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# Information Aggregation and Preference Heterogeneity in Committees*

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**Abstract**

This paper is concerned with the efficiency of information aggregation in a committee whose members have heterogeneous preferences over a binary decision variable. In a first stage, agents may exchange private (decision-relevant) information which is assumed to be verifiable. Then they reach a decision via majority voting. We study different information environments and identify conditions under which full information aggregation is possible. In particular, if preferences are common knowledge and each committee member is endowed with information full information aggregation is possible despite preference heterogeneity.

**Keywords:** Information aggregation, committee decisions, preference heterogeneity
**JEL:** D72, D78, D82

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# 1 Introduction

Decision-making entities are often comprised of agents who represent different interests. The most obvious example of such a decision-making institution is the government in a representative democracy. If the consequences of the decision are uncertain, the quality of the decision benefits from exchanging information prior to making a choice. However, in case committee members' interests are not completely aligned, we cannot take information exchange for granted. Should we worry about preference heterogeneity interfering with information aggregation? This is the question the paper is concerned with.

The idea that committee members pool private information relevant for the decision, and therewith make use of a broader information base than a single decision maker could access, dates back at least to Condorcet (1785). The advantage of involving a higher number of informed agents in the decision process is intuitive if the committee indeed makes use of the committee members' private information. However, if committee members' interests are not completely aligned, this cannot be taken for granted.

A recent game-theoretic literature has shown that we cannot take efficient information aggregation for granted even if preferences are perfectly aligned (e.g. Austen-Smith and Banks, 1996; for a survey see Gerling et al., 2005). The reason is that the individual voter cares only about his vote when it is pivotal. Obviously, there exist equilibria in which no single vote ever affects the outcome and voters do not use their information. Austen-Smith and Banks (1996) show (in a voting setting without communication) that the exploitation of all available information is generally not possible in equilibrium. But Condorcet's jury theorem may still apply to strategically acting agents with completely aligned preferences (McLennan, 1998), since they play a common interest game.

Conflicting interests among committee members may limit their ability to pool their information efficiently. When preferences are heterogeneous, it is not straightforward to decide how decision quality should be measured. One such measure used in the literature (e.g. Feddersen and Pesendorfer 1997, 1998; Gerardi 2000) is the extent of information aggregation, i.e. the probability with which the collective decision would be the same if all the information was common knowledge. In this paper, we will also use this benchmark. Full information aggregation is desirable if committee members agree on which decision to make if the state of the world is known. Then, preferences are heterogeneous in the sense that voters differ with respect to their 'thresholds of doubt', i.e. with respect to how convinced a voter must be that a certain alternative is the correct choice in order to support that alternative.

In voting games without communication, full information aggregation requires that private information is transmitted via individual votes. If preferences are too heterogeneous, then full information aggregation via majority voting is impossible because beliefs concentrate around the threshold of doubt of the politically decisive voter, the median preference type. Voters whose thresholds are too far away from the median do not condition their votes on information (see Feddersen and Pesendorfer 1997).

On the other hand, committee members – at least in small committees – generally have access to another instrument to pool their information other than individual votes: they may exchange views prior to making a decision. In this regard, the information aggregation potential of committees is still not well understood. Most papers restrict attention to voting and neglect the role of communication within the committee. Exceptions are Coughlan (2000), Doraszelski et al. (2003), Gerardi and Yariv (2003a, b), and Austen-Smith and Feddersen (2002). Coughlan (2000) and Gerardi and Yariv (2003b) deal with committees composed of agents with homogeneous preferences for which complete information aggregation is possible because agents share a common goal.

The papers by Doraszelski et al. (2001) and Austen-Smith and Feddersen (2002) indicate that in committees with heterogeneous preferences, information aggregation is severely limited even if pre-vote communication is allowed. In these papers, information is soft and preferences are assumed to be private information. In such a setting, agents with extreme preferences always make statements which favor their preferred decision. Hence, in equilibrium, the information content of a statement is limited. Austen-Smith (1990a,b) studies information transmission in an agenda-setting game where preferences are common knowledge and information is soft. Information transmission is possible only if preferences are sufficiently aligned.

Chwe (1999), Persico (2004), Gerardi et al. (2005), and Chwe (2006) propose information eliciting by means of distorting the decision or manipulating agents' payoffs via a bet on the state of the world. In this paper, we are not interested in optimal mechanisms. Instead, we want to study a widely used one: discussions followed by majority voting. We identify conditions for the existence of equilibria in which information is fully aggregated, i.e in which the decision is the same as if all signals were common knowledge.

We follow the existing literature and model the decision procedure as a deliberating process followed by a voting phase, but we study a different information environment. We assume that the decision-relevant private information is verifiable. That is, committee members are assumed to be aware of facts which can be proven. This assumption corresponds well to committees whose members are experts on the issue. An example would be a board of examiners who have to propose a candidate for a grant. Decision-relevant facts could for instance be the candidate's performance in individual examiners' courses which is verifiable, but private information.[1] We provide more examples for our setting in the next section. Transmission of verifiable information has been studied e.g. in Milgrom and Roberts (1986) and Banerjee and Somanathan (2001), where an informed party or several informed parties try to influence a decision maker by revealing information. In contrast to these papers, informed parties participate in the decision process. Moreover, their preferred decision may also depend on the other agents' information. One could presume that players are able to force each other to reveal verifiable information. In our model, there is no means for players to affect each other's payoffs except for the decision they collectively make.

In the basic model, we assume that preferences are common knowledge. This assumption is well in accordance with situations in which decision makers are elected in order

[1] It is likely that in reality, there is soft information on top of that. In this paper, we deal only with the aggregation of hard information. As soft information communication games always have babbling equilibria, we could assume that if there was soft information which somebody tried to communicate, nobody would listen.

to represent different interests (like in a representative democracy), or with situations in which members are sent into the committee as a representative of an affected group (like in hiring committees). Our analysis provides a more optimistic view on the information aggregation potential of heterogeneous committees than previous work does. We show that an equilibrium exists in which information is perfectly aggregated. This is not the case in the games studied by Doraszelski et al. (2001) and Austen-Smith and Feddersen (2002). The reason is that in our set-up, committee members may be able to perfectly deduce the information of a voter who does not reveal it voluntarily. By not communicating his private information, a voter reveals that he possesses information he does not want to reveal. This contains exactly the same information as revealing the information itself. If information is soft (see also Austen-Smith (1990a,b)) the option to report false information destroys the opportunity to credibly transmit this information if it is indeed the truth.

Private information concerning preferences and soft information concerning the quality of the decision are good assumptions for novel and rare decision situations, whereas the approach in our basic model is well in accordance with committees consisting of experts who know each other, or whose interests can be inferred from their role in the committee. Examples are representative governments, hiring committees, or boards of directors.

We extent our basic model into two directions and derive conditions for full information aggregation in each extended set-up. First, we allow for the possibility that some agents are not endowed with decision-relevant information. Moreover, we examine an environment in which preferences are private information. Last, we combine these two modifications and consider a framework in which preferences are private information and in which there is the possibility that agents are not endowed with information.

In the modified versions of the model, full information aggregation is possible only if the preference parameter range is restricted. If committee members are not endowed with information with certainty, full information revelation in the communication stage is possible if and only if interests are completely aligned. The reason is that committee members with information which is unfavorable for their favorite decision can pretend to have no information. Strongly biased committee members prefer to conceal such information. However, we can show that there exists an equilibrium in which every committee member reveals at least one type of information, and each type of information is revealed by more than half of the committee members (if they possess such information) for cases in which the probability of receiving information is high enough.

If preferences are private information, there exists an equilibrium in which information is completely revealed if preference diversity is not too extreme. Committee members are uncertain about the majority's preferred alternative. As it is possible that the majority's interests are aligned with their own, committee members have an incentive to provide information. This is supported by a belief system with the property that information revelation does not harm in cases in which the majority's preferred decision deviates from one's own. Uncertainty about the majority's preferences may provide incentives for information revelation if committee members do not possess information with certainty. There exist equilibria with full information revelation for preference parameter constellations for which this is not the case if preferences are common knowledge.

The paper is organized as follows. In the next section the model setup is presented. We derive the full information aggregation result of the basic model in Section 3. In Section 4 the extensions to the basic model are analyzed and conditions under which full information aggregation is possible are derived. In the final section we conclude and outline possible directions for future research.

## 2 The basic model

A collective decision $x \in \{a, b\}$ is made by majority voting without abstentions in a committee consisting of $n$ members. For the ease of exposition (to avoid ties), let $n$ be an odd number. Utility from the decision is state-dependent. There are two possible states of nature $\omega \in \{A, B\}$, and uncertainty about its realization. Ex ante both states are equally likely.

Each agent $i$ receives a signal $\sigma_i \in \{\alpha, \beta\}$ which is correlated with the true state of the world:

$$prob\,\{\sigma_i = \alpha\,|\omega = A\} = prob\,\{\sigma_i = \beta\,|\omega = B\} = q,\ \tfrac{1}{2} < q < 1\ \forall i.$$

The signals are drawn independently conditional on the state. A signal contains verifiable information. Prior to voting, there is the possibility to communicate within the committee. Verifiability of information implies that committee members cannot invent information: they can either report the information they are endowed with or stay silent.

Examples for this decision environment are the following:

- $x \in$ {stick to the status quo, implement a reform}; $\omega \in$ {the reform causes higher costs than benefits, the benefits outweigh the costs}; $\sigma_i \in$ {presumptive evidence for either state: a certain group looses surely but little, the reform worked in a neighbor state, etc.}

- $x \in$ {hire a new researcher, not}; $\omega \in$ {researcher is brilliant, researcher is mediocre}; $\sigma_i \in$ {presumptive evidence for either state: researcher has a joint paper in a leading journal; researcher performed badly at a conference, etc.}

- $x \in$ {conviction of a defendant; acquittal}; $\omega \in$ {defendant is guilty, defendant is innocent}; $\sigma_i \in$ {presumptive evidence for either state: defendant would have had a good reason to commit the crime, defendant has never been conspicuous so far, etc.}

The timing is as follows:

1. Nature draws the state of the world and an imperfect signal for every agent.

2. Agents may reveal their signal to the other agents.

3. Agents vote. The alternative which receives the most votes is implemented.

The solution concept is Perfect Bayesian Nash equilibrium. That is, at each possible node of the game in which a player is asked to take an action, the action is required to be a best response to the other players' strategies given the beliefs, and beliefs shall be consistent with equilibrium strategies.

## 2.1 Agents

Agents derive state-dependent utility from the collective decision, $U_i = u_i(x, \omega)$. They are Bayesians and seek to maximize expected utility taking into account all available information. Let $p_i(\omega = A)$ denote the probability which agent $i$ assigns to state of the world $A$ given the information available to him.

Agent $i$'s expected utility from decision $a$ is:

$$p_i(\omega = A)\, u_i(a, A) + (1 - p_i(\omega = A)) u_i(a, B),$$

and from $b$:

$$(1 - p_i(\omega = A))\, u_i(b, B) + p_i(\omega = A)\, u_i(b, A).$$

Throughout the analysis, we assume a certain degree of homogeneity in preferences, which ensures that the desirability of decision $a$ weakly increases in the probability that state $A$ realized for each agent.[2]

**Assumption 1** $u_i(a, A) + u_i(b, B) - u_i(a, B) - u_i(b, A) > 0\ \forall i.$

Agent $i$ prefers the implementation of $a$ over $b$, iff

$$p_i(\omega = A) > \frac{u_i(b, B) - u_i(a, B)}{u_i(a, A) + u_i(b, B) - u_i(a, B) - u_i(b, A)}. \tag{1}$$

Denote $\theta_i = \frac{u_i(b,B) - u_i(a,B)}{u_i(a,A) + u_i(b,B) - u_i(a,B) - u_i(b,A)}$. Arrange the names of agents $i = 1, \ldots n$ such that $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_n$, and denote the median type, $\theta_{\frac{n+1}{2}}$, with $\theta_m$. Following (among others) Feddersen and Pesendorfer (1998), $\theta_i$ is called $i$'s threshold of doubt. Agent $i$ prefers $a$ over $b$ if and only if he assesses the probability that the state of the world is $A$ higher than his threshold of doubt. Agents $i : \theta_i < 0$ prefer decision $a$ in both states of the world, and agents $j : \theta_j > 1$ prefer decision $b$ in both states of the world. The present paper allows for a larger preference parameter range than most of the existing literature

[2] If Assumption 1 does not hold, the number of voters who prefer decision $a$ over decision $b$ may not be monotone in the probability that state $A$ realized. The analysis then requires to consider all possible shapes which this relationship may have.

(e.g. Fedderson and Pesendorfer (1998) or Doraszelski et al. (2001)), where attention is restricted to the case $\theta_i \in [0, 1]$.

Agents $l : \theta_l \in [0, 1]$ agree which decision should be made under certainty. Hence there are incentives to pool private information in order to get a better estimate about the true state of the world. However, heterogeneous thresholds of doubt potentially cause disagreement at the time the decision has to be taken. Therefore, agents may not want to reveal their information if this could cause the politically decisive voter to vote against their preferred alternative.

Preferences are common knowledge. We preclude the implementation of transfer schemes. Reasons for this restriction are that either (i) there exists no authority which is able to collect the transfers after the decision was implemented and the state of the world was learned, (ii) the state of the world is not verifiable, and/or (iii) individual rationality and budget constraints cannot be satisfied simultaneously.[3]

## 2.2 Information Processing

As utility is state-dependent, agents would like to condition their choice on the state of the world. The state of the world is not observable, but correlated with individual signals. Agents use the information about the realization of the signals for updating their beliefs concerning the realization of the state of the world. Firstly, each agent observes a signal, which alters his beliefs about the state and about the distribution of signals held by the other committee members. Secondly, agents observe the communication outcome and therewith the realization of a subset of the signals. Moreover, they are able to interpret the actions of those committee members who did not reveal their information. Last, each agent is aware of the fact that in equilibrium his vote affects the outcome only for particular realizations of the other voters' signals.

Suppose for the moment that the realization of private information $\sigma_i,\ i = 1, \ldots n$

[3] If we could align preferences over the collective decision by a payoff manipulation $u_i(x, \omega)$ via a transfer scheme $\{t_i(x, \omega), x = a, b; \omega = A, B; i = 1, \ldots n\}$, beneficial information aggregation would be no problem.

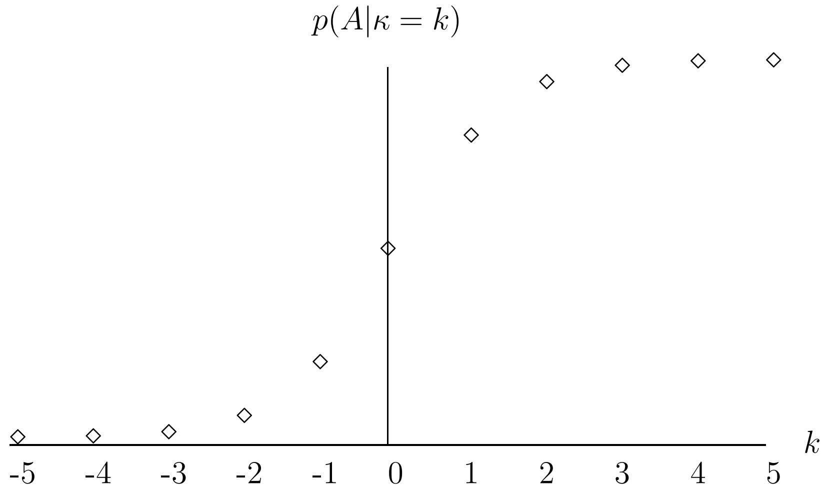


Figure 1: Probability that the state of the world is $A$ given the evidence $k$.

is common knowledge. Since the information environment is symmetric (i.e. $p(A) = 1/2,\ prob(\sigma_i = \alpha|\omega = A) = prob(\sigma_i = \beta|\omega = B)\ \forall i$), the only information which matters for updating beliefs with respect to the realization of the state of the world is the difference between the number of $\alpha$-signals and $\beta$-signals.[4] Denote this random variable with $\kappa$. We have $\kappa = \sum_{i=1}^{n} 1_{\sigma_i=\alpha} - 1_{\sigma_i=\beta}$, where $1_{\sigma_i=\hat{\sigma}} = 1$ if $\sigma_i = \hat{\sigma}$ and 0 else. Bayesian updating yields:

$$p\left(A\,|\kappa = k\right) = \frac{q^k}{q^k + (1-q)^k}. \tag{2}$$

Figure 1 depicts the probability that the state of the world is $A$ given there are $k$ more $\alpha$-signals than $\beta$-signals.[5]

If the realization of some $\sigma_j,\ j = 1, \ldots n$, (and hence of $\kappa$) is not known, agent $i$ has to form beliefs $\mu_i(\sigma_j),\ i \neq j$, with respect to these realizations, incorporating all available information. We denote with $\kappa_{-i}$ the difference between the number of $\alpha$-signals and $\beta$-signals held by committee members except for $i$. The beliefs held with respect to the

[4]See Appendix.

[5]Note that (as $n$ is an odd number) $\kappa$ assumes even values with positive probability only if some agents do not receive information, see Sections 4.1 and 4.3.

other agents' signals can be transformed into a belief regarding $\kappa_{-i}$.[6]

$$\mu_i(\kappa_{-i}=k)=\begin{cases}\sum\limits_{J\subseteq\{1,...n\}\setminus\{i\}:|J|=\frac{n-1+k}{2}}\ \prod\limits_{j\in J}\mu_i(\sigma_j=\alpha)\prod\limits_{l\in\{1,...n\}\setminus\{J\cup i\}}\mu_i(\sigma_l=\beta), & \\ & \text{for } k\in E\{-n+1,...,n-1\} \\ 0, & \text{for } k\notin\{-n+1,...,n-1\},\end{cases} \tag{3}$$

where $E\{x,...,y\}$ denotes the set of even integers between (including) $x$ and $y$. Taking his own signal and $\kappa=\kappa_{-i}+1_{\sigma_i=\alpha}-1_{\sigma_i=\beta}$ into account, $i$'s belief regarding $\kappa$ is given by:

$$\mu_i(\kappa=k)=\begin{cases}\mu_i(\kappa_{-i}=k+1), & \text{if } \sigma_i=\alpha\\ \mu_i(\kappa_{-i}=k-1), & \text{if } \sigma_i=\beta.\end{cases} \tag{4}$$

Agent $i$'s belief regarding the state of the world is then given by:

$$p_i(\omega=A)=\sum_{k\in O\{-n,...,n\}}\mu_i(\kappa=k)p\left(A\,|\kappa=k\right), \tag{5}$$

where $O\{-n,...n\}$ is the set of odd integers between $-n$ and $n$, and $\mu_i(\kappa)$ is updated whenever the agent receives new information. There are several stages at which the beliefs can be updated.[7] Firstly, the agent learns his own signal $\sigma_i$. Secondly, the other agents' observed communication actions contain information. Moreover, agents may be able to deduce information through equilibrium voting strategies. They care only about their vote when it is pivotal, hence they update their beliefs using the information contained in this event. If the event that $i$ is pivotal never occurs in equilibrium, basically any beliefs can be assigned, provided that they do not contradict Bayes' Law.

Figure 2 depicts a possible path of belief-updating for an agent in a committee with five agents. The illustrations represent agent $i$'s beliefs with respect to $\kappa$ at different stages of the game. Note that positive probability is assigned only to odd values for $\kappa$ because there is an odd number of signals. Ex ante, the probability that $k_\alpha$ agents receive $\alpha$-signals and $k_\beta=n-k_\alpha$ agents receive $\beta$-signals is given by $\binom{n}{k_\alpha}(\frac{1}{2}q^{k_\alpha}(1-q)^{k_\beta}+\frac{1}{2}q^{k_\beta}(1-q)^{k_\alpha})$.

[6]See Appendix.

[7]For convenience, we use the term 'updating' even if the updating does not change an agents' assessment.

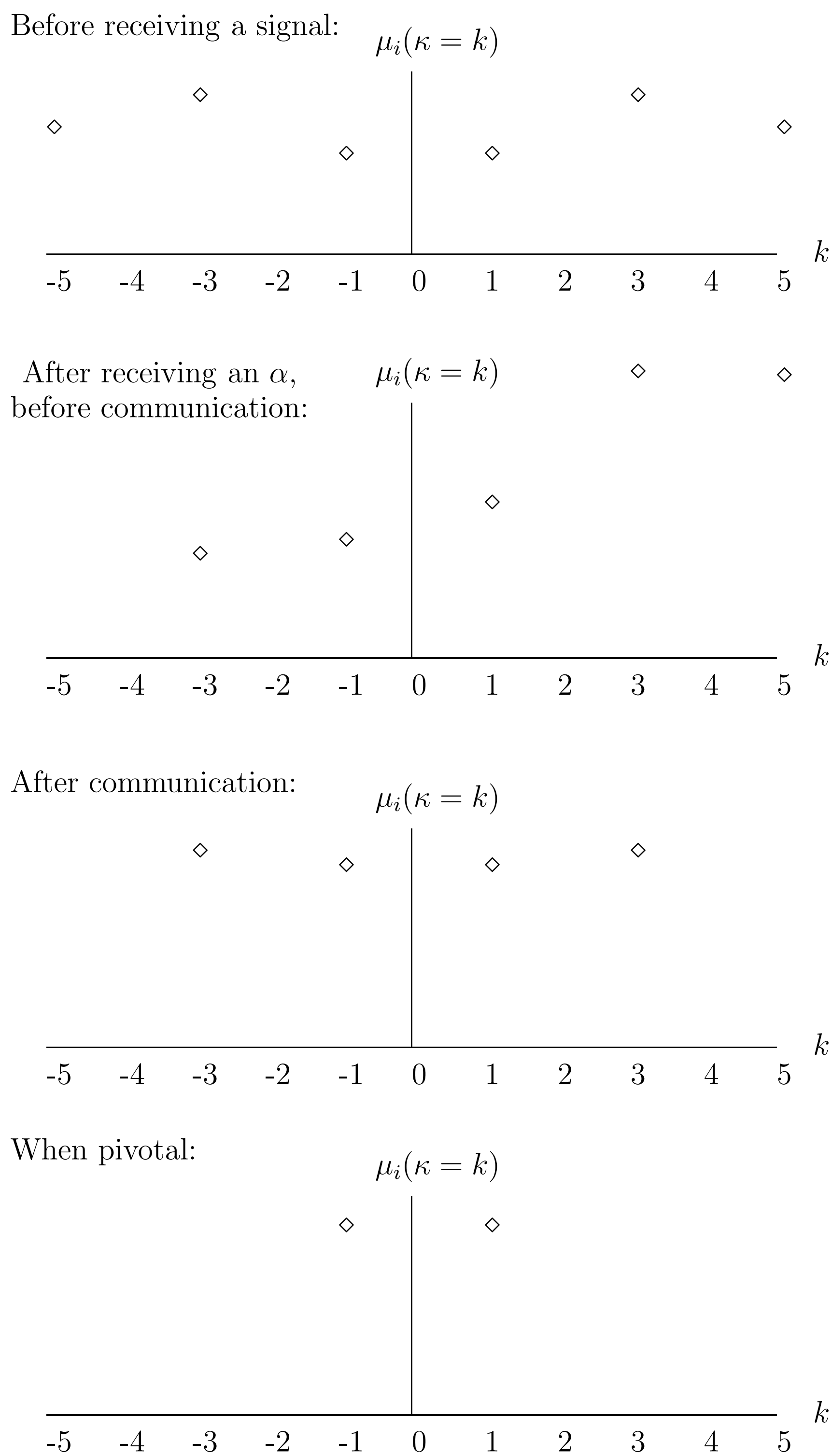


Figure 2: Possible path of updating in the course of the game with 5 players.

That is, prior to the receipt of information, probability is assigned to $\kappa = k$ according to:

$$\binom{n}{\frac{n+k}{2}}\left(\frac{1}{2}q^{\frac{n+k}{2}}(1-q)^{\frac{n-k}{2}} + \frac{1}{2}q^{\frac{n-k}{2}}(1-q)^{\frac{n+k}{2}}\right),$$

where $k \in O\{-n, \dots n\}$.

After having learned that his signal is $\alpha$, $i$ assigns positive probability only to $\kappa \in O\{-n+2, \dots n\}$, and assigns probability to $\kappa = k$ according to:

$$\frac{\binom{n-1}{\frac{n+k}{2}-1}\left(q^{\frac{n+k}{2}}(1-q)^{\frac{n-k}{2}} + q^{\frac{n-k}{2}}(1-q)^{\frac{n+k}{2}}\right)}{\sum_{k' \in O\{-n+2,\dots n\}}\binom{n-1}{\frac{n+k'}{2}-1}\left(q^{\frac{n+k'}{2}}(1-q)^{\frac{n-k'}{2}} + q^{\frac{n-k'}{2}}(1-q)^{\frac{n+k'}{2}}\right)}.$$

Suppose that $i$ was shown one $\beta$ in the communication stage, and that those who stayed silent planned to do so for both types of signals.[8] Then, $i$ can exclude $\kappa = 5$ and conclude $p_i(\omega = A) = 1/2$. At the stage of voting, he again updates his beliefs, assigning positive probability only to those $\kappa$ which – given the voting strategy profile – may render his vote decisive. Suppose that the voter who revealed the $\beta$-signal votes for $b$, that one of the remaining voters votes for $a$ irrespective of his information, and that the other two voters vote informatively, that is each of them votes for $a$ if his signal is $\alpha$, and votes for $b$ if his signal is $\beta$. Then, $i$'s vote is decisive if and only if those who vote informatively have opposing signals.

## 2.3 The communication stage

The agents are allowed to reveal the signal they received from nature prior to the voting stage. As a signal contains verifiable information, agents cannot lie about their information. A communication strategy $\gamma_i$ for an agent $i$ is a plan whether to report his information ($\sigma_i$) or to remain silent ($s$) for each signal he may receive. Communication takes place simultaneously and is observed by all voters.[9]

[8] If a voter $j$ reveals an $\alpha$-signal with a higher probability than a $\beta$-signal, beliefs formed after observing $j$'s silence would take this into account, assigning higher probability to $\sigma_j = \beta$.

[9] The full information aggregation result (Proposition 1) does not hinge upon the assumption of simultaneous communication.

Let $C$ denote the set of possible outcomes of the communication stage. An outcome of the communication stage is denoted $c = (c_1, c_2, \dots c_n)$, where $c_i \in \{\alpha, \beta, s\}$ $\forall i = 1, \dots n$. Denote with $\mathcal{A}(c)$ the set of agents who revealed $\alpha$-signals, $\mathcal{A}(c) = \{i \in \{1, \dots n\} : c_i = \alpha\}$. Define $\mathcal{B}(c)$ and $\mathcal{S}(c)$ analogously. The most important summaries of the information provided in the communication stage are the number of revealed $\alpha$-signals, $k_\alpha(c) = |\mathcal{A}(c)|$, the number of revealed $\beta$-signals, $k_\beta(c) = |\mathcal{B}(c)|$, and the number of unrevealed signals, $k_s(c) = |\mathcal{S}(c)|$. Denote with $k(c) = k_\alpha(c) - k_\beta(c)$ the number of revealed $\alpha$-signals in excess of the number of $\beta$-signals. We call $k(c)$ *evidence*, and say that the communication stage produced evidence for $A$ if $k(c) > 0$, evidence for $B$ if $k(c) < 0$ and no evidence if $k(c) = 0$.

Having observed $j$'s communication action, $i$ updates his belief regarding the realization of $j$'s signal. Denote with $\mu_i(\sigma_j = \alpha | c_j = \hat{c})$ the probability which $i$ assigns to $\sigma_j = \alpha$ given $j$'s communication action $\hat{c}$. Because communication strategies are restricted to either truthful revelation or no revelation, we have that $\mu_i(\sigma_j = \alpha | c_j = \alpha) = 1$, and $\mu_i(\sigma_j = \alpha | c_j = \beta) = 0$. Beliefs regarding the signals of voters $j : j \in \mathcal{S}(c)$ must be consistent with these agents' communication strategies along the equilibrium path, and have to respect Bayes' Rule off the equilibrium path.

## 2.4 The voting stage

Agents vote simultaneously and without abstentions for an alternative to be implemented. The alternative which gets the most votes is implemented. Every agent takes into account all the information available to him, that is the own signal $\sigma_i$, the communication outcome $c$, and what he can learn through equilibrium play. In particular, agents base their votes on being pivotal.

A voting strategy $v_i$ for agent $i$ is a plan for which alternative to vote, for all possible outcomes of the communication stage and for each signal he may receive. Allowing for mixed voting strategies, we have $v_i : \{\alpha, \beta\} \times C \rightarrow [0, 1]$, where $v_i(.)$ denotes the probability to vote for $a$.

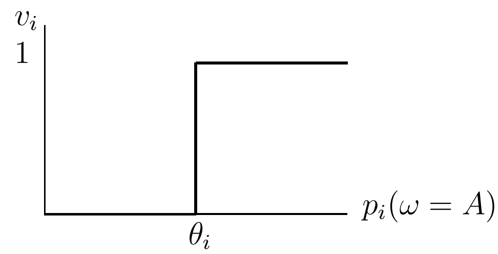


Figure 3: Bayesian sincere voting strategy

**Definition 1** *A voting strategy which satisfies*

$$v_i\,(.) = \begin{cases} 1, & \textit{if}\; p_i\,(\omega = A) > \theta_i \\ 0, & \textit{if}\; p_i\,(\omega = A) < \theta_i \end{cases}$$

*is called* *<u>Bayesian sincere voting</u>.*

Recalling (1), it is easy to see that a Bayesian sincere voting strategy maximizes $i$'s expected utility. Moreover, if the voting strategy profile allows the event that $i$ is pivotal to occur, the beliefs $\mu_i(\kappa)$ are well-defined and Bayesian sincere voting is the only utility-maximizing strategy. However, if an agent is never pivotal, any voting action is utility-maximizing, since none has an effect. Therefore, Bayesian sincere voting is always in the best response set at the stage of voting for every player. Figure 3 depicts a Bayesian sincere voting strategy for voter $i$. Assume again that the realization of the signals – and hence the realization $k$ of $\kappa$ – is common knowledge. Then, using Bayesian sincere voting strategies, agents $i$ with thresholds of doubt $\theta_i < p(A|\kappa = k)$ vote for $a$, and agents $j : \theta_i > p(A|\kappa = k)$ vote for $b$. An agent's threshold of doubt reflects how much evidence for state of the world $A$ must be presented to the agent such that he supports alternative $a$. For convenience, we assume that the voter with the median preference type is never indifferent between $a$ and $b$ if the realization of $\kappa$ is known with certainty. This assumption is made without loss of generality to avoid case differentiations. Alternatively, if he is indifferent between the two alternatives, we could restrict attention to equilibria in which the median preference type takes a particular action, say vote for $a$.

**Assumption 2** $\exists k_m \in \{-n, \dots n\}$ *such that* $p(A|\kappa = k_m) < \theta_m < p(A|\kappa = k_m + 1)$.

# 3 Full information aggregation

Both, preferences over the set of alternatives and beliefs regarding the state of the world may differ among voters. Voter $i$ with preferences $\theta_i$ does not want alternative $a$ to be implemented as long as he assesses the probability that the state of the world is $A$ to be smaller than $\theta_i$. Communication allows for the possibility to influence the politically decisive voter – and consequently the collective decision – in one's own favor. Whether or not a voter has the possibility to influence the decision in his favor depends on the kind of information he is endowed with. Thus, the sheer possibility to communicate may cause information exchange, even if the committee members do not talk to each other.

It is easy to see that an equilibrium exists in which the information is fully revealed and taken into account by every voter when making the voting choice. The result is stated in the following proposition.

**Proposition 1** *The following strategy profile is a Perfect Bayesian Nash equilibrium of the communication-and-voting-game: All agents reveal their information in the communication stage. All agents assign probability 1 to* $\kappa = k(c)$*. Agents whose thresholds of doubt are smaller than* $p(A|\kappa = k(c))$ *vote* $a$*, the other agents vote* $b$*. If* $c_i = s$ *is observed, and* $\theta_i \geq \theta_m$*, voters* $j \neq i$ *assign probability 1 to* $\sigma_i = \alpha$ *and play voting strategies as if* $i$ *had revealed* $\alpha$*. If* $c_i = s$ *is observed, and* $\theta_i < \theta_m$*, voters* $j \neq i$ *assign probability* 0 *to* $\sigma_i = \alpha$ *and play voting strategies as if* $i$ *had revealed* $\beta$*.*

**Proof.** We have to distinguish two cases: (i) $k_m$ is odd, and (ii) $k_m$ is even. Consider agent $i : \theta_i \geq \theta_m$. The only deviation in the communication stage which has an effect is to conceal $\beta$. In case (i), this deviation will change the outcome only if $\kappa_{-i} = k_m+1$. If $i$ sticks to his equilibrium communication action, all voters believe (know) that $\kappa = k_m$, and assess the probability that the state of the world is $A$ to be $p(A|\kappa = k_m)$. As $\theta_m > p(A|\kappa = k_m)$, a majority votes for $b$. If $i$ conceals $\beta$, the other agents believe that $\kappa = k_m + 2$ and consequently that the probability that the state of the world is $A$ is $p(A|\kappa = k_m + 2)$. $\theta_m < p(A|\kappa = k_m + 1)$, hence a majority votes for $a$. In case (ii), $i$'s revelation affects the outcome only if $\kappa_{-i} = k_m$. If $i$ sticks to his equilibrium communication action, all voters believe that $\kappa = k_m - 1$, and a majority votes for $b$. If $i$ conceals $\beta$, the other agents believe that $\kappa = k_m + 1$ and consequently a majority votes for $a$. Since $\theta_i \geq \theta_m$, agent $i$ prefers $b$ over $a$ in case his deviation has an effect. Thus, $i$ has no incentive to deviate in the communication stage. The voting strategy is Bayesian sincere. Thus, there exists no profitable deviation. A similar argument applies to the case $\theta_i < \theta_m$. Note that beliefs are consistent with Bayes' Rule. Q.E.D.

The intuition for the result is as follows. Full information revelation is possible because committee members may apply the following reasoning: I know who you are, I know what you want, and I know that you know something. If you don't tell me what you know, then I suppose that you have information which is unfavorable for your favorite decision because otherwise you would have told me.

In equilibrium, every agent has an incentive to reveal at least one type of information, given he possesses it: Agents who are more biased towards $a$ than the median preference type prefer alternative $a$ whenever the majority prefers $a$. Hence, they have an incentive to reveal $\alpha$-signals in order to convince the majority of alternative $a$. Agents who are biased towards $b$ have an incentive to provide $\beta$-signals. By not revealing what he knows, an agent reveals that he does not know anything he wants to reveal. As all agents know something, staying silent reveals exactly the same information as the revelation of the information itself, and amounts to be a superfluous action.

# 4 Extensions to the basic model

In this section, we modify the basic setting along two dimensions and identify conditions under which full information revelation in the communication stage is possible. First we allow for the possibility that agents do not possess decision-relevant information with a certain probability. In this case, agents with unfavorable information can pool with agents who have no information. Hence, it may not always be possible to perfectly deduce the information endowment of a committee member who does not talk. Next, we consider the case that preferences are private information. Then it is impossible to assume 'worst news' in case of a player's silence because it is private information what would be bad news. In both modifications of the basic model, full information aggregation is possible only if committee members' preferences are sufficiently aligned. However, there may still be a large range of preference parameters for which full information aggregation is possible. Interestingly, if there is the possibility that agents do not receive information, then a full information aggregation equilibrium exists for a larger preference parameter range if preferences are private information then if they are common knowledge.

We identify conditions for the existence of equilibria in which all signals are revealed in the communication stage and call these equilibria *full information revelation equilibria*. One could presume that the existence of such an equilibrium is not necessary for full information aggregation, as non-revealed information may still be aggregated in the voting stage. We will show that if the extended model does not have a full information revelation equilibrium, then there neither exists a full information aggregation equilibrium.

## 4.1 Possibility of receiving no signal

In the basic model, full information aggregation is possible because committee members may apply the reasoning "I know what you want, and I know that you know something. If you don't tell me what you know, then I suppose that you have information which is unfavorable for your favorite decision." Here, we eliminate the "I know that you know something" part from this line of argumentation. We assume that each committee member receives a signal with probability $\delta < 1$. If a voter is not endowed with information, we denote $\sigma_i = \emptyset$. We assume that it is impossible to verify $\sigma_i = \emptyset$, and that this event is equally likely in both states of the world. That is, the message spaces remain the same as in the basic model for those agents who receive information, whereas those who do not receive information are restricted to staying silent.

We modify nature's moves in the following way:

1. Nature determines the set of agents $\Delta \subseteq \{1, \dots n\}$ that she will endow with signals.

2. Nature draws the state of the world.

3. Nature draws a signal for each agent $i \in \Delta$.

Suppose that full information revelation as in Proposition 1 was part of an equilibrium in this game as well. Then, in this equilibrium, if a voter $i$ does not reveal information in the communication stage, the only admissible belief for the other voters is to assign probability 1 to $\sigma_i = \emptyset$. In an equilibrium in which all the available information is revealed in the communication stage, $\mu_i(\kappa)$ assigns probability 1 to $\kappa = k(c)$ for all agents $i$. Hence, it is rational for voter $i$ to vote for $a$ if $p(A|\kappa = k(c)) > \theta_i$, and to vote for $b$ if $p(A|\kappa = k(c)) < \theta_i$.

Consider agent $i$, $\sigma_i = \alpha$, and assume that agents $-i$ reveal any signal they receive from nature, and hold the belief that if $c_i = s$, then $\sigma_i = \emptyset$ with probability 1. Agent $i$ is pivotal with signal $\alpha$ if and only if $\kappa_{-i} = k_m$. In this case the revelation would cause a majority to vote for $a$, while a majority votes for $b$ given only the evidence $\kappa_{-i}$. If agents $-i$ expect $i$ to reveal any information nature endows him with, they think he has no information if he stays silent, and assign probability 1 to $\kappa = k_m$ in case $i$ deviates. A concealment would not be noticed. However, whether or not he reveals his own signal to the other agents, $i$ would know that $\kappa = k_m + 1$, and assign probability $p(A|\kappa = k_m + 1)$ to state of the world $A$. Hence, he prefers the revelation of $\alpha$ if and only if $\theta_i < p(A|\kappa = k_m + 1)$. With the same line of reasoning, we conclude that an agent $i : \sigma_i = \beta$ reveals his signal if and only if all the other agents reveal their signals if $\theta_i > p(A|\kappa = k_m)$.

**Proposition 2** *Consider a communication-and-voting-game in which each voter is endowed with information with probability $\delta < 1$. There exists a full information revelation equilibrium if and only if $\theta_i \in [p(A|\kappa = k_m), p(A|\kappa = k_m + 1)]\ \forall i$.*

Full information revelation is possible if and only if all committee members agree with the median preference type which decision should be made given the presented evidence, i.e. if there is essentially no preference heterogeneity. However, there may still be considerable information aggregation even in the presence of strongly diverging interests. Suppose that there is an agent $i : \theta_i < p(A|\kappa = k_m)$. Agent $i$ disagrees with the majority only insofar as they implement $b$ in some cases in which $i$ would better like $a$. This is why $i$ does not want to show a $\beta$-signal if he is endowed with it. However, agent $i$ might be willing to reveal an $\alpha$-signal. The following proposition states that the existence of an equilibrium with considerable information aggregation is guaranteed if $\delta$ is high enough.

**Proposition 3** *Consider a communication-and-voting-game in which each voter is endowed with information with probability $\delta < 1$. There is a lower bound on $\delta$, $\delta' < 1$, such that for all $\delta > \delta'$, there is a $k$ with $|k| < |k_m|$ such that, an equilibrium exists in which at least $\frac{n+1}{2}$ agents reveal $\alpha$-signals if they are endowed with them, at least $\frac{n+1}{2}$ agents reveal $\beta$-signals if they are endowed with them, and all agents vote for $a$ if $k(c) \geq k+1$ and vote for $b$ if $k(c) \leq k$.*

We present the proof of Proposition 3 in the appendix. We show that given the beliefs induced by a certain strategy profile, it is possible to construct this strategy profile such that the strategies are mutually best responses and beliefs are indeed correct. For small values of $\delta$, the construction is possible for some preference parameter constellations, but cannot be applied for the general case. The reason is that as $\delta$ goes to zero, beliefs conditional on providing decisive information converge to $p(\omega = A|\kappa = k)$ and $p(\omega = A|\kappa = k+1)$, but are also sensitive to the communication strategy profile. Hence, we cannot guarantee that an equilibrium exists in which equilibrium beliefs concentrate around the median preference type. The equilibrium identified in Proposition 3 has the property that more than half of the agents reveal $\alpha$-signals if they are endowed with them, and more than half of the agents reveal $\beta$-signals if they are endowed with them. The decision rule is such that less extreme evidence has to be presented in order to change the decision as is necessary for the median preference type to change his mind in the full information case. The reason is that the revelation of an additional $\alpha$-signal affects beliefs in two ways, directly via the correlation, and indirectly because less unrevealed information remains.

As $\delta \to 1$, $k \to k_m/2$ for the class of equilibria identified in Proposition 3. Note that for $\delta = 1$, if half of the agents reveal either type of information, and the communication stage yields evidence $k(c)$, it follows that $\kappa = 2k(c)$. Hence, for $\delta = 1$, the decision will be $a$ if and only if $\kappa > k_m$, which implies full information aggregation as in Proposition 1 also in this class of equilibria. Although we may have considerable information aggregation in the absence of a full information revelation equilibrium, full information aggregation is impossible for $\delta < 1$.

**Proposition 4** *Consider a communication-and-voting-game in which each voter is endowed with information with probability $\delta < 1$. If a full information revelation equilibrium does not exist, then there exists no full information aggregation equilibrium.*

**Proof.** Full information aggregation requires the following. If $\kappa \leq k_m$, then there must be at least $\frac{n+1}{2}$ $b$-votes. If $\kappa > k_m$, then there must be at least $\frac{n+1}{2}$ $a$-votes. This in turn is feasible only (i) if a voter who did not reveal $\beta$ votes $b$ and would have voted $a$ if $\sigma_i = \emptyset$, and (ii) a voter who did not reveal $\alpha$ votes $a$ and would have voted $b$ if $\sigma_i = \emptyset$. As beliefs must be consistent, an agent who votes $b$ anticipates that in case his vote is pivotal, then $\kappa = k_m$, and a pivotal agent who votes $a$ infers that $\kappa = k_m + 1$. Hence, the required voting strategies are consistent with equilibrium play only if $\theta_i \in [p(A|\kappa = k_m), p(A|\kappa = k_m + 1)]$ $\forall i$. Then a full information revelation equilibrium exists. Q.E.D.

## 4.2 Private information concerning preferences

In this section, we assume that not only individual signals but also preferences are private information. Therewith, we eliminate the "I know what you want"-part from the line of argumentation underlying Proposition 1. Agent $i$'s preference parameter $\theta_i$ is drawn according to a commonly known probability function $\phi(\theta_i)$, which is assumed to be identical for all $i = 1, \ldots n$. The realization of $\theta_i$ is $i$'s private information. We denote the distribution function of individual types with $\Phi(\theta_i)$.

Nature's moves are now as follows:

1. Nature draws the agents' types.

2. Nature draws the state of the world.

3. Nature draws a signal for each committee member.

We are interested in conditions for the existence of a full information revelation equilibrium. Clearly, agents with types above 1 (respectively below 0) would never reveal information which makes the choice of $a$ (respectively $b$) more likely. If all the other agents reveal their information and vote Bayesian sincerely, the revelation of an $\alpha$-signal (respectively the revelation of a $\beta$-signal) necessarily has this effect (if any). Hence, in order that a full information revelation equilibrium exists, the support of $\phi(\theta_i)$ must be bounded. That is, there must exist $\theta_{min} > 0$, $\theta_{max} < 1$ such that $\theta_i \in [\theta_{min}, \theta_{max}]\,\forall i$. In particular, for every $i$, there must be an integer $k_{\theta_i} \in E\{-n+1, \ldots, n-1\}$ such that $\theta_i \in [p(A|\kappa = k_{\theta_i} - 1), p(A|\kappa = k_{\theta_i} + 1)]$, i.e. such that agent $i$ prefers decision $a$ for all $\kappa_{-i} > k_{\theta_i}$ and prefers decision $b$ for all $\kappa_{-i} < k_{\theta_i}$. For $\kappa_{-i} = k_{\theta_i}$, he prefers $a$ if his own signal is $\alpha$, and $b$ if his own signal is $\beta$.

In the following, we assume the existence of a full information revelation equilibrium in which all agents vote Bayesian sincerely (which implies full information aggregation). If all the information is revealed during the communication stage, then $p_i(\omega = A) = p(A|\kappa = k(c))\;\forall i$, and hence the agents' Bayesian sincere voting strategies are unique. In case of remaining uncertainty about decision-relevant information, Bayesian sincere voting strategies are determined by the belief system $\mu$. Hence, the incentive to reveal information - and therewith full information revelation in equilibrium - hinges upon the beliefs agents hold in case of a deviation (i.e. a concealment of information). To derive conditions for the existence of a full information revelation equilibrium, we specify beliefs $\mu$ which best support the full information revelation equilibrium. It suffices to specify these beliefs for the case that a single agent conceals his information in the communication stage.

Given that preferences are private information, it does not make sense to condition the beliefs in case of $i$'s silence on $i$'s name. However, beliefs can be conditioned on the communication outcome $k(c)$. Denote with $\mu_{-i}(k(c)) = \mu_{-i}\left(\kappa = k(c) + 1|\kappa_{-i} = k(c), c_i = s\right)$ the out-of-equilibrium-belief agents $j \neq i$ assign to $\sigma_i = \alpha$ in case of $i$'s silence given the communication outcome $k(c)$.

| $\kappa_{-i}$ | revelation of $\alpha$ | has an effect iff $\theta_m \in$ | benefits/ harms $i$ iff |
|---|---|---|---|
| $< k_{\theta min}$ | no effect | - | - |
| $k_{\theta min}$ | $b \rightarrow a$ | $[p(A\|\mu(k_{\theta min}))$,<br>$p(A\|\kappa = k_{\theta min} + 1)]$ | $k_{\theta_i} < k_{\theta min} + 1$<br>$k_{\theta_i} > k_{\theta min} + 1$ |
| $\hat{k}$ | $b \rightarrow a$ | $[p(A\|\mu(\hat{k}))$,<br>$p(A\|\kappa = \hat{k} + 1)]$ | $k_{\theta_i} < \hat{k} + 1$<br>$k_{\theta_i} > \hat{k} + 1$ |
| $k_{\theta max}$ | $b \rightarrow a$ | $[p(A\|\kappa = \mu(k_{\theta max}))$,<br>$p(A\|\kappa = k_{\theta max} + 1)]$ | $k_{\theta_i} < k_{\theta max} + 1$<br>$k_{\theta_i} > k_{\theta max} + 1$ |
| $> k_{max}$ | no effect | - | - |
| $\kappa_{-i}$ | revelation of $\beta$ | has an effect iff $\theta_m \in$ | benefits/ harms $i$ iff |
| $< k_{\theta min}$ | no effect | - | - |
| $k_{\theta min}$ | $a \rightarrow b$ | $[p(A\|k_{\theta min} - 1)$,<br>$p(A\|\mu(k_{\theta min}))]$ | $k_{\theta_i} > k_{\theta min} - 1$<br>$k_{\theta_i} < k_{\theta min} - 1$ |
| $\hat{k}$ | $a \rightarrow b$ | $[p(A\|\kappa = \hat{k} - 1)$,<br>$p(A\|\mu(\hat{k}))]$ | $k_{\theta_i} > \hat{k} - 1$<br>$k_{\theta_i} < \hat{k} - 1$ |
| $k_{\theta max}$ | $a \rightarrow b$ | $[p(A\|\kappa = k_{\theta max} - 1)$,<br>$p(A\|\mu(k_{\theta max}))]$ | $k_{\theta_i} > k_{\theta max} - 1$<br>$k_{\theta_i} < k_{\theta max} - 1$ |
| $> k_{max}$ | no effect | - | - |

Figure 4: Possible effects of $i$'s revelation given information revelation by agents $-i$.

To see how these beliefs best support a full information revelation equilibrium, consider the possible effects of $i$'s revelation of the two types of signals given agents $-i$ reveal their signals, and given beliefs $\mu_{-i}(k(c))$. These are illustrated in Figure 4. In this figure beliefs with respect to the state of the world given the communication outcome $c$ and information revelation by all agents but $i$ are (with slight abuse of notation) denoted with $p(A|\mu(k(c)) = (1-\mu_{-i}(k(c)))p(A|\kappa = k(c)-1) + \mu_{-i}(k(c))p(A|\kappa = k(c)+1)$. Consider agent $i : \theta_i = \theta_{min}$. Whenever this agent's revelation of an $\alpha$-signal has an effect, this effect is beneficial for agent $i$. The reason is that (as $i$ is most biased towards $a$) whenever the majority prefers $a$ over $b$ given all the available information, then $i$ likes $a$ better than $b$ as well. However, the revelation of a $\beta$-signal can have a beneficial effect for $i$ only if $\kappa_{-i} = k_{\theta_{min}}$. For $\kappa_{-i} < k_{\theta_{min}}$, all agents agree that $b$ is the best choice, regardless of $i$'s signal. For all $\kappa_{-i} > k_{\theta_{min}}$ the revelation of a $\beta$-signal can only harm $i$. Given the realization of the other agents' signals, $\kappa_{-i} = \hat{k}$, the revelation of a $\beta$-signal will change the majority decision from $a$ to $b$ if and only if the median of the preference types $\theta_m$ realized within the interval $[p(A|\kappa = \hat{k}-1), (1-\mu_{-i}(\hat{k}))p(A|\kappa = \hat{k}-1) + \mu_{-i}(\hat{k})p(A|\kappa = \hat{k}+1)]$, that is if the Bayesian sincere voting strategies for the majority prescribe to vote for $b$ in case $i$ reveals a $\beta$-signal, and prescribe to vote for $a$ given the beliefs $\mu_{-i}(k(c))$ if agent $i$ conceals his information. Note that the probability that the median voter's preference type realizes in the relevant range is highest, and hence the incentive to reveal a $\beta$-signal is strongest for agent $i : \theta_i = \theta_{min}$ if out-of-equilibrium-beliefs assign probability 1 to $\kappa = k_{min} + 1$ if $\kappa_{-i} = k_{min}$. Note also that given this belief, $i$'s revelation of an $\alpha$-signal has no effect on expected utility for $\kappa_{-i} = k_{min}$.

We now quantify the effect of information revelation versus information concealment in a full information revelation equilibrium on $i$'s expected utility. First, we fix the realizations of the random variables and suppose that the revelation of a $\beta$-signal changes the majority decision from $a$ to $b$, given $\kappa_{-i} = \hat{k}$. The effect on $i$'s expected utility is:

$$\begin{aligned}(1 - p(A|\kappa = \hat{k}-1))(u_i(b,B) - u_i(a,B)) - p(A|\kappa = \hat{k}-1)(u_i(a,A) - u_i(b,A)) \\ = (u_i(b,B) - u_i(a,B) + u_i(a,A) - u_i(b,A))(\theta_i - p(A|\kappa = \hat{k}-1)), \qquad (6)\end{aligned}$$

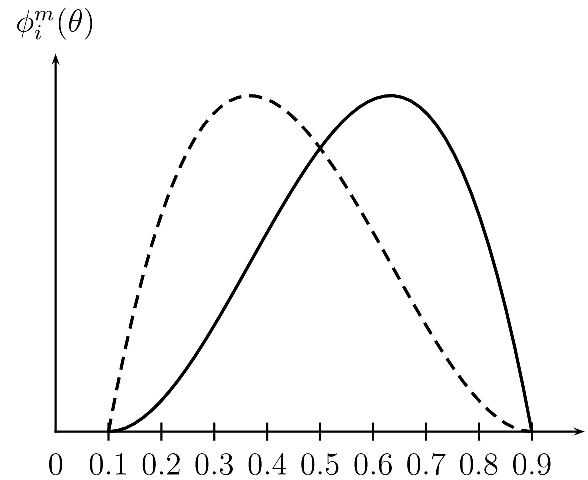


Figure 5: $\phi_i^m(\theta)$, for $n = 5$, $\phi(\theta) = U[0.1, 0.9]$, $\theta_i = 0.9$ (solid), and $\theta_i = 0.1$ (dashed).

which is positive for $(\theta_i > p(A|\kappa = \hat{k} - 1))$ and proportional to $(\theta_i - p(A|\kappa = \hat{k} - 1))$. This implies that whenever the smallest preference type gains from the revelation of a $\beta$-signal, then every other type gains as well. Symmetrically, whenever the highest preference type gains from the revelation of an $\alpha$-signal, then every other type does so.

The probability which agent $i$ assigns to the effect (6) on his expected utility (i.e. the probability assigned to the joint events $\kappa = \hat{k} - 1$ and $\theta_m \in [p(A|\kappa = \hat{k} - 1), (1 - \mu_{-i}(\hat{k}))p(A|\kappa = \hat{k} - 1) + \mu_{-i}(\hat{k})p(A|\kappa = \hat{k} + 1)]$) depends on his own private information, $\sigma_i$ and $\theta_i$.

Let $\phi_i^m(\theta')$ denote the probability which agent $i$ assigns to the event that the median voter has a type $\theta'$ given his own preference type $\theta_i$. It depends on his own type $\theta_i$ because $i$ is part of the sample drawn from $\phi(\theta)$. $\phi_i^m(\theta')$ is given by (17)–(19) which are stated in the appendix and depicted in Figure 5 for $n = 5$, and a uniform distribution on $[0.1, 0.9]$ of individual preference types. The figure shows $\phi_i^m(\theta)$ for $\theta_i = 0.1$ and $\theta_i = 0.9$.

Let $\mu_i(\kappa_{-i} = k|\sigma_i)$ denote the probability which agent $i$ assigns to the event that the realization of the other agents' signals yield $\kappa_{-i} = k$ given his own signal $\sigma_i$. Note that an agent with an $\alpha$-signal assigns a higher probability to high realizations of $\kappa_{-i}$ than an agent with a $\beta$-signal because individual signals are correlated via the state of the world.

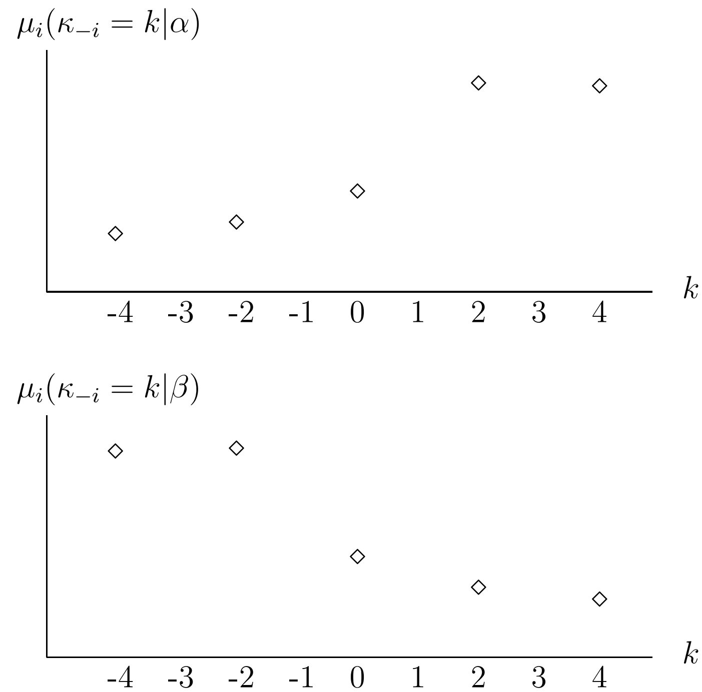


Figure 6: Beliefs regarding the other agents' signals given the own signal ($n = 5$, $q = 0.8$).

$\mu_i(\kappa_{-i} = k|\sigma_i)$ is given by (20) and (21) in the appendix and depicted in Figure 6 for $n = 5$ and $q = 0.8$. The figure shows $\mu_i(\kappa_{-i} = k|\sigma_i)$ for $\sigma_i = \alpha$ and $\sigma_i = \beta$

Given that all agents $-i$ reveal their information and hold beliefs $\mu(k(c))$, the effect of $i$'s revelation of a $\beta$-signal on expected utility is proportional to:

$$\sum_{k=k_{\theta_{min}}}^{k_{\theta_{max}}} \mu_i(\kappa_{-i} = k|\beta)(\theta_i - p(A|\kappa = k-1)) \int_{p(A|\kappa=k-1)}^{(1-\mu_{-i}(k))p(A|\kappa=k-1)+\mu_{-i}(k)p(A|\kappa=k+1)} \phi_i^m(\theta)d\theta, \tag{7}$$

and the effect of the revelation of an $\alpha$-signal on $i$'s expected utility is proportional to:

$$\sum_{k=k_{\theta_{min}}}^{k_{\theta_{max}}} \mu_i(\kappa_{-i} = k|\alpha)(p(A|\kappa = k+1) - \theta_i) \int_{(1-\mu_{-i}(k))p(A|\kappa=k-1)+\mu_{-i}(k)p(A|\kappa=k+1)}^{p(A|\kappa=k+1)} \phi_i^m(\theta)d\theta. \tag{8}$$

As argued above, (8) is unambiguously positive for $\theta_i = \theta_{min}$. Similarly, (7) is positive for $\theta_i = \theta_{max}$. In the following we derive conditions under which (7) is non-negative for $\theta_i = \theta_{min}$, and (8) is non-negative for $\theta_i = \theta_{max}$.

(7) is non-negative for $\theta_i = \theta_{min}$ iff

$$\mu_i(\kappa_{-i} = k_{\theta_{min}}|\beta)(\theta_{min} - p(A|\kappa = k_{\theta_{min}} - 1)) \int\limits_{p(A|\kappa=k_{\theta_{min}}-1)}^{(1-\mu_{-i}(k_{\theta_{min}}))p(A|\kappa=k_{min}-1)+\mu_{-i}(k_{\theta_{min}})p(A|\kappa=k_{\theta_{min}}+1)} \phi^m_{min}(\theta)d\theta$$

$$\geq \sum_{k=k_{\theta_{min}}+2}^{k_{\theta_{max}}} \mu_i(\kappa_{-i} = k|\beta)(p(A|\kappa = k-1) - \theta_{min}) \int\limits_{p(A|\kappa=k)}^{(1-\mu_{-i}(k))p(A|\kappa=k-1)+\mu(k)p(A|\kappa=k+1)} \phi^m_{min}(\theta)d\theta, \tag{9}$$

and (8) is non-negative for $\theta_i = \theta_{max}$ iff

$$\mu_i(\kappa_{-i} = k_{\theta_{max}}|\alpha)(p(A|\kappa = k_{\theta_{max}} + 1) - \theta_{max}) \int\limits_{(1-\mu_{-i}(k_{\theta_{max}}))p(A|\kappa=k_{\theta_{max}}-1)+\mu_{-i}(k_{\theta_{max}})p(A|\kappa=k_{\theta_{max}}+1)}^{p(A|\kappa=k_{\theta_{max}}+1)} \phi^m_{max}(\theta)d\theta$$

$$\geq \sum_{k=k_{\theta_{min}}}^{k_{\theta_{max}}-2} \mu_i(\kappa_{-i} = k|\alpha)(\theta_{max} - p(A|\kappa = k)) \int\limits_{(1-\mu_{-i}(k))p(A|\kappa=k-1)+\mu_{-i}(k)p(A|\kappa=k+1)}^{p(A|\kappa=k+1)} \phi^m_{max}(\theta)d\theta, \tag{10}$$

where $\phi^m_{min}(\theta)$ is given by (19), $\phi^m_{max}(\theta)$ is given by (17), $\mu_i(\kappa_{-i} = k|\beta)$ is given by (21), and $\mu_i(\kappa_{-i} = k|\alpha)$ is given by (20).

Obviously, (9) and (10) are necessary conditions for the existence of a full information revelation equilibrium. Note that in case agent $i$'s expected utility is higher if he reveals $\beta$ ($\alpha$) than if he conceals the information, then the same is true for agent $j : \theta_j > \theta_i$ ($\theta_j < \theta_i$) because $j$ benefits relatively more and looses relatively less than agent $i$ whenever the revelation has an effect. Hence, conditions (9) and (10) are also sufficient for the existence of a full information revelation equilibrium. That is, for the existence of a full information revelation equilibrium in which agents vote Bayesian sincerely, it suffices to make sure that the type who is most biased towards alternative $a$ is willing to reveal a $\beta$-signal and that the type who is most biased towards alternative $b$ is willing to reveal an $\alpha$-signal.

If $k_{\theta_{min}} = k_{\theta_{max}}$, then (9) and (10) hold for any $\mu_{-i}(k(c))$, as the left-hand-sides are zero. It should be intuitively clear that there is no incentive to conceal information in this case because $k_{\theta_{min}} = k_{\theta_{max}}$ implies that there is essentially no preference heterogeneity, i.e. voters agree on the mapping of information into the decision.

If $k_{\theta_{min}} < k_{\theta_{max}}$, then the right-hand-side of (9) increases, and the left-hand-side of (10) decreases in $\mu_{-i}(k_{min})$. Hence, out-of-equilibrium beliefs $\mu_{-i}(k(c))$ best support the full information revelation equilibrium if $\mu_{-i}(k_{\theta_{min}}) = 1$. Similarly, out-of-equilibrium-beliefs best support the full information revelation equilibrium if $\mu_{-i}(k_{\theta_{max}}) = 0$.

This observation yields sufficient conditions for the existence of a full information revelation equilibrium, stated in Proposition 5.

**Proposition 5** *Consider a communication-and-voting-game in which the preference parameters $\theta_i$ are private information.*
*(i) There exists a full information revelation equilibrium if the preference parameters are drawn from $[p(A|\kappa = k-1), p(A|\kappa = k+3)]$ for all agents $i$ and some even integer $k \in \{-n+1, \ldots, n-3\}$.*
*(ii) There exists a full information revelation equilibrium if the preference parameters are drawn from $[p(A|k_{\theta_{min}} - 1), p(A|k_{\theta_{min}} + 1)] \cup [p(A|k_{\theta_{max}} - 1), p(A|k_{\theta_{max}} + 1)]$ for all agents $i$ and some even integers $k_{\theta_{min}}, k_{\theta_{max}} \in \{-n+1, \ldots, n-1\}$.*

Hence, full information aggregation is possible in heterogeneous committees if preference heterogeneity is not too severe. For $q = 0.8$, a sufficient condition for the existence of a full information revelation equilibrium is that preference types are drawn from the interval $[0.2, 0.985]$. That is, the committee may have members who need to be at least 80% sure that the state of the world is $B$ in order to support alternative $b$ as well as agents who need to be 98.5% sure that the state of the world is $A$ in order to support decision $a$ (and any preference type in between). Moreover, full information aggregation is possible regardless of the quality of the signal if preference types assume only two values between 0 and 1, provided that there exist realizations of the signals for each type which convinces him of either alternative (this can be achieved by increasing the number of committee members).

If the conditions stated in Proposition 5 hold, out-of-equilibrium-beliefs can be defined such that a revelation has an effect only if the preferences of the median voter are aligned with the own preferences. However, even if information revelation has unfavorable effects in some cases, the effect may still be favorable in expectation, such that (9) and (10) hold for more heterogeneous preferences (as measured by $\theta_{max} - \theta_{min}$) than in part (i) of

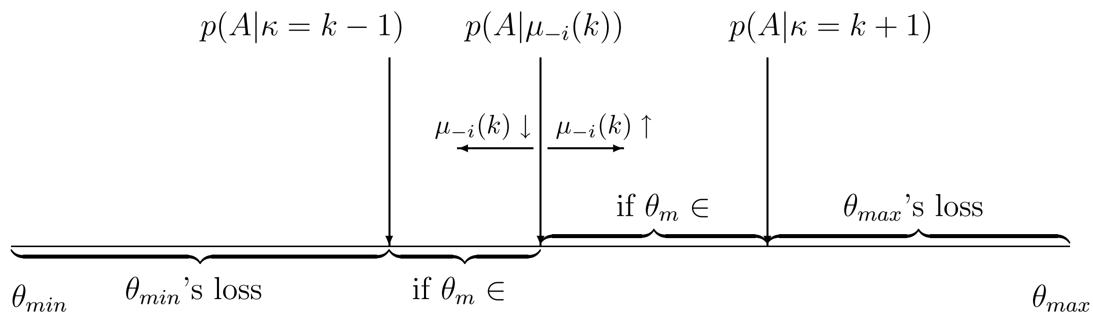


Figure 7: $\mu_{-i}(k)$ determines the probabilities with which $i$ looses from signal revelation.

the proposition and more general preference distributions than in part (ii). It is hard to tell in general, which out-of-equilibrium-beliefs best support a full information revelation equilibrium. Figure 7 illustrates how the out-of-equilibrium-beliefs affect the incentives for information revelation for the most biased committee members for $\kappa_{-i} = k \neq k_{\theta_{min}}, k_{\theta_{max}}$. By decreasing $\mu_{-i}(k)$ for some $k \neq k_{min}, k_{max}$, the incentive to reveal a $\beta$-signal increases for type $\theta_{min}$ because the probability that he will incur the loss $p(A|\kappa = k-1) - \theta_{min}$ decreases. At the same time, the incentive for type $\theta_{max}$ to reveal an $\alpha$-signal decreases because he will incur the loss $\theta_{max} - p(A|\kappa = k+1)$ with a higher probability. The smaller (higher) $k$, the higher the loss incurred by type $\theta_{max}$ ($\theta_{min}$) in case the revelation of a $\beta$-signal ($\alpha$-signal) has an effect. From this point of view, $\mu_{-i}(k)$ should be high for small realizations of $\kappa_{-i}$, and low for high values. However, type $\theta_{min}$, endowed with a $\beta$-signal, assigns less probability to both events than type $\theta_{max}$ with an $\alpha$-signal, (i) the realization of a high $\kappa_{-i}$, and (ii) the realization of the median type in the relevant range (see Figures 5 and 6). Which of the two forces is stronger depends on the parameters of the model. As the necessary conditions for the existence of a full information revelation equilibrium can only be derived with knowledge of the most favorable out-of-equilibrium-beliefs, they cannot be stated without further specification of the model. We provide an example in the appendix, where we show that the sufficient conditions for the existence of a full information revelation equilibrium are not necessary.

We can again exclude other full information aggregation equilibria, if the full information revelation equilibrium does not exist.

**Proposition 6** *Consider a communication-and-voting-game in which the preference parameters $\theta_i$ are private information. If a full information revelation equilibrium does not exist, then there exists no full information aggregation equilibrium.*

**Proof.** A necessary condition for a strategy profile to be a full information aggregation equilibrium is that the decision is responsive to each signal. This requires informative voting by those who did not reveal their signals. Then, all $a$-voters draw the same inferences out of being pivotal, and all $b$-voters draw the same inferences when they are pivotal. Hence, there is a $k$ such that $p(A|\kappa = k-1) \leq \theta_i \leq p(A|\kappa = k+1)$ $\forall i$. A full information revelation equilibrium exists. Q.E.D.

If preferences are private information, the preference parameter range for which a full information revelation equilibrium exists is smaller than in the common knowledge case (where it is unbounded), but larger than in the case in which committee members are informed about other members' preferences but can pretend to have no information. The reason is that the median voter may agree even with the most biased committee members. In the next section, we show that this effect may allow for a larger preference parameter range in case committee members can pretend to have no information.

### 4.3 Private preferences, possibility of receiving no signal

If there is the possibility of receiving no signal, we cannot support the full information revelation equilibrium with out-of-equilibrium-beliefs anymore, because no action can be identified as being out-of-equilibrium. As pointed out in Section 4.1, in a full information revelation equilibrium the only admissible belief regarding $i$'s signal when observing $c_i = s$ is $\sigma_i = \emptyset$. The necessary and sufficient conditions for the existence of a full information revelation equilibrium are – analogously to (9) and (10):

$$\mu_i(\kappa_{-i} = k_{min}+1|\beta)(\theta_{min} - p(A|\kappa = k_{min})) \int_{p(A|\kappa=k_{min})}^{p(A|\kappa=k_{min}+1)} \phi_{min}^m(\theta)d\theta$$

$$\geq \sum_{k=k_{min}+2}^{k_{max}+1} \mu_i(\kappa_{-i} = k|\beta)(p(A|\kappa = k-1) - \theta_{min}) \int_{p(A|\kappa=k-1)}^{p(A|\kappa=k)} \phi_{min}^m(\theta)d\theta,$$

$$\mu_i(\kappa_{-i} = k_{max}|\alpha)(p(A|\kappa = k_{max}+1) - \theta_{max}) \int\limits_{p(A|\kappa=k_{max})}^{p(A|\kappa=k_{max}+1)} \phi^m_{max}(\theta)d\theta$$

$$\geq \sum_{k=k_{min}}^{k_{max}-1} \mu_i(\kappa_{-i} = k|\alpha)(\theta_{max} - p(A|\kappa = k+1)) \int\limits_{p(A|\kappa=k)}^{p(A|\kappa=k+1)} \phi^m_{max}(\theta)d\theta,$$

and $\theta_{min} \in [p(A|k_{min}), p(A|k_{min}+1)]$ and $\theta_{max} \in [p(A|k_{max}), p(A|k_{max}+1)]$.

Note that the necessary condition for the existence of a full information revelation equilibrium in case preferences are common knowledge, $k_{min} = k_{max}$, (see Proposition 2) is sufficient here. As in the previous section, committee members are uncertain about the median voter's preferences. If the probability that the majority has the same interests as one's own is high enough, committee members have an incentive to reveal their information even if there are (potentially) conflicts of interest. We illustrate this possibility by means of an example.

Consider a committee with three members, and suppose each of them receives a signal from nature with probability $\delta$. If an agent receives a signal, the signal is correct with probability 0.8. Preference parameters $\theta_i$ are drawn from $[0.2+\epsilon, 0.8-\epsilon]$ according to a uniform distribution. Suppose agents 1 and 2 reveal their information. If both reveal an $\alpha$-signal, then all agents agree the decision should be $a$ regardless of agent 3's information. If both reveal a $\beta$-signal, then all agents agree the decision should be $b$ regardless of agent 3's information. Suppose agent 3 is endowed with a $\beta$-signal. This information will be pivotal if (i) $\theta_m \in [0.2, 0.5]$ and (a) agents 1 and 2 reveal different signals, or (b) none of the other agents received information, or (ii) $\theta_m \in [0.5, 0.8]$ and one of the other agents revealed an $\alpha$-signal and the other agent did not receive information. If $\theta_3 \in [0.2, 0.5]$, the revelation of the $\beta$-signal is beneficial in case (i), but not in case (ii). If $\theta_3 \in [0.5, 0.8]$, the revelation is beneficial in both cases. Agent $3 : \theta_3 = 0.2 + \epsilon$ (and hence all other types) has an incentive to reveal a $\beta$-signal if

$$\begin{aligned} \epsilon * \tfrac{3}{4}(\delta^2 * 2 * 0.8 * 0.2 + (1-\delta)^2) &> (0.3-\epsilon) * \tfrac{1}{4} * 2\delta(1-\delta)2 * 0.8 * 0.2 \\ \Leftrightarrow \epsilon &> \frac{\delta(1-\delta)}{5\delta^2 + \frac{125}{8}(1-\delta)^2 + \frac{10}{3}\delta(1-\delta)}. \end{aligned}$$

Because of the symmetry of the parameter constellation, the condition for the revelation of an $\alpha$-signal for preference type $0.8 - \epsilon$ is the same.

For values of $\delta$ close to 0 or 1, a full information revelation equilibrium exists for very small $\epsilon$. Independently of $\delta$, the existence of a full information revelation equilibrium in this example is guaranteed if $\theta_{min} = 0.25$, and $\theta_{max} = 0.75$. Note that for this preference parameter range, there exists no full information revelation equilibrium in case preferences are common knowledge. Summing up, we have the following proposition.

**Proposition 7** *Consider two communication-and-voting-games* $\Gamma^c, \Gamma^p$ *in which each voter is endowed with a signal with probability* $\delta < 1$*. The games are identical except that in* $\Gamma^c$*, preferences are common knowledge, and in* $\Gamma^p$*, preferences are private information.*
*(i) If* $\Gamma^c$ *has a full information revelation equilibrium, then* $\Gamma^p$ *has a full information revelation equilibrium.*
*(ii) There are parameter constellations such that* $\Gamma^p$ *has a full information revelation equilibrium, whereas no full information revelation equilibrium exists for* $\Gamma^c$*.*

For $\delta < 1$ the existence of a full information revelation equilibrium hinges upon voluntary information revelation by the players. As we have shown, the incentive to do so may be greater in the light of uncertainty about the majority's preferences. For $\delta = 1$, the event that a committee member does not reveal information arises in a full information revelation equilibrium only of the equilibrium path. Information revelation can be supported by out-of-equilibrium beliefs. If preferences are common knowledge, out-of-equilibrium beliefs can be conditioned on the preferences of the deviating player, whereas in the private information case, this is impossible.

# 5 Conclusion

This paper provides a first step towards the analysis of committees who deal with verifiable information, and whose members have conflicting interests. We identified conditions under which all decision-relevant information is revealed at the communication stage and taken into account at the stage of voting. If preferences are common knowledge, and every committee member is endowed with information with certainty, then there exists an equilibrium with these properties independently of the extent of preference heterogeneity.

If preferences are private information, then there exists a full information revelation equilibrium if preference heterogeneity is not too severe. If preferences are common knowledge, but agents are endowed with information only with a certain probability, then full information aggregation is possible if and only if all voters agree how the information should be mapped into a decision, i.e. if there are no conflicts of interest. Moreover, if there is the possibility of receiving no signal then preferences being private information allows for a larger extent of preference heterogeneity than the common knowledge case. The reason is the possibility that the majority can have the same interests as oneself.

Our results may be used to analyze the quality of collective decisions in several extended frameworks. First of all, it is worth studying incentives for information acquisition. In the present paper, information comes for free and in the basic model every committee member possesses private information with certainty. The impossibility to lie about the realization of private signals allows the committee to deduce a member's information even if this member does not want the committee to be aware of it. This could weaken incentives to acquire information in the first place beyond the usually found free riding problem.

If preferences are homogeneous 'enough', we can expect efficient information aggregation. However, information aggregation may be a problem if preferences are too heterogeneous (if there is the possibility that some agents are not endowed with information and/or preferences are private information). Agents might want to exclude agents who have preferences which are too distinct from their own from communication, while sharing their information with more like-minded. There may be demand for a device which allows agents to match into a homogeneous subgroup in order to pool information more efficiently. It would be interesting to compare the efficiency of a system in which information is pooled within subgroups (which may be interpreted as political parties) who are represented by a single voice in the voting stage to the efficiency of information aggregation within a direct democracy.

We used the simple majority rule as the decision mechanism. For homogenous preferences, there exists a unique best decision rule (Costinot and Kartik, 2006). An interesting extension would be to take a mechanism design perspective in a setting with heterogenous preferences. Suppose for instance an alternative needs a fraction $q > 1/2$ of the votes in order to be implemented. If no alternative gets this fraction, then the status quo is maintained. Then, information $\alpha$ is pivotal in two cases: for changing the decision from status quo to $a$, and for changing it from $b$ to status quo. Full information revelation might be possible for parameter ranges for which it is impossible using simple majority rule. The optimal mechanism must trade-off the provision of incentives to reveal information versus the risk of maintaining the status quo to often.

In our model, individual signals are – conditional on the state of the world – independent random variables. This is a good assumption if committee members have different areas of expertise. In other cases, it might be more appropriate to allow for the possibility that the information contained in the agents' signals partially overlaps. An example would be the hiring committee, where some of the candidates' characteristics are more easily observable than others. The information environment could be modeled as a set of verifiable signals, containing information about the alternatives at hand, out of which nature draws a subset for each committee member. In such a setting (again referring to the hiring committee), it would be particularly interesting to allow for a manipulation of nature's moves (influenced by the candidates' actions) and to study the interaction with committee members' information acquisition efforts.

# Appendix

**Derivation of Equation (2).**

Given $k_\alpha$ $\alpha$-signals and $k_\beta$ $\beta$-signals, Bayesian updating yields:

$$\begin{aligned} p(\omega = A|\kappa = k_\alpha - k_\beta) &= \frac{\frac{1}{2}q^{k_\alpha}(1-q)^{k_\beta}}{\frac{1}{2}q^{k_\alpha}(1-q)^{k_\beta} + \frac{1}{2}q^{k_\beta}(1-q)^{k_\alpha}} \\ &= \frac{q^{k_\alpha - k_\beta}}{q^{k_\alpha - k_\beta} + (1-q)^{k_\alpha - k_\beta}} \end{aligned}$$

Defining $k = k_\alpha - k_\beta$ gives us:

$$p\left(A\,|\kappa = k\,\right) = \frac{q^k}{q^k + (1-q)^k}.$$

**Derivation of Equation (3).**

There are exactly $k$ more $\alpha$-signals than $\beta$-signals ($k$ possibly negative) within the group of voters except for $i$ if there are exactly $\frac{n-1+k}{2}$ $\alpha$-signals, and (the residuum) $n-1-\frac{n-1+k}{2}$ $\beta$-signals. We have to sum up all these cases:

$$\mu_i(\kappa_{-i} = k) = \begin{cases} \sum\limits_{J\subseteq\{1,\ldots n\}\setminus\{i\}:|J|=\frac{n-1+k}{2}} \prod\limits_{j\in J} \mu_i(\sigma_j = \alpha) \prod\limits_{l\in\{1,\ldots n\}\setminus\{J\cup i\}} \mu_i(\sigma_l = \beta), \\ \qquad\qquad \text{for } k \in E\{-n+1,\ldots,n-1\} \\ 0, \qquad\qquad \text{for } k \notin \{-n+1,\ldots,n-1\}. \end{cases}$$

**Proof of Proposition 3.**

First note that in a unanimous voting strategy profile (conditional on the communication outcome) as in the potential equilibrium, no single vote has an effect on the outcome. Hence there exists no profitable deviation at the voting stage. Moreover, the voting strategy profile has the property that no private information (information which was not revealed in the communication stage) will be aggregated in the voting stage. The collective decision depends only on the evidence presented in the communication stage: $x = a$, if $k(c) \geq k + 1$, and $x = b$ else.

In the following, we can take the decision rule as given. Note that the revelation of an $\alpha$-signal can only have the effect to change the decision from $b$ to $a$, and vice versa for the revelation of a $\beta$-signal.

Agent $i$ has an incentive to reveal an $\alpha$-signal if and only if he believes that $p_i(\omega = A) \geq \theta_i$ conditional on the event that his revelation changes the decision from $b$ to $a$, i.e. conditional on the evidence being $k$ without his revelation. He has an incentive to reveal a $\beta$-signal if and only if he believes that $p_i(\omega = A) \leq \theta_i$ conditional on the event that his revelation changes the decision from $a$ to $b$, i.e. conditional on the evidence being $k+1$ without his revelation.

$\gamma^*$ is a communication equilibrium (given the decision rule) iff

(i) $\forall i : \gamma_i^*(\sigma_i) = \begin{cases} \sigma_i, & \text{for } \sigma_i = \alpha \\ s, & \text{for } \sigma_i = \beta \end{cases}$ :

$$p_i(\omega = A|\alpha_i^{piv}) \geq \theta_i, \text{ and } p_i(\omega = A|\beta_i^{piv}) > \theta_i,$$

(ii) $\forall i : \gamma_i^*(\sigma_i) = \sigma_i$ :

$$p_i(\omega = A|\sigma_i = \alpha_i^{piv}) \geq \theta_i \geq p_i(\omega = A|\beta_i^{piv}), \text{ and}$$

(iii) $\forall i : \gamma_i^*(\sigma_i) = \begin{cases} s, & \text{for } \sigma_i = \alpha \\ \sigma_i, & \text{for } \sigma_i = \beta \end{cases}$ :

$$p_i(\omega = A|\alpha_i^{piv}) < \theta_i, \text{ and } p_i(\omega = A|\beta_i^{piv}) \leq \theta_i,$$

where $p_i(\omega = A|\hat{\sigma}_i^{piv})$ denotes the probability $i$ assigns to $\omega = A$ conditional on the event that his signal $\hat{\sigma}_i$ is pivotal for the decision, taking as given the communication strategies of agents $-i$ and the decision rule. We will have a closer look at $p_i(\omega = A|\hat{\sigma}^{piv})$. It is convenient to introduce some further notation.

Consider a (pure) communication strategy profile $\gamma$. Denote $\mathcal{N}_\beta(\gamma) = \{i : \gamma_i(\beta) = \beta\}$, $n_\beta(\gamma) = |\mathcal{N}_\beta(\gamma)|$, $\mathcal{N}_\alpha(\gamma) = \{i : \gamma_i(\alpha) = \alpha\}$, $n_\alpha(\gamma) = |\mathcal{N}_\alpha(\gamma)|$. Denote with $k_{-i}(c)$ the evidence provided by agents $-i$ in the communication stage.

Given communication strategies $\gamma_{-i}$, $k_{-i}(c) = k$ happens if (and only if) there are $k+l$ $\alpha$-signals within the group of committee members other than $i$ planning to reveal $\alpha$, i.e. agents $j \in \mathcal{N}_\alpha(\gamma^*) \setminus \{i\}$, and $l$ $\beta$-signals within the group of committee members (other than $i$) planning to reveal $\beta$ (agents $j \in \mathcal{N}_\beta(\gamma^*) \setminus \{i\}$), for all $l = \max\{0, -k\}, \ldots, \min\{n_\alpha(\gamma^*) - k - 1_{\gamma_i^*(\alpha)=\alpha}, n_\beta(\gamma^*) - 1_{\gamma_i^*(\beta)=\beta}\}$, where $1_x = 1$ if $x$ is true and 0 else. Abbreviate $L(\gamma) = \{\max\{0, -k\}, \ldots, \min\{n_\alpha(\gamma^*) - k - 1_{\gamma_i^*(\alpha)=\alpha}, n_\beta(\gamma^*) - 1_{\gamma_i^*(\beta)=\beta}\}$ An $\alpha$-signal is pivotal in state $A$ with probability

$$\begin{aligned} prob\{\alpha_i^{piv}|\omega = A\} \quad = \quad & \sum_{l \in L(\gamma^*)} \binom{n_\alpha(\gamma^*) - 1_{\gamma_i^*(\alpha)=\alpha}}{k+l} (\delta q)^{k+l} (1 - \delta q)^{n_\alpha(\gamma^*) - 1_{\gamma_i^*(\alpha)=\alpha} - k - l} \\ & * \binom{n_\beta(\gamma^*) - 1_{\gamma_i^*(\beta)=\beta}}{l} (\delta(1-q))^l (1 - \delta(1-q))^{n_\beta(\gamma^*) - 1_{\gamma_i^*(\beta)=\beta} - l}. \end{aligned} \tag{11}$$

In state $B$, an $\alpha$-signal is pivotal with probability

$$prob\{\alpha_i^{piv}|\omega = B\} = \sum_{l\in L(\gamma^*)} \binom{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}}{k+l}(\delta(1-q))^{k+l}(1-\delta(1-q))^{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}-k-l} * \binom{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}}{l}(\delta q)^l(1-\delta q)^{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}-l}. \quad (12)$$

Using Bayes' Rule, we have

$$\begin{aligned} p_i(\omega = A|\alpha_i^{piv}) &= \frac{p(\omega=A|\sigma_i=\alpha)prob\{\alpha_i^{piv}|\omega=A\}}{p(\omega=A|\sigma_i=\alpha)prob\{\alpha_i^{piv}|\omega=A\}+(1-p(\omega=A|\sigma_i=\alpha))prob\{\alpha_i^{piv}|\omega=B\}} \\ &= \frac{qprob\{\alpha_i^{piv}|\omega=A\}}{qprob\{\alpha_i^{piv}|\omega=A\}+(1-q)prob\{\alpha_i^{piv}|\omega=B\}} \\ &= \frac{1}{1+\left(\frac{1-q}{q}\right)^{k+1}\left(\frac{1-\delta(1-q)}{1-\delta q}\right)^{n_\alpha(\gamma^*)-n_\beta(\gamma^*)-k-1_{\gamma_i^*(\alpha)=\alpha}+1_{\gamma_i^*(\beta)=\beta}}}. \quad (13) \end{aligned}$$

Analogously, $k_{-i}(c) = k+1$ happens if there are $k+1+l$ $\alpha$-signals within the group of committee members other than $i$ planning to reveal $\alpha$, i.e. agents $j \in \mathcal{N}_\alpha(\gamma^*) \setminus \{i\}$, and $l$ $\beta$-signals within the group of committee members (other than $i$) planning to reveal $\beta$ (agents $j \in \mathcal{N}_\beta(\gamma^*) \setminus \{i\}$), for all $l \in L(\gamma^*) \setminus \{-k\ ; n_\alpha(\gamma^*) - k - 1_{\gamma_i^*(\alpha)=\alpha}\}$. Denote $L'(\gamma^*) = L(\gamma^*) \setminus \{-k\ ; n_\alpha(\gamma^*) - k - 1_{\gamma_i^*(\alpha)=\alpha}\}$.

A $\beta$-signal is pivotal in state $A$ with probability

$$prob\{\beta_i^{piv}|\omega = A\} = \sum_{l\in L'(\gamma^*)} \binom{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}}{k+1+l}(\delta q)^{k+1+l}(1-\delta q)^{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}-k-1-l} \quad (14)$$
$$* \binom{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}}{l}(\delta(1-q))^l(1-\delta(1-q))^{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}-l}.$$

In state $B$, a $\beta$-signal is pivotal with probability

$$prob\{\beta_i^{piv}|\omega = B\} = \sum_{l\in L'(\gamma^*)} \binom{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}}{l}(\delta q)^l(1-\delta q)^{n_\beta(\gamma^*)-1_{\gamma_i^*(\beta)=\beta}-l} \quad (15)$$
$$* \binom{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}}{k+1+l}(\delta(1-q))^{k+1+l}(1-\delta(1-q))^{n_\alpha(\gamma^*)-1_{\gamma_i^*(\alpha)=\alpha}-k-1-l}$$

Using Bayes' Rule, we have

$$\begin{aligned} p_i(\omega = A|\beta_i^{piv}) &= \frac{p(\omega=A|\sigma_i=\beta)prob\{\beta_i^{piv}|\omega=A\}}{p(\omega=A|\sigma_i=\beta)prob\{\alpha_i^{piv}|\omega=A\}+(1-p(\omega=A|\sigma_i=\beta))prob\{\alpha_i^{piv}|\omega=B\}} \\ &= \frac{(1-q)prob\{\beta_i^{piv}|\omega=A\}}{(1-q)prob\{\beta_i^{piv}|\omega=A\}+(1-q)prob\{\beta_i^{piv}|\omega=B\}} \\ &= \frac{1}{1+\left(\frac{1-q}{q}\right)^{k}\left(\frac{1-\delta(1-q)}{1-\delta q}\right)^{n_\alpha(\gamma^*)-n_\beta(\gamma^*)-k-1_{\gamma_i^*(\alpha)=\alpha}+1_{\gamma_i^*(\beta)=\beta}-1}}. \quad (16) \end{aligned}$$

We have that $p_i(\omega = A|\beta_i^{piv}) < p_i(\omega = A|\alpha_i^{piv})\ \forall i$. Hence, given the decision rule $k$, in any communication equilibrium (in pure strategies) each voter reveals at least one type of signal. Note that $p_i(\omega = A|\hat{\sigma}_i^{piv})$ is ceteris paribus higher (i) the higher $k$, (ii) the lower $n_\alpha(\gamma)$, and (iii) the higher $n_\beta(\gamma)$. Note also that $p_i(.)$ are the same for $i$ (given $k$) for communication profiles $\gamma'$ and $\gamma''$ if $n_\alpha(\gamma') - n_\beta(\gamma') = n_\alpha(\gamma'') - n_\beta(\gamma'')$ and $\gamma_i' = \gamma_i''$. Further note that $p_i(\omega = A|\hat{\sigma}_i^{piv}) = p_j(\omega = A|\hat{\sigma}_j^{piv})$ if $\gamma_i = \gamma_j$.

Consider a communication profile $\gamma$. Denote the belief $p_i(\omega = A|\alpha_i^{piv})$ of an agent who reveals both types of signals (if endowed with them), i.e. agent $i : \gamma_i(\sigma_i) = \sigma_i$ with $p_\alpha(\gamma)$ and $p_i(\omega = A|\beta_i^{piv})$ with $p_\beta(\gamma)$. Similarly, denote the beliefs of an agent who reveals $\alpha$ and conceals $\beta$, i.e. agent $i : \gamma_i(\alpha) = \alpha, \gamma_i(\beta) = s$ with $p_\alpha^{\beta-conc}(\gamma)$ and $p_\beta^{\beta-conc}(\gamma)$, respectively. Denote the beliefs of agent $i : \gamma_i(\alpha) = s, \gamma_i(\beta) = \beta$ with $p_\alpha^{\alpha-conc}(\gamma)$ and $p_\beta^{\alpha-conc}(\gamma)$, respectively. It is easy to verify that $p_{\hat{\sigma}}^{\beta-conc}(\gamma) > p_{\hat{\sigma}}(\gamma) > p_{\hat{\sigma}}^{\alpha-conc}(\gamma), \hat{\sigma} = \alpha, \beta$. Concerning the position of $p_\alpha^{\alpha-conc}(\gamma)$ and $p_\beta^{\beta-conc}(\gamma)$, we have to distinguish three cases:

(i) If $\delta < \delta'(q)$, we have

$$p_\beta^{\alpha-conc}(\gamma) < p_\beta(\gamma) < p_\beta^{\beta-conc}(\gamma) < p_\alpha^{\alpha-conc}(\gamma) < p_\alpha(\gamma) < p_\alpha^{\beta-conc}(\gamma),$$

(ii) if $\delta'(q) < \delta < \delta''(q)$, we have

$$p_\beta^{\alpha-conc}(\gamma) < p_\beta(\gamma) < p_\alpha^{\alpha-conc}(\gamma) < p_\beta^{\beta-conc}(\gamma) < p_\alpha(\gamma) < p_\alpha^{\beta-conc}(\gamma),$$

(iii) if $\delta > \delta''(q)$, we have

$$p_\beta^{\alpha-conc}(\gamma) < p_\alpha^{\alpha-conc}(\gamma) < p_\beta(\gamma) < p_\alpha(\gamma) < p_\beta^{\beta-conc}(\gamma) < p_\alpha^{\beta-conc}(\gamma),$$

where $\delta'(q) = \frac{1-\left(\frac{q}{1-q}\right)^{1/3}}{1-q\left(1+\left(\frac{q}{1-q}\right)^{1/3}\right)}$ and $\delta''(q) = \frac{1-\left(\frac{q}{1-q}\right)^{1/2}}{1-q\left(1+\left(\frac{q}{1-q}\right)^{1/2}\right)}$. The three cases are depicted in Figure 8. In case (i), the information contained in any committee member's silence plays a minor role, because the endowment with information is relatively unlikely. Hence, beliefs are mainly determined by revealed information and the own signal. As the endowment with information becomes more likely, communication strategies of the other committee members gain importance whereas the own information endowment becomes relatively unimportant for the beliefs. As $\delta \to 1$, $p_\alpha - p_\beta \to 0$. To see why this is the case,

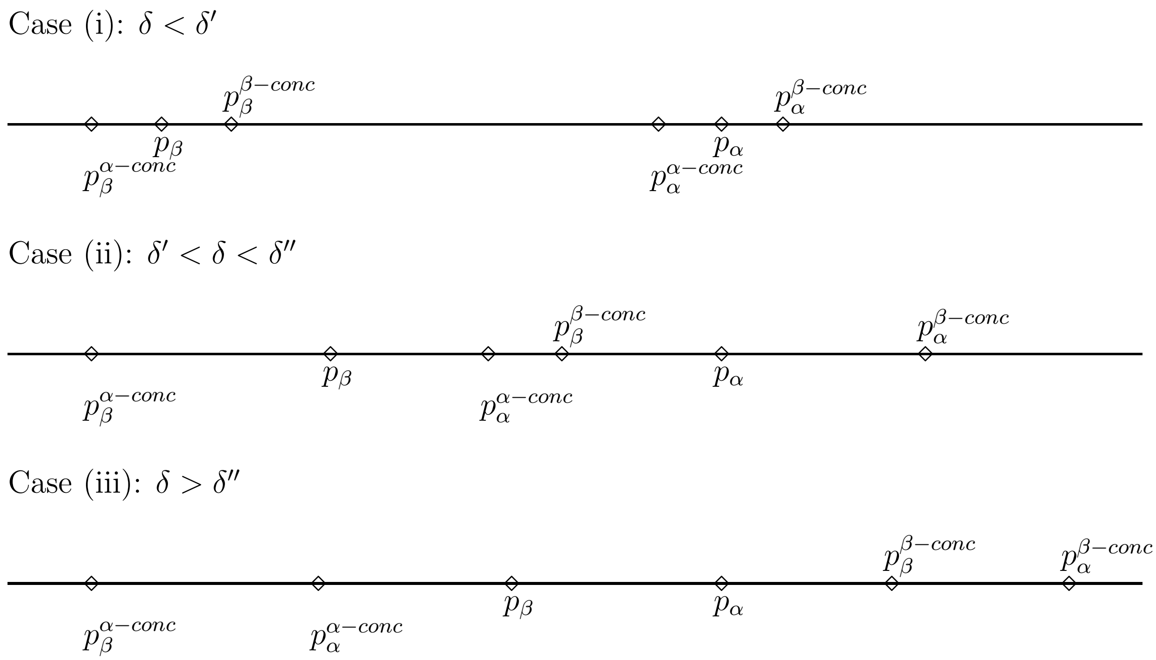


Figure 8: Structure of the committee members' beliefs.

suppose that $\delta = 1$ and suppose that $n_\alpha$ agents other than $i$ reveal $\alpha$ and $n_\beta$ agents other than $i$ reveal $\beta$. Agent $i$'s $\alpha$ is pivotal if $k+l$ $\alpha$-signals and $l$ $\beta$-signals are revealed – which implies that $(n-1-n_\alpha-l)$ $\alpha$-signals and $(n-1-n_\beta-k-l)$ $\beta$-signals are concealed. Hence, $i$ can infer that $\kappa = 2k + n_\beta - n_\alpha - 1$. He makes the same inference if he is pivotal with a $\beta$-signal. The two situations differ in that there must be an agent who has an $\alpha$ in the former, and a $\beta$ in the latter case. As $i$ has a $\beta$ in the former and an $\alpha$ in the latter case, $\kappa$ is inferred to be the same.

A communication profile $\gamma$ is a communication equilibrium if

$(i)$ every agent reveals at least one type of signal,

$(ii)$ $\forall i : \theta_i < p_\alpha^{\alpha-conc}(\gamma^*) : \gamma_i^*(\alpha) = \alpha$,

$(iii)$ $\forall i : \theta_i > p_\beta^{\beta-conc}(\gamma^*) : \gamma_i^*(\beta) = \beta$,

$(iv)$ $\forall i : \theta_i > p_\alpha(\gamma^*) : \gamma_i^*(\alpha) = s$, and

$(v)$ $\forall i : \theta_i < p_\beta(\gamma^*) : \gamma_i^*(\beta) = s$.

We proof existence by constructing communication profiles together with a decision rule $k$ such that conditions (i)-(v) are met for cases (ii) and (iii). For case (i), existence is not guaranteed.

Consider first case (iii). Let $k$ be the integer for which:

$$\frac{1}{1+\frac{1-\delta q}{1-\delta(1-q)}\left(\frac{(1-q)(1-\delta q)}{q(1-\delta(1-q))}\right)^k} \leq \theta_m \leq \frac{1}{1+\frac{1-q}{q}\left(\frac{(1-q)(1-\delta q)}{q(1-\delta(1-q))}\right)^k}.$$

The decision rule $k$ is chosen in such a way that the median preference type is willing to reveal both types of signals if there are as many other agents revealing $\alpha$ as there are revealing $\beta$.[10] The above conditions (i)-(v) hold for the following communication profile: $\forall i : \theta_i < \theta_m : \gamma_i(\alpha) = \alpha, \gamma_i(\beta) = s$; $\forall i : \theta_i > \theta_m : \gamma_i(\alpha) = s, \gamma_i(\beta) = \beta$; $i : \theta_i = \theta_m : \gamma_i(\alpha) = \alpha, \gamma_i(\beta) = \beta$. We have $n_\alpha(\gamma) = n_\beta(\gamma) = \frac{n+1}{2}$. Hence $\gamma$ is an equilibrium communication profile with the property stated in the proposition.

Consider case (ii). Let the decision rule $k$ be such that $p_\alpha^{\alpha-conc} \leq \theta_m \leq p_\beta^{\beta-conc}$ for $n_\alpha = n_\beta$.[11] The following communication profile is an equilibrium: $\forall i : \theta_i < \theta_m : \gamma_i(\alpha) = \alpha, \gamma_i(\beta) = s$; $\forall i : \theta_i > \theta_m : \gamma_i(\alpha) = s, \gamma_i(\beta) = \beta$; $i : \theta_i = \theta_m : \gamma_i(\alpha) = \alpha, \gamma_i(\beta) = \beta$. Again, we have $n_\alpha(\gamma) = n_\beta(\gamma) = \frac{n+1}{2}$.

Consider case (i). Choose $k$ such that $p_\beta^{\beta-conc} \leq \theta_m \leq p_\alpha^{\alpha-conc}$ if $n_\alpha = n_\beta$. Construct the communication profile as follows: First, let agents $i : \theta_i \leq p_\alpha^{\alpha-conc}$ reveal $\alpha$ and agents $i : \theta_i \geq p_\beta^{\beta-conc}$ reveal $\beta$, and let all all other information be concealed. Note that $n_\alpha, n_\beta \geq \frac{n+1}{2}$. If $n_\alpha = n_\beta$, the communication profile is an equilibrium. If $n_\alpha > n_\beta$, modify the communication profile for agents $i : p_\beta \leq \theta_i \leq p_\beta^{\beta-conc}$: let $\min\{n_\alpha - n_\beta, |\{i : p_\beta \leq \theta_i \leq p_\beta^{\beta-conc}\}|\}$ of them reveal $\beta$ in addition to the revelation described above. The new communication profile is an equilibrium with the properties stated in the proposition if $n_\alpha - n_\beta \leq |\{i : p_\beta \leq \theta_i \leq p_\beta^{\beta-conc}\}|$. However, we cannot guarantee existence of such a

[10] As $\frac{1}{1+\frac{1-\delta q}{1-\delta(1-q)}\left(\frac{(1-q)(1-\delta q)}{q(1-\delta(1-q))}\right)^{k+1}} > \frac{1}{1+\frac{1-q}{q}\left(\frac{(1-q)(1-\delta q)}{q(1-\delta(1-q))}\right)^k}$, such a $k$ might not exist. In this case, we can find a $k$ together with $|n_\alpha - n_\beta| = 1$ such that the above inequalities hold. The following arguments are analogous, hence we restrict ourselves to the case that $\theta_m$ is such that a $k$ exists for which both inequalities hold.

[11] If such a $k$ does not exist, it exists for $|n_\alpha - n_\beta| = 1$ or 2.

communication equilibrium in general.

It remains to be shown that $|k| < |k_m|$. To see this, note that in equilibrium, $\theta_m \in [p_\beta, p_\alpha]$. Remember that $\theta_m \in \left[\frac{1}{1+\left(\frac{1-q}{q}\right)^{k_m}}, \frac{1}{1+\left(\frac{1-q}{q}\right)^{k_m+1}}\right]$. Given decision rule $k$, for $\delta \to 0$, $p_\beta$ and $p_\alpha$ converge to $\frac{1}{1+\left(\frac{1-q}{q}\right)^{k}}$ and $\frac{1}{1+\left(\frac{1-q}{q}\right)^{k_m+1}}$, respectively. For $\delta \to 1$, they converge to $\frac{1}{1+\left(\frac{1-q}{q}\right)^{2k-n_\alpha-n_\beta+1}}$. Hence, in the equilibria which we consider $|k|$ is at most $|k_m|$ and the lower the higher $\delta$. Q.E.D.

**Probability agent $i$ assigns to $\theta_m = \theta'$.**

For $\theta' < \theta_i$ :

$$\phi_i^m(\theta') = (n-1)\binom{n-2}{\frac{n-1}{2}}\Phi(\theta')^{\frac{n-1}{2}}(1-\Phi(\theta'))^{\frac{n-3}{2}}\phi(\theta')d\theta', \tag{17}$$

for $\theta' = \theta_i$ :

$$\phi_i^m(\theta_i) = \binom{n-1}{\frac{n-1}{2}}\Phi(\theta_i)^{\frac{n-1}{2}}(1-\Phi(\theta_i))^{\frac{n-1}{2}}, \tag{18}$$

and for $\theta' > \theta_i$ :

$$\phi_i^m(\theta') = (n-1)\binom{n-2}{\frac{n-1}{2}}\Phi(\theta')^{\frac{n-3}{2}}(1-\Phi(\theta'))^{\frac{n-1}{2}}\phi(\theta')d\theta'. \tag{19}$$

**Probability agent $i$ assigns to $\kappa_{-i}$ given $\sigma_i$.**

For $\sigma_i = \alpha$:

$$\mu_i(\kappa_{-i} = k'|\alpha\} = \frac{\binom{n-1}{\frac{n-1+k'}{2}}\left(\left(\frac{q}{1-q}\right)^{\frac{k'}{2}}q + \left(\frac{1-q}{q}\right)^{\frac{k'}{2}}(1-q)\right)}{\sum\limits_{\hat{k}\in E\{-n+1,\dots n-1\}}\binom{n-1}{\frac{n-1+\hat{k}}{2}}\left(\left(\frac{q}{1-q}\right)^{\frac{\hat{k}}{2}}q + \left(\frac{1-q}{q}\right)^{\frac{\hat{k}}{2}}(1-q)\right)}, \tag{20}$$

and for $\sigma_i = \beta$ :

$$\mu_i(\kappa_{-i} = k'|\beta) = \frac{\binom{n-1}{\frac{n-1+k'}{2}}\left(\left(\frac{q}{1-q}\right)^{\frac{k'}{2}}(1-q) + \left(\frac{1-q}{q}\right)^{\frac{k'}{2}}q\right)}{\sum\limits_{\hat{k}\in E\{-n+1,\dots n-1\}}\binom{n-1}{\frac{n-1+\hat{k}}{2}}\left(\left(\frac{q}{1-q}\right)^{\frac{\hat{k}}{2}}q + \left(\frac{1-q}{q}\right)^{\frac{\hat{k}}{2}}(1-q)\right)}, \tag{21}$$

where $E\{-n+1, \dots n-1\}$ is the set of even numbers between (including) $-n+1$ and $n-1$, and $\mu_i(\kappa_{-i} = k'|\sigma_i) = 0$ for odd values $k'$.

**Necessary and sufficient condition for full information revelation: Example.**

Consider a committee with three members, each of whom receives a signal which is correct with probability 0.8. We know from Proposition 5 that a full information revelation equilibrium exists if the preference parameters are drawn from $[0.2, 0.985]$ or from $[0.015, 0.8]$. Suppose preferences are drawn from a uniform distribution which is symmetric with respect to 1/2. We identify the minimum $\theta_{min}$ (and therewith the maximum $\theta_{max}$) for which a full information revelation equilibrium exists. Existence is guaranteed for $\theta_{min} \geq 0.2$, and obviously, we must have $\theta_{min} > 0.015$, otherwise the most biased types' preferred alternative does not depend on the realization of the signals. Hence, we consider the case $0.015 < \theta_{min} < 0.2$ As outlined above, out-of-equilibrium-beliefs best support a full information equilibrium, if (i) probability 1 is assigned to $\kappa = -1$ in case $c_i = s$ and $k(c) = -2$, and (ii) probability 1 is assigned to $\kappa = 3$ in case $c_i = s$ and $k(c) = 2$. Because of symmetry, out-of-equilibrium-beliefs for the case $c_i = s$ and $k(c) = 0$ assign equal probability to $\kappa = -1$ and $\kappa = 1$. Again because of symmetry, existence is guaranteed if type $\theta_{min}$ has an incentive to reveal a $\beta$-signal. Type $\theta_{min}$ has an incentive to reveal a $\beta$-signal if

$$\begin{aligned}
&(1/2 * 0.8^3 + 1/2 * 0.2^3)(\theta_{min} - 0.015)\Phi(0.2)(1 - \Phi(0.2)) \geq \\
&(1/2 * 0.8 * 0.2^2 + 1/2 * 0.2 * 0.8^2)(0.2 - \theta_{min})(\Phi(0.5) - \Phi(0.2)(1 - \Phi(0.5)) \\
\Leftrightarrow\ & 0.26(\theta_{min} - 0.015)(0.2 - \theta_{min})(0.8 - \theta_{min}) > 0.08(0.2 - \theta_{min})(0.5 - (0.2 - \theta_{min}))0.5 \\
\Leftrightarrow\ & \theta_{min} \geq 0.10446.
\end{aligned}$$

Note that the sufficient condition stated in Proposition 5 allows for potential conflicts of interests (as measured by $\theta_{max} - \theta_{min}$) of 0.785 for this example, whereas the necessary and sufficient condition allows for 0.79.

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