be more risk averse than Bolivia and Bolivian endowments and American endowments might be positively correlated. Theorem 1 does not allow for any of these possibilities.

5 Persistent gains to risk pooling

This leads us to consider generalizations of Theorem 1. What really is a sound basis for the presumption that peso debt is better for Bolivia than dollar debt? Example 1 shows that it is not enough that peso debt is more convenient for Bolivia to pay than dollar debt. One must also take into account which is more attractive to America. One cannot simply drop the risk neutrality hypothesis from Theorem 1.

A natural conjecture is that if peso debt is *mutually* more attractive than dollar debt solely on the basis of second period preferences, i.e. in terms of their effect on the risk profiles of America and Bolivia, then the peso equilibrium will be better than the dollar equilibrium. Unfortunately, Example 1 also shows that this conjecture is false.

Example 1 *Consider quadratic utilities given by $\gamma_1 = \gamma_2 = 0.5$ and*

$$v^A(x) = u^A(x) = x - \frac{1}{100}x^2 \text{ and } v^B(x) = u^B(x) = x - \frac{1}{1000}x^2$$

Suppose that endowments are given by

$$(e_0^A, e_1^A, e_2^A) = (10, 6, 6) \text{ and } (e_0^B, e_1^B, e_2^B) = (1, 4, 10)$$

Note that dollar debt is riskless, so the initial endowments satisfy the endowment condition of theorem 1. However, both countries are better off at the dollar equilibrium than at the peso equilibrium! The equilibrium utilities with the dollar-bond are given by $(U^A, U^B) = (10.4422, 2.7487)$ while the equilibrium utilities with the peso-bond are given by $(U^A, U^B) = (10.3213, 2.2724)$. ■

For an infinitesimal debt, America values peso debt relative to dollar debt more than Bolivia does, in Example 1. At the initial endowments America is locally risk neutral, and values any infinitesimal trade according to its expected value, while Bolivia is risk averse and willing to sell a promise for less than its expected value whenever this trade reduces the variance of second period consumption, i.e.

$$\begin{aligned}
\frac{MU^A \cdot \tilde{e}^A}{MU^A \cdot \tilde{e}^B} &= \frac{(1 - 6/50, 1 - 6/50) \cdot (6, 6)}{(1 - 6/50, 1 - 6/50) \cdot (4, 10)} = .857 \\
\frac{MU^B \cdot \tilde{e}^A}{MU^B \cdot \tilde{e}^B} &= \frac{(1 - 4/500, 1 - 10/500) \cdot (6, 6)}{(1 - 4/500, 1 - 10/500) \cdot (4, 10)} = .859
\end{aligned}$$

Thus in Example 1 peso debt is mutually more attractive than dollar debt, yet dollar debt is Pareto superior. To understand how this could be, notice that the quantity of debt issued by Bolivia (and therefore held by America) depends primarily on Bolivia's desire to obtain wealth sooner in exchange for wealth later, and much less on Bolivia's preferences about future risk. At the first stages of lending, America is relatively more risk tolerant for exposure to Bolivian GNP fluctuations, but as America takes on more of it (and Bolivia correspondingly less of it), America becomes relatively less tolerant. Yet America continues to take on still more Bolivian debt because the different time preferences of the two countries justifies it. But quite soon America cannot bear any more exposure to the risky Bolivian debt, and the exploitation of intertemporal gains to trade is curtailed. With dollar debt, America can tolerate much more exposure, and so more of the intertemporal gains to trade can be reaped.

In generalizing Theorem 1, we have in mind country A as a large country and B as a smaller developing country. B is borrowing from A because it is relatively poor at present. We wish to assume that in addition to this intertemporal source of gains to trade, there is also a risk sharing source of gains to trade, that *persists* even after A absorbs the small debt of country B. Risk neutrality guaranteed that persistence, but it can be obtained from weaker hypotheses.

Inspection of the proof of Theorem 1 reveals a simple idea: no matter how much peso debt Bolivia has given up, there is an advantageous trade in which Bolivia gives up more peso debt in exchange for dollar debt from America. Since this does not involve any trade-off of consumption across time, we associate it with risk pooling, and since it must remain true even after Bolivia gives up some debt, we call it a persistent gains to risk pooling assumption:

(Persistent Gains to Risk Pooling). At any second period allocation $(\tilde{x}^A, \tilde{x}^B) = (\tilde{e}^A + \alpha d^B, \tilde{e}^B - \alpha d^B) \gg 0$, $\alpha \geq 0$, there exist $\gamma_1, \gamma_2 > 0$ such that

$$\sum_{s=1}^{S} \gamma_s u_A(x_s^A + \gamma_1 d_s^B - \gamma_2 d_s^A) \geq \sum_{s=1}^{S} \gamma_s u_A(x_s^A)$$

and

$$\sum_{s=1}^{S} \gamma_s u_B(x_s^B - \gamma_1 d_s^B + \gamma_2 d_s^A) \geq \sum_{s=1}^{S} \gamma_s u_B(x_s^B).$$

The persistent gains to risk pooling hypothesis implies that both countries would gain if A gave up a fraction of last period endowment in exchange for an appropriate fraction of B's last period endowment. Moreover, this risk sharing opportunity persists even after B transfers some of its endowment to A.

One way to ensure the persistent gains to risk pooling hypothesis, without making any assumptions on utility, is to suppose

(Endowment Generated Persistent Gains to Risk Pooling). $\tilde{e}^A$ and $\tilde{e}^B$ are inversely monotonically related, and aggregate endowment $\tilde{e}^A + \tilde{e}^B$ is inversely monotonically related to $\tilde{e}^B$.

This also is consistent with the idea that America is much bigger than Bolivia. The following lemma shows the connection between these two assumptions.

Lemma 1 *Endowment Generated Persistent Gains to Risk Pooling implies Persistent Gains to Risk Pooling.*

Proof.
The covariance of two random variables, one of which has zero mean is given by their expected product. Hence

$$\sum_{s=1}^{S} \gamma_s u_A'(x_s^A)\left(\frac{e_s^A}{\|\tilde{e}^A\|_1} - \frac{e_s^B}{\|\tilde{e}^B\|_1}\right) = \mathrm{Cov}_\gamma \left(u_A'(\tilde{x}^A), \left(\frac{\tilde{e}^A}{\|\tilde{e}^A\|_1} - \frac{\tilde{e}^B}{\|\tilde{e}^B\|_1}\right) \right) < 0$$

whenever $\tilde{x}^A$ is co-monotonic with $\tilde{e}^A$ (and inversely monotonic with $\tilde{e}^B$). Similarly,

$$\sum_{s=1}^{S} \gamma_s u_B'(x_s^B)\left(\frac{e_s^A}{\|\tilde{e}^A\|_1} - \frac{e_s^B}{\|\tilde{e}^B\|_1}\right) = \mathrm{Cov}_\gamma \left(u_B'(\tilde{x}^B), \left(\frac{\tilde{e}^A}{\|\tilde{e}^A\|_1} - \frac{\tilde{e}^B}{\|\tilde{e}^B\|_1}\right) \right) > 0$$

whenever $\tilde{x}^B$ is co-monotonic with $\tilde{e}^B$ (and inversely monotonic with $\tilde{e}^A$). Therefore

$$\frac{\sum_{s=1}^S \gamma_s u_A'(x_s^A) e_s^A / \|\tilde{e}^A\|_1}{\sum_{s=1}^S \gamma_s u_A'(x_s^A) e_s^B / \|\tilde{e}^B\|_1} < 1 < \frac{\sum_{s=1}^S \gamma_s u_B'(x_s^B) e_s^A / \|\tilde{e}^A\|_1}{\sum_{s=1}^S \gamma_s u_B'(x_s^B) e_s^B / \|\tilde{e}^B\|_1},$$

which proves the lemma. ■

The persistent gains to risk pooling hypothesis covers other cases as well.

Theorem 2 *Assume that Persistent Gains to Risk Pooling holds. Let* $(\pi(d^A), (x^h(d^A), \theta^h(d^A))_{h=A,B})$ *be an equilibrium for the dollar debt economy* (E, d_A)*, and let* $(\pi(d^B), (x^h(d^B), \theta^h(d^B))_{h=A,B})$ *be an equilibrium for the peso economy* (E, d_B)*. Then for at least one agent* $h \in \{A, B\}$*,* $U^h(x^h(d^B)) \geq U^h(x^h(d^A))$*.*

In fact, let $d_\lambda = \frac{\lambda\tilde{e}^A + (1-\lambda)\tilde{e}^B}{\|\lambda\tilde{e}^A + (1-\lambda)\tilde{e}^B\|_2}$*, and let* $(\pi(\lambda), (x^h(\lambda), \theta^h(\lambda))_{h=A,B})$ *be the corresponding equilibrium. Then if* $\lambda_1 > \lambda_2 \geq 0$*, for some* $h \in \{A, B\}$*,* $U^h(x^h(\lambda_2)) \geq U^h(x^h(\lambda_1))$*.*

The proof is deferred to the appendix. The proof proceeds by showing that with persistent gains to risk pooling, any allocation achievable via dollar debt can be Pareto-dominated by an allocation achievable via peso debt, without even changing first period consumption. Theorem 2 then follows from the constrained efficiency of equilibrium with one consumption good.

By assuming risk-neutrality for the lender, Theorem 1 focused attention on the borrower, since the lender will necessarily be indifferent to the currency of the debt. Theorem 2, on the other hand, treats the lender and borrower symmetrically, guaranteeing that at least one of them is better off with peso debt, but making no presumption that it is Bolivia that benefits from peso debt. In fact, Theorem 1 is not so asymmetric; it really concludes that peso debt Pareto dominates dollar debt. In generalizing Theorem 1 it will be important to see whether Bolivia gains from peso debt because everyone gains, or whether peso debt enables Bolivia to gain at the expense of America.

To answer this question we must make assumptions about utilities. We shall see that when utilities are quadratic, persistent gains to risk pooling is enough to guarantee that peso debt is better for Bolivia; but only because peso debt is also better for America. For linear-concave utilities, we will show that it is America, not Bolivia, that always prefers Bolivia to issue peso debt (when there are endowment generated persistent gains to risk sharing). Bolivia might be worse off with peso debt. In the following example, endowment generated persistent gains to risk pooling holds, yet America gains from peso debt and Bolivia loses.

Example 2 *There are two states, identical probabilities* $\gamma_1 = \gamma_2 = 0.5$*. Endowments are given by* $e^B(0) = 1, e^B(1) = 4, e^B(2) = 9$ *and* $e^A(0) = 1, e^A(1) = 8, e^A(2) = 3$ *. We assume that both agents have linear-concave utility functions with second period utility given by*

$$u_A(x) = 2\frac{x^{-2}}{-2}, \quad u_B(x) = \frac{x^{-4}}{-4}$$

Utility levels in the dollar equilibrium are $(U^A, U^B) = (0.9420, 1.0103)$ while in the peso equilibrium they are $(U^A, U^B) = (0.9701, 1.0091)$. The price of the bond is 0.426 in the dollar equilibrium but only 0.219 in the peso equilibrium; the expected return in the dollar equilibrium is equal to 2.2 while it the peso equilibrium it rises to 4.3. ■

An explanation for example 2 is given in Section 7.

6 Quadratic utility

6.1 The welfare correspondence

Suppose now that both agents have quadratic utility functions of the form

$$\begin{aligned} U^h(x) &= v_h(x_0) + \sum_{s=1}^{S} \gamma_s u_h(x_s) \\ v_h(x) &= \alpha^h x - \frac{1}{2}\beta^h x^2, \ h = A, B \\ u_h(x) &= a^h x - \frac{1}{2}b^h x^2, \ h = A, B \\ \frac{\beta^A}{b^A} &= \frac{\beta^B}{b^B} = k \end{aligned}$$

where the constants $\alpha^h > 0$, $a^h > 0$ and constants $b^h > 0$ and $\beta^h \geq 0$ are assumed to guarantee that utility is increasing in the relevant range. The last condition holds whenever agents have time independent utility $v_h = u_h$, or time dependent utility with the same discount factor, $u^h = \delta v^h$ or linear quadratic utility $\beta^h = 0$.

The impatience of an agent, h, at a vector x is defined by $\frac{v_h'(x_0)}{\sum_{s=1}^{S} \gamma_s u_h'(x_s)}$. It will be increased by multiplying a^h and b^h by some constant $\delta < 1$. The Arrow-Pratt risk aversion of an agent at the point $\bar{x}$,

$$-\frac{u_h''(\bar{x})}{u_h'(\bar{x})} = \frac{b^h}{a^h - b^h \bar{x}}$$

can be increased by raising b^h by $\epsilon > 0$ and a^h by $\epsilon \bar{x}$, without affecting marginal utility at $\bar{x}$.

The next theorem shows that the common presumption that peso debt benefits Bolivia and dollar debt benefits America is wrong, at least when utilities are quadratic.

Theorem 3 *With quadratic utilities, there is (at most) one interior equilibrium corresponding to each economy (E, d). Furthermore, the interior equilibria arising from all economies $\{(E, d) : d \in \mathcal{S}^{S-1}\}$ are Pareto comparable. In fact, the welfare graph, Graph$(\mathcal{W}_E^0)$ lies on a straight line.*

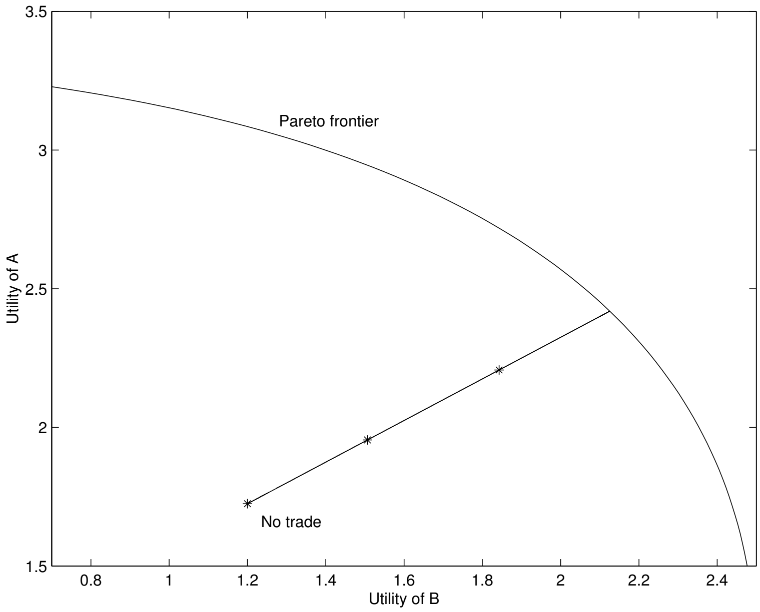

FIGURE 3: A welfare graph for quadratic utility

Proof. Define a pre-equilibrium for (E, d) as as a tuple $(\pi, (x^h, \theta^h)_{h=A,B})$ for which the first order conditions for utility maximization hold, and demand equals supply (but for which we might have x^h negative). Any interior equilibrium must be a pre-equilibrium. In a pre-equilibrium,

$$(\alpha^h - \beta^h x_0^h)\pi = (a^h \tilde{1} - b^h \tilde{x}^h) \cdot_\gamma d \equiv \sum_{s=1}^{S} \gamma_s (a^h - b^h x_s^h) d_s.$$

Dividing both sides by $b^h > 0$,

$$(\frac{\alpha^h}{b^h} - \frac{\beta^h}{b^h} x_0^h)\pi = (\frac{a^h}{b^h} \tilde{1} - \tilde{x}^h) \cdot_\gamma d.$$

Hence adding over the countries and using $k = \beta^h / b^h$ for all h gives

$$\begin{aligned}
\sum_h (\frac{\alpha^h}{b^h} - \frac{\beta^h}{b^h} x_0^h)\pi &= \sum_h (\frac{a^h}{b^h} \tilde{1} - \tilde{x}^h) \cdot_\gamma d \\
(\alpha - k e_0)\pi &= (a\tilde{1} - \tilde{e}) \cdot_\gamma d
\end{aligned}$$

where

$$\begin{aligned}
\alpha &\equiv \sum_h \frac{\alpha^h}{b^h} \\
a &\equiv \sum_h \frac{a^h}{b^h} \\
e_0 &\equiv \sum_h e_0^h = \sum_h x_0^h \\
\tilde{e} &\equiv \sum_h \tilde{e}^h = \sum_h \tilde{x}^h
\end{aligned}$$

where the last two equalities hold because of market clearing. It follows that

$$\pi = [(a\tilde{1} - \tilde{e}) \cdot_\gamma d]/(\alpha - ke_0) = \omega \cdot_\gamma d$$

where $\omega \equiv (a\tilde{1} - \tilde{e})/(\alpha - ke_0)$ does not depend on d. Thus pre-equilibrium price π is a linear function of d. In particular there is at most a single pre-equilibrium, and hence a single interior equilibrium.

The effect on price π of a change in dividends $d \to d + \Delta$ is easy to compute from the above formula: $D_\Delta \pi = \omega \cdot_\gamma \Delta$. From the envelope theorem, the effect on utility is also easy to compute. It is the same as the effect on utility if the agent did not change his portfolio θ^h, and simply paid the extra price for buying the same portfolio and then consumed the extra dividends.

$$\begin{aligned}
D_\Delta U^h &= \{-(\alpha^h - \beta^h x_0^h)[D_\Delta \pi] + (a^h\tilde{1} - b^h\tilde{x}^h) \cdot_\gamma \Delta\}\theta^h \\
&= \theta^h\{-(\alpha^h - \beta^h x_0^h)\omega + (a^h\tilde{1} - b^h\tilde{x}^h)\} \cdot_\gamma \Delta
\end{aligned}$$

Dividing by $b^h\theta^h$ and adding over all h gives

$$\begin{aligned}
\sum_h \frac{1}{b^h\theta^h} D_\Delta U^h &= \sum_h \{-(\frac{\alpha^h}{b^h} - \frac{\beta^h}{b^h} x_0^h)\omega + (\frac{a^h}{b^h}\tilde{1} - \tilde{x}^h)\} \cdot_\gamma \Delta \\
&= \{-(\alpha - ke_0)\omega + (a\tilde{1} - \tilde{e})\} \cdot_\gamma \Delta \\
&= 0 \cdot_\gamma \Delta = 0.
\end{aligned}$$

Finally, let us note that since there are only two agents, $\theta^A = -\theta^B$, hence we conclude that

$$\frac{1}{b^A} D_\Delta U^A = \frac{1}{b^B} D_\Delta U^B$$

for all dividends d and all perturbations Δ. We conclude that in pre-equilibrium, utilities vary along a straight line in utility space as d varies. A fortiori, the same holds for interior equilibria. ■

Figure 3 shows a typical welfare graph for quadratic utility. The equilibrium utilities $(\bar{U}^A, \bar{U}^B)$, achieved with Arrow-Debreu dividends $\bar{d}$ Pareto dominate all other equilibria. The autarky equilibrium utilities, (U_0^A, U_0^B), achieved with d^0 are Pareto-dominated by all other equilibria.

The theorem shows that

$$\frac{\bar{U}^A - U_0^A}{b^A} = \frac{\bar{U}^B - U_0^B}{b^B}.$$

However it does not say anything about where on the line $\mathcal{W}(d^A)$ lies relative to $\mathcal{W}(d^B)$.

Combining theorems 2 and 3 gives a sufficient condition under which both America and Bolivia will be better off with peso debt.

Corollary 1 *If utility is quadratic and persistent gains to risk pooling holds, then peso debt Pareto dominates dollar debt, if both equilibria are interior.*

6.2 Linear quadratic utility

The presumption has been that because dollar debt exacerbates crises, it must be better to borrow through peso debt (in the absence of central bank uncertainty). But from an ex ante perspective, this ignores the preferences of lenders. It is imaginable, for example, that Bolivia has a riskier GNP than America, but that Bolivians are more tolerant to additional risk than Americans. Paying in pesos might be better for Bolivians but so much worse for Americans that equilibrium interest rates would need to be so high that even Bolivia is worse off with peso debt. Indeed we will show that with linear quadratic utilities, if America has riskless endowments, then peso debt is Pareto-worse when America is sufficiently risk averse and Bolivia is sufficiently impatient.

For the linear quadratic case we can completely characterize the welfare rankings of dividends $d \in \mathcal{S}^{S-1}$. This therefore gives us sufficient conditions for the Pareto superiority of dollar debt.

Assume now that agents' utilities are given by

$$\begin{aligned} U^h(x) &= x_0 + \sum_{s=1}^{S} \gamma_s [a^h x_s - \frac{1}{2} b^h x_s^2] \\ &= x_0 + \sum_{s=1}^{S} \gamma_s u^h(x_s) \end{aligned}$$

Theorem 4 *Suppose lenders and borrowers have linear-quadratic utilities. Given two dividends $d^1, d^2 \in \mathcal{S}^{S-1}$, suppose that A is the lender ($\theta^A > 0$) in the equilibria for (E, d^1) and (E, d^2), and that both equilibria are interior. Then the equilibrium allocation with d^2 will Pareto-dominate the allocation with d^1, if and only if*

$$\sum_{s=1}^{S} \gamma_s (u_A'(e_s^A) - u_B'(e_s^B))(d_s^2 - d_s^1) > 0$$

Proof.

The utility of an asset purchase of θ^h is

$$V^h(\theta^h) = e_0^h - \pi\theta^h + \sum_{s=1}^{S} \gamma_s [a^h(e_s^h + \theta^h d_s) - \frac{1}{2} b^h (e_s^h + \theta^h d_s)^2]$$

It follows that

$$
\begin{aligned}
&V^A(\theta^A) + V^B(-\theta^A) = \\
&(e_0^A + \sum_s \gamma_s a^A e_s^A - \tfrac{1}{2}\sum_s \gamma_s b^A (e_s^A)^2) + (\theta^A \sum_s a^A \gamma_s d_s - \theta^A \sum_s \gamma_s b^A e_s^A d_s) - \tfrac{1}{2}(\theta^A)^2 \sum_s \gamma_s d_s^2 b^A + \\
&(e_0^B + \sum_s \gamma_s a^B e_s^B - \tfrac{1}{2}\sum_s \gamma_s b^B (e_s^B)^2) - (\theta^A \sum_s a^B \gamma_s d_s - \theta^A \sum_s \gamma_s b^B e_s^B d_s) - \tfrac{1}{2}(\theta^A)^2 \sum_s \gamma_s d_s^2 b^B
\end{aligned}
$$

Using $\sum_{s=1}^{S} \gamma_s d_s^2 = 1$, we obtain

$$
\begin{aligned}
&V^A(\theta^A) + V^B(-\theta^A) = \\
&U^A(e^A) + \theta^A u_A'(\tilde{e}^A) \cdot_\gamma d - \tfrac{1}{2}(\theta^A)^2 b^A + U^B(e^B) - \theta^A u_B'(\tilde{e}^B) \cdot_\gamma d - \tfrac{1}{2}(\theta^A)^2 b^B = \\
&U^A(e^A) + U^B(e^B) + \theta^A(u_A'(\tilde{e}^A) - u_B'(\tilde{e}^B)) \cdot_\gamma d - \tfrac{1}{2}(\theta^A)^2(b^A + b^B)
\end{aligned}
$$

We know that with linear quadratic utilities (pre) equilibrium will maximize the sum of utilities. Differentiating the last expression for the sum of utilities, and solving for θ^A to make the expression zero gives equilibrium

$$\theta^A = \frac{(u_A'(\tilde{e}^A) - u_B'(\tilde{e}^B)) \cdot_\gamma d}{b_A + b_B}$$

Plugging equilibrium θ^A into the last expression gives

$$V^A(\theta^A) + V^B(-\theta^A) = U^A(e^A) + U^B(e^B) + \frac{1}{2}\frac{[(u_A'(\tilde{e}^A) - u_B'(\tilde{e}^B)) \cdot_\gamma d]^2}{b^A + b^B}.$$

A change of dividends which increases the sum of both agents' utilities must make at least one agent better off. But Theorem 3 implies that then both agents must be better off. ■

Theorem 4 is remarkable since it shows that with linear-quadratic utilities, one can rank different assets just by the knowledge of marginal utility at individual endowments, knowing only that equilibrium consumptions are interior. Peso debt is Pareto better than dollar debt if and only if, starting from their initial endowments, there is an incentive for an infinitesimal trade in which America buys $(d^B - d^A)$ from Bolivia. Another proof of Theorem 4 is given in Section 7.

We call the situation in which

$$\sum_{s=1}^{S} \gamma_s (u_A'(e_s^A) - u_B'(e_s^B))(d_s^B - d_s^A) > 0$$

the **Peso Surplus Condition**. Theorem 4 proves that with linear quadratic utility, peso debt Pareto dominates dollar debt if and only if the peso surplus condition holds.

The condition is guaranteed by persistent gains to risk pooling and interiority of equilibrium. It is not implied by gains to risk pooling at the endowment point alone.

Theorem 4 does not hold if first period utility is also quadratic. This is illustrated in Example 1.

6.3 When dollar debt is riskless

An interesting special case of Theorem 4 occurs when American endowments are riskless, $\tilde{e}^A = k\tilde{1}$ for some $k > 0$, and when $\tilde{e}^B$ is risky. Since $\|x\|_2 > \|x\|_1$, whenever x is not constant, it follows that

$$\sum_{s=1}^{S} \gamma_s u'_A(e_s^A)(d_s^A - d_S^B) = u'_a(k) \sum_{s=1}^{S} \gamma_s (1 - \frac{e_s^B}{\|\tilde{e}^B\|_2}) > 0$$

Hence if Bolivia is sufficiently impatient (which is achieved by multiplying u'_B by $\delta < 1$) then we necessarily get that

$$\sum_{s=1}^{S} \gamma_s [u'_A(e_s^A) - u'_B(e_s^B)](d_s^A - d_s^B) > 0.$$

By Theorem 4, this implies that dollar debt is Pareto better than peso debt, provided that both equilibria are interior. Taking America close to risk neutral, we seem to get a contradiction to Theorem 1, which shows that peso debt is necessarily better for Bolivia. The paradox is resolved by noticing that equilibrium will not be interior. Interiority can be guaranteed by ensuring that the trade θ^A is sufficiently small, which in turn is guaranteed if b^A is very big.

Corollary 2 *Suppose*

$$u^h(x) = x_0 + \sum_{s=1}^{S} \gamma_s [a^h x_s - \frac{1}{2} b^h x_s^2], \quad h = A, B$$

If American endowments are riskless (so that $e_s^A = k$ for all $s = 1, ..., S$) and Bolivia's endowments are risky, then dollar debt Pareto dominates peso debt if Bolivian impatience and American risk aversion are sufficiently high. More precisely, if we replace (a^B, b^B) by $(\delta a^B, \delta b^B)$ and (a^A, b^A) by $(a^A + k\epsilon, b^A + \epsilon)$, then there is $\bar{\delta} > 0$ and $\bar{\epsilon} > 0$ such that for all $\delta \leq \bar{\delta}$ and $\epsilon \geq \bar{\epsilon}$, both equilibria are interior and dollar debt Pareto dominates peso debt.

This corollary captures the general equilibrium intuition discussed in the introduction. Bolivia, stimulated by the attractive form of peso debt, will be led to borrow more and more, driving up the interest rate, and making itself worse off than it would have been with dollar debt.

7 Who Does Peso Debt Really Help ? The Linear-Concave Case

When utilities are quadratic, there is agreement between A and B about which currency to use as debt. When utilities are not quadratic, disagreement might arise. In that case, who stands to gain by switching from dollar debt to peso debt ? As Example 2 suggests, it is paradoxically America, not Bolivia, that gains from peso debt if persistent gains to risk pooling holds.

We shall show that in the linear-concave case, whenever $u'''_h \geq 0$ for both $h = A$ and $h = B$, America must gain from the switch from dollar debt to peso debt, while Bolivia might gain or lose, provided that there are endowment generated persistent gains to risk sharing. The reason is that the price of debt must move unfavorably for Bolivia, and if this effect is big enough, it can dwarf the gain Bolivia receives from being able to pay in pesos. America benefits both from receiving peso debt, and from the favorable price move. This conclusion is confirmed by Example 2 (where aggregate supply is constant across states, so there are endowment generated persistent gains to risk pooling).

To prove this result, we introduce our argument by reexamining the linear-quadratic case from a different point of view.

7.1 The Linear-Quadratic Case

Assume that agents' first period utility is linear in consumption, i.e., $U^h(x) = x_0 + \sum_{s=1}^{S} \gamma_s u_h(x_s)$. In this case, we can give our problem an easy intuitive interpretation.

The agents' marginal utilities for the asset can be associated with their demand and supply curves for the asset. The infinitesimal surplus from trading an infinitesimal amount of the asset with dividend d is

$$\text{infinitesimal surplus} = \sum_{s=1}^{S} \gamma_s [u'_A(e_s^A) - u'_B(e_s^B)] d_s.$$

Furthermore, the utility of final consumption for the borrower is equal to the producer surplus (plus a constant equal to the utility of the initial endowment allocation), and the utility of final consumption for the lender is given by the consumer surplus (plus a constant equal to the utility of the initial endowment allocation). We can easily examine how the producer and consumer-surplus changes as the asset structure changes. As the dividend-promise becomes less onerous for the borrower, the supply curve shifts outward as indicated in Figure 4. The demand curve of the lender may shift in or out, depending on whether the new asset dividend is more or less attractive to him.

The peso surplus condition implies that the move from dollar debt to peso debt spreads the demand and supply curves further apart at the initial endowments. But the equilibrium price also moves, and the change could be so great that the borrower loses in the end, even if total surplus increases. We shall see below that when utilities are linear quadratic, the marginal utilities are linear and so supply and demand curves are linear. By normalizing dividends d so that $\|d\|_2 = 1$, we guarantee that the slope of the supply curve is equal to the quadratic coefficient b^B, irrespective of the dividend (and similarly for the demand curve). Thus changing from dollar debt to peso debt causes parallel shifts in the supply and demand curves, as illustrated in Figure 4. If the shifts in demand and supply are both parallel, then from elementary economics we know that total surplus must increase whenever infinitesimal surplus increases. Furthermore, both parties must gain whenever total surplus increases, confirming Theorem 4. We make this intuition rigorous below.

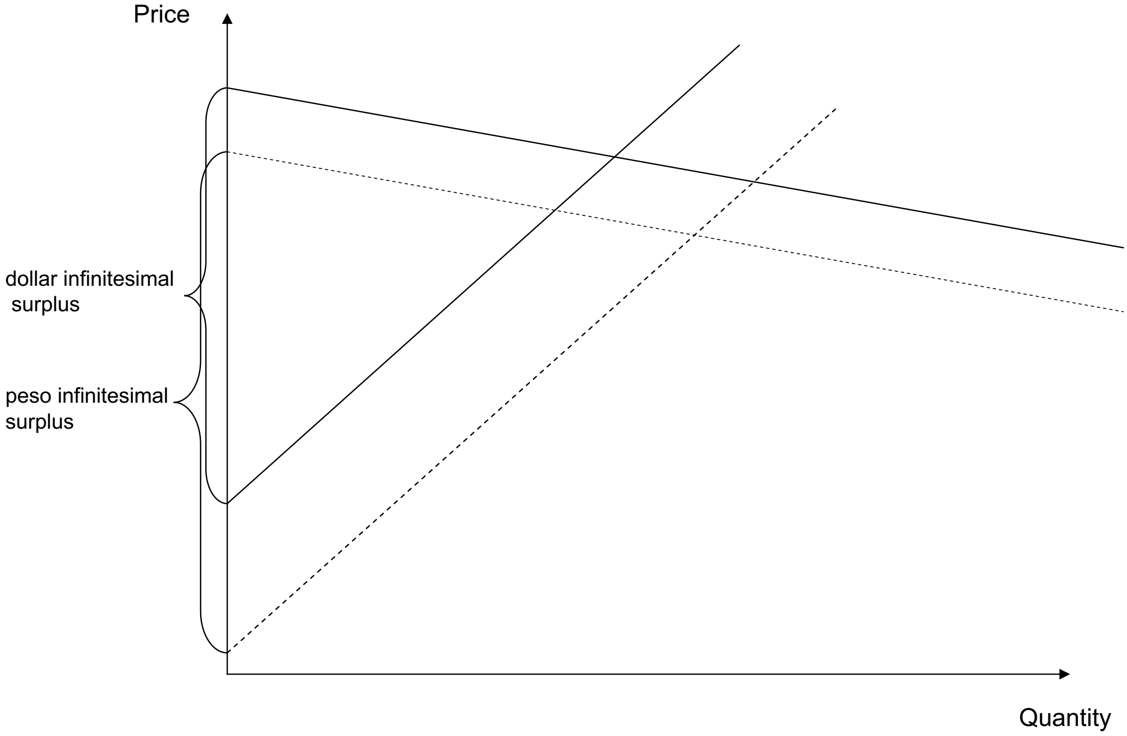

FIGURE 4: Supply and demand in the linear-quadratic case

7.1.1 Another proof of Theorem 4:

The utility of an asset purchase of θ^h is

$$e_0^h - \pi\theta^h + \sum_{s=1}^{S} \gamma_s [a^h(e_s^h + \theta^h d_s) - \frac{1}{2} b^h (e_s^h + \theta^h d_s)^2]$$

The marginal utility at time 2 of one more unit of the asset is then

$$MU^h = \sum_{s=1}^{S} \gamma_s [a^h d_s - b^h (e_s^h + \theta^h d_s) d_s]$$

Hence

$$\frac{dMU^h}{d\theta} = -(sign\theta^h) \sum_{s=1}^{S} \gamma_s b^h d_s^2 = -(sign\theta^h) b^h$$

where we have used the normalization $\sum_{s=1}^{S} \gamma_s d_s^2 = 1$. Supply and demand are indeed linear.

The change in marginal utility from an infinitesimal change in the direction Δ of the asset dividend d is given by

$$\begin{aligned} D_\Delta MU^h &= \sum_{s=1}^{S} \gamma_s[a^h\Delta_s - b^h(e_s^h + \theta^h d_s)\Delta_s - b^h(\theta^h\Delta_s)d_s] \\ &= \sum_{s=1}^{S} \gamma_s[a^h - b^h e_s^h]\Delta_s \\ &= \sum_{s=1}^{S} \gamma_s u_h^{'}(e_s^h)\Delta_s \end{aligned}$$

Again we have used the fact that $\|d\|_2$ is maintained constant, so $0 = D_\Delta \sum_{s=1}^{S} \gamma_s d_s^2 = 2\sum_{s=1}^{S} \gamma_s d_s \Delta_s$. Notice that the change in marginal utility does not depend on θ, hence a change in assets creates a parallel move in the demand and supply curves for the asset. Furthermore, by integrating infinitesimal changes we conclude that a discrete change from d^A to d^B causes a parallel shift of

$$\Sigma^h \equiv \sum_{s=1}^{S} \gamma_s u_h^{'}(e_s^h)(d_s^B - d_s^A) \text{ for each } h = A, B.$$

Equilibrium trade then changes by $(\Sigma^A - \Sigma^B)/(b^A + b^B)$, and so total surplus changes by $\frac{1}{2}(\Sigma^A - \Sigma^B)^2/(b^A + b^B)$. Thus total welfare increases if and only if the demand and supply curve move further apart, that is if

$$\Sigma \equiv \Sigma^A - \Sigma^B = \sum_{s=1}^{S} \gamma_s[u_A^{'}(e_s^A) - u_B^{'}(e_s^B)](d_s^B - d_s^A) > 0.$$

It follows from elementary economics (or from Theorem 3) that if total welfare increases each agent's welfare must increase as well. ■

7.2 Why America Gains and Bolivia Might Lose Switching from Dollar Debt to Peso Debt: The Linear-Concave Case

Let us return to the more general case of linear-concave utilities, dropping our restriction that u^h is quadratic. Supply and demand curves are typically not linear and the shifts of demand and supply are typically not parallel. They depend on the third derivative of the utility functions.

Let us impose the relatively weak assumption that $u_h^{'''} \geq 0$, which is satisfied by most of the popular von Neumann–Morgenstern utilities. This allows us to derive the following theorem.

Theorem 5 *Suppose that utilities are linear-concave with $u_h^{'''} \geq 0$, for both A and B, and suppose endowments satisfy the endowment generated persistent gains to risk pooling hypothesis. Suppose also that for all $d = \frac{\alpha d^A + (1-\alpha)d^B}{\|\alpha d^A + (1-\alpha)d^B\|_2}$, $0 \leq \alpha \leq 1$, the resulting equilibrium is interior. Then a switch from d^A to d^B necessarily makes America better off.*

To prove the theorem, observe that an infinitesimal change in dividends has two effects on welfare: the direct effect of paying in a different asset, holding previous trade levels fixed, and the indirect effect induced by a change in the equilibrium price. The change in the equilibrium price can be obtained by linearizing marginal utility (the supply and demand curves) around the equilibrium and then moving them in parallel according to how much marginal utility is affected by the dividend change at the original equilibrium trade levels. A change, therefore, that depresses marginal utility for both agents, without directly affecting utility, will depress the price and help America and hurt Bolivia, since America is the buyer and Bolivia the seller.

Suppose that at the original equilibrium consumption, the marginal utility of peso debt were higher to America than the marginal utility of dollar debt, and the reverse were true in Bolivia. Then an infinitesimal change from dollar debt towards peso debt would help both countries directly. In addition, the demand and supply curves would tend to move further apart, since holding previous consumption fixed, the marginal utility of the new asset is higher to America and lower for Bolivia.

But marginal utility also moves because previous consumption is now changed at the original trade levels (more peso consumption for America and less for Bolivia). We shall show that this last change lowers marginal utility for both countries, without affecting utility directly, provided that $u_h''' \geq 0$. Thus the price is depressed and Bolivia might lose.

More formally, the marginal utility to Bolivia from not selling another promise of d (after having already sold θ promises) is:

$$MU^B(\theta) = \sum_{s=1}^{S} \gamma_s u_B'(e_s^B - \theta d_s) d_s$$

This also describes Bolivia's supply curve for promises. The slope of the supply curve is given by

$$\sigma^B(\theta) = -\sum_{s=1}^{S} \gamma_s u_B''(e_s^B - \theta d_s) d_s^2$$

Similarly, the American demand curve is

$$MU^A(\theta) = \sum_{s=1}^{S} \gamma_s u_A'(e_s^A + \theta d_s) d_s$$

and the absolute value of its slope is

$$\sigma^A(\theta) = -\sum_{s=1}^{S} \gamma_s u_A''(e_s^A + \theta d_s) d_s^2$$

The change in marginal utilities from an infinitesimal change Δ in the asset payouts is

$$\begin{aligned}
\Sigma^B &= D_\Delta MU^B(\theta) = -\theta \sum_{s=1}^{S} \gamma_s u_B''(e_s^B - \theta d_s) d_s \Delta_s + \sum_{s=1}^{S} \gamma_s u_B'(e_s^B - \theta d_s) \Delta_s \\
\Sigma^A &= D_\Delta MU^A(\theta) = \theta \sum_{s=1}^{S} \gamma_s u_A''(e_s^A + \theta d_s) d_s \Delta_s + \sum_{s=1}^{S} \gamma_s u_A'(e_s^A + \theta d_s) \Delta_s
\end{aligned}$$

The following lemma allows us to sign the terms involving second derivatives.

Lemma 2 *Suppose the endowment generated gains to risk pooling hypothesis holds, and that*

$$0 \ll \theta d = \lambda\tilde{e}^A + \mu\tilde{e}^B + \nu\tilde{1} \ll \tilde{e}^B, \quad \lambda, \mu, \nu \geq 0.$$

Suppose that

$$\Delta = \beta\tilde{e}^B - \alpha\tilde{e}^A - \gamma\tilde{1}, \quad \alpha, \beta, \gamma \geq 0$$

and suppose that $\sum_{s=1}^{S} \gamma_s d_s \Delta_s = 0$. *Then*

$$-\theta \sum_{s=1}^{S} \gamma_s u_B''(e_s^B - \theta d_s) d_s \Delta_s \leq 0$$

and

$$\theta \sum_{s=1}^{S} \gamma_s u_A''(e_s^A + \theta d_s) d_s \Delta_s \leq 0.$$

Proof of the lemma.

Observe that the random variable Δ_s has expectation zero with respect to the positive measure $\gamma_s d_s$. Since $u_A''' \geq 0$, the random variable $u_A''(e_s^A + \theta d_s)$ is monotonic in $e_s^A + \theta d_s$. From the endowment generated gains to risk pooling hypothesis, and the assumption that $0 \ll \theta d = \lambda\tilde{e}^A + \mu\tilde{e}^B + \nu\tilde{1} \ll \tilde{e}^B, \quad \lambda, \mu, \nu \geq 0$, we know that $e_s^A + \theta d_s$ varies co-monotonically with e_s^A and hence inversely with Δ_s. The covariance of any two inversely monotonic random variables, with respect to any positive measure, must be negative. Hence $\theta \sum_{s=1}^{S} \gamma_s u_A''(e_s^A + \theta d_s) d_s \Delta_s \leq 0$. The proof for B is analogous (the sign is reversed because $\tilde{e}^B - \theta d$ is co-monotonic with Δ but then reversed again because of the $-\theta$). ∎

Proof of the theorem.

From elementary economics, we know the change in price is

$$D_\Delta \pi = \frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B}.$$

By the envelope theorem, the change in utility coming entirely from the difference in dividend payoffs (ignoring the price effect) is

$$\begin{aligned} \bar{D}_\Delta U^B(\theta) &= -\theta \sum_{s=1}^{S} \gamma_s u_B'(e_s^B - \theta d_s)\Delta_s \\ \bar{D}_\Delta U^A(\theta) &= \theta \sum_{s=1}^{S} \gamma_s u_A'(e_s^A + \theta d_s)\Delta_s \end{aligned}$$

By the envelope theorem the total change in utility is just the sum of these two effects,

$$D_\Delta U^h(\theta) = \theta^h D_\Delta \pi + \bar{D}_\Delta U^h(\theta), \quad h = A, B.$$

Thus the terms in Lemma 2 affect marginal utility without affecting utility.

Letting $\Sigma = \Sigma^A - \Sigma^B$, and $\sigma = \sigma^A + \sigma^B$, the total change in utility (including the price change) from an infinitesimal change in dividends of Δ is

$$\begin{aligned}
D_\Delta U^B(\theta) &= \theta D_\Delta \pi + \bar{D}_\Delta U^B(\theta) \\
&= \theta \left[\frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B} \right] - \theta \sum_{s=1}^{S} \gamma_s u'_B(e^B_s - \theta d_s)\Delta_s \\
&= \theta[\frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B} - \Sigma^B] + \theta\Sigma^B - \theta \sum_{s=1}^{S} \gamma_s u'_B(e^B_s - \theta d_s)\Delta_s \\
&= \theta[\frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B} - \Sigma^B] - \theta^2 \sum_{s=1}^{S} \gamma_s u''_B(e^B_s - \theta d_s) d_s \Delta_s \\
&= \frac{\theta\Sigma}{\sigma}\sigma^B - \theta^2 \sum_{s=1}^{S} \gamma_s u''_B(e^B_s - \theta d_s) d_s \Delta_s \\
D_\Delta U^A(\theta) &= -\theta D_\Delta \pi + \bar{D}_\Delta U^A(\theta) \\
&= -\theta \left[\frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B} \right] + \theta \sum_{s=1}^{S} \gamma_s u'_A(e^A_s + \theta d_s)\Delta_s \\
&= \theta[-\frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B} + \Sigma^A] - \theta\Sigma^A + \theta \sum_{s=1}^{S} \gamma_s u'_A(e^A_s + \theta d_s)\Delta_s \\
&= \theta[\Sigma^A - \frac{\Sigma^A \sigma^B + \Sigma^B \sigma^A}{\sigma^A + \sigma^B}] - \theta^2 \sum_{s=1}^{S} \gamma_s u''_A(e^A_s + \theta d_s) d_s \Delta_s \\
&= \frac{\theta\Sigma}{\sigma}\sigma^A - \theta^2 \sum_{s=1}^{S} \gamma_s u''_A(e^A_s + \theta d_s) d_s \Delta_s
\end{aligned}$$

The terms $\frac{\theta\Sigma}{\sigma}\sigma^B$ and $\frac{\theta\Sigma}{\sigma}\sigma^A$ represent the change in utility that would arise if the shifts in demand and supply were parallel. They both have the same sign as Σ. But by Lemma 2

$$-\theta^2 \sum_{s=1}^{S} \gamma_s u''_A(e^A_s + \theta d_s) d_s \Delta_s \geq 0 \geq -\theta^2 \sum_{s=1}^{S} \gamma_s u''_B(e^B_s - \theta d_s) d_s \Delta_s.$$

Hence if $D_\Delta U^B \geq 0$ then necessarily $D_\Delta U^A \geq 0$. By Theorem 2, one of the terms must be greater or equal to zero whenever Δ is inversely monotonically related to $\tilde{e}^A + \theta d$. Thus $D_\Delta U^A \geq 0$ for any θd and Δ satisfying the conditions of Lemma 2.

Now take $d = \frac{\lambda \tilde{e}^A + \mu \tilde{e}^B}{\|\lambda \tilde{e}^A + \mu \tilde{e}^B\|_2}$, $\lambda, \mu \geq 0$, and take $\Delta = \beta \tilde{e}^B - \alpha \tilde{e}^A$, where $\alpha \geq 0, \beta \geq 0$ are chosen so that $d \cdot_\gamma \Delta = 0$ and $\|\Delta\|_2 = 1$. Then Δ is tangent to $\mathcal{S}^{S-1}$ and points in the direction from d to d^B. Clearly Δ is inversely monotonically related to $\tilde{e}^A + \theta d$. So the conditions of Lemma 2 are satisfied. Integrating over d around the circle from d^A to d^B then gives welfare gains for A. ■

The proof of Theorem 5 can be used to prove the following corollary (except now Δ can be taken to be of the form $\beta d^B - \gamma \tilde{1}$).

Corollary 3 *Under the conditions of Theorem 5, moving from $d = \tilde{d} = \tilde{1}$ to peso debt makes America better off.*

8 Pareto-Improving Credit Controls

So far we have assumed that the purchasing power of a dollar in Bolivian goods is simply proportional to American output. In reality, it depends on many other factors. To capture some of this, we now assume that the purchasing power of the dollar in Bolivia increases when Bolivian output declines and that this effect becomes stronger the more debt Bolivia issues.

This assumption is surely consistent with the facts, at least in crises when developing countries have issued so much dollar debt that they are close to defaulting. One then typically observes that the purchasing power of the dollar soars as local income collapses.

We model this by assuming that d depends on aggregate debt:

(Debt Level Dependent Dollar Hypothesis).

$$d = d(\theta^A) = \frac{g(\theta^A)\tilde{e}^A + (1 - g(\theta^A))\tilde{e}^B}{\|g(\theta^A)\tilde{e}^A + (1 - g(\theta^A)\tilde{e}^B)\|_2}$$

where $g : \mathbb{R} \to \mathbb{R}$ is assumed to be a smooth and strictly increasing function of trades, θ^A, with $d(\theta^A) \gg 0$ for all θ^A.

The dependence of the real payoffs from dollar debt on the quantity of debt gives a rationale for credit controls. By limiting the amount of dollar debt that is issued, the government can affect the real payoffs of the asset, and thus improve welfare.

Indeed, the American government in our model might also be sympathetic to capital controls because they can improve American welfare. Contrary to popular opinion, the interests of both lender and borrower are often aligned.

We consider credit controls in the form of taxes on transactions, i.e. we assume that there are tax rates τ_B on selling an asset and τ_A on buying an asset. We also introduce transfers T_B to borrowers and T_A to lenders which satisfy

$$T_B = |\theta^B|\tau_B, \quad T_A = |\theta^A|\tau_A.$$

With $\theta^B < 0$, the agents' first period budget constraints then become:

$$x_0^B = e_0^B + T_B - (\pi - \tau_B)\theta^B \text{ and } x_0^A = e_0^A + T_A - (\pi + \tau_A)\theta^A.$$

Since there is a continuum of each type of agent, individual choices of θ^h do not affect transfers.

Theorem 6 *Assume linear quadratic utilities and the debt level dependent dollar hypothesis. If America is the lender, if the persistent gains to risk pooling assumption holds, and if the equilibrium is interior, then taxes of $\tau_A = \epsilon b^B$ per unit of asset purchases, and $\tau_B = \epsilon b^A$ per unit of asset sales are Pareto improving for small enough $\epsilon > 0$.*

Proof. Let $(\pi, (c_0^h, \tilde{c}^h, \theta^h)_{h=A,B})$ be the original dollar equilibrium for the original economy, E, and let $(\underline{\pi}, (\underline{c}_0^h, \underline{\tilde{c}}^h, \underline{\theta}^h)_{h=A,B})$ be the equilibrium for the economy $\underline{E}$ after the imposition of the taxes $(\tau_A, \tau_B) = (\epsilon b^B, \epsilon b^A)$. Let $d = d(\theta^A)$ be the original dollar debt payoffs in terms of Bolivian goods, and let $\underline{d} = d(\underline{\theta}^A)$ be the dollar payoffs with new trade levels.

First we argue that trade goes down:

$$\underline{\theta}^A = -\underline{\theta}^B < \theta^A = \theta^B.$$

Suppose otherwise. Then, since taxes decrease trade, trade would also have increased in the equilibrium of the economy $\underline{E}^*$ when the dollar payoffs are *fixed* at $\underline{d}$, but no tax is imposed. But we know from the second proof of Theorem 4 that utility must then be higher for both agents in $\underline{E}^*$ than in E. But then, by Theorem 2, $g(\underline{\theta}^A) < g(\theta^A)$, a contradiction to $\underline{\theta}^A > \theta^A$ and the monotonicity of g.

With $\underline{\theta}^A < \theta^A$, we know that $g(\underline{\theta}^A) < g(\theta^A)$, so by Theorem 2 at least one agent is better off in $\underline{E}^*$ then in E, and so by Theorem 3 both agents are better off in $\underline{E}^*$ then in E.

We shall now argue from the envelope theorem that for very small ϵ, the utilities in $\underline{E}$ are equal (up to a first order) to the utilities in $\underline{E}^*$, and hence strictly higher than the utilities in E. Notice that the equilibrium in $\underline{E}$ is identical to the equilibrium that would result if in the economy $\underline{E}^*$ (with fixed dividends $\underline{d}$) taxes of ϵb^B and ϵb^A were imposed. Notice further that with the chosen taxes supply and demand move precisely so that the price $\underline{\pi}$ in $\underline{E}$ net of taxes must be identical to the equilibrium price $\underline{\pi}^*$ in $\underline{E}^*$. The loss of income from the tax is just compensated by the transfers $T_A = \underline{\theta}^A \epsilon b^B$ and $T_B = -\underline{\theta}^B \epsilon b^A$. Finally, if ϵ is very small, there is no additional (first order) loss in utility from the reduction in trade $\underline{\theta}^A = -\underline{\theta}^B < \theta^A = \theta^B$. ■

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9 Appendix

We prove several lemmas to facilitate the proof of theorem 2.

Given an arbitrary dividend $d \in \mathbb{R}^S$, we say that an allocation (y^A, y^B) is achievable via d if there is some θ with

$$\begin{aligned} y_0^A + y_0^B &= e_0^A + e_0^B \\ \tilde{y}^A &= \tilde{e}^A + \theta d \\ \tilde{y}^B &= \tilde{e}^B - \theta d \end{aligned}$$

The following constrained efficiency theorem is standard, with one good per state.

Lemma 3 *Any equilibrium allocation to an economy (E, d), with only one consumption good per state, is constrained Pareto-optimal, i.e., there is no allocation (y^A, y^B) achievable via d which Pareto dominates an equilibrium allocation $(x^A(d), x^B(d))$ of (E, d).*

In the following lemmas we maintain all the assumptions of Theorem 2. For any $x \in \mathbb{R}_+^S$, denote $\sum_{s=1}^S \gamma_s u_h(x_s)$ by $\tilde{U}^h(x)$, for any $h \in \{A, B\}$.

Lemma 4 *Given any second period allocation $(c) = (c^A, c^B) \in \mathbb{R}_{++}^{2S}$ with $c^A = \tilde{e}^A + \alpha d^B$, $c^B = \tilde{e}^B - \alpha d^B$, there are no $\gamma_1, \gamma_2 > 0$ such that*

$$\tilde{U}^A(c^A - \gamma_1 d^B + \gamma_2 d^A) > \tilde{U}^A(c^A) \text{ and } \tilde{U}^B(c^B + \gamma_1 d^B - \gamma_2 d^A) > \tilde{U}^B(c^B).$$

Proof. By the assumption of persistent gains to risk-sharing, there must exist an allocation $\hat{c}^A = c^A - \gamma_1 d^B + \gamma_2 d^A$ and $\hat{c}^B = c^B + \gamma_1 d^B - \gamma_2 d^A$, $\gamma_1, \gamma_2 > 0$, which is Pareto-dominated by (c), i.e. $\tilde{U}^h(\hat{c}^h) < \tilde{U}^h(c^h)$ for both $h = 1, 2$.

Suppose there is an allocation $(\underline{c})$ with $\underline{c}^A = c^A - \alpha_1 d^B + \alpha_2 d^A$ and $\underline{c}^B = c^B + \alpha_1 d^B - \alpha_2 d^A$ for some $\alpha_1, \alpha_2 > 0$ such that $\tilde{U}^h(\underline{c}^h) > \tilde{U}^h(c^h)$ for both $h = 1, 2$. If $(\underline{c})$ Pareto-dominates (c), so will $\lambda \underline{c} + (1-\lambda)c$ for $\lambda \in [0,1)$. But this implies that we can scale (α_1, α_2) by $\frac{\gamma_1}{\alpha_1}$, and hence there has to be a $\delta = \alpha_2 \frac{\gamma_1}{\alpha_1} > 0$ such that $c^A - \gamma_1 d^B + \delta d^A$ and $c^B + \gamma_1 d^B - \delta d^A$ Pareto dominates (c). Since it Pareto-dominates (c) it must also Pareto-dominate $\hat{c}$. By strict monotonicity, this is impossible. Either $\delta \leq \gamma_2$ in which case $c^A - \gamma_1 d^B + \delta d^A \leq \hat{c}^A$ or $\delta > \gamma_2$ in which case $c^B + \gamma_1 d^B - \delta d^A < \hat{c}^B$. A contradiction. ■

Lemma 5 *Let* $\underline{d} = \lambda d^A + (1-\lambda) d^B$, *for any* $\lambda > 0$. *Let* $(x) \in \mathbb{R}^{2S}_{++}$ *satisfy* $x^A = \tilde{e}^A + \alpha \underline{d}$, $x^B = \tilde{e}^B - \alpha \underline{d}$. *Then there exists an allocation* $(z^A, z^B) \in \mathbb{R}^{2S}_{+}$ *with* $z^A = \tilde{e}^A + \gamma d^B$, $z^B = \tilde{e}^B - \gamma d^B$ *for some* γ *such that* (z) *Pareto-dominates* (x), *i.e., such that*

$$\tilde{U}^A(z^A) > \tilde{U}^A(x^A) \text{ and } \tilde{U}^B(z^B) > \tilde{U}^B(x^B).$$

Proof. Denote by $(c^A, c^B) \in I\!R^{2S}_{+}$ the unique allocation which satisfies $\tilde{U}^A(c^A) = \tilde{U}^A(x^A)$ and $c^A = \tilde{e}^A + \lambda d^B$, $c^B = \tilde{e}^B - \lambda d^B$ for some $\lambda > 0$ (this allocation must exist by monotonicity). By Lemma 4, (c) cannot be Pareto-dominated by (x) or by any point on the line connecting (c) with (x). By strict concavity, it must then hold that $\tilde{U}^B(c^B) > \tilde{U}^B(x^B)$. By monotonicity there exists the allocation, (z), which Pareto-dominates (x). ■

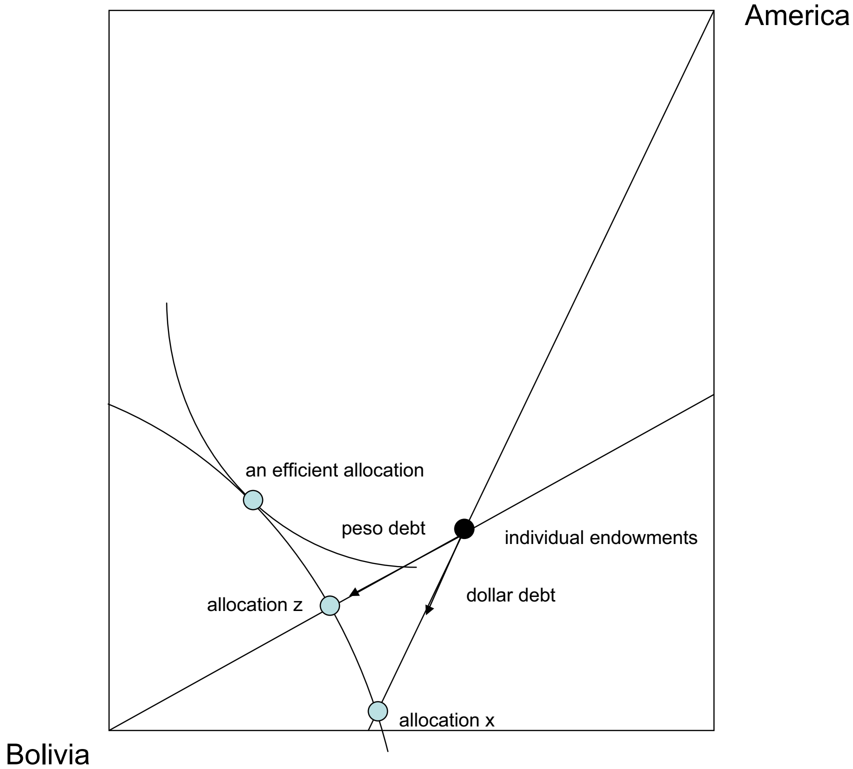

FIGURE 5: The Edgeworth-box

Figure 5 illustrates lemmas 4 and 5. The gains to risk pooling assumption requires that the contract curve lies above the line connecting the origin to Bolivian endowments. With this, the proof of Theorem 2 is straightforward:

Proof of Theorem 2. By Lemmas 4 and 5 there must be an allocation $z \in \mathbb{R}^{2S}_{+}$, $z^A = \tilde{e}^A + \gamma d^B$, $z^B = \tilde{e}^B - \gamma d^B$ which Pareto-dominates dominates $\tilde{x}$. Therefore

$$U^h(x_0^h, z^h) > U^h(x_0^h, \tilde{x}^h),$$

for $h = A, B$ and $(x_0^h, z^h)_{h=A,B}$ Pareto-dominates the allocation (x). But if $(x_0^h, z^h)_{h=A,B} \neq y$, since $(x_0^h, z^h)_{h=A,B}$ is a feasible allocation for the economy $E = ((e^h, u^h)_{h=A,B}, \beta, \pi, f)$ and by Lemma 3, there must be at least one agent h for which $U^h(y^h) > U^h(x_0^h, z^h)$. If we replace d^A by arbitrary $\underline{d} = \lambda d^A + (1-\lambda) d^B \gg 0$ for any $\lambda > 0$ exactly the same argument shows that any allocation achievable via $\underline{d}$ can be Pareto-dominated by an allocation achievable via d^B. But then we can repeat the same argument using $\underline{d}$ in place of d^B, giving Theorem 2. ■