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Dinámicas del Estado del bienestar

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Welfare state dynamics

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Abstract

The goal in the present paper is twofold, to shed some light on endogenous dynamic of the welfare states, and to provide a procedure to select among several equilibria. To this end, a dynamic model is presented, in which private agents are assumed to be "locked" to current decisions for a while. If "frictions" are large enough, the economy might exhibit more than one stable Pareto-rankable stationary state. Equilibrium paths would then be determined by history. The economy might become "stuck" at an inferior stationary state, with too much insurance and too little effort.

Resumen

El objetivo de este documento es doble, arrojar alguna luz sobre la dinámica endógena de los Estados del Bienestar y aportar un procedimiento para seleccionar entre varios equilibrios. Con este fin, se presenta un modelo dinámico en el cual los agentes se suponen "atados" por un tiempo a sus decisiones de hoy. Si las "fricciones" son suficientemente grandes, la economía podría presentar varios estados estacionarios estables y ordenables en el sentido de Pareto. Los senderos de equilibrio serían determinados entonces por la historia. La economía podría terminar empantanada en un estado estacionario inferior, con demasiado seguro y muy bajo esfuerzo.

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1. Introduction

Welfare states evolve along time. Both governments and private agents adapt to changing external conditions. Technological progress, for instance, induces the government to modify its policies. But welfare states experience also some endogenous dynamics. Institutions and policies designed in a time might later induce changes in private agents behavior that, in turn, demand new adjustments to economic policies and institutions. It is sometimes argued that welfare systems that were successful years ago might now be facing difficulties do to changes in private agents behavior. The question arises whether the switch in citizens behavior has been endogenously determined by the policy. Yet, welfare states are usually analyzed in static terms (Lindbeck, 1994).

Some recent models of the welfare state exhibit multiple Pareto-rankable equilibria, with the potentially interesting ability of explaining the diversity of experiences observable in the real world, in a unified framework (Forteza, 1994, 1995a and b). However, at the same time, these models raise some difficult issues related to coordination of expectations and equilibrium selection. How can agents correctly anticipate other agents decisions when more than one equilibrium are possible? Which is the predicted outcome?

The goal in the present paper is twofold, to shed some light on endogenous dynamic of the welfare states, and to provide a procedure to select among several equilibria. To this end, a dynamic version of the model in Forteza (1994) is developed. The society consists of a large number of agents that produce one single output exerting effort. They choose effort levels to maximize discounted expected utility. Agents dislike effort, yet they might work hard for higher effort means higher probability of getting high income. There is a benevolent government that redistributes income in order to reduce agents exposition to risk. In so doing, the government generates a link between all individual producers, and the possibility of strategic complementarity arises. An agent working hard, not only raises his probability of getting high income, but also modifies other agents incentives to work hard. Indeed, when agents switch from low to high effort, aggregate output rises and, hence, the government might decide to modify the redistribution scheme, affecting other individuals incentives. Thus, each agent optimal choice depends on other agents choices, and multiple equilibria might arise.

A change in the incentives system might not provoke an immediate switch in agents actions. Habits, norms, attitudes, necessary reorganization in production processes, and a variety of adjustment costs, are some of the many possible reasons why responses might take time (Lindbeck, 1994). Lindbeck's hypothesis of sluggish welfare state dynamics is formally analyzed in the present paper with the help of some analytical tools borrowed from recent literature in dynamic game theory. Following Matsui and Matsuyama (1995), it is assumed that agents cannot switch effort at any instant. Opportunities to choose effort arrive randomly, following Poisson processes, and hence individual choices are asynchronized. The model exhibits thus a dynamic with frictions, in which the distribution of the strategies over the population evolves gradually.

It is shown in this paper that the stationary states of the dynamic model are the Nash equilibria of the original static version. If the degree of friction is high enough, more than one stable stationary state might exist, and the economy will exhibit history-dependent equilibrium paths. In particular, the low-effort-full-insurance equilibrium of the static game is shown to be a stable stationary state of its dynamic counterpart under appropriate conditions. Thus, the economy might indeed become "stuck" at an inferior stationary state. Different mixed-strategies stationary states are shown to have very different stability properties. Some are unstable, looking as highly unlikely. But other mixed-strategies stationary states might be stable, and thus the economy might remain close to them indefinitely. This is remarkable, for mixed-strategies equilibria have been considered as extremely fragile, when not lacking a meaning at all (Rubinstein, 1991).

The literature on equilibrium selection and dynamic games have been growing steadily in recent years. There are today many competing models in the field, and no simple criterion to choose between them (the by now extense literature is surveyed in Harsanyi and Selten, 1988; Mailath, 1992; Sargent, 1993). The increasing interest in models exhibiting coordination failures has recently fostered a number of research programs on equilibrium selection and dynamic in this particular class of models (Cooper, 1994; Moore, 1995; Matsui and Matsuyama, 1995). Moore (1995) analyzes the dynamic associated with learning. Under the assumption of rational expectations, the model is unable to predict one single outcome among several possible equilibria. The indeterminacy is solved by substituting the hypothesis of rational expectations for the assumption that agents do not know the parameters of the economy and learn them in much the same way as an econometrician would do.

Cooper (1994) and Matsui and Matsuyama (1995) introduce dynamic in former static games. Cooper proposes an equilibrium selection mechanism in which history creates a focal point. Without providing a formal foundation for it, Cooper argues that this mechanism can be motivated in terms of best response dynamics, i.e. in terms of the dynamic generated by agents playing their short run best response to the current environment. Matsui and Matsuyama study dynamics with frictions. Agents are assumed perfectly rational, but unable to switch actions at any instant. The dynamic generated in this way allows the authors to select among two strict Nash equilibria in 2x2 bimatrix games. This setting provides also a foundation for Cooper type of dynamic. The best response dynamic that serves to motivate Cooper's selection criterion emerges as a particular case of the more general class of dynamics analyzed by Matsui and Matsuyama.

In the present paper, analytical tools from Matsui and Matsuyama (1995) are borrowed. Similar assumptions are made in terms of agents being perfectly rational, but constrained to commit to their choice in the short run. However, the static game on which the dynamic model is built is more complex than that in Matsui and Matsuyama, so that the application is non trivial. Moreover, some results that are drawn in this paper have very different economic implications than those in Matsui and Matsuyama. They single out one equilibrium, the risk-dominant one, as the unique likely equilibrium in the long run. In this paper, on the contrary, the point is made that more than one equilibrium might remain in the long run, including some mixed strategies equilibria.

The model is presented in section 2. In section 3 the stability properties of the stationary states are analyzed. Section 4 ends the paper with some concluding remarks.

2. The model

The economy is populated by a large number of anonymous identical agents living for ever (alternatively, "dynasties"), that produce one single perishable good out of "effort". Individual output is stochastic, and the probability of getting high output is higher when agents decide to put in high effort. Thus, even though agents dislike effort, they might choose to work hard in order to increase their expected individual output. For simplicity, both output and effort are assumed to take just two values each: high output (X), low output (x), high effort (H), and low effort (L). The probability of getting high output when the agent has chosen effort a = {H,L} is denoted by P(a). Agents are risk averse. Their preferences over consumption and effort are represented by an expected utility function, which is increasing and concave in consumption and decreasing in effort.

There is a benevolent government that redistributes output aiming at reducing agents' exposition to risk. Lacking a commitment technology, and having a wide range of instruments to redistribute income, the government is assumed to choose in each moment the disposable income (which coincides with consumption in this setting) of each individual agent. Having computed aggregate output, it decides how to split the cake. The government does not observe individual effort, but it does observe individual output, so that it can condition disposable income on it. In instant 't', the government associates consumption 'w_t' to low individual output, and 'W_t' to high individual output. It provides "insurance" by making the gap between disposable income of agents getting high output and agents getting low output smaller than the corresponding gap in output. Full insurance is provided when consumption is uniform across states of nature (w_t=W_t). Formally, the government is assumed to choose pairs (w_t,W_t) in order to maximize the summation of the present value of agents expected utilities in an infinite horizon. It is not myopic, but since it can pick a completely different consumption allocation in each instant, with no links between periods, its intertemporal program can be split up in a continuum of instantaneous static optimization programs (point wise optimization). As a consequence, the static government reaction function presented in Forteza (1994, 1995a and b) still holds in the dynamic setting. [2]

Following Matsui and Matsuyama (1995), it is assumed that private agents must commit to actions for some time. Opportunities to switch actions arrive randomly, following Poisson processes with mean arrival rate 'm'. Thus, once an agent has made a choice, he will be "locked" to it for a period that lasts on average '1/m'. Poisson processes are assumed independent across agents, so that, by the law of large numbers, there is no aggregate uncertainty. The strategy distribution in the society at time 't' can be described by 'q_t', the proportion of individuals committed to high effort at that time. Because of these assumptions, 'q_t' is continuous in time and its rate of change belongs to the interval $[- m.q_t, m(1-q_t)]$.

Agents are assumed fully rational. Unlike in the recent related literature on evolutionary games, players are highly sophisticated in this model. [3] They maximize the present value of the whole stream of expected utility. Expectations are rational. Strategic behavior by part of private agents is ruled out: agents are too many and too small. The effects of individual decisions on 'q' are negligible by the same reason. And finally, there is no state variable to link periods. Still, current

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[2] It should be noticed, however, that in this version of the model the government is assumed to observe individual output along time while it is still rewarding agents according to current output. If it could check output in each instant, as it is assumed, it would be able to condition consumption on the whole stream of individual output realizations in the past. Thus, even though output in each instant is assumed to take just two values, the cumulated production in an interval becomes a continuous variable. The government might then want to make the consumption scheme continuous. This possibility has been assumed away in this paper. The assumption is actually irrelevant if, as in Forteza (1995b), utility functions are assumed additively separable, for then the government is not interested in making any difference on the basis of effort. When utility functions are not additively separable, however, the assumption that the government rewards agents according to current output simplifies the analysis.

[3] G. Mailath (1992) emphasizes as a distinctive feature of evolutionary games the assumption that players are not sophisticated, in the sense that they do not take into account all the consequences of their actions when making decisions.

choices have lasting consequences, for agents cannot switch actions at every moment.

Consider an agent having the opportunity to choose effort in 't'. His instantaneous expected utility in 't' is given by:

$$V(q_t,a) = P(a).u(W_{(q_t)},a) + (1-P(a)).u(w_{(q_t)},a)$$

The probability that he is still locked to that decision in 't+s' is: exponential(-ms). Thus, assuming a constant discount rate θ, the present value as of time 't' of the expected instantaneous utility in 't+s' *associated* with a choice made in 't' is:

$$V(q_{t+s},a).e^{-(m+\theta)s}$$

In picking an effort level, rational agents should consider the whole stream of expected utilities associated to it. Thus, agents are assumed to solve the following program:

$$\underset{a \in (H,L)}{Maximize}\ V_t(a) = (m+\theta)\int_0^{\infty} V(q_{t+s},a).e^{-(m+\theta)s}.ds \qquad \textbf{(3)}$$

Hence, q_t is an equilibrium path from q_0 if its right-hand derivative exists and satisfies

$$\frac{\partial^{+} q_t}{\partial t} \in \begin{cases} m.(1-q_t) & if \quad V_t(H) > V_t(L) \\ [-m.q_t\ ,\ m.(1-q_t)] & if \quad V_t(H) = V_t(L) \\ -m.q_t & if \quad V_t(H) < V_t(L) \end{cases} \qquad \textbf{(4)}$$

for all $t \in [0,\infty)$. Equation (4) states that all agents currently exerting low effort (high effort), if given the opportunity, switch to high effort (low effort), when $V_t(H)>V_t(L)$ (<). Agents are indifferent and hence randomize, when $V_t(H) = V_t(L)$.

According to equations (3) and (4), the dynamic of q_t depends (in a non trivial way) on the difference of instantaneous expected utilities associated with high and low effort, from present to the infinite future. In turn, these differences depend on q_{t+s}. The welfare policy game admits many different mappings of 'q' on instantaneous expected utility differentials. Nevertheless, for the sake of concreteness, but with some loss of generality, the analysis that follows will be referred to the example in figures 1 and 2.

Insert figure 1

Insert figure 2

The incentive line (IL) in figure 1 divides the w-W space in two regions. To the west (east), the current instantaneous expected utility associated with high effort is higher (lower) than with low effort. In other words, high effort is the best *short run* response to the west of IL. Government's strategy is summarized by its reaction function (GRF). It indicates the consumption pair the government chooses for each value of 'q'. Points A, B and C are the equilibria of the corresponding static game (see Forteza 1994). Nobody is working hard (q=0) and the difference of expected instantaneous utilities is strictly negative in point C. If the proportion of agents working hard rises, the government modifies the consumption allocation moving upwards on the GRF. When the proportion of hard workers reaches q_B, the instantaneous expected utilities associated with high and low effort coincide. For values of 'q' larger than q_B but smaller than q_A the current instantaneous expected utility is higher for an agent working hard. The difference is again zero at q_A, and it becomes negative again for larger values of 'q'.

Notice that, unlike in the static version of the game presented in Forteza (1994), agents do not necessarily prefer to work hard in the region to the west of the incentive line, nor to work little in the opposite region. *Current* instantaneous expected utility differential has a definite sign in these regions, but agents know they will be locked to current choices for a while, so that they care about current *as well as future* instantaneous expected utilities. Suppose, for instance, that the proportion of individuals working hard is currently some number slightly larger than q_B, and agents expect that 'q' will reduce in the future. Then, agents having the opportunity to choose actions today might prefer to pick low effort, even though hard effort currently yields higher instantaneous expected utility.

3. Stability properties of the stationary states

The dynamic system (3) and (4) driven by the instantaneous expected utilities differential represented in figure 2 has three stationary states: q=0, q_B and q_A. Not by chance, these points are the Nash equilibria of the associated static game. [4]

Proposition 1: q is a stationary state of the dynamics (3) and (4) if and only if q is a Nash equilibrium of the associated one-shot game.

Proof: see the appendix.

The aim of this section is to study the stability properties of these equilibria.Before proceeding, it seems convenient to introduce some definitions of stability especially adapted to the particularities of the case. The dynamic system (3) and (4) generally admits multiple equilibrium paths from one initial condition, so that standard definitions might be ambiguous. Definitions (i) and (ii) below are due to Matsui and Matsuyama (1995), while (iii) and (iv) are extensions, based on standard theory of dynamic systems (Perko, 1991).

Definitions:

(i) q 0 [0,1] is **accessible** from q' 0 [0,1] if there is an equilibrium path from q' that converges to q. q is **globally accessible** if it is accessible from any q' 0 [0,1].

(ii) q 0 [0,1] is **absorbing** if there is a neighborhood of q, Q, such that any equilibrium path from Q converges to q.

(iii) a closed invariant set *A* 0 [0,1] is an **absorbing or attracting set** of the dynamics (3) and (4) if there is some neighborhood Q of *A* such that any equilibrium path from Q converges to *A*.

(iv) q 0 [0,1] is **fragile** if it is not included in any absorbing set.

The first point to be made is that the low-effort-full-insurance equilibrium might be absorbing (for appropriate parameter values). Thus, a country that reaches this equilibrium (or a point close to it), might continue there for ever. The intuition is as follows. Agents having to choose effort when 'q' is close to zero know that current expected instantaneous utility is higher if they choose low effort. Yet, they wonder whether in the future, when they will probably be still locked to current decisions, the proportion of agents working hard could have risen enough to make high effort a better choice today. If the answer were affirmative, the economy might escape from the "bad" equilibrium. However, if frictions were relatively high, agents would know that the velocity of change of the proportion of hard workers will be slow. Thus, even in the most optimistic scenario (in which all agents having the opportunity to switch actions chose high effort), the economy would stay in the

[4] This assertion, in a slightly more general version, is proved in the appendix.

region in which low effort is the best short-run response for a long period. Unless agents were extremely patient, they would prefer to choose low effort.

Proposition 2: The stationary state q = 0 is absorbing, if the degree of friction (δ=θ/m) is above certain threshold.

Proof: if $V_t(H) < V_t(L)$ for all feasible q_t in a neighborhood of 0, then the equilibrium path would be unique and would converge to q = 0 for q_0 sufficiently close to 0. The strategy of the proof is to show that this is indeed the case when the degree of friction is high enough.

The highest accessible q_{t+s} from q_t is:

$$q_{t+s} = 1 - (1-q_t)\cdot e^{-ms}$$

Consider a point in the region in which low effort is the best short-run response: $q_t \in [0,q_B)$. The minimum time for a feasible path to leave this region($s^-_{B,t}$) is:

$$s^-_{B,t} = \ln\left(\frac{1-q_B}{1-q_t}\right)^{-\frac{1}{m}}$$

Then, the above mentioned condition can be written as:

$$\begin{aligned} & V_t(H) - V_t(L) = \\ & = (m+\theta)\cdot\left[\int_0^{s^-_{B,t}} \Delta V(q_{t+s})\cdot e^{-(m+\theta)\cdot s}\cdot ds + \int_{s^-_{B,t}}^{\infty} \Delta V(q \right. \end{aligned} \quad (7)$$

where $\Delta V(q_{t+s}) = V(q_{t+s},H) - V(q_{t+s},L)$ is the function represented in figure 2.

The first integral in the RHS of (7) is strictly negative for q_t close enough to 0. Moreover, it is strictly smaller than the initial expected utility differential, which is already negative:

$$(m+\theta)\cdot\int_0^{s^-_{B,t}} \Delta V(q_{t+s})\cdot e^{-(m+\theta)s}\cdot ds < (m+\theta)\cdot\Delta V(q_t) < \quad (8)$$

The second integral in the RHS of (7) might be positive, but it is "small" for some values of m and θ. Indeed, $\Delta V(q_{t+s})$ is bounded, so that the supremum exists; call it ΔV. Then:

$$(m+\theta)\cdot\int_{s_{B,t}}^{\infty}\Delta V(q_{t+s})\cdot e^{-(m+\theta)s}\cdot ds < \Delta V\cdot\left(\frac{1-q_B}{1-q_t}\right)^{\frac{m+\theta}{m}} \qquad \textbf{(9)}$$

It follows from (7), (8) and (9) that:

$$V_t(H) - V_t(L) < (m+\theta)\cdot\Delta V(q_t) + \Delta V\cdot\left(\frac{1-q_B}{1-q_t}\right)^{\frac{m+\theta}{m}} \qquad \textbf{(10)}$$

and q = 0 is absorbing if:

$$(m+\theta)\cdot\Delta V(q_t) + \Delta V\cdot\left(\frac{1-q_B}{1-q_t}\right)^{\frac{m+\theta}{m}} < 0 \qquad \textbf{(11)}$$

for any q_t in a neighborhood of 0. This inequality must hold for at least some values of m and θ. Indeed, it is easy to show that:

(i) for each θ>0, there are m*(θ)>0 and m**(θ)>0 such that for all m' ∈(0,m*(θ)] and for all m''∈(m**(θ),∞) inequality (11) holds for any q_t in a neighborhood of 0, i.e. q = 0 is absorbing;

(ii) for each m>0, there is a θ*>0 such that for all θ>θ*(m) inequality (11) holds for any q_t in a neighborhood of 0, i.e. q = 0 is absorbing. Conditions (i) and (ii) imply that q=0 is absorbing if the degree of friction (δ=θ/m) is high enough. **QED**.

There might be other absorbing equilibria. However, when the low-effort equilibrium is globally accessible, other equilibria are necessarily fragile. Proposition 3 provides sufficient conditions for the low-effort equilibrium to be globally accessible.

Proposition 3: The stationary state q=0 is globally accessible if:

$$\int_{q_B}^{q_A}\Delta V(q)\cdot q^{*}\cdot dq < -\int_{0}^{q_B}\Delta V(q)\cdot q^{*}\cdot dq \qquad \textbf{(12)}$$

where: $\Delta V(q) = V(q,H) - V(q,L)$

$\delta = \theta/m$ = degree of friction.

Proof: A proof of this proposition consists in showing that a feasible trajectory from $q_t = 1$ to $q = 0$ is an equilibrium path of the dynamics (3) and (4), if condition (12) holds. Consider the following path:

$$q_{t+s} = q_t . e^{-ms} \tag{13}$$

First, notice that the discounted sum of expected utilities differential along such path can be rewritten as:

$$V_t(H) - V_t(L) = (1+\delta)\, q_t^{-(1+\delta)} \int_0^{q_t} \Delta V(q) . q^{\delta} . dq \tag{14}$$

Indeed, equation (13) implies that:

i) when $s = 0$, $q_{t+s} = q_t$;

ii)when $s \to \infty$, $q_{t+s} \to 0$; and

iii)

$$dq_{t+s} = -\, q_t . e^{-ms} . m . ds$$

Using (i) to (iii) in $V_t(a)$, as defined in (3), yields (14).

The orbit (13) will be an equilibrium path if $V_t(H) - V_t(L) \leq 0$ for all $q_t \in [0,1]$. Consider qs in each of the following two regions $[0,q_B]$, and $(q_B,1]$.

i) $q_t \in [0,q_B]$. The condition $V_t(H) - V_t(L) \leq 0$ necessarily holds, for $\Delta V(q) \leq 0$ in this region (see figure 2).

ii) $q_t \in (q_B,1]$. Now:

$$V_t(H) - V_t(L) = (1+\delta)\, q_t^{-(1+\delta)} \int_0^{q_t} \Delta V(q) . q^{\delta} . dq \leq$$

$$\leq (1+\delta)\, q_t^{-(1+\delta)} \left[\int_0^{q_B} \Delta V(q) . q^{\delta} . dq + \int_{q_B}^{q_A} \Delta V(q) . q^{\delta} . dq \right]$$

which is non-positive if condition (12) holds. **QED.**

This proposition admits an intuitive interpretation. Because of frictions, 'q_t' makes no jumps, i.e. it is a continuous function of time. Hence, any path from qs above q_A to q=0 must pass through the region in which high effort is the best *short-run* response (q_B,q_A). But this region might "capture" equilibrium paths, in the sense that any equilibrium path that reaches it from above might stop moving downwards. The utility loss of choosing low effort in that region might be too high for agents to do it. Agents having the opportunity to switch actions when the proportion of individuals working hard is close to q_A might thus prefer to choose high effort. In this case, the equilibrium path would stop moving downwards, so that q=0 would not be accessible from initial points above q_A.

Equation (12) provides a sufficient condition for q=0 to be globally accessible. When it holds, the "high-effort" region is not able to "capture" any feasible equilibrium path that reaches it; at least some paths might escape. Not surprisingly, this condition holds if the "high-effort" region is small enough (q_A-q_B below a threshold), or the expected gains of working hard are relatively minor ones ($\Delta V(q)$ is positive but small in that region, compared with relatively large negative values out of this region). In these cases, incentives for agents to keep on working hard might not be strong enough, and a wave of pessimism might drive the economy down to the low stationary state.

If the low stationary state is globally accessible, other stationary states are fragile. Thus, if inequality (12) is a sufficient condition for q=0 to be globally accessible, the reverse is a necessary condition for other stationary states to be absorbing. Proposition 3 means, therefore, that the "high effort" region should be large enough or appealing enough, for a stationary state other than q=0 to be absorbing. Notice also that for large degrees of friction the reverse of (12) must hold. Hence, when the economy exhibits large frictions, a necessary condition for stationary states in mixed strategies to be absorbing is fulfilled.

If condition (12) did not hold, there might be other absorbing stationary states. However, it is shown in what follows that the remaining stationary states exhibit very different stability conditions. While B is never absorbing, A might be in the interior of an absorbing set. The intuition is quite clear. Any small deviation from B creates incentives for further deviations, while small deviations from A might (depending on the parameter values) create incentives to return to A.

Proposition 4: The stationary state q_B is fragile.

Proof: Consider the following paths:

$$q_{t+s} = q_t \cdot e^{-ms}$$

with $q_t \in [0,q_B]$. They converge to q=0 as time goes to infinite. These are equilibrium paths, for they satisfy the condition $V_t(H)- V_t(L)\leq 0$. Thus, q_B is fragile. **QED.**

Proposition 5: The stationary state q_A is in the interior of an absorbing set if the degree of friction (δ) is high enough. The absorbing set of q_A can be approached to a first order by the following expression:

$$\left[q_A - \frac{1-q_A}{1+\delta} \;,\; q_A + \frac{q_A}{1+\delta} \right]$$

Proof: In a neighborhood of q_A, the instantaneous expected utilities differential can be approached by a linear expression:

$$V(q_t,H) - V(q_t,L) = \Delta V(q_t) \approx \Delta V'(q_A).(q_t - q_A)$$

where:

$$\Delta V'(q_A) = \frac{d\Delta V(q_A)}{dq_t} < 0$$

This last inequality corresponds to figure 2. The sign of this derivative defines in general the A-type equilibria, while the opposite holds for the B-type equilibria.

Any feasible path from q_t satisfies:

$$q_t . e^{-ms} \leq q_{t+s} \leq 1-(1-q_t).e^{-ms}$$

If q_t is not too far from q_A , then:

i)

$$\Delta V_t = V_t(H) - V_t(L) = -(\mu+\delta)\int_0^{\infty} \Delta V'(q_A)\cdot(q_A - q_{t+s})\cdot e^{-(\mu+\delta)s}ds >$$

$$> -(\mu+\delta)\int_0^{\infty} \Delta V'(q_A)\cdot(q_A - q_t\cdot e^{-\mu s})\cdot e^{-(\mu+\delta)s}ds =$$

$$= -(\mu+\delta)\cdot\Delta V'(q_A)\cdot\left(\frac{q_A}{\mu+\delta} - \frac{q_t}{2\mu+\delta}\right)$$

and thus:

$$\Delta V_t > 0 \quad \Rightarrow \quad \frac{d^{+}q_t}{dt} > 0 \quad if \quad q_A + \frac{q_A}{1+\ast} > q_t$$

ii)

$$\Delta V_t = -(\mu+\delta)\int_0^{\infty} \Delta V'(q_A)\cdot(q_A - q_{t+s})\cdot e^{-(\mu+\delta)s}ds <$$

$$< -(\mu+\delta)\int_0^{\infty} \Delta V'(q_A)\cdot(q_A - 1 + (1-q_t)\cdot e^{-\mu s})\cdot e^{-(\mu+\delta)s}ds =$$

$$= -(\mu+\delta)\cdot\Delta V'(q_A)\cdot\left(-\frac{1-q_A}{\mu+\delta} + \frac{1-q_t}{2\mu+\delta}\right)$$

and thus:

$$\Delta V_t < 0 \quad \Rightarrow \quad \frac{d^{+}q_t}{dt} < 0 \quad if \quad q_t > q_A - \frac{1-q_A}{1+\ast}$$

If δ is large, the absorbing set is a small region around q_A, and the linearization of $\Delta V(q_t)$ is a good approximation in a neighborhood of this region. **QED.**

It is interesting to notice that the class of dynamics analyzed in this paper contains as a limiting case the best response dynamic. This case arises when the degree of friction tends to infinite. Hence, propositions 2 to 5 imply that if agents play short-run best responses, the model exhibits two absorbing stationary states. It is known also that these points are Pareto rankable (Forteza 1994 and 1995a), so that the economy might become "stuck" at a "bad" equilibrium, characterized by too much insurance and too little effort.

Best response dynamic is often associated with bounded rationality (Mailath, 1992). Agents playing their short-run best response might be myopic. In the present setting, however, this type of behavior appears as a rational strategy of highly sophisticated agents. Rational agents strategies might look like short run best responses without actually being so, if frictions are so large that the distribution of strategies in the population moves extremely slowly. Then, in practice, the only relevant distribution (q) is the current one.

The stationary states that are absorbing for high enough frictions are also evolutionary stable strategies, while the fragile stationary state is not an evolutionary stable strategy. Small deviations from q=0 and q_A make deviators to do worse off than their peers that are still playing the original strategy. In the terminology of evolutionary game theory, the "mutants" perform worse, so that they tend to die out. On the contrary, small deviations from q_B are profitable, and so they are selfreinforcing. The similarities between the analysis in this paper and evolutionary theory should not be overemphasized, though. Stationary states q=0 and q_A are necessarily evolutionary stable, while they might not be absorbing. In this sense, the criterion used in the present paper to say that an equilibrium is "stable" is more demanding than the criterion commonly used in evolutionary game theory.

It seems remarkable that the mixed strategies stationary state 'A' might be absorbing. Exactly at point A, agents are indifferent between both effort levels, so that individuals having the opportunity to switch effort will likely pick the "wrong" strategies. But then, a small deviation from q_A will take place and in the near future, agents will have incentives to move again towards the stationary state. Therefore, states like this might be observable -and even likely- in the real world.

4. Concluding remarks

The dynamic with frictions analyzed in this paper have some points in common with the idea of "hazardous" and "virtuous" dynamics in the welfare state introduced by Lindbeck (1994). An economy moving upwards along an equilibrium path would be in a "virtuous" dynamic. Along such path, agents decisions and welfare policies change in a mutually reinforcing manner, and social welfare improves (in a Pareto sense). A "hazardous" or "vicious" dynamic would correspond to a downward equilibrium path. Agents having the opportunity to choose actions would then switch to low effort, so that economic performance in the aggregate would gradually deteriorate. Economic policies would respond reducing further the incentives to work hard. Social welfare would gradually decrease, and in the end the economy would eventually reach a Pareto inferior stationary state.

It is interesting to notice that, if the analysis in this paper proves correct, "social norms" might have long lasting consequences, even though agents are *directly* constrained by norms only in the short run. Social norms might produce frictions that reduce the velocity of change of social behavior, and in this way, alter the economic performance not just in the short run - as presumed by Lindbeck -, but also eventually in the medium and long run. Indeed, as it was shown in previous sections, a relatively high degree of friction is necessary for a high-effort stationary state to be absorbing. An economy lacking "good social norms" might move too fast for agents to feel absolutely confident that the system is stable. Then, a wave of pessimism, provoked by any even minor shock, might push the economy downwards, unleashing a "vicious" circle.

Appendix

Proposition 1: q is a stationary state of the dynamics (3) and (4) if and only if q is a Nash equilibrium of the associated one-shot game.

Proof: i) The "if" part states that, whenever q is a Nash equilibrium of the static game, $\{q_t = q \,,\ \forall t\}$ is an equilibrium path of the class of dynamics defined by (3) and (4). Along these paths, the present value of expected utilities associated with a choice is equal to the current expected utility:

$$V_t(a_i) = V(q,a_i)\cdot(m+\rho)\int_0^\infty e^{-(m+\rho)s}\cdot ds = V(q,a_i)$$

From propositions 1 and 2 in Forteza (1994), the Nash equilibria of the static game are q = 0 and $\{q^* / V(q^*,H)=V(q^*,L)\}$. In the example drawn in figures 1 and 2 of this paper, $q^* = \{q_B, q_A\}$. In the first case, $\{q_t = 0 \,,\ \forall t\}$ is indeed an equilibrium path, for:

$$V_t(H) = V(0,H) < V(0,L) = V_t(L) \quad \Rightarrow \quad \frac{d^+q_t}{dt} = -m\cdot q_t = 0$$

In the case of mixed strategies equilibria, $\{q_t = q^* \,,\ \forall t\}$ is also an equilibrium path of the dynamics (3) and (4), since:

$$V_t(H) = V(q^*,H) = V(q^*,L) = V_t(L) \quad \Rightarrow \quad \frac{d^+q_t}{dt} = 0$$

ii) ("only if") q_t is a stationary state if either: a) $q_t = 1$ and $V_t(H)>V_t(L)$; b) $q_t \in (0,1)$ and $V_t(H)=V_t(L)$; or $q_t = 0$ and $V_t(H)<V_t(L)$. But (a) cannot hold for:

$$q_t = 1 \Rightarrow V_t(H) = V(1,H) < V(1,L) = V_t(L)$$

The remaining candidates are the Nash equilibria of the static game. **QED**.

References

Cooper, Russell and Andrew John (1988): "Coordinating coordination failures in keynesian models", **Quarterly Journal of Economics,** vol CIII, 3, 441-463.

Cooper, Russell (1994): "Equilibrium selection in imperfectly competitive economies with multiple equilibria", **The Economic Journal,** 104 (September), 1106-1122.

Forteza, Alvaro (1994): "On the political economy of the welfare state". **Working paper 5/94**, Departamento de Economía, Universidad de la República, Uruguay.

Forteza, Alvaro (1995a): "Multiple equilibria in the welfare state: heterogenous population". **Working paper**, Departamento de Economía, Universidad de la República, Uruguay.

Forteza, Alvaro (1995b): "Multiple equilibria in the welfare state: costly policies"**. Working paper**, Departamento de Economía, Universidad de la República, Uruguay.

Harsanyi, J.C. and R. Selten (1988): **A general theory of equilibrium selection in games**, MIT Press, Cambridge, MA.

Lindbeck, Assar (1994): "Hazardous welfare state dynamics. Endogenous habits and norms". **Working paper, Scandinavian Journal of Economics conference: "The future of the welfare state".**

Mailath, George (1992): "Introduction: symposium on evolutionary game theory", **Journal of Economic Theory** 57, 259-277.

Matsui, Akihiko and Kiminori Matsuyama (1995): "An approach to equilibrium selection", **Journal of Economic Theory 65**, 415-434.

Perko, Lawrence (1991): **Differential equations and dynamical systems.** Texts in applied mathematics. Springer-Verlag, New-York.

Rubinstein, Ariel (1991): "Comments on the interpretation of game theory". **Econometrica**, 59(4), 909-924.

Sargent, Thomas J. (1993): **Bounded rationality in macroeconomics.** Oxford University Press.