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Resumen

La incapacidad del gobierno de comprometerse a no ayudar a los "infortunados" podría provocar distorsiones excesivas en el estado del bienestar. Las consideraciones relativas a los incentivos, que son cruciales en los seguros estándar en presencia de riesgo moral, no tienen lugar en este caso. Como consecuencia, el gobierno podría proveer demasiado seguro. Aun así, podrían tener lugar equilibrios con seguro incompleto y esfuerzo por encima del mínimo. En este artículo se exploran dos posibles razones para la multiplicidad de equilibrios, que la utilidad marginal del consumo esté asociada positivamente con el esfuerzo y que la política económica sea costosa. Se muestra que los equilibrios pueden ser ordenables en el sentido de Pareto. Utilizando herramientas analíticas desarrolladas recientemente en la literatura de juegos dinámicos (Matsui and Matsuyama, 1995), se analizan las condiciones de estabilidad de diferentes equilibrios.

Abstract [2]

Excess distortions in the welfare state might be the consequence of the government's lack of ability to commit not to help "unlucky" agents. Incentive considerations that are crucial in standard insurance in the presence of moral hazard, plays no role in this case. As a consequence, the government might provide too much insurance. Still, equilibria with incomplete insurance and above-minimum effort might arise. Two possible reasons for multiple equilibria are explored in the paper, namely that marginal utility of consumption is positively associated with effort and that economic policy is costly. It is shown that the equilibria can be Pareto rankable. Borrowing analytical tools from recent developments in dynamic games (Matsui and Matsuyama, 1995), stability conditions of different equilibria are analyzed.

JEL: D82, H30.

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[2] Suggestions from Torben Andersen were very useful in revising this paper. The usual disclaimer applies.

1. Introduction

The current political concern about the distortions that the welfare state might be introducing in the system of incentives has recently motivated an increasing academic interest in the subject. Atkinson and Mogensen (1993) conducted an empirical research program aimed at assessing the actual consequences of welfare policies in the job market. In Sweden, a commission appointed by the swedish government, analyzed several welfare policies and recommended deep reforms (Lindbeck et al, 1993). The Scandinavian Journal of Economics organized a meeting about the future of the welfare state (1994 special issue).

It has long been known that welfare policies introduced some distortions in the economic system (Barr, 1992), but the current concern is based on the increasing feeling that the distortions are very often outweighing the benefits. Forteza (1994) presents a simple model aimed to show why a government that maximizes the expected utility of private agents providing insurance might provoke too many distortions. The basic story is that a government that does not have the ability to commit not to help "unlucky" agents induces individuals to free ride on the government's concern, exerting little effort. The economy ends up in a situation of too much insurance and too little effort, compared with what would be the case if the government had the commitment capacity.

Forteza (1994) assumes an homogeneous population. Still, the model exhibits mixed-strategies equilibria in which private agents take different actions. There might be a relatively successful welfare state, in which individuals would work hard most of the time and apply for social benefits just from time to time (or equivalently in the model, most individuals would work hard the whole time). There might also be less successful welfare states, represented by equilibria with lower probabilities of working hard and more applications for social benefits.

These are expectations-driven equilibria, in the sense that agents would behave according to the "good" equilibrium if they expected that other individuals would do the same. For that reason, "social norms" might play an important role in determining the performance of the welfare state, as it has been recently suggested in the scandinavian countries (Lindbeck, 1994). Accordingly, evolution of welfare states might be associated with the evolution of social norms and beliefs.

The multiplicity of equilibria might account for the diversity that is observable in the real world, both across countries and across time. In this sense, rather than being a weakness of the model, multiple equilibria might be part of its strength and richness. However, multiple equilibria in the above mentioned model are relatively fragile, for all of them save one are in mixed strategies, and this concept has been under severe criticism (Rubinstein, 1991). In the present setting, mixed strategies admit a simple interpretation as proportions of the population that are playing pure strategies. Still, if all individuals are identical, why should some of them pick up high effort and others low effort? Moreover, nothing compels agents to choose the "right" randomization. Thus, these equilibria do not look likely. Then the only equilibrium that would rest is the low-effort-pure-strategies one. The model would still provide a simple story to account for excess distortions, but it would hardly capture the diversity of the welfare states. Therefore, one of the aims of the present paper is to explore further the multiplicity issue, trying to determine possible reasons for the diversity that is observable in the real world.

One of the extensions presented in the paper allows for heterogeneous population. The problem with the mixed strategies equilibria in the homogeneous population model stems from the fact that all agents become indifferent between working hard or soft under the same conditions. It is shown in section 2 that a bit of heterogeneity between agents is enough to generate multiple equilibria in pure strategies.

A second extension presented in section 3 relaxes the assumption of absolute lack of commitment capacity by part of the government. To that end, costs of implementing economic policies are introduced. Not surprisingly, these costs might cause incomplete insurance. Less obviously, they might provoke multiple equilibria, including two equilibria in pure strategies.

In both versions of the model, welfare comparisons are performed, and conditions for equilibria to be Pareto rankable are provided. There is thus a precise sense in which it can be said that some welfare states might be "better" than others.

Some of the static equilibria might be just unlikely theoretical possibilities. Unstable equilibria do not seem convincing candidates to represent any relevant case observable in the world. Thus, stability analysis could help in selecting among several equilibria. Besides, the Nash equilibria of the static game might be the stationary states of a dynamical model with many possible equilibrium paths. If the velocity of the economy moving along those paths were not too high, and shocks that disturb the system took place relatively often, it would be more likely to observe points on these paths than points corresponding to stationary states. The third extension of the basic model that is pursued in this paper rests on recent developments in the theory of dynamic games (Matsui and Matsuyama, 1995). Agents are assumed perfectly rational but unable to change actions at any instant, so that there is some degree of friction. It is shown in section 4 that there might be more than one "absorbing" equilibrium, so that history dependent paths cannot be ruled out.

2. A model with heterogeneous agents

Consider an economy populated by a continuum of individuals indexed by 'i', ranging in the real interval [0,1]. All of them produce the same consumption good, incurring in effort (a_i), which, for the sake of simplicity, can take just two values: high (H) and low (L) effort (H>L). Each agent gets an amount X with probability $P(a_i)$ and x with probability $(1-P(a_i))$. Just to fix ideas, assume X > x so that $P(a_i)$ is the probability of "being lucky". This probability is a function of the individual's action. The probability of getting the good outcome is higher when the agent chooses to put in high effort (P(H) > P(L)). Probabilities of different individuals are independent, so that, by the law of large numbers, there is no aggregate risk. Ex ante, individual output is only probabilistically known while total output is known with certainty.

Individuals seek to maximize expected utility functions, which are increasing and concave in consumption ('W' in the good state and 'w' in the bad state; u'>0, u''<0) and decreasing in effort. Population might be heterogeneous in that, loosely speaking, individuals with higher index are less willing to work hard, under the same conditions. A relatively simple way of formally introducing this idea is to assume that expected utility is equal for all agents save for an additive term in effort:

$$V_i(w_i,W_i,a_i) = P(a_i).u(W_i,a_i) + (1-P(a_i)).u(w_i,a_i) - i.a_i \quad \textbf{(1)}$$

So that agents with higher index exhibit higher disutility of effort. The population is distributed over types according to a distribution function F(i). If the whole probability mass is concentrated in one single index value, the population is homogeneous. Nobody knows others type, but the distribution function over types is common knowledge.

It is immediate that there will be some room for insurance in this economy. Without it, i's expected utility is given by (1) with W_i=X and w_i=x. If instead, individuals were offered a zero-cost-full-insurance scheme, agent i would get $W_i = w_i = P(a_i)X + (1 - P(a_i))x$ in both states of nature. Due to risk aversion, expected utility with insurance is higher, for each action. In the presence of perfect private insurance, agents would be able to fully diversify risk. But, if insurance markets were incomplete, citizens might give politicians a mandate to provide insurance.

Consider now the timing. Unlike private companies, which must set the conditions of the insurance contract before agents choose actions, the government can reoptimize in any moment. Unless it can commit the insurance policy in advance, the government will face an incentive compatibility constraint. In what follows, it is assumed that the government does not have the commitment ability, so that it behaves discretionally. The timing of the game includes two stages. In the first stage, all private agents simultaneously pick effort levels, and in the second stage, the government chooses consumption allocations.

2.1. The second stage: the government reaction function (GRF)

The government is assumed to maximize the summation of individuals' expected utilities. The redistributive conflict has been somehow solved, and a diligent politician is in office with the exclusive aim of maximizing individuals' utilities. The weight of each agent in the government objective function reflects the outcome of the political process that solved the redistributive conflict. None of the results bellow bear upon the simplifying assumption of uniform weights. Using taxes and transfers, the government determines consumption allocations in both states of nature. Unlike effort and the agent's type, individual output is observable, so that the government can make consumption allocations contingent on it. Agents receive W when they get high output and w when they get low output.

In the discretionary regime, when the government plays, private agents have already picked up effort levels, and production has taken place. Each individual belongs now to one of the following four groups: 1) those that worked hard and got high output; 2) those that worked hard, but got low output; 3) those that did not work hard, and still got high output; and 4) those that did not work hard and got low output. Calling 'q' the proportion in the population of individuals that worked hard, the ex-post government objective function can be written as:

$$\begin{aligned} & q.P(H).u(W,H) + q.(1-P(H)).u(w,H) + \\ & \quad + (1-q).P(L).u(W,L) + (1-q).(1-P(L)).u(w,L) \end{aligned} \qquad \textbf{(2)}$$

Calling 'N' the proportion in the population of individuals that got high output, the whole economy resources constraint can be written as:

$$X^A \equiv N.X + (1-N).x \geq N.W + (1-N).w \qquad \textbf{(3)}$$

The benevolent government chooses (w,W) to maximize (2) subject to (3). It follows from the first order conditions that:

$$\lambda(q) = \frac{q.P(H)}{N}.u_1(W,H) + \frac{(1-q).P(L)}{N}.u_1(W,L) = \frac{q.(1-P(H))}{1-N}.u_1(w,H) + \frac{(1-q).(1-P(L))}{1-N}.u_1(w,L) \qquad \textbf{(4)}$$

where $u_1(.)$ stands for the first derivative in the first argument and λ(q) is the Kuhn-Tucker multiplier. The statistic 'q' is not directly observable, but it can be inferred from the number of lucky agents:

$$N = P(H).q + P(L).(1-q) \qquad \textbf{(5)}$$

The government's first order conditions admit a simple bayesian interpretation. Factors multiplying marginal utilities in equation (4) are government posterior beliefs about individual effort, conditional on the observation of individual outputs, when the priors are q and 1-q. Remember that P(H) is a short for the probability of high output conditional on high effort, so that a more explicit notation could be: Prob(X/H). Remember also that N is the proportion of agents that got high output, so that it could be noted as: Prob(X). Finally, the proportion in the whole population of individuals that have worked hard can be written as Prob(H). Thus, posteriors are given by:

$$Prob(H/X,q) = \frac{Prob(X/H).Prob(H)}{Prob(X)} = \frac{P(H).q}{N}$$

where Prob(H/X,q) is the probability that someone that got high output has worked hard when the unconditional probability of observing high effort is Prob(H). Similar interpretations hold for the other conditional probabilities. Using these observations, equation (4) can be written as:

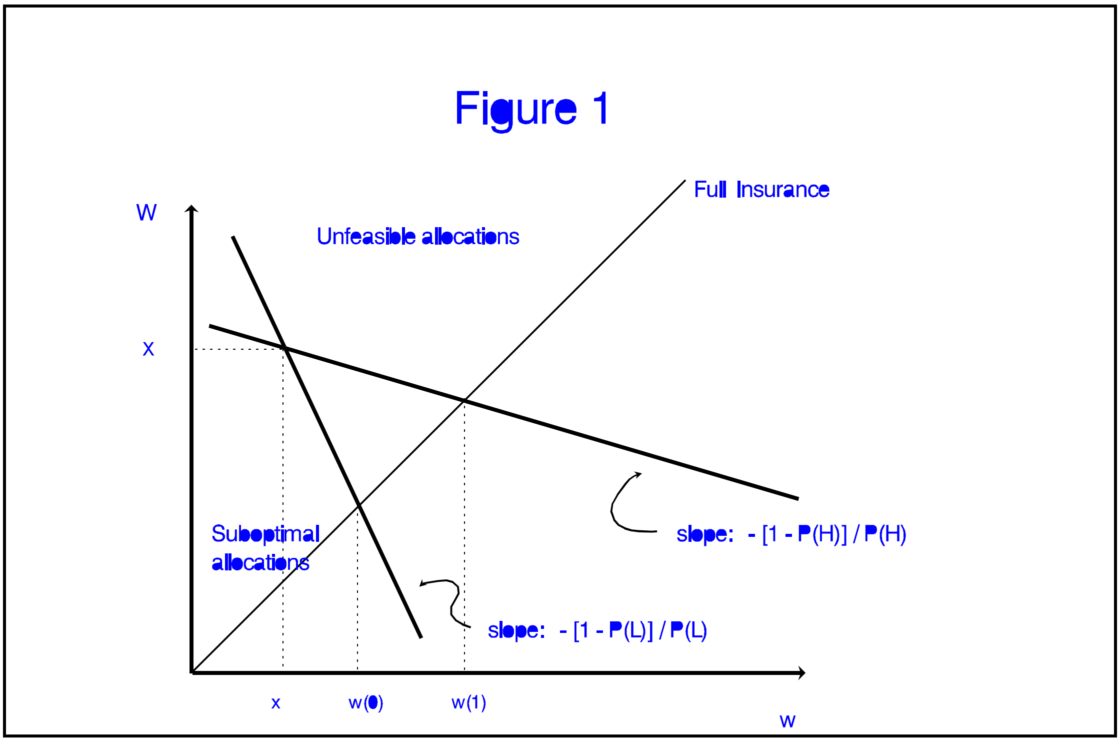
$$Prob(H/X,q).u_1(W,H) + (1-Prob(H/X,q)).u_1(W,L) = Prob(H/x,q).u_1(w,H) + (1-Prob(H/x,q).u_1(w,L) \qquad \textbf{(7)}$$

Equation (7) says that the government maximizes social welfare with given actions when the expected marginal utilities of consumption of the lucky and the unlucky are equal. The expectations operator is conditional on the relevant information available for the government.

By virtue of the implicit function theorem, the system of equations (3), (4) and (5) determines a continuous mapping from q to the pairs (w,W). This is the **government's reaction function**. The existence and some properties of this function are formally stated and proved in the following proposition.

Proposition 1: Existence and properties of the government reaction function (GRF).

i) There is a continuous function mapping from the proportion in the population of individuals that worked hard (q) to consumption allocations (w,W) such that the government optimizes social welfare ex-post.
ii) The image of the GRF in the (w,W) space lies in the cones determined by the resource constraints for N = P(L) and N = P(H) (see figure 1).



iii) If either nobody worked hard (q = 0) or everybody did work hard (q = 1), the government provides full insurance, with consumption equal to per capita output:

$$
\begin{aligned}
w(q=0) &= W(q=0) = P(L).X + (1-P(L)).x \\
w(q=1) &= W(q=1) = P(H).X + (1-P(H)).x
\end{aligned}
\qquad (8)
$$

iv) If there exist consumption levels $w^* \in [w(0), w(1)]$ for which marginal utilities of consumption of individuals that worked hard and individuals that worked soft coincide, then the consumption allocations (w^*,w^*) lie in the image of the GRF. In particular, if utility functions are additively separable in consumption and effort, the GRF is the segment of the full insurance line with extremes w(0) and w(1).
v) If marginal utilities of consumption for high and low effort do not coincide for any $w \in [w(0), w(1)]$, then the GRF lies on one side of the full-insurance line. If $u_1(w,H) > u_1(w,L)$, the GRF lies to the west of the full-insurance line (W > w). If instead $u_1(w,H) < u_1(w,L)$, the GRF lies to the east of the full-insurance line (W < w).
vi) If marginal utilities of consumption cross each other for some $w^* \in [w(0), w(1)]$, the GRF lies to the west (east) of the full-insurance line in the intervals in which marginal utility of consumption is higher (lower) when agents have worked hard.
vii) Other things equal, the more risk averse agents are, the smaller the consumption gap between lucky and unlucky (W-w) on the GRF, i.e. the "closer" the GRF is to the full-insurance line. The larger the marginal utility differential, the larger the difference (W-w) on the GRF, i.e. the "farther" the GRF lies from the full-insurance line.
viii) The image of the GRF has positive slope in the space (w,W), for points to the southeast of (x,X), if and only if condition (9) is fulfilled [3].

$$\begin{aligned} -N.\lambda(q).GARA(w,q).[P(H)-P(L)].(X-W+w-x) &< GMUD(q) \\ &< (1-N).\lambda(q).GARA(W,q).[P(H)-P(L)].(X-W+w-x) \end{aligned} \qquad \textbf{(9)}$$

where the functions GARA(.) - the government measure of absolute risk aversion -, and GMUD(.) - the government measure of marginal utility differentials- are defined as follows:

$$GARA(w,q) = \alpha(w,q).\left(\frac{-u_{11}(w,H)}{u_1(w,H)}\right) + (1-\alpha(w,q)).\left(\frac{-u_{11}(w,L)}{u_1(w,L)}\right)$$

$$\textit{where:} \quad 0 \le \alpha(w,q) = \frac{q.(1-P(H)).u_1(w,H)}{(1-N).\lambda(q)} \le 1 \qquad \textbf{(10)}$$

[3] Nothing precludes in this version of the model that the GRF has some points in the northwest of (x,X). In this case, the inequalities are reversed. However, this case does not seem very likely, so it will not be analyzed in detail.

and:

$$GMUD(q) = \frac{P(H).P(L).(1-N)}{N}[u_1(W,H) - u_1(W,L)] - \\ - \frac{[1-P(H)].[1-P(L)].N}{1-N}[u_1(w,H) - u_1(w,L) \quad \textbf{(11)}$$

ix) The slope of the image of the GRF in the space (w,W) is positive in the points in which the GRF crosses the full-insurance line.

Proof:

i) Using (5) in (3) and (4):

$$[P(H).q + P(L).(1-q)].(X-W) + \\ + [1-(P(H).q + P(L).(1-q))].(x-w) = 0 \quad \textbf{(12)}$$

$$\frac{q.P(H).u_1(W,H) + (1-q).P(L).u_1(W,L)}{P(H).q + P(L).(1-q)} - \\ - \frac{q.(1-P(H)).u_1(w,H) + (1-q).(1-P(L)).u_1(w,L)}{1-[P(H).q + P(L).(1-q)]} = 0 \quad \textbf{(13)}$$

This system can be more compactly written as:

$$\begin{bmatrix} F^1(w,W,q) \\ F^2(w,W,q) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \textbf{(14)}$$

By virtue of the implicit function theorem, a sufficient condition for the existence of a continuous function mapping from q to (w,W) is that:

$$J \equiv Det \begin{bmatrix} \frac{\partial F^1}{\partial w} & \frac{\partial F^1}{\partial W} \\ \frac{\partial F^2}{\partial w} & \frac{\partial F^2}{\partial W} \end{bmatrix} \neq 0$$

which is indeed the case:

$$J = -\frac{1-N}{N}.[q.P(H).u_{11}(W,H) + (1-q).P(L).u_{11}(W,L)] -$$

$$- \frac{N}{1-N}.[q.(1-P(H)).u_{11}(w,H) + (1-q).(1-P(L)).u_{11}(w,L)] > 0$$

since: i) $0 < P(H), P(L) < 1$; ii) $0 \leq q \leq 1$; iii) $0 < N < 1$; iv) $u_{11}(.) < 0$.

ii) First notice that $P(L) \leq N \leq P(H)$. Second, notice that the government chooses a consumption allocation **on** the resources constraint that corresponds to the observed N, which is the straight line passing through (x,X), with slope -(1-N)/N . Points to the east of this line in the (w,W) space are not feasible, while points to the west are suboptimal.

iii) Equations (3) and (4) imply that if q = 0, then N = P(L) and $u_1(w(0),L) = u_1(W(0),L)$, so that: w(0)= W(0)= P(L).X + (1-P(L)).x . Also, if q = 1, then N = P(H) and $u_1(w(1),H) = u_1(W(1),H)$, so that: w(1)= W(1)= P(H).X + (1-P(H)).x .

iv) If $u_1(w^*,H) = u_1(w^*,L)$, then the point (w*,w*) satisfies equation (4). If $w^* \in [w(0), w(1)]$, there must be an $N^* \in [P(L), P(H)]$ and, correspondingly, a $q \in [0,1]$ such that the resources constraint holds:

$$P(L).X + [1-P(L)].x \leq w^* = N^*.X + [1-N^*].x \leq$$
$$\leq P(H).X + [1-P(H)].x \qquad \textbf{(17)}$$

Thus, point (w*,w*) satisfies equations (3) and (4), so that it must belong to the image of the GRF.

v) Consider the following Taylor series expansion:

$$u_1(W,H) = u_1(w,H) + (W-w).u_{11}(w',H) \quad , \quad w' \in (w,W)$$

$$u_1(W,L) = u_1(w,L) + (W-w).u_{11}(w'',L) \quad , \quad w'' \in (w,W)$$

in the FOC (equation (4)):

$$(W-w) = \frac{[Pr(H/X,q)-Pr(H/x,q)].[u_1(w,H)-u_1(w,L)}{Pr(H/X,q).\frac{-u_{11}(w',H)}{u_1(w',H)}.u_1(w',H) + [1-Pr(H/X,q)].\frac{-u_{11}(}{u_1(}} \tag{19}$$

and thus:

$$sign(W-w) = sign[u_1(w,H) - u_1(w,L)] \tag{20}$$

vi) This result is immediate from equation (20).

vii) These results follow immediately from equation (19).

viii) Differentiating (14):

$$\begin{bmatrix} \frac{dw}{dq} \\ \frac{dW}{dq} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F^1}{\partial w} & \frac{\partial F^1}{\partial W} \\ \frac{\partial F^2}{\partial w} & \frac{\partial F^2}{\partial W} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F^1}{\partial q} \\ \frac{\partial F^2}{\partial q} \end{bmatrix} = -\frac{1}{J} \begin{bmatrix} \frac{\partial F^2}{\partial W}.\frac{\partial F^1}{\partial q} - \frac{\partial F^1}{\partial W}.\frac{\partial F^2}{\partial q} \\ -\frac{\partial F^2}{\partial w}.\frac{\partial F^1}{\partial q} + \frac{\partial F^1}{\partial w}.\frac{\partial F^2}{\partial q} \end{bmatrix} \tag{21}$$

so:

$$\frac{dW}{dw}\bigg|_{GRF} = \frac{-\frac{\partial F^2}{\partial w}.\frac{\partial F^1}{\partial q} + \frac{\partial F^1}{\partial w}.\frac{\partial F^2}{\partial q}}{\frac{\partial F^2}{\partial W}.\frac{\partial F^1}{\partial q} - \frac{\partial F^1}{\partial W}.\frac{\partial F^2}{\partial q}}$$

Computing the partial derivatives, and using again the FOCs and the definitions given in equations (10) and (11), it can be shown that the slope of the government reaction function in the w-W space is given by:

$$\frac{dW}{dw}\bigg|_{GRF} = \frac{-\lambda(q).GARA(w,q).[P(H)-P(L)].(X-W+w-x) \; -}{-\lambda(q).GARA(W,q).[P(H)-P(L)].(X-W+w-x) \; +} \tag{23}$$

The slope of the GRF is positive if and only if the numerator and the denominator in the above expression have the same sign. Consider points of the GRF in the southeast of the non-intervention point (x,X): the term (X-W+w-x) is then positive. For these points, the numerator and the denominator cannot both be positive, since GMUD(q) must be negative for the numerator to be positive, while the denominator cannot be positive in that case. Thus, for the points on the GRF located in the southeast of (x,X), the slope of the GRF is positive if and only if both the numerator and the denominator are negative:

$$-\lambda(q).GARA(w,q).[P(H)-P(L)].(X-W+w-x) < \frac{GMUD(q)}{N}$$

$$-\lambda(q).GARA(W,q).[P(H)-P(L)].(X-W+w-x) < -\frac{GMUD(q)}{1-N}$$

Condition (9) follows immediately. It is easy to check that the inequalities are reversed for points in the northwest of (x,X).

ix) From property iv), it is known that: $u_1(w^*,H) = u_1(w^*,L)$. This equality, in turn, implies that GMUD(q) = 0. But then, inequalities (9) must hold for the point (w*,w*), for the term in the left is negative while the term in the right is positive.

QED

2.2.- The first stage: the private sector reaction function

In the first stage, private agents choose effort to maximize expected utility. They know the government reaction function, so that they can, in principle, anticipate the government's policy. However, as it was argued above, the discretionary policy depends on what private agents do. Thus, in order to make a rational choice, each agent has to foresee other agents decisions.

For each possible pair (w,W), individuals' decisions about effort generate an aggregate outcome that can be summarized in certain proportions of the population working hard. If the population is smoothly distributed across types, this correspondence is a continuous single valued function. It will be called **the private sector reaction function.** This result is formally stated and proved in the following proposition.

Proposition 2: (Existence of a continuous private sector reaction function). Associated with any feasible consumption pair *that is expected by all agents*, there is a set of proportions of the population willing to work hard. In other words, under shared beliefs about consumption allocations, there is a correspondence that maps from (w,W) to a set of qs. If the distribution function of the population across types exhibits no atoms, this correspondence is a continuous single valued function.

Proof: For any pair (w,W) expected by all agents, there are three possibilities. First, agents with any $i \in [0,1]$ choose to work hard. This will be the case if:

$$P(H).u(W,H) + (1-P(H)).u(w,H) - i.H \; > \; P(L).u(W,L) + (1-P(L)).u(w,L) - i.L \quad \text{for all } i \in [0,1] \tag{25}$$

Second, agents with any i choose to work little (the converse of (25) holds). Third, there is an $i \in [0,1]$ such that individuals with index i are indifferent [4].

$$P(H).u(W,H) + (1-P(H)).u(w,H) - i.H \; = \; P(L).u(W,L) + (1-P(L)).u(w,L) - i.L \quad \text{for an } i \in [0,1] \tag{26}$$

In the first two cases, the correspondence is indeed univocal. In the third case, individuals i might choose any probability of working hard. However, if the distribution function exhibits no atoms, individuals of type i are negligible and the correspondence is again single valued. This completes the proof of the existence of the private sector reaction function.

Continuity is obvious in the first two cases. In the third one, equation (26) implies the existence of a continuous function i(w,W) (implicit function theorem). Finally, if there are no atoms in F(i), there is a continuous function mapping from i(w,W) to q and hence from (w,W) to q. To see why, notice first that individuals with indexes $i' \in [0,i(w,W))$ strictly prefer to work hard while individuals with indexes $i'' \in (i(w,W),1]$ strictly prefer to work little. Thus,

$$q = \int_0^{i(w,W)} dF(i') = F(i(w,W)) \tag{27}$$

which was assumed continuous (no atoms). QED.

It is worth emphasizing that the existence of a private sector reaction function hinges on the assumption that all agents expect the same consumption pair.

2.3. - The equilibria

A **discretionary equilibrium** is a set of consumption allocations and individual probabilities of working hard such that: i) both the government and private agents are optimizing, taking others' strategies as given; and

[4] With the utility function (1) there cannot be more than one i such that, with a given pair (w,W), the individual is indifferent between high and low effort. This uniqueness is not necessary true for more general utility functions.

ii) private agents' forecasts about other agents choices are on average correct. Condition ii), in particular, implies shared beliefs about the proportion of hard workers in the population and, therefore, about (w,W).

Corollary of propositions 1 and 2: Under the maintained assumptions of shared beliefs and no atoms in the distribution of the population across types, there is a continuous function mapping from the proportion of individuals that work hard (q) to itself. The **discretionary equilibria** of the game must be fixed points of this mapping. Brouwer's fixed-point theorem implies that at least one equilibrium exists.

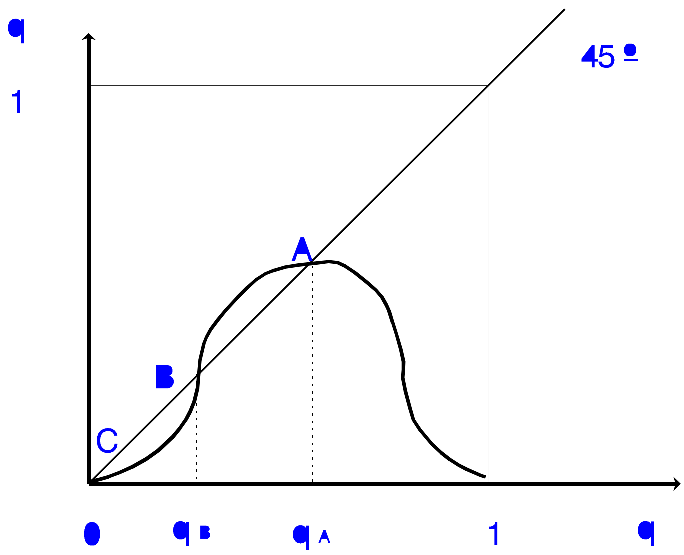
Proposition 3: The discretionary policy game always exhibits an equilibrium with full insurance and all agents putting in low effort (zero probability of working hard). There is no equilibrium in which all agents work hard. This is true for any distribution of the population across types, including the cases in which there are atoms.

Proof: If all agents did the same, either choosing low or high effort with certainty, the government would provide full insurance. Indeed, equations (4) and (5) imply that the government provides full insurance (w = W) whenever the proportion of individuals working hard (q) is either 0 or 1. But if the government provided full insurance, all agents would choose low effort. This completes the proof. Notice that it was not necessary to make any special assumption about the distribution function F(i). **QED**.

Proposition 4: The discretionary policy game *might* exhibit equilibria with incomplete insurance and some, but not all, agents working hard. If the population is completely heterogeneous, in the sense that F(i) exhibits no atoms, the incomplete-insurance equilibria *must* be in pure strategies -save for a negligible part of the population that might play mixed strategies. If instead the population is not completely heterogeneous, the incomplete-insurance equilibria *might* be in mixed strategies. In particular, if the population is completely homogeneous, i.e. the whole probability mass is concentrated in one index value, the incomplete-insurance equilibria might have the whole population randomizing.

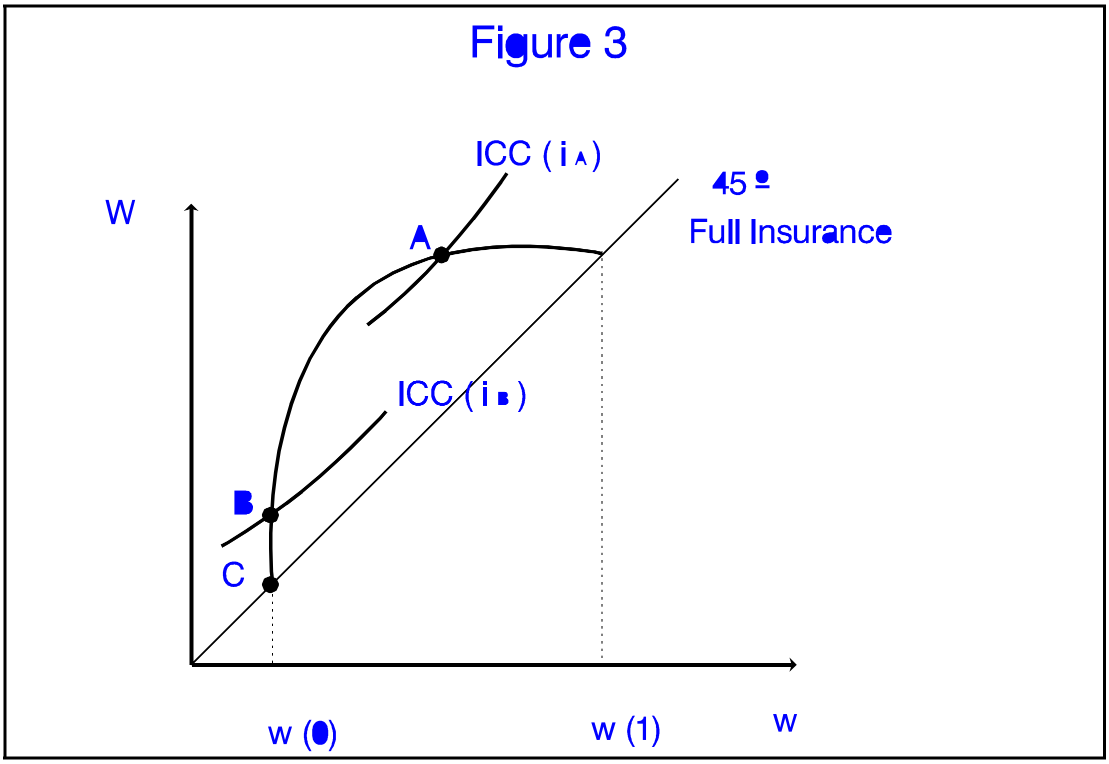
Proof: The first part of the proposition establishes just the possibility that incomplete-insurance equilibria exist; thus, it is enough to show an example. Consider the case of a distribution function with no atoms. Then, the corollary of propositions 1 and 2 says that there is a continuous mapping from q to itself. The equilibria in pure strategies mentioned in this proposition correspond then to the crosses of this mapping with the 45º line for positive values of q. This function might lie below the 45º line for all q other than 0, in which case the game has only one equilibrium. Figure 2 presents an example with three equilibria.

Figure 2



There might be some individuals randomizing, but in equilibria like A and B in figure 2, they represent a negligible part of the population. Almost the whole population is playing pure strategies.

The example depicted in figure 2 can be conveniently represented in the (w,W) space (see figure 3). The 45º line is the full insurance line. For an individual of type i, there is a set of (w,W) for which the individual is indifferent between both effort levels. This set is called the ICC-line of individuals of type i (for the incentive compatibility constraint holds as an equality in that locus). Agents choose to work hard when they expect consumption pairs located to the left of their respective ICC-line. It can be shown that ICC-lines of individuals with lower i are located to the right. An A-equilibrium is represented by the cross of the ICC-line of type i^A with the GRF in the point $q^A = F(i^A)$. A B-equilibrium has a smaller proportion of the population working hard, ($q^B<q^A$), with which an $i^B < i^A$ is associated. Point C is the full-insurance equilibrium in which nobody work hard.



If the population is homogeneous, either all individuals strictly prefer an effort level or all individuals are indifferent. In the first case, by proposition 3, only low effort is an equilibrium. In the second case, being all individuals indifferent between both effort levels, all of them might choose mixed strategies. **QED**.

An interesting point about proposition 4 is that a bit of heterogeneity in the population allows for equilibria **in pure strategies** with more than minimum effort. Forteza (1994) has already shown that, with an homogeneous population, there might be multiple equilibria. But all the equilibria with more than minimum effort are then in mixed strategies, and this is an equilibrium concept that has been heavily criticized (Rubinstein, 1991). The equilibria of proposition 4 are free from this criticism.

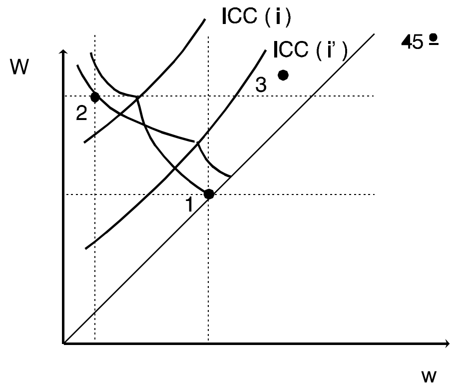
Proposition 5: The game exhibits only one equilibrium - the low-effort-full-insurance equilibrium of proposition 3 - if either the utility function is additively separable in consumption and effort or, being non separable, still the marginal utility of consumption is never higher, for each consumption level, when agents worked hard than when they worked little.

Proof: If the utility function is additively separable, the GRF lies on the full-insurance line (see proposition 1 iv). If the marginal utility of consumption is never higher for a hard worker, the GRF lies to the east of the full-insurance line (see proposition 1 v). In both cases, the mapping introduced in the corollary of propositions 1 and 2 is a flat line that coincides with the horizontal axis. Thus, there cannot be equilibria other than the full-insurance one. **QED.**

Proposition 6: Welfare comparisons. If the population is homogeneous, the equilibria are Pareto rankable from an ex-ante perspective. If the population is heterogeneous, equilibria might not be fully Pareto rankable. A sufficient condition for an equilibrium to be Pareto superior to another equilibrium is that consumption is not smaller in any state of nature. Thus, a sufficient condition for equilibria to be fully Pareto rankable is that the government's reaction function is monotonically increasing in q. Necessary and sufficient conditions for monotonicity are provided in proposition 1 viii. With monotonicity, equilibria with higher effort (higher q) are Pareto superior.

Proof: If the population is homogeneous, all agents get the same expected utility in equilibrium. It follows that equilibria must be Pareto rankable in this case. If the population is heterogeneous, some agents might prefer an equilibrium while others prefer another equilibrium. For an example, see figure 4 below.

Figure 4



Agent i prefers point 1 to point 2, while agent i' prefers point 2 to point 1. However, both prefer point 3 to either 1 or 2. Indeed, point 3 includes no less than points 1 and 2 in each state of nature. Any point to the northeast of a consumption allocation in the (w,W) space is preferred by everybody. This is also true for an equilibrium allocation. Finally, if the government reaction function gradient is non negative, equilibria with higher proportion of the population working hard yield higher consumption in each state of nature. Thus, in this case, all equilibria are rankable, being preferred those with higher effort. **QED**

The Pareto dominated equilibria are coordination failures à la Cooper and John (1988). All agents would be better off if they could coordinate on an equilibrium with higher effort, but no player in this game can do that, not even the government. The multiplicity of equilibria stems from strategic complementarity that goes through bayesian inference. When someone works harder, the likelihood that each one has worked hard conditional on his output rises. The government might then change the consumption allocation in such a way that the expected marginal utility of effort rises. Thus, one agent working hard might provide incentives for others to do the same.

Proposition 7: There is a neighborhood of the low-effort-full-insurance equilibrium in which all agents put in low effort. In other words, if q is close to zero, the consumption pair chosen by the government will not induce any agent to put in high effort.

Proof: Equations (4) and (5) imply that:

$$\lim_{q \to 0} W - w = 0 \qquad \textbf{(28)}$$

Equation (1) implies that all agents - even those with i = 0 -
require that consumption in the good state of nature is strictly larger than in the bad state in order to put in high effort. Thus, there must be a positive q such that for any $q' \in [0,q]$ all agents choose minimum effort. **QED**.

Corollary: The mapping from q to itself lies below the 45º line in a neighborhood of the origin (figure 2).

Proposition 3 implies that the mapping from q to itself introduced in the first corollary includes points (0,0) and (1,0). The first point establishes the existence of the low- effort-full-insurance equilibrium and the second establishes the impossibility of a high-effort-full-insurance equilibrium. It was shown, in proposition 4, that there might be other equilibria with a proportion of the population working hard strictly larger than zero and lower than one. Proposition 6 contains the conditions for the equilibria to be Pareto rankable. Proposition 7 and its corollary will prove important for the stability of the full-insurance-low-effort equilibrium.

3. A model with costly welfare policies

It has been assumed so far that welfare policies are costless. In the real world, however, there are costs associated with redistributive policies. The analysis presented in this section suggests that the costs of implementing welfare policies might have important consequences for the performance of the welfare state. Not surprisingly, as in other policy games, the existence of costs of changing government decisions might be welfare improving, since they enhance the commitment ability of the government. Also, these costs might provide a simple explanation for the fact that, in the real world, governments do not fully eliminate disparities between the lucky and the unlucky. Even in highly distorted economies, corresponding to the low-effort equilibrium of the previous section, private agents would face some degree of risk. Yet, there might be overinsurance in the sense that agents are facing less risk than what is ex-ante socially optimal. Less obvious, the fact that welfare policies are costly constitutes a source of coordination failures in the welfare state. There might be "good" equilibria in which agents work hard, average performance is good, and a small amount of resources is spent in welfare policies, and other "bad" equilibria in which agents work little, there is a higher number of unlucky, and thus a relatively large amount of resources is spent in the welfare state.

Assume that a proportion 'c' of the redistributed income is dissipated in the process. The whole economy resources constraint is then: [5]

$$N.X + (1-N).x - c.[N(X-W) + (1-N)(w-x)] \geq N.W + (1-N).w \qquad \textbf{(29)}$$

where: $0<c<1$.

The government maximizes (2) subject to (29). The first order conditions yield:

$$\frac{Prob(H/X,q).u_1(W,H) + (1-Prob(H/X,q)).u_1(W,L)}{Prob(H/x,q).u_1(w,H) + (1-Prob(H/x,q).u_1(w,L)} = \frac{1-c}{1+c} \qquad \textbf{(30)}$$

The left hand side in (30) is the marginal rate of substitution between consumption in the good state and consumption in the bad state of nature *expected by the government*. The right hand side is the marginal rate of transformation implicit in the distribution technology. If the welfare policy were costless, the marginal rate of transformation would be one. Costly welfare policies imply that the pool of unlucky agents receive less than one additional unit of output per unit withdrawn from the lucky, i.e. the marginal rate of transformation is less than one. The larger the costs of the policy, the smaller the marginal rate of transformation, and thus the smaller the marginal rate of substitution. A smaller marginal rate of substitution means a larger gap between consumption of the lucky and the unlucky, i.e. less insurance. In summary, the government might provide incomplete insurance ex-post simply because providing insurance is a costly activity in the margin. [6]

The following proposition formally states and proves the above mentioned results. In order to fully separate this reason for incomplete insurance from the one analyzed in the previous section, it will be assumed that the utility function is additively separable. Thus, marginal utility of consumption of hard workers and soft workers is the same, and the government would not want to let the hard workers consume more than the soft workers, had the policy been costless. It is easy to extend the proposition to the more general case in which marginal utility of consumption of hard workers is not smaller than marginal utility of soft workers.

Proposition 8: Assume that the utility functions are additively separable in consumption and effort. Welfare

---

[5] The assumption that costs are proportional to the amount redistributed is not crucial for the results that follow. What is crucial is that the costs of the policy are non decreasing in "taxes" levied on the lucky (X-W) and on "subsidies" distributed to the unlucky (w-x), being increasing in at least one of them. Some of the results that follows would not hold if the costs of the welfare state were just fixed costs, independent of the amount redistributed. It is not difficult to analyze that case, but the one described in this section seems both more realistic and more interesting.

[6] Simple as it is, the point should not be oversimplified: the government might still provide full insurance with costly policies if all the costs were fixed. In this case, once the government has decided to incur in the costs of mounting the system, costs would not be a reason to provide less than full insurance.

policies are costly, so that the whole economy resources constraint is (29). Then, the discretionary policy involves incomplete insurance.

Proof: Using the separability assumption in (30):

$$\frac{u_1(W)}{u_1(w)} = \frac{1-c}{1+c} < 1 \quad \Rightarrow \quad w < W \tag{31}$$

QED.

Notice that inequality (31) holds for all N, so that the image of the government reaction function in the w-W space lies to the west of the full insurance line when welfare policies are costly (in the margin) and utility functions are additively separable. This observation is crucial, for it implies that multiple equilibria might arise, even when utility functions are additively separable. Furthermore, the new set of equilibria differs from the one analyzed in the previous section in some important aspects. As it was already pointed out, there is no longer an equilibrium with full insurance. Also, there might be an equilibrium in which everybody work hard, i.e. a pure-strategies-high-effort equilibrium, something that was ruled out in the case of costless welfare policies.

In order to explore some of these possibilities, keeping the analysis relatively simple, it will be assumed in the rest of this section that the population is homogeneous and the utility functions are of the constant relative risk aversion type and separable:

$$u(w,a) = \begin{cases} \frac{w^{1-\gamma}}{1-\gamma} - a, & for \quad \gamma>0, \ \gamma\neq 1 \\ \ln w - a, & for \quad \gamma>0, \ \gamma = 1 \end{cases} \tag{32}$$

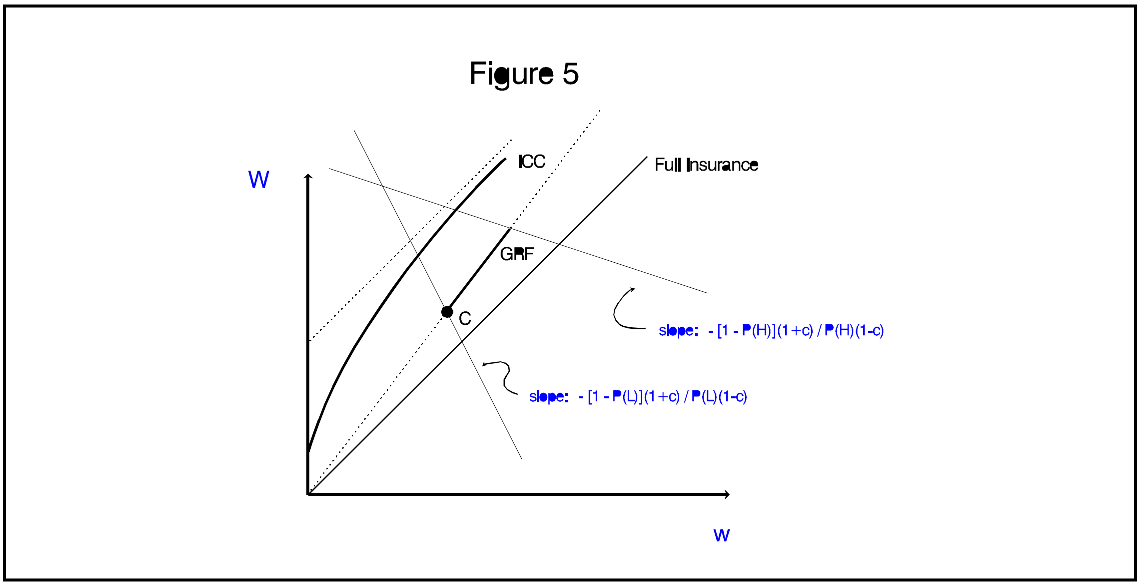
Explicit functional forms for the government reaction function (33) and the agents incentive compatibility constraints ((34) and (35)) are obtained for this case:

$$\frac{W}{w} = \left(\frac{1+c}{1-c}\right)^{\frac{1}{\gamma}} > 0 \tag{33}$$

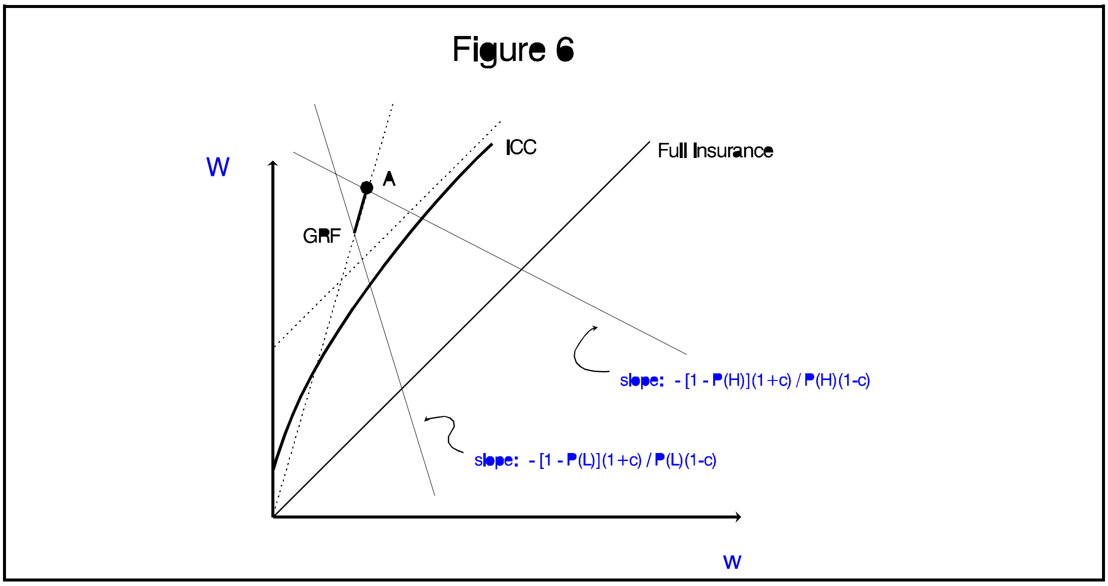
$$W = \left[w^{1-\gamma} + (1-\gamma)\frac{H-L}{P(H)-P(L)}\right]^{\frac{1}{1-\gamma}}, \quad for \quad \gamma\neq 1 \tag{34}$$

$$W = w + e^{\frac{H-L}{P(H)-P(L)}}, \quad for \quad \gamma = 1 \tag{35}$$

Some possible equilibria configurations are presented below: [7]
i) One equilibrium in pure strategies, low effort and incomplete insurance.



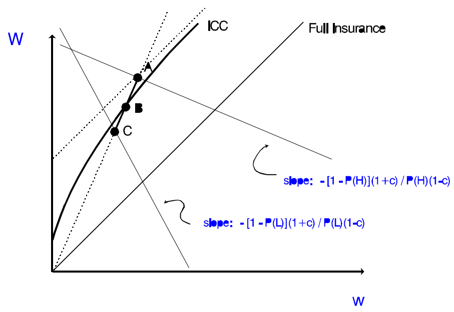
ii) One equilibrium in pure strategies, high effort and incomplete insurance.



[7] The three examples assume $\gamma < 1$. The incentive compatibility constraint is a straight line when $\gamma=1$ and it is convex when $\gamma > 1$. Other configurations of the equilibria arise in these cases.

iii) Three equilibria. 'A' is an equilibrium in pure strategies and high effort. 'B' is an equilibrium in mixed strategies. 'C' is an equilibrium in pure strategies and low effort.

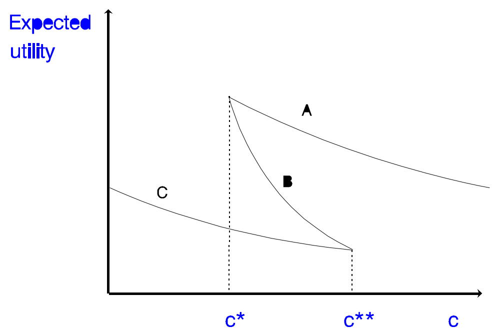
Figure 7



By virtue of proposition 6, these three equilibria are Pareto rankable, with 'A' dominating 'B', and 'B' dominating 'C'. Thus, 'B' and 'C' can be typified as coordination failures.

Which of these three possible equilibria configurations arises depends on the parameter 'c' (maintaining the assumption of CRRA utility functions with γ less than 1). It can be shownthat expected utility in equilibrium is a set valued function of the parameter 'c', as represented in figure 8:

Figure 8



where c* and c** correspond to the bifurcations.

As in other policy games, making government actions costly might be welfare improving. For instance, expected utility can be raised by an increase in 'c', starting in the low equilibrium associated with c=c**. But if the cost 'c' is made larger, expected utility decreases.

These results are summarized and proved in the following proposition.

Proposition 9: Assume the population is homogeneous and the utility function is both additively separable in consumption and effort and of the constant relative risk aversion type with elasticity of substitution larger than 1 (equation (32), $\gamma<1$). Assume the point of no insurance (x,X) lies to the west of the ICC line. Then, the discretionary policy game with costly welfare policy exhibits no less than one and no more than three Nash equilibria, all of them with incomplete insurance. More specifically, there exist threshold values for the cost parameter, c* and c**, such that:
i) For 0 < c < c*, there is one equilibrium in pure strategies and low effort (figure 5).
ii) For c = c* and for c = c**, there are two equilibria in pure strategies, one with high and the other with low effort.
iii) For c* < c < c**, there are three equilibria: a pure-strategies-low-effort equilibrium, a mixed-strategies equilibrium and a pure-strategies-high-effort equilibrium (figure 7).
iv) For c** < c, there is one equilibrium in pure strategies and high effort (figure 6).
v) Expected utility *in each equilibrium* is a decreasing function of the cost parameter 'c'.
vi) For each cost 'c', equilibria are fully Pareto rankable, with higher expected utility associated with higher proportion of the population working hard.

Proof: Points (i) to (iv) summarize results presented in the previous discussion. Point (v) requires a separate proof. Expected utility in equilibrium can be written as:

$$L[w(c),W(c),c] = P(a)u(W(c))+(1-P(a))u(w(c))-a + \lambda.[(1-c)N(X-W(c)) + (1+c)(1-N)(x-w(c))]$$

where a = H, L. Notice that this expression is the lagrangian of the government's optimization program in equilibrium (it is not out of equilibrium, though). This is selfevident in pure strategies equilibria, for taking q=0,1 in the government's program yields the above expression. In the case of mixed strategies equilibria, this expression is obtained by substituting the individuals incentive compatibility constraints in the government's lagrangian. This observation allows to use the envelope theorem, so that indirect effects of changes in 'c' going through w and W can be disregarded:

$$\frac{dL}{dc}(w(c),W(c),c) = -\lambda.[N(X-W) + (1-N)(w-x)] < 0$$

Finally, point (vi) follows from the monotonicity of the government reaction function (see proposition 6). **QED.**

The model provides a simple story for development traps in middle and high income countries. Assume that in an initial state of development, welfare policies are highly costly (c>c**). As the country develops, the government manages to provide less costly welfare systems and welfare increases. But, as the marginal costs of the policy approach the critical value c**, the possibility arises that further reductions in these costs provoke a decrease in welfare. If 'c' continues decreasing and reduces below c*, the risk of a welfare loss becomes a certainty. Thus, sooner or later the country falls in a development trap. A distinctive feature of this story is that, unlike other development traps reported in the literature, the overinsurance trap requires some degree of development to take place. The state must be relatively sophisticated for people to feel safe under its protection, something that looks unlikely in the poorest countries. Thus, overinsurance seems to be a middle and high income countries disease.

4. Dynamics with rational agents and frictions.

Economic policy and private economic activity in the real world are ongoing processes. Both the government and private agents are continually making economic decisions and their interactions develop dynamically. Thus, the question arises whether the one-shot game analyzed in the previous sections is an appropriate model of these activities. In this section, an extension of the basic model to a dynamic environment is presented. Policies are assumed costless and population is homogeneous. Most of the results of the one-shot game can be extended to the dynamic setting, while some new complementary results referring to stability and evolution of the welfare state are obtained.

In order to build a meaningful dynamic model, two points must be carefully reconsidered: timing and objectives. In the static game, it was assumed that private agents play simultaneously first and the government plays afterwards. Agents decide how much effort to exert, and afterwards the government redistributes the outcome. "Effort" here stands for a broad set of actions, but all of them share one important property: once agents have chosen actions, they must stick to them for some time. The government in turn has many instruments to affect individual income, so that by one way or another it can modify individuals disposable income relatively fast. This different velocity of reaction is captured in the one-shot game by assuming that the government plays after the private agents. Turning to dynamics, the natural way of modelling the same stylized facts is to allow the government to choose actions more often than private agents. This will remain a key assumption in what follows. There is nothing so fundamental instead about the assumption made in the static game that private agents play simultaneously. In fact, in the dynamic setting, staggering might be more realistic for many particular applications. Specially so since the welfare policy involves and links the whole society. Thus, in this section private decisions will be considered asynchronized.

The government chooses in each moment a pair (w_t,W_t) in order to maximize the present value of agents expected utilities in an infinite horizon. It is not myopic, but since it can pick a completely different consumption allocation in each instant, with no links between periods, its intertemporal program can be split up in a continuum of instantaneous static optimization programs (point wise optimization). As a consequence, the government reaction function presented in proposition 1 still holds in the dynamic setting.

It should be noticed, however, that in this version of the model the government is assumed to observe individual output along time while it is still rewarding agents according to current output. If it could check output in each instant, as it is assumed, it would be able to observe infinite realizations of the stochastic process that represents individual uncertainty in a non infinitesimal but arbitrarily small period. This observation period could be made as small as wanted to make the probability that a switch of effort has taken place within it arbitrarily small. Then, the government would be able to infer effort with almost certainty. This possibility has been assumed away in this paper, for the costs of recording that information would be extremely high. The fact still remains, however, that allowing the government to recall individual output might modify the dynamic of the model. If the assumption of perfect recall might be unrealistic, it should be acknowledged that the other extreme of no recall that has been adopted in this paper is not less so.

Following Matsui and Matsuyama (1995), it will be assumed that private agents must commit to actions for some time. Opportunities to switch actions arrive randomly, following Poisson processes with mean arrival rate 'm'. Thus, once an agent has made a choice, he will be "locked" to it for a period that lasts on average 'm'. Poisson processes are assumed independent across agents, so that there is no aggregate uncertainty. The strategy distribution in the society at time 't' can be described by 'q_t', the proportion of individuals committed to high effort at that time. Because of these assumptions, 'q_t' is continuous in time and its rate of change belongs to the interval [- $m.q_t$, $m(1-q_t)$]. The government in turn can switch actions continuously, and it has no commitment capacity at all [8].

Agents are assumed fully rational. Unlike in the recent related literature on evolutionary games, players are highly sophisticated in this model. [9] They maximize the present value of the whole stream of expected utility. Expectations are rational, and as for aggregate variables there is perfect foresight. Strategic behavior by part of private agents is assumed away: agents are too many and too small. The effects of individual decisions on 'q' is negligible by the same reason. And finally, there is no state variable to link periods. Still, current choices have lasting consequences, for agents cannot switch actions at every moment.

Consider an agent having the opportunity to choose effort in 't'. His instantaneous expected utility in 't' is given by [10]:

$$V(q_t,a_i) = P(a_i).u(W_{(q_t)},a_i) + (1-P(a_i)).u(w_{(q_t)},a_i)$$

The probability that he is still locked to that decision in 't+s' is: exponential(-ms). Thus, assuming a constant discount rate δ, the present value as of time 't' of the expected instantaneous utility in 't+s' *associated* with a choice made in 't' is:

[8] This is an extreme assumption indeed. But it seems a useful one, for the point is precisely to show some of the possible consequences of the lack of commitment ability. In the same spirit, it is assumed in this section that welfare policies are costless.

[9] G. Mailath (1992) emphasizes as a distinctive feature of evolutionary games the assumption that players are not sophisticated, in the sense that they do not take into account all the consequences of their actions when making decisions. This means that agents are assumed boundedly rational (Sargent (1993)).

[10] For the sake of simplicity, population is assumed homogeneous.

$$V(q_{t+s},a_i).e^{-(m+\theta)s}$$

In picking an effort level, rational agents should consider the whole stream of expected utilities associated to it.

$$\underset{a_i \in (H,L)}{Maximize}\ V_t(a_i) = (m+\theta)\int_0^\infty V(q_{t+s},a_i).e^{-(m+\theta)s}.ds \tag{40}$$

Hence, q_t is an equilibrium path from q_0 if its right-hand derivative exists and satisfies

$$\frac{d^+q_t}{dt} \in \begin{bmatrix} m.(1-q_t) & if & V_t(H)>V_t(L) \\ [-m.q_t\ ,\ m.(1-q_t)] & if & V_t(H)=V_t(L) \\ -m.q_t & if & V_t(H)<V_t(L) \end{bmatrix} \tag{41}$$

for all $t\in[0,\infty)$. Equation (41) states that all agents currently exerting low effort (high effort), if given the opportunity, switch to high effort (low effort), when $V_t(H)>V_t(L)$ (<). Agents are indifferent and hence randomize, when $V_t(H) = V_t(L)$.

Proposition 10: q is a stationary state of the dynamics (40) and (41) if and only if q is a Nash equilibrium of the one-shot game.

Proof: i) The "if" part states that, whenever q is a Nash equilibrium of the static game, $\{q_t=q\ ,\ \forall\ t\}$ is an equilibrium path of the class of dynamics defined by (40) and (41). Along these paths, the present value of expected utilities associated with a choice is equal to the current expected utility:

$$V_t(a_i) = V(q,a_i).(m+\theta)\int_0^\infty e^{-(m+\theta)s}.ds = V(q,a_i)$$

From propositions 3 and 4, the Nash equilibria of the static game are q = 0 and $\{q^*/V(q^*,H)=V(q^*,L)\}$. In the first case, $\{q_t=0\ ,\ \forall\ t\}$ is indeed an equilibrium path, for:

$$V_t(H) = V(0,H) < V(0,L) = V_t(L) \Rightarrow \frac{d^+q_t}{dt} = -m.q_t = 0$$

In the case of mixed strategies equilibria, $\{q_t=q^*\ ,\ \forall\ t\}$ is also an equilibrium path of the dynamics (40) and (41), since:

$$V_t(H) = V(q^*,H) = V(q^*,L) = V_t(L) \;\Rightarrow\; \frac{d^+q_t}{dt} = 0$$

ii) ("only if") q_t is a stationary state if either: a) $q_t = 1$ and $V_t(H)>V_t(L)$; b) $q_t \in (0,1)$ and $V_t(H)=V_t(L)$; or $q_t = 0$ and $V_t(H)<V_t(L)$. But (a) cannot hold for:

$$q_t = 1 \;\Rightarrow\; V_t(H) = V(1,H) < V(1,L) = V_t(L)$$

The remaining candidates are the Nash equilibria of the static game. **QED**.

Definitions:

(i) $q \in [0,1]$ is **accessible** from $q' \in [0,1]$ if there is an equilibrium path from q' that converges to q. q is **globally accessible** if it is accessible from any $q' \in [0,1]$.

(ii) $q \in [0,1]$ is **absorbing** if there is a neighborhood of q, Q, such that any equilibrium path from Q converges to q.

(iii) a closed invariant set $A \in [0,1]$ is an **absorbing or attracting set** of the dynamics (40) and (41) if there is some neighborhood Q of *A* such that any equilibrium path from Q converges to *A*.

(iv) $q \in [0,1]$ is **fragile** if it is not included in any absorbing set. [11]

Proposition 11: If q=0 is the unique Nash equilibrium of the static game, then it is absorbing, globally accessible and the basin of attraction is the whole range [0,1].

Proof: If q=0 is the unique Nash equilibrium, then $V(q,H)<V(q,L)$ for all q and $V_t(H)<V_t(L)$. Thus, the equilibrium path from any $q \in [0,1]$ is in fact unique, and is represented by the following expression:

$$q_{t+s} = q_t.e^{-ms} \quad \textbf{(46)}$$

so that it approaches zero as time goes to infinite. **QED.**

Proposition 12: When the static game exhibit multiple equilibria, there are positive thresholds m*(θ), m**(θ) and θ(m) such that q = 0 is absorbing if any of the following conditions is fulfilled: (i) $m \in (0,m^*(\theta)]$, (ii) $m \in (m^{**}(\theta),\infty)$, (iii) $\theta>\theta^*(m)$.

Proof: if $V_t(H)< V_t(L)$ for all feasible q_t in a neighborhood of 0, then the equilibrium path would be unique and would converge to q = 0 for q_0 sufficiently close to 0. The strategy of the proof is to show that this is indeed the case when any of the above mentioned conditions is fulfilled.

[11] Definitions (i) and (ii) are due to Matsui and Matsuyama (1995). Definitions (iii) and (iv) are extensions, based on standard theory of dynamic systems (Perko, 1991).

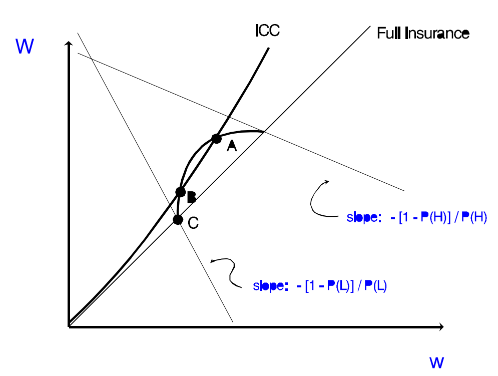
The highest accessible q_{t+s} from q_t is:

$$q_{t+s} = 1 - (1-q_t).e^{-ms}$$

$$s^{-}_{B,t} = \ln\left(\frac{1-q_B}{1-q_t}\right)^{-\frac{1}{m}}$$

Therefore, the minimum time ($s^{-}_{B,t}$) in which a feasible path from q_t in the low-effort region leaves the region is:

Figure 9

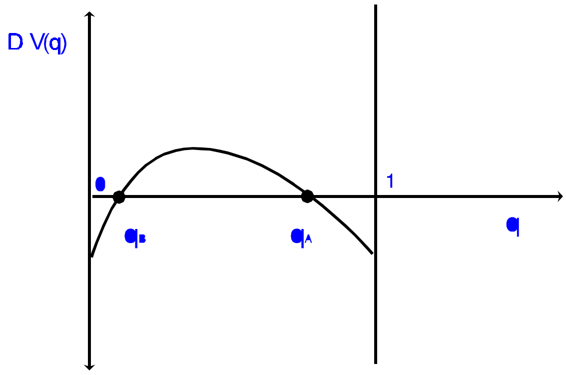


Then, the above mentioned condition can be written as:

$$V_t(H) - V_t(L) = (m+\theta).\left[\int_0^{s^{-}_{B,t}} \Delta V(q_{t+s}).e^{-(m+\theta).s}.ds + \int_{s^{-}_{B,t}}^{\infty} \Delta V(q_{t+s}).e^{-(m+\theta).s} \right. \qquad \textbf{(49)}$$

where $\Delta V(q_{t+s}) = V(q_{t+s},H) - V(q_{t+s},L)$. The function $\Delta V(q_{t+s})$ corresponding to the case depicted in figure 9 is presented in figure 10.

Figure 10



The first integral in the RHS of (49) is strictly negative for q_t close enough to 0. Moreover, it is strictly smaller than the initial expected utility differential, which is already negative:

$$(m+\theta).\int_0^{s_{B,t}^-} \Delta V(q_{t+s}).e^{-(m+\theta)s}.ds < (m+\theta).\Delta V(q_t) < 0$$

The second integral in the RHS of (49) might be positive, but it is "small" for some values of m and θ. Indeed, $\Delta V(q_{t+s})$ is bounded, so that the supremum exists; call it ΔV. Then:

$$(m+\theta).\int_{s_{B,t}^-}^{\infty} \Delta V(q_{t+s}).e^{-(m+\theta)s}.ds < \Delta V.\left(\frac{1-q_B}{1-q_t}\right)^{\frac{m+\theta}{m}}$$

and q = 0 is absorbing if:

$$V_t(H) - V_t(L) < (m+\theta).\Delta V(q_t) + \Delta V.\left(\frac{1-q_B}{1-q_t}\right)^{\frac{m+\theta}{m}} < 0 \qquad \textbf{(52)}$$

for any q_t in a neighborhood of 0. This inequality must hold for at least some values of m and θ. Indeed, it is easy to show that:

(i) for each θ>0, there are m*>0 and m**>0 such that for all $m' \in (0,m^*(\theta)]$ and for all $m'' \in (m^{**}(\theta),\infty)$ inequality (52) holds for any q_t in a neighborhood of 0, i.e. q = 0 is absorbing;

(ii) for each m>0, there is a θ*>0 such that for all θ>θ*(m) inequality (52) holds for any q_t in a neighborhood of 0, i.e. q = 0 is absorbing. **QED.**

There might be other absorbing equilibria. However, when the low-effort equilibrium is globally accessible, other equilibria are necessarily fragile. Proposition 13 provides sufficient conditions for the low-effort equilibrium to be globally accessible (necessary conditions for mixed-strategies equilibria not to be fragile). For the sake of simplicity, proposition 13 and the following ones will be referred to the case represented in figures 9 and 10. Nevertheless, extensions to other cases with more Nash equilibria in the static game are straightforward.

Proposition 13: Consider the static game represented by figures 9 and 10. q=0 is globally accessible if:

$$\int_{q_B}^{q_A} \Delta V(q).q^{\delta}.dq < -\int_{0}^{q_B} \Delta V(q).q^{\delta}.dq \qquad \textbf{(53)}$$

where: δ = θ/m = degree of friction.

Proof: A proof of this proposition consists in showing that the feasible trajectory (46) from $q_t = 1$ is an equilibrium path of the dynamics (40) and (41), if condition (53) holds. First, notice that the discounted sum of expected utilities differential along such path can be rewritten as:

$$V_t(H)-V_t(L) = (1+\delta)q_t^{-(1+\delta)} \int_0^{q_t} \Delta V(q).q^{\delta}.dq$$

This orbit will be an equilibrium path if $V_t(H)- V_t(L)\leq 0$ for all $q_t \in [0,1]$. Consider qs in each of the following two regions $[0,q_B]$, and $(q_B,1]$.

i) $q_t \in [0,q_B]$. The condition $V_t(H)- V_t(L)\leq 0$ necessarily holds, for $\Delta V(q)\leq 0$ in this region (see figure 10).

ii) $q_t \in (q_B,1]$. Now:

$$V_t(H)-V_t(L) = (1+\delta)q_t^{-(1+\delta)} \int_0^{q_t} \Delta V(q).q^{\delta}.dq \leq (1+\delta)q_t^{-(1+\delta)} \left[\int_0^{q_B} \Delta V(q).q^{\delta}.dq + \int_{q_B}^{q_A} \Delta V(q).q^{\delta}.dq \right]$$

which is non-positive if condition (53) holds. **QED.**

Proposition 14: Consider the static game represented by figures 9 and 10. The Nash equilibrium q_B is fragile.

Proof: Consider the paths represented by (46) with $q_t \in [0,q_B]$. They converge to q=0 as time goes to infinite. These are equilibrium paths, for they satisfy the condition $V_t(H)- V_t(L)\leq 0$. Thus, q_B is fragile. **QED.**

Proposition 15: The Nash equilibrium q_A is in the interior of an absorbing set if the degree of friction (δ) is high enough. The absorbing set of q_A can be approached to a first order by the following expression:

$$\left[q_A - \frac{1-q_A}{1+\delta} \; , \; q_A + \frac{q_A}{1+\delta} \right]$$

Proof: In a neighborhood of q_A, the instantaneous expected utilities differential can be approached by a linear expression:

$$\Delta V(q_t) \approx \Delta V'(q_A).(q_t - q_A)$$

where:

$$\Delta V'(q_A) = \frac{d\Delta V(q_A)}{dq_t} < 0$$

This last inequality corresponds to figure 10. The sign of this derivative defines in general the A-type equilibria, while the opposite holds for the B-type equilibria.

Any feasible path from q_t satisfies:

$$q_t.e^{-ms} \leq q_{t+s} \leq 1-(1-q_t).e^{-ms}$$

If q_t is not too far from q_A , then:

i)

$$\Delta V_t = -(m+\theta)\int_0^\infty \Delta V'(q_A).(q_A - q_{t+s}).e^{-(m+\theta)s}ds \leq$$

$$\leq -(m+\theta)\int_0^\infty \Delta V'(q_A).(q_A - q_t.e^{-ms}).e^{-(m+\theta)s}ds = -(m+\theta).\Delta V'(q_A).\left(\frac{q_A}{m+\theta} - \frac{q_t}{2m+\theta}\right)$$

and thus:

$$\Delta V_t \leq 0 \;\Rightarrow\; \frac{d^+q_t}{dt} \leq 0 \quad if \quad q_A + \frac{q_A}{1+\delta} \leq q_t$$

ii)

$$\Delta V_t = -(m+\theta)\int_0^\infty \Delta V'(q_A).(q_A - q_{t+s}).e^{-(m+\theta)s}ds \geq$$

$$\geq -(m+\theta)\int_0^\infty \Delta V'(q_A).(q_A - 1 + (1-q_t).e^{-ms}).e^{-(m+\theta)s}ds = -(m+\theta).\Delta V'(q_A).\left(-\frac{1-q_A}{m+\theta} + \frac{1-q_t}{2m+\theta}\right)$$

and thus:

$$^{a}V_t \ \$ \ 0 \quad Y \quad \frac{d^{\%}q_t}{dt} \ \$ \ 0 \quad if \quad q_t \ \# \ q_A \ \& \ \frac{1\&q_A}{1\%\delta}$$

If δ is large, the absorbing set is a small region around q_A, so that the linearization of $^{a}V(q_t)$ is a good approximation to the non-linear function in a neighborhood of this region. **QED.**

This proposition is rather vague in that the threshold δ is not determined, and the basin of attraction has unknown limits. But, on the other hand, it is very general: for any utility function such that mixed-strategies equilibria exist, there are degrees of friction that make an A equilibrium part of an absorbing set. Besides, it is not difficult to determine sufficient conditions for a certain region around q_A to be in the basin of attraction. It is straightforward to show, for instance, that when δ6 4, q_A becomes absorbing and the basin of attraction is $(q_B,1]$. However, no attempt is made in the present paper to provide more precise quantitative results on the stability of the equilibria, for it seems likely that these results will be very sensitive to specificities that will vary widely between different applications of the general model.

5. Summary and conclusions

1. A government that does not have the ability to commit to an incomplete insurance scheme might provide too much insurance, provoking more distortions than what would be ex-ante optimal. Optimizing ex-post, the government does not take into account incentive considerations when choosing the redistribution schemes. This is the basic source of the overinsurance. Still, it might provide incomplete insurance for reasons other than incentives. In the present paper, two different complementary stories that account for partial insurance are proposed. The first one hinges on the possibility that the marginal utility derived from consumption be higher when agents worked hard. Then, if getting a high output is a "good" signal for the government, in the sense that it is more likely that an agent that got high output has worked hard, the government might let lucky agents enjoy higher consumption. The second explanation of incomplete insurance rests on the existence of costs of implementing economic policies. If government intervention is a costly activity, the government might not provide full insurance simply because it is expensive to do it. These two "forces" might thus alleviate the credibility problem of a benevolent government that lacks a "commitment technology".

2. Both differences in marginal utility of hard and soft workers, and costs of implementing policies, can separately cause not only incomplete insurance, but also multiple equilibria. Assuming costless policies, the effects of marginal utility differentials on equilibria configuration were analyzed in the first section of this paper. Under these assumptions, there is a minimum-effort-full-insurance equilibrium in which realized output conveys no information about effort and consequently the government does not make any difference between lucky and unlucky. But there might also be equilibria in which high output is a good signal and thus the government associates higher consumption to higher output.

3. If the population is assumed homogeneous, and policies are costless, the above minimum effort equilibria are

in mixed strategies. This concept has been heavily criticized (Rubinstein, 1991), and indeed in many economic applications mixed strategies equilibria look highly unlikely. Nevertheless, it is argued in the paper that the above minimum effort equilibria should not be ruled out in the context of the costless welfare policy game. On one hand, it is shown that there might be more than one pure-strategies equilibrium, if the simplifying assumption of homogeneous population is dropped. Allowing for a bit of heterogeneity is enough to generate the possibility of multiple equilibria in pure strategies. On the other hand, it is shown that even if the population is assumed homogeneous, some mixed strategies equilibria might be "robust", looking as plausible -and even likely- outcomes of the welfare policy game. Indeed, in the present context, unlike in most other economic applications, the concept of mixed strategies has a simple interpretation as distributions of pure strategies in the population. Thus, in this model, mixed strategies equilibria are immune against 'Rubinstein critique'. Furthermore, some mixed strategies equilibria might attract equilibrium paths in a dynamic setting with frictions, so that it would not be unlikely to observe the economy around them.

4. The dynamic of strategies was analyzed in a setting similar to that in Matsui and Matsuyama (1995). If frictions are not too large, only pure strategies equilibria can be absorbing. Other equilibria are fragile, and can thus be ruled out as unlikely. But there might be more than one absorbing set in the presence of large enough frictions. Then, the distribution of pure strategies in the population would evolve towards these sets, and would not depart much from them, if the previous history were not too far from them. In this case, the dynamic with frictions is unable to single out one equilibrium, but it still provides a history-dependent criterion for equilibrium selection. In this respect, the analysis has the same flavor of the selection criterion recently proposed by Cooper (1994).

5. Costly policies might significantly alter the equilibria configuration of the welfare policy game. The low-effort equilibrium in pure strategies will still exist, though associated now with incomplete insurance, if marginal cost of the policy is not too high. Equilibria with above minimum effort arise when the policy costs exceed a threshold. Unlike in the costless policy game analyzed before, there is now an equilibrium in which everybody work hard, i.e. a high-effort equilibrium in pure strategies. If the costs of the policy are too high, the low-effort equilibrium might collapse.

6. The equilibria of the welfare policy game might be Pareto rankable, so that there is a meaningful sense in which it can be said that some equilibria are better. Moreover, under some plausible conditions, all the equilibria are Pareto rankable, the higher average effort the better. In this case, the static welfare policy game is a coordination game, and low-effort equilibrium is a coordination failure in the sense of Cooper and John (1988).

7. The above mentioned results suggest that multiplicity of equilibria in the welfare policy game might capture the diversity observable in the performance of different countries in the real world. Some countries have been "more successful" than others in implementing the welfare state, and they have been so for several decades. Accordingly, some equilibria involve less distortions than others, and both might be stable. Thus, multiplicity of equilibria, far from being a weakness of the model or the result of an incomplete theory, seems to play an important positive role in capturing regularities present in the real world. It seems possible, for instance, to interpret the overinsurance equilibrium as a development trap. Interestingly enough, this trap would be more likely in middle and high income countries than in the poorest ones. The latter would be relatively immune against this "disease", simply because governments would not be able to help people that have fallen in disgrace.

8. No attempt is made in this paper to empirically test the model, but it is suggested that plausible "stories" that resemble the actual experience of some countries can be told using relatively simple versions of the model. It seems possible, for example, to reproduce in this framework some of the striking singularities of the modern history of countries like Argentina and Uruguay.

9. Needless to say, there are sources of multiple equilibria and coordination failures in the welfare state that are not analyzed in the present paper. One potentially important and worth mentioning here is the government's monitoring capacity. An interesting example is provided by Ljundquist and Sargent (1994). Analyzing unemployment benefits in Sweden, they argue that government's control over the unemployed was a major factor in the low rate of unemployment prior to the crisis in the early 1990s. Afterwards, when unemployment rates rose significantly, the ability of the government to monitor unemployed decreased. Thus the incentives for unemployed to actively search for jobs and accept offers reduced when the rate of unemployment increased.

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