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Endogenous differential mortality, non monitored effort and optimal non linear taxation

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Endogenous differential mortality, non monitored effort and optimal non linear taxation

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Abstract

This paper studies the normative problem of redistribution among individuals who can influence their longevity through a non-monetary effort but have different taste for effort. As benchmarks, we first present the laissez-faire and the first best. In the first best, the level of effort is always lower than in the laissez-faire as the social planner takes into account the consequences of higher survival on the budget constraint. However, since we suppose that effort is private and non-monetary (like exercising), it is reasonable to think that the social planner has no control over it. Thus, we modify our framework and assume for the rest of the paper that effort is determined by the individual while the social planner only allocates consumptions. Under full information with non monitored effort, early consumption is preferred to future consumption and the high-survival individual obtains higher future consumption. Under asymmetric information, the distortion is identical for the low-survival individual while the direction of the distortion for the high-survival individual is ambiguous. We finally show how to decentralize these allocations through a perfect annuity market and (positive or negative) taxes on annuities.

Keywords: annuities, effort, differential mortality, non linear taxation.

JEL Classification: H21, H23, H55, I12

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1 Introduction

Is our life expectancy predetermined? To what extent can we modify it? Several factors may determine human longevity. It surely depends on intrinsic characteristics (such as gender or heredity) or on environmental and sociocultural factors. However, individuals certainly have some control over their life expectancy and may be able to modify it either through monetary investments (for example, undergoing expensive surgery) or through non-monetary ones. In this latter case, health improving effort can equivalently be exercising, dieting, living a healthy life, sleeping 8 hours per night, etc.. To illustrate this point, Kaplan et al. (1987) show that little or no physical activity is associated with higher mortality risks at all ages. In a more recent study, Okamoto (2006) also finds a significantly positive relationship between leisure time engaged in sports and the increase in life expectancy at 65 of Japanese men. More evidence on the relationship between physical activity and life expectancy can also be found in Ferucci et al. (1999), Franco et al. (2005, 2006).

Relating these questions to current Social Security debates, the factors and the consequences of life extension are certainly a matter of public concern. For instance, several empirical studies (such as Coronado et al. 2000, Liebman 2001 and Bommier et al. 2006) find that life duration differentials reduce intragenerational redistribution since there is no link between the amount of per period pension benefits and expected length of life. For instance, individuals with lower income obtain higher replacement rates, yet redistribution is partly neutralized due to positive correlation between life expectancy and income. These studies then highlight the importance of considering redistribution in a life-cycle framework.

From a theoretical point of view, the contributions of Bommier et al. (2007a, b) deal with this relation between life duration and Social Security benefits. They study the optimal pension design when individuals differ in their life expectancy which they assume to be exogenous. Their paper's focus is on two main points. A first technical problem of how to model life expectancy in optimal income taxation problems: the paper demonstrates that the "double additivity assumption" (i.e. a utilitarian social planner and additively separable individuals' preferences) implies that individuals exhibit temporal risk neutrality, which leads to very specific and questionable conclusions in terms of redistribution.[1] Their second point concerns optimal taxation results when relaxing the assumption of additive preferences. They find that in the first best optimum, long-lived individuals should obtain lower per period consumption and work longer while short-lived individuals should be compensated for their unluckiness by getting higher per period consumption and retiring earlier.

Quite the opposite, this paper proposes a framework in which individuals' longevity is only the result of a private (and costly) effort, taking other possible determinants as fixed, and studies the optimal redistribution problem. We assume a two-period model in which individuals survive to the second period with a probability which depends on the level of effort they make in the first period. Individuals may yet differ in their taste for efforts so that they end up having different survival probabilities. As opposed to Eeckhoudt and Pestieau (2006) and Becker and Philipson (1998), we also assume that individuals' effort is always non-monetary so that the social planner cannot influence it directly. We also assume that lifetime utility is additively separable so that individuals exhibit temporal risk neutrality in our model. As opposed to Bommier et al. (2007a, b), we decided to retain to this standard formulation in order to emphasize the role of private efforts on the optimal allocation when it is determined by the individual prior to the allocation of consumptions by the social planner.[2]

Under these assumptions, we first present the laissez-faire as a benchmark case and second we study a (hypothetical) first best problem in which the social planner allocates consumptions and effort levels. The ensuing result is that the optimal level of effort is smaller in the first best than in the laissez-faire. This

[1] On the notion of temporal risk aversion, see Bommier (2006b).

[2] As it is shown in Bommier (2006a-b) and in Bommier et al. (2007a-b), assuming both additively separable utility functions and a utilitarian social welfare function implies that individuals exhibit temporal risk neutrality which leads to the equalisation of consumptions between individuals with different length of life. Relaxing the assumption of additivity accross time in the lifetime utility function would surely change our results.

is due to the fact that in the first best, the social planner takes into account that effort changes the survival probability which in turn modifies the resource constraint.[3] However, since effort is non monetary, the social planner cannot decentralize this first best optimum through, e.g. taxation. This is why in the following, we resort to a *constrained* first best in which it is assumed that individuals choose ex-ante the level of their effort.

Thus, in this modified framework, the social planner influences effort only through the allocation of consumptions. We find that under full information, future consumption is always lower than present consumption as a way to make individuals exert less effort and make it tend to its first best level. We also demonstrate that under specific assumptions on the form of the survival probability, second period consumption is higher for the individual with higher taste for effort so that the optimal allocation transfers resources from low-survival individuals toward high-survival ones. Finally, we study the problem looking at asymmetric information, when the social planner cannot observe tastes for effort and effort levels. In this case, the distortion is identical to the full information one for the low-survival individual so that it is still optimal to encourage early consumption for this individual. On the contrary, for the high-survival individual, the trade-off between present and future consumption is modified due to the introduction of the incentive constraint. Indeed, under asymmetry of information with non monitored effort, two effects are playing in opposite directions for this individual. On the one hand, the social planner wants to encourage early consumption relative to future consumption so that the individual exerts lower effort (as under full information with non monitored effort) and on the other hand, it wants to discourage early consumption so as to satisfy incentive compatibility constraints (in this case, a low survival individual willing to mimic a high survival individual would obtain too high a level of future consumption). Using numerical examples, we find that the overall effect for this individual crucially depends on the gap between individuals' types and on the elasticity of per

[3] These findings are closely related to those of Becker and Philipson (1998) who studied the optimal trade-off between the quantity and the quality of life when health expenditures modify the length of life.

period utility with respect to consumption.

We also study how to decentralize these optima through a perfect annuity market. Under full information and moral hazard constraints, a tax on annuity is optimal for both types of individuals but his level is higher for the individual with low-taste for effort. Under asymmetric information, it turns out that in some cases, a subsidy is desirable for the individual with high-taste for effort.

This paper is constructed as follows. In Section 2, we present the model and derive the laissez-faire and first best problems. In Section 3, we present a modified framework with full information and in which the social planner has no direct control on individuals' effort while Section 4 sets out the results found under asymmetric information. Section 5 then gives numerical examples. The last section concludes.

2 The Model

2.1 Individuals' types and preferences

We consider a stationary population composed of two groups of individuals, indexed by $i = 1, 2$ who have different tastes for effort γ^i and represent a proportion n^i of the population. Individuals may live for two periods, each of them with length normalized to 1. The first time period is certain while individuals survive to the second period only with probability $\pi(e) \in [0, 1]$ which depends on their effort level e. We assume that $\pi(.)$ takes the same form for both individuals and that $\pi'(.) > 0$, $\pi''(.) < 0$.[4] The individual's effort is made in first period and it is assumed to be non-monetary (such as exerting sport, dieting, living a healthy life) so that it does not enter in the individual's budget constraint. However, exerting an effort creates some disutility which depends on the individual's taste for effort; this total utility cost is represented by $\gamma^i e$ and γ^i can equivalently be seen as the intensity of effort disutility. Regarding these previous assumptions, individuals with different tastes for effort may eventually differ in their survival probability.

[4] These assumptions on the shape of the survival function are standard (see Eeckhoudt and Pestieau, 2007). Later in Section 3, we make more precise assumptions on the form of the survival function.

Individuals also derive utility from consumption at each period and we denote c and d, the level of consumption at first and at second periods respectively. Setting the discount and interest rates equal to zero, the expected lifetime utility of an individual with taste for effort γ^i is simply given by:

$$U\left(c,d,e,\gamma^i\right)=\pi\left(e\right)\left[u\left(c\right)+u\left(d\right)-\gamma^i e\right]+\left(1-\pi\left(e\right)\right)\left[u\left(c\right)-\gamma^i e\right]$$

In our framework, there is no bequest motive so that if the individual dies at the beginning of the second period, his utility is zero. The above utility function simplifies to

$$U^i\left(c,d,e\right)=u\left(c\right)+\pi\left(e\right)u\left(d\right)-\gamma^i e \quad (1)$$

where for ease of notation, we denote $U^i\left(c,d,e\right)\equiv U\left(c,d,e,\gamma^i\right)$. Per period utility of consumption, $u\left(c\right)$ and $u\left(d\right)$ are such that $u'\left(.\right)>0$ and $u''\left(.\right)<0$.

For the rest of the paper, we assume that $\gamma^1>\gamma^2$ so that type-1 individuals are "bad"-type individuals (the ones with high disutility of effort or equivalently with low taste for effort) while type-2 individuals are "good"-type individuals.

2.2 The laissez-faire

We assume that individuals invest all their savings on a perfect annuity market. An individual with taste for effort γ^i determines optimal levels of savings as well as his optimal level of effort by solving the following problem:

$$\begin{aligned}\max_{s^i,e^i} U^i\left(c^i,d^i,e^i\right) &= u\left(c^i\right)+\pi\left(e^i\right)u\left(d^i\right)-\gamma^i e^i\\ &\text{s.to}\ \left\{\begin{array}{c} c^i=w-s^i\\ d^i=Rs^i\end{array}\right.\end{aligned}$$

where $s^i\geq 0$ is the amount of savings made in the first period and R is the rate of return from annuitized savings obtained in the second period. We assume that the initial wealth endowment w is exogenous and identical for any individual. Rearranging first order conditions yield

$$\frac{\pi\left(e^i\right)u'\left(d^i\right)}{u'\left(c^i\right)} = \frac{1}{R} \quad (2)$$

$$\pi'\left(e^i\right)u\left(d^i\right) = \gamma^i \quad (3)$$

Condition (2) gives the trade-off between present and future consumptions. Always assuming that insurers can perfectly observe individuals' survival probability and that the market for annuities is actuarially fair, the return from the annuity is set such that $R = 1/\pi\left(e^i\right)$ where the interest rate is implicitely assumed to be zero. Then $u'\left(c^i\right) = u'\left(d^i\right)$ and individual's consumption is smoothed across periods. Yet, if savings are taxed, their return is $R' = (1-t)/\pi\left(e^i\right)$ with $t \in [0,1]$ a linear tax rate; in this case, $R' < 1/\pi\left(e^i\right)$ and first period consumption is higher than second period consumption. We shall use this result in the following sections.

Condition (3) defines the individual's preferred level of effort $e^*\left(\gamma^i, d^i\right)$ as a function of both the level of his second period consumption and of his taste for effort. It states that at the optimal level, the expected marginal utility of increased life expectancy must be equal to marginal disutility of effort.[5] Note also that in the laissez-faire, the individual takes the annuity return as given so that he does not take into account that by choosing a specific level of effort, he changes the return of the annuity and thus his budget set. Indeed, since R depends on $\pi\left(e^i\right)$, increasing e^i increases the individual survival chance which in turn decreases the return of the annuity. In the laissez-faire, the individual only takes into account the first direct effect and the level of effort is too high compared to the optimal one.[6]

We now present the results on the laissez-faire allocation:

Proposition 1 *When the annuity market is actuarially fair, the laissez-faire allocation of a type i individual is such that:*

(i) consumption is smoothed across periods: $c^i = d^i$,

(ii) $e^*\left(\gamma^i, d^i\right)$ *is decreasing in* γ^i *and increasing in* d^i.

[5] Under our assumptions, the second order condition $\pi''(e)\,u(d) < 0$ is always satisfied.

[6] This imperfection was first highlighted by Becker and Philipson (1998). When choosing their longevity effort, individuals face a free rider problem; each individual from a same longevity risk category decides of their longevity effort without taking into account that in overall, it affects the annuity price.

This proposition directly follows from the above optimality conditions. In point (ii), we logically find that individuals with low taste for effort always exert lower level of effort, for a given level of second period consumption. As we prove in Appendix A, the level of effort also increases with the level of future consumption. Indeed, if second period consumption is high, the individual has more incentives to exert higher effort and to increase his survival probability.

We finally compare individuals' laissez-faire allocations. Since we assumed w to be identical between individuals, individuals with different types end up with identical expected lifetime consumption, defined as $c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i = w \; \forall i$. But, as consumption is smoothed across periods, the only possible allocations $\left(c^1, d^1\right)$ and $\left(c^2, d^2\right)$ which satisfy this equality are such that $c^1 = d^1 > c^2 = d^2$ with $e^*\left(\gamma^1, d^1\right) < e^*\left(\gamma^2, d^2\right)$. First consumption is then higher for the high-disutility individual and he prefers to concentrate consumption in the first period of his life since he has lower chances to enjoy any consumption in the following period.

2.3 The first best problem

We assume that the social planner is utilitarian and that he perfectly observes individuals' types. The economy is assumed to be in a steady state equilibrium and the social planner can lend or borrow at a zero interest rate in order to balance the budget at any given period. The resource constraint of the economy is thus:

$$\sum_{i=1,2} n^i \left(c^i + \pi\left(e^i\right) d^i\right) \leq w \tag{4}$$

where $\left(c^i, d^i\right)$ is the consumption allocation of an individual of type $i = 1, 2$ and e^i is his effort level. The social planner chooses consumption paths as well as effort levels in order to maximize

$$\sum_{i=1,2} n^i \left[u\left(c^i\right) + \pi\left(e^i\right) u\left(d^i\right) - \gamma^i e^i\right]$$

subject to (4). First order conditions of this problem yield:

$$\frac{\pi\left(e^i\right)u'\left(d^i\right)}{u'\left(c^i\right)} = \pi\left(e^i\right) \tag{5}$$

$$\pi'\left(e^i\right)u\left(d^i\right)\left[1-\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] = \gamma^i \tag{6}$$

for any $i = 1, 2$. Obviously, condition (5) states that consumption should be equalized across time and between agents with different types. The second condition defines the optimal level of effort, $e^*\left(\gamma^i, d^i\right)$.[7] The expression $u'\left(d^i\right)d^i/u\left(d^i\right)$ is assumed to be lower than 1 which is standard in the economic literature that studies the welfare benefits related to longevity extension and ensures that the value of a statistical life is sufficiently large.[8] Here, the individual level of effort is strictly positive only when life is worth living, i.e. when this elasticity is lower than 1; otherwise, the optimal level of effort is zero. Moreover, by comparison with the laissez-faire expression (3), the first best allocation now includes an additional term, $-u'\left(d^i\right)\pi'\left(e^i\right)d^i$. This corresponds to the effect of effort on the budget set; in the first best, the social planner takes into account that a higher effort might decrease second period consumption possibilities through a tightened resource constraint. In the first best, the optimal level of effort is such that at this level, the marginal gain in utility due to increased survival probability is equal to total marginal cost of effort (i.e. marginal disutility of effort and marginal decrease in utility due to smaller consumption possibilities). Comparing (3) with (6), we obtain the following proposition:

Proposition 2 *For any type of individual, the first best level of effort is lower than the laissez-faire one.*

Hence, the effect of longevity-enhancing behavior on the budget set is now accounted for, which was not the case in the preceding laissez-faire section. This result is in line with Becker and Philipson (1998) who studied the trade-off between the quantity (i.e. longer lifetime) and the quality (i.e. less resources per

[7] The second order condition is always satisfied under our assumptions.

[8] See Murphy and Topel (2003) and Becker et al. (2005).

period) of life and how individuals' attitude toward life extension affects mortality contingent claims. Note however that for very high values of a statistical life (equivalently, small levels of $u'\left(d^i\right) d^i / u\left(d^i\right)$), first best levels of effort tend to the laissez faire ones; in this case, nothing is more important than being alive and the effect on the resource constraint only has a marginal impact. Our second set of results is summarized hereafter:

Proposition 3 *The first best allocation is characterized by*

(i) $c^i = d^i = \bar{c}\ \forall i$,

(ii) $e^*\left(\gamma^1, \bar{c}\right) < e^*\left(\gamma^2, \bar{c}\right)$ *for any given* $\bar{c}$ *and* $\gamma^1 > \gamma^2$.

Point *(i)* directly follows from the assumption of no pure time preferences and from the *double additivity assumption* (i.e. both additively separable individual preferences and a utilitarian social planner); this jointly leads to the equalization of consumptions across periods and between agents with different survival chances in the first best.[9] The social planner also requires lower effort from the individual with lower taste for effort so that he has smaller survival probability. Since the expected lifetime consumption of an individual of type γ^i is equal to $c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i$, one finds that the expected consumption is higher for the high-taste individual and the first best optimum transfers resources from low-taste (low-survival) individuals toward high-taste (high-survival) ones.

Finally, this first best allocation cannot be decentralized through a tax-and-transfer scheme since effort is non-monetary. This is why in the following section, we resort to a *constrained* first best in which the social planner lets individuals choose their effort level and can eventually influence it through the allocation of consumptions.

[9] As shown in Bommier (2006) and Bommier et al. (2007a, b), this assumption implies that individuals are risk neutral toward the length of life so that a utilitarian social planner corrects for intra-period inequality but does not take into account inter-period inequality (and thus, the fact that one individual might live longer than the other).

3 Full information with non monitored effort

3.1 The optimum

Since effort takes a non monetary form in our framework, it is reasonable to assume that the social planner has no control on it. Thus, we now assume that the social planner only allocates consumptions, knowing that it may have consequences on individuals' choice of effort. In this section, the social planner perfectly observes individuals' types.

The timing of the problem is the following one. First, the social planner allocates consumptions and second, individuals choose their level of effort. Proceeding by backward induction, we first determine the optimal level of effort and then solve the social planner's problem. In this framework, for any $i = 1, 2$, the optimal level of effort $e^*\left(\gamma^i, d^i\right)$ is defined by (3) and satisfies point (i) of Proposition 1.

The social planner then chooses consumption paths of individuals with types γ^1 and γ^2 taking into account that effort depends on individual's type and on second period consumption. This amounts to solve the following problem:

$$\begin{aligned} &\max_{c^i, d^i} \sum_{i=1,2} n^i \left[u\left(c^i\right) + \pi\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) - \gamma^i e^*\left(\gamma^i, d^i\right)\right] \\ &\text{s.to } \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right] \leq w \end{aligned}$$

First order conditions with respect to c^i and d^i are respectively:

$$u'\left(c^i\right) = \lambda \tag{7}$$

$$u'\left(d^i\right) = \lambda\left[1 + \frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)} d^i \frac{de^*\left(\gamma^i, d^i\right)}{dd^i}\right] \tag{8}$$

Comparing these two conditions and recalling that $de^*\left(\gamma^i, d^i\right)/dd^i > 0$ for any given γ^i, we now find that with non monitored effort, present consumption should be higher than future consumption (since the expression inside brackets is greater than 1). The explanation is related to the optimal level of effort. As mentioned in Proposition 2, the first best level of effort should be lower than in the laissez-faire due to the effect of effort on the resource constraint. Thus, one way to make individuals exert less effort is to provide them with less second period consumption. This limits the increase of individuals' survival probability and thus the negative impact of effort on the resource constraint. This result is summarized in the following proposition:

Proposition 4 *Under full information with non monitored effort, $c^i > d^i$ for any individual with type γ^i.*

We further study how consumptions should be allocated between individuals with different tastes for effort. First, from condition (7), we obtain that first period consumption is equalized between individuals; this directly follows from our assumptions on individual preferences and on the social welfare function but also from the fact that first period consumption has no impact on effort levels. On the contrary, second period consumption might be differentiated depending on our assumptions on survival probabilities and on per period utility. Rewriting condition (8) in terms of elasticities of substitution, we obtain

$$u'\left(d^i\right) = \lambda\left[1 + \varepsilon_{\pi(e^*(\gamma^i,d^i))} \times \varepsilon_{e^*(\gamma^i,d^i),d^i}\right] \tag{9}$$

where $\varepsilon_{\pi(e^*(\gamma^i,d^i))}$ and $\varepsilon_{e^*(\gamma^i,d^i),\, d^i}$ are respectively the elasticity of the survival probability with respect to effort and the elasticity of effort with respect to future consumption and have the following expressions:

$$\begin{aligned}
\varepsilon_{\pi(e^*(\gamma^i,d^i))} &= \frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right) e^*\left(\gamma^i,d^i\right)}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)} \\
\varepsilon_{e^*(\gamma^i,d^i),\, d^i} &= \frac{de^*\left(\gamma^i,d^i\right)}{dd^i}\frac{d^i}{e^*\left(\gamma^i,d^i\right)} \\
&= -\frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)}{\pi''\left(e^*\left(\gamma^i,d^i\right)\right) e^*\left(\gamma^i,d^i\right)}\frac{u'\left(d^i\right) d^i}{u\left(d^i\right)}
\end{aligned}$$

Note that in the last line, we have replaced for the expression of $de^*\left(\gamma^i,d^i\right)/dd^i$. Surprisingly, one finds that if $\pi\left(e^*\left(\gamma^i,d^i\right)\right)$ has constant elasticity with respect to effort, not only $\varepsilon_{\pi(e^*(\gamma^i,d^i))}$ is equal to a constant but also $\varepsilon_{e^*(\gamma^i,d^i),\, d^i}$ depends only on future consumption levels and not on individuals' types.[10] Thus, in this

[10]Looking at expressions (3) (12) in Appendix A, this result depends on the (linear) form of the effort disutility. Assuming convex disutility may yield different result.

special case, (9) is independent of γ^i and future consumption is equalized between individuals. Yet, if the elasticity of the survival probability is non constant, the result is ambiguous and d^1 might be greater or lower than d^2. This is stated formally in the following proposition.

Proposition 5 *Consider two groups of individuals with types γ^1 and γ^2 such that $\gamma^1 > \gamma^2$. Under full information with non monitored effort,*

(i) First period consumption is equalized between individuals, $c^i = \bar{c}\ \forall i$.

(ii) Second period consumption is such that

- *if the survival probability has constant elasticity with respect to effort, $d^1 = d^2$,*
- *otherwise, $d^1 \gtrless d^2$ if and only if*

$$\varepsilon_{\pi(e^*(\gamma^1,d^1))} \times \varepsilon_{e^*(\gamma^1,d^1),d^1} \lesseqgtr \varepsilon_{\pi(e^*(\gamma^2,d^2))} \times \varepsilon_{e^*(\gamma^2,d^2),d^2}$$

Obviously, our results strongly depend on the form of the survival probability. In the following, we consider cases in which the survival probability has decreasing elasticity with respect to effort.[11] Assuming some specific functional forms for the survival probability, we show in Appendix that second period consumption should be higher for the high-taste-for-effort individual:

Proposition 6 *If individuals' survival probability can be modelled as $\pi(e) = \log e$ or $\pi(e) = e/(1+e)$, the full information optimum with non monitored effort yields that $d^1 < d^2$.*

The social planner then rewards the individual with high taste for effort by giving him higher level of second period consumption; equivalently, he gives more to the individual who is more likely to survive. The effort for the high-taste individual as well as his survival probability are then higher. Thus, expected lifetime consumptions are such that $\bar{c} + \pi\left(e^*\left(\gamma^2, d^2\right)\right) d^2 > \bar{c} + \pi\left(e^*\left(\gamma^1, d^1\right)\right) d^1$

[11] For instance, it is proven that in the case of physical activity, there exists an optimal level of effort above which additionnal effort may effectively decrease survival chances.

and the optimal allocation with non-monitored effort transfers resources from the low-taste for effort individual toward the high-taste for effort individual. Note that this result is still valid for constant elasticity of substitution of the survival probability but the level of the transfer is lower than under decreasing elasticity.

3.2 Decentralization

The above optimum can be decentralized through lump sum transfers from the low-taste toward the high-taste-for-effort individual (equivalently from the low-survival toward the high-survival individual). Moreover, individual's savings are invested on a perfect annuity market so that comparing (7) and (9) with (2), we find that the decentralisation of this optimum is achieved throught the taxation of annuitized savings. If the elasticity of the survival probability with respect to effort is constant, the level of this tax, t^i is identical between individuals with different for effort and equal to:[12]

$$t^i = \frac{\varepsilon_{\pi(e^*(\gamma^i,d^i))} \times \varepsilon_{e^*(\gamma^i,d^i),d^i}}{1 + \varepsilon_{\pi(e^*(\gamma^i,d^i))} \times \varepsilon_{e^*(\gamma^i,d^i),d^i}}$$

Otherwise, whether $t^1 \gtrless t^2$ depends on the form of the survival probability. Under specific forms for the survival probability such that it has decreasing elasticity of substitution (as in Proposition 6), one obtains that this tax is higher for the individual with low taste of effort

$$t^1 = \frac{\varepsilon_{\pi(e^*(\gamma^1,d^1))} \times \varepsilon_{e^*(\gamma^1,d^1),d^1}}{1 + \varepsilon_{\pi(e^*(\gamma^1,d^1))} \times \varepsilon_{e^*(\gamma^1,d^1),d^1}} > t^2 = \frac{\varepsilon_{\pi(e^*(\gamma^2,d^2))} \times \varepsilon_{e^*(\gamma^2,d^2),d^2}}{1 + \varepsilon_{\pi(e^*(\gamma^2,d^2))} \times \varepsilon_{e^*(\gamma^2,d^2),d^2}}$$

4 Asymmetric information with non monitored effort

4.1 Theoretical results

In this section, we assume that the social planner neither observes individuals' tastes for effort nor their levels of effort.[13] Using results of Proposition 5, it

[12]For example, if $\pi(e) = e^{\varepsilon}$ with elasticity of substitution ε, the tax is simply equal to

$$t^i = \frac{\frac{\varepsilon}{\varepsilon-1}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}}{\left[1 + \frac{\varepsilon}{\varepsilon-1}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right]}$$

[13]Note that the social planner observes survival probabilities ex-post but it does not give additional information on types since survival can always be due to luck and not because the

is straightforward that with constant elasticity of the survival probability with respect to effort, the first best allocation is implementable under asymmetric information. On the contrary, if the survival probability has decreasing elasticity and if the social planner proposes first best bundles, the individual with low taste for effort (type-1 individual) has interest in claiming to have high taste for effort and to enjoy higher consumption d^2.[14] Then, to avoid mimicking behavior, we add an incentive constraint to the preceding problem such that:

$$\begin{aligned} \max_{c^1,d^1,c^2,d^2} \quad & \sum_{i=1,2} n^i \left[u\left(c^i\right) + \pi\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) - \gamma^i e^*\left(\gamma^i, d^i\right)\right] \\ & \text{s.to} \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right] \leq w \\ & \text{s.to} \begin{array}{c} u\left(c^1\right) + \pi\left(e^*\left(\gamma^1, d^1\right)\right) u\left(d^1\right) - \gamma^1 e^*\left(\gamma^1, d^1\right) \geq \\ u\left(c^2\right) + \pi\left(e^*\left(\gamma^1, d^2\right)\right) u\left(d^2\right) - \gamma^1 e^*\left(\gamma^1, d^2\right) \end{array} \end{aligned}$$

The trade-offs between two-period consumptions for type 1 and type 2 are then:

$$\frac{u'\left(d^1\right)}{u'\left(c^1\right)} = \left(1 + \frac{\pi'\left(e^*\left(\gamma^1, d^1\right)\right)}{\pi\left(e^*\left(\gamma^1, d^1\right)\right)} d^1 \frac{de^*\left(\gamma^1, d^1\right)}{dd^1}\right) \tag{10}$$

$$\frac{u'\left(d^2\right)}{u'\left(c^2\right)} = \left(1 + \frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)} d^2 \frac{de^*\left(\gamma^2, d^2\right)}{dd^2}\right) \times \left[\frac{1 - \frac{\mu}{n^2}}{1 - \frac{\mu}{n^2}\frac{\pi(e^*(\gamma^1, d^2))}{\pi(e^*(\gamma^2, d^2))}}\right] \tag{11}$$

where μ is the Lagrange multiplier associated to the incentive constraint. The trade-off between present and future consumptions is equivalent to the full information case for type-1 individual. Quite the opposite, for the individual with taste for effort γ^2, it is distorted downward since the expression inside brackets is lower than 1. Our results are summarized hereafter:

Proposition 7 *Assume two groups of individuals with taste for effort* γ^1 *and* γ^2 *such that* $\gamma^1 > \gamma^2$. *Under asymmetric information with non monitored effort,*

(i) there is no distortion at the top for the individual with type 1 *and* $c^1 > d^1$,

individual lied on his type.

[14] Since effort and consumption are positively correlated, an increase in consumption not only increases direct utility derived from consumption but also survival and total effort disutility. However, using the envelop theorem, it is easy to prove that marginal utility with respect to future consumption is always positive. Thus, type-1 individual has always interest in lying on his type under decreasing elasticity.

(*ii*) *the trade off between two-period consumptions is distorted downward for the individual with type* 2 *and* $c^2 \gtrless d^2$.

Then, under asymmetric information, the distortion for the individual with low taste for effort is equivalent to the full information case. The social planner simply imposes a distortion on this individual so as to make him exert lower effort, as in the full information case. This is *kind of* "no distortion at the top" result for the individual who would like to lie on his type.

On the contrary, the individual with high taste for effort now faces an additional distortion, due to the introduction of the incentive constraint so that the marginal rate of substitution is distorted downward compared to the full information case. Thus, under asymmetric information, the high taste for effort individual faces two distortions with countervailing effects. Indeed, in (11), the first expression inside parenthesis is greater than 1 and is related to the effect of effort on the resource constraint; as under full information, the social planner wants to induce lower level of effort by encouraging present consumption relative to future consumption. Afterward, we call it the *effort effect.* On the other hand, the second expression inside brackets is lower than 1 and is related to the incentive constraint. Under asymmetric information, it is desirable to encourage future consumption for this individual. This makes the low-taste-for-effort individual less interested in the allocation proposed to the high-taste individual as it would provide him with too high a level of future consumption relative to present consumption. It is a way to relax an otherwise binding self-selection constraint. Further on, we call it the *incentive effect.*

Thus, if the effort effect dominates the incentive effect, early consumption should still be encouraged relatively to future consumption for individual 2; yet, the difference between present and future consumption will be lower than in the full information case. It is also possible that future consumption is preferred if the incentive effect dominates the effort effect.

Finally, we study how to implement these results. Comparing (10) and (11) with their laissez-faire counterparts, we find that this optimum can be decentralized through lump sum transfers and taxes on annuities. Using the same reasoning as in the preceding section, these taxes have the following expressions:

$$t^1 = \frac{\varepsilon_{\pi(e^*(\gamma^1,d^1))} \times \varepsilon_{e^*(\gamma^1,d^1),d^1}}{1+\varepsilon_{\pi(e^*(\gamma^1,d^1))} \times \varepsilon_{e^*(\gamma^1,d^1),d^1}} > 0$$

$$t^2 = \frac{\left(1+\varepsilon_{\pi(e^*(\gamma^2,d^2))} \times \varepsilon_{e^*(\gamma^2,d^2),d^2}\right) \times \left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{\pi(e^*(\gamma^1,d^2))}{\pi(e^*(\gamma^2,d^2))}}\right] - 1}{\left(1+\varepsilon_{\pi(e^*(\gamma^2,d^2))} \times \varepsilon_{e^*(\gamma^2,d^2),d^2}\right) \times \left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{\pi(e^*(\gamma^1,d^2))}{\pi(e^*(\gamma^2,d^2))}}\right]} \gtrless 0$$

This tax is identical to the full information case for the low-taste-for-effort individual. On the contrary for the high-taste individual, whether he faces a positive or negative tax now depends on the direction of the overall distortion. If total distortion goes upward (i.e. the effort effect dominates the incentive effect), a positive tax on annuities is desirable but it is lower than under full information since the incentive effect partly neutralizes the effort effect. Early consumption is still favoured for this individual. On the opposite, if the distortion goes downward, he will benefit from a subsidy (negative tax) on annuities and future consumption will be encouraged for this individual. A positive or negative tax on annuities is way to make the problem incentive compatible under asymmetric information.

In the next subsection, we simulate our model and study how the direction of the overall distortion depends on the distance between types $\left(\gamma^1 - \gamma^2\right)$ and on the elasticity of per period utility with respect to consumption.

4.2 Numerical examples

Consider the following specifications for the various components of our model. Types $\left(\gamma^1, \gamma^2\right)$ are distributed on $]0,1]$ and we set $w = 10$. We first consider three possible cases: a case where types are very close, a case where $\gamma^1 = 2\gamma^2$ and a case where types are very different. The utility function has the following form, $u(c) = c^\varepsilon/\varepsilon$ with constant elasticity, ε. For the moment, we fix $\varepsilon = 0.2$. The survival probability is modeled as $\pi(e) = e/(1+e)$ which ensures that it is always lower than one and that it has decreasing elasticity. In the following table, we present a first set of results under full information (FI) and asymmetric information (AI):

types (γ^1, γ^2)		c	d	e	$\pi(e)$	$\frac{u'(d^i)}{u'(c^i)}$
$(1, 0.9)$	individual 1 FI	6.28141	5.8404	1.66765	0.625139	1.05996
	AI	6.95551	6.47464	1.6953	0.628983	1.05899
	individual 2 FI	6.28141	5.8738	1.81356	0.644578	1.05514
	AI	5.60584	5.2548	1.7824	0.640598	1.05309
$(1, 0.5)$	individual 1 FI	6.092	5.6623	1.6594	0.623976	1.06026
	AI	6.85537	6.38039	1.69135	0.628439	1.05912
	individual 2 FI	6.092	5.828	2.77183	0.734877	1.03608
	AI	5.32801	5.20183	2.7292	0.731846	1.01936
$(1, 0.1)$	individual 1 FI	5.80636	5.39381	1.64652	0.622145	1.06073
	AI	6.69415	6.22867	1.68488	0.627544	1.05935
	individual 2 FI	5.80636	5.70996	7.41683	0.88119	1.01348
	AI	4.92035	5.08828	7.32036	0.879813	0.973509

Table 1: Optimal allocations with non monitored effort

This table confirms the findings of Proposition 6 that under full information, future consumption is always higher for type-2 individuals. These results are yet more interesting when studying the optimal allocations under asymmetric information. First period consumption is not smoothed anymore between individuals and type-1 individual (with low taste for effort) always gets higher levels of first period consumption as well as future consumption; he also exerts lower level of effort but has lower survival probability. In the last column, we present the level of the distortion under both full and asymmetric information. As expected, early consumption is always encouraged for type-1 individual. For the individual with high taste for effort (type-2 individual), we find that the level of the distortion under asymmetric information is always lower than under full information but whether it is greater or lower than 1 depends on the distance between the γs. For very close levels of the γs and for a ratio of 2, this distortion is still greater than 1; thus the effort effect dominates the incentive effect and early consumption is still preferred. On the opposite, when individuals' types are very different, this distortion is lower than 1 so that it is desirable to encourage future consumption relatively to early consumption.

In order to verify our conjectures on the possible link between distance in types and the size of the distortion (whether it is greater or smaller than 1), we last check the level of the overall distortion for type-2 individual, $u'\left(d^2\right)/u'\left(c^2\right)$ by fixing $\gamma^1 = 1$ but making γ^2 decrease. We also compute it for different values of ε:

γ^2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$\varepsilon = 0.1$	0.9971	0.9727	0.9480	0.9230	0.8973	0.8707	0.8424	0.8112	0.7738
$\varepsilon = 0.2$	1.0531	1.0451	1.0368	1.0283	1.0194	1.0099	0.9996	0.9879	0.9735
$\varepsilon = 0.5$	1.2007	1.1798	1.1596	1.1399	1.1203	1.1006	1.0803	1.0587	1.0336

Table 2: Distortion levels for type-2 individual under asymmetric information

This table yields two types of comments related to both the levels of γ^2 and of ε. First, let us recall that the smaller is $u'(c)\,c/u(c) = \varepsilon$, the higher is the value of life so that the laissez-faire level of effort tends to the optimal one (as we showed in Section 2.3) and there is no need to correct for effort. Thus, under asymmetric information, the distortion in (11) is always smaller than 1 since the so-called effort effect does not play any role and there exists only one type of distortion which is related to the incentive constraint. In this case, future consumption is always encouraged for type-2 individual. On the contrary, if ε is high, the value of life is relatively small so that first best effort is very different from its laissez faire level. In this case, the effort effect might dominate the incentive effect and the distortion in (11) is likely to be greater than 1. This results in encouraging relatively more early consumption.

It is also interesting to see that the distance between types plays a crucial role in determining whether the distortion is greater or lower than 1 under asymmetric information. Indeed, for a given ε, the higher is the gap between types, the smaller is the level of the distortion for a type-2 individual and the more likely it is to be lower than one. For instance, when $\varepsilon = 0.2$, the level of the distortion is first greater and then lower than one as γ^2 decreases. Hence, as long as types are not very different, the effort effect dominates and present consumption is preferred while if types are very different, it will be the reverse. The explanation is the following one. For a given ε, when the distance in types is small, first best allocations $\left(c^i, d^i\right)$ of type-1 and type-2 are almost identical so that the incentive effect might be less "constraining" than the effort effect under asymmetric information. In such a case, the effort effect dominates the incentive effect and the distortion is greater than 1. On the opposite, when types are very different, type-2 individual obtains much higher level of second period consumption than type-1 under full information so that the incentive effect will be more constraining than the effort effect; in this case, the incentive effect dominates and the overall distortion is lower than 1. It results in encouraging future consumption for the individual with high-taste for effort under asymmetric information.

5 Concluding Remarks

This paper studies the problem of redistribution among individuals who can influence their longevity by exerting efforts. We assume that these efforts are non-monetary (such as physical activity) so that the social planner has no direct control on it. Thus, in our framework, the social planner only allocates consumptions while the individual chooses his level of effort depending on his taste for effort and on the expected level of future consumption. We first highlight the trade-off between the quantity and the quality of life by explaining the relations between survival probabilities, effort and the return of annuities. On the one hand, higher effort increases expected length of life but it also decreases consumption possibilities through a decrease in the return of the annuity. We showed that in the laissez-faire, the effort level is higher than in the first best, because in the former, the individual does not integrate the consequence of higher effort over his budget constraint. Thus, under full information and non monitored effort, we find that early consumption is encouraged relative to future consumption as a way to make individuals exert less effort. Under asymmetry of information, the distortion in the trade-off between present and future consumption is identical to the full information case for the individual with low taste for effort. However, for the high-taste individual, the distortion is lower than under full information as the incentive effect partly neutralizes the effort effect. Future consumption may even be preferred to present consumption for this individual when individuals have very different tastes for effort. In order to clarify these results, the following table provides a summary

	Consumptions	Effort	Survival Probability
Laissez Faire	$c^1 = d^1 > c^2 = d^2$		
First Best	$c^1 = d^1 = c^2 = d^2$		
Full Information MH			
Constant Elasticity of $\pi(.) \rightarrow$	$c^1 = c^2 > d^1 = d^2$	$e^1 < e^2$	$\pi^1 < \pi^2$
Decreasing Elasticity of $\pi(.) \rightarrow$	$d^1 < d^2 < c^1 = c^2$		
Assym.Info MH	$c^1 > c^2, d^1 > d^2, c^1 > d^1$		
Decreasing Elasticity	$c^2 \gtrless d^2$		

Table 3: Recapitulative table

We also analyzed how to decentralize these optima through a tax-and-transfer scheme. Under full information, a positive tax on annuities is desirable for both types of individuals. Under asymmetry of information, a tax on annuities is still desirable for the low-taste-for-effort individual but whether the tax should be positive or negative for the high-taste-for-effort individual depends on the distance between types and on the elasticity of per period utility with respect to consumption.

There are several directions in which the model could be extended. For instance, we have neglected the fact that life expectancy depends on intrinsic characteristics (such as gender) and that efforts and genetics may be correlated. Adding this additional characteristic, how would our model be modified? Moreover, we assume additively separable preferences which imply temporal risk neutrality. Relaxing this assumption may modify substantially our results. Answering these questions is on our research agenda.

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Appendix

A Laissez Faire

Proof of Proposition 1: Fully differentiating (3), we find that effort decreases with γ^i and increases with d^i:

$$\begin{aligned} \frac{de^*\left(\gamma^i, d^i\right)}{d\gamma^i} &= \frac{1}{\pi''\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right)} < 0 \\ \frac{de^*\left(\gamma^i, d^i\right)}{dd^i} &= -\frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right) u'\left(d^i\right)}{\pi''\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right)} > 0 \end{aligned} \tag{12}$$

B First best

Proof of Proposition 3: Using (5), we obtain point (i) of Proposition 3. Point (ii) is obtained from full differentiation of (6):

$$\frac{de^i}{d\gamma^i} = \frac{\gamma^i}{\pi''\left(e^i\right)\left[u\left(d^i\right) - u'\left(d^i\right) d^i\right]} < 0$$

C Full information with moral hazard

C.1 Proof of Proposition 5

The Lagrangian of the problem is

$$\begin{aligned} \pounds = &\sum_{i=1,2} n^i \left[u\left(c^i\right) + \pi\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) - \gamma^i v\left(e^*\left(\gamma^i, d^i\right)\right)\right] \\ &+\lambda \left[w - \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right]\right] \end{aligned}$$

where λ is the Lagrange multiplier associated to the resource constraint. First order conditions with respect to c^i and d^i are:

$$u'\left(c^i\right) - \lambda = 0 \tag{13}$$

$$\begin{aligned} &\left[\begin{array}{c} \pi'\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) \\ -\gamma^i v'\left(e^*\left(\gamma^i, d^i\right)\right) - \lambda \pi'\left(e^*\left(\gamma^i, d^i\right)\right) d^i \end{array}\right] \frac{de^*\left(\gamma^i, d^i\right)}{dd^i} \\ + \quad &\pi\left(e^*\left(\gamma^i, d^i\right)\right)\left[u'\left(d^i\right) - \lambda\right] = 0 \end{aligned} \tag{14}$$

Replacing for (3) into (14) and rearranging terms, one obtains

$$\pi\left(e^*\left(\gamma^i, d^i\right)\right) u'\left(d^i\right) - \pi\left(e^*\left(\gamma^i, d^i\right)\right) \lambda \left[1 + \frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)} d^i \frac{de^*\left(\gamma^i, d^i\right)}{dd^i}\right] = 0$$

Replacing for $\varepsilon_{\pi(e^*(\gamma^i,d^i))}$ and $\varepsilon_{e^*(\gamma^i,d^i),\ d^i}$ in the above expression, one obtains (9). Replacing for (12) in the above expression, we obtain

$$u'\left(d^i\right) - \lambda\left[1 - \frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)^2}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)\pi''\left(e^*\left(\gamma^i, d^i\right)\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] = 0 \qquad (15)$$

Let assume that $\pi(e)$ *has constant elasticity* denoted ε with $\varepsilon < 1$ and $e \in [0,1]$. Thus,

$$u'\left(d^i\right) - \lambda\left[1 - \frac{\varepsilon}{\varepsilon - 1}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] = 0$$

so that first order condition on d^i is independent of individual's types and $d^1 = d^2$ for $\gamma^1 > \gamma^2$.

C.2 Proof of Proposition 6

Assume that $\pi(e) = \log e$ with $e \in [1, \exp]$. In this case,

$$\frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)^2}{\left(\pi\left(e^*\left(\gamma^i, d^i\right)\right)\pi''\left(e^*\left(\gamma^i, d^i\right)\right)\right)} = -\frac{1}{\log e^*\left(\gamma^i, d^i\right)}$$

and (15) is equal to

$$\begin{aligned} u'\left(d^i\right) - \lambda\left[1 + \frac{1}{\log e^*\left(\gamma^i, d^i\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] &= 0 \\ u'\left(d^i\right) - \lambda\left[1 + \frac{1}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] &= 0 \qquad (16) \end{aligned}$$

Using the implicit function theorem, one has

$$sign\left(\frac{dd^i}{d\gamma^i}\right) = sign\left(\frac{\partial\left(u'\left(d^i\right) - \lambda\left[1 + \frac{1}{\pi(e^*(\gamma^i,d^i))}\frac{u'(d^i)d^i}{u(d^i)}\right]\right)}{\partial\gamma^i}\right)$$

where the expression on the right hand side is equal to

$$\lambda\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)^2}\frac{de^*\left(\gamma^i, d^i\right)}{d\gamma^i}$$

which is negative since $de^*\left(\gamma^i, d^i\right)/d\gamma^i < 0$. Thus, $d^1 < d^2$ for $\gamma^1 > \gamma^2$.

Assume now that $\pi(e) = e/(1+e)$. In this case,

$$\frac{\pi'\left(e^*\left(\gamma^i, d^i\right)\right)^2}{\left(\pi\left(e^*\left(\gamma^i, d^i\right)\right)\pi''\left(e^*\left(\gamma^i, d^i\right)\right)\right)} = -\frac{1}{2e^*\left(\gamma^i, d^i\right)}$$

and (15) is equal to

$$u'\left(d^i\right) - \lambda \left[1 + \frac{1}{2e^*\left(\gamma^i, d^i\right)} \frac{u'\left(d^i\right) d^i}{u\left(d^i\right)}\right] = 0$$

Again using the implicit function theorem, one obtains that $dd^i/d\gamma^i < 0$ so that $d^1 < d^2$ for $\gamma^1 > \gamma^2$.

D Asymmetric information with moral hazard

The Lagrangian of the problem under asymmetric information can be written as

$$\begin{aligned}
\pounds\left(c^1, c^2, d^1, d^2\right) &= \sum_{i=1,2} n^i \left[u\left(c^i\right) + \pi\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) - \gamma^i e^*\left(\gamma^i, d^i\right)\right] \\
&\quad + \lambda \left\{ w - \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right] \right\} \\
&\quad + \mu \left\{ \begin{array}{c} u\left(c^1\right) + \pi\left(e^*\left(\gamma^1, d^1\right)\right) u\left(d^1\right) - \gamma^1 e^*\left(\gamma^1, d^1\right) \\ -u\left(c^2\right) - \pi\left(e^*\left(\gamma^1, d^2\right)\right) u\left(d^2\right) + \gamma^1 e^*\left(\gamma^1, d^2\right) \end{array} \right\}
\end{aligned}$$

First order conditions of the second best problem simplify to:

$$\begin{aligned}
& u'\left(c^1\right)\left(1 + \frac{\mu}{n^1}\right) = \lambda \\
& u'\left(c^2\right)\left(1 - \frac{\mu}{n^2}\right) = \lambda \\
& u'\left(d^1\right)\left(1 + \frac{\mu}{n^1}\right) = \lambda \left[1 + \frac{\pi'\left(e^*\left(\gamma^1, d^1\right)\right)}{\pi\left(e^*\left(\gamma^1, d^1\right)\right)} \frac{de^*\left(\gamma^1, d^1\right)}{dd^1} d^1\right] \\
& u'\left(d^2\right)\left(1 - \frac{\mu}{n^2} \frac{\pi\left(e^*\left(\gamma^1, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\right) = \lambda \left[1 + \frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)} \frac{de^*\left(\gamma^2, d^2\right)}{dd^2} d^2\right]
\end{aligned}$$

So that the trade-offs between two-period consumptions for each type of individual are equal to

$$\begin{aligned}
\frac{u'\left(d^1\right)}{u'\left(c^1\right)} &= \left(1 + \frac{\pi'\left(e^*\left(\gamma^1, d^1\right)\right)}{\pi\left(e^*\left(\gamma^1, d^1\right)\right)} d^1 \frac{de^*\left(\gamma^1, d^1\right)}{dd^1}\right) \\
\frac{u'\left(d^2\right)}{u'\left(c^2\right)} &= \left(1 + \frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)} d^2 \frac{de^*\left(\gamma^2, d^2\right)}{dd^2}\right) \times \left[\frac{1 - \frac{\mu}{n^2}}{1 - \frac{\mu}{n^2} \frac{\pi\left(e^*\left(\gamma^1, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)}}\right]
\end{aligned}$$

Since $\pi\left(e^*\left(\gamma^1, d^2\right)\right) < \pi\left(e^*\left(\gamma^2, d^2\right)\right)$, the expression inside brackets is always lower than 1.

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