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Income Levels and Income Growth.
Some New Cross-Country Evidence
and Some Interpretative Puzzles

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Income levels and income growth: Some new cross-country evidence and some interpretative puzzles[*]

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Abstract

This work brings together two distinct pieces of evidence concerning, at *macro* level, international distributions of incomes and their dynamics, and, at *micro* level, the size distributions of firms and the properties of their growth rates. Moreover, we take into the picture an intermediate level of observation, namely the statistical properties of sectoral growth.

First, our empirical analysis provides a fresh look at the international distributions of incomes and growth rates by investigating more closely the relationship between the two entities and the statistical properties of the growth process.

Second, we try to identify those statistical properties which are invariant with respect to the scale of observation (country or firm) as distinct from those that are instead scale specific. This exercise puts forward a few major interpretative challenges regarding the correlating processes underlying the statistical evidence.

Keywords: international distribution of income, international growth rates, scaling laws, growth volatility, exponential tails

JEL classification: C10, C14, O11

1   Introduction

This paper brings together two distinct ensembles of evidence concerning, *first*, international distributions of income and their dynamics, and, *second*, the micro-economic evidence on size distributions of firms and the properties of their growth rates.

Such an exercise entails two major interpretative questions concerning:

(i) the relationships between the distributions of the relevant entities (e.g. countries or firms) and the properties of the growth process;

(ii) the identification of the properties that appear to be invariant *vis-à-vis* the scale of observation and those that conversely are scale specific.

With respect to the first question, this work links with the stream of studies in growth empirics concerning the international divergence/convergence properties of income (for thorough reviews see Durlauf and Quah (1999) and Temple (1999)). We take a fresh look at the macro-evidence on the distribution of income levels and growth

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rates, based on a longer time series from the updated Penn World Tables, version 6.1, investigate the shape of the distribution of incomes, test for multi-modality and study the rank mobility of countries.

Next, we study the properties of growth rates and their dependence upon possible conditioning factors including income levels and the size of the economies. In order to address the second question, we compare distributions and growth processes at the two levels of observation, namely countries and firms. In particular, we apply to output growth rates some non-parametric analyses recently used for the investigation of firm growth rates. As we shall see, one finds striking similarities in the growth processes which hold across levels of observation. In turn, such statistical properties hint at the ubiquitous presence of some *correlating mechanisms* which withstand aggregation from firms to sectors to countries.

In what follows, we start with a brief overview of the two existing sets of micro and macro evidence on the statistical properties of the distribution of 'size' and growth rates (Section 2). In Section 3 we describe the data and the variables of interest for our empirical analysis. Section 4 investigates the international distribution of income, while in Section 5 we turn to the statistical properties of the distribution of growth shocks. Next, we consider the properties of *sectoral* distributions and dynamics (Section 6). Finally, Section 7 offers a discussion of the interpretative challenges stemming from the empirical evidence and puts forward a few conjectures. Section 8 concludes.

2 The 'size' of firms, the 'size' of countries and their growth processes: some background evidence

Let us begin by bringing side by side two streams of literature which have been rarely connected with each other, addressing the statistical properties of *firm* sizes and growth, on the one hand, and those of *country* (income) sizes and growth on the other. Start with the former level of observation.

2.1 The micro-evidence on firm size and firm growth rates

The statistical properties of the size distribution of firms and of their growth rates have been the objects of interest of a longstanding stream of empirical literature dating back to the seminal contributions of Gibrat (1931), Steindl (1965), Hart and Prais (1956), Simon and Bonini (1958). These pioneering insights and the more recent evidence (for a broad discussion cf. Marsili (2001)) all indicate a generic right-skewness of the distribution of firm size over quite wide supports, wherein fewer large firms co-exist with

many more firms of smaller size. However, the overall shape of the size distributions differs sensibly when disaggregated at, say, 3- or 4-digit levels.[1] Indeed, the precise shape of such distributions varies a great deal across sectors, and sometimes displays also two or more modal values.

A tricky issue regards in particular the properties of the upper tail of the distribution and its 'fatness'. The evidence so far seems to suggest that at sectoral level such tails are at least log-normal and sometimes Pareto-distributed.[2] Stronger evidence, however, corroborates Paretian tails at the *aggregate* manufacturing level: indeed, this might be a puzzling property of the aggregation process itself (cf. Dosi et al. (1995) for some conjectures and some corroborating simulation results).

The statistical literature on size distributions is closely linked with the studies of the statistical properties of the process of growth at the firm level. One of the longstanding issues relates to the validation of the so-called Law of Proportionate Effect (as originally presented in Gibrat (1931)).[3] This null hypothesis states that firm growth rates are realizations independent of size. Under this assumption the limit distribution of size is log-normal. The available evidence indeed does not lend support to any systematic dependence of growth rates on the initial size of firms. At the same time most analyses display a violation of the Gibrat hypothesis in that the *variance* of growth rates *does* depend (negatively) on size.

Moreover, recent studies including Stanley et al. (1996) and Bottazzi et al. (2004), have shifted the focus toward the analysis of the overall *distribution of firm growth rates*. Firm growth rates robustly follow a Laplacian distribution: that is, they are not distributed as Normal variables, but display instead exponential tails. This result by itself sheds new light on the nature of the process of firm growth. If growth rates are markedly non-Gaussian, then one has to strongly reject the hypothesis that growth proceeds over time as the cumulation of small uncorrelated shocks. Interestingly, this stylized fact holds both at the level of the whole manufacturing and at sectoral level, independently of the degrees of statistical disaggregation (as far as one can go given the available data).

In a nutshell, the micro statistical evidence robustly displays: (i) persistent skewed distributions in firm sizes (and similarly skewed distributions in relative produc-

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[1]Cf. Bottazzi and Secchi (2004) on US manufacturing data and Bottazzi et al. (2004) on Italian data.

[2]Pareto distributions yield a cumulative distribution which in a double logarithmic space displays a linear relation between probabilities and values of the variable itself (e.g. the size of firms). A different but germane formulation taking ranks rather than probabilities goes under the heading of Zipf Law.

[3]Within an extensive literature cf. Ijiri and Simon (1977), Hymer and Pashigian (1962), Hall (1987), Evans (1987) and the discussions in Sutton (1997).

tivities and degrees of innovativeness, which we will not review here[4]); (ii) widespread differences across sectors in the shapes of the size distribution themselves; and, at the same time, (iii) no robust relation between initial size and subsequent rates of growth (except possibly for the smallest firms); (iv) a variability of growth rates themselves which often appears to fall with firm size[5]; (v) utterly robust evidence on a Laplacian distribution of firm growth rates, which appears to hold across sectors, across countries and across periods of observation.

Given all that, what is the matching evidence concerning countries? It is straightforward that firms and countries differ in many crucial respects. *First*, and most obviously, firms may easily enter and subsequently die. This is much more unlikely for countries which 'enter' and 'die' under much more infrequent events of revolution and conquest. *Second*, firms within distinct markets are subject to competitive pressures which inevitably correlate their performances. The size of one firm's market share in any particular market means the fall of other firms' shares. As we shall conjecture below, the very process of market competition is likely to contribute to the explanation of the observed statistical structure of firms' growth rates. This is not necessarily so for countries as a whole. Of course, it trivially holds that if some countries grow more than others their share in world income will grow and vice versa. However, there is no *a priori* reason to expect that country growth rates should yield statistical properties similar to those displayed by micro-economic entities undergoing reciprocal competitive pressures. Countries do not necessarily compete as firms do. In fact they might well coordinate in order to achieve higher *common* rates of growth. Under all these qualifications, let us consider the macro, cross-country evidence.

2.2 The macro-evidence on the international distribution of income levels and growth rates

An insightful new set of contributions has recently added to the empirics of international growth, shedding new light on the statistical distributions of income levels and their change, if any, over time (see Quah (1996, 1997), Durlauf and Quah (1999) Bianchi (1997), Jones (1997), Paap and van Dijk (1998)).

Let us start, somewhat symmetrically to the foregoing micro-evidence, from the distributions of the levels of per capita incomes. While it is impossible to discuss at any depth the secular evidence, just notice, *first*, that the *mean per capita incomes* have shown roughly exponential increases since the "Industrial Revolution" in all countries which have been able to join it, and, *second*, that the variance across countries has

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[4]For discussions, cf. Nelson (1981), Bartelsman and Doms (2000), Marsili (2001) and Dosi (2005).

[5]Some empirical exceptions may be found in the Italian evidence in Bottazzi et al. (2004).

correspondingly exploded (more on all this, from different angles, in Bairoch (1981), Maddison (2001), Dosi, Freeman and Fabiani (1994)). Given these long-term tendencies, the foregoing stream of analyses, largely concerning the post World War II period, finds that the distribution of income levels has been moving over the years to a bi-modal shape which indicates a process of 'polarization' of countries into two groups characterized by markedly different income levels. Clearly this witnesses also against any prediction of a tendency towards global convergence of all countries to a common income level.[6]

Even superficial comparisons between firm-level and country-level distributions of 'sizes' (which should be properly understood as 'total incomes of firms or countries' and 'per capita incomes') reveal suggestive analogies concerning, at the very least, (i) the skewness of distributions; (ii) the large width of their supports; and, (iii) high persistence over time of relative rankings.

So far, the statistical properties of *country growth rates* have been much less investigated (insightful exceptions include Canning et al. (1998) and Lee et al. (1998)). Indeed, such properties, and their possible analogies with firm-level processes of growth are major topics to their own right which we shall address below.

3   The variables

We measure the per capita income of a country i in year t, say y_{it}, by the country's per capita GDP at constant prices and constant exchange rates. The data source are the Penn World Tables, version 6.1 (see Heston et al. (2002)) for 111 countries in the years 1960-1996.[7]

Let Y_{it} be the aggregate income. This variable is a proxy for the actual 'size' of a national economy. However, a twin interest of ours lies in the level of economic development of countries. This is primarily captured by the measure of *per capita* income. Here we will consider both total and per capita GDP measures and compare the empirical analyses using the two alternative variables.

Call s_{it} and S_{it} the logarithmic version of the two measures of income (be it per

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[6]Indeed, bimodality is a property that cannot be detected if one only considers the moments of a given distribution, as it is ultimately the case when one performs a regression analysis. Instead, estimating the whole distribution offers the opportunity to inspect the statistical properties of the entire set of observations. The estimation of the empirical density is done by Quah (1996) through kernel smoothing in order to analyze the overall shape of the distribution of income levels. Bianchi (1997) used kernel density estimations to construct statistical tests for multi-modality in the international distribution of income.

[7]See the Appendix for details on the construction of our balanced panel.

capita GDP or total GDP):

$$s_{it} = \log(y_{it})$$
$$S_{it} = \log(Y_{it})$$
(1)

Define the percentage growth rates in income as the logarithmic differences:

$$g_{it} = s_{it} - s_{i,t-1}$$
$$G_{it} = S_{it} - S_{i,t-1}$$
(2)

In order to compare the properties of country-specific variables over time, let us 'de-trend', i.e. "wash away" any trend common to all countries in a given year. For this purpose we consider 'normalized' (log) incomes defined by:

$$s_{it}^* = \log(y_{it}) - \overline{\log(y_t)}$$
$$S_{it}^* = \log(Y_{it}) - \overline{\log(Y_t)}$$
(3)

and calculate normalized year-by-year growth rates as:

$$g_{it}^* = s_{it}^* - s_{i,t-1}^*$$
$$G_{it}^* = S_{it}^* - S_{i,t-1}^*$$
(4)

We refer to these last variables as the *growth shocks* of interest.[8] Notice that Canning et al. (1998) only considers total GDP in its analysis of the distribution of international growth rates, while we include here two different measures of national income.

4   The distribution of levels of income

Let us go back to the evidence telegraphically introduced above. How is income distributed across the countries worldwide? Has this distribution changed over the years? As already mentioned, a set of recent contributions which have addressed this question (cf. Bianchi (1997), Jones (1997), Paap and van Dijk (1998), Quah (1997), Durlauf and Quah (1999)) all show how the distribution of relative per capita income have changed from a unimodal shape to a two-humped shape from 1960 to 1988, the years covered by data from Penn Tables version 5.1.[9]

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[8]The reader should be aware that we use the word 'shock' in tune with a common jargon of practitioners of statistics: however, the terminology does not involve any commitment on the 'exogeneity' of the event itself. In fact, 'shocks' are endogenously generated by the very process of country growth.

[9]A point to keep in mind is that all these works use per capita GDP data, while Jones (1997) opts instead for the measure of GDP per worker.
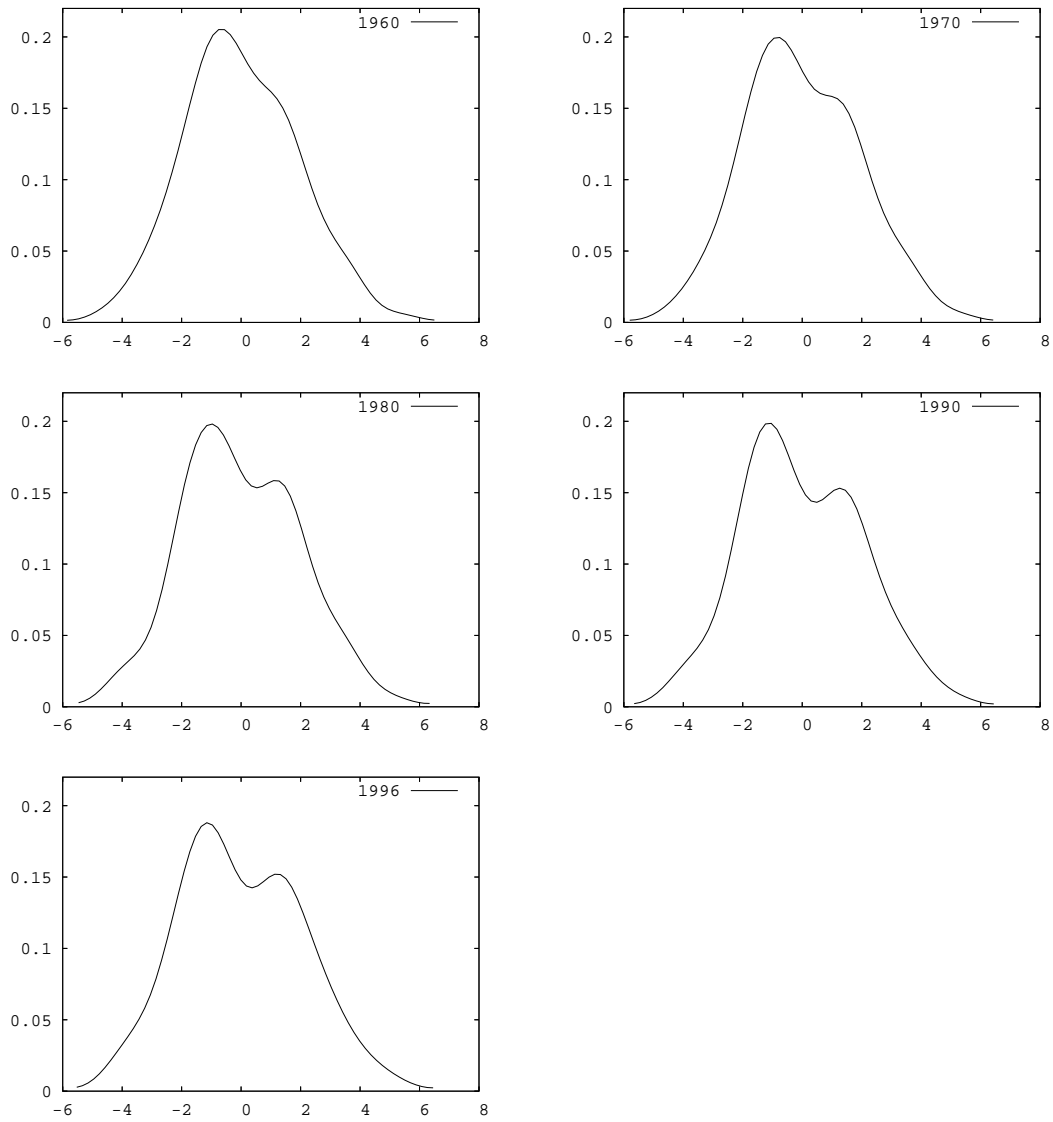
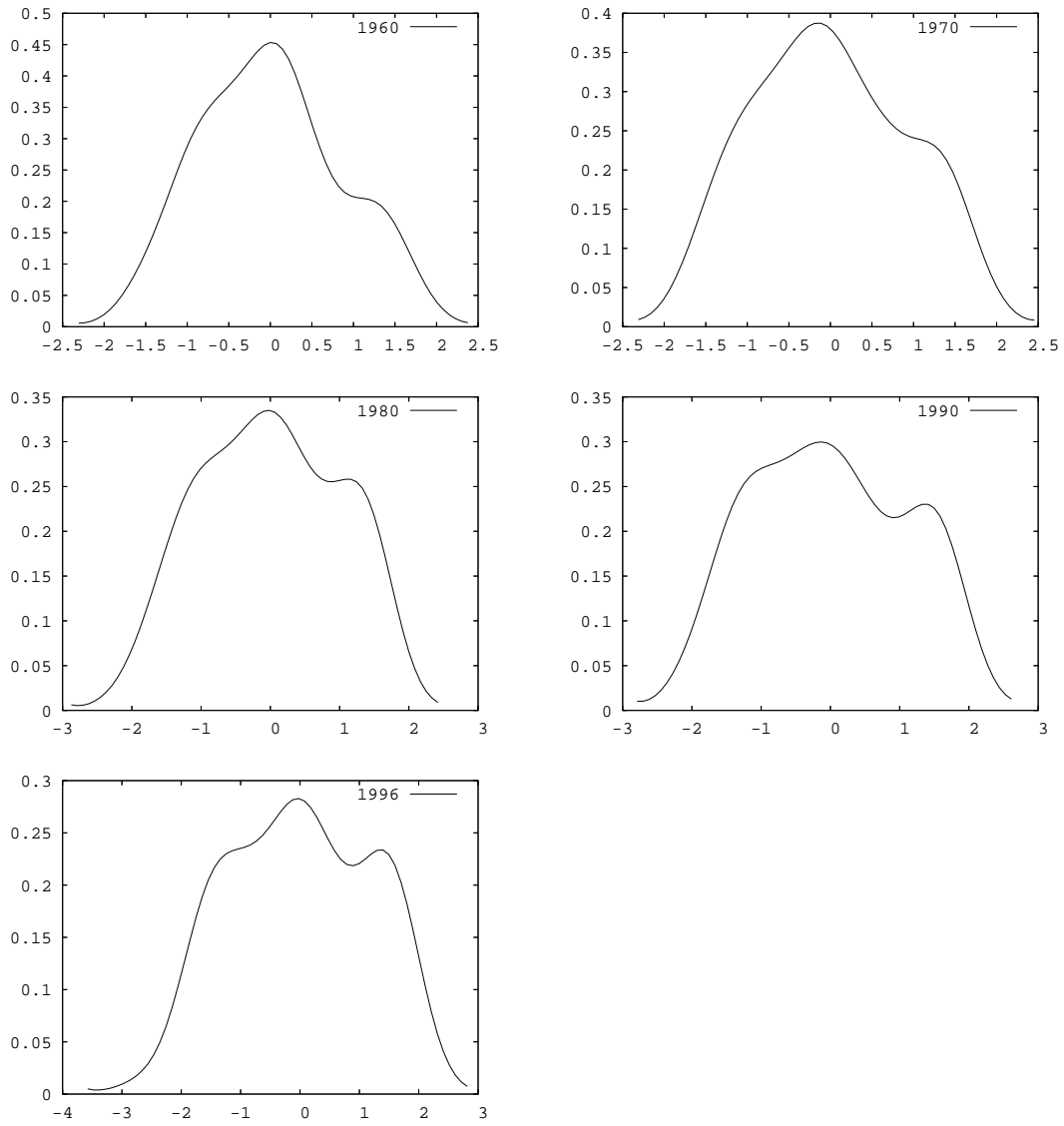Figure 1: Kernel estimation of the empirical density of (log) normalized income S^*.

Figure 2: Kernel estimation of the empirical density of (log) normalized per capita income s^*.

Here, let us consider the time series available from Penn Tables version 6.1 and estimate the kernel density for the distribution of normalized income and normalized per capita income. Following the standard notation, the kernel density estimator for a sample of data $\{x_i\}_{i=1:n}$ is defined as:

$$\widehat{f}(x) = \frac{1}{nh} \sum_i^n K(\frac{x - x_i}{h}) \tag{5}$$

where K is the chosen kernel function and h the kernel bandwidth. This non-parametric estimation procedure sensitively depends on the choice of the kernel bandwidth. The larger the chosen bandwidth, the smoother the estimated density.

In order to get a preliminary look at the distributions, let us select a bandwidth with the rule of thumb proposed in Silverman (1986). These exploratory plots suggest that the estimated densities become less and less unimodal over the years. Figures 2 for per capita data show that the distribution could have been already bimodal in 1960 and that it might have gone towards a three-humps shape after 1996.

There are a few statistical procedures to test for the number of modes of a given empirical distribution. Bianchi (1997) first used the non-parametric procedure presented in Silverman (1981) to test for multi-modality of the cross-country distributions of per capita income for the years 1970-1989. More recently, Henderson et al. (2006) apply to our same data set two different tests and suggest that the income distribution could have been multi-modal throughout the whole period.

Let us perform multi-modality tests on our longer time series. The Silverman test is based on kernel estimation and relies on the calculation of critical kernel bandwidths for the appearance of a given number of modes m. Call $h_c(m)$ the critical bandwidth such that for any bandwidth $h > h_c(m)$ the density displays less than m modes, while for any $h < h_c(m)$ the modes are at least $m + 1$. Any $h_c(m)$ may be used as a statistic to test the hypothesis $H_0 : m$ modes vs $H_1 :$ more than m modes. The actual p-value of the test can be calculated via bootstrapping. When $\hat{p}_c(m) < \alpha$, where α stands for the significance level of the test, one can reject the null hypothesis that the distribution has m modes and not more. This test is known to have a bias towards conservatorism, in the sense that it leads to rejection in fewer cases than other tests would. A procedure to correct this shortcoming has been proposed in Hall and York (2001) for the unimodality test and it allows to calculate corrected actual p-values for a given significance level of the test.[10]

Bianchi (1997) already discusses some of the problems involved in using a fully

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[10]For a discussion on the advantages and the shortcomings of the Silverman test see Henderson et al. (2006). The main shortcoming appears to be the fact that Silverman test is not nested and it may thus yield inconclusive results.

non-parametric technique. In particular he points out that this kind of test may fail to detect multiple modes when modes are not well separated. For the particular instance of GDP data, this may indeed be the case when one considers logarithmic transformations of the GDP data. The log transformation is in fact a smoothed version of the actual data and possible modes in the distribution will appear closer to each other than in the actual data. To avoid this problem Bianchi suggests taking non-logarithmic transformations, such as the per capita income relative to the sum of all incomes. Let us then define:

$$z^*_{i,t} = \frac{y_{i,t}}{\sum y_{i,t}} \tag{6}$$

We report the outcome of our multi-modality Silverman tests on this specific income measure to make our results comparable with Bianchi's findings. Table 1 shows estimates for key years and for all years in the transition phase from unimodality to bimodality regime. We choose a significance of $\alpha = 0.1$, a reasonable significance level for this type of data. Scores that lead to rejection of the statistical hypothesis are highlighted in italics, the results for the unimodality test include the Hall-York correction. We indeed confirm that the assumption of bimodality can not be rejected at a 10% level, even since 1970. [11]

The result of bimodality hints at the apparent emergence of distinct clusters of countries. It is further supporting evidence against the hypothesis of global convergence of countries to a common steady state level. Instead, it provides evidence that countries tend to polarize in 'relatively rich' and 'relatively poor' countries.

At the same time, the other part of the story, as discussed in Quah (1997), is that the same shape of a given distribution may conceal very different intra-distribution dynamics. Is it the case that poor countries have been converging to a common income level and rich countries to their own high level of income, or the two modes are also the result of shifting in ranking between poor and rich countries? The issue at stake is the respective weight of *persistence* and *mobility* of countries inside the distribution. Quah (1997) finds evidence that the period 1960-1988 has been characterized by high persistence of relative rankings with low (*albeit* positive) transition probabilities between the 'poor' and 'rich' clubs (and *vice versa* too). The main events contributing to mobility are the 'growth miracles' of countries like Hong Kong, Singapore, Japan, Korea and Taiwan. 'Growth disasters' include some sub-Saharan African countries and other impressive dramas such as that of Venezuela which was among the richest

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[11]We should mention that recent work (Henderson et al. (2006) and Goerlich Gisbert (2003)) has discussed the opportunity of weighting the income variables by population, but we do not address this type of analysis in our work here.

Table 1: Results from multi-modality tests: critical bandwidths from Gaussian kernel estimates and corresponding significance score from smoothed bootstrap test (B=1000 replications) for the variable z^*.

| Year	$h_c(1)$	$p_c(1)$	$h_c(2)$	$p_c(2)$	$h_c(3)$	$p_c(3)$
1960	0.0034	0.284	0.0026	0.370	0.0023	0.103
1965	0.0035	0.196	0.0026	0.352	0.0021	0.110
1966	0.0038	0.109	0.0026	0.392	0.0021	0.149
1967	0.0039	*0.061*	0.0027	0.284	0.0022	0.106
1968	0.0035	0.188	0.0028	0.239	0.0020	0.232
1969	0.0035	0.161	0.0023	0.524	0.0019	0.324
1970	0.0039	*0.049*	0.0029	0.186	0.0015	0.655
1971	0.0041	*0.024*	0.0030	0.147	0.0015	0.503
1972	0.0042	*0.015*	0.0027	0.257	0.0015	0.557
1973	0.0042	*0.014*	0.0024	0.417	0.0012	0.894
1974	0.0042	*0.011*	0.0024	0.349	0.0015	0.445
1975	0.0043	*0.003*	0.0019	0.501	0.0015	0.455
1980	0.0043	*0.005*	0.0016	0.861	0.0015	0.512
1985	0.0050	*0.000*	0.0018	0.576	0.0015	0.404
1990	0.0053	*0.000*	0.0025	0.262	0.0021	0.077
1996	0.0047	*0.011*	0.0026	0.404	0.0022	0.134
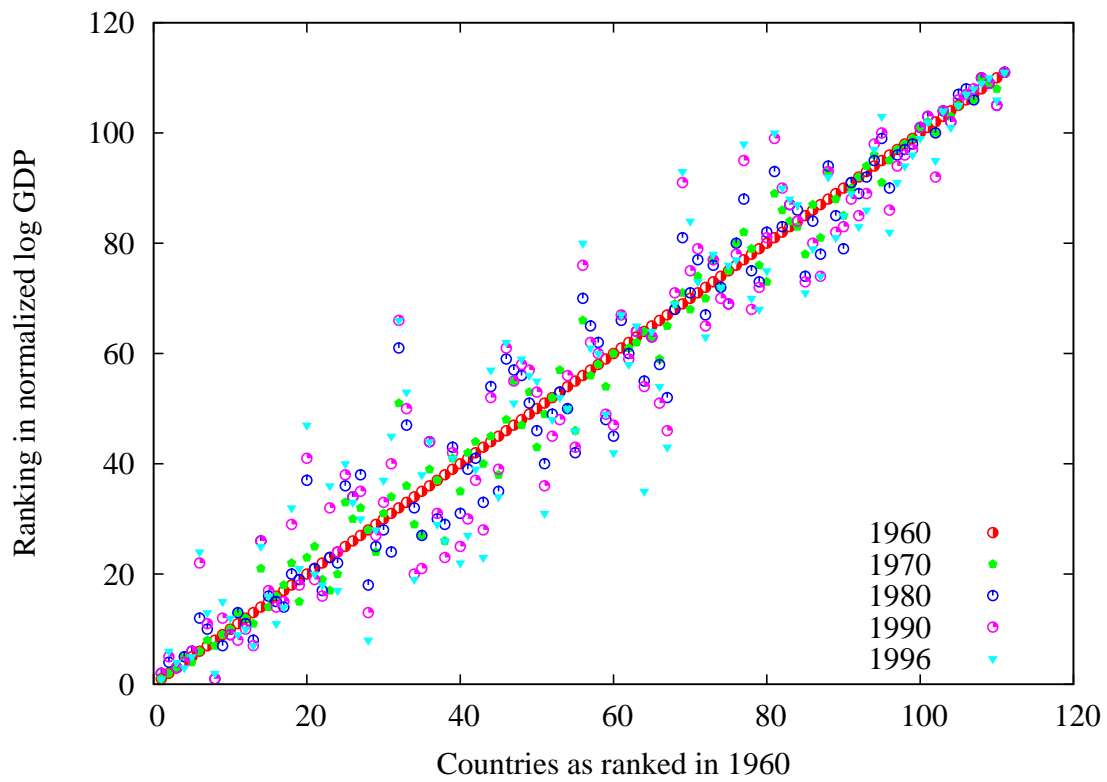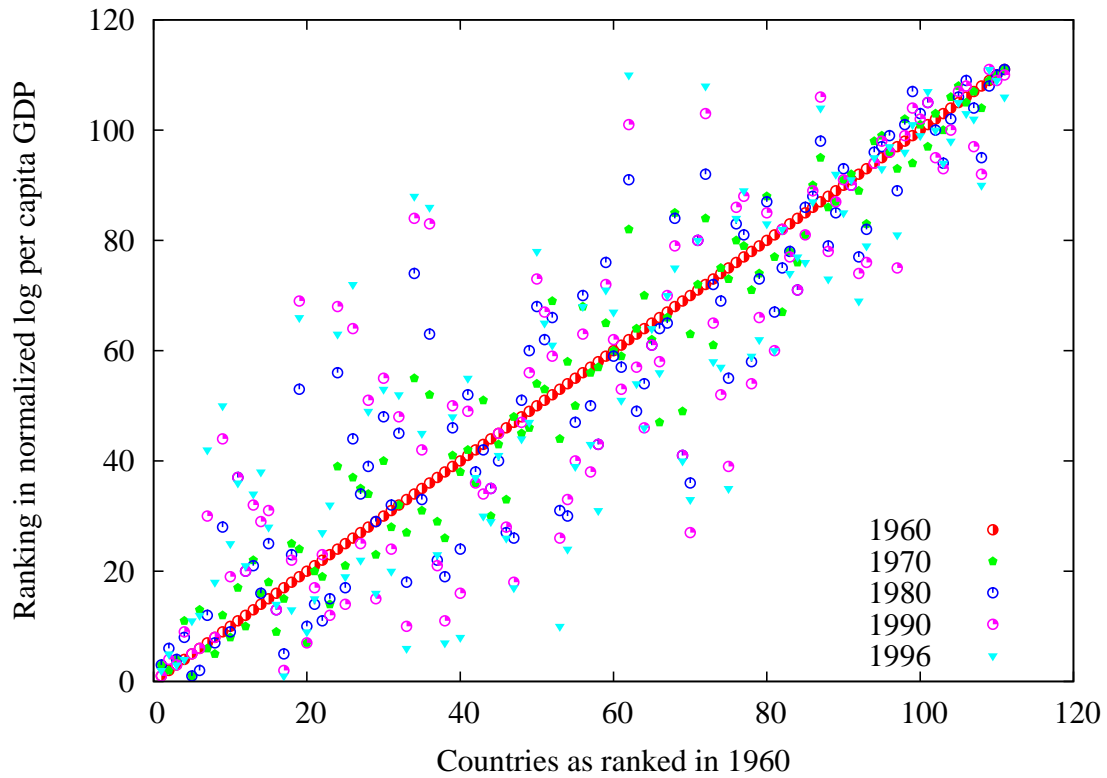
Figure 3: Changes in ranking of countries with respect to per capita income (top) and total income (bottom).

countries in 1960 and has since dramatically fallen in the 'poor' countries club.

With the new data at hand, we re-assess the evidence concerning intra-distribution dynamics by looking at changes in the rankings of countries with respect to income until 1996. We present here a quite simple measure of intra-distribution dynamics. More elaborate estimations using Markov transition matrices can be found in Quah (1997) and in the recent Fiaschi and Lavezzi (2003). Figure 3 provides an impressionistic picture of changes in relative ranking for five chosen years, namely 1960, 1970, 1980, 1990 and 1996. One can clearly see that the bottom and top part of the plot display high persistence in rankings. Countries that ranked very poorly and countries that were ranked at the top in 1960, both keep their position. Most of the action takes place for countries in the middle rankings, with both up- and down-shifting even if of limited magnitude. Interestingly one also observes a higher 'turbulence' when rankings are taken on per capita income.

The result of bi-modality provides descriptive evidence that cannot be uncovered from regression analysis, but does not *per se* shed any light on the determinants of the cross-country distribution. Part of the interpretation involves the analysis of the appropriate conditioning variables which might account for the emergence of separate 'clubs' (Quah (1997)). Together, important circumstantial evidence is bound to come also from the investigation of the statistical properties of growth rates. This is what we shall do in the following.

5 The statistical properties of growth shocks

5.1 Preliminary analysis

Let us now turn to the distribution of the GDP growth rates. We first plot the moments of the (non-normalized) growth rates g_{it} (Figure 4). The evolution of the average growth rate hints to two distinct phases, reasonably separated by the year 1973. This major discontinuity is well known to appear in most economic time series. Also here we find that the years before 1973 are characterized by a somewhat higher average level of growth, and a lower mean value thereafter. The standard deviation is stable across all sample years, which implies that in fact the coefficient of variation of rates is higher after 1973.

Notwithstanding these discontinuities in growth patterns let us nonetheless begin by studying the properties of de-trended growth dynamics over the whole post World War II period. Following the procedure also used in Canning et al. (1998), we pool together the normalized observations for all years and countries and we obtain a sample of $111 * 36$ observations, large enough to support robust statistical analysis.

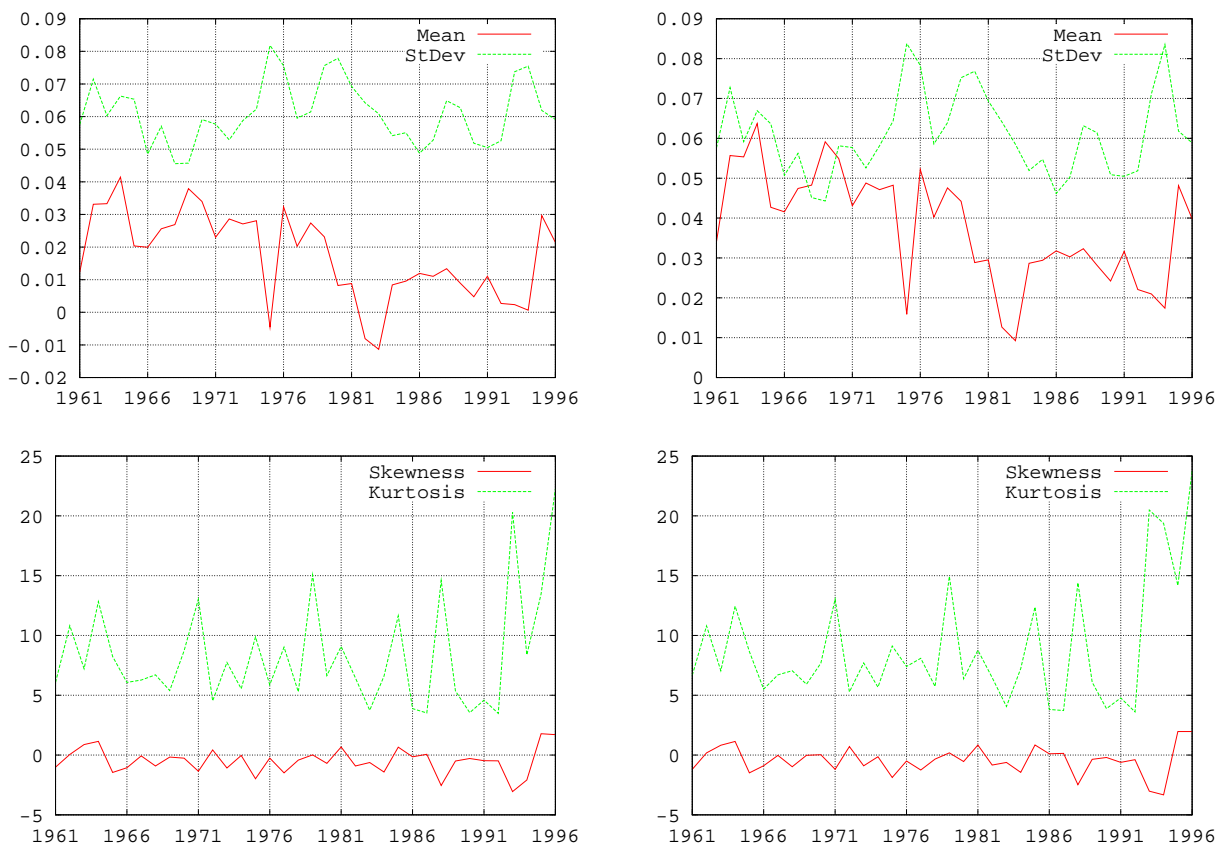Figure 4: Evolution in time of the moments of the distribution of growth rates. Left panels refer to g_{it}, right panels to G_{it}.
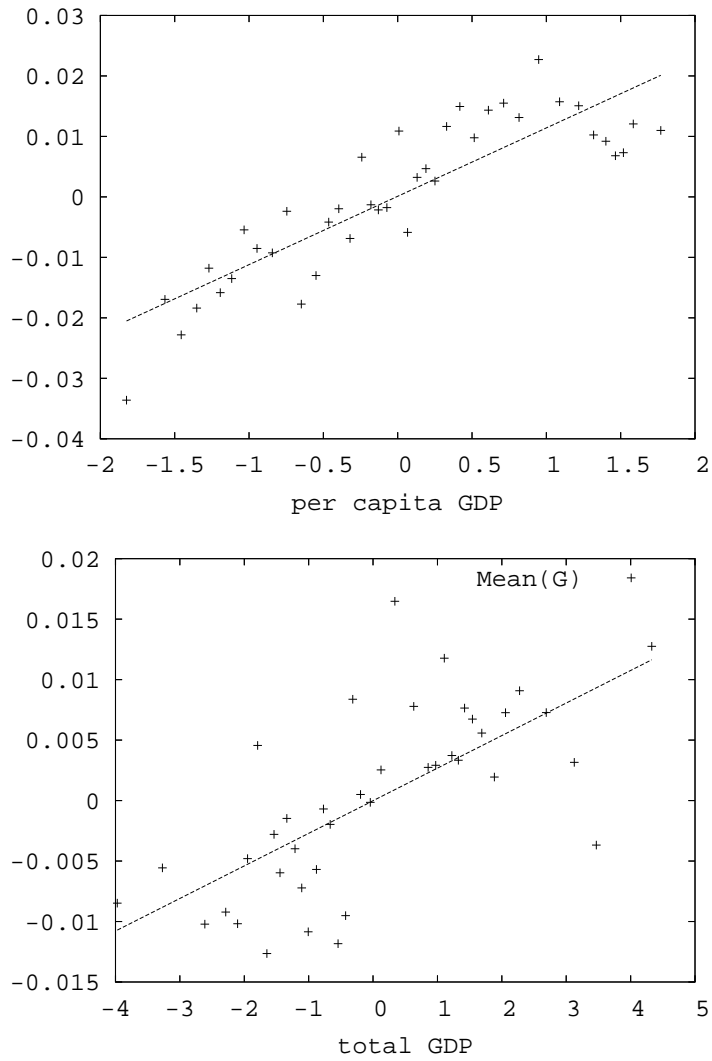
Figure 5: The relation between average growth rate and income level for different income classes. Linear fits are also shown. The top plot refers to per capita variables (slope= 0.01131 ± 0.00104), the bottom one to total income ones (slope=0.00269 ± 0.00047).

As a preliminary point let us ask whether higher or lower income countries are characterized on average by (relatively) higher growth rates.

We group countries into 40 equally populated groups with respect to income s^* (or S^*) and calculate the mean annual growth rate g^* (or G^*) in each income class. Indeed, we find a statistically significant and positive correlation between the average growth rates and levels of income, be it the total or the per capita income (Figure 5). Larger and more developed (i.e. with higher per capita incomes) countries are characterized, on average, by a higher growth performance.

The interpretation of the two relations offers quite different insights. When we look at per capita income data the result that richer countries display on average higher growth rates can be read as straightforward evidence for divergence and polarization of countries into two classes of 'very rich' and 'very poor' countries. Such piece of evidence does indeed suggest the existence of some form of dynamic increasing returns in production and in the accumulation of technological knowledge. However notice that the relation for per capita data seems to be more of a parabolic rather than a linear nature: for the highest levels of per capita income the relation is not significant or even becomes negative.

Conversely, the positive relation between average growth rate and total domestic income hints at structural effects of the sheer size of an economy similar to 'static' economies of scale.[12]

5.2 The volatility of growth rates

Are higher income countries characterized by less volatile growth rates? Some recent evidence (see for example Pritchett (2000) and Fiaschi and Lavezzi (2004)) shows that the volatility of growth rates is much higher for developing countries than for industrialized ones. Throughout the process of development the levels of per capita GDP obviously increase. Together, reductions in the dispersion of growth performance may also be taken as an indication that countries move on more stable growth paths.

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[12]It should be clear that the possible scale effects that we identify here do not necessarily bear any direct relation with the scale effect which has been the object of controversy among 'new growth' theorists, as discussed in Jones (1999). One of the questionable predictions by the first wave of 'new growth' models was the presence of a scale effect on the steady state growth according to which an increase in the total population, and thus in the available specialized labor force, proportionally increased the long run per capita growth. In some subsequent models the scale effect has shifted to the level of per capita income, rather than its long run growth rate. In our strictly 'inductive' analysis here of course we do not make any commitment on the existence of a steady state rate of growth: simply, the statistical relations between income and growth appear to suggest some forms of increasing returns.
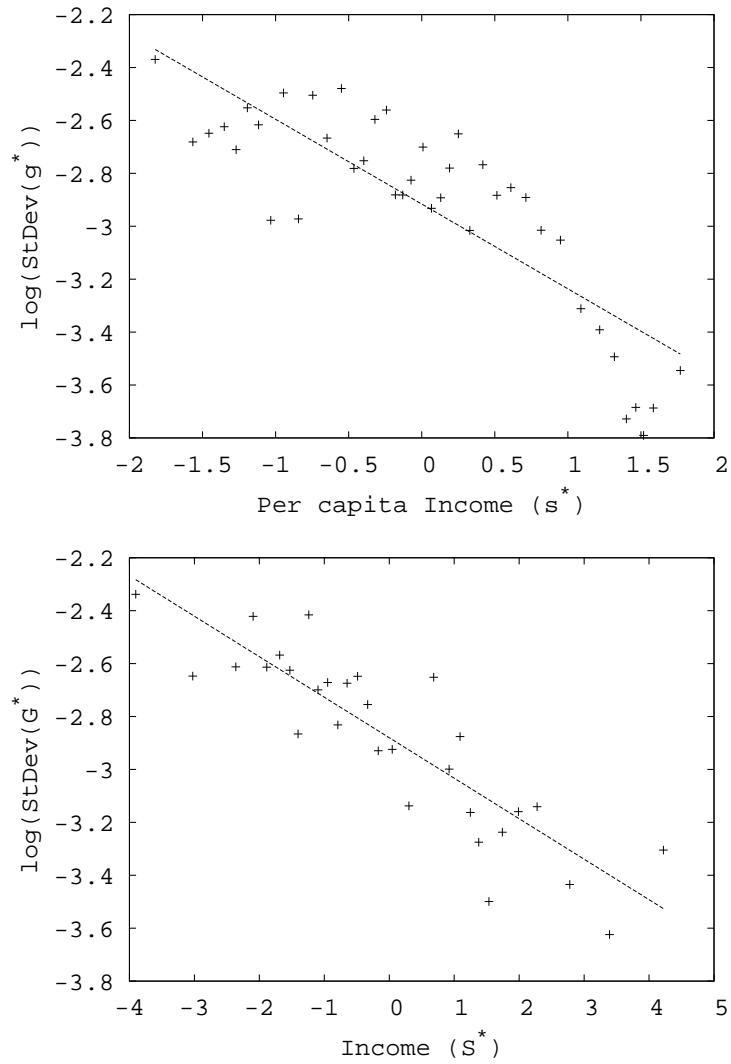
Figure 6: The relation between the logarithm of the volatility of growth rates and the levels of income. The relation is fitted via a linear regression, with estimated slopes of $c = -0.32 \pm 0.03$ (per capita) and $d = -0.15 \pm 0.02$ (total).

We again group countries by size, calculate the standard deviation of the normalized growth shocks and associate it to the central value of income in each class. Here, we uncover a negative relation between the log standard deviation of growth rates and the level of per capita income. In other words the volatility of growth rates scales with income as a power law. The estimated slope for a linear fit equals $c = -0.32 \pm 0.04$.

We also confirm on our extended dataset the scaling relation found in Canning et al. (1998) and Lee et al. (1998) for aggregate GDP data. The slope of a fitted line, however, is in this case much lower, $e = -0.15 \pm 0.02$. This may in fact tell us that a 'strong' scaling relation holds only when one considers the level of economic development, as proxied by per capita income. Growth performances are less volatile for *more developed* countries. The sheer size of an economy is relatively less relevant. Fiaschi and Lavezzi (2004) confirm a negative relation between growth volatility and the size of an economy. Their work tries to explain growth volatility with a set of country variables including total GDP, the share of the agriculture sector as a proxy for the structure of the economy and trade openness. Interestingly, they find in their sample that per capita income does not play a significant role when the mentioned variables are considered.

5.3   The distribution of growth shocks

One way to deal with the 'size effect' on the average growth rate is to group countries by their level of income in three classes: Low, Medium and High (per capita) GDP. This same procedure is used in Canning et al. (1998) and Lee et al. (1998), who also recognize different growth distributions for countries characterized by different size in terms of total income. We further normalize the growth rates in each group and then proceed by plotting their empirical histograms. (We show in figures 7 and 8 only the Small and Large Income classes, since the Medium one always lies in between.)

We refine the description of the properties of the distribution of growth rates by fitting on the empirical densities a general family of distributions, the set of Subbotin densities. The idea of fitting Subbotin densities to distributions of growth rates is introduced in Bottazzi and Secchi (2003).

The functional form of the Subbotin family is given by:

$$f(x) = \frac{1}{2ab^{\frac{1}{b}}\Gamma(1 + \frac{1}{b})} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^b} \tag{7}$$

where the parameter a controls the standard deviation and b is a parameter which determines the shape of the distribution. Note that for a value $b = 2$ the distribution turns out to be a Normal one, while for $b = 1$ the distribution is Laplacian, also know
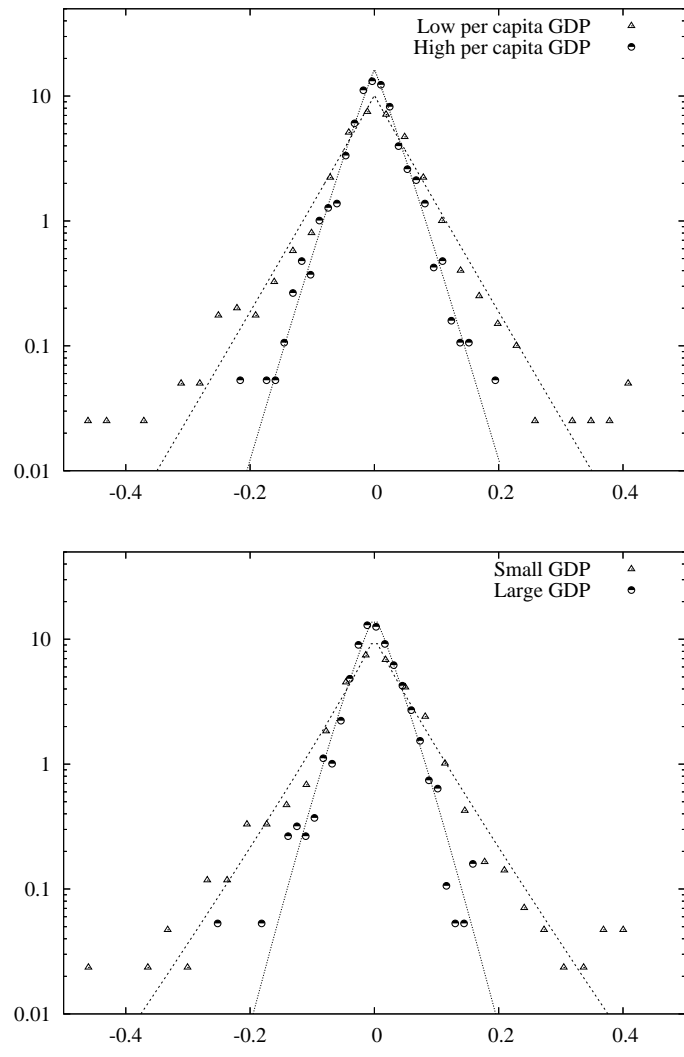
Figure 7: The empirical distribution of growth rates of per capita income (top) and income (bottom) for two income classes, Low and High.

as Double Exponential. As b gets smaller, the tails get heavier and the peak of the density becomes more pronounced. For $b = 0$ the distribution is degenerate in the mean. We fit the family of density using a maximum likelihood procedure (for details see Bottazzi (2004)).

The empirical distribution of the growth rates is quite well fitted by a Subbotin density with a b-parameter close to 1, hence the distribution is approximately Laplacian (Figure 7)[13]. Note that if growth residuals were Normal the fitted curve would be a parable ('bell shape') in a logarithmic scale. On the contrary, we find that the distribution of growth rates is markedly non-Gaussian and closer to a Laplacian density which displays a 'tent shape' in the log scale. Note also that the distribution is nearly invariant when we consider sub-periods in the overall sample years.

Further, notice that the plots in Figure 7 reveal a sensibly different width of the distribution for low income and high income countries, which one should expect given the dependence of the dispersion of growth rates upon a country's income level shown in the previous section. Let us then exploit the two linear relations, estimated as in Figure 6. We re-scale growth rates as follows:

$$
\begin{aligned}
\hat{g}_{it} &= g_{it}^* / exp(cs_{it}^* + d + \tfrac{1}{2v^2})) \\
\hat{G}_{it} &= G_{it}^* / exp(eS_{it}^* + f + \tfrac{1}{2w^2}))
\end{aligned}
\tag{8}
$$

where v^2 and w^2 are unbiased estimators of the variance of residuals in the linear regressions between the (log) standard deviation of growth rates and the (log) size measure.

Figure 8 shows the distribution of growth rates after rescaling. With this procedure we eliminate any possible size effect on the dispersion of the distribution. The Laplacian shape of the distribution is confirmed: growth shocks are markedly not Gaussian.

Still the two distributions for the two income classes coincide only for the central part of the observations. They differ on the tails, which suggests that controlling for the effects of the level of income on the first two moments of the growth rates is not enough to fully characterize the structure of growth shocks.[14] Possibly, higher moments play a role. In any case observations at the extremes seem to be crucial in shaping the

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[13]The estimation is done on the normalized growth rates, thus the parameter μ of the Subbotin is always set to zero.

[14]Note that this result continues to hold also if one fits the data with distributions characterized by heavy tails. Indeed, we tried fitting the family of 'stable distributions' (which includes Cauchy and Lèvy ones) to check whether the gap between the distribution of re-scaled growth rates for the different income classes was due to an unsatisfactory fit of the Subbotin on the tails. We find that the gap between the estimated distributions for the two classes is not eliminated. Moreover, heavy tailed distributions do not provide an overall better fit to the data.
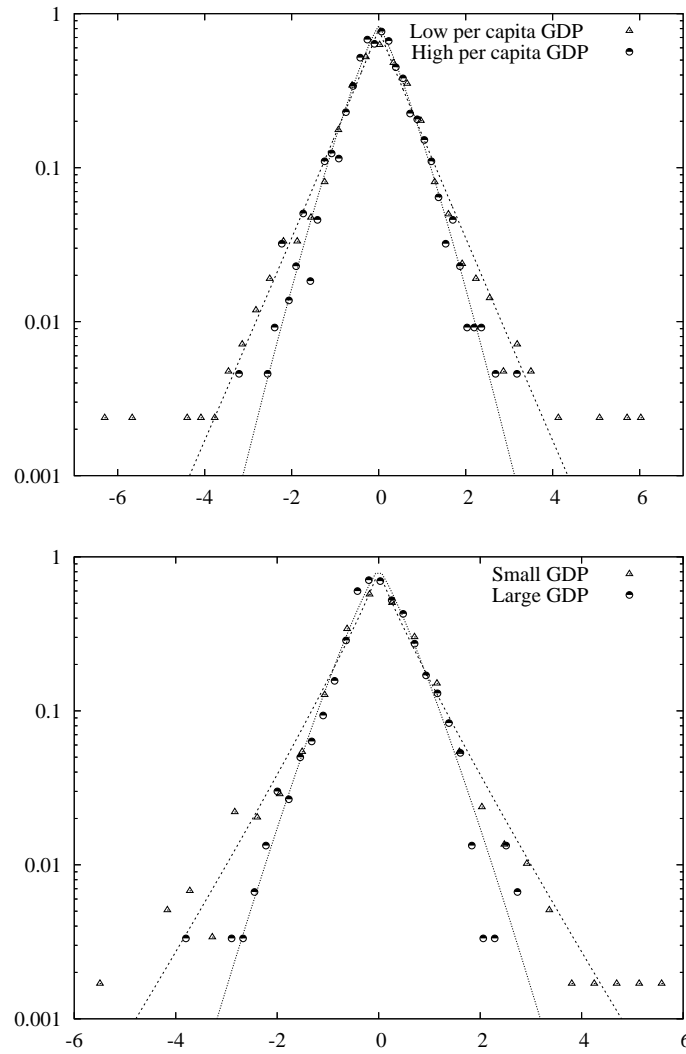
Figure 8: The distributions of re-scaled growth rates for two income classes.

| | Growth rates | | Rescaled growth rates | |
Income classes	b	a	b	a
Low per capita GDP	0.9829	0.049758	0.9729	0.006895
	(0.0014)	(0.000048)	(0.0013)	(0.000006)
High per capita GDP	1.0644	0.029598	1.2029	0.004162
	(0.0015)	(0.000028)	(0.0017)	(0.000004)
Small GDP	0.9323	0.050318	0.9308	0.002454
	(0.0013)	(0.000049)	(0.0013)	(0.000002)
Large GDP	1.1885	0.030904	1.2092	0.001519
	(0.0017)	(0.000028)	(0.0018)	(0.000001)

Table 2: Estimated Subbotin parameters for the distributions of growth rates. Standard errors are reported in parenthesis.

distribution of growth rates. Another way of putting it is by saying that particularly high and particularly low growth performance rather than being considered simply as outliers, might in fact play a crucial role in the international process of growth.

5.4 Conditioning on openness to trade

As mentioned, a few empirical studies have been looking for underlying conditioning variables that may explain 'convergence clubs', that is groups of countries that appear to follow a similar growth path. Here, let us explore whether a measure of a country openness to trade is able to provide a useful conditioning scheme to support different growth rates regimes.

We exploit the variable 'Openness to trade' in the Penn World Tables 6.1, calculated as the ratio of the value of the sum of exports and imports over the value of GDP.[15]

Let us begin by studying the existence of scaling relations for both the mean and the dispersion of growth rates in different trade classes. Figure 9 shows how both relationships in the different groups overall confirm the findings of Section 5.2. The average growth rate continues to be positively correlated with the level of per capita income. While the relationship is the same for countries characterized by Low and Medium openness, we find a higher correlation for High openness countries. One way to read this result is via the conjecture that those countries that are more open to trade are, *ceteris paribus*, in a better position to benefit from dynamic increasing returns stemming from the access to the world markets. Conversely the same countries do not show any statistically significant difference in volatility profiles (right panel of Figure 9).

We also looked at the relation between the growth volatility in per capita data and the total GDP as a measure for the size of an economy (Figure 10). The reduction in dispersion for bigger economies holds for the whole data set as well as for each of the 'trade openness' classes. Putting it another way, the evidence suggests that, overall, the patterns of insertion as such of an economy into the world system of trade do not seem to exert any major impact either on the mean or on the volatility of growth rates – as distinct from the effects of structural characteristics of economies, such as size and level of economic development.

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[15]The variable is quite volatile, since it strongly depends also on short-run volatility in exchange rates, thus we take a backward moving average over 4 years.
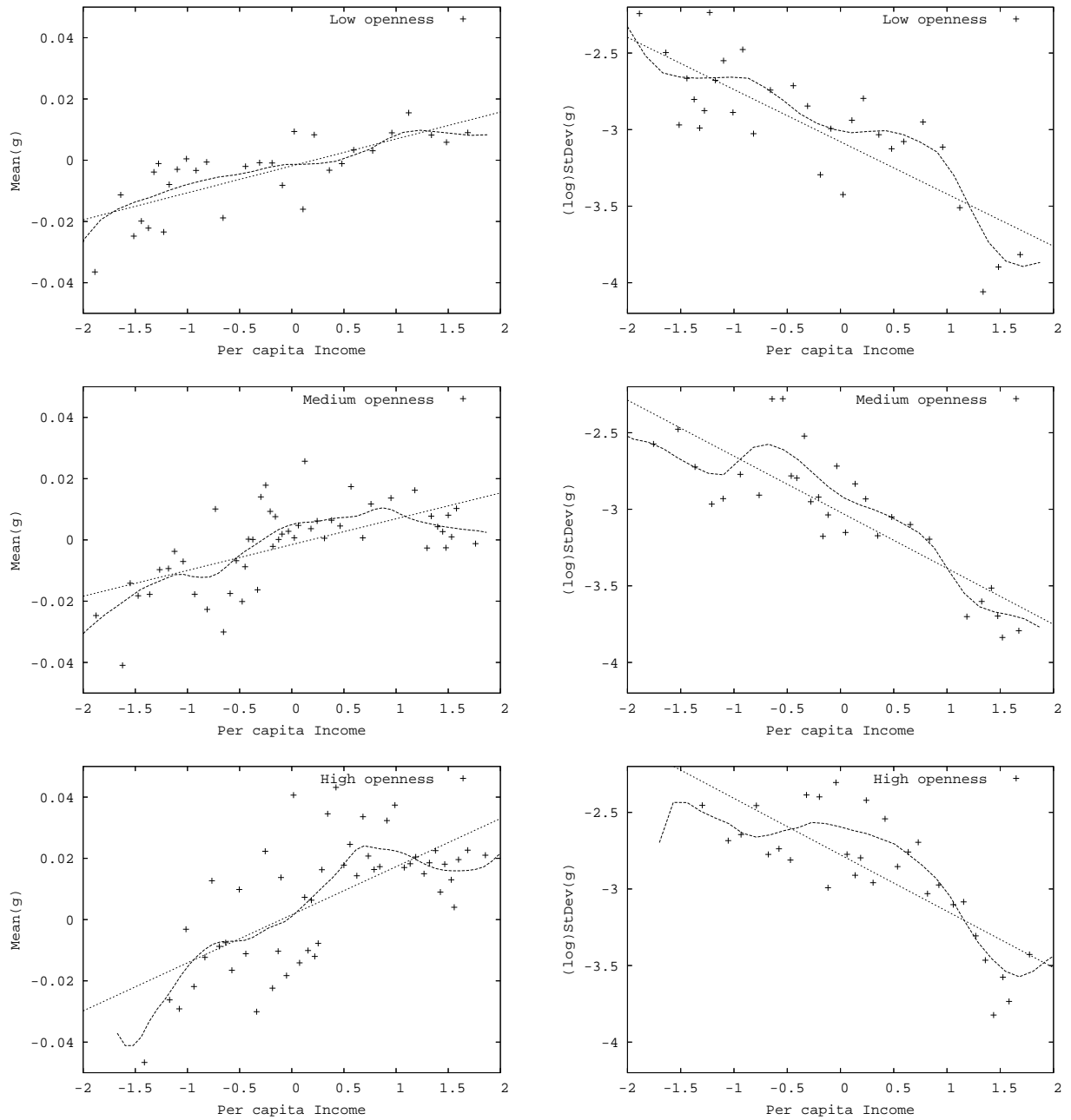
Figure 9: Scaling of growth rates (left panels) and growth volatility (right panels) in trade classes. Per capita data.
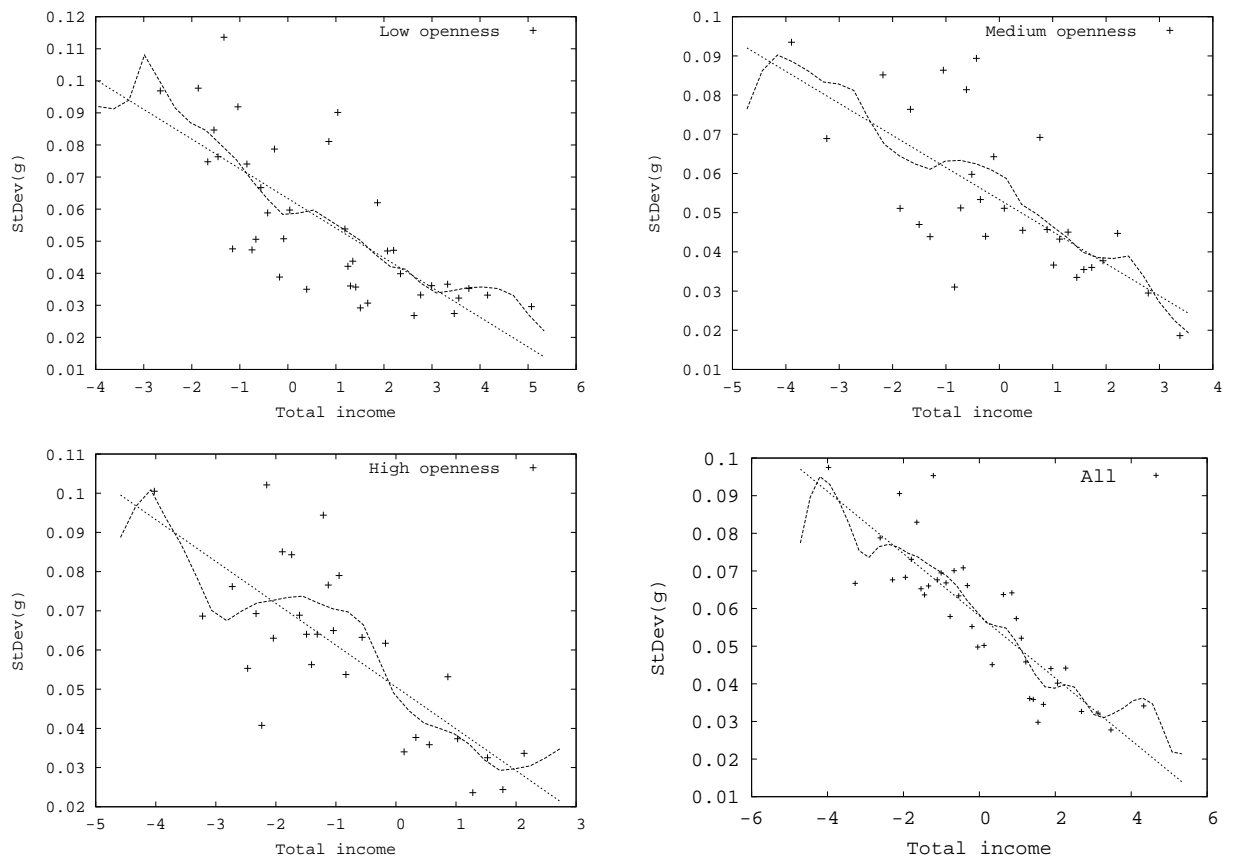
Figure 10: Scaling of the dispersion of per capita growth rates with respect to the total size of the economy for the 3 trade classes and for the whole set of observations.

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6 The sectoral process of growth

We have recalled at the beginning of this work some characteristics of the process of growth of firms and then we have moved to the level of observation of whole economies, trying to identify similarities and differences between the micro and macro levels of observation. However, economies are composed by many distinct sectors of activity linked with each other by a thread of supply/demand relations and flows of technological knowledge. Moreover, individual sectors are also the *locus* of competition wherein firms interact with each other. Thus, the dynamics of sectoral value added represent a meaningful intermediate level of observation between individual firms and whole countries. Let us investigate also at this level the statistical properties of growth.

In order to do it, we make use of the 60 Industry Database compiled at Groningen Growth and Development Centre (GGDC (2005)). Our sample covers the years 1979-2002 and includes information on the value added of different sectors in 20 countries.[16] We consider data for 52 sectors defined at a 2-digit ISIC-Rev.2 classification level and covering manufacturing and services (SIC codes range from 15 to 95).

Let $g_{i,j,t}$ be the (logarithmic) growth rate of the valued added at constant 1995 prices of sector j of country i between $t-1$ and t. For each country, we calculate normalized sectoral growth shocks as:

$$h_{i,j,t} = g_{i,j,t} - g_{i,t} \tag{9}$$

where the normalization is done using the average growth rate across sectors in the chosen country and year.

As a preliminary analysis, we performed Kolmogorov-Smirnov tests for each country to check whether the distribution of the normalized sectoral growth rates can be considered stationary in all years. The tests never reject the null hypothesis that the data for the different years come from the same distribution. Given these results, we pool all data together and study the distribution of sectoral growth rates in each country of our sample.

Figure 11 shows the empirical distribution for six countries. We fit on each distribution the Subbotin family of densities. We also fit an asymmetric version of the Subbotin family where the key parameters a and b are allowed to vary in the left and right side of the distribution. Indeed, the empirical distributions reveal for a few

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[16]Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, South Korea, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Taiwan, UK, and US. We are aware of the limitations of considering a rather small and relatively homogenous set of countries. At the same time, this dataset provides reliable and comparable data which would hardly be available on a larger set of countries with a sufficient disaggregation.
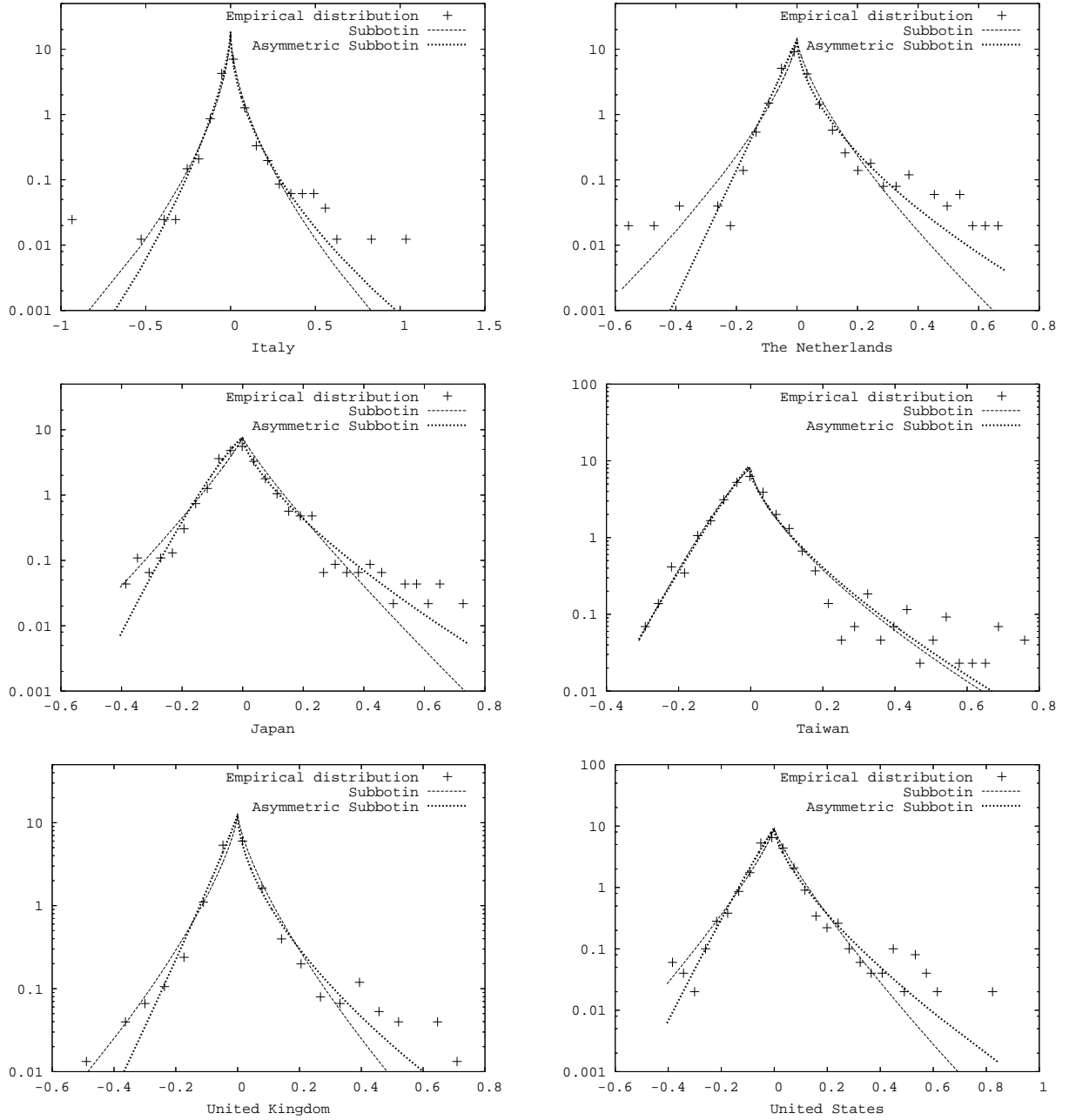
Figure 11: Empirical distribution of the normalized sectoral growth rates fitted with the Subbotin distribution and the Asymmetric Subbotin distribution for a selection of countries.

Table 3: Estimated coefficients of the Subbotin density for the distribution of sectoral growth rates in 20 countries and in some chosen years. Standard errors for the estimates are reported in parenthesis.

	Fit with Subbotin		Fit with Asymmetric Subbotin			
Country	b	a	b_{left}	b_{right}	a_{left}	a_{right}
Australia	0.618 (0.000)	0.050 (0.000)	0.695	0.568	0.050	0.052
Austria	0.643 (0.000)	0.057 (0.000)	0.823	0.520	0.055	0.060
Belgium	0.649 (0.000)	0.072 (0.000)	0.721	0.593	0.071	0.074
Canada	0.757 (0.001)	0.061 (0.000)	0.925	0.654	0.060	0.063
Denmark	0.717 (0.001)	0.084 (0.000)	0.837	0.601	0.083	0.085
Spain	0.603 (0.000)	0.047 (0.000)	0.733	0.507	0.045	0.050
Finland	0.666 (0.000)	0.071 (0.000)	0.901	0.519	0.069	0.077
France	0.723 (0.001)	0.051 (0.000)	0.924	0.595	0.049	0.053
Greece	0.675 (0.000)	0.053 (0.000)	0.852	0.616	0.054	0.056
Ireland	0.803 (0.001)	0.093 (0.000)	0.980	0.694	0.092	0.097
Italy	0.586 (0.000)	0.041 (0.000)	0.675	0.529	0.041	0.043
Japan	0.885 (0.001)	0.071 (0.000)	1.219	0.709	0.070	0.074
South Korea	0.753 (0.001)	0.087 (0.000)	0.939	0.583	0.083	0.091
The Netherlands	0.714 (0.001)	0.044 (0.000)	0.988	0.576	0.043	0.047
Norway	0.644 (0.000)	0.079 (0.000)	0.668	0.624	0.079	0.080
Portugal	0.597 (0.000)	0.071 (0.000)	0.679	0.548	0.069	0.075
Sweden	0.797 (0.000)	0.069 (0.000)	0.917	0.701	0.073	0.065
Taiwan	0.812 (0.001)	0.066 (0.000)	1.184	0.660	0.066	0.071
UK	0.720 (0.001)	0.050 (0.000)	0.942	0.591	0.049	0.053
US	0.831 (0.001)	0.060 (0.000)	1.084	0.699	0.060	0.063

countries a clear right skewness: allowing asymmetry in the fitted density improves the fit significantly.

Interestingly, the distributions, again, are generally Laplacian, albeit with significant cross-country differences in both the shape and the width (cf. Table 3 for the estimated Subbotin parameters). In quite a few cases the estimated shape parameter b is significantly less than one, indicating that the tails of the distribution are even 'fatter' than those yielded by an exponential distribution. Moreover, the asymmetries between right and left tails with bigger positive shocks appears to be interesting in its own right, possibly hinting at the occurrence of those large 'spurs' of growth which Schumpeter suggested to be associated with the diffusion of major innovations.

These findings complement those of Sapio and Thoma (2006) who undertake a

similar exercise on US data from the NBER Manufacturing Productivity Database on 458 industries, finding a value of b of the Subbotin distribution, on average very close to one, that is, as discussed above, the signature of exponential tails of the distribution.

7 Some interpretations and concluding remarks

Let us put together the evidence and the puzzles stemming from the presented empirical analysis and propose a few tentative interpretations and conjectures.

7.1 Candidates for an explanation of the tent-shaped distribution of country growth rates

A first robust stylized fact, to recall, is that growth rates at the level of countries follow a Laplacian distribution. This property robustly holds also for subsets of countries and for different observational periods. Developed and less developed countries remarkably show the same exponential structure in their growth rates even after accounting for their different dispersion in growth performance. A first puzzle arises if we compare the invariance of this property with the evolution of the distribution of incomes. We have seen how this distribution changes over time starting from an approximately unimodal shape and getting later to an evident bimodality for which we have provided novel evidence. How does this relate to the invariance in the distribution of growth rates?

Remarkably, the distributional invariance of growth is a statistical feature analogous to that found with respect to corporate growth rates: see Stanley et al. (1996) and Bottazzi and Secchi (2004) on US business firms, Bottazzi et al. (2001) on the international pharmaceutical industry and Bottazzi et al. (2004) on Italian firms. All these quite diverse data sets robustly display Laplacian distributions of growth rates. Moreover, we have shown, exponential distributions of relative growth rates apply also at the 'intermediate' level whereby the sectors of activity within any one economy are the units of observation.

In the industrial organization literature, a common interpretation of the growth process builds on a baseline stochastic model of growth of a given unit of observation (e.g. a firm). If the growth process proceeded as the result of the cumulation in time of independent growth shocks one would find the growth residuals g_{it}^* to be Normally distributed and, thus, only representing 'noise'. Instead, one finds a very specific structure for the distribution of growth rates, which forces to reject the null hypothesis that growth is simply the outcome of adding independent shocks over time. Thus, one has to search for explanations of the growth process which admit that the 'elementary'

growth shocks are actually correlated with each other. And, indeed, such explanations ought to account for the scale invariance of such property, since correlation mechanisms in the growth process appear at all levels of observation, from firms to sectors to countries.

This scale invariant regularity is thus in need of a convincing economic explanation. Ultimately two diverse (but possibly complementary paths) seem to be available for the modeler.

(i) A known statistical result refers to the property that a mixture of a small number of Normal distributions produces fat-tailed distributions (see Lindsay (1995)). Thus, a tent-shape distribution can be seen as a mixture of Normal distributions given an appropriate parameterization. Mixtures are in principle an appealing tool for understanding the tent-shape distribution of growth rates because one can envision mixtures of mixtures of mixtures, capturing different scales of observation. Also, one could think of relating the components of the mixture to groups of countries representing different convergence clubs (see Durlauf, Kourtellos and Minkin (2001)). Nevertheless, such a statistical exercise still demands an interpretation of the underlying processes of growth yielding the purported distributional mixtures.

(ii) A distinct interpretative strategy tries to explicitly take into account what we know about micro-processes of growth, in particular acknowledging basic correlating mechanisms in the processes of market competition, together with the lumpiness of major competitive events. Recent research in macroeconomics has proposed a few models where aggregate GDP fluctuations are explained by micro-shocks at firm or sector level (e.g. Bak et al. (1993)). In these models the micro-shocks aggregate in a non-trivial way: instead of being diluted by the aggregation process, under certain circumstances they amplify and form the basis for the structure of macro-shocks. In this vein, Gabaix (2005) shows how a major part of aggregate growth shocks can be accounted for by the growth of the top 100 firms in a country.

In a similar vein, the exponential tails of the distribution of firm growth rates are explained in Bottazzi and Secchi (2006) with a minimal probabilistic model which couples a mechanism capturing forms of increasing returns (more successful firms tend to catch more business opportunities) together with competitive forces (firms compete for market shares). One could think of elaborating a similar multi-country model (keeping however in mind the different nature of inter-firm *vs* inter-country competition and complementarities).

Evolutionary agent-based models are also good candidates within this second style of modeling (see Dosi and Winter (2002), Silverberg and Verspagen (2005) and Verspagen (2005) for discussions of such a literature in a perspective pioneered by Nelson and

Winter (1982)). Indeed, preliminary exercises on the grounds of the model in Dosi, Fagiolo and Roventini (2005), wherein macro-dynamics are nested into heterogenous boundedly rational firms, show its ability to reproduce the tent-shape distributions of firm *and* country growth rates.

7.2 Scaling of the growth volatility

The other stylized fact highlighted by our analysis is the existence of a negative relation between the dispersion of growth rates and the level of *per capita* income. Moreover the volatility scales with income as a power law. Its estimated coefficient for per capita data, $c = -0.32$, is much higher than the $e = -0.15$ estimated with aggregate income data. This seems to suggest that the 'true' scaling relation does not hold for size as such, as measured by the gross product of an economy, but it characterizes *in primis* the level of development of a country. The structural effect of the total size of an economy plays a role, but the stability of growth performances for high income countries stands out more strongly when the income measure pertains to per capita incomes rather than the sheer size of countries.

Amaral et al. (2001) and Lee et al. (1998) propose to interpret the scaling relation by reference to a benchmark model of 'complex organizations'. The idea is to view an economic organization, i.e. a country in our specific instance, as made up of different units of identical size. Then two opposite situations might occur. If all units grew independently then the volatility of growth rates would fall as a power law with coefficient -0.5 (a result of the law of large numbers, as suggested already in Hymer and Pashigian (1962)). Conversely, if the composing units were perfectly correlated there would be no relation between the volatility of growth shocks and size, so we would find a slope of 0.

The estimated coefficients, lying in between 0 and -0.5 may be taken, in fact, as an indicator of the overall 'complexity', or better the inner inter-relatedness of the economic organization under study. If we translate this into our cross-country analysis, we may take the negative relation between the volatility of growth rates and the level of income as evidence of the importance of the internal interdependencies of any national economy. Indeed, the way income is generated in a country via input-output relations among the different sectors may be a candidate for explaining the degree of 'internal correlation' which produces the observed stylized fact.

Together, scaling relations clearly depend on the number of activities (or "lines of business") within the entity under consideration (e.g. a country or a firm). Keeping this in mind, one may offer a possible explanation for the different observed slopes of the scaling relations, which could be the following. Economic development is likely

to be correlated with the density of economic activities or, put it another way, with the number of different economic sectors in which a country is active in. Hence, in line with the evidence, richer countries, characterized by a higher number of relevant economic activities, would display less variable growth rates, while poorer countries embodying fewer activities would be more volatile in their growth performances.[17] There is yet another analogy here with the micro level: as Bottazzi et al. (2001) find, the standard deviation of growth rates declines with the number of sub-markets where firms operate.[18]

Finally, we used the scaling relation to re-scale growth rates and we showed how the re-scaled distributions for the different income classes collapse only for the central part of the observations. One may interpret this result in terms of the statistical relevance of the very best and very worst performing countries. Indeed, these observations seem to ultimately shape the distribution of growth rates across countries. The finding suggests that also the higher moments of the growth rates distribution, in addition to mean and volatility, depend on the level of income.

The scaling relations analyzed in this work concerned both to the average and the dispersion of growth rates. A *caveat* to keep in mind when dealing with such scaling laws, as Brock (1999) suggests, is that, "Most of them are 'unconditional objects', i.e. they only give properties of stationary distributions, e.g. 'invariant measures', and hence cannot say much about the dynamics of the stochastic process which generated them. ... Nevertheless, if a robust scaling law appears in data, this does restrict the acceptable class of conditional predictive distributions somewhat." (p.426).

8 Concluding remarks

The evidence presented in this work suggests indeed striking invariances in the processes of growth which hold at different levels of observation, from firms, to sectors, to whole countries. This work has discussed new statistical results on output growth rates which are in line with what has been found in the recent literature on firm growth rates. The common exponential properties of growth rates which they share mark widespread correlating mechanisms which aggregation does not dilute.

A puzzling question regards precisely the nature of such mechanisms which might well be different across levels. For example, one may reasonably conjecture that at micro level 'lumpy' technological events, idiosyncratic increasing returns, together with

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[17]Along these lines, see also Harberger (1998) for some insights.

[18]See also Bottazzi (2001) for a branching model of corporate diversification able to account for such an evidence.

the inter-dependences induced by the very competitive process, may robustly account for the 'tent-shape' distribution of growth shocks. Conversely, at country level, it might well be due to, again, some forms of increasing returns together with the inter-sectoral propagation of technological and demand impulses.

One way ahead in order to disentangle the underlying mechanisms involves, as Brock (1999) suggests, the joint consideration of scaling laws with other types of statistical evidence which may provide conditioning schemes useful to refine the evidence on the data generating process. And here precious insights are likely to come by linking the evidence on growth with that on the processes of arrival of technological and organizational innovations.

Appendix

The country variables used in the analysis are taken from the most recent version of the Penn World Tables (Heston et al. (2002)). Version 6.1 extends the previous Version 5.6 by providing data until 1998 for most countries. The benchmark year has been changed from 1985 to 1996. We choose to perform our analysis on a balanced panel of 111 countries whose variables of interest are available for all years between 1960 and 1996. The most notable exclusions of countries from the database are for entities that have undergone some political transformation affecting the definition of their own borders, such as Germany and former-USSR. Nevertheless, the remaining sample appears to be quite representative.

Table A.1 provides a list of the 111 countries included in the balanced panel.

Table A.1: List of countries included in our balanced panel.

Code	Country	Code	Country	Code	Country
AGO	Angola	GBR	United Kingdom	NER	Niger
ARG	Argentina	GHA	Ghana	NGA	Nigeria
AUS	Australia	GIN	Guinea	NIC	Nicaragua
AUT	Austria	GMB	Gambia, The	NLD	Netherlands
BDI	Burundi	GNB	Guinea-Bissau	NOR	Norway
BEL	Belgium	GNQ	Equatorial Guinea	NPL	Nepal
BEN	Benin	GRC	Greece	NZL	New Zealand
BFA	Burkina Faso	GTM	Guatemala	PAK	Pakistan
BGD	Bangladesh	GUY	Guyana	PAN	Panama
BOL	Bolivia	HKG	Hong Kong	PER	Peru
BRA	Brazil	HND	Honduras	PHL	Philippines
BRB	Barbados	HTI	Haiti	PNG	Papua New Guinea
BWA	Botswana	IDN	Indonesia	PRT	Portugal
CAF	Central African Rep.	IND	India	PRY	Paraguay
CAN	Canada	IRL	Ireland	ROM	Romania
CHE	Switzerland	IRN	Iran	RWA	Rwanda
CHL	Chile	ISL	Iceland	SEN	Senegal
CHN	China	ISR	Israel	SGP	Singapore
CIV	Cote d'Ivoire	ITA	Italy	SLV	El Salvador
CMR	Cameroon	JAM	Jamaica	SWE	Sweden
COG	Congo, Rep. of	JOR	Jordan	SYC	Seychelles
COL	Colombia	JPN	Japan	SYR	Syria
COM	Comoros	KEN	Kenya	TCD	Chad
CPV	Cape Verde	KOR	Korea, Rep. of	TGO	Togo
CRI	Costa Rica	LKA	Sri Lanka	THA	Thailand
CYP	Cyprus	LSO	Lesotho	TTO	Trinidad Tobago
DNK	Denmark	LUX	Luxembourg	TUR	Turkey
DOM	Dominican Rep.	MAR	Morocco	TWN	Taiwan
DZA	Algeria	MDG	Madagascar	TZA	Tanzania
ECU	Ecuador	MEX	Mexico	UGA	Uganda
EGY	Egypt	MLI	Mali	URY	Uruguay
ESP	Spain	MOZ	Mozambique	USA	USA
ETH	Ethiopia	MRT	Mauritania	VEN	Venezuela
FIN	Finland	MUS	Mauritius	ZAF	South Africa
FJI	Fiji	MWI	Malawi	ZAR	Congo, Dem. Rep.
FRA	France	MYS	Malaysia	ZMB	Zambia
GAB	Gabon	NAM	Namibia	ZWE	Zimbabwe

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