Congestion, risk aversion and the value of information*

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Abstract

Information about traffic conditions is conveyed to drivers by radio and variable message signs, and more recently available via the Internet and Advanced Traveler Information Systems (ATIS). This has spurred research on how travelers respond to information, how much they are likely to benefit from it and how much they are willing to pay for it. We analyze the decisions of drivers whether to acquire information and which route to take on a simple congested road network. Four information regimes are considered: *No information*, *Free information* which is publicly available at no cost, *Costly information* which is publicly available for a fee, and *Private information* which is available free to a single individual. We find that *Private information* is individually more valuable than either *Free* or *Costly information*, while the benefits from *Free* and *Costly information* cannot be ranked in general. We also find that *Free* or *Costly information* can decrease the expected utility of drivers who are sufficiently risk-averse.

1 Introduction

For decades information about driving conditions and traffic delays has been provided to drivers by radio and variable message signs. More recently, information has become available via the Internet and Advanced Traveler Information Systems (ATIS). The advent of modern communications technology has spurred research on how travelers respond to information, how much they are likely to benefit from it, and how much they are willing to pay for it. Because information affects individual travel decisions, individual travel decisions collectively affect travel conditions, and travel conditions determine what information should be conveyed, a complex set of interdependencies exists.

Due to these complexities much of the research on ATIS has considered simple road networks and focused on just one or two dimensions of travel behaviour. Furthermore, it is often assumed that travelers seek to minimize their expected travel costs. In a context where route choice is a decision variable this implies that travelers choose a route with the lowest expected travel time. But it is unrealistic to assume that expected travel time is the only criterion for route choice (Abdel-Aty et al., 1997), and a number of recent studies (e.g. Bates et al, 2001; Lam and Small, 2001; Brownstone and Small, 2005; Small et al, 2005; De Palma and Picard, 2005) have uncovered convincing empirical evidence that travelers dislike not only travel time, but also uncertainty about travel time. De Palma and Picard (2006a, 2006b) incorporate travel time uncertainty into the analysis of information systems with endogenous route choice by assuming that travelers are risk averse and seek to maximize their expected utility where utility is a decreasing and concave function of travel time. However, De Palma and Picard do not analyze the decisions of drivers whether to acquire information or the welfare impacts of information as a function of its cost.

This paper builds on De Palma and Picard (2006a, 2006b) by assuming that information is costly and considering the decisions of drivers whether to purchase it. The paper therefore spans the literature on demand for information with endogenous route choice with risk-neutral drivers (e.g. Yang, 1998; Lo and Szeto, 2002) and the literature that adopts a utility-theoretic approach with risk-averse drivers but does not analyze demand for information (e.g.

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Kobayashi, 1994; Yin and Ieda, 2001; Yin et al., 2004). In the model individuals have a choice between a "safe" route and a "risky" route with a capacity that fluctuates unpredictably from day to day. Individuals are risk averse and weigh average travel time and variability of travel time when choosing a route. Individuals differ in their degrees of risk aversion so that each trades off expected travel time and variability of travel time at a different rate.

Four information regimes are considered. In the *No information* regime drivers do not have day-specific information and base their route-choice decisions on the unconditional probability distribution of states. The second regime is one of *Free information* in which all drivers receive free and accurate information about travel conditions each day. With this information they can predict the Wardrop equilibrium and the travel times that will prevail on each route. In the third regime of *Costly information* drivers can purchase accurate information about travel conditions for a fee. The analysis focuses on how the purchase decision depends on the fee and the individual degree of risk aversion, as well as on how the route-choice decisions of informed drivers affect the expected utility of drivers who do not purchase information. The final information regime is one of *Private information* in which information is made available free to a single individual.

Two results stand out. The first is that *Private information* is always beneficial to a driver relative to *No information*, and the individual benefit exceeds the benefit from *Free information* or *Costly information*. The second, and more, notable finding is that *Free information* or *Costly information* leaves sufficiently risk-averse drivers worse off even though they may be willing to pay for the information given that other drivers have acquired it.

The paper is organized as follows. Section 2 lays out the model and explains the measurement of welfare change using compensating variation. As a benchmark, Section 3 analyzes the equilibria and welfare properties of the information regimes when drivers are risk-neutral. Section 4 derives equilibria for the *No information* and *Free information* regimes when drivers are risk averse, and analyzes the positive and normative impacts of *Free information*. Section 5 — which constitutes the heart of the paper — conducts a parallel analysis for *Costly information*. Section 6 considers *Private information* and compares its benefits with the benefits of *Free information* and *Costly information*. Section 7 presents a numerical example that illustrates the theoretical results and conveys a sense of the magnitude of the welfare impacts. Section 8 concludes.

2 The model

This section presents the model of De Palma and Picard (2006b), defines the information regimes and explains the measurement of welfare change. De Palma and Picard's (2006b) notation is modified and simplified where feasible to streamline extensions in later sections.

2.1 Assumptions

In the model a mass N of drivers travels from a common origin to a common destination. N is fixed and exogenous. Each driver has to choose between a safe route, S, with a deterministic and known travel time t_S, and a risky route, R, with a stochastic travel time T_R which depends on the state. There are two states: a good state denoted "−" with a travel time t_R^-, and a bad state denoted "+" with a larger travel time $t_R^+ > t_R^-$. For brevity the two states will hereafter be called *Good* days and *Bad* days. The probability of a *Bad* day is $p \in (0,1)$ which is assumed fixed and exogenous. Thus:

$$\mathbb{P}\left(T_R = t_R^+\right) = p, \text{ and } \mathbb{P}\left(T_R = t_R^-\right) = 1 - p.$$

There is congestion on route S, as well as on route R on *Bad* days, in the sense that travel time on route j increases with the number N_j of users who choose route j, $j = R, S$. However, there is no congestion on route R on *Good* days: t_R^- is constant. To formalize:

Assumption 1 *Travel time $t_S(N_S)$ is continuous and strictly increasing in N_S. On Good days travel time on route R is a constant t_R^-. On Bad days, travel time is a continuous and strictly increasing function $t_R^+(N_R)$ of N_R.*

On *Good* days, usage of the two routes is N_R^- and N_S^-, and on *Bad* days it is N_R^+ and N_S^+. Three further assumptions about travel times are made. First, on *Good* days route R is faster than S even with $N_S = 0$. Second, on *Bad* days travel time on R is always longer than on *Good* days even with $N_R = 0$. Finally, if all drivers choose one route on a *Bad* day then it is slower than the other (unused) route. Therefore:

Assumption 2 *The following inequalities hold:*

$$t_R^- < t_S(0) < t_R^+(N) \text{ and } t_R^- < t_R^+(0) < t_S(N).$$

No assumption is made about the relative magnitudes of $t_S(0)$ and $t_R^+(0)$.

Assumptions 1 and 2 are illustrated in Figure 1. The number of drivers taking route R, N_R, is measured in the usual way to the right from the left-hand vertical axis, while the number taking route S, N_S, is measured to the left from the right-hand axis. Travel times are measured on the vertical axes. Functions $t_R^+(N_R)$ and $t_S(N_S)$ intersect where $N_R > 0$ and $N_S > 0$ and lie wholly above the horizontal line at t_R^-.

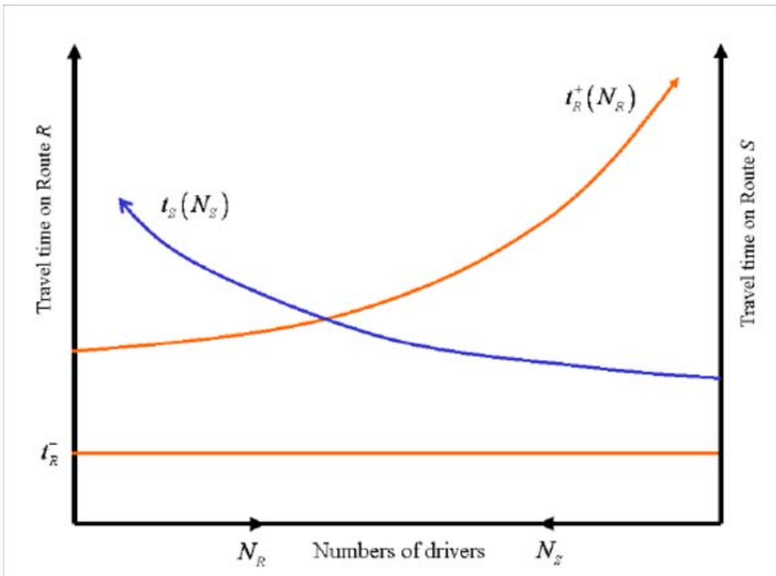

Figure 1: Travel time functions

Drivers' preferences are specified in:

Assumption 3 *Drivers' preferences are represented by a differentiable utility function $U(t;\theta)$ where $\theta \geq 0$ is the risk aversion parameter. For $\theta = 0$, $U(t;0) = -t$. For $\theta > 0$ and $t > 0$, $U(t;\theta)$ is strictly decreasing and strictly concave in t. For $\theta > 0$, $\partial U(t;\theta)/\partial\theta$ is a strictly concave function of U and $\underset{\theta\to\infty}{Lim}\ U(t_2;\theta)/U(t_1;\theta) = \infty$ for $t_2 > t_1 > 0$.*

It is straightforward to check (see Appendix 1) that Assumption 3 is satisfied for Constant Relative Risk Aversion (CRRA) preferences and Constant Absolute Risk Aversion (CARA) preferences. For CRRA the utility function is $U(t;\theta) = -\frac{t^{1+\theta}}{1+\theta}$, and for CARA it is $U(t;\theta) = \frac{1-\exp(t\theta)}{\theta}$. CARA preferences will be adopted in the numerical example of Section 7.

Expected utility on a route is $\mathbb{E}U(T;\theta) = pU(T^+;\theta) + (1-p)U(T^-;\theta)$, where T^+ denotes travel time on the route on *Bad* days, and T^- denotes travel time on *Good* days. The distribution of θ in the population is described in:

Assumption 4 *The risk aversion parameter θ has a continuous distribution over an interval $\mathcal{I}$ (with either $\mathcal{I} = \mathbb{R}^+$ or $\mathcal{I} = \left[0;\theta^M\right] \subset \mathbb{R}^+$ for $\theta^M \in [0,\infty)$). The distribution is characterized by the strictly increasing cumulative distribution function $F(\theta) \in [0;1]$, $\forall\, \theta \in \mathcal{I}$, and by the density $f(\theta) > 0$, $\forall\, \theta$ in the interior of $\mathcal{I}$. If the distribution is bounded then $\mathbb{E}U\left(T_R(N);\theta^M\right) < U\left(t_S(0);\theta^M\right)$.*

The last assumption of Assumption 4 implies either that expected travel time is larger on R when all drivers select R, or that θ^M is sufficiently large. This assumption is necessary for an interior equilibrium to exist. Otherwise, all the users would select R when the state is unknown.[1]

2.2 Information regimes

Four information regimes will be considered that differ according to drivers' knowledge of the state before they make a route choice:

No information (Z):[2] drivers know the probability p, but not the actual state.

Free information (F): all drivers are informed about the state at no cost.

Costly information (C): all drivers can learn the state at a cost π.

Private information (I): a single driver learns the state at no cost.

For each information regime, all drivers are assumed to know all the parameters of the model (values of p and t_R^-, congestion functions $t_S(.)$ and $t^+(.)$, distribution of θ, price of information π), and to be able to compute the equilibrium solution.[3] With *Private information* the equilibrium is the same as with *No information* since the route choice of one (atomless) driver does not affect traffic conditions.

In the case of *Costly information* it is assumed that the price of information is equivalent to an increase in travel time. By assuming that the value of time is constant and normalized to unity, expected utility can be written $\mathbb{E}U(T+\pi;\theta)$. For all regimes except *Free information* the division of traffic between routes is characterized by:

Proposition 1 *On Bad days, in the No information, Costly information and Private information regimes traffic N_R^+ on R is such that $t_R^+(N_R^+) > t_S(N_S^+)$.*

Proof. Assumptions 1 and 2 imply that route R is preferred by all users to route S on *Good* days even if everyone takes R. If R were also preferred to S on *Bad* days, then R would be preferred to S whatever the state. But this implies $N_R^+ = N$ and $t_R^+(N) \leq t_S(0)$, which contradicts Assumption 2. ■

Depending on their preferences and on π, in the *Costly information* regime drivers choose between three strategies:

- *Strategy R* (n_R drivers): Do not pay for information and choose route R in both states.
- *Strategy S* (n_S drivers): Do not pay for information and choose route S in both states.
- *Strategy I* (n_I drivers): Pay for information. Given Assumption 2 and Proposition 1, the best choice when informed is route R on *Good* days and route S on *Bad* days.

The following conservation laws apply:

$$n_R + n_S + n_I = N_R^+ + N_S^+ = N_R^- + N_S^- = N.$$

Traffic N_j^q on route j when the state is q is therefore as given in Table 1:

Route j \ State q	*Bad* days ($q =$ "$+$")	*Good* days ($q =$ "$-$")
$j = R$	$N_R^+ = n_R$	$N_R^- = n_R + n_I = N - n_S$
$j = S$	$N_S^+ = n_S + n_I = N - n_R$	$N_S^- = n_S$

Table 1: Route split in *Costly information* regime

[1] No similar assumption is necessary when θ is distributed over $\mathbb{R}^+$ since the assumption $t_R^+(N) > t_S(0)$ guarantees that a sufficiently risk averse driver prefers route S to route R.

[2] Superscript Z (zero information) is used for the *No information* regime rather than N to avoid confusion with the number of drivers.

[3] Equivalently, and less restrictively, it can be assumed that all drivers know π and the probability distribution of travel times on the two routes conditional on the information regime and whether they are informed.

The strategies, the numbers of drivers choosing each strategy, their route choices and expected utilities in the four information regimes are summarized in Table 2. Note that Strategies R and S are equivalent to a route choice in both states. With Strategy I, drivers select route R on *Good* days and route S on *Bad* days.[4]

Strategy	Numbers of drivers	*Bad* days	*Good* days	Expected utility
		No information		
R	n_R^Z	R	R	$pU\left(t_R^+\left(n_R^Z\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$
S	n_S^Z	S	S	$U\left(t_S\left(n_S^Z\right);\theta\right)$
		Free information		
R	$n_R^{F+}=N_R^+$	R	R	$pU\left(t_R^+\left(n_R^{F+}\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$
I	$n_I^{F+}=N_S^+$	S	R	$pU\left(t_S\left(n_I^{F+}\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$
		Costly information		
R	n_R^C	R	R	$pU\left(t_R^+\left(n_R^C\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$
S	n_S^C	S	S	$pU\left(t_S\left(n_S^C+n_I^C\right);\theta\right)+(1-p)\,U\left(t_S\left(n_S^C\right);\theta\right)$
I	n_I^C	S	R	$pU\left(t_S\left(n_S^C+n_I^C\right)+\pi;\theta\right)+(1-p)\,U\left(t_R^-+\pi;\theta\right)$
		Private information		
R	n_R^Z	R	R	$pU\left(t_R^+\left(n_R^Z\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$
S	n_S^Z	S	S	$U\left(t_S\left(n_S^Z\right);\theta\right)$
I	Single driver	S	R	$pU\left(t_S\left(n_S^Z\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right)$

Table 2: Strategies, numbers of drivers, route choices and expected utilities

2.3 Measuring welfare change

Willingness to pay is measured in economics using compensating variation and equivalent variation (see Varian 1992, Chapter 10). The compensating variation is defined to be the amount an individual is willing to pay for a change to take place, whereas equivalent variation is defined to be the amount an individual requires in order to be as well off as if a change takes place. Compensating variation (CV) will be adopted here since it is assumed when assessing the *Free*, *Costly* and *Private information* regimes that information is actually provided so that the change does take place. CV will be measured in time units in the same way that the cost of information is measured; thus it corresponds to the additional certain travel time that a driver is willing to incur for information. Since the value of time (VOT) is assumed to be constant, if desired CV can be translated into monetary units by multiplying the CV by the VOT.

Definition 1 *The individual compensating variation $CV^r(\theta)$, $r\in\{F,C,I\}$, corresponds to the additional time an individual with utility $U(.;\theta)$ is willing to incur for a transition from the No information regime to information regime r.*

The compensating variation for information depends, *a priori*, on who receives the information because this affects traffic equilibrium. Consequently, the CVs for *Free, Costly* and *Private information* all differ in general.

3 Equilibria with risk-neutral drivers

To develop a preliminary understanding of the model as well as to provide a benchmark against which to assess the implications of risk aversion, it is instructive to identify and characterize equilibria for the four information regimes when drivers are risk neutral.[5]

[4] With *Free information* all drivers are informed at no cost. Since, however, n_R^{F+} drivers take route R on *Bad* days they can be thought of as choosing Strategy R and ignoring the state.

[5] Risk neutrality is a limiting case of the model with a compact distribution of risk aversion with parameter θ^M in Assumption 4 set to zero.

3.1 Free information equilibrium

In the *Free information* regime all drivers choose route R on *Good* days and incur a travel cost $C^{F-} = t_R^-$. The equilibrium corresponds to point F_R^- in Figure 2 with coordinates $\left(N, t_R^-\right)$, where the subscript R on F_R^- indicates that the outcome is realized on Route R. On *Bad* days, drivers divide themselves between routes according to Wardrop's principle so that travel costs are equalized: $C^{F+} = t_R^+\left(n_R^{F+}\right) = t_S\left(n_S^{F+}\right) = t_S\left(N - n_R^{F+}\right)$. The corresponding point in Figure 2 is $F_{R,S}^+$ where the subscript R, S indicates that the outcome is realized on both routes. Expected travel costs are $\mathbb{E}C^F = (1-p)\,C^{F-} + pC^{F+} = (1-p)\,t_R^- + pt_R^+\left(n_R^{F+}\right)$.

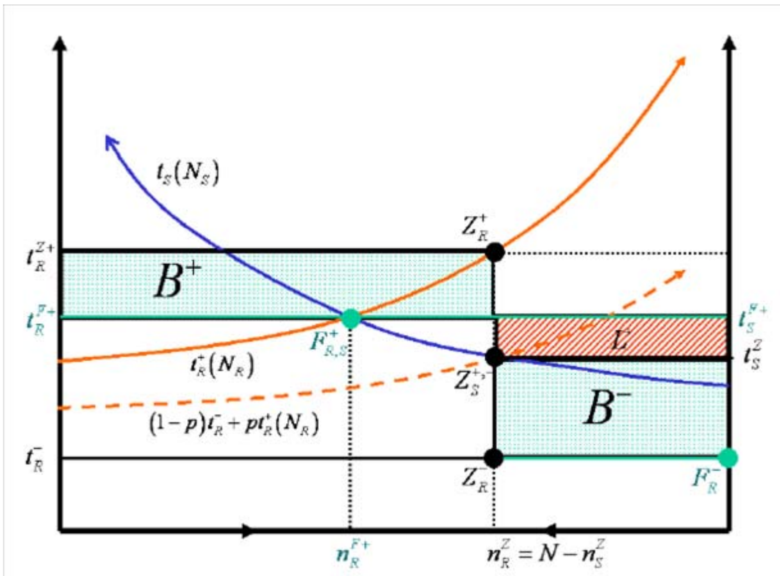

Figure 2: Effects of *Free information* with risk-neutral drivers

3.2 No information equilibrium and Private information

If drivers do not know the state and are risk neutral, they allocate themselves between routes so that expected travel costs are equal: $\mathbb{E}C^Z = (1-p)\,t_R^- + pt_R^+\left(n_R^Z\right) = t_S\left(N - n_R^Z\right)$.[6] Point $Z_S^{+,-}$ identifies the equilibrium for users of Route S. This outcome is certain because travel costs are the same in both states. Point Z_R^- identifies the outcome for users of Route R on *Good* days, and Z_R^+ the outcome on *Bad* days. The comparative statics properties of the equilibrium are derived in Appendix 2 and presented in Table 3, where a positive partial derivative is denoted by a " + " and a negative partial derivative by a " − ".

	p	N
n_R	$-$	$\in (0,1)$
n_S	$+$	$\in (0,1)$
$E{\cdot}C^Z$	$+$	$+$

Table 3: Comparative statics of *No information* equilibrium with risk neutral drivers

As expected, an increase in the probability of *Bad* days reduces usage of route R, increases usage of route S, and increases expected travel costs. If the number of drivers increases, usage of both routes increases as do expected travel costs.

[6] To obtain $n_S^Z = N - n_R^Z > 0$, the probability of *Bad* days must satisfy $p > \frac{t_S(0) - t_R^-}{t_R^+(N) - t_R^-}$.

With risk-neutral drivers the compensating variation for *Free information* is simply the difference in expected costs between the *No information* and *Free information* regimes: $CV^F = \mathbb{E}C^Z - \mathbb{E}C^F = p\left(t_R^+\left(n_R^Z\right) - t_R^+\left(n_R^{F+}\right)\right) > 0$. CV^F is positive because on *Bad* days fewer drivers take route R; i.e. $n_R^{F+} < n_R^Z$. The welfare gain can be decomposed into three parts. First, on *Good* days information benefits drivers who would otherwise take route S. This benefit is identified by the speckled area labeled B^- in the lower right of Figure 2. Second, on *Bad* days information benefits drivers who take route R as indicated by the speckled area B^+ in the upper left. Finally, information imposes a loss on drivers who take route S as shown by the lightly shaded area L^+ to the upper right.[7] However, this loss is outweighed by the benefits.[8]

Free information reduces expected travel time, as well as variability of travel time on route R. But it also raises travel time on *Bad* days on route S (since $t_S\left(n_S^{F+}\right) > t_S\left(n_S^Z\right)$) and results in travel time uncertainty for all drivers. This uncertainty is of no consequence *per se* for risk-neutral drivers, but it does matter with risk-averse drivers as will be shown in Section 4.

Before turning to *Costly information* consider the *Private information* regime. If a single driver is informed he can take route R on *Good* days, and route S on *Bad* days. The compensating variation for *Private information* is $CV^I = \mathbb{E}C^Z - \mathbb{E}C^I = p\left(t_R^+\left(n_R^Z\right) - t_S\left(n_S^Z\right)\right) > 0$. CV^I exceeds the compensating variation for *Free information* since $t_S\left(n_S^Z\right) < t_R^+\left(n_R^{F+}\right) = t_S\left(n_R^{F+}\right)$. Information is more valuable when it is private because the benefits of shifting to route S on *Bad* days are not dissipated by similar adjustments of other drivers. As will be seen, the advantage of *Private information* over *Free information* holds — and with greater force — when drivers are risk-averse.

3.3 Costly information equilibrium

Equilibrium for the *Costly information* regime with risk-neutral drivers is derived in Appendix 2. In order for some drivers to be willing to purchase information (Strategy I) the cost must be bounded above; otherwise the equilibrium is the same as with *No information*. Regardless of the cost of information some drivers choose not to purchase it and stick to route R (Strategy R). If the cost of information exceeds a lower bound then some drivers choose Strategy S; otherwise all drivers adopt either Strategy R or Strategy I. The comparative statics properties of equilibrium are presented in Table 4. Predictably, as the cost of information increases, n_I^C decreases and n_R^C and n_S^C increase. As the probability of *Bad* days increases, n_S^C rises and n_R^C falls. The effect on n_I^C is indeterminate. It turns out (see Appendix 2) that the derivative is positive when p is small, and negative when p is large. This is because travel time uncertainty is greatest for intermediate values of p. Perhaps surprisingly, n_S is independent of N.[9] As will be seen this result does not obtain with risk-averse drivers.

	π	p	N
n_R	$+$	$-$	$\in (0,1)$
n_S	$+$ if $n_S > 0$ 0 if $n_S = 0$	$+$ if $n_S > 0$ 0 if $n_S = 0$	0
n_I	$-$	? if $n_S > 0$ $+$ if $n_S = 0$	$\in (0,1)$
$\mathbb{E}C^C$	$+$	$+$	$+$

Table 4: Comparative statics of *Costly information* equilibrium with risk neutral drivers

Equilibrium with *Costly information* is depicted in Figure 3 for the case with $n_S^C > 0$. On *Good* days, Groups R and I take Route R and realize an outcome at point $C_{R,I}^-$. Group S incurs a higher cost at point C_S^-. On *Bad* days, Group R ends up at point C_R^+, while Groups S and I incur a lower cost at point $C_{S,I}^+$. Group I enjoys the best of both worlds, but pays a fee π for the privilege.

[7] Area B^- corresponds to what Zhang and Verhoef (2006) call decision-making benefits. Areas B^+ and L^+ correspond to what they call travel cost benefits (or costs).

[8] To see this geometrically note that area B^- (weighted by the probability of *Good* days) equals the area of the rectangle defined by points $Z_S^{+,-}$, Z_R^+, t_R^{Z+} and t_S^Z (weighted by the probability of *Bad* days) which exceeds area L^+. Area B^- thus outweighs area L^+, and area B^+ adds to the net benefit.

[9] To see why, note that the choice between strategies I and S depends on the cost of information relative to the travel time saved on *Good* days from taking route R. Since both the cost of information and travel time on R on *Good* days are constants, travel time on route S must also be constant and hence usage of S must be independent of N.

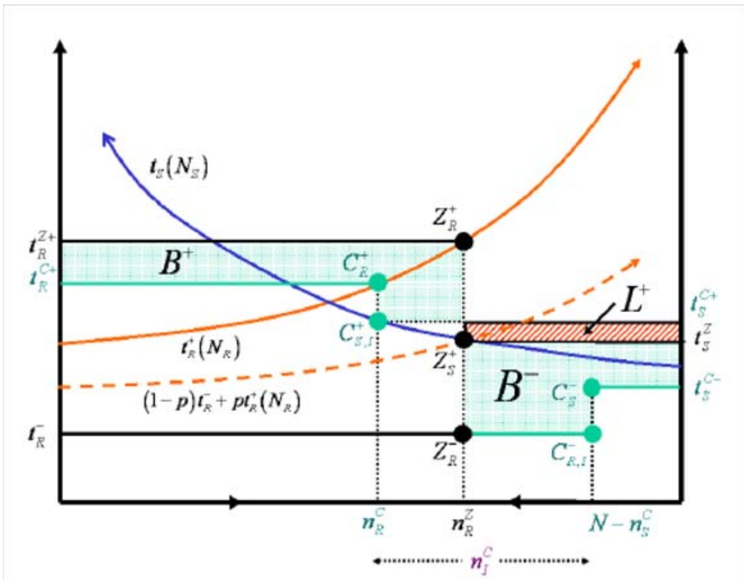

Figure 3: Effects of *Costly information* with risk-neutral drivers

Since expected travel costs are the same for all three groups, the compensating variation for *Costly information* can be computed for any of them. For Group R, expected costs are $\mathbb{E}C_R^C = (1-p)\,t_R^- + pt_R^+\left(n_R^C\right)$, and hence $CV^C = \mathbb{E}C^Z - \mathbb{E}C_R^C = p\left(t_R^+\left(n_R^Z\right) - t_R^+\left(n_R^C\right)\right) > 0$. CV^C is positive, but it is smaller than for *Free information* since $n_R^C > n_R^{F+}$. This is apparent in Figure 3 from the fact that area B^+ is thinner than in Figure 2. For Group S travel costs are higher on *Good* days than with *Free information* ($t_S^{C-} > t_R^-$), but lower on *Bad* days ($t_S^{C+} < t_S^{F+}$). Consequently, Group S suffers less variability in costs — which is advantageous if they are risk averse as in the general model.

4 Equilibrium for No information and Free information regimes with risk-averse drivers

This section establishes some properties of the *No information* and *Free information* regimes when drivers are risk averse, and examines the compensating variation for *Free information.* Some of the results generalize results derived in de Palma and Picard (2006b), and other results are new.[10]

4.1 Existence and uniqueness of equilibria

The equilibrium of the *Free information* regime is described by:

Theorem 1 *Consider the Free information regime. Under Assumptions 1 and 2:*
(a) On Good days there exists a unique boundary equilibrium: $(N_R^{F-} = N, N_S^{F-} = 0)$.
(b) On Bad days there exists a unique interior equilibrium: $(N_R^{F+} \in (0,N)\,, N_S^{F+} = N - N_R^{F+})$*, with* $t^+\left(N_R^{F+}\right) = t_S\left(N - N_R^{F+}\right)$.

Proof. See Appendix 3. ■

On *Good* days all drivers prefer route R. On *Bad* days both routes are used and all drivers are indifferent between them. The equilibrium is the same as when drivers are risk-neutral.

The individual route choice decisions in the *No information* regime are described by:

[10] De Palma and Picard (2006b) derive most of their results for CRRA and CARA preferences. The results here are derived under Assumption 3, which includes CRRA and CARA as special cases and permits more compact proofs.

Theorem 2 *Consider the No information regime. Under Assumptions 1-4, for any N_R such that $t_R^+(N_R) > t_S(N - N_R)$ and $pt_R^+(N_R) + (1-p)t_R^- < t_S(N - N_R)$:*

(a) There exists a unique risk aversion threshold $\tilde{\theta}^Z(p, N_R)$ such that $R \succ S \Leftrightarrow \theta < \tilde{\theta}^Z(p, N_R)$.

(b) $\tilde{\theta}^Z(p, N_R)$ is strictly decreasing in p and N_R,

Proof. See Appendix 4. ■

According to Theorem 2, whenever expected travel time is lower on R than on S a group of drivers with the least risk aversion prefer to use R, and the remainder prefer S. An obvious corollary of Theorem 2 is that the number of users on route R in the *No information* regime, n_R^Z, solves: $n_R^Z = NF\left(\tilde{\theta}^Z(p, N_R)\right)$.

Equilibrium in the *No information* regime is described by:

Theorem 3 *Consider the No information regime. Under Assumptions 1-4, there exists a unique equilibrium traffic volume on R, $n_R^Z(p) \in \left(n_R^{F+}, N\right)$, and a unique risk aversion threshold, equal to $\tilde{\theta}^Z\left(p, n_R^Z\right)$, which solve:*

$$\begin{cases} F^{-1}\left(n_R^Z/N\right) = \tilde{\theta}^Z\left(p, n_R^Z\right) \\ pU\left(t_R^+\left(n_R^Z\right); \tilde{\theta}^Z\left(p, n_R^Z\right)\right) + (1-p)\,U\left(t_R^-; \tilde{\theta}^Z\left(p, n_R^Z\right)\right) = U\left(t_S\left(N - n_R^Z\right); \tilde{\theta}^Z\left(p, n_R^Z\right)\right) \end{cases}. \tag{1}$$

Moreover, $n_R^Z(p)$ and $\tilde{\theta}^Z\left(p, n_R^Z\right)$ are decreasing in p.

Proof. See Appendix 5. ■

We now turn to a comparison of the *No information* and *Free information* regimes.

Theorem 4 *Under Assumptions 1-4:*

(a) With Free information, equilibrium traffic on route R on Bad days is less than traffic with No information: $n_R^{F+} < n_R^Z$.

(b) Free information reduces expected travel time for all drivers, and the reduction is larger for the most risk averse drivers ($\theta > \tilde{\theta}^Z\left(p, n_R^Z\right)$) than for the least risk averse drivers ($\theta < \tilde{\theta}^Z\left(p, n_R^Z\right)$): $pt_R^+\left(n_R^{F+}\right) + (1-p)\,t_R^- < pt_R^+\left(n_R^Z\right) + (1-p)\,t_R^- < t_S\left(N - n_R^Z\right)$.

(c) Free information reduces the variability of travel time for the least risk-averse drivers, and increases the variability of travel time for the most risk-averse drivers.

Proof. See Appendix 6. ■

The compensating variation for *Free information* equalizes individual expected utility in the *No information* and *Free information* regimes, and is therefore driver-specific. Recall that with *No information* the n_R^Z least risk-averse drivers choose route R and incur a random travel time:

$$T_R\left(n_R^Z\right) = \begin{cases} t_R^- \text{ on } \textit{Good} \text{ days} \\ t_R^+\left(n_R^Z\right) \text{ on } \textit{Bad} \text{ days} \end{cases},$$

and the $N - n_R^Z$ most risk averse drivers choose route S with deterministic travel time $t_S\left(N - n_R^Z\right)$. In the *Free information* regime, all users choose R on *Good* days (with travel time t_R^-) and are indifferent between R and S on *Bad* days since equilibrium traffic on R on *Bad* days, n_R^{F+}, is such that $t_R^+\left(n_R^{F+}\right) = t_S\left(N - n_R^{F+}\right)$. The individual compensating variations for *Free information* for groups R and S, $CV_R^F(\theta)$ and $CV_S^F(\theta)$, therefore solve:

$$\begin{array}{ll} R: & \begin{array}{c} pU\left(t_R^+\left(n_R^{F+}\right) + CV_R^F(\theta); \theta\right) + (1-p)\,U\left(t_R^- + CV_R^F(\theta); \theta\right) = \\ pU\left(t_R^+\left(n_R^Z\right); \theta\right) + (1-p)\,U\left(t_R^-; \theta\right) \quad \text{if } \theta < \tilde{\theta}^Z\left(p, n_R^Z\right), \end{array} \\ S: & \begin{array}{c} pU\left(t_R^+\left(n_R^{F+}\right) + CV_S^F(\theta); \theta\right) + (1-p)\,U\left(t_R^- + CV_S^F(\theta); \theta\right) = \\ U\left(t_S\left(n_S^Z\right); \theta\right) \quad \text{if } \theta > \tilde{\theta}^Z\left(p, n_R^Z\right). \end{array} \end{array} \tag{2}$$

The left-hand side of each equation in (2) is expected utility in the *Free information* regime with CV added to travel time, and the right-hand side is expected utility in the *No information* regime. The pair of equations in (2) is the counterpart to the equation $CV^F = \mathbb{E}C^Z - \mathbb{E}C^F$ for risk-neutral drivers.

According to Theorem 4, *Free information* reduces expected travel time for all drivers, and the reduction is larger for the most risk-averse drivers who choose route S. Compensating variation, however, exhibits a different pattern. Under Assumption 3, $CV_R^F(\theta)$ is an increasing function of θ, and $CV_S^F(\theta)$ is a decreasing function of θ. CV is therefore highest for drivers with risk aversion $\tilde{\theta}^Z(p, n_R^Z)$ who are indifferent between strategies R and S. Furthermore, CV is negative for drivers who are sufficiently risk averse. (This follows intuitively from the definition of $CV_S^F(\theta)$ and the fact that $t_R^+(n_R^{F+}) = t_S(n_S^{F+}) > t_S(n_S^Z)$ so that drivers who take the safe route fare worse on *Bad* days when information is provided.) These results are formalized in:

Proposition 2 *Under Assumptions 1-4:*

(a) The compensating variation for Free information is an increasing function of risk aversion for the least risk-averse drivers who take route R with No information, and a decreasing function of risk aversion for the most risk-averse drivers who take route S with No information.

(b) When the risk aversion parameter is distributed over $\mathbb{R}^+$, the compensating variation for Free information is negative for the most risk-averse users.

Proof. Part (a) is proved in Appendix 7. Part (b) is proved in Appendix 8. ■

The most risk averse drivers take the safe route in the *No information* regime. They gain and lose from *Free information*. They gain on *Good* days because they can save time by shifting to the risky route. But they lose on *Bad* days because some of the least risk averse drivers shift onto the safe route. This increases travel time on the safe route and also increases the variability of travel time that the most risk averse drivers experience. The benefit on *Good* days outweighs the loss on *Bad* days for drivers with intermediate levels of risk aversion. (Recall from Section 3 that this is true of all drivers if drivers are risk-neutral.) But for the *very most* risk averse drivers the benefit is outweighed by the loss.

Free information can be individually welfare-reducing because it can induce changes in driver behaviour that exacerbate congestion in a particular way. The expected private benefit that an individual derives from adjusting his or her route choice on the basis of daily travel conditions can be outweighed by the effects of adjustments by other drivers. As the next section demonstrates, this is also possible if drivers have to pay for information.

5 Equilibrium for Costly information regime with risk-averse drivers

The *Costly information* regime is more complicated than the *No information* regime since drivers can choose between three strategies (R, S and I) rather than just two (R and S). Similar to the approach taken for the *No information* regime, equilibrium will be derived in two steps. In the first step an individual driver's strategy choice for information acquisition and route selection is derived while holding fixed the numbers of drivers who adopt each strategy (i.e. n_R, n_S and n_I). For the second step the sets of drivers choosing each strategy and thus the equilibrium values of n_R, n_S and n_I are determined given the individual strategy choices derived in step 1.

5.1 Driver strategy choice with exogenous traffic

The first step is to derive an individual driver's choice between strategies R, S and I as a function of θ, the price of information, π, and traffic conditions (n_R, n_S, n_I).[11] If a driver is not informed the choice is restricted to strategies R and S. The preference ranking between R and S is described by the condition:

$$R \succ S \Leftrightarrow pU\left(t_R^+(n_R);\theta\right) + (1-p)\,U\left(t_R^-;\theta\right) > pU\left(t_S(n_S+n_I);\theta\right) + (1-p)\,U\left(t_S(n_S);\theta\right),$$

where the expected utilities of R and S are as given in Table 2. Rearranging terms, this condition can be written in terms of the difference in utilities:

$$R \succ S \Leftrightarrow \psi_{RS}(\theta; p, n_R, n_S, n_I) \equiv \begin{array}{l} p\left[U\left(t_R^+(n_R);\theta\right) - U\left(t_S(n_S+n_I);\theta\right)\right] \\ +(1-p)\left[U\left(t_R^-;\theta\right) - U\left(t_S(n_S);\theta\right)\right] \end{array} > 0.$$

Naturally, this condition does not depend on the price of information — although if π is small enough a driver may prefer strategy I to either R or S. The properties of the preference ranking are described in:

[11] Note that traffic conditions depend only on the numbers of drivers adopting each strategy, and not independently on the distribution of risk aversion within each group.

Proposition 3 *Under Assumptions 1-4:*
(a) There exists a unique risk aversion threshold $\hat{\theta}_{RS}(p, n_R, n_S, n_I)$ *such that* $R \succ S \Leftrightarrow \theta < \hat{\theta}_{RS}(p, n_R, n_S, n_I)$.
(b) For $\hat{\theta}_{RS} < \infty$, $\hat{\theta}_{RS}$ *is decreasing in* p *and* n_R, *and increasing in* n_S *and* n_I.

Proof. See Appendix 9. ■

The preference ranking between strategies R and I is described by the condition:

$$R \succ I \Leftrightarrow pU\left(t_R^+(n_R);\theta\right) + (1-p)U\left(t_R^-;\theta\right) > pU\left(t_S(n_S+n_I)+\pi;\theta\right) + (1-p)U\left(t_R^- + \pi;\theta\right),$$

which is equivalent to:

$$R \succ I \Leftrightarrow \psi_{RI}(\theta, \pi; p, n_R, n_S+n_I) \equiv \begin{array}{c} p\left[U\left(t_R^+(n_R);\theta\right) - U\left(t_S(n_S+n_I)+\pi;\theta\right)\right] \\ +(1-p)\left[U\left(t_R^-;\theta\right) - U\left(t_R^-+\pi;\theta\right)\right] \end{array} > 0.$$

Note that this condition depends on the combined numbers of drivers who choose Strategies S and I, $n_S + n_I$, but not independently on n_S and n_I. The properties of the preference ranking are described in:

Proposition 4 *Under Assumptions 1-4:*
(a) There exists a unique risk aversion threshold $\hat{\theta}_{RI}(\pi, p, n_R, n_S+n_I)$ *such that* $R \succ I \Leftrightarrow \theta < \hat{\theta}_{RI}(\pi, p, n_R, n_S+n_I)$.
(b) For $\hat{\theta}_{RI} < \infty$, $\hat{\theta}_{RI}$ *is decreasing in* p *and* n_R, *and increasing in* π *and* $n_S + n_I$.

Proof. See Appendix 10. ■

The preference ranking for the final pair of strategies, I and S, is:

$$I \succ S \Leftrightarrow pU(t_S(n_S+n_I)+\pi;\theta) + (1-p)U\left(t_R^-+\pi;\theta\right) > pU(t_S(n_S+n_I);\theta) + (1-p)U(t_S(n_S);\theta),$$

or

$$I \succ S \Leftrightarrow \psi_{IS}(\theta, \pi; p, n_S, n_I) \equiv \begin{array}{c} p\left[U(t_S(n_S+n_I)+\pi;\theta) - U(t_S(n_S+n_I);\theta)\right] \\ +(1-p)\left[U\left(t_R^-+\pi;\theta\right) - U(t_S(n_S);\theta)\right] \end{array} > 0.$$

The properties of the preference ranking are given in

Proposition 5 *Under Assumptions 1-4:*
(a) There exists a unique risk aversion threshold $\hat{\theta}_{IS}(\pi, p, n_S, n_I)$ *such that* $I \succ S \Leftrightarrow \theta < \hat{\theta}_{IS}(\pi, p, n_S, n_I)$.
(b) For $\hat{\theta}_{IS} < \infty$, $\hat{\theta}_{IS}$ *is decreasing in* π, p *and* n_I, *independent of* n_R, *of ambiguous sign in* n_S, *but increasing in* n_S *with* $n_S + n_I$ *held fixed.*

Proof. See Appendix 11. ■

Figure 4 depicts the combinations of (θ, π) for which drivers choose strategy R, S or I when traffic conditions are fixed. Along the boundary labelled $R \approx S$, $\psi_{RS} = 0$ and drivers are indifferent between R and S. The boundary between regions R and I where $R \approx I$, and the boundary between regions I and S where $I \approx S$, are interpreted similarly. The following lemmas establish the locations of the indifference curves as depicted in Figure 4.

Lemma 1 *When* $R \succ S$, *the curve* $R \approx I$ *lies below the curve* $I \approx S$. *When* $S \succ R$, *the curve* $R \approx I$ *lies above the curve* $I \approx S$.

Proof. Transitivity of preferences implies that, when $R \succ S$ and $I \approx S$, then $R \succ I$ and the curve $R \approx I$ is located at a lower π. Similarly, when $S \succ R$ and $I \approx S$, then $R \succ I$ and the curve $R \approx I$ is located at a higher π. ■

Lemma 1 confirms that the decision whether to become informed involves a comparison between I and R for the least risk-averse drivers, and a comparison between I and S for the most risk-averse drivers.

Lemma 2 *When* $\pi = \theta = 0$, $I \succ S$ *and either* $R \succ I$ *or* $I \succ R$ *depending on the value of* n_R.

Proof. When $\theta = 0$, $U(t;0) = -t$ by Assumption 3. If $\pi = 0$ in addition, then

$$\psi_{IS}(\theta = 0, \pi = 0; p, n_S, n_I) = (1-p)\left[t_S(n_S) - t_R^-\right] > 0, \text{ and}$$
$$\psi_{RI}(\theta = 0, \pi = 0; p, n_R, n_S+n_I) = p\left[t_S(n_S+n_I) - t_R^+(n_R)\right] \gtrless 0.$$

■

Propositions 3, 4 and 5 are summarized in:

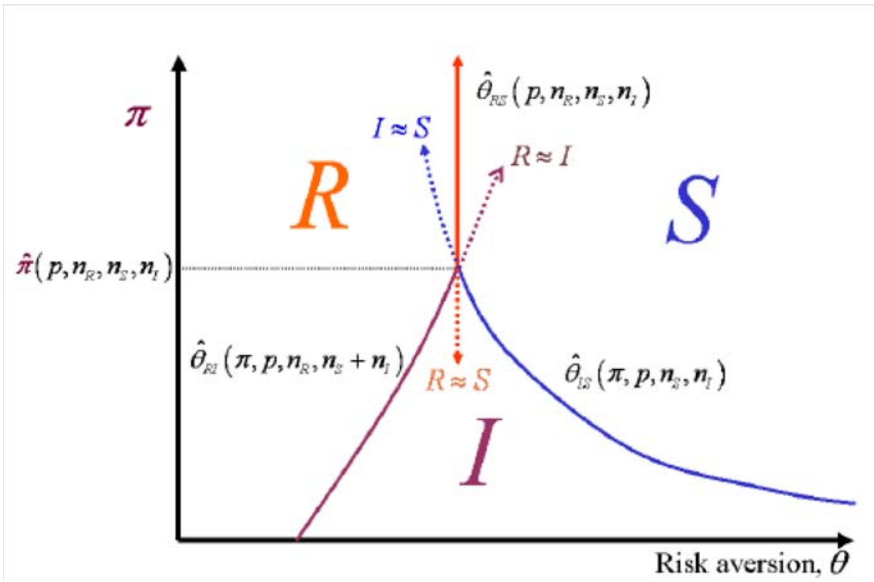

Figure 4: Strategy choice with *Costly information*

Proposition 6 *Under Assumptions 1-4, for any* (p, n_R, n_S, n_I) *there exists a unique price* $\widehat{\pi}(p, n_R, n_S, n_I) > 0$, *a unique risk aversion threshold* $\hat{\theta}_{RS}(p, n_R, n_S, n_I)$ *and two functions* $\hat{\theta}_{RI}(\pi, p, n_R, n_S + n_I)$ *and* $\hat{\theta}_{IS}(\pi, p, n_S, n_I)$ *respectively increasing and decreasing in* π *such that:*

$$\hat{\theta}_{RI}\left(\widehat{\pi}(p, n_R, n_S, n_I), p, n_R, n_S + n_I\right) = \hat{\theta}_{IS}\left(\widehat{\pi}(p, n_R, n_S, n_I), p, n_S, n_I\right) = \hat{\theta}_{RS}(p, n_R, n_S, n_I).$$

A driver selects strategy R if $\theta < \hat{\theta}_{RS}(p, n_R, n_S, n_I)$ *and* $\theta < \hat{\theta}_{RI}(\pi, p, n_R, n_S + n_I)$*; selects S if* $\theta > \hat{\theta}_{RS}(p, n_R, n_S, n_I)$ *and* $\theta > \hat{\theta}_{IS}(\pi, p, n_S, n_I)$*, and selects I if* $\theta > \hat{\theta}_{RI}(\pi, p, n_R, n_S + n_I)$ *and* $\theta < \hat{\theta}_{IS}(\pi, p, n_S, n_I)$*. For* $\hat{\theta}_{RS} < \infty$*,* $\hat{\theta}_{RS}$ *is decreasing in p and* n_R*, and increasing in* n_S *and* n_I*. For* $\hat{\theta}_{RI} < \infty$*,* $\hat{\theta}_{RI}$ *is decreasing in p and* n_R*, and increasing in* π *and* $n_S + n_I$*. For* $\hat{\theta}_{IS} < \infty$*,* $\hat{\theta}_{IS}$ *is decreasing in* π*, p and* n_I*, independent of* n_R*, of ambiguous sign in* n_S*, but increasing in* n_S *with* $n_S + n_I$ *held fixed.*

5.2 Driver strategy choice and equilibrium

Proposition 6 characterizes individual driver strategy choices with *Costly information* for given traffic conditions. The analysis now proceeds to the derivation of equilibrium values of n_R^C, n_S^C and n_I^C as functions of π. The first step is to establish a critical price for information above which no drivers choose to be informed.

Proposition 7 *Under Assumptions 1-4, for any probability* $p \in (0,1)$ *and any* $N > 0$*, there exists a unique price* $\pi^C(p, N)$*, a unique risk aversion threshold* $\theta_0^C(p, N)$*, and a unique traffic equilibrium* $(n_{R0}^C(p, N)$*,* $n_{S0}^C = N - n_{R0}^C(p, N)$*,*$n_{I0}^C = 0)$ *such that the three indifference curves* $R \approx S$*,* $R \approx I$ *and* $I \approx S$ *cross at* $\left(\theta_0^C, \pi^C\right)$*. Functions* $\theta_0^C(p, N)$ *and* $n_{R0}^C(p, N)$ *are decreasing in p.*

Proof. By Proposition 6, for any (p, n_R, n_S, n_I) there exists a unique price $\pi = \hat{\pi}(p, n_R, n_S, n_I)$ and a unique risk aversion threshold $\hat{\theta}_{RS}(p, n_R, n_S, n_I)$ such that a driver with risk aversion $\hat{\theta}_{RS}$ is indifferent between R, S and I. With $\pi = \hat{\pi}$, $n_I = 0$. The size of group R is $n_R = NF(\theta)$ with $\theta = \hat{\theta}_{RS}(p, n_R, n_S, n_I = 0)$. By Assumption 4, $F(\cdot)$ is continuous and strictly increasing in θ over its support so that n_R is a continuous and strictly increasing function of θ, with $n_R = 0$ at $\theta = 0$ and $\underset{\theta\to\infty}{Lim}\ n_R = N$. By Proposition 3, $\hat{\theta}_{RS}$ is continuous and decreasing with n_R, and continuous and increasing with n_S. And by Assumption 4, $\hat{\theta}_{RS}(p, n_R = N, n_S = 0, n_I = 0) < \infty$. Hence the pair of equations $n_R = NF(\theta)$ and $\theta = \hat{\theta}_{RS}(p, n_R, n_S, n_I = 0)$ has a unique solution $n_R = n_{R0}^C(p, N)$ and $\theta = \theta_0^C(p, N)$ with $n_{R0}^C = NF\left(\theta_0^C\right)$ and $\theta_0^C = \hat{\theta}_{RS}\left(p, n_{R0}^C, N - n_{R0}^C, 0\right)$. In addition, $\pi^C(p, N) = \hat{\pi}\left(p, n_{R0}^C, N - n_{R0}^C, 0\right)$.

To establish the comparative statics properties for p, consider $p' > p$. The curve $n = NF(\theta)$ is unchanged while the curve $\theta = \hat{\theta}_{RS}(p, n_R, n_S, n_I = 0)$ is decreasing in p by Proposition 3. Hence $\theta_0^C(p', N) < \theta_0^C(p, N)$ and $n_{R0}^C(p', N) < n_{R0}^C(p, N)$. ■

Proposition 7 establishes the existence of a critical price for information, $\pi^C(p, N)$, at and above which no driver chooses to be informed and the equilibrium with *Costly information* is therefore the same as with *No information*. Consequently, $\theta_0^C(p, N) = \tilde{\theta}^Z\left(p, n_R^Z\right)$. It remains to establish existence of equilibrium when $\pi < \pi^C(p, N)$.

Theorem 5 *For any probability $p \in (0, 1)$, any $N > 0$ and any price $\pi \geq 0$, there exists a unique equilibrium $n_R^C(\pi, p, N)$, $n_S^C(\pi, p, N)$, $n_I^C(\pi, p, N) = N - n_R^C(\pi, p, N) - n_S^C(\pi, p, N)$. If $\pi \geq \pi^C(p, N)$, the equilibrium is as described in Proposition 7 . If $\pi < \pi^C(p, N)$, there exist unique risk aversion thresholds, $\theta_A(\pi, p, N)$ and $\theta_B(\pi, p, N) > \theta_A(\pi, p, N)$, such that a driver with $\theta < \theta_A(\pi, p, N)$ selects Strategy R, a driver with $\theta \in (\theta_A(\pi, p, N), \theta_B(\pi, p, N))$ selects Strategy I, and a driver with $\theta > \theta_B(\pi, p, N)$ selects Strategy S. The comparative statics properties of n_R^C, n_S^C, n_I^C, θ_A and θ_B are given in Table* 5.

Proof. For $\pi = \pi^C(p, N)$ the equilibrium is as described in Proposition 7 with no one choosing strategy I. The same equilibrium clearly applies for $\pi > \pi^C(p, N)$. For $\pi < \pi^C(p, N)$ all three strategies R, S and I are selected. By Proposition 6, for any (p, n_R, n_S, n_I) there exists a unique price $\hat{\pi}(p, n_R, n_S, n_I)$ and two functions $\hat{\theta}_{RI}(\pi, p, n_R, n_S + n_I)$ and $\hat{\theta}_{IS}(\pi, p, n_S, n_I)$ such that for $\pi < \hat{\pi}$, $\hat{\theta}_{RI} < \hat{\theta}_{IS}$ and $n_R = NF\left(\hat{\theta}_{RI}\right)$, $n_I = N\left(F\left(\hat{\theta}_{IS}\right) - F\left(\hat{\theta}_{RI}\right)\right)$ and $n_S = N\left(1 - F\left(\hat{\theta}_{IS}\right)\right)$. The five unknowns $\left(n_R^C, n_S^C, n_I^C, \hat{\theta}_{RI}, \hat{\theta}_{IS}\right)$ can be solved in two steps. The first step is to solve n_R and $\hat{\theta}_{RI}$. The function $NF(\theta)$ is continuous and strictly increasing in θ over its support. By Proposition 4 or 6, $\hat{\theta}_{RI}$ is continuous, decreasing in n_R, and increasing in $n_S + n_I$. And $\hat{\theta}_{RI}(\pi, p, N, 0) < \infty$. Hence the pair of equations $n_R = NF(\theta)$ and $\theta = \hat{\theta}_{RI}(\pi, p, n_R, N - n_R)$ has a unique solution $n_R = n_R^C(\pi, p, N)$ and $\theta = \theta_A(\pi, p, N)$ with $n_R^C = NF(\theta_A)$ and $\theta_A = \hat{\theta}_{RI}\left(\pi, p, n_R^C, N - n_R^C\right)$.

The second step is to solve n_S^C, n_I^C, and $\hat{\theta}_{IS}$. Now $\hat{\theta}_{IS} \in \left[\hat{\theta}_{RS}, \infty\right)$ and $n_S^C + n_I^C = N - n_R^C$. The function $n_S = N(1 - F(\theta))$ is continuous and decreasing from $N - n_R^C$ to 0 as $\hat{\theta}_{IS}$ increases from $\hat{\theta}_{RS}$ to ∞. By Proposition 5 or 6, $\hat{\theta}_{IS}$ is a continuous and increasing function of n_S with $n_S + n_I$ held fixed. Hence the pair of equations $n_S = N(1 - F(\theta))$ and $\theta = \hat{\theta}_{IS}\left(\pi, p, n_S, n_I = N - n_R^C - n_S\right)$ has a unique solution $n_S = n_S^C(\pi, p, N)$ and $\theta_B = \hat{\theta}_{IS}\left(\pi, p, n_S^C, N - n_R^C - n_S^C\right)$. Finally, $n_I = n_I^C(\pi, p, N) = N - n_R^C - n_S^C$.

Comparative statics properties of the equilibrium are derived in Appendix 12. ■

	π	p	N
n_R	$+$	$-$	$\in (0, 1)$
n_S	?	$+$	?
n_I	?	?	?
θ_A	$+$	$-$	?
θ_B	?	$-$	?

Table 5: Comparative statics of *Costly information* equilibrium with risk-averse drivers

Comparison of Table 5 with Table 4 reveals three differences in the properties of equilibria with risk-averse drivers and risk-neutral drivers. First, if the cost of information is positive, sufficiently risk-averse drivers eschew buying it and adopt strategy S instead. By contrast, if drivers are risk neutral none choose strategy S if information is relatively cheap. Second, n_S is an increasing function of the number of drivers, N, whereas with risk neutrality it is independent of N. Third, several of the comparative statics results are ambiguous in sign including all three of the derivatives for n_I. Indeed, it is not possible to rule out that the number of drivers who purchase information is a (locally) increasing function of the price although it seems highly unlikely that this will be the case.

Given Proposition 7 and Theorem 5 it is possible to draw Figure 5, which is the counterpart to Figure 4 with $\left(n_R^C, n_S^C, n_I^C\right)$ set at their equilibrium values given in Theorem 5. Individual preference rankings for the information regimes are:

$$\begin{cases} R \succ S \Leftrightarrow \varphi_{RS}(\theta, \pi; p, N) > 0, \\ R \succ I \Leftrightarrow \varphi_{RI}(\theta, \pi; p, N) > 0, \text{ and} \\ I \succ S \Leftrightarrow \varphi_{IS}(\theta, \pi; p, N) > 0, \end{cases}$$

where

$$\begin{cases} \varphi_{RS}(\theta, \pi; p, N) = \psi_{RS}\left(\theta; p, n_R^C(\pi, p, N), n_S^C(\pi, p, N), n_I^C(\pi, p, N)\right), \\ \varphi_{RI}(\theta, \pi; p, N) = \psi_{RI}\left(\theta, \pi; p, n_R^C(\pi, p, N), n_S^C(\pi, p, N) + n_I^C(\pi, p, N)\right), \text{ and} \\ \varphi_{IS}(\theta, \pi; p, N) = \psi_{IS}\left(\theta, \pi; p, n_S^C(\pi, p, N), n_I^C(\pi, p, N)\right). \end{cases}$$

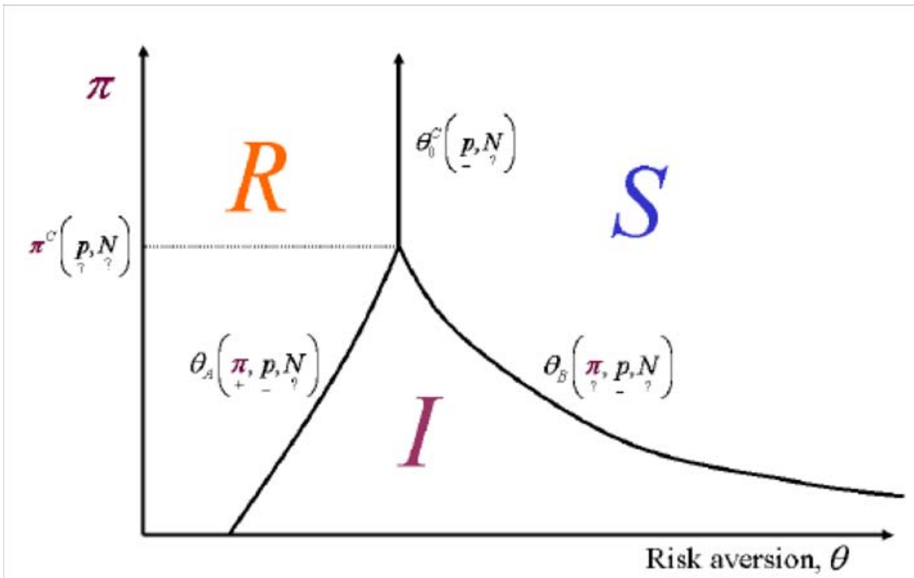

Figure 5: Strategy choice in *Costly information* equilibrium

The equilibrium threshold for $\pi \geq \pi^C(p, N)$ is $\theta_0^C(p, N) = \hat{\theta}_{RS}\left(p, n_{R0}^C, n_{S0}^C, 0\right)$, and for $\pi < \pi^C(p, N)$ the thresholds are

$$\begin{aligned} \theta_A(\pi, p, N) &= \hat{\theta}_{RI}\left(\pi, p, n_R^C(\pi, p, N), n_S^C(\pi, p, N) + n_I^C(\pi, p, N)\right), \text{ and} \\ \theta_B(\pi, p, N) &= \hat{\theta}_{IS}\left(\pi, p, n_S^C(\pi, p, N), n_I^C(\pi, p, N)\right). \end{aligned}$$

5.3 Welfare effects of Costly information

Four groups of drivers must be distinguished in assessing the welfare effects of *Costly information.* As shown in Figure 6, in order of increasing risk aversion these are: drivers who choose strategy R in both the *No information* and the *Costly information* regimes, drivers who choose R with *No information* and I with *Costly information*, drivers who choose S with *No information* and I with *Costly information*, and finally drivers who choose S in both regimes. The four groups will be called RR, RI, SI and SS respectively, and their compensating variations denoted CV_{RR}^C, CV_{RI}^C, CV_{SI}^C and CV_{SS}^C. Compensating variation is defined by a different equation for each group:

$$\begin{array}{ll} RR: & \begin{array}{c} pU\left(t_R^+\left(n_R^C\right) + CV_{RR}^C(\theta); \theta\right) + (1-p)\,U\left(t_R^- + CV_{RR}^C(\theta); \theta\right) = \\ pU\left(t_R^+\left(n_R^Z\right); \theta\right) + (1-p)\,U\left(t_R^-; \theta\right) \quad \text{if } \theta < \theta_A, \end{array} \\ RI: & \begin{array}{c} pU\left(t_S\left(n_S^C + n_I^C\right) + \pi + CV_{RI}^C(\theta); \theta\right) + (1-p)\,U\left(t_R^- + \pi + CV_{RI}^C(\theta); \theta\right) = \\ pU\left(t_R^+\left(n_R^Z\right); \theta\right) + (1-p)\,U\left(t_R^-; \theta\right) \quad \text{if } \theta_A < \theta < \theta_0^C, \end{array} \\ SI: & \begin{array}{c} pU\left(t_S\left(n_S^C + n_I^C\right) + \pi + CV_{SI}^C(\theta); \theta\right) + (1-p)\,U\left(t_R^- + \pi + CV_{SI}^C(\theta); \theta\right) = \\ U\left(t_S\left(n_S^Z\right); \theta\right) \quad \text{if } \theta_0^C < \theta < \theta_B, \end{array} \\ SS: & \begin{array}{c} pU\left(t_S\left(n_S^C + n_I^C\right) + CV_{SS}^C(\theta); \theta\right) + (1-p)\,U\left(t_S\left(n_S^C\right) + CV_{SS}^C(\theta); \theta\right) = \\ U\left(t_S\left(n_S^Z\right); \theta\right) \quad \text{if } \theta > \theta_B. \end{array} \end{array} \tag{3}$$

The left-hand side of each equation in (3) is expected utility with *Costly information*, and the right-hand side is expected utility with *No information*. Note that the cost of information, π, is added to CV_{RI}^{C} and CV_{SI}^{C} for groups RI and SI which purchase information so that their CVs are defined as net of π. As formalized in Prop. 8 below, $CV_{RR}^{C}(\theta)$ and $CV_{RI}^{C}(\theta)$ are increasing functions of θ, and $CV_{SI}^{C}(\theta)$ and $CV_{SS}^{C}(\theta)$ are decreasing functions of θ. Consequently, CV is highest for drivers who are indifferent between strategies R and S in the *No information* regime in the same way that CV for *Free information* is highest for the indifferent driver. CV is also negative for drivers who are sufficiently risk averse.

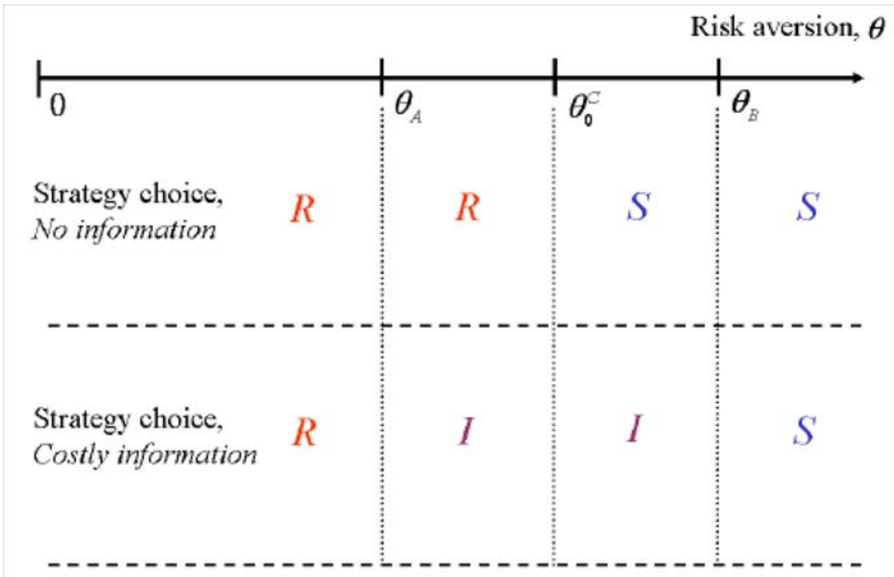

Figure 6: Strategy choices for *No information* and *Costly information* regimes

Compensating variation for *Costly information* and *Free information* can be ranked by comparing the defining equations in (2) and (3):

$$\begin{aligned}
&CV_{RR}^{C}(\theta) < CV_{R}^{F}(\theta) \text{ since } n_{R}^{C+} > n_{R}^{F+},\\
&CV_{RI}^{C}(\theta) \gtrless CV_{R}^{F}(\theta) \text{ depending on } \pi \text{ and the relative magnitudes of } t_S\left(n_S^C + n_I^C\right) \text{ and } t_R^{+}\left(n_R^{F+}\right),\\
&CV_{SI}^{C}(\theta) \gtrless CV_{S}^{F}(\theta) \text{ as } CV_{RI}^{C}(\theta) \gtrless CV_{R}^{F}(\theta),\\
&CV_{SS}^{C}(\theta) \gtrless CV_{S}^{F}(\theta) \text{ depending on the magnitudes of } t_R^{+}\left(n_R^{F+}\right),\ t_S\left(n_S^C + n_I^C\right),\ t_S\left(n_S^C\right) \text{ and } t_R^{-}.
\end{aligned}$$

Compensating variation for group RR is unambiguously smaller for *Costly information* than for *Free information*, but no general ranking is possible for the other three groups. These results are formalized in Prop. 8 which is the counterpart to Prop. 2 for *Free information*:

Proposition 8 *Under Assumptions 1-4:*

(a) The compensating variation for Costly information, $CV^{C}(\theta)$, *is an increasing function of risk aversion for the least risk-averse drivers who take route R with No information, and a decreasing function of risk aversion for the most risk-averse drivers who take route S with No information.*

(b) When the risk aversion parameter is distributed over $\mathbb{R}^{+}$, $CV^{C}(\theta) < 0$ *for the most risk-averse drivers.*

(c) $CV^{C}(\theta) < CV^{F}(\theta)$ *for the least risk-averse drivers who take route R in both the No information and Costly information regimes. For other drivers the ranking of* $CV^{C}(\theta)$ *and* $CV^{F}(\theta)$ *is ambiguous.*

Proof. Part (a) is proved in Appendix 7, and Part(b) is proved in Appendix 8. Part (c) was proved in the text. ■

6 Private information

As the cost of information rises towards $\pi^C(p, N)$ the number of drivers who purchase information approaches zero and information effectively becomes private in the sense that only a few drivers exploit it. As explained in Section 2, the *Costly information* equilibrium approaches the equilibrium with *No information* and the compensating variation for *Costly information* (gross of the cost) approaches the compensating variation for *Private information.* The compensating variations for *Private information* for Groups R and S, $CV_R^I(\theta)$ and $CV_S^I(\theta)$ respectively, are defined by the conditions:

$$\begin{array}{ll} R: & pU\left(t_S\left(n_S^Z\right)+CV_R^I(\theta);\theta\right)+(1-p)\,U\left(t_R^-+CV_R^I(\theta);\theta\right)= \\ & \quad pU\left(t_R^+\left(n_R^Z\right);\theta\right)+(1-p)\,U\left(t_R^-;\theta\right) \quad \text{if } \theta<\theta_{RS}, \\ S: & pU\left(t_S\left(n_S^Z\right)+CV_S^I(\theta);\theta\right)+(1-p)\,U\left(t_R^-+CV_S^I(\theta);\theta\right)= \\ & \quad U\left(t_S\left(n_S^Z\right);\theta\right) \quad \text{if } \theta>\theta_{RS}. \end{array} \tag{4}$$

Since $t_R^+\left(n_R^Z\right) > t_S\left(n_S^Z\right)$, $CV_R^I(\theta) > 0$, and since $t_S\left(n_S^Z\right) > t_R^-$, $CV_S^I(\theta) > 0$. The compensating variation for *Private information* is therefore unambiguously positive. Given Assumption 3, $CV_R^I(\theta)$ is an increasing function of θ, and $CV_S^I(\theta)$ is a decreasing function of θ so that — once again — compensating variation is highest for drivers with risk aversion θ_{RS} who are indifferent between strategies R and S with *No information*.

By comparing (4) with (2) and (3), it is clear that the compensating variation for *Private information* is larger than the compensating variation for either *Free information* or *Costly information*[12]:

$$\begin{array}{l} CV_R^I(\theta) > CV_R^F(\theta) \text{ and } CV_S^I(\theta) > CV_S^F(\theta) \text{ since } t_S\left(n_S^Z\right) < t_S\left(n_S^{F+}\right) = t_R^+\left(n_R^{F+}\right), \\ CV_R^I(\theta) > CV_{RR}^C(\theta) \text{ and } CV_R^I(\theta) > CV_{RI}^C(\theta) \text{ since } t_S\left(n_S^Z\right) < t_S\left(n_S^C\right) < t_R^+\left(n_R^C\right), \\ CV_S^I(\theta) > CV_{SI}^C(\theta) \text{ and } CV_S^I(\theta) > CV_{SS}^C(\theta) \text{ since } t_S\left(n_S^Z\right) < t_S\left(n_S^C+n_I^C\right) \text{ and } t_R^- < t_S\left(n_S^C\right). \end{array}$$

These results are formalized in

Proposition 9 *Under Assumptions 1-4:*

(a) The compensating variation for Private information is positive for all drivers: $CV^I(\theta) > 0$.

(b) The compensating variation for Private information exceeds the compensating variation for either Free information or Costly information: $CV^I(\theta) > CV^F(\theta)$ *and* $CV^I(\theta) > CV^C(\theta)$.

(c) The compensating variation for Private information is an increasing function of risk aversion for the least risk-averse drivers who take route R with No information, and a decreasing function of risk aversion for the most risk-averse drivers who take route S with No information.

Proof. Parts (a) and (b) are proved in the text. Part (c) is proved in Appendix 8. ■

Private information is more valuable to an individual than is *Costly information* or *Free information* because the benefits from selecting the quicker route diminish as more drivers exploit the information. Diminishing returns of this sort have been identified in a number of studies of ATIS with exogenous or endogenous market penetration. In the model here the negative external effects of information arise not only because the route choice decisions of informed drivers raise travel times for other informed drivers, but also because they contribute to uncertainty in travel times. This impact is especially pernicious for the most risk averse drivers who try to avoid uncertainty by sticking to the “safe” route. However, uninformed and less risk averse drivers who take the risky route benefit when informed drivers switch to the safer route on *Bad* days.

An interesting property of the model is that the compensating variation for *Private information* for the drivers with risk aversion θ_{RS} who value it most is strictly positive even in the limit as the probability of *Bad* days decreases to zero. This result is formalized in:

Proposition 10 *If Assumption 3 holds and the risk aversion parameter is distributed over* $\mathbb{R}^+$*, then as the probability of Bad days decreases to zero the maximum compensating variation for Private information approaches a limiting value of* $Min\left(t_S(0) - t_R^-, t_R^+(N) - t_S(0)\right) > 0$.

[12] Since *Free information* is a limiting case of *Costly information* as the cost of information approaches zero, the ranking of $CV^I(\theta)$ and $CV^F(\theta)$ is necessarily the same as the ranking of $CV^I(\theta)$ and $\underset{\pi\to 0}{Lim} CV^C(\theta)$.

Proof. See Appendix 13. ■

An intuitive explanation of sorts for Prop. 10 runs as follows. With *No information* a driver must choose between Strategy R and Strategy S. If he chooses R, and *Private information* then becomes available, he can occasionally save $t_R^+(N) - t_S(0)$ in travel time. Since this is a recovery from the worst state (a *Bad* day) an extremely risk-averse driver is willing to pay nearly $t_R^+(N) - t_S(0)$ for the information even though it will be exploited only rarely. If the driver chooses Strategy S instead, and *Private information* becomes available, he can almost always save $t_S(0) - t_R^-$ in travel time. Since this is a gain in the good state (a *Good* day) the driver is willing to pay nearly $t_S(0) - t_R^-$ for the information. Now the driver's expected utility with *Private information* is a given amount. Hence the driver will effectively choose between strategies R and S according to which willingness to pay is smaller. Hence the actual compensating variation for *Private information* is the lesser of $t_R^+(N) - t_S(0)$ and $t_S(0) - t_R^-$.

Prop. 10 contrasts with Propositions 2 and 8 which establish that the compensating variation for *Free information* and *Costly information* are negative for very risk-averse individuals. This highlights the contrast between the values of public and private information that was demonstrated by Hirshleifer (1971). It also suggests that there may be a niche demand for ATIS by highly risk-averse travelers even if travel conditions are fairly predictable.

7 Numerical example

The numerical example is representative of a commuting corridor. Travel time functions for the safe route and the risky route on *Bad* days, have a Bureau of Public Roads form[13]:

$$\begin{cases} t_S(n_S) = \tau_S \left(1 + \left(\frac{n_S}{K_S}\right)^b\right), \\ t_R^+(n_R) = \tau_R^+ \left(1 + \left(\frac{n_R}{K_R}\right)^b\right), \end{cases} \tag{5}$$

where τ_S and τ_R^+ are free-flow travel times, K_S and K_R are capacities, and b is a parameter. For the base case of the example the probability of a *Bad* day is $p = 0.2$ and the number of drivers is $N = 10,000$. Other parameter values are $\tau_S = 25$ min., $\tau_R^+ = 25$ min., $t_R^- = 20$ min., $K_S = 10,000$ per hr., $K_R = 8,000$ per hr. and $b = 2$. Drivers have CARA preferences and θ has a log-logistic distribution with parameter $\bar{\theta} = 2$; i.e. $F(\theta) = \theta / (\theta + \bar{\theta}) = \theta / (\theta + 2)$. [14] With these parameter values Assumptions 1-4 are all satisfied.

7.1 Base-case results

Summary statistics for the equilibria with the base-case parameterization are listed in Column 1 of Table 6.

7.1.1 No information

For the *No information* regime $n_R^Z = 6,654$: about two thirds of drivers (those with $\theta < \theta_{RS} = 3.98$) choose the risky route. Travel time on the risky route is $t_R^- = 20$ min. on *Good* days, and $t_R^{+Z} = 42.3$ min. on *Bad* days. Travel time on the safe route is $t_S = 27.8$ min. in both states. Expected travel time is $pt^- + (1-p)\, t_R^{Z+} = 24.46$ for group R, and 27.8 min. for group S. Group S therefore incurs more than three min. extra mean travel time for the privilege of travel time reliability.

7.1.2 Free information

With *Free information* all drivers take route R on *Good* days ($n_R^{F-} = 10,000$) whereas less than half of them do on *Bad* days ($n_R^{F+} = 4,444$). The 4:5 division of traffic between the two routes is independent of the distribution of risk preferences, and the set of drivers who take Route R on *Bad* days is indeterminate because travel time is known

[13] De Palma and Picard (2006b) adopt a different functional form.

[14] de Palma and Picard (2005) estimated the distribution of risk aversion for a sample of individuals taking morning trips in the Paris area. About 60 percent of the sample exhibited risk aversion. For this segment of the sample, absolute risk aversion was found to be relatively constant and therefore more consistent with CARA than with CRRA preferences. (CARA also has the advantage that it yields a closed-form expression for compensating variation, whereas CRRA does not.) A least-squares fit for the log-logistic parameter is $\bar{\theta} = 3.25$ per hr. A conservative value of $\bar{\theta} = 2$ per hr. is chosen for the base case here.

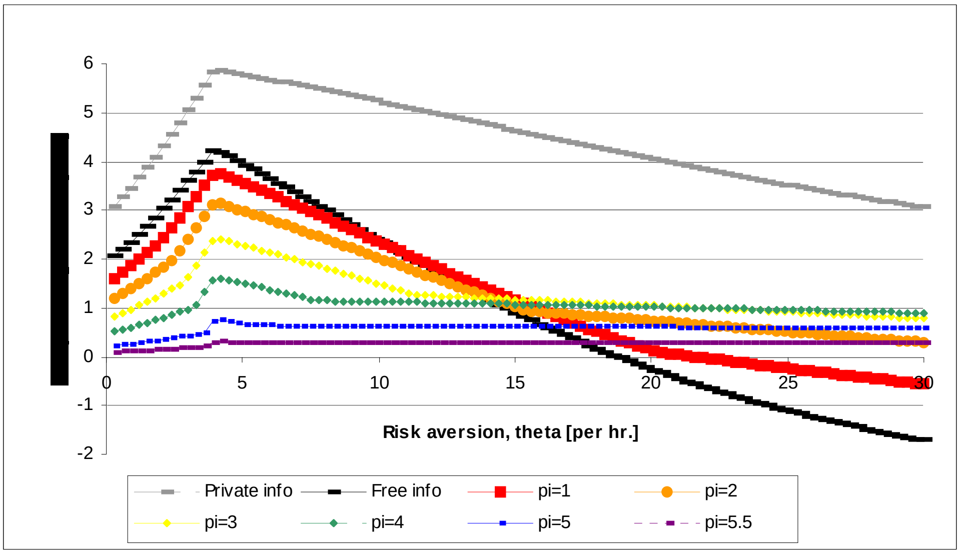

Figure 7: Compensating variations for *Free*, *Costly* and *Private information*

in advance. All drivers experience the same travel times of $t_R^- = 20$ min. on *Good* days, and $t_R^{F+} = t_S^{F+} = 32.72$ min. on *Bad* days. Expected travel time is 22.54 min.: a drop from the *No information* regime of 1.92 min. for Group R, and 5.26 min. for Group S. Average compensating variations are 2.60 min. for Group R, and 1.38 min. for Group S. Thus, although Group S experiences a much larger average reduction in travel time *as per* Theorem 4, it benefits less from *Free information* in terms of compensating variation.

Compensating variation for *Free information*, $CV^F(\theta)$, is plotted in Figure 7. (The other seven curves are discussed later.) Consistent with Prop. 2, $CV^F(\theta)$ reaches a maximum at $\theta_{RS} = 3.98$, and then drops — eventually below zero. Nearly 10 percent of the most risk averse users (with $\theta > 18.7$) and over a quarter of Group S end up worse off.

7.1.3 Costly information

The numerical counterpart to Figure 5 for the *Costly information* regime is shown in Figure 8. As the cost of information rises from 0 to the choke price of $\pi^C(p, N) = CV^I(\theta_{RS}) = 5.88$, the number of drivers purchasing information drops steadily to zero. Figure 9 shows how the fractions of drivers in each of the four groups evolve with π. As expected, the two groups that acquire information (RI and SI) decline steadily in size towards zero, whereas Groups RR and SS grow.

Figure 10 presents the compensating variation for *Costly information* by group. The two informed groups always fare better than do the uninformed, and their benefits increase as information becomes cheaper except when the price nears zero. As far as the two uninformed groups, the benefits to Group RR increase monotonically as information becomes cheaper. Group SS gains as well while information is expensive, but it loses out once information becomes cheaper. Nevertheless, the aggregate benefits for the two uninformed groups increase steadily because Group SS shrinks in size. Overall, the pattern is one in which most drivers benefit from information while a small minority of highly risk-averse drivers suffer appreciably.

Figure 7 compares the compensating variations for *Free* information, *Costly* information and *Private information* for several values of π. Consistent with Props. 2, 8 and 9, all three compensating variations reach their maxima at $\theta = \theta_{RS}$. And consistent with Prop. 9, $CV^I(\theta)$ exceeds $CV^F(\theta)$ and $CV^C(\theta, \pi)$ over the whole range of π. The behaviour of $CV^C(\theta, \pi)$ is rather complex. In the lower range of θ, $CV^C(\theta, \pi)$ is bounded between 0 and $CV^F(\theta)$ and decreases monotonically with π. For larger values of θ, $CV^C(\pi, \theta) > CV^F(\theta)$ because highly risk-averse drivers benefit from the fact that with *Costly information* fewer drivers shift to the safe route on *Bad* days. $CV^C(\pi, \theta)$ varies non-monotonically with π: it rises initially, but eventually declines towards zero as the

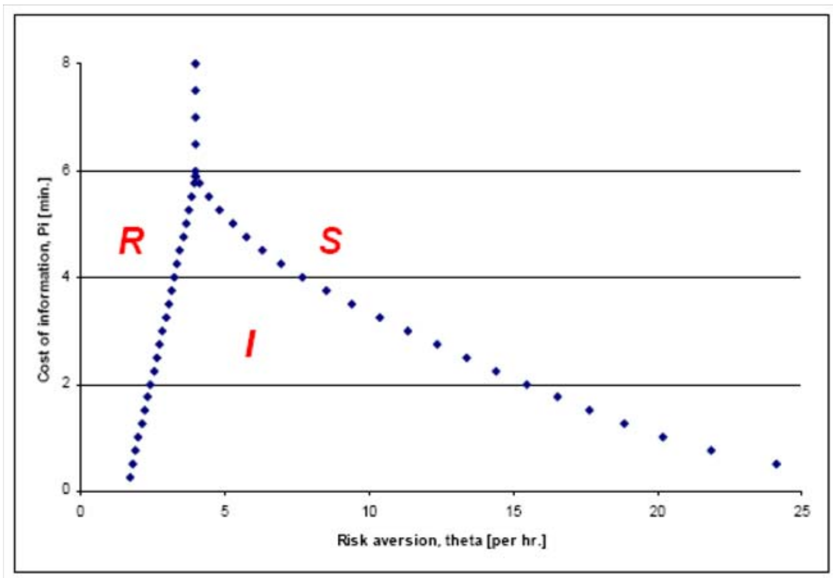

Figure 8: Strategy choice in *Costly information* equilibrium in numerical example

number of informed drivers diminishes and with it the potential for information to be beneficial. However, part (b) of Prop. 8 guarantees that some drivers are worse off as long as $\pi < \pi^C(p, N)$.

7.1.4 Sensitivity analysis

Probability of *Bad* days Figure 11 shows how the effects of *Free information* evolve with p (the base-case value of $p = 0.2$ is marked by the vertical dashed line). Over most of the range $p \in [0, 0.5]$ the mean travel time saving and compensating variation decrease for Group S, and increase for Group R. In the limit as $p \to 0$ the effects on Group R decrease to zero.

Distribution of risk aversion If parameter $\bar{\theta}$ of the log-logistic distribution is reduced to zero, the population degenerates to a set of N identical risk-neutral drivers such as considered in Section 3. Equilibria for this case are shown in Column 2 of Table 6. Compared to the base case, the fraction of drivers taking route R with *No information* increases from roughly 2/3 to 4/5, and the difference in travel times on R and S on *Bad* days is accentuated. Mean travel time savings and compensating variations all coincide at 3.46 min., and all drivers benefit from *Free information* by this amount. Figure 12 shows that as parameter $\bar{\theta}$ rises from 0 through the base case value of $\bar{\theta} = 2$ and upwards, mean travel time saving and CV_R^F fall slowly for Group R. In contrast, for Group S mean travel time saving rises and CV_S^F drops rather sharply so that the divergence between travel time saving and compensating variation is much sharper than for Group R. Consistent with this, the fraction of drivers made worse off by *Free information* rises from zero at $\bar{\theta} = 0$ to more than one fifth at $\bar{\theta} = 8$.

Other variations Raising the number of drivers from $N = 10,000$ to $N = 15,000$ (see Column 3 in Table 6) increases the effects of *Free information* as expected with the interesting exception that the mean compensating variation for Group S decreases slightly. This is attributable to the fact that with *Free information* travel time on the safe route varies by 8.25 min. $(42.36 - 34.11)$ min. compared to only 4.92 min. $(32.72 - 27.80)$ in the base case.

Reducing the capacity of route R in the bad state from $K_R = 8,000$ to $K_R = 4,000$ (see Column 4) results in similar travel times on *Bad* days with *No information*. But the benefits from *Free information* are generally larger because the variation in performance of route R is more pronounced.

In Column 5 parameter b of the travel time function is reduced from $b = 2$ to $b = 1$.[15] The main impacts of

[15] It is usually assumed that travel time is a strictly convex function of flow on a link, and a common choice is to set $b = 4$. However,

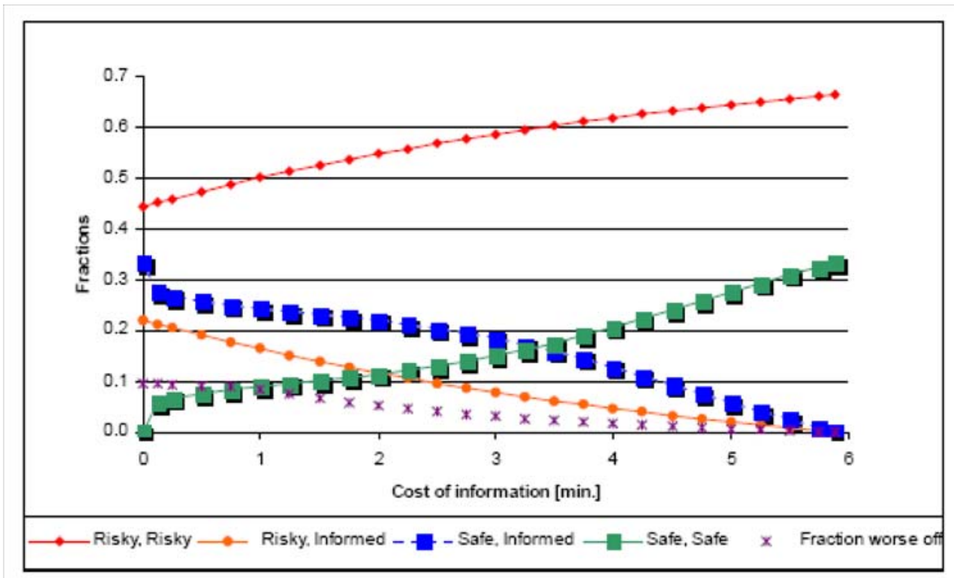

Figure 9: Route splits in numerical example

interest are to increase the divergence between mean travel time reduction and compensatng variation for group S, and to increase the fraction of drivers made worse off by *Free information*.[16]

In Column 6 the free-flow travel time on Route R, τ_R^+, is reduced from 25 min. to 20 min. to match the free-flow travel time on Good days, τ_R^-.[17] This leads to a considerably larger usage of Route R in both the *No information* and *Free information* regimes. The welfare effects of information are slightly smaller than in the base case, but follow the same pattern.

The final "extreme" case (with $N = 15,000$, $b = 1$, $p = 0.5$ and $\bar{\theta} = 8$) combines several of the one-way parameter variations in a direction designed to accentuate the adverse effects of *Free information*. Doing so does not affect greatly the average compensating variation for group R, but it does result in a negative average compensating variation for Group S ($CV_S^F = -0.21$) and losses for nearly 30 percent of all drivers. This illustrates rather dramatically that even with heavy congestion, a high probability of capacity loss, and a large proportion of highly risk-averse drivers, information is not necessarily very beneficial.[18]

travel time functions are typically specified in terms of static flows. When specified in terms of trips, the functional relationship can be approximately linear. In the case of Vickrey's bottleneck queuing model with identical travelers and linear schedule delay cost functions, the relationship is exactly linear (Arnott et al. 1998).

[16] Raising parameter b to $b = 4$ has the mirror image effect of reducing the fraction made worse off to about 4 percent.

[17] This violates the strict inequality $t_R^- < t_R^+(0)$ in Assumption 2, but does not invalidate the equilibrium.

[18] It is possible to construct examples in which the average compensating variation is negative and a majority of drivers lose. De Palma and Picard (2006b) do so by setting an extreme value of $\bar{\theta} = 120$ (per hr.) and using in place of (5) the travel time functions $t_S(n_S) = \tau_S\left(1 + \frac{n_S}{K_S}\right)^b$ and $t_R^+(n_R) = \tau_R^+\left(1 + \frac{n_R}{K_R}\right)^b$.

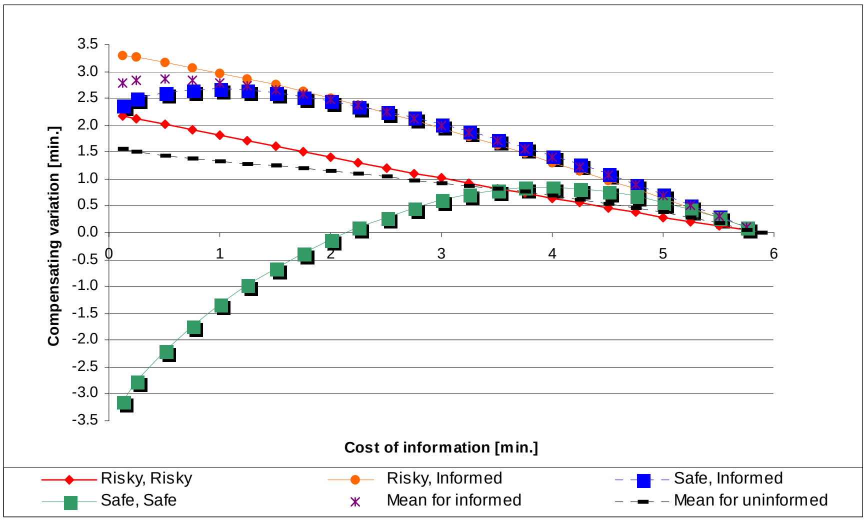

Figure 10: Compensating variation for *Costly information* in numerical example

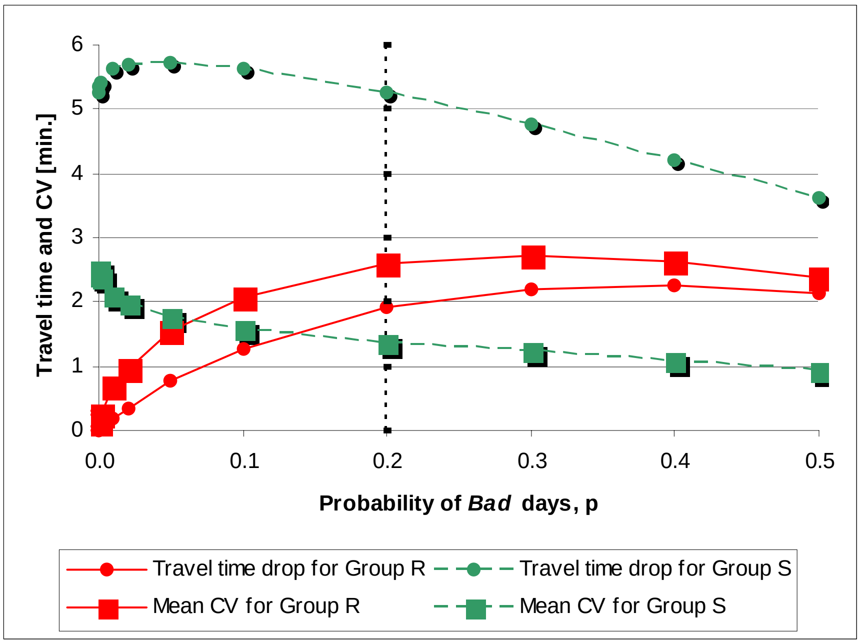

Figure 11: Effects of varying probability of *Bad* days in numerical example

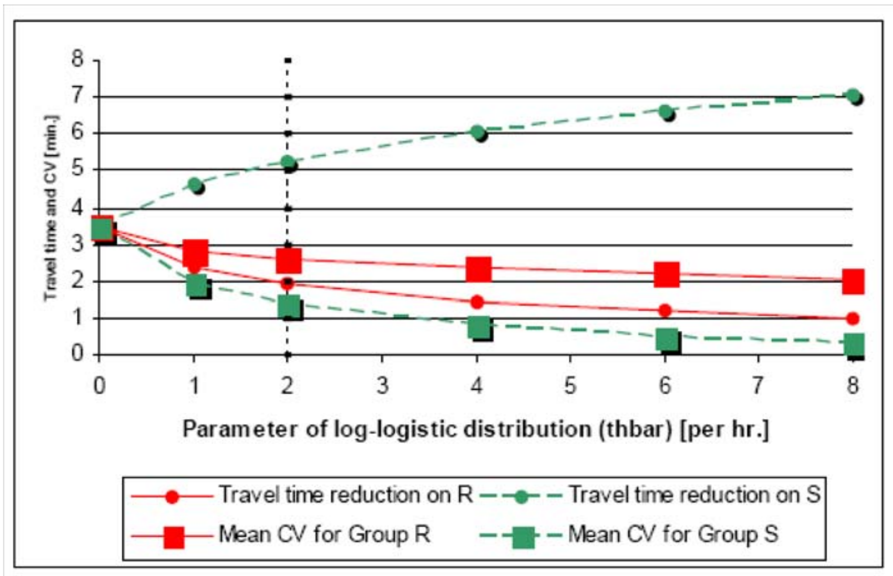

Figure 12: Effects of varying parameter $\bar{\theta}$ in numerical example

	1	2	3	4	5	6	7
	Base case	Risk neutral	$N =$ 15000	$K_R =$ 4000	$b = 1$	$\tau_R^+ = 20$	Extreme case†
			No information equilibrium				
n_R^Z	6,654	8,000	8,963	4,665	7,144	9,117	7,551
n_S^Z	3,346	2,000	6,037	5,335	2,856	883	7,449
t_R^{Z+}	42.30	50.00	56.38	59.01	47.32	37.45	48.60
t_S	27.80	26.00	34.11	32.11	32.14	26.60	43.62
			Free information equilibrium				
n_R^{F+}	4,444	4,444	6,667	2,857	4,444	5,607	6,667
n_S^{F+}	5,556	5,556	8,333	7,143	5,556	4,393	8,333
$t_R^{F+} = t_S^{F+}$	32.72	32.72	42.36	37.76	38.89	29.82	45.83
			Impacts of *Free information*				
Mean travel time red'n for group R	1.92	3.46	2.80	4.25	1.69	1.52	1.38
Mean travel time red'n for group S	5.26	3.46	9.64	8.56	8.36	4.63	10.71
Mean CV_R^F for group R	2.60	3.46	4.21	5.53	2.68	2.08	2.10
Mean CV_S^F for group S	1.38	3.46	1.31	3.92	0.29	1.50	-0.21
Mean CV^F for all drivers	2.19	3.46	3.05	4.67	1.99	1.94	0.96
Max. $CV^F(\theta_{RS})$	4.26	3.46	7.24	7.75	5.32	3.73	2.69
% drivers worse off	9.64	0	15.06	10.62	12.60	6.38	29.85
			Private information				
Max. $CV^I(\theta_{RS})$	5.88	4.80	10.39	9.32	8.54	4.89	4.83

Table 6: Effects of *Free information*: Sensitivity analysis

† $N = 15,000$, $b = 1$, $p = 0.5$, $\bar{\theta} = 8$

8 Conclusions and extensions

This paper has studied the information-acquisition and route-choice decisions of risk-averse drivers on a simple road network with one "safe" route and one "risky" route. Four information regimes are considered: *No information*, *Free information* — which is publicly available at no cost, *Costly information* — which is publicly available for a fee, and *Private information* — which is available free to a single individual.

Several general theoretical results are derived. First, it is drivers with intermediate levels of risk aversion who purchase information so that they can select the quickest route each day. The least risk-averse drivers remain uninformed and take the risky route every day, and the most risk-averse drivers take the safe route every day. This pattern mirrors a finding of Emmerink et al. (1996) that it is individuals with intermediate demands for travel who gain the most from information because travel is worthwhile for them under some conditions but not others.

Second, *Private information* is always beneficial to an individual driver relative to *No information*, and the benefit exceeds the benefits they derive from *Free information* or *Costly information*. Third, *Free information* and *Costly information* benefit drivers who are risk neutral or moderately risk averse. But very risk-averse drivers end up worse off even though some of them may be willing to pay for the information. A numerical example suggests that losers are likely to comprise a relatively small fraction of the population, but also that their losses as measured by compensating variation can be comparable to the highest gains of other drivers (cf. Figure 10).

The analysis could fruitfully be extended in various directions of which two will be mentioned. One is to examine more complex road networks. The two-route network with one safe route has the advantage of being amenable to analytical methods. And it is a natural choice to demonstrate the potential drawbacks of public information to highly risk-averse drivers. But real applications of ATIS in urban areas have to contend with multiple links and routes. Furthermore, the property of the model that information is most valuable to drivers with intermediate risk aversion is counterintuitive insofar as the benefits from information would seem, *a priori*, to be greatest for the most risk-averse individuals. In part, this result is driven by the existence of a safe route with superior "worst-case" prospects than the other route.

A second extension is to consider alternatives to the expected utility paradigm. Empirical evidence has been accumulating at least since Allais (1953) that contradicts expected utility theory, and in recent years Prospect Theory and other non-expected utilities have been applied in transportation research. Nevertheless, there are several reasons why these alternatives do not (at least yet) offer a clearly superior paradigm to expected utility theory for analyzing traveler decisions of the sort considered here. First, route-choice decision-making differs substantially from gambling on monetary values, and it is not obvious that similar behavioral patterns apply in the two contexts. Moreover, it is not clear what is an appropriate reference point for Prospect Theory (Avineri and Prashker, 2004). Second, route-choice decisions are made repeatedly for commuting and other routine trips, and it is plausible that as individuals become familiar with a particular environment their travel decisions will converge towards expected utility maximization.[19] Third, Avineri and Prashker (2005) found that Cumulative Prospect Theory failed to predict route-choice feedback-based decisions. There is a need for more empirical studies of risk aversion and route-choice decisions in the context of information provision — both in the field and in laboratory settings (see, for example, Helbing (2004) and Rapoport et al. (2005)).

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9 Appendixes

9.1 Appendix 1: Assumption 3, CRRA and CARA preferences

9.1.1 CRRA preferences

$U(t;\theta) = -\frac{t^{1+\theta}}{1+\theta}$ is differentiable and $U(t;0) = -t$. For $\theta > 0$ and $t > 0$, $U(t;\theta)$ is negative and strictly concave in t. $\frac{\partial U}{\partial \theta} = -\frac{(1+\theta)t^{1+\theta}\ln t - t^{1+\theta}}{(1+\theta)^2} = -\frac{t^{1+\theta}}{1+\theta}\left(\ln t - \frac{1}{1+\theta}\right)$. Now $t^{1+\theta} = -(1+\theta)U$, and $\ln t = \frac{\ln(-(1+\theta)U)}{1+\theta}$. Hence $\phi(U) \equiv \frac{\partial U}{\partial \theta} = U\frac{\ln(-(1+\theta)U)-1}{1+\theta}$, $\frac{\partial \phi}{\partial U} = \frac{\ln(-(1+\theta)U)}{1+\theta}$, and $\frac{\partial^2 \phi}{\partial U^2} = \frac{1}{(1+\theta)U} < 0$ for $t > 0$. Finally, $\underset{\theta\to\infty}{Lim}$ $\frac{U(t_2;\theta)}{U(t_1;\theta)} = \underset{\theta\to\infty}{Lim}\left(\frac{t_2}{t_1}\right)^{1+\theta} = \infty$ for $t_2 > t_1 > 0$.

9.1.2 CARA preferences

$U(t;\theta) = \frac{1-\exp(\theta t)}{\theta}$ is differentiable and $U(t;0) = -t$. For $\theta > 0$ and $t > 0$, $U(t;\theta)$ is negative and strictly concave in t. $\frac{\partial U}{\partial \theta} = \frac{-\theta + t\exp(\theta t)+\exp(\theta t)-1}{\theta^2}$.Now $\exp(\theta t) = 1-\theta U$, and $t = \frac{\ln(1-\theta U)}{\theta}$. Hence $\phi(U) \equiv \frac{\partial U}{\partial \theta} = \frac{-\theta(1-\theta U)\frac{\ln(1-\theta U)}{\theta}+1-(1-\theta U)}{\theta^2} = \frac{-(1-\theta U)\ln(1-\theta U)+\theta U}{\theta^2}$. $\frac{\partial \phi}{\partial U} = \frac{\theta\ln(1-\theta U)+\theta-\theta}{\theta^2} = \frac{\ln(1-\theta U)}{\theta}$, and $\frac{\partial^2\phi}{\partial U^2} = -\frac{1}{(1-\theta U)} < 0$. Finally,
$\underset{\theta\to\infty}{Lim}\frac{U(t_2;\theta)}{U(t_1;\theta)} = \underset{\theta\to\infty}{Lim}\frac{1-\exp(\theta t_2)}{1-\exp(\theta t_2)} = \underset{\theta\to\infty}{Lim}\exp(\theta(t_2-t_1)) = \infty$ for $t_2 > t_1 > 0$.

9.2 Appendix 2: Comparative statics properties of equilibria with risk-neutral drivers

9.2.1 No information regime

In the *No information* equilibrium the division of traffic, n_R^Z and n_S^Z, equalizes expected travel costs between routes: $\mathbb{E}C^Z = (1-p)t_R^- + pt_R^+(n_R^Z) = t_S(n_S^Z)$, where $n_S^Z = N - n_R^Z$. Comparative statics properties of the equilibrium are derived by totally differentiating this condition:

$$\frac{\partial n_R^Z}{\partial p} = \frac{t_R^- - t_R^+(n_R^Z)}{\frac{\partial t_S(N-n_R^Z)}{\partial n_S}+p\frac{\partial t_R^+(n_R^Z)}{\partial n_R}} < 0,\ \frac{\partial n_R^Z}{\partial N} = \frac{\frac{\partial t_S(N-n_R^Z)}{\partial n_S^Z}}{\frac{\partial t_S(N-n_R^Z)}{\partial n_S}+p\frac{\partial t_R^+(n_R^Z)}{\partial n_R}} \in (0,1),$$

$$\frac{\partial E\cdot C^Z}{\partial p} = -\frac{\partial t_S(N-n_R^Z)}{\partial n_S}\frac{\partial n_R^Z}{\partial p} > 0,\ \frac{\partial E\cdot C^Z}{\partial N} = \frac{\partial t_S(N-n_R^Z)}{\partial n_S}\left(1-\frac{\partial n_R^Z}{\partial N}\right) > 0.$$

9.2.2 Costly information regime

Equilibrium when $n_S^C > 0$ If strategies R, I and S are all adopted in the *Costly information* equilibrium, the numbers in each group, n_R^C, n_S^C and n_I^C, are determined by the accounting identity

$$n_R^C + n_S^C + n_I^C = N, \tag{6}$$

and two equal-cost conditions. First, the expected costs of Strategies I and R must be equal:

$$\pi + (1-p)t_R^- + t_S(n_S^C + n_I^C) = (1-p)t_R^- + pt_R^+(n_R^C). \tag{7}$$

Condition (7) can be written $\pi = p\left(t_R^+(n_R^C) - t_S(n_S^C + n_I^C)\right)$: the cost of information must balance the travel time saving gained by taking Route S rather than Route R on *Bad* days. Second, the expected costs of Strategies I and S must be equal:

$$\pi + (1-p)t_R^- + t_S(n_S^C + n_I^C) = (1-p)t_S(n_S^C) + pt_S(n_S^C + n_I^C). \tag{8}$$

Condition (8) can be written $\pi = (1-p)\left(t_S(n_S^c) - t_R^-\right)$: the cost of information must balance the travel time saving gained by taking Route R rather than Route S on *Good* days.

Totally differentiating (6), (7) and (8) one obtains

$\frac{\partial n_R^C}{\partial \pi} = \frac{1}{p\left(\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}\right)} > 0$, $\frac{\partial n_R^C}{\partial p} = \frac{t_S\left(n_S^C+n_I^C\right)-t_R^+\left(n_R^C\right)}{p\left(\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}\right)} < 0$,

$\frac{\partial n_R^C}{\partial N} = \frac{\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}}{\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}} \in (0,1)$; $\frac{\partial n_S^C}{\partial \pi} = \frac{1}{(1-p)\frac{\partial t_S\left(n_S^C\right)}{\partial n_S}} > 0$, $\frac{\partial n_S^C}{\partial p} = \frac{t_S\left(n_S^C\right)-t_R^-}{(1-p)\frac{\partial t_S\left(n_S^C\right)}{\partial n_S}} > 0$, $\frac{\partial n_S^C}{\partial N} = 0$;

$\frac{\partial n_I^C}{\partial \pi} = -\left(\frac{\partial n_R^C}{\partial \pi}+\frac{\partial n_S^C}{\partial \pi}\right) < 0$, $\frac{\partial n_I^C}{\partial p} = -\left(\frac{\partial n_R^C}{\partial p}+\frac{\partial n_S^C}{\partial p}\right) \overset{s}{=} (1-p)^2\frac{\partial t_S\left(n_S^C\right)}{\partial n_S} - p^2\left(\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}\right)$; $\frac{\partial n_I^C}{\partial N} = \frac{\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}}{\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}} \in (0,1)$.

Using these derivatives it is readily shown that $\mathbb{E}C^C$ is an increasing function of π, p and N. Note that the derivative $\frac{\partial n_I^C}{\partial p}$ switches sign from positive to negative as p increases.

Equilibrium when $n_S^C = 0$ If the cost of information is sufficiently small, the condition $I \approx S$ is not satisfied even with $n_S^C = 0$; i.e. $\pi < (1-p)\left(t_S(0) - t_R^-\right)$. If so, Condition (8) is not applicable and the equilibrium is derived using Conditions (6) and (7) with $n_S^C = 0$. The comparative statics properties of the model are the same as for $n_S^C > 0$ except that $\frac{\partial n_I^C}{\partial p} = -\frac{\partial n_R^C}{\partial p} = \frac{t_R^+\left(n_R^C\right)-t_S\left(n_S^C+n_I^C\right)}{p\left(\frac{\partial t_S\left(n_S^C+n_I^C\right)}{\partial n_S}+\frac{\partial t_R^+\left(n_R^C\right)}{\partial n_R}\right)} > 0$, so that the ambiguity in the sign of this derivative is eliminated.

9.3 Appendix 3: Proof of Theorem 1

Part (a) follows from the inequality $t_S(0) > t_R^-$. For Part (b) define $\Psi(n_R) \equiv t_R^+(n_R) - t_S(N - n_R)$. By Assumption 1, $\Psi(n_R)$ is a continuous and strictly increasing function of n_R. By Assumption 2, $\Psi(0) = t_R^+(0) - t_S(N) < 0$, and $\Psi(N) = t_R^+(N) - t_S(0) > 0$. Hence there exists a unique $n_R^{F+} \in (0, N)$ such that $\Psi\left(n_R^{F+}\right) = t_R^+\left(n_R^{F+}\right) - t_S\left(N - n_R^{F+}\right) = 0$.

9.4 Appendix 4: Proof of Theorem 2

Part (a): Define $\psi(p, N_R; \theta) \equiv pU\left(t_R^+(N_R); \theta\right) + (1-p)U\left(t_R^-; \theta\right) - U\left(t_S(N - N_R); \theta\right)$.
$\psi(p, N_R; 0) = -\left[pt_R^+(N_R) + (1-p)t_R^-\right] + t_S(N - N_R) > 0$. Given $t_R^+(N_R) > t_S(N - N_R)$ and Assumption 3, $\underset{\theta\to\infty}{Lim}\ \psi(p, N_R; \theta) < 0$. Given Assumption 3, $\psi(\cdot)$ is a continuous function of θ. Hence there exists at least one $\tilde{\theta}^Z$ such that $\psi\left(p, N_R; \tilde{\theta}^Z\right) = 0$ and hence $R \approx S$. Let Let $\mathbb{E}_i$ denote the expectations operator for route i, $i \in \{R, S\}$. Then $\psi = \mathbb{E}_R U(t;\theta) - \mathbb{E}_S U(t;\theta)$, and $\frac{\partial\psi}{\partial\theta} = \mathbb{E}_R\frac{\partial U(t;\theta)}{\partial\theta} - \mathbb{E}_S\frac{\partial U(t;\theta)}{\partial\theta}$. By Assumption 3, $\partial U/\partial\theta$ is a strictly concave function of U. Hence by Jensen's inequality

$$\frac{\partial\psi(p, N_R; \theta)}{\partial\theta}\Big|_{\theta=\tilde{\theta}^Z(p,N_R)} < 0\ . \tag{9}$$

This proves that $\tilde{\theta}^Z(p, N_R)$ is unique. Furthermore,

$$\psi(p, N_R; \theta) \gtreqless 0 as \theta \lesseqgtr \tilde{\theta}^Z(p, N_R)\,. \tag{10}$$

Part (b): Consider $p' > p$. Since $t_R^+(N_R) > t_S(N - N_R)$, $\psi\left(p', N_R; \tilde{\theta}^Z(p, N_R)\right) > 0$. Given (9) it follows that $\tilde{\theta}^Z(p', N_R) < \tilde{\theta}^Z(p, N_R)$; hence $\tilde{\theta}^Z(p, N_R)$ is strictly decreasing in p.

Now consider $\acute{N}_R > N_R$. Since $t_R^+(N_R)$ is increasing in N_R and $t_S(N - N_R)$ is decreasing in N_R, $\psi\left(p, \acute{N}_R; \tilde{\theta}^Z(p, N_R)\right) > 0$. By similar reasoning it follows that $\tilde{\theta}^Z(p, N_R)$ is strictly decreasing in N_R.

9.5 Appendix 5: Proof of Theorem 3

On the one hand, since the cdf $F(\theta)$ is continuous and strictly increasing for $\theta \in \mathcal{I}$, $n(\theta) = NF(\theta)$ defines an increasing relationship, with $n(0) = 0$ and $n(\theta) \underset{\theta\to\infty}{\to} N$. On the other hand, Theorem 2 implies that $\tilde{\theta}^Z(p, N_R)$ defines a decreasing relationship between θ and n, with $n(\theta) \in [0, N]$. The two curves therefore cross exactly once, which defines $n_R^Z(p)$ and $\tilde{\theta}^Z(p, N_R)$. Consider the function

$$\Omega(p, n; \theta) \equiv pU\left(t_R^+(n); \theta\right) + (1-p)U\left(t_R^-; \theta\right) - U\left(t_S(N-n); \theta\right).$$

According to Assumption 2, $t_R^- < t_R^+(n)$, so $U\left(t_R^+(n); \theta\right) - U\left(t_R^-; \theta\right) < 0$ and $\frac{\partial\Omega}{\partial p} < 0$. Decreasing utility and Assumption 1 imply that $U\left(t_R^+(n); \theta\right)$ is decreasing in n and $U\left(t_S(N-n); \theta\right)$ is increasing in n, so $\frac{\partial\Omega}{\partial n} < 0$. Finally, for any relevant (p, n), Ω is locally decreasing in θ at point $\left(p, n, \tilde{\theta}_p^Z(n)\right)$ because, according to Theorem 2, $\Omega\left(p, n, \tilde{\theta}^Z(p, n)\right) = 0$, $\Omega(p, n; \theta) > 0$ for any $\theta < \tilde{\theta}^Z(p, n)$ and $\Omega(p, n; \theta) < 0$ for any $\theta > \tilde{\theta}^Z(p, n)$. As a result, $\frac{\partial\tilde{\theta}^Z(p, n_R^Z)}{\partial p} = -\frac{\partial\Omega}{\partial p} / \frac{\partial\Omega}{\partial\theta} < 0$, and $\frac{\partial n_R^Z}{\partial p} = \frac{\partial n_R^Z}{\partial\theta}\frac{\partial\tilde{\theta}^Z(p, n_R^Z)}{\partial p} = Nf\left(\tilde{\theta}^Z\right)\frac{\partial\tilde{\theta}^Z(p, n_R^Z)}{\partial p} < 0$.

9.6 Appendix 6: Proof of Theorem 4

Part (a): By Theorem 3, in the *No information* regime

$$pU\left(t_R^+\left(n_R^Z\right); \tilde{\theta}^Z\right) + (1-p)U\left(t_R^-; \tilde{\theta}^Z\right) = U\left(t_S\left(N - n_R^Z\right); \tilde{\theta}^Z\right) \text{ where } \tilde{\theta}^Z > 0. \tag{11}$$

Given $t_R^+\left(n_R^Z\right) > t_R^-$, it follows that

$$t_R^+\left(n_R^Z\right) > t_S\left(N - n_R^Z\right). \tag{12}$$

With *Free information*:

$$t_R^+\left(n_R^{F+}\right) = t_S\left(N - n_R^{F+}\right). \tag{13}$$

Inequality (12) and equation (13) imply $n_R^{F+} < n_R^Z$.

Part (b): The goal is to show $t_R^+\left(n_R^{F+}\right) + (1-p)t_R^- < pt_R^+\left(n_R^Z\right) + (1-p)t_R^- < t_S\left(N - n_R^Z\right)$. The first inequality follows from Part (a). The second inequality follows from equation (11) and $\tilde{\theta}^Z > 0$.

Part (c): Travel times in the *No information* and *Free information* regimes are given in the following table:

Row	Regime	Group	*Good* days	*Bad* days
1	*No information*	R	t_R^-	$t_R^+\left(n_R^Z\right)$
2	*No information*	S	$t_S\left(N - n_R^Z\right)$	$t_S\left(N - n_R^Z\right)$
3	*Free information*	R and S	t_R^-	$t_R^+\left(n_R^{F+}\right)$

That travel time variability decreases for Group R follows from Rows 1 and 3, and $t_R^- < t_R^+\left(n_R^{F+}\right) < t_R^+\left(n_R^Z\right)$. That travel time variability increases for Group S follows from Rows 2 and 3, and $t_R^- < t_S\left(N - n_R^Z\right) < t_R^+\left(n_R^{F+}\right)$.

9.7 Appendix 7: Compensating variation and degree of risk aversion

Let travel time in the *No information* regime (Z) be T_1 on *Good* days, and T_2 on *Bad* days. And let travel time in the *Free information* (F), *Costly information* (C), or *Private information* regime (I) be T_3 on *Good* days, and T_4 on *Bad* days. The compensating variation $CV^r(\theta)$ for information regime r, $r \in \{F, C, I\}$, is defined by the condition

$$\Psi(\theta) \equiv \mathbb{E}U^r\left(CV^r(\theta); \theta\right) - \mathbb{E}U^Z(\theta) = 0,$$

or

$$(1-p)U\left(T_3 + CV^r(\theta); \theta\right) + pU\left(T_4 + CV^r(\theta); \theta\right) - (1-p)U\left(T_1; \theta\right) - pU\left(T_2; \theta\right) = 0.$$

Two generic cases cover all the cases considered in the text.

9.7.1 Case 1: $T_1 = T_3 < T_4 < T_2$

This case covers CV_R^F, CV_{RR}^C, CV_{RI}^C and CV_R^I. Compensating variation for these cases is guaranteed to be positive. With $CV^r(\theta) > 0$, $U_1 > U_3 > U_4 > U_2$, and *No information* induces a mean-preserving spread of utility relative to information. Now $\frac{\partial\psi}{\partial\theta} = \mathbb{E}_r \frac{\partial U(t;\theta)}{\partial\theta} - \mathbb{E}_Z \frac{\partial U(t;\theta)}{\partial\theta}$. By Assumption 3, $\partial U/\partial\theta$ is a strictly concave function of U. Hence by Jensen's inequality $\frac{\partial\psi}{\partial\theta}\big|_{CV^r(\theta)} > 0$. Given $\frac{\partial\psi}{\partial CV^r(\theta)} < 0$, it follows that $CV^r(\theta)$ is an increasing function of θ for Case 1.

9.7.2 Case 2: $T_3 < T_1 = T_2 \leq T_4$

This case covers CV_S^F, CV_{SI}^C, CV_{SS}^C and CV_S^I. Compensating variation is **not** guaranteed to be positive. But whether or not it is positive, after compensating variation is added to travel times $U_3 > U_1 = U_2 > U_4$. Information therefore induces a mean-preserving spread of utility relative to *No information*. Hence by Jensen's inequality $\frac{\partial\psi}{\partial\theta}\big|_{CV^r(\theta)} < 0$, and $CV^r(\theta)$ is a decreasing function of θ for Case 2.

9.8 Appendix 8: Compensating variation is negative for sufficiently risk-averse drivers

For brevity let t^Z denote (certain) travel time with *No information*, t_G travel time on *Good* days with information, and $t_B > t_G$ travel time on *Bad* days with information. $CV(\theta)$ is defined by the condition

$$pU(t_B + CV(\theta);\theta) + (1-p)U(t_G + CV(\theta);\theta) - U(t^Z;\theta) = 0,$$

or

$$G(t;\theta) \equiv U(t^Z;\theta)\left[\underbrace{\frac{pU(t_B+t;\theta) + (1-p)U(t_G+t;\theta)}{U(t^Z;\theta)}}_{\equiv H(t;\theta)} - 1\right] = 0 \text{ at } t = CV(\theta).$$

If $t < t^Z - t_B$ then $\underset{\theta\to\infty}{Lim} H(t;\theta) = 0$, and $\underset{\theta\to\infty}{Lim} G(t;\theta) = \infty$. If $t > t^Z - t_B$ then $\underset{\theta\to\infty}{Lim} H(t;\theta) = \infty$, and $\underset{\theta\to\infty}{Lim} G(t;\theta) = -\infty$. Therefore $\underset{\theta\to\infty}{Lim} CV(\theta) = t^Z - t_B$. Now $t^Z = t_S(N - n_R^{Z+})$. With *Free information*, $t_B = t_S(N - n_R^{F+})$ and $\underset{\theta\to\infty}{Lim} CV(\theta) = t_S(N - n_R^{Z+}) - t_S(N - n_R^{F+}) < 0$. With *Costly information*, $t_B = t_S(N - n_R^{C+})$ and $\underset{\theta\to\infty}{Lim} CV(\theta) = t_S(N - n_R^{Z+}) - t_S(N - n_R^{C+}) < 0$.

9.9 Appendix 9: Proof of Proposition 3

Part (a): The proof follows closely the proof of Theorem 2. $R \gtreqless S \iff \psi_{RS}(\theta; p, n_R, n_S, n_I) \gtreqless 0$. If $R \succ S$ for all $\theta \in [0,\infty)$ then $\hat{\theta}_{RS} = 0$. If $R \prec S$ for all $\theta \in [0,\infty)$ then $\hat{\theta}_{RS} = \infty$. If neither preference ranking holds for all θ then, by continuity of ψ_{RS} in θ, there exists at least one $\hat{\theta}_{RS}$ such that $\psi_{RS}(\hat{\theta}_{RS};\cdot) = 0$. Now $\psi_{RS} = \mathbb{E}_R U(t;\theta) - \mathbb{E}_S U(t;\theta)$, and $\frac{\partial\psi_{RS}}{\partial\theta} = \mathbb{E}_R \frac{\partial U(t;\theta)}{\partial\theta} - \mathbb{E}_S \frac{\partial U(t;\theta)}{\partial\theta}$. By Assumption 3, $\partial U/\partial\theta$ is a strictly concave function of U, and hence by Jensen's inequality

$$\frac{\partial\psi_{RS}}{\partial\theta}\Big|_{\theta=\hat{\theta}_{RS}} < 0, \tag{14}$$

and $\hat{\theta}_{RS}$ is unique. Furthermore,

$$\psi_{RS}(\theta; p, n_R, n_S, n_I) \gtrless 0 \text{ as } \theta \lessgtr \hat{\theta}_{RS}(p, n_R, n_S, n_I). \tag{15}$$

Part (b): Consider $p' > p$. Since $t_R^- < t_S(n_S)$ and $t_R^+(n_R) \geq t_S(n_S + n_I)$,

$$U\left(t_R^-; \hat{\theta}_{RS}(p, n_R, n_S, n_I)\right) > U\left(t_S(n_S); \hat{\theta}_{RS}(p, n_R, n_S, n_I)\right), \text{ and}$$
$$U\left(t_R^+(n_R); \hat{\theta}_{RS}(p, n_R, n_S, n_I)\right) \leq U\left(t_S(n_S + n_I); \hat{\theta}_{RS}(p, n_R, n_S, n_I)\right).$$

Therefore $\psi_{RS}\left(\hat{\theta}_{RS}(p,n_R,n_S,n_I);p',n_R,n_S,n_I\right)<0$. Given (14) it follows that

$$\hat{\theta}_{RS}(p',n_R,n_S,n_I)<\hat{\theta}_{RS}(p,n_R,n_S,n_I).$$

This proves that $\hat{\theta}_{RS}$ is decreasing in p. The comparative statics properties for n_R, n_S and n_I are derived similarly using the respective inequalities

$$\begin{aligned}
\psi_{RS}\left(\hat{\theta}_{RS}(p,n_R,n_S,n_I);p,n'_R,n_S,n_I\right)&<0 \text{ for } n'_R>n_R,\\
\psi_{RS}\left(\hat{\theta}_{RS}(p,n_R,n_S,n_I);p,n_R,n'_S,n_I\right)&>0 \text{ for } n'_S>n_S,\\
\psi_{RS}\left(\hat{\theta}_{RS}(p,n_R,n_S,n_I);p,n_R,n_S,n'_I\right)&>0 \text{ for } n'_I>n_I.
\end{aligned}$$

9.10 Appendix 10: Proof of Proposition 4

Part (a): The proof follows the proof of Prop. 3(a) by replacing regime S by regime I.
Part (b): Consider $p'>p$. Since $t_R^-<t_R^-+\pi$ and $t_R^+(n_R)\geq t_S(n_S+n_I)$,

$$\begin{aligned}
&U\left(t_R^-;\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I)\right)>U\left(t_R^-+\pi;\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I)\right), \text{ and}\\
&U\left(t_R^+(n_R);\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I)\right)\leq U\left(t_S(n_S+n_I);\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I)\right).
\end{aligned}$$

Hence $\psi_{RI}\left(\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I),\pi;p',n_R,n_S+n_I\right)<0$. The counterpart to Condition (15) is

$$\psi_{RI}(\theta,\pi;p,n_R,n_S+n_I)\gtreqless 0 \text{ as } \theta\lesseqgtr\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I). \quad (16)$$

It follows that

$$\hat{\theta}_{RI}(\pi,p',n_R,n_S+n_I)<\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I).$$

The comparative statics properties for n_R, π and n_S+n_I are derived similarly using the respective inequalities

$$\begin{aligned}
\psi_{RI}\left(\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I),\pi;p,n'_R,n_S+n_I\right)&<0 \text{ for } n'_R>n_R,\\
\psi_{RI}\left(\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I),\pi';p,n_R,n_S+n_I\right)&>0 \text{ for } \pi'>\pi,\\
\psi_{RI}\left(\hat{\theta}_{RI}(\pi,p,n_R,n_S+n_I),\pi;p,n_R,n'_S+n'_I\right)&>0 \text{ for } n'_S+n'_I>n_S+n_I.
\end{aligned}$$

9.11 Appendix 11: Proof of Proposition 5

Part (a): The proof follows the proof of Prop. 3(a) by replacing regime R by regime I.
Part (b): The comparative statics properties for π, p, n_I, n_S, and n_S with n_S+n_I held constant are derived using the respective inequalities

$$\begin{aligned}
\psi_{IS}\left(\hat{\theta}_{IS}(\pi,p,n_S,n_I),\pi';p,n_S,n_I\right)&<0 \text{ for } \pi'>\pi,\\
\psi_{IS}\left(\hat{\theta}_{IS}(\pi,p,n_S,n_I),\pi;p',n_S,n_I\right)&<0 \text{ for } p'>p,\\
\psi_{IS}\left(\hat{\theta}_{IS}(\pi,p,n_S,n_I),\pi;p,n_S,n'_I\right)&<0 \text{ for } n'_I>n_I,\\
\psi_{IS}\left(\hat{\theta}_{IS}(\pi,p,n_S,n_I),\pi;p,n'_S,n_I\right)&\gtreqless 0 \text{ for } n'_S>n_S,\\
\psi_{IS}\left(\hat{\theta}_{IS}(\pi,p,n_S,n_I),\pi;p,n'_S,n'_I\right)&> \text{ for } n'_S>n_S \text{ and } n'_S+n'_I=n_S+n_I
\end{aligned}$$

The last three inequalities are readily established if it assumed that all relevant functions are differentiable. For the third inequality:

$$\frac{\partial\psi_{IS}}{\partial n_I}=p\left[\underbrace{\frac{\partial U(t_S(n_S+n_I)+\pi;\theta)}{\partial t}-\frac{\partial U(t_S(n_S+n_I);\theta)}{\partial t}}_{-}\right]\underbrace{\frac{\partial t_S(n_S+n_I)}{\partial N_S}}_{+}<0,$$

where the term in square brackets is negative because $U(\cdot)$ is strictly concave in t. For the fourth inequality:

$$\frac{\partial \psi_{IS}}{\partial n_S} = p\left[\underbrace{\frac{\partial U\left(t_S\left(n_S+n_I\right)+\pi;\theta\right)}{\partial t} - \frac{\partial U\left(t_S\left(n_S+n_I\right);\theta\right)}{\partial t}}_{-}\right]\underbrace{\frac{\partial t_S\left(n_S+n_I\right)}{\partial N_S}}_{+}$$
$$-\left(1-p\right)\underbrace{\frac{\partial U\left(t_S\left(n_S\right);\theta\right)}{\partial t}}_{-}\underbrace{\frac{\partial t_S\left(n_S\right)}{\partial N_S}}_{+} \gtrless 0.$$

For the fifth inequality

$$\frac{\partial \psi_{IS}}{\partial n_S} - \frac{\partial \psi_{IS}}{\partial n_I} = -\left(1-p\right)\underbrace{\frac{\partial U\left(t_S\left(n_S\right);\theta\right)}{\partial t}}_{-}\underbrace{\frac{\partial t_S\left(n_S\right)}{\partial N_S}}_{+} > 0.$$

9.12 Appendix 12: Comparative statics of Costly information equilibrium

The five endogenous variables $\{n_R, n_S, n_I, \theta_A, \theta_B\}$ are determined by the five equations

$$n_R = NF\left(\theta_A\right), \tag{17}$$

$$n_I = N\left(F\left(\theta_B\right) - F\left(\theta_A\right)\right), \tag{18}$$

$$n_S = N\left(1 - F\left(\theta_A\right)\right), \tag{19}$$

indifference between Strategies R and I at $\theta = \theta_A$:

$$pU\left(t_R^+\left(n_R\right);\theta_A\right) + \left(1-p\right)U\left(t_R^-;\theta_A\right) = pU\left(t_S\left(N-n_R\right)+\pi;\theta_A\right) + \left(1-p\right)U\left(t_R^- + \pi;\theta_A\right), \tag{20}$$

and indifference between Strategies I and S at $\theta = \theta_B$:

$$pU(t_S(N-n_R)+\pi;\theta_B) + (1-p)\,U\left(t_R^- + \pi;\theta_B\right) = pU(t_S(N-n_R);\theta_B) + (1-p)\,U(t_S(n_S);\theta_B). \tag{21}$$

Variable n_I does not appear in eqns. (20) or (21), and hence can be solved *ex post* using eqn. (18). To simplify notation, index the six travel times numerically in order of increasing magnitude:

$$\underbrace{t_R^-}_{t_1} < \underbrace{t_R^- + \pi}_{t_2} < \underbrace{t_S(n_S)}_{t_3} < \underbrace{t_S(n_S+n_I)}_{t_4} < \underbrace{t_S(n_S+n_I)+\pi}_{t_5} < \underbrace{t_R^+\left(n_R\right)}_{t_6},$$

and define $m_{ij} \equiv \frac{\partial U(t_i;\theta_j)}{\partial t} < 0$, $c_i \equiv \frac{\partial t_i}{\partial n_i} > 0$, $v_{ij} \equiv \frac{\partial U(t_i;\theta_j)}{\partial \theta} < 0$ and $U_{ij} \equiv U\left(t_i;\theta_j\right) < 0$, $i = 1, ..., 6$, $j = A, B$.[20] Eqns. (17), (19), (20) and (21) can then be written:

$$n_R - NF\left(\theta_A\right) = 0, \tag{22}$$

$$n_S - N\left(1 - F\left(\theta_B\right)\right) = 0, \tag{23}$$

$$p\left(U_{6A} - U_{5A}\right) + \left(1-p\right)\left(U_{1A} - U_{2A}\right) = 0, \tag{24}$$

$$p\left(U_{4B} - U_{5B}\right) + \left(1-p\right)\left(U_{3B} - U_{2B}\right) = 0. \tag{25}$$

Equations (22) and (24) are separable in n_R and θ_A. The total differentials are

$$\begin{bmatrix} 1 & -Nf\left(\theta_A\right) \\ p\left(m_{5A}c_5 + m_{6A}c_6\right) & p\left(v_{6A} - v_{5A}\right) + \left(1-p\right)\left(v_{1A} - v_{2A}\right) \end{bmatrix}\begin{bmatrix} dn_R \\ d\theta_A \end{bmatrix} =$$

[20] Note that $c_1 = c_2 = 0 < c_3 \leq c_4 = c_5$.

$$\left[\begin{array}{ccc} 0 & 0 & F(\theta_A) \\ pm_{5A}+(1-p)\,m_{2A} & U_5-U_6+U_1-U_2 & pm_{5A}c_5 \end{array}\right]\left[\begin{array}{c} d\pi \\ dp \\ dN \end{array}\right].$$

Define $\Delta_A \equiv p(v_{6A}-v_{5A})+(1-p)(v_{1A}-v_{2A})+p(m_{5A}c_5+m_{6A}c_6)\,Nf(\theta_A)<0$. The comparative statics effects for n_R are:

$$\frac{\partial n_R}{\partial \pi}=\Delta_A^{-1}\left[\underbrace{pm_{5A}+(1-p)\,m_{2A}}_{-}\right]Nf(\theta_A)>0, \tag{26}$$

$$\frac{\partial n_R}{\partial p}=\Delta_A^{-1}\left[\underbrace{U_5-U_6+U_1-U_2}_{+}\right]Nf(\theta_A)<0, \tag{27}$$

$$\frac{\partial n_R}{\partial N}=\Delta_A^{-1}\left\{F(\theta_A)\left[\underbrace{p(v_{6A}-v_{5A})+(1-p)(v_{1A}-v_{2A})}_{-}\right]+Nf(\theta_A)\underbrace{pm_{5A}c_5}_{-}\right\}\in(0,1). \tag{28}$$

Given (22), (26) and (27),

$$\frac{\partial \theta_A}{\partial \pi}>0, \text{ and } \frac{\partial \theta_A}{\partial p}<0. \tag{29}$$

$$\frac{\partial \theta_A}{\partial N}=\Delta_A^{-1}p\left[m_{5A}c_5(1-F(\theta_A))-m_{6A}c_6F(\theta_A)\right]\stackrel{s}{=}?, \tag{30}$$

where $\stackrel{s}{=}?$ indicates that the sign is indeterminate. Equations (23) and (25) can be used to derive the comparative statics for n_S and θ_B by substituting in the comparative statics derivatives just derived for n_R. The total differentials are

$$\left[\begin{array}{cc} 1 & Nf(\theta_B) \\ (1-p)\,m_{3B}c_3 & p(v_{4B}-v_{5B})+(1-p)(v_{3B}-v_{2B}) \end{array}\right]\left[\begin{array}{c} dn_S \\ d\theta_B \end{array}\right]=$$

$$\left[\begin{array}{ccc} 0 & 0 & 1-F(\theta_B) \\ \begin{array}{c} pm_{5B}+(1-p)\,m_{2B} \\ +p(m_{4B}c_4-m_{5B}c_5)\frac{dn_R}{d\pi} \end{array} & \begin{array}{c} U_5-U_4+U_3-U_2 \\ +p(m_{4B}c_4-m_{5B}c_5)\frac{dn_R}{dp} \end{array} & \begin{array}{c} p(m_{5B}c_5-m_{4B}c_4)\cdot \\ \left(1-\frac{dn_R}{dN}\right) \end{array} \end{array}\right]\left[\begin{array}{c} d\pi \\ dp \\ dN \end{array}\right].$$

Define $\Delta_B \equiv p(v_{4B}-v_{5B})+(1-p)(v_{3B}-v_{2B})-(1-p)\,m_{3B}c_3Nf(\theta_B)>0$. The comparative statics derivatives for n_S are:

$$\frac{\partial n_S}{\partial \pi}=-\Delta_B^{-1}\left[\underbrace{pm_{5B}+(1-p)\,m_{2B}}_{-}+p\underbrace{(m_{4B}c_4-m_{5B}c_5)}_{+}\underbrace{\frac{\partial n_R}{\partial \pi}}_{+}\right]Nf(\theta_B)\stackrel{s}{=}?, \tag{31}$$

$$\frac{\partial n_S}{\partial p}=-\Delta_B^{-1}\left[\underbrace{U_5-U_4+U_3-U_2}_{-}+p\underbrace{(m_{4B}c_4-m_{5B}c_5)}_{+}\underbrace{\frac{\partial n_R}{\partial p}}_{-}\right]Nf(\theta_B)>0, \tag{32}$$

$$\frac{\partial n_S}{\partial N}=\Delta_B^{-1}\left\{\begin{array}{l}\underbrace{[p(v_{4B}-v_{5B})+(1-p)(v_{3B}-v_{2B})]}_{+}(1-F(\theta_B)) \\ -p\underbrace{(m_{5B}c_5-m_{4B}c_4)}_{-}\underbrace{\left(1-\frac{\partial n_R}{\partial N}\right)}_{+}Nf(\theta_B)\end{array}\right\}>0. \tag{33}$$

Given (23), (31) and (32)

$$\frac{\partial \theta_B}{\partial \pi} \overset{s}{=} ?, \text{ and } \frac{\partial \theta_B}{\partial p} < 0.$$

Given (23) and (33)

$$\frac{\partial \theta_B}{\partial N} = \Delta_B^{-1} \left[\underbrace{p\underbrace{(m_{5B}c_5 - m_{4B}c_4)}_{-}\left(1 - \frac{\partial n_R}{\partial N}\right)}_{+} - (1-p)\underbrace{m_{3B}c_3}_{-}(1 - F(\theta_B)) \right] \overset{s}{=} ?$$

$$\frac{\partial n_I}{\partial \pi} = -\left(\underbrace{\frac{\partial n_R}{\partial \pi}}_{+} + \underbrace{\frac{\partial n_S}{\partial \pi}}_{?} \right) \overset{s}{=} ?,$$

$$\frac{\partial n_I}{\partial p} = -\left(\underbrace{\frac{\partial n_R}{\partial p}}_{-} + \underbrace{\frac{\partial n_S}{\partial p}}_{+} \right) \overset{s}{=} ?,$$

$$\frac{\partial n_I}{\partial N} = 1 - \left(\underbrace{\frac{\partial n_R}{\partial N}}_{+} + \underbrace{\frac{\partial n_S}{\partial N}}_{+} \right) \overset{s}{=} ?$$

9.13 Appendix 13: Proof of Proposition 10

The proof has a similar structure to the proof of Proposition 3. By Proposition 9, $CV^I(\theta)$ is maximal at $\theta = \theta_{RS}$, where θ_{RS} is defined by the condition of indifference between Strategies R and S:

$$pU\left(t_R^+\left(n_R^Z\right);\theta_{RS}\right) + (1-p)\,U\left(t_R^-;\theta_{RS}\right) = U\left(t_S\left(N - n_R^Z\right);\theta_{RS}\right),$$

or

$$p\frac{U\left(t_R^+\left(n_R^Z\right);\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)} + (1-p)\frac{U\left(t_R^-;\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)} = 1. \tag{34}$$

Now $\underset{p\to 0}{Lim} t_R^+\left(n_R^Z\right) = t_R^+(N)$, $\underset{p\to 0}{Lim} t_S\left(N - n_R^Z\right) = t_S(0)$, $\underset{p\to 0}{Lim} \theta_{RS} = \infty$, and $\underset{p\to 0}{Lim} \frac{U\left(t_R^-;\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)} = \underset{p\to 0}{Lim} \frac{U\left(t_R^-;\theta_{RS}\right)}{U\left(t_S(0);\theta_{RS}\right)} = 0$ since $t_R^- < t_S(0)$. Hence by (34)

$$p\frac{U\left(t_R^+\left(n_R^Z\right);\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)} \underset{p\to 0}{\to} 1. \tag{35}$$

$CV^I(\theta_{RS})$ is defined by the condition

$$pU\left(t_S\left(N-n_R^Z\right) + CV^I(\theta_{RS});\theta_{RS}\right) + (1-p)\,U\left(t_R^- + CV^I(\theta_{RS});\theta_{RS}\right) \\ -U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right) = 0,$$

or

$$G\left(t;p,\theta_{RS}\left(p\right)\right)\equiv U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)\left[\underbrace{p\frac{U\left(t_S\left(N-n_R^Z\right)+t;\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)}}_{\equiv H(t;p,\theta_{RS}(p))}+\underbrace{\left(1-p\right)\frac{U\left(t_R^-+t;\theta_{RS}\right)}{U\left(t_S\left(N-n_R^Z\right);\theta_{RS}\right)}}_{\equiv J(t;p,\theta_{RS}(p))}-1\right]=0 \text{ at } t=CV^I\left(\theta_{RS}\right).$$

There are two cases to consider.

9.13.1 Case 1: $t_S\left(0\right)-t_R^-<t_R^+\left(N\right)-t_S\left(0\right)$.

If $t<t_S\left(0\right)-t_R^-$ then $\underset{p\to 0}{Lim}J\left(\cdot\right)=0$. And given $t_S\left(0\right)+t<t_S\left(0\right)+t_R^+\left(N\right)-t_S\left(0\right)=t_R^+\left(N\right)$, $\underset{p\to 0}{Lim}H\left(\cdot\right)=0$ by (35) and $\underset{p\to 0}{Lim}G\left(\cdot\right)=+\infty$.
If $t>t_S\left(0\right)-t_R^-$ then $\underset{p\to 0}{Lim}J\left(\cdot\right)=+\infty\Longrightarrow\underset{p\to 0}{Lim}G\left(\cdot\right)=-\infty$. Therefore $\underset{p\to 0}{Lim}CV^I\left(\theta_{RS}\right)=t_S\left(0\right)-t_R^-$.

9.13.2 Case 2: $t_R^+\left(N\right)-t_S\left(0\right)<t_S\left(0\right)-t_R^-$.

If $t<t_R^+\left(N\right)-t_S\left(0\right)$ then $\underset{p\to 0}{Lim}J\left(\cdot\right)=0$, $\underset{p\to 0}{Lim}H\left(\cdot\right)=0$ by (35), and $\underset{p\to 0}{Lim}G\left(\cdot\right)=+\infty$.
If $t>t_R^+\left(N\right)-t_S\left(0\right)$ then $\underset{p\to 0}{Lim}H\left(\cdot\right)=+\infty$ and $\underset{p\to 0}{Lim}G\left(\cdot\right)=-\infty$. Therefore $\underset{p\to 0}{Lim}CV^I\left(\theta_{RS}\right)=t_R^+\left(N\right)-t_S\left(0\right)$.
Cases 1 and 2 combined imply $CV^I\left(\theta_{RS}\right)\underset{p\to 0}{\to}Min\left(t_S\left(0\right)-t_R^-,t_R^+\left(N\right)-t_S\left(0\right)\right)$.