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MODELLING DYNAMIC STORAGE FUNCTION IN COMMODITY MARKETS: THEORY AND EVIDENCE

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Modelling Dynamic Storage Function in Commodity Markets: Theory and Evidence

Luca Pieroni [(*)] ***and Matteo Ricciarelli*** [(**)]

Abstract

Stockholding decisions in agricultural commodity markets represent a source for strengthening risk management techniques related to future markets development. In this work, we propose a generalised dynamic approach to obtain a consistent stockholding decision rule, in which the cash and storage markets are modelled simultaneously. The qualitative dynamic investigation of the storage function is carried out by considering two heterogeneous categories of agents - processors and speculators – who are responsible of fluctuations of the spot price equation. Using the U.S. corn market data, empirical estimations are statistically robust and economically coherent with the theory.

JEL Classification Number: *G13; L71, Q40*

Keywords: Spot and Storage Markets, Convenience Yield, Inventories.

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1 Introduction

As a “stylized fact”, the mechanism that determines the spot price in agricultural commodity markets is affected by the length of the production process; in fact, the economic agents choose the production plans prior to the production yield, so that the expectation shaping mechanism is relevant for the investment decisions. Moreover, the industrial activity to transform the agricultural products adds precautionary behaviour, leading the economic agents to store inventories even if they do not have speculative opportunities.

Although we refer to the competitive storage model (Deaton and Laroque, 1992; Chambers and Bailey, 1996), we emphasize the role played by heterogeneous agents in determining the equilibrium in the storage market. In this paper, we use the short-run commodity market framework to investigate simultaneously the impact of a dynamic equilibrium model for prices and stocks. In analogy to dynamic investment theory developed by Tobin (1963) and Hayashi (1982), we make use of the discrete net change in stock as a co-state variable, in order to obtain the dynamic relationship between storage and cash markets. As will be clarified in the remainder of the paper, to achieve a more realistic version of the storage function, we need to describe the heterogeneous behaviour of the agents, by invoking the co-existence of two categories, processors and speculators, which have different aims with regards to the stockholding accumulation activity. In addition to the speculative demand for stocks we also consider processors, whose inventories are accumulated for precautionary motives related to their activity[1]. These different motivations driving the storage behaviour are displayed by the storage market, in which the (unobservable) price could be extracted by the marginal storage value (λ), including the marginal convenience yield as a (dependent) variable which reflects the precautionary behaviour of economic agents (Pindyck,1993).

Our modelling strategy exhibits some attractive features. First, instead of modelling the depreciation rate of the storage as exclusively dependent on physical factors, more generally we consider it as an intertemporal depreciation rate of the storage affected by potential institutional changes. Moreover, we solve the link between cash and storage markets, by introducing a double constraint: *i)* a market clearing equation to obtain the spot price and *ii)* a motion law for inventories as the process which drives the accumulation in the storage market. Consequently, differently from Pindyck (2002), we propose to introduce a dynamic investment framework based on Tobin’s *q*, deriving consistent stockholding decision rule. Thus, our main results arise directly from the continuous optimisation program, in which the closed form for spot price and the marginal convenience yield equations are assessed by empirical estimation.

The rest of the paper is organized as follows. In Section *2*, the theoretical framework is presented by fully characterizing the agricultural commodity market, taking care to explain the different roles attributed to storage depreciation in our model. In Section *3*, the stockholding decisions are accounted for in a dynamic perspective. In particular, in the optimization problem is investigated in Section 3.1 while, in Section *3.2*, the dynamic interactions among state variables are exploited by phase diagrams. In Section *4*, the unknown parameters to model simultaneously spot price and marginal convenience yield equations are estimated. In Section *5* the data are described by checking the data generating process of the variables, in order to correctly specify the econometric model. The Three Stages Least Squares (*3SLS*) estimations for spot price and the marginal convenience yield equations and an assessment of the performances of the model to match the observed data are reported. The statistical robustness of the estimated parameters and the diagnostic tests allow us to derive a model which is coherent with the economic theory. Finally, in section 6 we present our concluding remarks.

2. The Theoretical Framework in Agricultural Commodity Markets

In this section we present the structural relationships which characterize a primary commodity market. Even if an unified model for either industrial or agricultural commodities could be desirable, the impossibility to specify a common model is claimed in several works. In particular, it is argued that both production and storage accumulation of a metal differ widely from the production and storage accumulation of an agricultural commodity. Comparing the features of both industrial and agricultural markets, Gilbert (1995, p. 390) pointed out that, in contrast to the metal markets, the production of an agricultural commodity is carried out over the time, whereas the consumption does not require to be planned in advance[2]. For these reasons, we focus our attention on the agricultural commodity markets and, consequently, the structural equations that will be presented in the remainder of the section aim to model this subset of primary commodities.
An interesting question to be investigated is what leads the stockholding decisions and how the storage affects simultaneously the spot price, since we are convinced that the inclusion of the information embedded into the agent behaviour in a *virtual* storage market can improve the model of fitting the observed spot price. In doing so, it seems to be appropriate to decompose the availability in Deaton and Laroque (1992) into two control variables - production and stock level - related to the spot and storage markets.
Theoretically, a profit maximization program, with respect to the production variable, determines the spot price in the cash market. In fact, this maximization, subject to the

market clearing constraint, ensures the derivation of the equilibrium price (Routledge, Seppi and Spatt, 2000). Conversely, the maximization with respect to the stockholding level and under the motion law for the storage, gives the expression for the storage marginal value, *i.e.* the shadow price of the storage market. In this section we analytically specify the microeconomic relationships in the cash market by explicitly investigating the fluctuations in the supply and/or demand side, while in next section we will show the convenience to store is not only determined by speculative opportunities, but the introduction of another equation allows us to capture also the economic and/or non-economic benefits which are obtained by holding a unit of storage.

Analytically, in order to link the maximization process to the classical storage model (Williams and Wright, 1991), we reproduce the functional physical components of market clearing and use production (*Q(t)*) to find a cost function by inversion. A theoretical approach is used to derive the supply function from behavioural relationships which describe the interactions and expectations of the production system. The set of behavioural relationships in the production from which we move is based on the following equation:

$$Q(t)^* = \left(HA(t)^*\right)\left(YD(t)^*\right) \tag{1}$$

The quantity produced stems from the interaction between the harvested area $\left(HA(t)^*\right)$ and the yield of that area $\left(YD(t)^*\right)$. In order to account for microeconomic foundations of production decisions, we take the logarithm transformation in (1) and separate the harvested area and yield (Chambers and Bailey, 1996). The following structural relationship is introduced to explain the underlying processes for the harvested area:

$$HA(t) = \ln\left(HA(t)^*\right) = \beta_0 + \beta_1 AP(t) + \upsilon(t) \tag{2}$$

where $AP(t)$ is the extension of the cultivated corn area in terms of planted area; the $AP(t)$ variable is assumed to be positively and linearly linked to the current prices of the own commodity ($Pm(t)$):

$$AP(t) = \ln AP(t)^* = \kappa_0 + \kappa_1 Pm(t) + \zeta(t) \tag{3}$$

Including (3) in (2), we get an explicative relationship for the harvested area:

$$HA(t) = \beta_0 + \beta_1\kappa_0 + \beta_1\kappa_1 Pm(t) + \beta_1\zeta(t) + \upsilon(t) \tag{4}$$

The yield equation $YD(t)^*$ is specified as follows:

$$YD(t) = \ln YD(t)^* = \omega_0 + \omega_1 Ctd(t) + \varpi(t) \tag{5}$$

where $\left(Ctd(t)\right)$ is a measure of the unit cost of inputs, while $\varpi(t)$ is the *i.i.d* stochastic component in which the weather index and the effects of technological changes in agricultural production are included.. The final specification of the production equation is obtained by including (4) and (5) in (1), reordering the parameters:

$$Q(t) = \ln Q(t)^* = \alpha^* + \beta_1^* Ctd(t) + \beta_2^* Pm(t) + \eta^*(t) \quad (6)$$

where the next equalities has been set:

$$\alpha^* = \beta_0 + \beta_1 \kappa_0 + \omega_0$$

$$\beta_1^* = \omega_1, \beta_2^* = \beta_1 \kappa_1$$

$$\eta(t) = \upsilon(t) + \beta_1 \zeta(t) + \xi(t) + \varpi(t)$$

Equation (6) is a well-done synthesis of two different processes of agricultural commodity production. On the one hand, we find that the allocation process of the farmer in 6 depends positively on the spot price of commodity while, on the other hand, the unitary cost of production reduces the supply potentiality. Therefore, it is worth noting that in this production equation the *i.i.d.* hypothesis survives in the $\eta(t)$ term[3].

Some important relationships are derived by net-export demand considering separately the export and import demands of the commodity[4]. We provide the equation describing the export demand:

$$X(t) = \psi_0 + \psi_1 EXC(t) + e(t) \quad (7)$$

We do not model the imports, *M(t)*, since they are negligible with respect to the exports for many U.S. commodity markets, such as the corn market. Therefore, the net-export equation coincides with the export demand equation and we can write the following behavioural relationship:

$$NX_t = X(t) - M(t) = \psi_0 + \psi_1 EXC(t) + e(t) \quad (8)$$

On the demand side, building up the model is relatively straightforward. A commonly adopted framework is to assume that consumption does not react to the expected price, but is much more sensitive to the current price. This hypothesis is due to the peculiarity of agricultural commodities for which the expectations driving the setup of production plans make most of the consumption plans inelastic. Therefore, processors relinquish a forward-looking perspective, choosing an optimal consumption period by period (Revoredo, 2000). The consumption equation is specified as follows:

$$Con(t) = \overline{Con} + \alpha_1 Pm(t) + u(t) \quad (9)$$

In such a specification, we assume that consumption fluctuates around a stable level $(\overline{Con})$ and deviations from that value are time justified by factors that depend upon either internal or external demand for the commodity. Moreover, it is worth noting that current price is a factor which simultaneously influences internal and storage demand and, hence, including stocks in this function seems to be redundant. Once consumption and net exports are defined, we can collapse these definitions into the demand:

$$D(t) = Con(t) + NX(t) = d_0 + d_1 Pm(t) + d_2 EXC(t) + v(t) \quad (10)$$

where:

$$
\begin{aligned}
d_0 &= \overline{Con} + \psi_0 \\
d_1 &= \alpha_1 \\
d_2 &= \psi_1 \\
v_t &= u_t + e_t
\end{aligned}
\tag{11}
$$

This variable is an indicator of the (internal and external) demand coming from the cash market.

The model is completed by the market clearing condition, which establishes the balance between the internal availability (Deaton and Laroque, 1992) and demand components: *i)* the cash market demand, *D(t)*, and *ii)* the storage market component, *FS(t)*.

The market clearing constraint is given by:

$$Q(t) + (1-\delta)FS(t-1) = D(t) + FS(t) \tag{12}$$

where δ represents the depreciation of the stock when we go from one period to another and/or directly linked to inefficiencies which arise contemporaneously with the carryover activity. Market clearing constraint ensures that the current production and incoming inventories (*i.e.* internal availability) balance time by time internal/external consumption and demand for storage. In previous works δ has been considered to be a measure of the physical storage depreciation or the waste related to spoilage in the agricultural commodity market crucially dependent upon technological and natural variables (Deaton and Laroque, 1992; Routledge, Seppi and Spatt, 2000). Although in crop field commodities (corn, soybean, wheat), the physical depreciation rate is not large, it cannot be neglected. Therefore, the assumption of a time-invariant δ in Deaton and Laroque (1992) seems to be unrealistic and prevents us from searching for the dynamics which drive stockholding decisions. An source of this time-changes, that affect the depreciation variable is due to the governance of the agricultural markets, *i.e.* institutional factors. In particular, we remark the U.S. Government has provided the farmers with a loan rate[5], so that the stockholding decision to accumulate inventories and the depreciation rate have been influenced. In the model the policy intervention are linked to the depreciation rate, since the different technologies in private and public stockholding functions lead to different levels of productivity, when the liberalization policies were introduced.

Once equipped with this theoretical background, we can specify the intertemporal depreciation rate of the storage as a function related to either technological or institutional factors:

$$\delta = \delta\left[STH(t), IF(t) \right] \tag{13}$$

where *STH(t)* and *IF(t)* are respectively the storage technology and the institutional factors. In particular, δ can be limited by the development of storage technology, whereas the institutional factors are modelled by the inclusion of the loan rate which is essentially a proxy of government intervention in the agricultural commodity markets.

3 Stockholding Decision in a Dynamic Perspective

In the last two decades, many studies have investigated the dynamics of commodity prices (Deaton and Laroque, 1992, 1996; Miranda and Rui, 1995; Ng and Ruge-Murcia, 1997, Pindyck**,** 1994; Gilbert**,** 1995). As pointed out, the stockholding decisions affecting the path of the spot price seems to be the cause of the poor fit performances of empirical models (Ng and Ruge-Murcia, 1997). A general specification of competitive storage model is used in empirical works the inclusion of the stock function inside of the availability and hiding the economic mechanism which leads the economic agents to the stockholding decisions.

The empirical pitfalls of competitive storage models is generated from the inclusion of the stock function inside of the availability variable since merging cash and storage markets avoids to discover how the supply-demand interaction takes place in the storage market. In particular, it prevents to discover the size of the precautionary behaviour of economic agents to process the primary commodity. Consequently, the use of the stock-outs as explanations of the evident difficulty of capturing the observed path of the spot price (Deaton and Laroque, 1992) is not completely correct because the stock-outs are themself an effect of a complex economic behaviour in the market, in which the cause is the precautionary intention of avoiding stock-outs. Thus, the exclusion of the precautionary motives could represent a source of the misspecification able to explain the empirical autocorrelation problems as in Deaton and Laroque (1992).

In contrast to this approach we focus our attention on the interaction between demand and supply in a *virtual* storage market simultaneously to the cash market. We relinquish the non-linearity framework to explain the spot price path, even if stock-out can be empirically justified by looking at more disaggregated data and we concentrate our attention on an extended model, in which the storage market is able to explain endogenously the path in the spot price. We admit the storage market is populated by agents that in addition to the speculative motive have behaviour related to precautionary motives of industrial consumers, since they want to reduce the cost, avoid stock-out and, generally, facilitate their own production processes (Pindyck, 1992).

The mechanism in action which leads the model can be summarized as follows: once taken the spot price, which is determined in the spot market by the interaction between demand and supply, the agents decide simultaneously their stockholding plans in the storage market. We remark that, even if the amount of storage is the only observable variable for the storage market, it contains heterogeneous stockholding behaviour which depends on the different aims of agents.

In order to derive our specification, we need of some assumptions on the spot market without restricting the generality of the model.

These assumptions concern the dual relationship between production (*Q(t)*) and cost function (*C(t)*) and the economic hypotheses about the market structure. First, we know that:

$$C(t) = C[Q(t)] \tag{14}$$

and, consequently:

$$\frac{\partial C}{\partial Q(t)} = C'[Q(t)] = mc(t) \tag{15}$$

where *mc(t)* is the direct marginal cost of production.

Second, it is reasonable to assume market competitiveness. In light of this, we find that the equilibrium spot market condition is provided by $mc(t) = Pm(t)$.

3.1 The Determinants of Stockholding Decision

In stockholding commodity markets, the uncertainty of production and transformation process plays a relevant role since it can modify the agent's decisions. Thus, analogously to the Net Present Value model (hereafter, *NPV*), an agent decides to invest or not by the comparison of the discounted benefit flows and the cost of the investment under uncertainty[6]. This principle has been used to investigate stockholding decisions; Revoredo (2000) summarizes that *the markets record positive storage only if the discounted expected price for the next period is greater than all the costs which an individual has to endure in order to store the commodity, i.e.* physical storage cost including the current price. Even if the inclusion of the current price may represent an attempt to model the opportunity cost, improving the econometric performance of this model with respect to the investment neoclassical models (Pindyck, 1991), this strategy is not enough to explain the stylized facts affecting both price and stock series (Ng and Ruge-Murcia, 1997).

In this section, we characterize a different approach to carry out optimal and dynamically consistent stockholding decision rule, which can be obtained from dynamic programming. As can be seen in the equations which underlie production, the economic agents can control the amount of the production through the cultivated area. In fact, even if it can also be affected by some unpredictable variables, such as the precipitation index, the production can be, at least, partially controlled by the agents. Moreover, by including an extended cost function, we can take both production and stockholding level as control variables. We remark that this model represents a break with respect to previous works (Deaton and Laroque, 1992; Gilbert, 1995), in which the availability has been considered such as a state variable.

The economic agents, under the previous competitive market conditions, maximize the profit function constraint at the market clearing condition. The objective function is given by:

$$\Pi\left(FS(t),Q(t)\right)=\int_{0}^{\infty}e^{-IRt}\left[Pm(t)D(t)-TC(t)\right] \tag{16}$$

where *TC* is the total cost function. Following Pindyck (2002), we specify the *TC* through the separation of three different components: *i)* production costs; *ii)* marketing costs, *i.e.* the cost of re-planning production to avoid stock-out and facilitate deliveries and *iii)* storage cost.

To figure out the production cost function, we recall the competitiveness of the market (15) and solve equation (6) with respect to *Pm(t)*. By this way, we derive an inverse function :

$$mc=Pm(t)=c_0+c_1Q(t)+c_2Ctd(t)+\eta(t) \tag{17}$$

in which:

$$c_0=\frac{\alpha^*}{\beta_2^*};c_1=\frac{1}{\beta_2^*};c_2=\frac{\beta_1^*}{\beta_2^*};\eta(t)=\frac{\eta^*(t)}{\beta_2^*}$$

By integrating the (17) with respect to *Q(t)*, we obtain the following production cost function which takes on the following quadratic form:

$$C\left(Q(t)\right)=\left(c_0+\eta(t)\right)Q(t)+\frac{1}{2}c_1Q(t)^2+c_2Ctd(t)Q(t) \tag{18}$$

in which $\eta(t)$ is a shock affecting the amount of fixed costs, *i.e.* it captures the technological shock in the cost function. *Ctd(t)* is the per unit production cost including the cost for wage, irrigation and fertilizers.

In our model, in order to describe the complementary behaviour of the processors, we term marketing cost ,*Φ(.)*. As reported above, this cost function typically reflects precautionary behaviour which characterizes the activity of processors with inelastic goods. The marketing cost function [$\Phi\left(FS(t),Pm(t)\right)$] has the following property:

$$\phi(t)=-\frac{\partial\Phi}{\partial FS(t)} \tag{19}$$

i.e. the marginal convenience yield (ϕ), which is the derivative of the marketing cost function with respect to the stock level with a change in the sign.

The storage cost function is assumed to be linear in the stock level (Miranda and Rui, 1997):

$$SC\left(FS(t)\right) = kFS(t) \tag{20}$$

where k is the cost of carry which is assumed to be invariant over time. The cost of carry (k) is a portion of the total storage cost and includes the physical storage cost (the per unit cost of the warehouse) and the foregone interest.

Therefore, collecting all the total cost function components, we can define:

$$TC(t) = C\left(Q(t)\right) + \Phi\left(FS(t), Pm(t)\right) + SC\left(FS(t)\right) \tag{21}$$

The maximization problem of the economic agent is:

$$\max \int_0^\infty e^{-IRt}\left\{Pm(t)D(t) - TC(t)\right\}dt \tag{22}$$

This is subject to two different constraints: the market clearing condition and the stock accumulation equation. The market clearing condition has already been presented among the structural equations, whereas the stock accumulation process is the following, where for brevity, we omit specifying the relationship for δ assumed in the above section:

$$F\dot{S}(t) = FS(t) - (1-\delta)FS(t-1) - \delta FS(t) \tag{23}$$

Such a formalisation[7] of the motion law for the stock level is worth looking at in depth. It tells us that the change in stocks is given by: *i)* the stockholding level at time *t* (*FS(t)*) which results from the agents' decision rule; *ii)* the net stock availability (*i.e.* initial condition) for period *t* (*(1-d)FS(t-1)*) and *iii)* the wasted stock ($\delta FS(t)$) related to inefficiencies in the stock carryover, *i.e.* amount of commodity which is ruined and/or wasted during the carryover[8].

In order to solve the maximization problem, we employ the Hamiltonian technique. The Hamiltonian for this task is:

$$H = e^{-IRt}\left\{\begin{array}{l}\left[\begin{array}{l}Pm(t)\left(Q(t) + (1-\delta)FS(t-1) - FS(t)\right) - (c_0 + \eta(t))Q(t) - \frac{1}{2}c_1 Q(t)^2 - \\ c_2 Ctd(t)Q(t) - \Phi\left[FS(t)\right] - kFS(t)\end{array}\right] + \\ \lambda(t)\left[FS(t) - (1-\delta)FS(t-1) - \delta FS(t)\right]\end{array}\right\} \tag{24}$$

Recalling equation (19), the first order conditions with respect to the production and stock level are:

$$\frac{\partial H}{\partial Q(t)} = 0 \Rightarrow Pm(t) = c_0 + c_1 Q(t) + c_2 Ctd(t) + \eta(t)$$
$$\frac{\partial H}{\partial FS(t)} = 0 \Rightarrow \lambda(t) = \frac{Pm(t) - [\phi(t) - k]}{1 - \delta(STH(t), Loan(t))} \tag{25}$$

The *F.O.C.* with respect to the stockholding level provides an expression for the marginal storage value ($\lambda(t)$) and is strictly linked to the spread between the current market price and the *marginal net convenience yield*. It represents the shadow price at which a stockholder is willing to sell a unit of storage and depends on the overall unit cost for stock (i.e. the price that an individual has to pay in order to purchase a bushel of commodity in the spot price plus the physical storage cost, k, and the marginal convenience yield which accounts for all the benefits arising from a unit of storage.

It is worth noticing that the storage depreciation, as defined in (13), affects the marginal storage value. If this parameter is close to one, the storage depreciation increases the marginal storage value, because it includes a risk premium due to the probability of losing a fraction of the stored commodity. Investigating the meaning of the storage depreciation rate in terms of marginal storage value, we consider price and marginal convenience yield to be fixed at their equilibrium values:

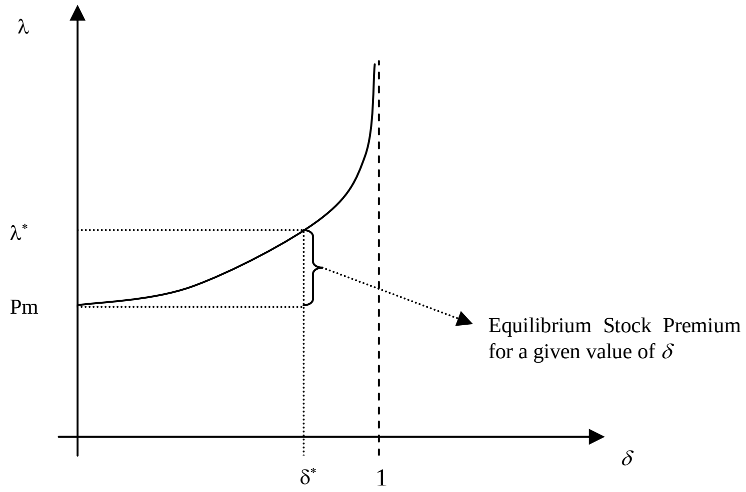
$$Pm(t) = \overline{Pm}$$
$$\phi(t) = \bar{\phi} = k \tag{26}$$

and, hence, (24) for *λ(t)* simplifies into:

$$\bar{\lambda} = \frac{\overline{Pm}}{1 - \delta} \tag{27}$$

In Figure 1, the behaviour of the marginal storage value is plotted at the equilibrium with a varying storage depreciation rate. It is graphically worth noting that if the storage depreciation rate is zero, then the equilibrium marginal storage value equals the equilibrium value for price, *i.e.* with a perfect storage technology the equilibrium marginal storage value coincides with the equilibrium price. There is no place for a risk premium due to the imperfection in the storage mechanism. If the storage depreciation rate approaches one (the upper bound of its domain), the equilibrium marginal storage value tends to infinity, *i.e.* the value of a unitary increase in stockholding level is very high given an inefficient storage technique. We have seen in the previous section that the stored level of the commodity can be affected by institutional factors, such as the indirect taxes, interest policy and/or by the role (directly) played by government's stock on the overall inventory accumulation. Hence, government decisions can drive the agents to obtain a better allocation.

Figure 1. - The role of δ in the marginal storage value



The *F.O.C.* with respect to production can be further manipulated by replacing the definition for *Q(t)* which comes from the market clearing equation and involves the structural representations of the internal/external demand. This sequence of substitutions gives us the reduced form of the price equation:

$$Pm(t) = \delta_0 + \delta_1 \Delta FS(t) + \delta_2 EXC(t) + \delta_3 Ctd(t) + \xi(t) \tag{28}$$

where we have set the following equalities:

$$\delta_0 = \frac{c_0}{1-c_1\alpha_1} + \frac{c_1}{1-c_1\alpha_1} d_0 ; \delta_1 = \frac{c_1}{1-c_1\alpha_1} ; \delta_2 = \frac{c_1 d_1}{1-c_1\alpha_1} ; \delta_3 = \frac{c_2}{1-c_1\alpha_1}$$
$$\xi(t) = \frac{1}{1-c_1\alpha_1}\left[\eta(t) + c_1 v(t)\right] \tag{29}$$

Therefore, the equilibrium price is intimately related to the change in stocks and to a pair of exogenous variables such as the exchange rate and unit cost of production.
The co-state condition provides the motion law for the marginal value of stocks, *i.e.* $\lambda(t)$.

$$-\frac{\partial H}{\partial \Delta FS(t)} = \frac{d}{dt}\left[\lambda(t)e^{-IRt}\right] \Rightarrow \dot{\lambda}(t) = (IR-1)\lambda(t) + Pm(t) \tag{30}$$

In order to derive the econometric form for the marginal convenience yield equation, we specify a quadratic equation for the total marketing cost function (Φ):

$$\Phi(Pm(t), FS(t), T(t)) = c_0 + \left[-a_0 - a_1 E\left[Pm(t)\right] - a_3 T(t)\right] FS(t) - \frac{a_2}{2} FS(t)^2 \qquad (31)$$

in which, in empirical part, *E[Pm(t)]* is the U.S. corn spot expected price, *FS(t)* is the stockholding level and *T(t)* is a deterministic time trend. While the price and stock variables affect the inventory accumulation in the storage markets, the time trend could include forms of positive autocorrelation linked with the smoothness of the storage process. Given the definition in (31), we exploit (19) and obtain the following equation which gives us an estimable relationship for the marginal convenience yield:

$$\phi(t) = a_0 + a_1 E\left[Pm(t)\right] + a_2 FS(t) + a_3 T(t) + e(t) \qquad (32)$$

Equation (32) is the relationship by which we can explain how the marginal convenience yield arises in commodity markets. In particular, it is related to a measure of the expected price whose impact on the marginal convenience yield is expected to be positive: an increasing expected price leads individuals to accumulate increasing levels of storage in order to face the uncertainty due to the price variability. The sign of the parameter related to the storage level depends on the level of the stock accumulation in the system: as pointed out by Susmel and Thompson (1997), the marginal convenience yield progressively decreases when the storage level goes to infinity. Therefore, the sign of such a parameter is expected to be negative when the market records low stock levels, whereas it approaches zero when the stock level is high.

Recalling Tobin (1969), we can define a ratio on which the stockholding decision rule can be grounded. This relevant parameter is:

$$q(t) = \frac{\lambda(t)}{Pm(t)} \qquad \textit{where } Pm(t)=mc(t) \qquad (33)$$

where the denominator is the replacement cost of a stored unit of corn which, under the assumption of competitiveness in the cash market, equals the spot price. Then, market records a positive storage if *q(t)* is greater than one, whereas market decreases its stocks with $q(t)<1$. The *q(t)* function links the marginal storage value to the current price, including it in stockholding functions. Moreover, it is worth noting that the optimal stockholding decision rule summarized into *q(t) depends upon various parameters that come from the market environment* (Pindyck, 1991). Finally, we can conclude that the stockholding decisions are made by comparing the marginal storage value carried out in the profit maximization. This ratio is a powerful tool in surveying the stockholding behaviour of the agents in the storage market.

By invoking again the equilibrium in the spot price and evaluating *q(t)* in that point we obtain an instrument for investigating the equilibrium stockholding decisions:

$$\bar{q} = \frac{\bar{\lambda}}{\overline{\overline{Pm}}} = \frac{1}{1-\delta} \tag{34}$$

Given *q(t)* definition and the equilibrium conditions, without the price uncertainty the stockholding decisions are made by only looking at the storage depreciation rate.

3.2 Dynamics

In this section, we want to shed some light on the short run adjustment process of spot and storage markets, by investigating the behaviour of the relevant variables and the effects of exogenous shocks. Firstly, in order to obtain a qualitative representation of the relevant variables, we differentiate the Tobin's-*q(t)* in equation (33) and we derive the isocline in (*ΔFS(t), q(t)*)-space. Since *q(t)* leads the storage decisions given the initial storage, the representation of the isocline for *q(t)* lies in the (*ΔFS(t), q(t)*)-space. In fact, positive change in stocks are recorded when *q(t)* is greater that one. Differentiating in the time equation (33), we achieve the following expression:

$$\dot{q}(t) = \frac{\dot{P}m(t) - \dot{\phi}(t)}{(1-\delta)Pm(t)} - q(t)\frac{\dot{P}m(t)}{Pm(t)} \tag{35}$$

Setting $\dot{q}(t) = 0$ and solving for *q(t)*, we obtain:

$$q(t) = \frac{1}{1-\delta}\left[\frac{\dot{P}m(t) - \dot{\phi}(t)}{\dot{P}m(t)}\right] \tag{36}$$

The equation (36) has been compacted for clarifying purposes; it represents the relationship that links *q(t)* to the changes in stocks through the equilibrium price. In order to depict the isocline for *q(t)*, we have to check its sloping. Since we have that:

$$\frac{\partial q(t)}{\partial \Delta FS(t)} = \frac{1}{1-\delta}\left(\frac{\delta_1 a_3}{\dot{P}m(t)^2}\right) = \begin{cases} > 0 & \Leftrightarrow \quad \delta_1 > 0 \\ < 0 & \Leftrightarrow \quad \delta_1 < 0 \end{cases} \tag{37}$$

The following proposition ensures that the isocline for *q(t)* in Figure 2 is upward sloping.

Proposition 1: Assuming that the structural parameters are: $c_1>0$ and $\alpha_1<0$, we have that δ_1 is always positive.

Proof: The thesis arises directly by applying the hypotheses on the structural parameters to the definition of δ_1 given in (29). In fact, by evaluating the equation (32) in its own equilibrium (*i.e.* $P\dot{m}(t) = F\dot{S}(t) = 0$), we have that:

$$\frac{\partial \phi(t)}{\partial t} = a_3 > 0 \qquad (38)$$

Therefore, the slope of the isocline of *q(t)* depends on the sign of δ_1 as defined in (29). In particular, the slope of this isocline is determined by two structural parameters. From equations (9), we know that

$$\alpha_1 = \frac{\partial Con(t)}{\partial Pm(t)} < 0 \qquad (39)$$

whereas, from equation (18), the marginal production cost is:

$$\frac{\partial C[Q(t)]}{\partial Q(t)} = (c_0 + \eta(t)) + c_1 Q(t) + c_2 Ctd(t) \qquad (40)$$

is an increasing function in *Q(t)*, *i.e.* $c_1 > 0$.

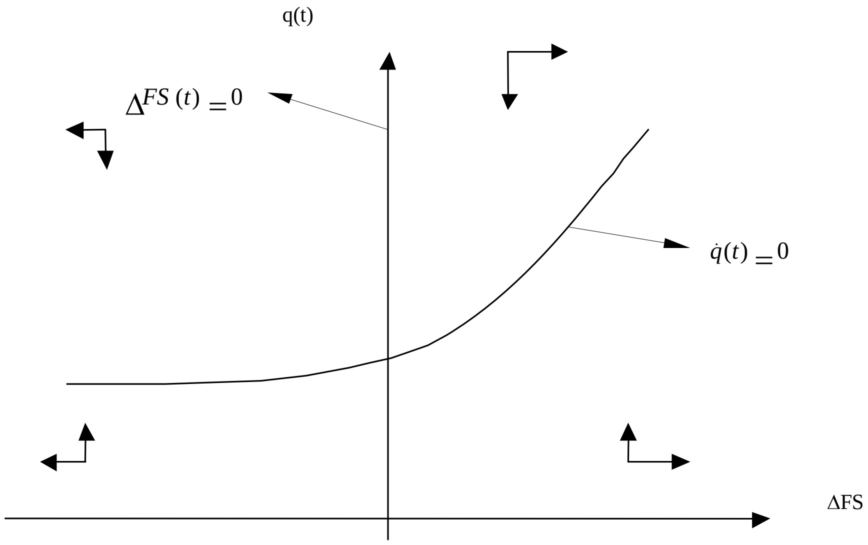
Q.E.D.

From *Proposition 1*, we find that *q(t)* is an increasing function of *ΔFS(t)*. This result is coherent with the definition of *q(t)* and reported in the phase-diagram *(Figure 2);* as we can see, when we consider a point which lies in the portion of the plane above the isocline for *q(t)*, we have that, in order to restore the equilibrium, we record positive change in stocks. The point in which the isocline for *q(t)* crosses the vertical axis corresponds to the value of *q(t)=1*.

To complete the qualitative analysis, we have to obtain the isocline for *ΔFS(t)*. Since it is defined as the locus where *ΔFS(t)=0*, this isocline naturally corresponds to the vertical axis.

It is worth noticing that the phase-diagram does not exhibit any stable branch. This means that the adjustment process only takes place along the isocline: whenever, we are out of the isocline, the adjustment paths diverge from the equilibrium. Therefore, facing with a shock to the exogenous variables, the adjustment path jumps from the old to the new isocline, in order to move along it to the new equilibrium point. This issue is the topic of the next subsection.

Figure 2 – Phase Diagram



3.2.1 An example of a positive shock in the unitary production cost

In commodity markets, we may be interested in explaining the reaction of market variables to a shock on some exogenous variables. In particular, in this work, a short-run perspective is assumed, implying that the fine-tuning of the market takes place only through the price (Gilbert, 1995). The disequilibria between production and consumption are ruled out by the short-run role played by the stocks. Indeed, from figure 2, we can observe that the short-run equilibrium is characterized by $\Delta FS(t)=0$ and, from the market clearing equation, this means that production perfectly matches the demand. Hence, from the underlying assumption, an exogenous shock in production and/or demand, affect transitorily the dynamic patterns by the price adjustment.

From these arguments, the relevant investigations concern the exogenous variables which directly or indirectly affect the spot price. These variables from the equation (28) are the exchange rate (*EXC(t)*) and the unitary production cost (*Ctd(t)*).

In order to clarify the exposition, we consider a shock which increases the unitary production cost. In analogy with the comparative static of the single-commodity model (Ridler and Yandle, 1972; Gilbert, 1989), we investigate the consequences of a shock on the unitary production cost in terms of price and stock accumulation adjustments. Thus, computing the derivative of the isocline for *q(t),* with respect to the unitary production cost, we have:

$$\frac{\partial q(t)}{\partial Ctd(t)} = \frac{1}{1-\delta}\left(\frac{\delta_3 a_3}{\dot{P}m(t)^2}\right) \qquad (41)$$

The sign of (41) is claimed by the following proposition.

Proposition 2: Assuming that the structural parameters are: $c_2>0$ and $\alpha_1<0$, we have that δ_3 is always positive.

Proof: The proof arises directly by recalling the hypotheses of Proposition 1 and applying them to the definition of δ_3 in (29).

Q.E.D.

Therefore, we can assess that an increase in the per unit production cost shifts upward the isocline for *q(t)*. The dynamic adjustment in the spot and storage equations is represented in Figure *3*. In particular, Figure 3.*a* unifies the results obtained in propositions 1 and 2, whereas Figure 3.*b* shows the consequences of an exogenous shock in terms of equilibrium price. The basic relationship of figure 3.*b* is obtained in the (*ΔFS(t), Pm(t)*)-space, by rearranging the expression for *Pm(t)* in the first stage of maximization. This result is coherent with the empirical evidence that price adjustments depend on short- run dynamics of stock (Pieroni and Ricciarelli, 2005). The relationship linking corn price and change in stocks takes the following linear form:

$$Pm(t) = K + \delta_1 \Delta FS(t) + \varepsilon(t) \qquad (42)$$

where *K* includes all the exogenous variables evaluated at their equilibrium levels and the error term. In other words:

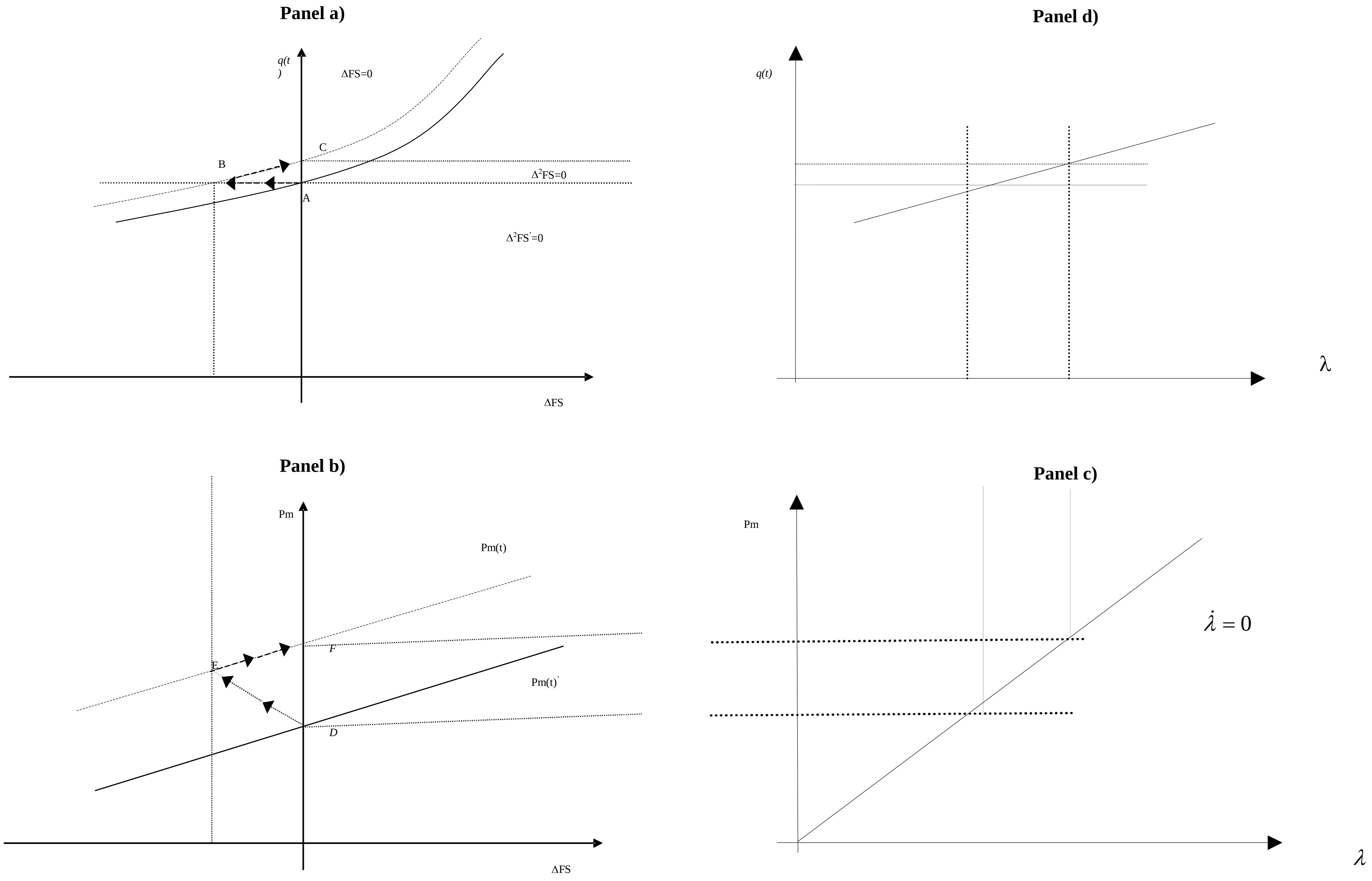
$$K = K(EXC, Ctd) \qquad (43)$$

In this space, the equilibrium price is located on the intersection between the price-stock line and the vertical axis, *i.e.* the equilibrium price is achieved only when the change in stocks is zero as shown in figure *3.a).* In both the figures, the equilibrium corresponds to the interception between the isocline (or the straight line in figure 3.*b*) and the vertical axes.

The effects over the equilibrium of a positive shock of the unitary cost of production in the state variables *q(t), Pm(t),* and $\Delta FS(t)$ are shown in figures 3.*a)* and 3.*b);* either the *q(t)* (Point C) or *Pm(t)* (point F) shift upward to achieve a new equilibrium point.

Since the increase in the unitary production cost reflects proportionally an increase in all costs which underlie the production process (*i.e.* wages, fertilizers, machineries etc.), generally it causes a worsening of the production conditions and, hence, a cut in the amount produced. The market clearing equation has to hold and, in turn, the market records a negative change in stocks (from A to B)[9].

Figure 3- Exogenous shock and market adjustments



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Simultaneously, the negative change in stocks causes a gradual increase (from D to E, in figure 3.*b)* in the spot price (*Pm(t)*) which is explained by the pressure of stable demand on the shocked supply side. We remark that E point in figure 3.*b* is outside of the equilibrium, since the spot price is smaller than new equilibrium point F.
The inventory adjustments are driven by the *Tobin-q* (from B to C, in figure 3.*a*); in fact, a lack of inventories with respect to the equilibrium level magnifies the convenience (marginal benefits) to accumulate stocks. This increase in inventories goes on until the equilibrium. Simultaneously, in the figure *3.b)*, by the positive relationship between *q(t)* and *Pm(t)*, we obtain an adjustment process of *Pm(t)* (from E to F). In fact, the agents put in their own storehouses inventories which they have previously sold out, they induce a further progressive increase in the spot price. Furthermore, we notice that, in figure *3.b)*, the movement along the new equilibrium relationship takes place much more gradually than the abrupt jump from D to E. Such a difference, the adjustment mechanism can be explained by the coexistence of heterogeneous agents in the markets. The share of processors that for precautionary reasons frictionally adjust towards the F point could be explain some empirical evidence of smoother patterns of the spot price.
In order to clarify the nexus between storage and cash market, we derive (figure 3.*c*) the isocline for the marginal storage value (*i.e.* the price for which stockholders are willing to sell inventories) in the (*λ(t), Pm(t)*)- space. The analytical derivation of the explicit functional relationship linking cash and storage market exploits the motion law that arises from the co-state condition (30), *i.e.* for the marginal storage value. Hence, we can depict the isocline $\dot{\lambda}(t) = 0$ in the (*Pm(t), λ(t)*) space. Replacing *λ(t)* with the second equation contained in (25) and setting $\dot{\lambda}(t) = 0$, we see an explicit relationship of the variation of the marginal storage value in terms of price and marginal storage value. Then, solving for *Pm(t)* yields, we have:

$$Pm(t) = (1 - IR)\lambda(t) \tag{44}$$

We remark that the slope of the linear relationship plotted in figure *3.c)* is crucially affected by the interest rate, which represents the opportunity cost to maintain inventories.

Finally, in figure 3.*d)*, we graphically derive the shadow supply curve in the storage market: by the equation (33) in which the $\partial q/\partial\lambda > 0$, we find a positive willingness to accumulate inventories when the marginal value of the storage increases.

4 Pre-estimation remarks

In this section, we deal with some econometric issues which are essential in the estimation of the model. In particular, these concern variables included into both the price equation (28) and the marginal convenience yield equation (32).

In order to obtain an empirical specification, we need to estimate the storage depreciation rate and calculate a monotonic transformation for the marginal convenience yield. Moreover, the recursive algorithms are extracted to assess the *q-Tobin* and future prices.

The storage depreciation rate (δ) which is an empirical variable it is involved in the marginal storage value (equation (25)). The theoretical relationship underlying the storage depreciation rate has to satisfy particular features: *i)* it takes values in the interval [0, 1]; *ii)* it has to be related to the storage technology (*STH(t)*) which could include institutional factors (*IF(t)*). Thus, in our work, storage depreciation is not assumed to be constant. The estimation of δ, which cannot be directly observed, is carried out as follows[10]. First, we set $\delta=0.05$: this value is considered to be plausible after looking at simulation exercises performed in many works (Deaton and Laroque, 1992). Second, we obtain the series for the wasted stock as follows:

$$Wasted\ Stock = \delta FS(t) \quad \text{where} \quad \delta=0.05 \tag{45}$$

As also recall above, we can estimate the wasted stocks function considering its dependence by the storage technology and institutional variables that contains information about the switching of the market intervention. In the corn market, we have used as proxy of the change in the loan rate during the 80s, that can independently affect the dependent variable by a different degree of efficiency. Thus, assuming a linear relationship, we can estimate the next equation by *OLS*:

$$\delta FS(t) = \beta_1 STH(t) + \beta_2 Loan(t) + \varsigma(t) \tag{46}$$

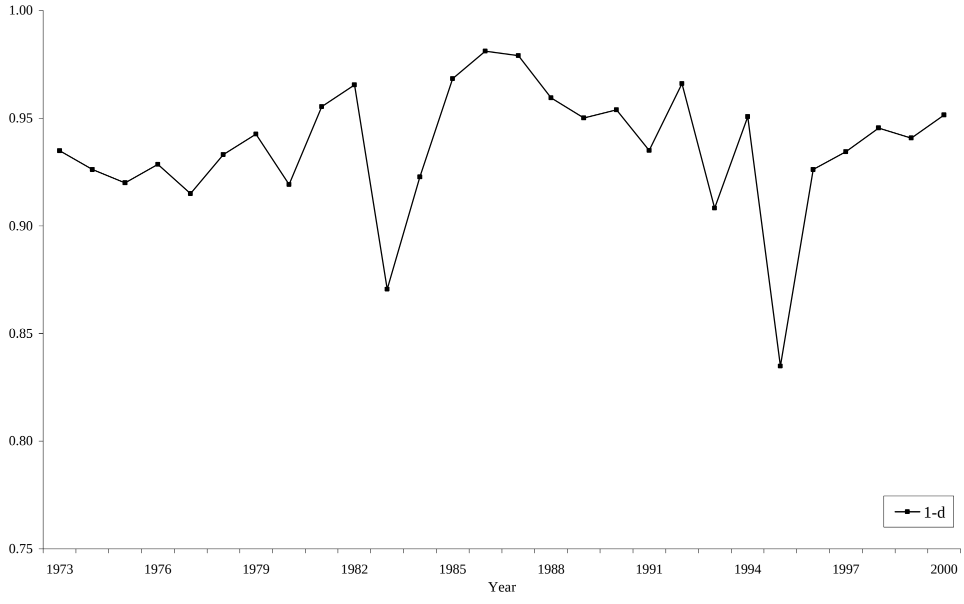
The appropriateness of such a regression is ensured by the high R^2 value, 0.77, in which the parameter β_1 results negative and β_2 positive; in both they are statistically significant. Economically, these results show that an increase in technology reduces the depreciation rate, whereas possible reduction in the loan rate increases the efficiency in the markets by the substitution of obsolete storage technologies.

In equation (46), we obtain a measure of the path of δ over time extracting the fitted values for the wasted stock and, consequently, filter the estimated series for δ as follows:

$$\hat{\delta} = \frac{\hat{\delta FS}(t)}{FS(t)} \tag{47}$$

As shown in Figure 4, the storage depreciation rate (δ) values are stable enough in the whole sample except for two particular years (*i.e.* 1982 and 1995); we can observe that the *(1-δ)* is in average above *0.90*. This fact ensures that stockholding activity, perceived as investment (Kahn, 1981), is not affected by irreversibility and the last statement is strengthened by the high development of financial commodity markets since 1970s.

Figure 4 - The (1-δ) dynamic path



In order to complete the information background to estimate the model, we need to compute the marginal net convenience yield (ϕ'(t)). Following Pindyck (2002), we build the convenience yield series as follows:

$$\phi(t) - k = \phi(t)' = [1 + IR(t)]Pm(t) - Pf(t) \tag{48}$$

where *Pf(t)* is the future price with six months delivery. To carry out the series of the gross marginal rate, we have to estimate the per unit storage cost, which is assumed to be constant (Turnovsky, 1983). Therefore, we set:

$$k = \left| \min \phi(t)^{'} \right| \quad (49)$$

and, once the value for *k* is quantified, we immediately obtain the series for the gross marginal convenience yield:

$$\phi(t) = \phi(t)^{'} + k \quad (50)$$

The dynamic framework derived in Section 3 and, in particular, the definition of *q(t)* in (33) can be employed to forecast the stockholding level. The algorithm we propose is based on the observation that the ending stocks in *t-1* coincide with the starting stock in period *t*. Therefore, we can forecast the ending stocks in period t by applying the function *q(t)* which describes the evolution of stockholding level in *t* to the initial stock level. However, because the variables involved in *q(t)* are not determined at the beginning of the forecast period, we replace them with the expected value according to equations (28) and (32). Hence, we can write the following recursive algorithm:

$$\hat{F}S(t) = FS(t-1)\hat{q}(t) \quad (51)$$

in which:

$$\hat{q}(t) = \frac{\dfrac{\hat{P}m(t) - \hat{\phi}(t) + k}{1-\hat{\delta}}}{\hat{P}m} \quad (52)$$

The initial condition, *FS(t-1)*, is modulated by the expected *q(t)* and gives the value of the ending stocks in *t*. Therefore, given the initial condition, the ending stocks are affected by the variables of spot and storage markets. Moreover, replacing the variables with the expected value obtained from the equations resulting from an optimization program can be explained by rationality of the agents.

Other than carrying out the stock computation as shown in (51), we can also give an algorithm by which a fitted series can be obtained for the U.S. corn futures price. Using the empirical estimates for the spot price and gross convenience yield and considering the value assigned to the marginal storage cost, the computation for the futures is as follows:

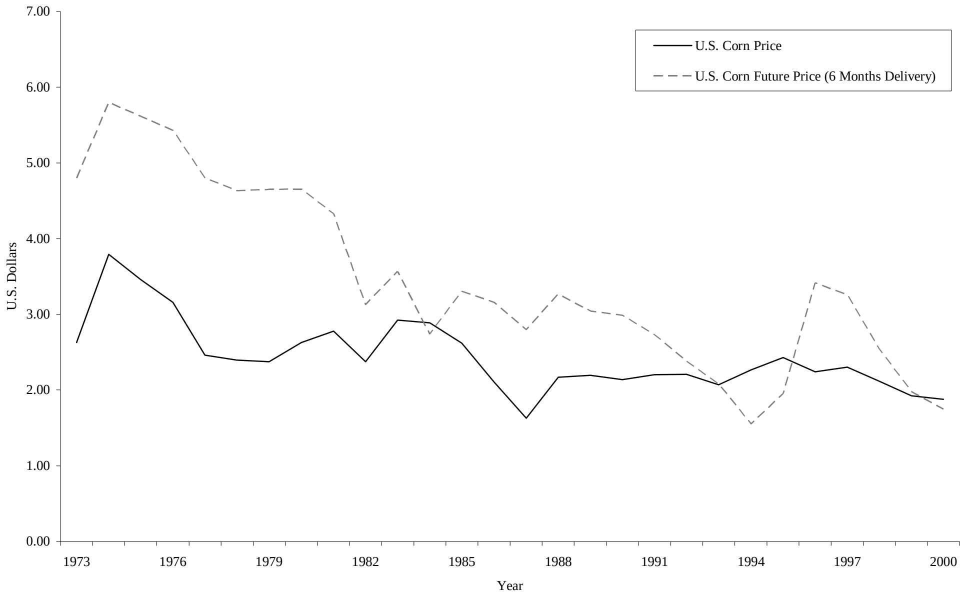
$$\hat{P}f(t) = \left[1 + IR(t)\right]\hat{P}m(t) - \hat{\phi}(t) + \hat{k} \quad (53)$$

By exploiting the estimates obtained from (28) and (32), we can implement (53) and find the estimates for the futures price. In the next Section, we present and comment on the dynamic properties of the series and the estimation results.

5 An empirical Illustration: Data and Results

The variable descriptions and sources of the U.S. corn market used in the estimation are reported in the Appendix A. Figure 5 describes the evolution of spot and futures corn price, in which the frequency of the sample period is represented by the annual time series (1973-2000). By exploring the variables, we found confirmation of the role of macroeconomic shocks that have strongly affected the value of expected and current corn prices in the 70s, and increased the risk premium included in the futures price. In particular, Gilbert and Perlman (1987) have suggested that the extraordinary events surrounding the first major oil price rise in 1973-4 have generated a non-valuable psychological impact on other primary commodity markets.

Figure 5- Future and Spot Price: A Comparison



Moreover, the evolution of the spot price shows a peak in the U.S. corn price in 1995. We remark that this evidence depends on the well-known crisis striking the world corn production along with the NAFTA agreement. Concerning the last one, we remark that few months later liberalization, the demand for corn import of Mexico from the U.S. doubled in comparison with the years immediately prior to the agreement.

Figure 6 proposes the paths of the production and stocks. It is relevant to notice that over the whole sample, the stock patterns (*FS(t)*) show a stable behaviour even if, as remarked above, the farm bill reform in the mid-80s has probably ignited the choice changes in the stock accumulation. Therefore, this behaviour seems to be transitory since the data show that the market agents have likely revised down their estimate and adjusted quickly the accumulation process of inventory.

Figure 6- Production and Stock behaviour in U.S. Corn Market

In order to specify robust empirical estimates, we investigate the dynamic properties of the series through the implementation of the unit root tests. The Augmented Dickey Fuller test (*ADF*) is implemented for current price (*Pm(t))*, marginal convenience yield ($\phi(t)$), discrete change in stocks (*ΔFS*) and stockholding level (*FS*) with optimal lags and presence/absence of deterministic components. We strengthen the *ADF* results in our sample by computing - under the null hypothesis of stationary variables - the *KPSS* test.

Table 1 shows the results. The estimation of the unit root is implemented with a constant for price (*Pm*), discrete change in stock (*ΔFS*) and stockholding level (*FS*), whereas the convenience yield (ϕ) is corrected for the significant relevance of the time trend that justifies the previous specification (32).

Table 1 - Unit root tests

	ADF			
	Pm	FS	DFS	f
Value of test statistic	-3.5139	-2.5947	-5.1576	-4.101
Optimal number of lags				
Akaike Info Criterion	1	0	0	0
Final Prediction Error	0	0	0	0
Hannan-Quinn Criterion	0	0	0	0
Schwarz Criterion	0	0	0	0

Asymptotic critical values: -3,43 (1%); -2,86(5%); -2,57(10%); Davidson et al. (1993)

	KPSS			
	Pm	FS	DFS	f
Value of test statistic	-0.17	0.2155	0.064	0.4322

Asymptotic critical values: 0,739 (1%); 0,463(5%); 0,347(10%); Kwiatkowski et al.(1992)

Two issues have to be discussed. Statistically, the optimal number of lags selected by the usual non-parametric criteria shows the consistency in annual data of a simple Dickey-Fuller test. Second, the current price and discrete changes in stock are stationary at a significance level of one percent, whereas, for the other variables, the performed tests are not fully informative. Indeed, in the stockholding level the *ADF* test rejects the non-stationarity hypothesis only at the ten percent level, whereas *KPSS* test does not reject stationarity. Conversely, the convenience yield shows an ambiguous result in the *KPSS* test, whereas it is consistent with the expectations in the *ADF* test. However, from the empirical results, we prefer to reject the unit root of (*Pm*) and (ϕ) and specify the simultaneous model (28) and (32) in levels.

In table 2, the theoretical expected signs of the independent variables in the (observable) price equation linked with the preliminary *OLS* estimations are shown. The results indicates a positive and significant impact of a unitary stock changes (δ_1) in the price equation, in this way conforming the adjustment dynamic anticipated in the paragraph *3.2.1*. Moreover, the theoretical positive impact on commodity prices of exchange rate and unit production costs parameters (δ_2,δ_3) are confirmed by the data, even if for the exchange rate variable a statistically significant relationship is rejected.

Table 2: Theoretical Expectations and Preliminary Analysis on the Price Equation

	Expected sign	Regression coefficient
$\frac{\partial Pm}{\partial \Delta Fs} = \delta_1$	+	0.3615 (0.0639)
$\frac{\partial Pm}{\partial EXC} = \delta_2$	-	-0.2795 (0.1580)
$\frac{\partial Pm}{\partial Ctu} = \delta_3$	+	0.6301 (0.0004)

In order to estimate the crucial relationships in both cash and storage markets, by (28) and (32) respectively, the instrumental variable estimator (*IV*) can be computed by using a three-stage least squares (*3SLS*) procedure. Preliminary, an estimation in two steps is obtained to show the bias in the *OLS* estimator. In the first step, the fitted values of the *OLS* regression of the corn price equation (28) are outlined. Then *2SLS* estimations are obtained by the *OLS* regression of the marginal convenience yield equation (32), in which the fitted price equation is used as a proxy for an expected variable in both the other exogenous variables. It is worth noticing that the two-step procedure does not necessarily give a correct estimator of the variance, since the bias depends on the residual in the first stage (Pesaran, 1987, 1990).

A parsimonious specification of the marginal convenience yield equation (32) is carried out and a formal test that the stock level variable (*FS*) has a non-significant impact for the precautionary behaviour implemented. The statistical distribution *F* is used as an efficient approximation of the asymptotic distribution in a finite sample (Harvey and Peters, 1990), testing the null hypothesis that H_0: $FS=0$. The result of the hypothesis test is $F_{(1,24)} = 1.155$ *(p-value=0.293)* does not reject the restriction in the marginal convenience equation. In contrast to the general expectations, the *FS* is non-significant and this result in the corn market has to be explained. The basic theory of storage reports that shifting the supply in response to relative scarcity and assuming a convex convenience function for high level of stocks, a large change in stocks is associated with a small change in the marginal convenience yield. Consequently, large quantities can be shifted intertemporally and the current or permanent shocks do not change so much between spot and forward prices. On the other hand, a high level of inventory reduce the impact in the variance of the spot price, such that the low volatility is not significant to explain precautionary behaviour in the markets.

The first column in Table 3 reports the estimated parameters of price and parsimonious marginal convenience yield equations. In accordance with the impact on primary commodity markets of the oil price shock a dummy for the 1974 year (*D74*) is included in the price equation. The parameters of the equation price are robust to the sign changes and statistically significant. In particular, the ΔFS variable has got a positive impact on the price equation. Thus, the empirical results are in accordance with the aforementioned proposition *1* which proved the positive impact on the spot price. It is worth remarking that the consistence with the theoretical sign expectations is confirmed by adding the introduction of the oil shock dummy, leading to recover the statistical significance of the exchange rate. In fact, we find a confirmation that excluding the effects of the oil shock in the spot price of other commodity markets, the exchange rate variable is able to quickly adjust to the monetary shocks (Gilbert, 1989).

The estimated coefficients of the marginal convenience yield equation are all positive and consistent with a well-behaved marginal value of storage function. The preliminary statistics tests for structural stability considers the post-sample residuals and notes that high estimation residuals in 1984 could justify the inclusion of a dummy to account for the possible shocks. As aforementioned above, the agricultural policy changes could have increased the demand of stocks for precautionary reasons. Since we have considered the depreciation rate as an instrument to make the institutional changes endogenous, the policy change dummy will be used below for the sensitivity analysis.

In order to interpret the coefficients, we recall that, when the expected price increases, the value of the option to produce a marginal unit of the commodity is expected to increase. The estimation of the parameter is 0.81; thus, an unitary increase in the corn expected price leads to an increase in the marginal convenience yield of the *81%*. The time trend assures that the unknown adjustment process - sticky process - in the precautionary stockholding of corn reduces the misspecification issues linked with the simultaneity between spot price and marginal net convenience yield equations. The high significance of the time trend parameter indicates the importance of the short-run adjustment for precautionary motives, even if it does not provide explicitly to characterize the economic behaviour. It is worth noticing that the misspecification of the short-run along with the high autocorrelation (Deaton, Laroque, 1992), could be responsible for the failure of the empirical models. In order to account for simultaneity of the price as well as contemporaneous correlation of the disturbances in the equation system, the three-stage least squares (*3SLS*) is used as an instrumental variable method[11]. We use the exogenous variable and the constant as an instrument for the estimation. The results are shown in the second column of Table 3. It is important to remark that size of coefficients of the two estimations are closed and fully consistent with the theoretical results. For this reason we can use the previous interpretation concerning the impact of the variables in the spot price and marginal convenience yield.

In column 3 the estimation is carried out by inserting the dummy for demand of stock consequently to the policy reform in the 1984 (*D84*).

Table 3: Estimation Results

Parameter	2SLS	3SLS	3SLSD
δ_1	1.28E-04	1.37E-04	1.45E-04
	[2.164]	*[2.671]*	*[3.317]*
δ_2	0.6292	0.517	0.596
	[2.765]	*[2.560]*	*[3.047]*
δ_3	102.255	109.484	104.420
	[6.802]	*[8.228]*	*[8.216]*
δ_4	1.219	1.133	1.113
	[4.388]	*[4.566]*	*[4.908]*
a_0	-1.528	-1.378	-1.319
	[-1.904]	*[-1.375]*	*[-1.463]*
a_1	0.744	0.685	0.655
	[2.738]	*[2.080]*	*[2.183]*
a_2	0.113	0.112	0.109
	[8.609]	*[6.973]*	*[8.011]*
a_3			1.785
			[2.320]
Price Eqn.	$R^2 = 0.716$	$R^2 = 0.711$	$R^2 = 0.712$
ConvenienceYield Eqn.	$R^2 = 0.777$	$R^2 = 0.749$	$R^2 = 0.801$

The results of estimation show a high and significant parameter for the dummy, even if the lack between fitted and observed data in the marginal convenience yield is maintained for the following three years with the fitted marginal convenience yield values greater than the observed data. Thus, the announced farm bill reform has likely leaded towards public information, such that the agents have quickly adjusted their stockholding choices and anticipated, in a rational way, the market changes, completing the learning process in the smoothest way possible.
Table 3 also contains the statistical index for evaluating the fits and diagnostic tests on estimated equations.
The statistical R^2 index for *3SLSD* is high enough either in the marginal convenience yield equation (0.80), or in the price equation (0.71). In both price and marginal convenience yield equations, the heteroskedasticity is tested by univariate LM test; the

results are consistent with the lack of autocorrelation in both equations. In fact, we find that the hypothesis of omoskedasticity is not rejected with the p-value of *0.437* for price equation and *0.988* for the marginal convenience yield equation, respectively. The diagnostic test for the serial correlation is implemented in a multivariate framework, since the autocorrelation hypothesis is crucial to verify the observed patterns of the spot price in of agricultural commodity markets. The F statistic distribution of the autocorrelation test for finite sample is derived by Harvey (1990), in which the degrees of freedom account for a multivariate dimension. The empirical value of the statistic distribution ($F_{(4,24)}$) is *0.243 (p-value=0.912)* rejecting the first order autocorrelation hypothesis. In order to strengthen our results, we test the second order autocorrelation hypothesis. Also, in this case, the $F_{(8,31)}$ statistic distribution strongly rejects the misspecification hypothesis (*F* is *0.484* and the *p-value=0.85*).

It is worth noticing that the diagnostic tests are in contrast with Chambers and Bailey (1996) and Deaton Laroque (1992; 1996) results that find a substantial persistence of the series in primary commodity, marginally reduced when serially correlated shocks to the production are included. Conversely, our empirical autocorrelation tests are supported from the Ng and Ruge-Murcia results (1997), in which the inventory holding has a prominent role in determining the time series of the commodity price process. Since the marginal convenience yield is modelled to compensate for the expected loss of processors when the basis is below carrying charges, to take into account the lack of adjustment to the private information or the possibility of volatility, the specification (32) is an isomorphic model that is able to explain the persistence in the commodity data starting from the stock-out, assuming a decreasing probability of explanation of agricultural price spikes when a precautionary function is introduced (Pindyck, 2002).

In Figure 7, the comparison between the observed and fitted spot price equation is illustrated. As we can see, the oil shock is not completely accounted by the *D74*, so that we also have a slight persistence of residual in the two following years, while after the spot price is well-suited by our model (mid-80s). Thus, we can conclude that the shock caused by the agricultural reform is effectively accounted for the price equation.

The derivation of *q(t)* shows us the movements of inventories subject to the behaviour of the agents (Figure 8). As a stylised fact a downward trend exists to accumulate inventory that in recent years has lead to the *q(t)* to be less than one. In fact, the high macroeconomic uncertainty generated in the 70s the values of *q(t)* to such an extent as to induce a reaction in the economic agents by requesting a higher risk premium and, in turn, increase the futures price which embodies the risk premium (Fama and French, 1987).

Figure 7- Current and Fitted Values for the U.S. Corn Spot Price

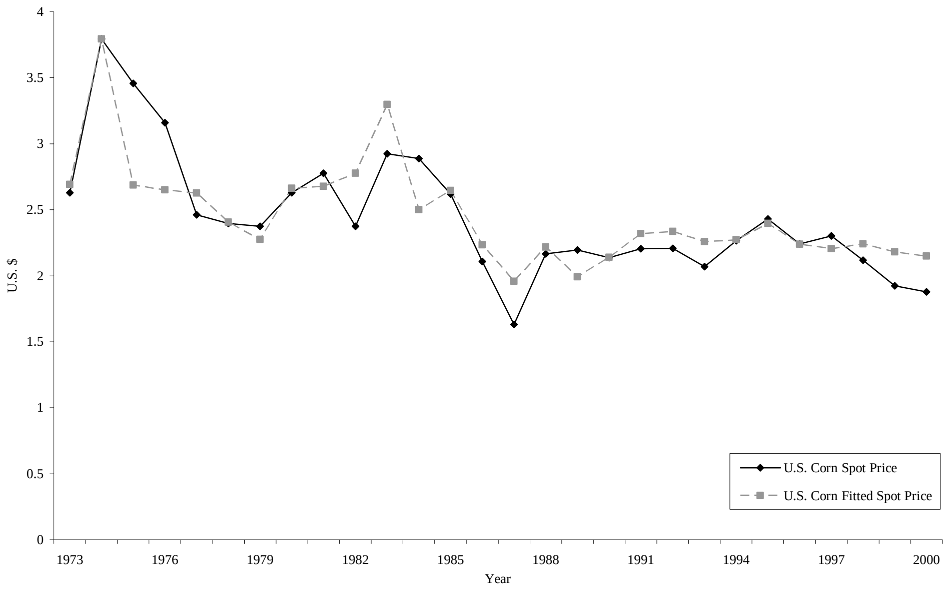
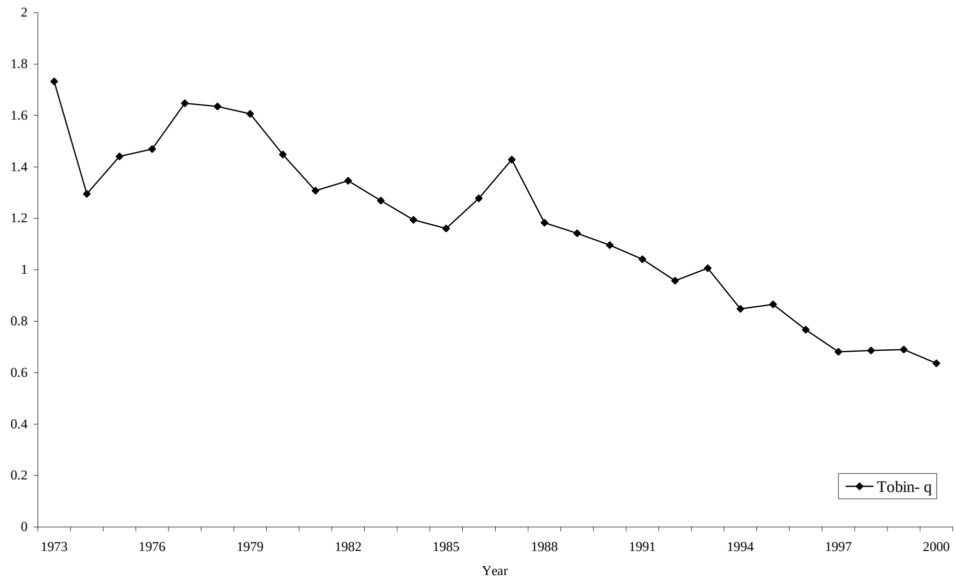
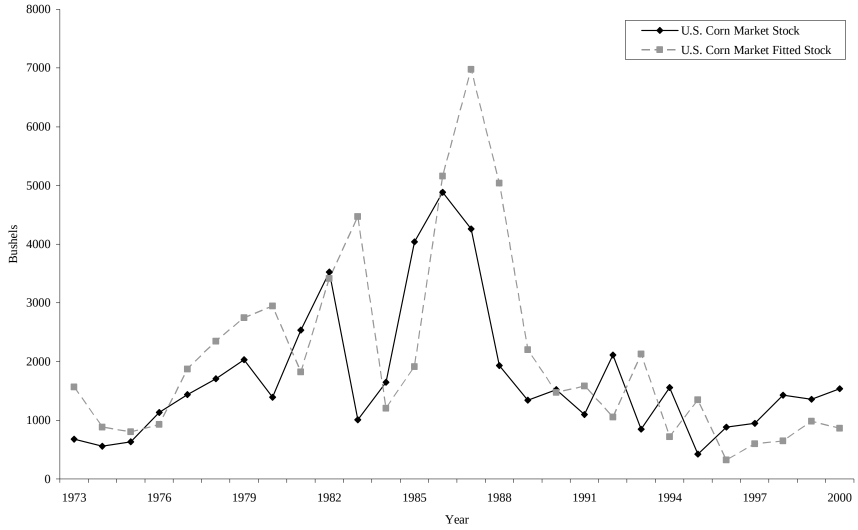


Figure 8 – The Computation for the q(t) Ratio



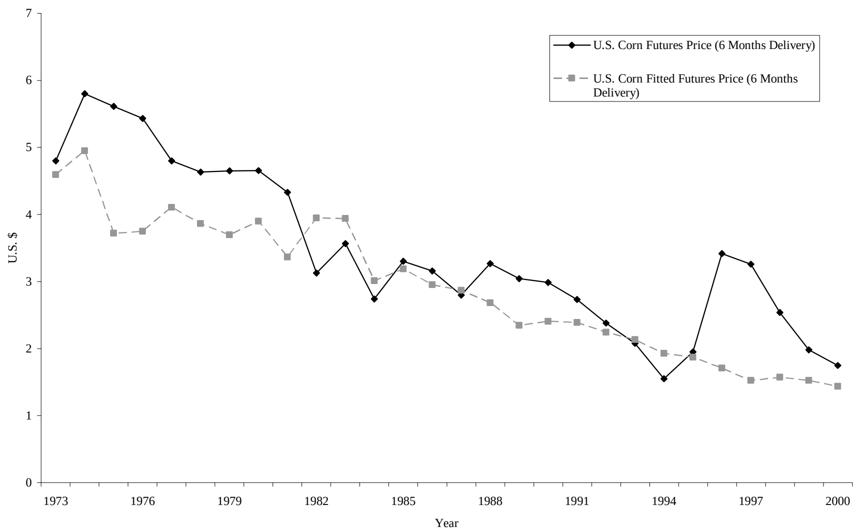
By using the algorithm described in Section 4, we obtain the graph in Figure 9, in which the fitted stock values are compared with the observed stock series. The stockholding dynamics captured by our model shows that as the government changes in the corn market are overvalued during the period from 1986 to 1988. We remark that when the speculative motives are considered along with the precautionary explanation in the storage market, the estimated model could introduce a source of misspecification. In fact, while the precautionary stockholdings are important to explain much of the behaviour in the observed data, especially for shocks deriving from other markets or unanticipated shocks of production, the partial dynamic adjustment leads to an over-accumulation of inventory when policy changes are announced and the information in the market is complete.

Figure 9 - Stock Computation



Finally, in Figure 10 a powerful result is achieved by using equation (53) to carry out a fitted values series which matches the six-month delivery futures price. It is worth noticing that our expected price captures, excluding in the 1996, the dynamic in the futures price, thus verifying the statistical robustness of the model and the economic coherence.

Figure 10 - A Computation for the U.S. Corn Futures Price (Six Months Delivery)



6 Concluding remarks

The speculative stockholding models currently used to dynamically solve price equations in commodity markets yield unsatisfactory results to trace changes in current or expected prices.

The aim of this paper is to extend the interactions between agents in the spot market to the stockholding decisions made by the storage market, in which the precautionary reasons are explicitly incorporated to account for realistic aspects of the production process and financial markets that include the potential persistence of commodity prices.

In order to investigate the determinants of the price function in the corn market, a continuous optimization of the total discounted profit function has been proposed in which, simultaneously, the production and stock levels are considered the control variables which lead both spot and storage markets. The precautionary behaviour of the processors has been modelled by the marginal convenience yield function, that extends the previous literature specifications in three directions. Firstly, the storage market has been considered to be populated by both speculators and processors. Dynamic simulations show that the shocks have an instantaneous effect on speculator behaviour, whereas the frictional adjustments of processors could be useful to model the significant degree of serial correlation in the agricultural prices. A criterion based on the Tobin idea has been proposed deriving recursively the predictions for stocks and futures prices. Secondly, the linkage between the cash and storage markets is given by the net change in stocks. This variable is also relevant since it included a storage depreciation rate time-varying influenced by technological arguments and institutional factors.

In the empirical part, we use annual data of the U.S. corn market to validate the model. The estimations are carried out by the two and three stage least squares in order to account for endogeneity in the prices function. The estimated parameters are statistically robust and coherent with the economic theory either in price or in marginal convenience yield equations. Moreover, the introduction of a function for precautionary behaviour of economic agents in the model accounts for the persistence observed in the data. Finally, we show that the stockholding behaviour is fully driven by the *q-ratio* variable.

In conclusion, it is worth outlining two aspects. The precautionary stockholding equation, that includes the frictional adjustment, can lead to an over-accumulation of inventory when the information in the market is complete. In our sample, the farm bill reform in the mid-80s probably ignited the choice changes in the stock accumulation but the market agents revised down their estimate and quickly adjusted the accumulation process of inventory. Secondly, while the net change of stocks represents the link between spot and storage market, in this framework the stock level represents an indirect test for volatility. In the corn storage market, the high level of stocks leads to non-significant impact on the marginal convenience yield.

Appendix A – Data and Statistical Sources

Data Table – Variables and Statistical Sources

U.S. Corn Market Data (Sample Period: 1973-2000)		
Variable	**Symbol**	**Statistical Source**
Production Cost	*Ctd*	U.S. National Service for Economic Research
Spot Price	*Pm*	U.S. National Service for Economic Research
Production	*Q*	U.S. National Statistical Service for Agriculture
Imports	*M*	U.S. National Statistical Service for Agriculture
Exports	*X*	U.S. National Statistical Service for Agriculture
Consumption	*Con*	U.S. National Statistical Service for Agriculture
Stocks	*FS*	U.S. National Statistical Service for Agriculture
Storage Technology	*STH*	U.S. National Statistical Service for Agriculture
Loan Rate	*Loan*	U.S. National Statistical Service for Agriculture
Interest Rate	*IR*	U.S. Federal Reserve Bank
Exchange Rate	*EXC*	U.S. Federal Reserve Bank
Future Price (6 months)	*Pf*	U.S. Federal Reserve Bank

Footnotes

[1] Different from Revoredo (2000), we show that the behaviour of the agents can not be enclosed in a unique and over-simplified decision rule concerning the stock accumulation.

[2] It means that, in the metal markets, spot price matches the supply to the change in demand, whereas in agricultural markets, the spot price matches the demand to the change in supply.

[3]Conversely, Ng and Ruge-Murcia (1997) use the precipitation index to explicitly model the stochastic process.

[4] Export demand *X(t)* is represented by a behavioural equation with the goal of analyzing the relationship of this variable with the world corn price, *Pw(t)*. We add an equation taking into account the world demand and world supply excesses (*Wd(t)* and *Ws(t)*) and we add a relationship where the world corn price is dependent on the exchange rate, *EXC(t)*. The equations are reported below:

$$Ws(t) = \lambda_1 + \lambda_2 Pw(t) + s(t)$$

$$Wd(t) = \lambda_0 - \lambda_3 Pw(t) + d(t)$$

$$X(t) = Wd(t) - Ws(t) + x(t)$$

$$Pw(t) = v_0 + v_1 EXC(t) + p(t)$$

Solving for X_t in the set of equations, we find the (7).

[5] The loan rate is the unitary price (per bushel, per quintal or per pound) at which the Government loans money to farmers in order to let them store the product at the end of the period, giving them the opportunity to sell it in the next period.

[6] Generally, the NPV principle does not provide dynamically consistent decision rule whether agents face with irreversibility and uncertainty on investments (Pindyck, 1991). However, the irreversibility in the commodity storage markets does not explicitly arise. Indeed, the irreversibility is ruled out from commodity storage either for the low depreciation affecting the stored commodity from one period to another or for the intensive development of commodity exchange markets which increase the investment liquidity.

[7] The motion law can also be re-written by aggregating some terms in the right hand side. In particular, defining $\Delta FS=FS(t)-(1-\delta)FS(t-1)$, we get: $\dot{FS}(t) = \Delta FS(t) - \delta FS(t)$. However, in a continuous time setup, the use of $\Delta(.)$ operator can generate some misunderstandings.

[8] For example, some units of a storable commodity could be crashed and/or get wet during transportation from the market to the warehouse and, in turn, partially affect the agents' stockholding choice.

[9] In point B, the corresponding change in stocks is such that it satisfies the market clearing equation, *i.e.* the supply- demand disequilibrium is balanced by the change in stocks.

[10] The estimation results from this procedure are available upon request to the Authors.

[11] We have estimated the simultaneous model by non-linear least squares estimator. Since the equations are linear in their parameters this amounts to standard three-stage least squares.

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