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The patterns of output growth of firms and countries: new evidence on scale invariances and scale specificities

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Abstract

This work brings together two distinct pieces of evidence concerning, at the *macro* level, international distributions of incomes and their dynamics, and, at the *micro* level, the size distributions of firms and the properties of their growth rates.

First, our empirical analysis provides a new look at the international distributions of incomes and growth rates by investigating more closely the relationship between the two entities and the statistical properties of the growth process.

Second, we identify the statistical properties that are invariant with respect to the scale of observation (country or firm) as distinct from those that are scale specific. This exercise proposes a few major interpretative challenges regarding the correlating processes underlying the statistical evidence.

Keywords: international distribution of income, international growth rates, firm growth, scaling laws, growth volatility, exponential tails

JEL classification: C10, C14, O11

1 Introduction

This paper brings together two distinct ensembles of evidence concerning, *first*, international distributions of growth rates in aggregate and per capita income, and, *second*, the micro-economic evidence on the distributions of firm growth rates.

Such an exercise entails two major interpretative questions concerning:

(i) the relationships between the distributions of the relevant entities (e.g. countries or firms) and the properties of the growth process;

(ii) the identification of the properties that appear to be invariant *vis-à-vis* the scale of observation and those that conversely are scale specific.

With respect to the first question, this work links with the stream of studies in growth empirics concerning the international divergence/convergence properties of income (for thorough reviews see Durlauf and Quah (1999) and Temple (1999)).

Together, we study the properties of growth rates and their dependence upon possible conditioning factors including income levels and the size of the economies. To address the second question, we compare distributions and growth processes at the two levels of observation, namely countries and firms. In particular, we apply to output growth rates some non-parametric analyses recently used for the investigation of firm growth rates. As we shall see, one finds striking similarities in the growth processes that hold across levels of observation. In turn, such statistical properties hint at the ubiquitous presence of some *correlating mechanisms* that survive aggregation from firms to sectors to countries.

A number of recent studies, including Fagiolo et al. (2007) and Maasoumi et al. (2007), have begun to address the properties of the whole distributions of international growth rates. The relevance of these contributions, to which our work connects in a complementary fashion, is twofold. First, the emerging evidence of non-normality of growth rates, has important implications for growth theory in that it challenges all modeling exercises (such as the generality of Real Business Cycle models, among others) that run on the assumption of normally distributed growth shocks. The explicit account of fat-tailed distributions in the growth process adds realism and, likely, predictive power to the models themselves. Second, more detailed evidence on the statistical properties of growth rates contributes to the understanding of the possible generating mechanisms underlying economic growth and of the processes of diffusion of technological and demand shocks. In fact, as we argue below, further insights can be gained from the identification of those properties of growth rates that are scale invariant, from firms to countries, as distinct from those that are scale-specific. In this respect, the evidence is that the distribution of growth *rates* of outputs or incomes, is quite robustly invariant in its shape, from firms to countries. Conversely, the distributions of the *levels* of the same quantities, that is, the size of firms, the size of countries and their per capita incomes, do not present apparent scale- and time-invariances. Moreover, regarding specifically per capita incomes, the contemporary observations on bimodal, or possibly even tri-modal, distributions add to the challenge of linking the evidence on the distributions of levels and growth rates.

In what follows, we start with a brief overview of the existing micro evidence on the statistical properties of the distribution of firm sizes and growth rates (Section 2). In Section 3 we describe the data and the variables of interest for our, more country-focused, analysis. Section 4 provides a reassessment of the cross-country evidence on the distribution of levels of income. Section 5 investigates the statistical properties of the distribution of growth shocks and their relation to the international distribution of incomes. Finally, Section 6 offers a discussion of the interpretative challenges stemming from the empirical evidence and puts forth a few conjectures.

2 The 'size' of firms, the 'size' of countries and their growth processes: some background evidence

With the purpose of bringing together two streams of literature which have rarely been connected to each other, namely those addressing the statistical properties of *firm* sizes and growth, on the one hand, and those of *country* (income) sizes and growth on the other, let us start with the former level of observation.

2.1 The micro-evidence on firm size and firm growth rates

The statistical properties of the size distribution of firms and of their growth rates have been the objects of inquiry of a longstanding stream of empirical literature dating back to the seminal contributions of Gibrat (1931), Steindl (1965), Hart and Prais (1956), Simon and Bonini (1958). These pioneering insights and the more recent evidence (for a broad discussion cf. Marsili (2001)) all indicate a generic right-skewness of the distribution of firm size over quite wide supports, wherein fewer large firms co-exist with many more firms of smaller size. However, the overall shape of the size distributions differs sensibly when disaggregated at, say, 3- or 4-digit levels.[1] Indeed, the precise shape of such distributions varies a great deal across sectors, and sometimes displays also two or more modal values.

A tricky issue is related to the properties of the upper tail of the distribution and its 'fatness'. The evidence so far suggests that at the sectoral level such tails are generally skewed, and sometimes lognormal or Pareto-distributed.[2] Stronger evidence, however, corroborates Paretian tails only at the *aggregate* manufacturing level: indeed, this might be a puzzling property of the aggregation process itself (cf. Dosi et al. (1995) for some conjectures and some corroborating simulations). More disaggregated evidence, say at 2- or 3- digit sectoral observations, most often yields 'badly behaved' profiles which, despite always maintaining skewness in size distributions, display heterogenous, sometimes bi- or tri-modal distributions (cf. Bottazzi et al. (2007) on the Italian evidence).

The statistical literature on size distributions is closely linked with the studies of the statistical properties of the process of growth at the firm level. One of the

[1]Cf. Bottazzi and Secchi (2003b) on US manufacturing data and Bottazzi et al. (2007) on Italian data.

[2]Pareto distributions yield a cumulative distribution which in a double logarithmic space displays a linear relation between probabilities and values of the variable itself (e.g. the size of firms). A different but germane formulation taking ranks rather than probabilities goes under the heading of Zipf Law.

longstanding issues relates to the validation of the so-called Law of Proportionate Effect (as originally presented in Gibrat (1931)).[3] This null hypothesis states that firm growth rates are realizations independent of size. Under this assumption the limit distribution of size is log-normal. The available evidence does not support any systematic dependence of growth rates on the initial size of firms. At the same time most analyses display a violation of the Gibrat hypothesis in that the *variance* of growth rates *does* depend (negatively) on size.

Moreover, recent studies including Stanley et al. (1996) and Bottazzi et al. (2007), have shifted the focus toward the analysis of the overall *distribution of firm growth rates*. The latter robustly follow a Laplacian distribution: that is, they are not distributed as Normal variables, but instead display exponential tails. This finding by itself sheds new light on the nature of the process of firm growth. If growth rates are markedly non-Gaussian, then one has to strongly reject the hypothesis that growth is the result of the accumulation of small uncorrelated shocks. Interestingly, this stylized fact holds both at the level of the whole manufacturing and at sectoral level, independently of the degrees of statistical disaggregation (as far as one can go given the available data).[4]

In a nutshell, the micro statistical evidence strongly displays: (i) persistent skewed distributions in firm sizes (and similarly skewed distributions in relative productivities and degrees of innovativeness, which we will not review here[5]); (ii) widespread differences across sectors in the shapes of the size distribution themselves; and, at the same time, (iii) no robust relation between initial size and subsequent rates of growth (except possibly for the smallest firms); (iv) a variability in growth rates themselves which often appears to fall with firm size; (v) robust evidence on a Laplacian distribution of firm growth rates, which appears to hold across sectors, across countries and across periods of observation. Figure 1 illustrates the sectoral heterogeneity in the size distribution together with the invariance in the Laplacian shape of the growth rates distribution.

Given these findings, what is the matching evidence concerning countries? It is clear that firms and countries differ in many crucial respects. *First*, and most obviously, firms may easily enter and subsequently die. This is much more unlikely for countries which 'enter' and 'die' under much more infrequent events of revolution and conquest. *Second*, firms within distinct markets are subject to competitive pressures

[3]Within an extensive literature cf. Ijiri and Simon (1977), Hymer and Pashigian (1962), Hall (1987), Evans (1987) and the discussions in Sutton (1997).

[4]In the recent literature, the description of the properties of the distribution of growth rates has relied, on the fitting of a general family of distributions, the set of Subbotin densities including the Laplacian distribution (cf. Bottazzi and Secchi (2003a)); see also below.

[5]For discussions, cf. Nelson (1981), Bartelsman and Doms (2000), Marsili (2001) and Dosi (2007).

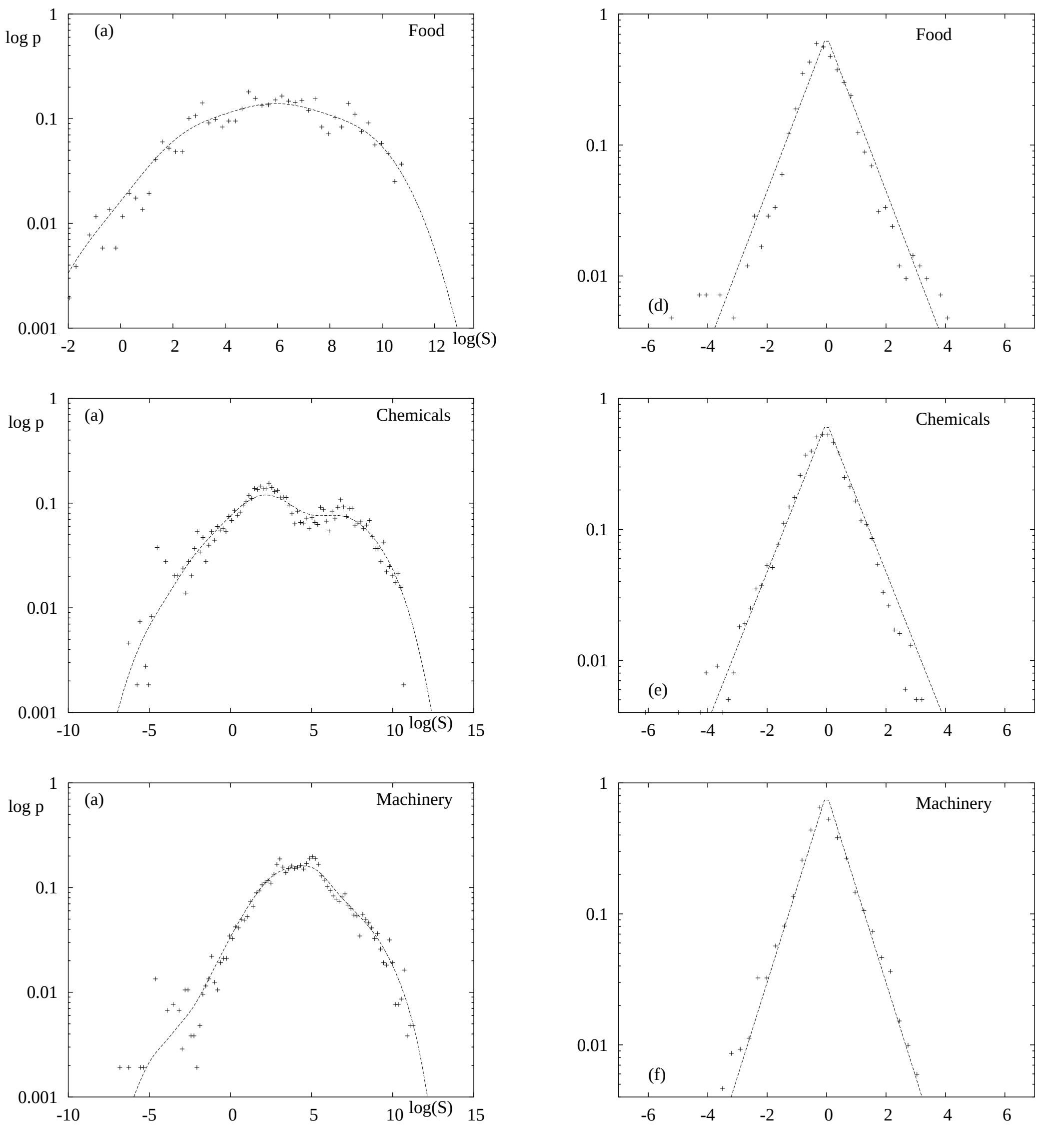


Figure 1: Size distributions and growth rate distributions: evidence on US Manufacturing firms from Bottazzi and Secchi (2003b). Left plots show the kernel estimation of the empirical density of the size distribution of firms in three illustrative industries (a), while right plots show the corresponding fitted Subbotin density of the distribution of growth rates (d-e-f). The industries are Food, Chemicals and Machinery and they are simply selected for the sake of illustration.

that inevitably correlate with their performances. An increase in the size of one firm's market share in any particular market means the fall of other firms' shares. As we conjecture below, the very process of market competition is likely to contribute to the observed statistical structure of firms' growth rates. This is not necessarily the case for countries as a whole. It trivially holds true that if some countries grow more than others their share in world income will grow and vice versa. However, there is no *a priori* reason to expect that country growth rates should yield statistical properties similar to those displayed by micro-economic entities undergoing reciprocal competitive pressures. Countries do not necessarily compete as firms do. In fact they might well co-ordinate in order to achieve higher *common* rates of growth. With these qualifications in mind, let us consider the macro, cross-country evidence.

3 The variables

We measure the per capita income of a country i in year t, say y_{it}, by the country's per capita GDP at constant prices and constant exchange rates. The data source are the Penn World Tables, version 6.1 (see Heston et al. (2002)) for 111 countries for 1960-1996.[6]

Let Y_{it} be the aggregate income. This variable is a proxy for the actual 'size' of a national economy. Another variable of interest is the level of economic development of the various countries. This is primarily captured by the measure of *per capita* income. Here we will consider both total and per capita GDP measures and compare the empirical analyses using the two alternative variables.

To identify the country-specific properties of our variables over time, let us 'de-trend' by "washing away" any component common to all countries in a given year. For this purpose we consider 'normalized' (log) incomes defined by:

$$\begin{aligned} s_{it} &= \log(y_{it}) - \overline{\log(y_t)} \\ S_{it} &= \log(Y_{it}) - \overline{\log(Y_t)} \end{aligned} \tag{1}$$

and calculate normalized year-by-year logarithmic growth rates as:

$$\begin{aligned} g_{it} &= s_{it} - s_{i,t-1} \\ G_{it} &= S_{it} - S_{i,t-1} \end{aligned} \tag{2}$$

We refer to these last variables as the *growth shocks* of interest.[7] Notice that

[6] See the Appendix for details on the construction of our balanced panel.

[7] The reader should be aware that we use the word 'shock' in tune with a common jargon of practitioners of statistics: however, the terminology does not involve any commitment to the 'exogeneity' of the event itself. In fact, 'shocks' are endogenously generated by the very process of country growth.

Canning et al. (1998) only consider total GDP in their analysis of the distribution of international growth rates, while we include here two different measures of national income.

4 The distribution of levels of income

Let us start, somewhat symmetrically to the foregoing micro-evidence, from the distributions of the levels of per capita incomes. An insightful new set of contributions has recently been added to the empirics of international growth, shedding new light on the statistical distributions of income levels and their change, if any, over time (see Quah (1996, 1997), Durlauf and Quah (1999) Bianchi (1997), Jones (1997), Paap and van Dijk (1998)). While it is not possible to discuss in any depth the secular evidence, notice, *first*, that the *mean per capita incomes* have shown roughly exponential increases since the "Industrial Revolution" in all countries that have been able to join it, and, *second*, that the variance across countries has correspondingly exploded (more on this, from different perspectives, in Bairoch (1981), Maddison (2001), Dosi, Freeman and Fabiani (1994)). Given these long-term tendencies, the foregoing stream of analyses, largely concerning the post World War II period, finds that the distribution of income levels has been moving over the years to a bi-modal shape indicating a process of 'polarization' of countries into two groups characterized by markedly different income levels. Clearly this testifies against any prediction of a tendency towards global convergence of all countries to a common income level.

Let us consider the time series available from Penn Tables version 6.1 and estimate the kernel density for the distribution of normalized income and normalized per capita income. Following the standard notation, the kernel density estimator for a sample of data $\{x_i\}_{i=1:n}$ is defined as:

$$\widehat{f}(x) = \frac{1}{nh} \sum_{i}^{n} K(\frac{x - x_i}{h}) \quad (3)$$

where K is the chosen kernel function and h the kernel bandwidth. This non-parametric estimation procedure depends on the choice of the kernel bandwidth. The larger the chosen bandwidth, the smoother the estimated density.

To get a graphical impression of the distributions, let us select a bandwidth with the rule of thumb proposed in Silverman (1986). The exploratory plots in Figures 2 and 3 suggest that the estimated densities become less and less unimodal over the years. The emergence of bimodality is more evident in the case of per capita income than for total income. Figure 3 for per capita data shows that the distribution could have been already bimodal in 1960 and that it might have gone towards a three-humps

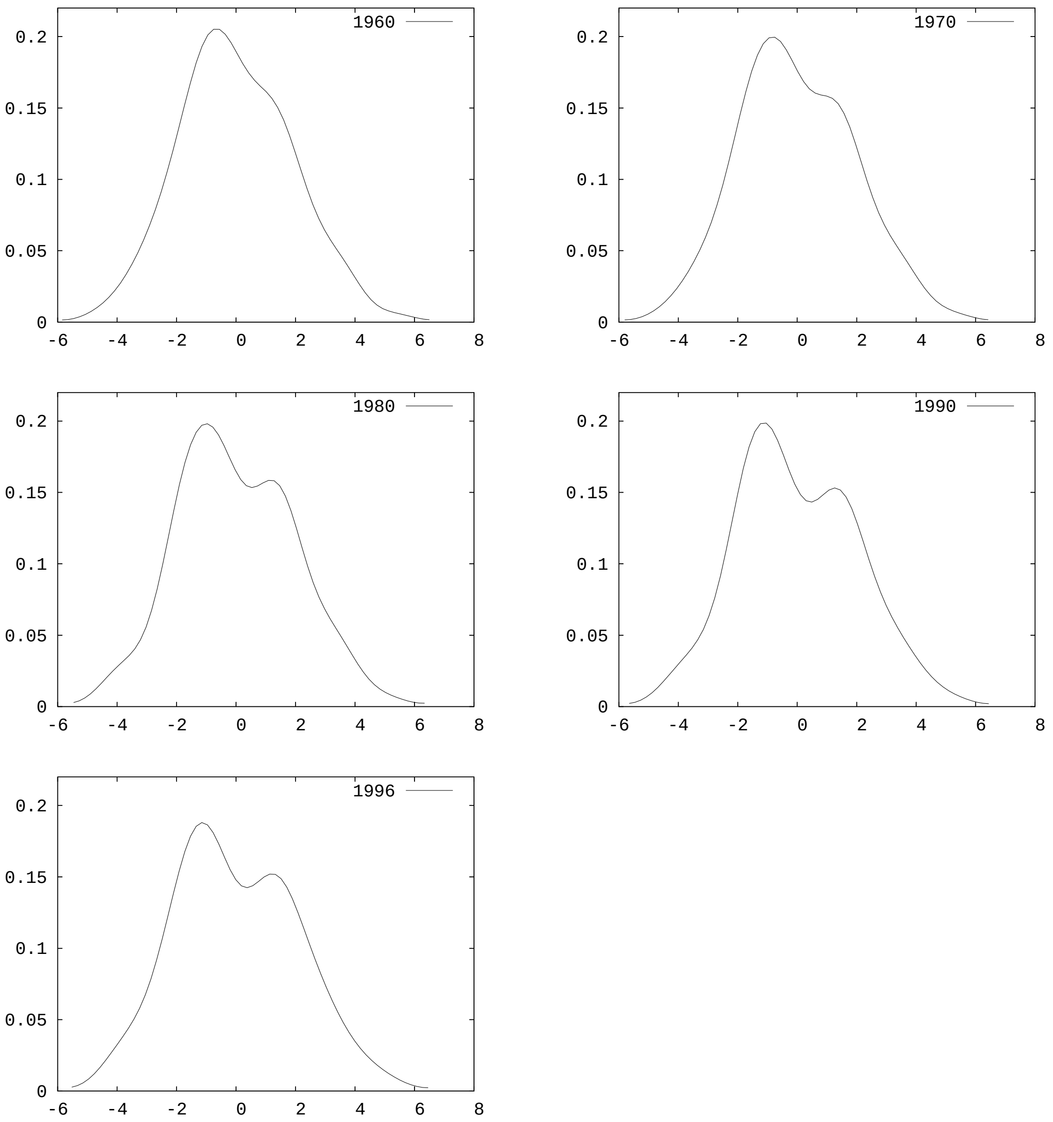


Figure 2: Kernel estimation of the empirical density of (log) normalized income S, different years.

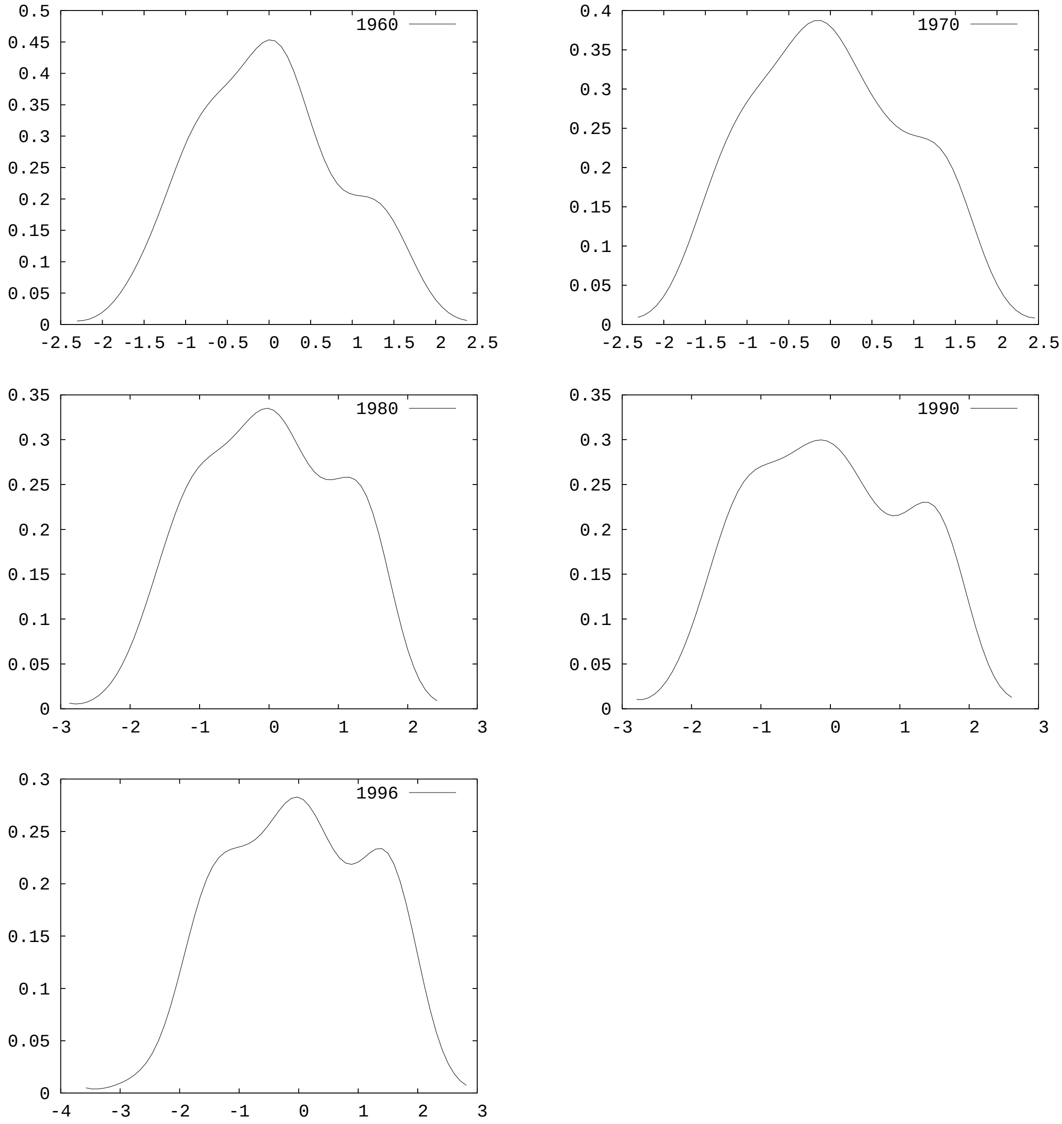


Figure 3: Kernel estimation of the empirical density of (log) normalized per capita income s, different years.

shape after 1996. Formal multi-modality tests following the procedure introduced by Silverman (1981) and first applied to income data by Bianchi (1997), confirm the presence of bimodality starting from 1970.

Let us perform multi-modality tests on our longer time series. The Silverman test is based on kernel estimation and relies on the calculation of critical kernel bandwidths for the appearance of a given number of modes m. Call $h_c(m)$ the critical bandwidth such that for any bandwidth $h > h_c(m)$ the density displays less than m modes, while for any $h < h_c(m)$ the modes are at least $m+1$. Any $h_c(m)$ may be used as a statistic to test the hypothesis H_0 : m modes *vs* H_1 : more than m modes. The actual p-value of the test can be calculated via bootstrapping. When $\hat{p}_c(m) < \alpha$, where α stands for the significance level of the test, one can reject the null hypothesis that the distribution has m modes and not more. This test is known to have a bias towards being conservative, in the sense that it leads to rejection in fewer cases than other tests would. A procedure to correct this shortcoming has been proposed in Hall and York (2001) for the unimodality test and it allows to calculate corrected actual p-values for a given significance level of the test.[8]

Bianchi (1997) discusses some of the problems involved in using a fully non-parametric technique. In particular he points out that this kind of test may fail to detect multiple modes when modes are not well separated. For the particular instance of GDP data, this may be the case when one considers logarithmic transformations of the GDP data. The log transformation is a smoothed version of the actual data and possible modes in the distribution will appear closer to each other than in the actual data. To avoid this problem Bianchi suggests taking non-logarithmic transformations, such as the per capita income relative to the sum of all incomes. Let us then define:

$$z^*_{i,t} = \frac{y_{i,t}}{\sum y_{i,t}} \tag{4}$$

We report the outcome of our multi-modality Silverman tests on this specific income measure to make our results comparable with Bianchi's findings. Table 1 shows estimates for selected years and for all years in the transition phase from unimodality to a bimodality regime. We choose a significance of $\alpha = 0.1$, a reasonable level for this type of data. Scores that lead to rejection of the statistical hypothesis are highlighted in italics. The results for the unimodality test include the Hall-York correction. We confirm that the assumption of bimodality can not be rejected at a 10% level, even since 1970.

[8]For a discussion on the advantages and the shortcomings of the Silverman test see Henderson et al. (2007). The main shortcoming appears to be the fact that Silverman test is not nested and it may thus yield inconclusive results.

Table 1: Results from multi-modality tests: critical bandwidths from Gaussian kernel estimates and corresponding significance score from smoothed bootstrap test (B=1000 replications) for the variable z^*.

Year	$h_c(1)$	$p_c(1)$	$h_c(2)$	$p_c(2)$	$h_c(3)$	$p_c(3)$
1960	0.0034	0.284	0.0026	0.370	0.0023	0.103
1965	0.0035	0.196	0.0026	0.352	0.0021	0.110
1966	0.0038	0.109	0.0026	0.392	0.0021	0.149
1967	0.0039	*0.061*	0.0027	0.284	0.0022	0.106
1968	0.0035	0.188	0.0028	0.239	0.0020	0.232
1969	0.0035	0.161	0.0023	0.524	0.0019	0.324
1970	0.0039	*0.049*	0.0029	0.186	0.0015	0.655
1971	0.0041	*0.024*	0.0030	0.147	0.0015	0.503
1972	0.0042	*0.015*	0.0027	0.257	0.0015	0.557
1973	0.0042	*0.014*	0.0024	0.417	0.0012	0.894
1974	0.0042	*0.011*	0.0024	0.349	0.0015	0.445
1975	0.0043	*0.003*	0.0019	0.501	0.0015	0.455
1980	0.0043	*0.005*	0.0016	0.861	0.0015	0.512
1985	0.0050	*0.000*	0.0018	0.576	0.0015	0.404
1990	0.0053	*0.000*	0.0025	0.262	0.0021	0.077
1996	0.0047	*0.011*	0.0026	0.404	0.0022	0.134

Henderson et al. (2007) also discuss an alternative test, the DIP test, and find evidence for multi-modality since 1960. Their result indicates that the evidence on multi-modality depends on the test used. Still, all evidence suggests that the last decades have been characterized by multi-modality in international income levels, indicating a process of 'club convergence' (Quah (1996)).

The results on bi-modality provide descriptive evidence that cannot be uncovered from regression analysis, but does not shed any light on the determinants of the cross-country distribution *per se*. Part of the interpretation involves the analysis of the appropriate conditioning variables that might account for the emergence of separate 'clubs' (Quah (1997)). Together, important circumstantial evidence is bound to also come from the investigation of the statistical properties of growth rates. This is discussed in the following section.

Even superficial comparisons between firm-level and country-level distributions of 'sizes' (which should be properly understood as 'total incomes of firms or countries' and 'per capita incomes') reveal suggestive analogies concerning, at the very least, (i) the skewness of distributions; (ii) the large width of their supports; and, (iii) high persistence over time of relative rankings.

So far, the statistical properties of *country growth rates* have been much less investigated (insightful exceptions include Canning et al. (1998), Lee et al. (1998) and Maasoumi et al. (2007)). Indeed, such properties, and their possible analogies with firm-level processes of growth are major topics to their own right which we shall address below.

5 The statistical properties of growth shocks

5.1 Preliminary analysis

We begin by plotting the moments of the (non-normalized) growth rates g_{it} and G_{it} (Figure 4). The evolution of the average growth rate hints to two distinct phases, reasonably separated by the year 1973. This major discontinuity is well known to appear in most economic time series. Note also that the years before 1973 are characterized by a somewhat higher average level of growth, and a lower mean value thereafter. The standard deviation is stable across all sample years, implying that in fact the coefficient of variation of rates is higher after 1973.

Keeping in mind these discontinuities in the overall growth patterns we study the properties of de-trended growth dynamics over the post World War II period. Following the procedure used in Canning et al. (1998), we pool together the normalized observations for all years and countries and we obtain a sample of $111 * 36$ observations,

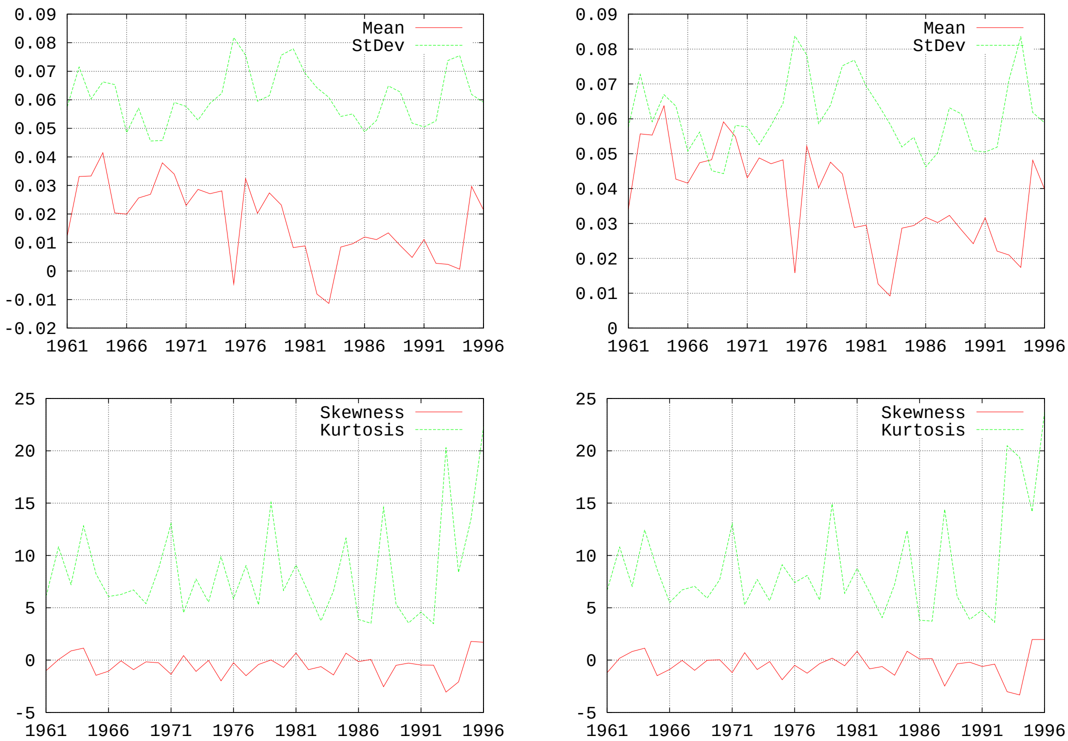


Figure 4: Evolution in time of the moments of the distribution of growth rates. Left panels refer to g_{it}, right panels to G_{it}.

large enough to support robust statistical analysis.

As a preliminary question let us ask whether higher or lower income countries are characterized on average by (relatively) higher/lower growth rates.

We group countries into 40 equally populated subsets ('bins') according to income s^* (or S^*) and calculate the mean annual growth rate g^* (or G^*) in each income class. We fit a linear relation to the observations and account for heteroskedasticity by using the White estimator (White (1980)). We find a statistically significant and *positive* correlation between the average growth rates and levels of income, both the total and the per capita income (Figure 5). Larger and more developed (i.e. with higher per capita incomes) countries are characterized, on average, by a higher growth performance.

The interpretation of the two relations offers quite different insights. When we look at per capita income data the result that richer countries display on average higher growth rates can be read as straightforward evidence for divergence and polarization of countries into two classes of 'very rich' and 'very poor' countries. Such evidence suggests the existence of some form of dynamic increasing returns in production and in the accumulation of technological knowledge. However, the relation for per capita

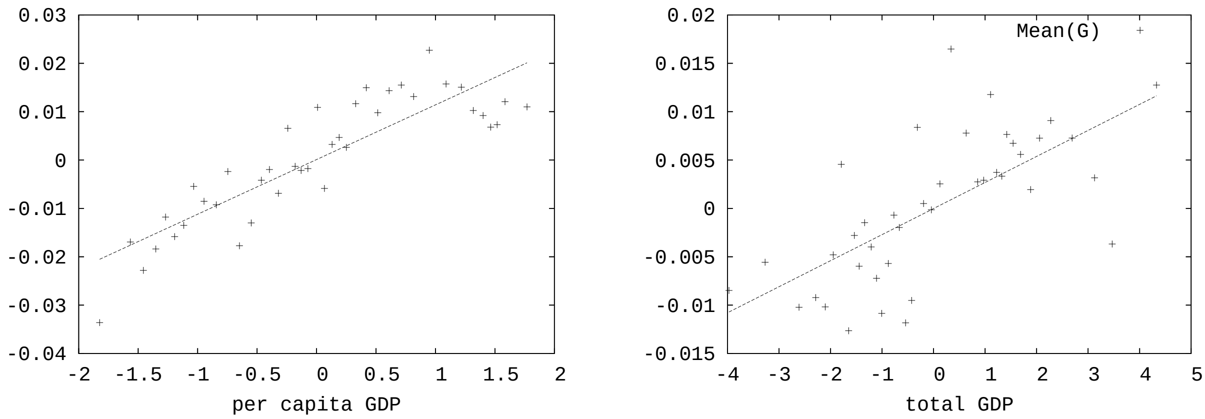


Figure 5: The relation between average growth rate and income level for different income classes. Linear fits are also shown. The left plot refers to per capita variables (slope= 0.0113 ± 0.0012), the right one to total income ones (slope=0.0027 ± 0.0004).

resembles more a parable than a straight line: for the highest levels of per capita income the relation is not significant or even becomes negative.

Conversely, the positive relation between average growth rate and total domestic income hints at structural effects of the sheer size of an economy similar to 'static' economies of scale.[9]

5.2 The volatility of growth rates

Are higher income countries characterized by less volatile growth rates? Recent evidence (see for example Pritchett (2000) and Fiaschi and Lavezzi (2005)) shows that the volatility of growth rates is much higher for developing countries than for industrialized ones. Throughout the process of development the levels of per capita GDP obviously increase. Together, reductions in the dispersion of growth performance may also be taken as an indication that countries move on more stable growth paths.

We again group countries by income, calculate the standard deviation of the

[9]It should be clear that the possible scale effects that we identify here do not necessarily bear any direct relation with the scale effect that has been the object of controversy among 'new growth' theorists, as discussed in Jones (1999). One of the questionable predictions by the first wave of 'new growth' models was the presence of a scale effect on the steady state growth according to which an increase in the total population, and thus in the available specialized labor force, proportionally increased the long run per capita growth. In some subsequent models the scale effect has shifted to the level of per capita income, rather than its long run growth rate. In our strictly 'inductive' analysis here we do not make any commitment to the existence of a steady state rate of growth: simply, the statistical relations between income and growth appear to suggest some forms of increasing returns.

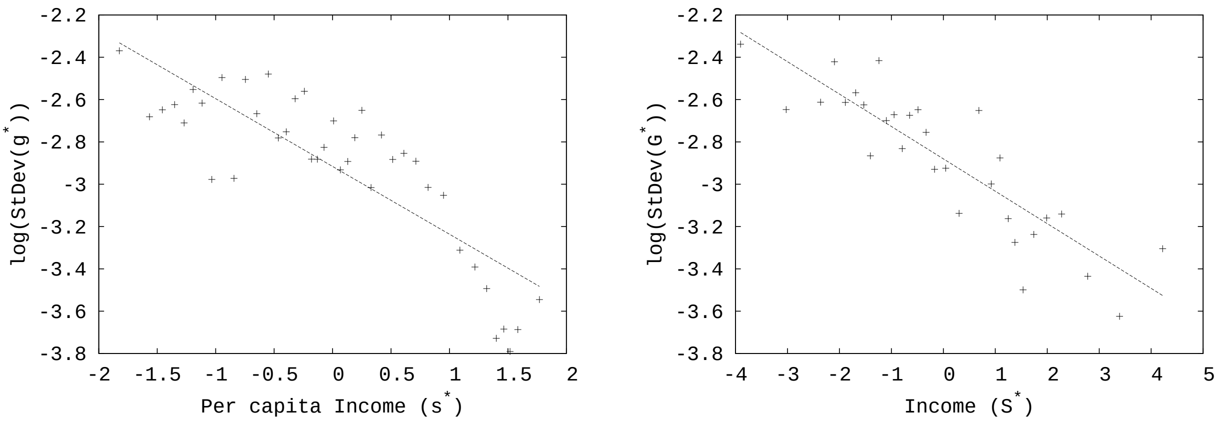


Figure 6: The relation between the logarithm of the volatility of growth rates and the levels of income.

normalized growth shocks and associate this with the central value of income in each class. Here, we uncover a negative relation between the log standard deviation of growth rates and the level of per capita income. In other words the volatility of growth rates scales with income as a power law.

The same scaling relation can also be studied by taking the following model as a starting point:

$$g_{it} = s_{it} - s_{i,t-1} = e^{\beta s_{i,t-1}} \epsilon_{i,t} \tag{5}$$

The scaling parameter β can then be estimated via non-linear regression, using numerical methods based on different optimization criteria, as suggested in Bottazzi et al. (2005). Depending on the underlying assumptions about the error terms ϵ_{it}: (i) non-linear LS (Least Squares) if $\epsilon_{it} \sim$ Normal; (ii) non-linear LAD (Least Absolute Deviation) if $\epsilon_t \sim$ Laplace.

Non-linear LAD estimates are the most precise. Assuming Laplace, heavy-tailed disturbances considerably improves the estimation performance and is fully consistent with the results in the next section about distributional shapes.

	Binned OLS	**Nonlinear LS**	**Nonlinear LAD**
Per capita GDP	-0.320 (0.036)	-0.295 (0.021)	-0.260 (0.008)
Total GDP	-0.152 (0.013)	-0.149 (0.011)	-0.134 (0.004)

Table 2: Estimated power-law scaling coefficients for the volatility of growth rates: binned OLS (40 bins), nonlinear LS, nonlinear LAD. Standard errors in parenthesis.

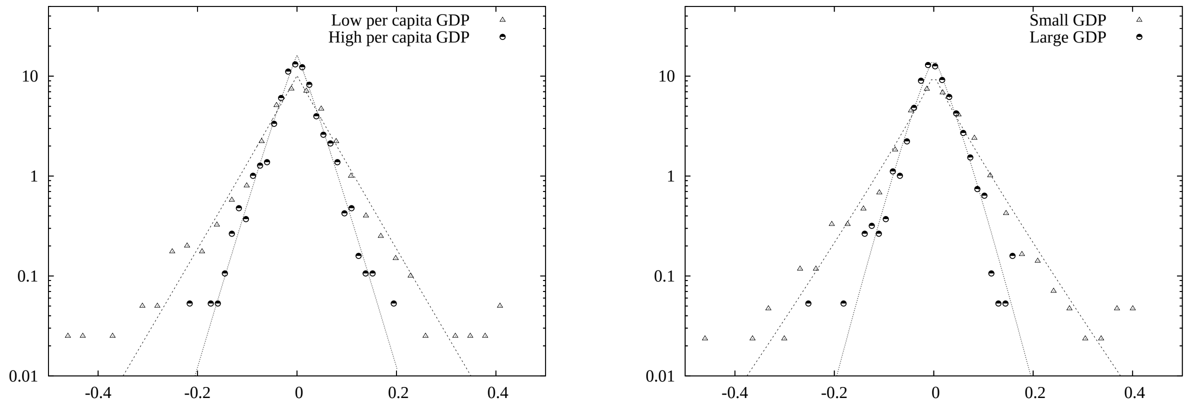


Figure 7: The empirical distribution of growth rates of per capita income (left) and income (right) for two income classes, Low and High.

The interesting result here is that the scaling coefficient for aggregate GDP data is much lower than that for per capita data. This may in fact tell us that a 'strong' scaling relation holds only when one considers the level of economic development, as proxied by per capita income. Growth performances are less volatile for *more developed* countries. The sheer size of an economy is relatively less relevant. [10]

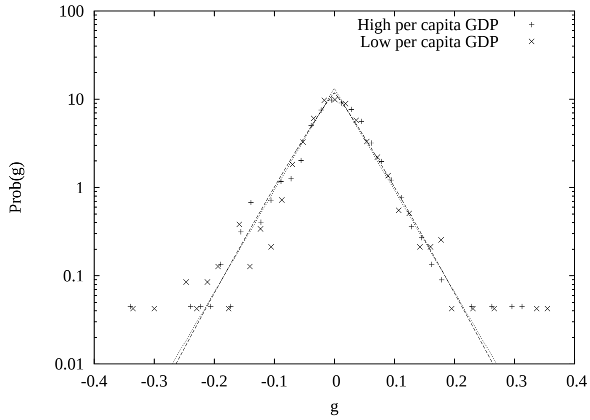
5.3 The distribution of growth rates

One way to deal with the 'size effect' on the average growth rate is to group countries by their level of income in three classes: Low, Medium and High (per capita) GDP. This same procedure is used in Canning et al. (1998) and Lee et al. (1998), who recognize different growth distributions for countries characterized by different size in terms of total income. We further normalize the growth rates in each group and then proceed by plotting their empirical histograms. (In figures 7 and 8 we show only the Small and Large Income classes, since the Medium one always lies in between.)

We refine the description of the properties of the distribution of growth rates by fitting on the empirical densities a general family of distributions, the set of Subbotin densities (cf. Bottazzi and Secchi (2003a), the original reference is Subbotin (1923)).

The functional form of the Subbotin family is given by:

[10] Fiaschi and Lavezzi (2005) confirm a negative relation between growth volatility and the size of an economy. Their work tries to explain growth volatility with a set of country characteristics including the share of the agriculture sector as a proxy for the structure of the economy and trade openness. They find in their sample that per capita income does not play a significant role when the mentioned variables are considered. However note that the share of agriculture in income correlates negatively with per capita income.



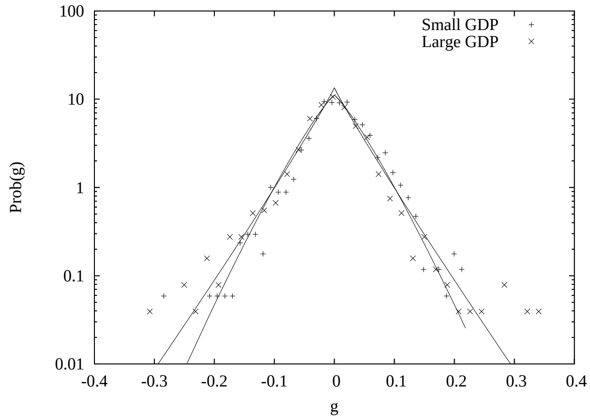


Figure 8: The distributions of re-scaled growth rates for two income classes.

$$f(x) = \frac{1}{2ab^{\frac{1}{b}}\Gamma(1+\frac{1}{b})} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^{b}} \tag{6}$$

where the parameter a controls the standard deviation and b is a parameter which determines the shape of the distribution. Note that for a value $b = 2$ the distribution turns out to be a Normal one, while for $b = 1$ the distribution is Laplacian, also known as Double Exponential. As b gets smaller, the tails get heavier and the peak of the density becomes more pronounced. For $b = 0$ the distribution is degenerate in the mean. We fit the family of density using a maximum likelihood procedure (for details see Bottazzi (2004)).

The empirical distribution of the growth rates is quite well fitted by a Subbotin density with a b-parameter close to 1, hence the distribution is approximately Laplacian (Figure 7)[11]. Note that if growth residuals were Normal the fitted curve would be a parable ('bell shape') in a logarithmic scale. On the contrary, we find that the distribution of growth rates is markedly non-Gaussian and closer to a Laplacian density, which displays a 'tent shape' in the log scale.

Further, notice that the plots in Figure 7 reveal a sensibly different width of the distribution for low income and high income countries, which one should expect given the dependence of the dispersion of growth rates upon a country's income level shown in the previous section. We then consider rescaled growth rates in the form of residuals from model 5, estimated with a non-linear LAD procedure.

Even after eliminating any possible size effect on the dispersion of the distribution, the Laplacian shape of the distribution is confirmed: growth shocks are markedly not

[11]The estimation is done on the normalized growth rates, thus the parameter μ of the Subbotin is always set to zero.

Income classes	Growth rates b	a	Rescaled growth rates b	a
Per capita GDP	0.9517	0.0407	1.0448	0.0411
	(0.0277)	(0.0008)	(0.031)	(0.0008)
Total GDP	0.9313	0.0398	1.0253	0.0402
	(0.0269)	(0.0008)	(0.0310)	(0.0008)
Low per capita GDP	0.9829	0.0498	1.0015	0.0376
	(0.0498)	(0.0017)	(0.0510)	(0.0013)
High per capita GDP	1.0644	0.0296	1.1845	0.0404
	(0.0549)	(0.0010)	(0.0627)	(0.0014)
Small GDP	0.9323	0.0503	0.9431	0.0384
	(0.0467)	(0.0018)	(0.0474)	(0.0014)
Large GDP	1.1885	0.0309	1.1845	0.0414
	(0.0629)	(0.0010)	(0.0627)	(0.0014)

Table 3: Estimated Subbotin parameters for the distributions of growth rates. Standard errors are reported in parenthesis. Rescaled growth rates are obtained using LAD estimates of the scaling coefficient.

Gaussian (Figure 8).

The distributions for the two income classes almost coincide in the case of per capita GDP, while they still differ on the tails in the case of total GDP growth rates (cf. the estimates of both Subbotin coefficient and their standard errors as reported in Table 3, the differences between the classes are statistically significant). The distributions differ mostly on the tails, suggesting that observations at the extremes are crucial in shaping the distributions themselves.[12] In fact, we checked also for the existence of scaling relations for higher moments of growth rates. Skewness positively scales with total GDP, but does not significantly scale with per capita GDP (see the estimates of the slopes of a linear scaling in Table 4). This confirms the wider differences in the tails for GDP growth rates, where asymmetries are higher for 'larger' countries. Note that in principle this should not occur if countries were simply 'tags' of ensembles of

[12]Note that this result continues to hold also if one fits the data with distributions characterized by heavy tails. Indeed, we tried fitting the family of 'stable distributions' (which includes Cauchy and Lèvy ones) to check whether the gap between the distribution of re-scaled growth rates for the different income classes was due to an unsatisfactory fit of the Subbotin on the tails. We find that the gap between the estimated distributions for the two classes is not eliminated. Moreover, heavy tailed distributions do not provide an overall better fit to the data.

	Skewness	**Kurtosis**
Per capita GDP	0.097 (0.140)	-1.319 (0.531)
Total GDP	0.068 (0.049)	-0.606 (0.206)

Table 4: Estimated slope coefficients for linear scaling relations of skewness and kurtosis of growth rates with respect to total and per capita GDP.

independent and divisible activities. On the contrary, the evidence hints at the influence of the sheer size of countries on the overall distribution of negative and positive growth rates.

Conversely, kurtosis, which is a measure of the 'peakness' of the distributions negatively scales with both per capita and total GDP, in line with the remaining gap between the two peaks of the rescaled distributions. At risk of over-theorizing, we would be tempted to suggest that there appears to be a 'dumpening effect' associated with both absolute size and levels of development. Hence, one may conjecture, the same technological and demand shock might have a much greater impact on growth of small, possibly more specialized, countries as compared to bigger/more diversified ones.

6 Some interpretative remarks

Let us weave together the threads of evidence of the foregoing discussion and of the puzzles stemming from the presented empirical analysis and propose a few tentative interpretations and conjectures.

6.1 Candidates for an explanation of the tent-shaped distribution of country growth rates

A first robust stylized fact is that growth rates, also at the level of countries, follow a Laplacian distribution. This property robustly holds also for subsets of countries and for different observational periods. Developed and less developed countries remarkably show the same exponential structure in their growth rates even after accounting for their different dispersion in growth performance. A first puzzle arises if we compare the invariance of this property with the evolution of the distribution of incomes. We have seen how this distribution changes over time starting from an approximately unimodal shape and acquiring later an evident bimodality for which we have provided novel evidence. How does this relate to the invariance in the distribution of growth rates?

Remarkably, the distributional invariance of GDP growth and per capita income growth rates is a statistical feature analogous to that found with respect to *corporate* growth rates. All the evidence robustly displays Laplacian distributions of growth rates.

In the industrial organization literature, a common interpretation of the growth process builds on a baseline stochastic model of growth of a given unit of observation (e.g. a firm). If the growth process proceeded as the result of the cumulation in time of independent growth shocks one would find the growth residuals g^*_{it} to be Normally distributed and, thus, only representing 'noise'. Instead, one finds quite structured processes generating growth rates, which forces to reject the null hypothesis that growth is simply the outcome of the sum of independent shocks. Thus, one has to search for explanations of the growth process which admit that the 'elementary' growth shocks are actually correlated with each other. And, indeed, such explanations ought to account for the scale invariance of such property, since correlation mechanisms in the growth process appear at all levels of observation, from firms to sectors to countries.[13]

This scale invariant regularity is thus in need of a convincing economic explanation. Ultimately two diverse (but possibly complementary paths) seem to be available for the modeler.

(i) A known statistical result refers to the property that a mixture of a small number of Normal distributions produces fat-tailed distributions (see Lindsay (1995)). Thus, a tent-shape distribution can be interpreted as a mixture of Normal distributions given an appropriate parameterization. Mixtures are in principle an appealing tool for understanding the tent-shape distribution of growth rates because one can envision mixtures of mixtures of mixtures, capturing different scales of observation. Also, one could think of relating the components of the mixture to groups of countries representing different convergence clubs (see Durlauf, Kourtellos and Minkin (2001)). Nevertheless, a fundamental qualification should be considered. Such a statistical exercise, as well as our 'compact' representation, both still demand an economic interpretation of the underlying processes of growth yielding either the purported distributional mixtures or, directly, a fat-tailed distributions of growth shocks.

(ii) A distinct interpretative strategy tries to explicitly interpret the observed non-Normal distributions taking into account what we know about micro-processes of growth, in particular acknowledging some basic correlating mechanisms in the processes of market competition, together with the lumpiness of major competitive events. At micro level, the exponential tails of the distribution of firm growth rates are explained

[13] On the sectoral evidence cf. Castaldi and Dosi (2004) and Castaldi and Sapio (2007). Both works find evidence of exponential tails for the value added growth rates of sectors at 3-digit and 4-digit level of aggregation.

in Bottazzi and Secchi (2006a) with a minimal probabilistic model which couples a mechanism capturing some forms of increasing returns (more successful firms tend to catch more business opportunities) together with competitive forces (firms compete for market shares). In fact, we conjecture, fat-tailed distributions of growth rates might turn out to be a quite generic property of a wide class of processes of industrial evolution. One could think of elaborating a similar multi-country model (keeping in mind the different nature of inter-firm *vs* inter-country competition and complementarities).

A further challenge is to show how the observed structure of micro-shocks underlies similar *macroscopic* distributions. Recent research in macroeconomics has proposed a few models where aggregate GDP fluctuations are explained by micro-shocks at firm or sector level. In these models the micro-shocks aggregate in a non-trivial way: instead of being diluted by the aggregation process, under certain circumstances they amplify and form the basis for the structure of macro-shocks. In this vein, Gabaix (2007) shows how a major part of aggregate growth shocks can be accounted for by the growth of the top 100 firms in a country. Conversely, on the theory side, Bak et al. (1993) and Durlauf (1994) model aggregate fluctuations as the outcome of the propagation of demand shocks through inter-linked sectors.

From a different angle, Delli Gatti et al. (2005) also goes in the direction of a micro-macro bridge, by relating the Double Exponential distribution of both firm and country growth rates to the skewed distribution of firm size in a model based on the interaction among heterogenous firms[14]. Quite overlapping evolutionary agent-based models are also good candidates within this style of modeling (see Silverberg and Verspagen (2005) for discussions of such a literature in a perspective pioneered by Nelson and Winter (1982)). Indeed, preliminary exercises on the grounds of the model in Dosi, Fagiolo and Roventini (2006), wherein macro-dynamics are nested into heterogenous boundedly rational firms, show its ability to reproduce the tent-shape distribution of firm *and* country growth rates.

6.2 Scaling of the growth volatility

The other stylized fact highlighted by our analysis is the existence of a negative relation between the dispersion of growth rates and the level of *per capita* income. Moreover the volatility scales with income as a power law. Its estimated coefficient for per capita data, $c = -0.32$, is much higher than the $e = -0.15$ estimated with aggregate income data. This seems to suggest that the 'true' scaling relation does not hold for size as

[14] The argument there, however, does not seem to be formally robust in so far as Gibrat-type growth can be proved to be inconsistent with Laplace shocks and stationary Pareto size distributions, see Bottazzi (2007).

such, as measured by the gross product of an economy, but it characterizes *in primis* the level of development of a country. The structural effect of the total size of an economy plays a role, but the stability of growth performances for high income countries stands out more strongly when the income measure pertains to per capita incomes rather than the sheer size of countries.

Amaral et al. (2001) and Lee et al. (1998) propose to interpret the scaling relation by reference to a benchmark model of 'complex organizations'. The idea is to view an economic organization, i.e. a country in our specific instance, as made up of different units of identical size. Then two opposite extreme scenarios may be contemplated. If all units grew independently then the volatility of growth rates would fall as a power law with coefficient -0.5 (a result of the law of large numbers, as suggested already in Hymer and Pashigian (1962)). Conversely, if the composing units were perfectly correlated there would be no relation between the volatility of growth shocks and size, so we would find a slope of 0.

The estimated coefficients, lying in between 0 and -0.5 may be taken, in fact, as an indicator of the overall 'complexity', or, better, the inner inter-relatedness of the economic organization under study. If we translate this into our cross-country analysis, we may take the negative relation between the volatility of growth rates and the level of income as evidence of the importance of the internal interdependencies of any national economy. Indeed, the patterns of income generation in a country via input-output relations among the different sectors may be a candidate for explaining the degree of 'internal correlation' which produces the observed stylized fact. Scaling relations clearly depend also on the number of activities (or "lines of business") within the entity under consideration (e.g. a country or a firm). Keeping this in mind, a possible explanation for the different observed scaling slopes could be the following. Economic development is likely to be correlated with the density of economic activities or, putting it another way, with the number of different economic sectors in which a country is active in. Hence, in line with the evidence, richer countries, characterized by a higher number of relevant economic activities, would display less variable growth rates, while poorer countries embodying fewer activities would be more volatile in their growth performances.[15] Yet another analogy can be made here with the micro level: as Bottazzi et al. (2001) find, the standard deviation of growth rates declines with the number of sub-markets where firms operate.[16]

[15] Along these lines, see also Harberger (1998) for some insights.

[16] See also Bottazzi and Secchi (2006b) for a branching model of corporate diversification able to account for such an evidence.

7 Conclusions

The evidence presented in this work suggests striking invariances in the processes of growth that hold at different levels of observation, from firms to whole countries. This work has discussed new statistical results on output growth rates that are in line with what has been found in the recent literature on firm growth rates. The scaling relations analyzed in this work concerned both to the average and the dispersion of growth rates. A *caveat* to keep in mind when dealing with such scaling laws, as Brock (1999) suggests, is that, "Most of them are 'unconditional objects', i.e. they only give properties of stationary distributions, e.g. 'invariant measures', and hence cannot say much about the dynamics of the stochastic process which generated them. ... Nevertheless, if a robust scaling law appears in data, this does restrict the acceptable class of conditional predictive distributions somewhat." (p.426).

The common exponential properties of growth rates mark widespread correlating mechanisms which aggregation does not dilute. A puzzling question relates to the nature of such mechanisms that might well be different across levels. For example, one may reasonably conjecture that at micro level 'lumpy' technological events, idiosyncratic increasing returns, together with the inter-dependences induced by the very competitive process, may robustly account for the 'tent-shape' distribution of growth shocks. Conversely, at country level, it might well be due to, again, some forms of increasing returns together with the inter-sectoral propagation of technological and demand impulses.

In any case, both micro and macro evidence supports the impressionistic Schumpeterian intuition that growth is not a smooth process but rather tends to proceed by "fits and starts". Granted that, the big ensuing challenge is to better understand why and how this is so.

One way to disentangle the underlying mechanisms involves, as Brock (1999) suggests, the joint consideration of scaling laws with other types of statistical evidence that may provide conditioning schemes useful to refine the evidence on the data generating process. Maasoumi et al. (2007) propose a new way of conditioning using non-parametric models that can be applied in a flexible way to growth data. We also expect that precious insights are likely to come by linking the evidence on growth with the processes of arrival of technological and organizational innovations.

The theorist faces symmetric challenges.

One of them regards the relationships between the properties of the distributions of growth *rates* - which appear to be robustly exponential, irrespectively of the time and the scale of observation,- and the distributions of the *levels* (for example, the size of firms, countries or per capita GDP), whose shapes appear to be much more time-and scale-specific. In this respect, a delicate, still largely unresolved, issue concerns the points of relative strength and weakness of two distinct heuristic strategies, namely, a first one attempting to derive the observed statistical properties from mixtures of underlying normally distributed variables, and, a second one focusing on the identification of possible generating processes which might directly yield the observed sample paths.

Another, related, challenge, concerns the ability of the models to account, at the same time, for both microscopic and macroscopic patterns of growth. Ultimately the emerging evidence on such patterns, some of which has been discussed in this work, ought to be an important yardstick to evaluate the robustness and success of different theoretical efforts aimed at modeling the growth dynamics of contemporary economies.

Appendix

The country variables used in the analysis are taken from the most recent version of the Penn World Tables (Heston et al. (2002)). Version 6.1 extends the previous Version 5.6 by providing data until 1998 for most countries. The benchmark year has been changed from 1985 to 1996. We choose to perform our analysis on a balanced panel of 111 countries whose variables of interest are available for all years between 1960 and 1996. The most notable exclusions of countries from the database are for entities that have undergone some political transformation affecting the definition of their own borders, such as Germany and former-USSR. Nevertheless, the remaining sample appears to be quite representative. Table A.1 provides a list of the 111 countries included in the balanced panel.

Table A.1: List of countries included in our balanced panel.

Code	Country	Code	Country	Code	Country
AGO	Angola	GBR	United Kingdom	NER	Niger
ARG	Argentina	GHA	Ghana	NGA	Nigeria
AUS	Australia	GIN	Guinea	NIC	Nicaragua
AUT	Austria	GMB	Gambia, The	NLD	Netherlands
BDI	Burundi	GNB	Guinea-Bissau	NOR	Norway
BEL	Belgium	GNQ	Equatorial Guinea	NPL	Nepal
BEN	Benin	GRC	Greece	NZL	New Zealand
BFA	Burkina Faso	GTM	Guatemala	PAK	Pakistan
BGD	Bangladesh	GUY	Guyana	PAN	Panama
BOL	Bolivia	HKG	Hong Kong	PER	Peru
BRA	Brazil	HND	Honduras	PHL	Philippines
BRB	Barbados	HTI	Haiti	PNG	Papua New Guinea
BWA	Botswana	IDN	Indonesia	PRT	Portugal
CAF	Central African Rep.	IND	India	PRY	Paraguay
CAN	Canada	IRL	Ireland	ROM	Romania
CHE	Switzerland	IRN	Iran	RWA	Rwanda
CHL	Chile	ISL	Iceland	SEN	Senegal
CHN	China	ISR	Israel	SGP	Singapore
CIV	Cote d'Ivoire	ITA	Italy	SLV	El Salvador
CMR	Cameroon	JAM	Jamaica	SWE	Sweden
COG	Congo, Rep. of	JOR	Jordan	SYC	Seychelles
COL	Colombia	JPN	Japan	SYR	Syria
COM	Comoros	KEN	Kenya	TCD	Chad
CPV	Cape Verde	KOR	Korea, Rep. of	TGO	Togo
CRI	Costa Rica	LKA	Sri Lanka	THA	Thailand
CYP	Cyprus	LSO	Lesotho	TTO	Trinidad Tobago
DNK	Denmark	LUX	Luxembourg	TUR	Turkey
DOM	Dominican Rep.	MAR	Morocco	TWN	Taiwan
DZA	Algeria	MDG	Madagascar	TZA	Tanzania
ECU	Ecuador	MEX	Mexico	UGA	Uganda
EGY	Egypt	MLI	Mali	URY	Uruguay
ESP	Spain	MOZ	Mozambique	USA	USA
ETH	Ethiopia	MRT	Mauritania	VEN	Venezuela
FIN	Finland	MUS	Mauritius	ZAF	South Africa
FJI	Fiji	MWI	Malawi	ZAR	Congo, Dem. Rep.
FRA	France	MYS	Malaysia	ZMB	Zambia
GAB	Gabon	NAM	Namibia	ZWE	Zimbabwe

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