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Report 1B: Package Bidding for Spectrum Licenses

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EXECUTIVE SUMMARY

The FCC was an innovator in adopting the rules of the simultaneous ascending-price auction for its sales of spectrum licenses. While these rules have performed well in the auctions conducted so far (and would perform even better with the design improvements suggested in our first report), there are two inherent limitations in any design that seeks to assign and price the licenses individually. First, such designs create strategic incentives for bidders interested in multiple licenses that are substitutes to reduce their demands for some of the licenses in order to reduce the final prices of the others; this is the *demand reduction* problem. Second, even if bidders behave non-strategically, there is a fundamental problem with the basic concept of individual-license pricing when licenses are complementary. In simultaneous ascending-price auctions, from a bidder's perspective this is the *exposure problem*. A bidder who is unsuccessful in bidding for a large package of licenses may be left with a partial package whose total price cannot be justified in the absence of those complementary licenses it failed to win. This problem is present in any auction mechanism that sells licenses individually, with no opportunity to bid on packages.

In principle, bidding for packages can resolve both problems, although package bidding introduces new difficulties that any successful design must overcome. Our report reviews the theory and evidence about package bidding, including both the advantages and the problems this format introduces. We also review the previous FCC auctions, arguing that the absence of strong complementarities among licenses contributed to their success. (The sole exception was the regional narrowband auction, which worked well because of the special pattern of only important synergies, which were those among regional licenses within a given spectrum band.)

For our analysis, we anticipate some of the likely characteristics of the environments in which package bidding may be useful in future spectrum auctions. We rely on these characteristics to recommend two auction designs that could be developed for initial trials of package bidding. These recommendations stem from the conclusions of our analysis. One main conclusion is that although a Vickrey auction minimizes incentives for strategic manipulation and provides the best assurance of an efficient outcome, it faces potentially severe problems of implementation. These are associated with the need to pre-specify a large number of potential combinations, the prospect that bidders will be uncomfortable with the novel Vickrey pricing rule, and the particularly difficult problem of formulating bids for the Vickrey auction when bidders face serious budget constraints. The second main conclusion is that allowance for package bidding in the simultaneous ascending auction in environments with weak complementarities introduces new possibilities for collusive behavior and involves a loss of efficiency. However, in environments with strong complementarities, these disadvantages may be offset by the elimination of the exposure problem.

In view of the Congressional mandate to seek ways to promote efficient use of the spectrum, we recommend two kinds of experiments, tailored to the kinds of auction environments that face the Commission. The first is a test of a modified Vickrey auction in a suitable auction environment. We also recommend a second experiment in which the standard simultaneous ascending-price auction is augmented to allow package bidding, based as closely as possible on the procedural rules for simultaneous ascending-price auctions of individual licenses.

In this report our task is confined to analyses of the merits of package bidding and the practical problems of implementation. In our next report, we will outline proposals for the details of the procedural rules and other aspects of implementing a practical design, as well as the software development that would be necessary.

Report 1B: Package Bidding for Spectrum Licenses

1. INTRODUCTION

The accumulated evidence now solidly establishes that the simultaneous ascending auction rules used in the FCC spectrum auctions led to significantly more efficient license assignments than older sets of rules used in the most comparable situations. The FCC rules allowed the bidders to perform approximate arbitrage among similar licenses within an auction, so licenses that are close substitutes sold for nearly identical prices. Comparing that performance with such comparable auctions as RCA's auction of leases on satellite transponders (in which the highest priced transponder sold for 35% more than the least expensive) or New Zealand's auction of TV licenses (in which the highest priced license sold for 300% more than the cheapest near-perfect substitute), the FCC auctions have set a new standard. The evidence about arbitrage is important, because it indicates that bidders were able to switch among close substitutes in response to relative price differences until late in the auction. The ability to shift demand among substitutes in response to competitor's bids is a *necessary* condition for achieving efficient license assignments.

The ability to shift demand among substitutes, however, is not a sufficient condition for efficiency, and even perfect arbitrage is consistent with the presence of other sources of inefficiency. Our previous report dealt with two such sources: collusion among bidders and transactional inefficiencies (slow pace and difficult bidding). This report deals with two others that might be addressed by designs using "*package bidding*."

The Demand Reduction Problem. The auction may create a strategic incentive for a large bidder to withhold some of its demand in order to depress prices (Ausubel and Cramton, 1996).

The Exposure Problem. The auction may require a bidder that tries to assemble a package of complementary licenses to incur the risk of winning only a partial package at a total price exceeding its value.

The economics literature includes two groups of papers that suggest that these problems might be mitigated through some form of package bidding. One group consists of theoretical papers and has its origins in the Nobel-prize winning work of William Vickrey (1961). The second consists of reports on laboratory experiments in which a package bidding scheme has often performed better than the tested alternatives (e.g., Ledyard, Noussair and Porter (1994)).

We do not wish to overstate the potential advantages of these improvements in spectrum auctions in general. The incentive for large bidders to reduce their demand for licenses in the simultaneous ascending auction is much the same as any large buyer's incentive to withhold demand in an ordinary market. If ordinary markets set the standard, one may well judge that the demand reduction problem is not serious. Also, the success of bidders in the regional narrowband auction in assembling nationwide collections of paging licenses in a single spectrum band establishes that the simultaneous ascending auction can *sometimes* allow the formation of efficient license combinations when there are complementarities. Nevertheless, as we argue below, in environments with significant complementarities, the simultaneous ascending auction cannot be expected to perform as well as it has in the FCC auction environments to date. Moreover, theory and experimental evidence indicate package bidding mechanisms might perform significantly better in these environments if the difficulties of implementation can be overcome. That provides ample reason to investigate package bidding.

2. TWO DEFICIENCIES OF NON-PACKAGE BIDDING

In this section, we describe in more detail the two problems identified above: demand reduction for substitutes and the exposure problem in bidding for packages of complementary licenses. Of these, the second is by far the more complex and perhaps more important. For that reason, we accord it more attention here.

In the two subsections that follow, we focus on these two problems, explaining how each may arise in simultaneous ascending auctions for individual licenses. The analysis of package bidding schemes and how they might mitigate one or both of these problems is reserved for section 3.

For the casual reader, the next two paragraphs provide a brief synopsis of our conclusions about the two problems. In both cases, a key benchmark is the Vickrey pricing rule, which requires each bidder to pay (to the Treasury) the imputed loss incurred by other bidders on account of the licenses that it wins. This contrasts with the more familiar "pay-your-bid" pricing rule in which each bidder pays precisely the prices it has offered for the packages it wins.

Demand Reduction. If licenses are mutual substitutes, a buyer seeking to acquire many licenses faces the same problem as a large buyer in any other market: its own large demand may raise prices overall. Anticipating that effect, the bidder may reduce its bids, thereby acquiring fewer licenses than would maximize its total value, but acquiring them at lower prices. This behavior can be mitigated by an auction design that, in effect, offers discounts to large buyers. Such discounts are a kind of nonlinear pricing, but in an auction context they can also be implemented via the Vickrey pricing rule. The discounts implicit in the Vickrey rule promote license assignments that are efficient in the sense that the bidders with the highest private values win.

(The case for using discounts to prevent inefficient demand reduction is weaker if encouraging entry by small bidders is a policy objective.)

Complementarities. Licenses are *complements* for a bidder when they are worth more together than separately. For example, bidders in the regional narrowband auction apparently regarded the regional licenses within each spectrum band as complements because the licenses could be combined to offer a nationwide paging service for which consumers would be willing to pay a premium. As shown below, the presence of significant complementarities can fundamentally change the character of the license assignment problem. When licenses are mutual substitutes, an auction is a mechanism for identifying an efficient license assignment and the prices that support it. With complements, there may be no prices supporting the efficient assignment, and the auctioning problem becomes the much harder one of finding prices for the many possible packages of licenses. The Vickrey pricing rule is one solution to this problem that works exactly in a certain class of environments. A carefully designed auction that invokes a pay-your-bid rule may identify approximately optimal prices for packages in a wider class of environments.

2.1 Demand Reduction

The problem of *demand reduction* is entirely analogous to the problem of withholding supply in monopoly theory. Corresponding to the concept of marginal revenue in that theory is the concept of marginal expenditure. The *marginal expenditure* to acquire a license is the price paid for the license *plus* the increase in the total amount paid for the other (substitute) licenses that the bidder acquires as a result of this purchase. In monopoly theory, a seller produces output just to the point where the marginal revenue associated with the last unit produced is equal to the marginal cost. The analogous condition in auction theory is that a profit-maximizing bidder will acquire additional licenses to the point where the marginal value of the last license equals the associated marginal expenditure (although the condition should be adjusted slightly for discrete units). In monopoly theory, the result is that the monopolist produces less output than it would at the efficient outcome. The analogous conclusion here is that a large bidder acquires fewer licenses than it would at the total value-maximizing license assignment.

In monopoly theory, the “market solution” to the inefficiency described above is price discrimination, which allows the monopolist to sell additional units of output without reducing the prices of its inframarginal units. In auction theory, the Vickrey auction solution is also a form of price discrimination, allowing a large bidder to acquire incremental licenses without bidding up the prices of its inframarginal licenses. Similarly, in Ausubel’s design of an ascending-price version of the Vickrey auction, a bidder is assigned a license at the current price whenever it could precipitate the close of the auction by refusing to raise its bid for that license. The implicit price discrimination in a Vickrey auction works better, in theory, than designs that rely on explicit quantity discounts, because explicit discounts can be set optimally only in the unrealistic case in which the seller knows each bidder's demand elasticity.

There may be public policy reasons to reject such a solution. One is the fact that licenses that are perfect substitutes may be sold at different prices; another is the potential bias such auctions create in favor of large firms, an outcome that could be inefficient if it encourages uneconomic mergers or expansions. Our formal analysis sets aside these considerations and treats maximizing the total value to bidders as the main objective of the auction design.

2.2 Complementarities

One conception of a spectrum auction is that it is a process to discover a license assignment and license prices that approximate those that would be found in the perfectly competitive markets of economic theory. Under standard assumptions, the theoretical analysis of markets tells us that the license assignment associated with a competitive equilibrium is efficient and hence maximizes the total value for the bidders. However, the presence of complementarities (so that the value of a package of licenses can exceed the sum of the values of the component licenses) combined with the inherent indivisibility of the licenses can negate this conception–because these combined conditions can imply that no competitive equilibrium exists! That is, there may be no prices at which the demand for each license exactly equals the number of units supplied. The remainder of this section is a detailed analysis showing that this problem is a fundamental one.

NON-EXISTENCE OF COMPETITIVE EQUILIBRIUM

Roughly, two licenses are *complements* to a bidder when the most the bidder would pay to acquire the pair is more than the sum of the most the bidder would pay to acquire each one individually. This is just a rough definition, because the bidder's willingness to pay may depend on the other licenses acquired or on their prices. To make our point simply, we suppress that refinement in this section and restrict attention to cases in which no bidder ever demands more than two licenses.

Example 1. Suppose there are two licenses, A and B. Suppose bidder 1 is willing to pay up to a for A, up to b for B, and up to $a+b+c$ for the pair. We represent these preferences by the value triplet ($a,b,a+b+c$). If the synergy value is positive, $c>0$, then the licenses are complements for this bidder and *c* is the maximum premium the bidder is willing to pay to acquire the pair. A problem arises when the same pair of licenses may be substitutes for a second bidder that competes effectively for the licenses individually, but not for the pair. For example, that bidder may be willing to pay up to $a+d$ for license A, $b+d$ for license B, and, $a+b+d$ for the pair, where $c/2<d<c$. In this case, the value-maximizing assignment assigns the pair of licenses to the first bidder. But, for this to be an outcome of a market equilibrium, the market clearing prices must be high enough to drive the second bidder's demands to zero. That is, the price of license A must be at least $a+d$ and that of license B must be at least $b+d$, so the sum of the two prices exceeds the first bidder's value of $a+b+c$. The first bidder would not want to buy the license pair at these prices, so there are no prices for the two licenses individually that clear the two markets simultaneously.

The choice of values for the first bidder was completely general and this example can be extended to any number of licenses. That establishes the following.

> ***Theorem****. (Milgrom, 1997) If licenses are complements for any bidder, then there is a profile of values for the other bidders in which licenses for those bidders are mutual substitutes such that no competitive equilibrium exists.*

To verify that this really is a problem arising from complementarity, we note the following theorem for contrast. In it, we say that bidders bid *straightforwardly* if, at each round, they make the minimum bid for the additional licenses they would wish to acquire if the auction were to end after that round.

> ***Theorem****. (Milgrom, 1997) If the licenses are mutual substitutes for all bidders then, despite the license indivisibility, a competitive equilibrium always exists. Moreover, if bidders in a simultaneous ascending auction bid straightforwardly and if the minimum increment is sufficiently small, then the final license assignment maximizes total value and it is a competitive equilibrium assignment.*

Of course, the prices may not converge to even an approximate competitive equilibrium if bidders withhold demand, as it is often in their interest to do. What the theorem indicates is that the failure to obtain efficiency results only from strategic considerations and not from (1) failure of existence of equilibrium prices or (2) incapacity of the auction process to reveal enough information to identify the equilibrium prices.

Our first problematic example above was one in which licenses can be substitutes for some bidders and complements for others. Thus, the success of the regional narrowband auction, despite the complementarities among regional paging licenses, might result from the situation that licenses to serve different regions, while sometimes technical complements, can never be technical substitutes. This leaves open the question of whether equilibrium prices may fail to exist in environments like that of the regional narrowband. However, even in the regional narrowband auction, it would be possible for bidders to want to form different combinations of regions, leading to similar non-existence problems. For an example along those lines, see Milgrom (1997). Here we give an example based on the experience of another spectrum manager, in which the fact that licenses exhibit technical complementarities does not imply that they avoid the problem of price-theoretic substitutability.

Example 2. Suppose that in some two-way wireless service, the outbound and return communication channels are to be auctioned separately. Each bidder is allowed only one pair of channels. Bidders use alternative technologies: two bidders require adjacent outbound and return bands and a third bidder requires that the bands be separated by one band. Four bands, A, B, C and D are being sold, with the restriction that the pair BC is not available. The bidder values for license pairs are shown in Table 1.

Table 1

	AB	CD	AC	BD
1	10	10	0	0
2	10	10	0	0
3	0	0	15	15

Analytically, the problem in this example is that, despite the technical complementarities that seem to characterize this problem, licenses A and C are price-theoretic substitutes for bidder 1 but complements for bidder 3. That creates a non-existence problem, as follows. The efficient assignment assigns the packages AB and CD to bidders 1 and 2 and attains a total value of 20. These assignments exclude bidder 3. Consequently, if there are to be competitive equilibrium prices for the four licenses, the total price of licenses A and C must be at least 15, and the same is true for licenses B and D. Then, the total price of the four licenses must be at least 30. But that is inconsistent with the licenses being purchased by bidders 1 and 2. We conclude that there is no competitive equilibrium in this example.

THE EXPOSURE PROBLEM

Related to the non-existence of equilibrium with complementarities is a problem that has become known as the *exposure problem*. In an auction where all licenses are substitutes, a bidder who bids straightforwardly would never wish to withdraw any bid or bids at the end of the auction. However, the same is not true when some licenses are complementary. In such a setting, a bidder who bids straightforwardly is exposed to the risk that it may fail to acquire some components of a package of complementary licenses. It may then find itself forced to pay more than its value for the set of licenses it does acquire. In that case, it would welcome the opportunity to withdraw some of its bids.

The exposure problem creates myriad opportunities for strategic bidding. On one hand, a bidder may bid cautiously in the early rounds of the auction, seeking to assess its chances of winning its desired package before committing to an aggressive strategy. On the flip side, a bidder whose competitor faces an exposure problem may wish to bid aggressively early in the auction to discourage its competitor from bidding on the other licenses in its package. In that way, it may hope to reduce competition and acquire some licenses at low prices.

To avoid exaggeration, it is well to remember that the exposure problem is most serious when the license valuations of individual bidders are close. When the winning bidders have ample margins compared to the marginal bidders who set prices, all of the problems with the simultaneous ascending auction are substantially reduced. Nevertheless, it is fair to conclude that the non-existence problem combined with the exposure problem make designing closely contested auctions for complementary goods much trickier than designing similar auctions for substitutes.

3. PACKAGE BIDDING IN THEORY

Two suggestions for package bidding have been analyzed in the economics literature. The first, introduced by William Vickrey (1961), was originally proposed as a solution to the problem of demand reduction and has since been extended and given a complete theoretical evaluation. The second, advocated by Ledyard, Noussair, and Porter (1994) among others, is a mechanism that has heretofore been subject to little or no theoretical evaluation but that has performed well in initial laboratory tests. This section reviews and extends the theory of these mechanisms.

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3.1 The Vickrey Auction

In the economics literature, the first important auction design to use package bids was that of William Vickrey, and was later extended and generalized by Groves and Clarke in their analyses of public goods problems. The "(generalized) Vickrey auction" provides a theoretical solution to the design of auctions with package bidding, in particular environments. These are ones with purely private values, no binding budget constraints, and with a bidder's payoff equal to the value of the licenses acquired minus the payments made.

The Vickrey auction (also known as the "pivot mechanism" or "pivot auction") is a one-shot sealed-bid auction in which each bidder makes a separate "bid" for every non-empty set of licenses. Thus, if there are N licenses, a complete bid is a list of 2^N-1 numbers. Any complete package-bidding auction would have this structure. What distinguishes the Vickrey auction are the additional rules that specify how these bids are used to determine the license assignment and the pricing rule that determines how much the winning bidders must pay.

The auctioneer first computes the license assignment that maximizes the total of the bid prices. This "total-bid-maximizing" license assignment is the one implemented by the Vickrey auction. Of course, this assignment will not generally maximize the bidders' total *values* unless their bids coincide with their values, but the Vickrey auction operates under the presumption that the bids are essentially declarations of values for packages.

For the payments, let V_n be the amount bid by bidder *n* for the combination of licenses it acquires in the total bid maximizing assignment, and let V_T be the total bids of all bidders for the packages of licenses acquired. For each bidder *n*, the auctioneer also computes the license assignment that *would* maximize the total bid *if* bidder *n* had bid zero (for every combination of licenses). Suppose the maximum total of the accepted bids in case *n* bids zero is X_n. In the Vickrey auction, bidder *n* is required to pay a price equal to $X_n-[V_T-V_n]$. If the bids coincide with the bidders' actual values, this formula would compute the total loss of value in the assignment to other bidders resulting from *n*'s bidding more than zero. Roughly, each winning bidder "pays the amount of its externality" imposed on other participants by its claim to the licenses it wins. Note that unlike the pay-your-bid pricing rule, the Vickrey pricing rule does not allow a bidder to know when it submits its bids the amount it will have to pay for any particular package it might win.

EQUILIBRIUM AND EFFICIENCY

The interest in the Vickrey auction comes mainly from the following two theorems.

> ***Theorem***. *(Vickrey, 1961) In the Vickrey auction, each bidder has a* ***dominant*** *strategy, that is, a strategy that maximizes its own payoff regardless of the bids made by the other bidders. The dominant strategy is to bid its actual value for every package. If each bidder bids according to its dominant strategy, then the assignment chosen by the Vickrey auction is the total value maximizing assignment, that is, the auction outcome is efficient.*

There is no other auction that can duplicate the full set of desirable properties of the Vickrey auction. Here is a more precise statement.

> ***Theorem***. *(Holmstrom, 1979) If the possible bidder values are any non-negative numbers, then the Vickrey auction is the essentially unique auction mechanism with dominant strategies and for which (1) the outcome is always efficient and (2) bidders with zero values earn precisely zero profits. ("Essentially unique" means that the other auctions that match the Vickrey auction's performance can be constructed from the Vickrey auction by re-labeling the strategies and adding new strategies that no bidder will ever use.)*

REVENUE COMPARISONS

Holmstrom's theorem concerns the possibility of getting efficient outcomes from an auction with *dominant* strategies for the bidders. Other scholars have studied alternative mechanisms with different solution concepts. For example, Ausubel (1997) proposed a dynamical implementation of the Vickrey auction that works with Nash equilibrium when all licenses are identical and bidders have declining marginal values for licenses. Broadening attention to Bayes-Nash equilibrium, there are other auctions that also lead to efficient equilibrium outcomes. This fact creates a logical possibility that there could be another auction with the same efficient equilibrium assignments as the Vickrey auction but with higher expected revenues for the seller. Indeed, such auctions do exist for some environments. However, in the auction models most commonly studied, no auction can improve on the Vickrey auction in this way, as shown by the following *revenue equivalence* theorem.

> ***Theorem***. *(Engelbrecht-Wiggans, 1988; Krishna and Perry, 1997) Consider any finite sets of licenses and bidders. For any bidder n, let n's valuation of any set of*

licenses S be given by $v_n(S)$ and suppose that these values are private information of the bidder. Suppose that these value vectors are drawn from distributions on a convex set and such that the efficient outcome is almost surely unique. Then any auction rules and associated (Bayes-Nash) equilibrium in which

- *only winning bidders pay, and*
- *the outcome is always efficient,*

have precisely

- *the same expected profit for each bidder conditional on its valuations, and*
- *the same expected revenue for the seller*

as the Vickrey auction.

This key theorem shows that, in the class of environments it studies, there is no auction design that is inherently superior to the Vickrey auction, either for the bidders or for the seller. Basically, if a design is to ensure efficiency then for the bidders and for the seller it is equivalent in terms of net profits and revenue.

The economics literature reflects some misunderstandings about the revenue-generating properties of the Vickrey auction. In his seminal paper, Vickrey himself concluded that the package form of his auction was impractical because it entailed too great a loss of revenue for the government. However, he offered no analysis to support that conclusion and, in general, his conclusion is incorrect.

Example 3. The following table offers an illustrative and robust example with two licenses and three bidders.

Table 2

	A	B	AB
1	10	10	19
2	8	8	12
3	5	5	10

In a simultaneous ascending auction with complete information, the equilibrium license prices in this example will each be 5 (or one increment higher), with bidders 1 and 2 each acquiring one license. The reason is that there is always demand for at least three licenses at any lower price, and any attempt by bidder 1 to acquire a second license will either fail or drive the price up to 8. Either outcome results in lower profits for bidder 1 compared to acquiring a single license for a price of 5, so bidder 1 finds it optimal to engage in *demand reduction* to keep the prices low. The total sales proceeds are then 10.

In contrast, in the Vickrey auction, if all bidders bid according to their dominant strategies, then bidder 1 would acquire both licenses. Its price will then be the maximum value of the licenses acquired by the *other* bidders when 1 does not participate – which in this case is 13 – minus the value of the licenses they acquire at the Vickrey auction outcome, which in this case zero. The Vickrey price of the package AB is therefore 13, which is greater than the sale proceeds of 10 obtained in the simultaneous ascending auction.

We find nothing unusual or atypical about this example. Indeed, one can construct examples in which the revenue from the Vickrey auction is many times larger than that of the simultaneous ascending auction, as well as examples in which the revenue is much smaller. On theoretical grounds, there is no valid general prediction about whether revenues from the Vickrey auction will be higher or lower than those from the simultaneous ascending auction.

PRICE DISCRIMINATION AND MERGER INCENTIVES

The Vickrey auction is not neutral in the way it determines prices for large (“merged”) and small (“unmerged”) bidders. To indicate the biases it introduces, we offer the following theorem.

> ***Theorem**. Suppose that when any two bidders A and B merge to form a single entity AB, their valuation for any set of licenses S is just the value attained when the licenses in S are assigned optimally between A & B, that is:*
>
> $$v_{AB}(S) = \max_{T \subseteq S} \{ v_A(T) + v_B(S \backslash T) \}.$$
>
> *If bidders A and B are replaced in a Vickrey auction by the merged bidder AB, then this merger will have no effect on either the assignment or on the prices paid by the other bidders. If all licenses are substitutes for all the bidders, then the Vickrey price paid by the merged entity AB will be (weakly) less than the sum of the Vickrey prices for A and B in the original auction with the unmerged bidders.*

However, if the licenses are *complements* for some bidders besides A and B, then the merger *may* result in a *higher* Vickrey price for AB than the sum of the Vickrey prices for A and B in the original auction. Example 4 in the next section illustrates that possibility.

THEORETICAL DEFICIENCIES OF THE VICKREY AUCTION

The analysis of the Vickrey auction in this section has been conducted within the usual model with no budget constraints or wealth effects and with purely private values. (*Purely private values* refers to a situation in no which bidder would ever be led to revise its own estimate of the value of a license or a package if it learned some of what its competitors know about the market covered by the license, markets covered by other licenses, equipment availability, new technologies, and so on.) The next two subsections deal separately with the difficulties raised by budget constraints and *common value* elements in the auction.

Budget Constraints

We begin by examining the Vickrey auction when bidders face a budget constraint. If no modifications are made to the auction rules and defaults are heavily penalized, then the effect of the constraint is to limit a bidder so that the highest price it can bid for any license or package does not exceed its budget.

For example, suppose only one bidder has values significantly exceeding its budget. Suppose that bidder has a budget of 10 and values of 12 and 18 for licenses A and B, respectively, and a value of 30 for the package. The unmodified Vickrey auction with this constraint provides no way for the bidder to express its relative value for packages, even when the budget constraint turns out not to be binding. Let ε be a small positive number and suppose the bidder bids (x,y,z) for the license packages (A,B,AB).

If licenses are to be assigned efficiently when the competitor truthfully bids (ε,6,6+ε) as well as when it truthfully bids (0,6+ε,6+ε), then it must be true that $y - x = 6$.

If licenses are to be assigned efficiently when the competitor truthfully bids (10-ε,20,30), then $x = 10$.

These are examples in which the budget constraint does not block the efficient allocation, yet the two requirements are quite inconsistent with the budget limitation: together, they imply that $y = 16$.

As a practical matter, the problem we are describing is a consequence of a bidder's uncertainty about the approximate prices of the licenses that it will win. If a Vickrey auction were to be conducted for Treasury bills (in which the extent of price uncertainty is very small compared to the value of the items being purchased), the practical effect of the problem we have just described would be negligible. However, in auctions for spectrum licenses, a bidder may expect a price to be X and yet need to allow for the possibility that it could be 2X. In such a situation, budget constraints represent a very serious limitation on the ability of a bidder to express its values correctly in a Vickrey auction.

One might hope to avoid the budget problem described above by allowing bidders to name budget amounts or “maximum exposures” in addition to their values. One might then have an algorithm to withdraw bids that cause the bidder to exceed its budget. However, such an algorithm would not only complicate the rule for evaluating the final license assignment, it would also introduce a strategic motive for bidders to modify their bids. Indeed, by increasing its bids on licenses it expects to lose, a bidder can sometimes force a competitor to withdraw bids on some packages, allowing the bidder to win those packages. The most desirable strategic properties of the Vickrey auction are unavoidably lost in applications in which the maximum value of licenses that can be bought with a given budget is highly uncertain.

Bidding subject to budget constraints is one of the most difficult areas of auction theory, and we have nothing more to add here about ways to mitigate this problem.

Private vs. Common Value Analyses

When there is uncertainty about technology and demand, as there certainly is for the new services involved in the spectrum auctions, *common value uncertainty* becomes an important factor. A bidder interested only in a license to serve Chicago may still learn something of value from observing others’ bids for licenses to serve New York and Los Angeles. Those license prices reflect the estimates of other bidders about the profitability of those licenses, and those estimates depend on many of the same variables – penetration rates, equipment availability, demand growth rates, and so on – that determine its valuation of the Chicago license. When common value elements are important, the absence of communication about prices during a single-round Vickrey auction makes it more likely that mis-estimation by bidders will impair the efficiency of the license assignment, compared to the simultaneous ascending auction.

Klemperer (1997) has recently analyzed a variation of the Vickrey model in which one bidder has a slight but known value advantage over the other bidders. His assumptions imply that, at equilibrium, the outcome is efficient and the winner predictable. That discourages other bidders from bidding aggressively and sometimes even from participating, resulting in much lower revenues for the seller. As evidence for the importance of this effect, Klemperer cites a certain

British takeover bidding contest as well as the bidding for the Los Angeles MTA license in the AB block PCS auction. He concludes that in such cases, an auction such as a standard sealed-tender auction, despite its possibly inefficient outcomes, may generate substantially more sales revenue than a Vickrey auction.

3.2 Simultaneous Ascending Auctions with Package Bidding

The second important suggestion for a package bidding mechanism comes from economic experiments. Tests have been conducted of a simultaneous ascending auction in which bidders can bid not only for individual licenses but also for packages of two or more licenses. The experimenters call their mechanism the Adaptive User Selection Mechanism (AUSM).

DESCRIPTION OF AUSM

AUSM is a particular version of the simultaneous ascending auction with package bids. A bid consists of a pair (S,b), where S identifies a set or *package* of licenses, and b is the price bid for that set. A collection of bids, say $\{(S_1,b_1)... (S_n,b_n)\}$, is called consistent if the sets $S_1...,S_n$ are pair-wise disjoint. Intuitively, this means that no license is demanded twice in a consistent collection of bids. At any point in the auction, the *standing high bids* are the prices offered in the consistent collection selected by an optimization procedure. As in the Vickrey auction, the consistent collection that is selected is the one that generates the highest total of the bid prices.

In AUSM, bidding takes place continuously in time and ends after a random amount of time. For example, bidding in a lab experiment may be closed after 8-10 minutes with the ending time uniformly distributed on that range, or when there has been no activity for a fixed amount of time, say 15 seconds. These parameters can be modified as appropriate to the particular auction environment. Unlike the FCC auction, AUSM makes no use of activity rules, relying on the nearness and randomness of the stopping time to encourage bidders to be active.

An important aspect of the design is the *standby queue* – a kind of electronic bulletin board on which bidders post offers that are not part of the standing high bids. Using the standby queue as their only means of communication, bidders can negotiate new consistent collections to displace the standing high bids. According to the usual rules, bids can be withdrawn from the standby queue without penalty, but standing high bids cannot be withdrawn.

The standby queue plays a critical role in providing the information necessary to help guide the search for efficient packages. Without a standby queue, the auction ends when no bidder is willing individually to make a bid for a set of licenses that offers a higher price than the sum of the bids it displaces. However, one cannot conclude that a proposed collection is efficient just from that information. Indeed, neither the auctioneer nor the bidders can necessarily identify a better collection if the information provided by the standby queue is excluded. In principle, if bidder rationality is unlimited and bidders bid straightforwardly, ignoring their strategic incentives, a *tatonnement* process based on a standby queue could converge to an efficient allocation.

In the double-auction market for emission permits run by ACE and the Pacific Stock Exchange, bidders are allowed to communicate outside the formal structure of the standby queue. The idea is that that permitting informal communication of this sort makes it easier for excluded bidders to form coalitions to compete with the standing high bidders, resulting in more efficient license assignments. Common sense suggests that such arrangements also facilitate collusion, particularly in one-sided auctions with substantial money at stake. (In two-sided auctions, like the emission-permits market, the possibility of cooperation on both sides of the market as well as across sides makes analysis difficult.) Also, in some experimental applications of AUSM, bidders are issued tokens that do not affect their payoffs but do induce a budget constraint, since this “inside money” is used in the bidding. Because the tokens are not convertible to cash, bidders have no incentive to collude to economize on the total price in terms of tokens. Consequently, results from the token experiments are not relevant evidence for assessing the likelihood of collusion in an auction such as the spectrum auction in which bids are denominated in actual dollars.

THEORETICAL ANALYSIS OF DYNAMIC PACKAGE BIDDING

We begin with a simple example of a resource allocation problem in which package bidding can be useful.

Example 4. There are three bidders 1, 2, and 3 and two licenses A and B. The bidders’ values for the individual licenses and for the pair are tabulated in Table 3. The last column of the table is the Vickrey payment for each bidder.

Table 3

Bidder	A	B	AB	Vickrey Price
1	10	10	10	5 = 25–20
2	20	20	20	15 = 25–10
3	0	0	25	0

Bidders 1 and 2 are individual bidders who regard the licenses as perfect substitutes and only wish to acquire a single license at a price of up to 10 or 20, respectively. Bidder 3 has no demand for an individual license, but is willing to pay up to 25 for the pair. This is a moderately hard resource allocation problem about which several interesting observations can be made.

No competitive equilibrium exists in this example. Arbitrage requires that the competitive equilibrium prices of A and B be equal, but there is no single price at which there is demand for exactly two licenses. (At a price below 12.5 there is a total demand for at least three licenses; at a price above 12.5 there is demand for just one license. At a price of 12.5 exactly, there is demand for either one license or three, but not for two.)

Even though the two licenses are perfect substitutes, the Vickrey mechanism imposes different prices on the two winning bidders. (This illustrates price discrimination in the Vickrey auction.)

The sum of the two Vickrey prices, which is 20, is insufficient to discourage bidder 3 from wanting to acquire the license pair AB. Hence, these are not market clearing prices.

With complete information, it is possible that the efficient allocation could result from a mechanism like AUSM, but only if bidder 2 forgoes its ability to force arbitrage and accept unequal prices for the perfect substitutes. Bidder 2 has to pay a higher price than bidder 1 in order for the sum of the individual bids to discourage bidder 3.

Bidders 1 and 2 face a *free rider problem* in coordinating their bids against bidder 3. Each of them benefits from an increased bid by the other.

If bidders 1 and 2 were to merge, the Vickrey price for the merged pair would be 25, which is higher than the sum 20 of the individual Vickrey prices. This illustrates the potential, mentioned earlier, for a merger to be disadvantageous to participants in a Vickrey auction when there are complementarities among licenses.

Can AUSM with a standby queue be expected to lead to efficient outcomes in an environment like this one but with incomplete information? Or is the free-rider problem too hard to allow them to displace 3 when they should? To investigate that, let us replace the value table shown above by the following generalized version:

Table 4

	A	B	AB
1	v_1	v_1	v_1
2	v_2	v_2	v_2
3	0	0	v_3

Suppose the random variables v_1, v_2 , and v_3 are independently and continuously distributed. In the Vickrey auction, bidders 1 and 2 are winners at the dominant strategy equilibrium only when $v_1+v_2 > v_3$ and in that case the prices they pay are v_3-v_2 and v_3-v_1, respectively. The total price they pay when they win is thus $2v_3-v_1-v_2$, which is less than v_3. This generalized example has the following implications.

Inefficiency: Using AUSM with the standby queue, bidders 1 and 2 can be expected to defeat bidder 3 only by paying prices that add up to more than v_3, which is more than the total Vickrey price. Thus, if AUSM could possibly lead to efficient outcomes at equilibrium, it would do so while generating lower payoffs for 1 and 2 than the Vickrey auction, which is impossible according to the "revenue equivalence theorem" (see section 3.1). Hence, it is impossible that AUSM with the standby queue generates efficient equilibrium outcomes. It is clear by inspection that AUSM always allows bidder 3 to win when $v_3 > v_1+v_2$, so the inefficiency must be that the package bidder 3 wins too often and the individual bidders 1 and 2 do not win often enough.

Price Levels: When bidders 1 and 2 do win in the foregoing model, the total price they pay is v_3, which is always more than the total Vickrey price of $2v_3-v_1-v_2$ because the bidders win only when $v_1+v_2 > v_3$. Conversely, when bidder 3 wins, the price it pays never exceeds the Vickrey price of v_1+v_2; it may be less if 1 and 2 cannot overcome their free rider problem. Using the Vickrey auction as the benchmark, the AUSM equilibrium prices are too high for 1 and 2 and too low for bidder 3 – which further explains the source of the inefficiency cited above.

Strategic Manipulations: On account of the foregoing biases, a bidder might use package bidding strategically to win more licenses even when the bidder sees no actual complementarities among licenses. It might do this because the package bids win more often and lead to lower average prices than individual bids.

Finally, AUSM raises unresolved questions about collusion. While some signaling to promote market sharing certainly appears to have occurred in the simultaneous ascending auction, it appears to have been difficult for bidders to communicate detailed proposals for dividing markets. The proposals included in our first report for improving the simultaneous ascending auction will make collusion even more difficult. Depending on the packages allowed, package bids would make such communication much easier. Indeed, such communication is the main motive for allowing proposals for package bids to be "negotiated" via standby queue. While market division of that sort may or may not be good for efficiency, it is very likely to reduce the seller's revenues.

DYNAMIC PACKAGE BIDDING WITH DISCOUNTS

The determinate biases in the AUSM design compared to the Vickrey auction suggest that it could be improved by altering the pricing rule to reduce the biases. For example, a rule might specify that winning bids on individual licenses receive a discount. Such a rule would (1) increase the likelihood that individual bids win, (2) reduce the likelihood that package bids win, (3) reduce the payments made by individual bidders when they win, and (4) increase the payments made by package bidders when they win. Each of these changes brings the outcome closer to the outcome of a Vickrey auction. To avoid some obvious gaming opportunities, the rule would need to be accompanied by another rule that disqualifies a bidder who has made a package bid from receiving a discount for subsequent bids on individual licenses in that package.

An obvious difficulty with introducing discounts is that one must decide how large the discounts will be. However, from the perspective of experimenting with auction designs, a system of discounts has the advantage that it nests two proposed auction designs. When the discount is zero, the payment rules resemble the AUSM rules. When the discount is so large that no bidder will offer a package bid, the auction reduces to the FCC's standard simultaneous ascending auction. Further, because bidders can always bid for individual licenses, bidders who wish to acquire large packages are no worse off in this auction than in the FCC's standard auction, regardless of the discount allowed. This suggests that there is little risk to experimenting with moderate discounts, say in the range of 10%-30%, which discourage frivolous packages and correct some biases while still creating a potentially valuable opportunity for bidders who find it important to acquire all the licenses in a package.

3.3 Alternatives to Package Bidding

In light of the potential difficulties with package bidding, it is useful to assess whether there are alternative ways to reduce the exposure problem. Two approaches have been used to date: withdrawals and license definition.

The primary intent of withdrawals is to reduce the exposure problem by allowing bidders to back out of failed aggregations. Two factors limit the use of withdrawals for this purpose. First, under the existing FCC rules, a withdrawal exposes the bidder to the possibility of a substantial withdrawal penalty if another bidder does not pick up the license. Second, the activity rule may prevent another bidder from picking up the license late in the auction. Both the withdrawal penalty and the activity rule are required to promote sincere bidding in the current FCC auction design, yet both limit the effectiveness of bid withdrawals.

In FCC auctions to date, few withdrawals appear to be made to back out of failed aggregations. Instead, the withdrawals appear to have been part of parking strategies or signaling strategies. When withdrawals do occur the ultimate withdrawal penalties tend to be small, although it is also common for the withdrawing bidder to re-acquire the license, especially when the withdrawal is late in the auction. Most of the evidence suggests that either the exposure problem was not important in the spectrum auctions to date, or that withdrawals were ineffective as a tool to back out of failed aggregations. Ausubel et al. (1997), in an analysis of the AB and C auctions, present evidence that supports the claim that the exposure problem was not severe. Bidders had modest synergies for adjacent licenses. They bid for these synergies and were often successful. They did not pay a substantial premium for adjacent licenses.

Withdrawal rules can be adjusted to make them more effective at reducing the exposure problem. For example, Evan Kwerel has suggested the following option in an auction where licenses have either a nationwide or local use. Bidders designate themselves as nationwide or local. Local bidders face the standard rules. Nationwide bidders must place individual bids for the whole nation, topping the local bidders by a particular premium. To limit the exposure problem, a nationwide bidder's withdrawal penalty is limited to, say, one bid increment and is calculated on an aggregate basis, so that licenses that sell for more than the nationwide bidder's bid contribute to reducing the overall penalty. Following a nationwide bidder's withdrawal, eligibilities of the other bidders is restored to their original levels.

The second approach to reduce the exposure problem is for the FCC to define licenses that package bands in ways that take account of expected complementarities. This works well when the preferences of the bidders are not too diverse with respect to optimal packages. The FCC can then package different bands differently to reflect the interests of the bidders. An example is selling the narrowband spectrum at several levels of aggregation (nationwide, regional, and MTA). When conducted this way the FCC should err on the side of too much disaggregation, as they did in the narrowband allocation, since bidders can (and did) form the more aggregated packages.

The number of "copies" of each kind of license can also affect the ability of the auction to generate efficient aggregations. If many copies are offered, prices become more predictable late in the auction, allowing bidders to decide more accurately when to exit.

4. EXPERIMENTAL EVIDENCE ABOUT AUSM

As best we can determine presently, the only prior uses of package bidding in auctions by government agencies were two auctions in New Zealand. One was apparently a Vickrey auction of forestry rights for timber on approximately 50 parcels, conducted in 1989 by the firm Fay Richwhite. The second was the privatization of the regional Radio New Zealand companies in which package bids were allowed, but we have not yet learned which pricing rule was used. Unfortunately, nothing has been published about these auctions, although we understand that the NZ Treasury might have records. The forestry auction is especially interesting because to select the winning packages it used an integer-programming code developed at Turnbull Partners in Sydney. Informal reports indicate that these auctions proceeded without difficulties; on the other hand, there is no indication of further efforts to employ package bidding in subsequent auctions.

In the remainder of this section, we summarize and evaluate the results from experimental laboratories that assess the desirability of package bidding in the AUSM format. Although the data are limited, the preliminary results are largely consistent with our theoretical analysis.

4.1 Credibility of Experimental Evidence

The external validity of experimental results remains a controversial topic among economists. Some economists are skeptical of virtually all experimental results, arguing that the behavior of students in laboratories is not a reliable guide to behavior in "real life." Some experimenters argue that experiments with real money at stake *are* real life and consequently that nothing in standard economic theory justifies expecting behavior in laboratories to be different from behavior in market settings.

In our view, experimental results represent real evidence about how certain people play the auction game. The reported behavior of subjects in experiments seems to lack the foresight and sophistication shown by some of the corporate bidders in the FCC auctions. We think it is unsurprising that bidders in very high stakes FCC auctions are more analytically sophisticated than subjects in experiments. This perspective guides our assessment of what can be usefully learned from laboratory experiments involving auctions.

Any auction rule that is gamed successfully in the a typical laboratory experiment is likely to be gamed successfully by bidders who take more time, play for greater stakes, and have access to expert consultants. Similarly, any mechanism simple enough for experimental subjects to understand and play skillfully is simple enough for the FCC's bidders. On the other hand, we should not be too quick to conclude that a mechanism that is not gamed successfully in a typical laboratory environment will survive gaming in the FCC environment, or that the absence of collusion in the laboratory will predict the same in the FCC auctions. For conclusions of that sort, a careful analysis of gaming opportunities is necessary. Both experiments and theoretical analysis can be useful guides for forecasting bidder behavior in complex situations.

4.2 Comparing Theoretical Predictions to Experimental Evidence

The following observations compare outcomes from two auction designs: a simultaneous ascending auction with individual bidding and a simultaneous ascending auction with package bidding and a standby queue.

Observation 1: The experimental evidence is consistent with the theoretical predictions about both the *exposure* problem with individual bidding (efficient packages are often missed) and the *free rider* problem with package bidding (inefficient packages are often formed). In the experiments, individual bidding never produces a package when it should not, but sometimes fails to do so when it should. Package bidding produces a package when it should, but sometimes when it should not.

Package bidding tends to favor package bidders. Small bidders can have a difficult time topping a large package bid, even when the small bidders' values exceed that of the package bidder. For example, Ledyard, Porter, and Rangel (1997), hereafter LPR, found in one set of experiments that an inefficient package bid won in 55% (12 of 22) of the cases where single-item bids should have won. The full package won in 100% (13 of 13) of the cases where it should have won.

Individual bidding tends to discourage packages. The full package won in only 14% (1 of 7) of the cases where it should have won. Part of this may be explained by demand reduction. However, the same incentives for demand reduction are present under package bidding, so it would be necessary to infer that demand reduction is more salient without package bidding. An alternative explanation is that bidders were reluctant to bid for packages due to the exposure problem.

Observation 2: Settings with large and inconsistent complementarities require package bidding to attain high levels of efficiency. In other settings, individual bidding performs well.

LPR found that package bidding was needed to get high levels of revenue and efficiency in settings requiring complex "fitting" of packages to attain efficiency. Prices on only individual items were insufficient to guide the bidders to the efficient packages.

In "easy" settings with downward sloping demands or modest complementarities, individual bidding yielded both high revenues and high efficiency. LPR have no evidence on how well package bidding performs in these easy settings, but they speculate that it may do worse than individual bidding by focusing attention on inefficient packages. Data on how well package bidding does in easy environments would be useful, since in practice it is difficult for the FCC to know whether the setting is easy or hard.

4.3 Comparisons to the FCC Auction Environments

The theory and experimental evidence agree that the desirability of package bidding depends on the auction environment, with complementarities (or, less precisely, "synergies") the crucial variable. The available evidence about the environments encountered in the FCC auctions to date suggest that none was the sort where package bidding is advantageous. All appeared to produce excellent results with bidding for individual licenses.

The following qualitative assessments are based on our own experiences in working with bidders as well as on published results (Ausubel et al. (1997) and Cramton (1995, 1997)).

1. Nationwide narrowband: Downward sloping demand for spectrum, so there is no complementarity arising from scale economies. Modest synergy (5 to 15%) from acquiring adjacent bands.

2. Regional narrowband: Synergy from building nationwide aggregation on same band. Synergy is obvious and consistent across bidders.

3. AB blocks: MTAs are too large to make synergies an important factor.

4. C block: At the BTA level, synergies are more important, but tended to be consistent across bidders. Major BTAs were sufficiently large to be administered as standalone entities. The exposure problem was greatly reduced by securing major BTAs first and then bidding for smaller neighboring BTAs.

5. DEF blocks: Similar to C block, since aggregation across bands was less important than aggregation across BTAs.

6. MDS: Encumbered spectrum. Like C block, but much more obvious who should win what.

7. SMR: Encumbered spectrum. PageNet valued nationwide aggregation on two bands.

8. DBS, DARS: No synergies. No opportunities to build packages.

There is neither experimental nor field data to suggest how auction outcomes would have changed if package bidding were allowed. Neither LPR nor others have tested package bidding in the “easy” settings of the FCC auctions to date. The ACE market for emissions credits differs in other ways: It is a two-sided double-auction market that brings together a handful of buyers and sellers. The process allows extensive communication and coordination, and may be best interpreted as a mechanism to facilitate negotiations among a few traders. The process used in Los Angeles is limited to just 5 bidding rounds. This is quite different from the one-sided auctions conducted by the FCC with hundreds of bidders and licenses. (AUSM has reportedly been used in one or more private auctions, but the application and the data are confidential.)

5. TOWARD AN FCC AUCTION EXPERIMENT WITH PACKAGE BIDDING

In this section we outline our initial recommendations regarding auction designs that are viable candidates for experimental trials. We focus on the main structural features, deferring until our next report the development of procedural details. Our intent is to focus on matching the auction design to an environment in which there is a reasonable prospect that the auction will perform well, with little risk to the government, while at the same time revealing considerable information about the advisability of extending the use of package bidding to a wider array of auctions. This requires consideration of several performance criteria, including efficiency and revenue from the outcome, as well as the practical aspects of implementation for both the FCC and the bidders.

5.1 Motivating Contexts and Possible Experimental Settings

We envision three possible contexts as the ones most likely for an initial trial of package bidding for spectrum licenses. The first two involve complementarities of a traditional sort, while the third entails a subtle mixture of complements and substitutes.

Structured complementarities. Among the structured complementarities, the most familiar are the hierarchical structures associated with spatial aggregations, such as licenses covering national and regional areas, MTAs, and BTAs. Another is associated with aggregating frequency bands. The defining characteristic of hierarchical structures is that whenever the spectrum bands or service areas of two licenses intersect, one subsumes the other.

Other kinds of special structures are also possible. For example, implementing a technology may require spectrum bands separated by a particular or sufficient distance in the spectrum. As another example, if business plans vary according to whether a core city is included, then the relevant packages may be all those that contain the core city and one or more licenses covering nearby areas.

In these cases, package bidding in an auction can be used to supplant administrative decisions on the extent of spatial and spectral aggregation. It can also replace the role of aftermarkets for assembling aggregations, as in the case of Nextel’s purchase of radio dispatch bands to construct a system for mobile telephony.

In a simultaneous ascending auction with package bidding, restrictions on license combinations amount to limiting the kinds of *complementarity* that bidders can express in their bids and give rise to a corresponding exposure problem. If a bidder regards different licenses or different packages as *substitutes*, however, it has no problem expressing those values straightforwardly by altering the packages for which it bids as prices change.

The most straightforward way to implement a Vickrey auction with restricted combinations is simply to ask bidders to submit values for every allowed license combination. However, to be effective, the allowed set of combinations may need to be much larger than in a simultaneous ascending auction, because it needs to account for substitutes as well as for complements. To illustrate this point, suppose a bidder has values of 10 each for licenses A and B and 15 for the pair AB. If bidding for the AB combination is not allowed, it cannot accurately express these values. Its profit-maximizing bid for license A depends on what outcome it expects in the bidding for license B.

There are some special circumstances in which the Vickrey auction does not require such additional packages. One possibility is that it is known *a priori* that none of the licenses or packages are substitutes. Another is that the pattern of substitution is known (for example, in the PCS broadband auction it was known that in each market the A and B band licenses were substitutes). In the second case, the mutual exclusivity of the licenses could be built into the structure of the auction, without any need for explicit recognition in the bids. Generally, the feasibility of describing the relevant packages in a way that is suitable for a Vickrey auction depends in a detailed way on the choice of environment.

Complex "fitting" problems. This is the context for which AUSM was originally developed. In the case of spectrum licensing, it arises when the most relevant combinations of complementary licenses are overlapping (non-hierarchical). Such a context arose in Australia and will arise in Mexico, where the bidders' choices between European and American standards could produce conflicts in the pairing of incoming and outbound bands. At this time it appears that package bidding to solve fitting problems will serve mainly to resolve issues that previously were addressed administratively via specifications of band plans and band-specific restrictions on the allowable technologies or standards.

Knapsack problems. The familiar example of this context is one where bidders are constrained by limited budgets. Standard complementarities are not the primary issue in this case. It is unclear whether such a setting is likely to arise for the FCC, but it might occur if limited subsidies or quotas were used to fulfill policy objectives; e.g., allocation of preferences to rural telephone companies, or to wireless services in competition with wireline services for provision of universal service.

The Vickrey auction is implementable in these scenarios only if the packages that can be won and the valuations of them are easily described, either because the number of licenses is small or because the dependencies are neatly structured. (If no restrictions are placed on the allowed packages then, in principle, the bid submissions require an exponentially large amount of data, and the optimization procedure must be capable of solving an integer programming problem with exponentially many variables and constraints.) The structured dependencies must include both substitutes and complements. In addition, the price uncertainty needs to be small enough to relegate the budget problems described in section 3.1 to minor status.

Simultaneous ascending auctions have the comparative advantage that only complementarity relations need to be described in the packages; there is no need for bidders to submit initially their entire lists of bids for all packages. Bidders can substitute among licenses or packages as prices evolve by revising their bids from round to round. Due to this practical advantage in implementation, most of the following discussion focuses on a simultaneous ascending auction with package bidding.

It is easiest to implement an auction that deals satisfactorily with the first context, in which any complementarities are well structured. The results of such an auction would be informative mostly about the validity of the theoretical predictions and the relative importance of the several influences that we have identified. Hierarchical structures, while the easiest to implement, seem also to be the most susceptible to collusive bidding, because bids for non-overlapping packages can be used to deliver clear proposals and counterproposals about how to divide licenses between bidders.

The second context provides the ideal setting for realizing the potential advantages of a simultaneous ascending auction with package bidding, without encountering its possible adverse effects. Moreover, the experiment could be highly informative about the ability of bidders to form complex packages that fit together. Implementation may be difficult if it requires software with elaborate integer-programming capabilities, but in the New Zealand forestry rights case the software was adequate to accomplish the optimizations required, so there is some hope that software development would not be a severe constraint on the design.

These considerations can be distilled into two opposing forces. On the one hand, a full Vickrey auction has desirable efficiency and incentive properties, but it is difficult to implement except in very well-structured environments. Also, the Vickrey pricing rule may be seen as problematic. On the other, a simultaneous ascending auction with package bidding has weaker efficiency and incentive properties, but it is easier to implement and the pricing rule is entirely standard. The key to examining the tradeoffs between these two designs is the relative importance of "negotiations" among the bidders to build packages. In the context of fitting problems, there is presumably no way to pre-specify the candidate packages according to any simple scheme, so the relative advantage may lie with an AUSM-style auction *cum* negotiation process. For the FCC, however, it is important to approach this design with considerable caution to avoid facilitating collusion.

5.2 Leveraging the FCC's Experience to Create an Experimental Prototype for Package Bidding

A prototype that conforms closely to the existing format for the spectrum auctions would have three important advantages. First, it obviates the need to re-design the full set of procedural rules, software interfaces and displays, etc. Second, bidders will be familiar with the main ingredients from prior experience or observation of the previous spectrum auctions. Finally, the results of a trial auction will be informative about the effects of package bidding without the confounding influence of other changes in the procedural rules. Because of these advantages, we focus attention on rules that are close to the existing simultaneous ascending auction rules. We also consider the Vickrey auction for its impressive theoretical advantages in specialized circumstances.

The selection of an application for the initial trial of package bidding must be done with great care. The most severe risk is that in situations with nonexistent or moderate synergies, the disadvantageous gaming that can accompany package bidding in a simultaneous auction could significantly reduce the FCC revenue without any gain in efficiency.

There are two basic ways to implement package bidding, depending on who assumes the responsibility for identifying the most valuable combinations. One method uses a standby queue or an analogous system to allow the bidders themselves to determine how their packages may fit together to defeat the current standing high bids. The second assigns the task of finding the most valuable feasible combination of packages to the auctioneer. Both systems require powerful software to select the efficient combination of packages. Even if the software's full capability is not always used in an auction, it must be capable of dealing with the most severe complexities likely to arise. For example, if there is no hierarchical or other similarly special structure for the packages, then the software must in principle have the capability of solving elaborate integer programming problems. Testing and ensuring the accuracy and reliability of such software will be extremely difficult.

Any dynamical system with package bidding involves serious risks of collusion, since a bid for a large package can often be used to communicate an understandable proposal about how licenses should be divided. Standby queues, to the extent that they allow bidders to propose packages without actually committing money to purchase them, exacerbate the problem. Bulletin boards with even more detailed information provide further opportunities to collude to keep prices low.

If standby queues are used at all, their use should be circumscribed to communications needed to solve the fitting problem. In particular, there should be no natural-language messages. Posted offers on the bulletin board should not specify more than contingent bids that in some way are purged of information conducive to collusive negotiations. In any case, the role of standby queues will need to be vetted by experts on possible conflicts with antitrust law. Our current projection is that it will be extremely difficult to devise a format for a standby queue that is not vulnerable to some degree of implicit collusion.

5.3 Basic Design Considerations for an Experimental Prototype for Package Bidding

The main decisions needed to implement an experimental package bidding auction can be reduced to a set of questions.

WILL THERE BE *RESTRICTIONS* ON THE ALLOWABLE PACKAGES?

If the number of licenses to be auctioned is small (say, ten or fewer), then there is little difficulty in reporting and evaluating bids for any package of licenses that the bidders may wish to suggest. However, when a much larger number of licenses is to be auctioned, restrictions on the allowable packages can be useful because they drastically reduce the complexity of both the government's problem in determining which packages are winning bids and the bidders' problems in determining which packages to bid on. Heuristically, the allowable packages in an auction design correspond to the seriously competing proposals in an administrative hearing. It is competition among these that determines the outcome.

When restrictions are to be employed, one must ask: what restrictions? Here one may seek bidder input, either through an administrative procedure or by allowing bidders to nominate license packages. It is difficult to offer general advice about the nomination process, since little can be said in the abstract about the best packages of licenses. The nomination process should be structured to allow bidders to name the most likely packages while still keeping the number of packages small or the overlap structure of the packages simple enough to be manageable by the software available to the FCC and to the bidders. Also, if a Vickrey auction is to be used, each bidder in the auction should be permitted to identify bids that are mutually exclusive.

WHAT IS TO BE THE FORM OF THE *PRICING RULE*?

With discriminatory pricing allowed, one can implement a version of the Vickrey auction in which each bidder is charged the opportunity cost or externality imposed on other bidders, thereby ensuring truthful declarations of valuations as a dominant strategy, resulting in an efficient allocation of licenses. However, these charges may have other properties that are undesirable from a legal or public relations perspective; e.g., a losing bidder might have offered more for a package that the sum of the charges paid by those bidders who won its components.

Alternatively, if a winning bidder pays the amount of its bid as in a standard simultaneous ascending auction, then the problem identified in the previous paragraph is avoided. However, as we have seen, this pricing rule has important disadvantages. It encourages strategic manipulation (bidding for packages for which no synergies exist), and results in the package bidders winning too often and at prices that are too low relative to the standard set by the Vickrey auction.

HOW WILL THE AUCTION *DYNAMICS* BE STRUCTURED?

This involves a choice among a single-iteration sealed-bid auction, multiple iterations, and continual opportunities to revise bids. The latter two may require innovative designs of activity rules, stopping rules, withdrawal rules, and other procedural features. There is, of course, also the possibility of allowing different formats for single-item bids and package bids; e.g., one version provides single sealed bids for packages and an iterative ascending-price auction for individual items.

WILL PACKAGE BIDS AND INDIVIDUAL LICENSE BIDS BE TREATED DIFFERENTLY?

The most germane prospect here is that, with a pay-your-bid pricing rule as in AUSM, a discount might be applied to winning bids for individual licenses. The reasons to apply such discounts were discussed in section 3.2 of this report. As experience is accumulated, there could even be discounts that vary by the coverage of the package bid.

5.4 Design Priorities

We distinguish between two categories of design criteria. One emphasizes the results of the auction; the other emphasizes the information extracted from the initial trial of package bidding.

EFFICIENT AUCTION OUTCOMES

The congressional mandate for the spectrum auctions explicitly sets efficiency as the primary objective, with revenue distinctly secondary. On the basis of this objective and the existing theory, the most promising avenue for substantial improvement is to adopt some version of a Vickrey auction. In the context of private values and no budget constraints for which the favorable theoretical analysis applies, the Vickrey auction is unique in its ability to assure efficient outcomes while simultaneously suppressing gaming opportunities. If an appropriate environment can be identified, the prospect of substantial gains could be realized.

Besides the question of environments, there are political and legal issues to be resolved before the FCC could decide to employ the Vickrey pricing rule. Besides the usual complaint that this rule is complex and does not assign equal prices to licenses that are close substitutes, it also creates the risk that some losing bidder will bid an amount for a package that exceeds the charges imposed on winners of the package components. (See the example in Table 3, in which the total Vickrey prices of licenses A and B is 20 but bidder 3 is willing to pay up to 25 for the package AB.) The acceptability of such outcomes and any associated risk of litigation must be resolved as a legal or policy matter within the FCC. Here, however, we take it as given that the current congressional mandate for experiments with package bidding provides the FCC with the requisite latitude to use the Vickrey pricing rule, at least on an experimental basis.

ACQUIRING PERFORMANCE INFORMATION

The Vickrey auction is relatively well understood on theoretical grounds and the main lessons are likely to be about the importance of the budget and common value issues described in section 3.1 and the about difficulties of implementation. One may also learn something about how the attitudes of bidders affect the implementability of the auction. The Vickrey auction has the dual properties that (1) bidders do not know when they bid how much they will have to pay (which is especially significant when there are binding budget limits) and (2) bidders have to reveal more information about their values than in other kinds of auctions. It is sometimes argued that this latter disadvantage is a source of substantial bidder resistance that is overlooked by the standard theory, but we consider that assertion to be still a conjecture, neither supported nor refuted by any systematic evidence.

By comparison, the pay-your-bid rule in a simultaneous ascending auction with package bidding has been largely overlooked by theorists and has received limited laboratory testing. It is possible that a trial auction by the FCC might provide substantial new insights into the functioning and outcomes associated with such a rule. If discounts for individual winning bids are employed, the experiments could yield useful information about the proper size of the discounts. Bidders' use of or failure to use, the discounts could reveal information about the extent of complementarities in their license valuations. And, if a standby queue is used, the experiment could yield useful information about how bidders in spectrum auctions use it to build packages.

The most severe risk seems to be the inherent prejudice in favor of the package bidders and the resulting prospect of strategic manipulation, inefficiency, and reduced revenue. On theoretical grounds it appears that this risk could be mitigated if the winning bids for individual licenses were discounted by a fixed amount, in a fashion similar to the preferences the FCC has implemented in other auctions.

5.5 Proposed Designs for Trial Auctions of Experimental Versions of Package Bidding

The primary objective of a trial auction is to examine the merits of allowing bidders to make offers for packages of complementary licenses. However, this objective is confounded by two other considerations. One is the *pricing rule*, since it matters greatly whether package bids are priced as in the Vickrey pricing rule, which eliminates strategic incentives, or on a pay-your-bid basis as in AUSM, which engenders substantial incentives for strategic manipulation, as well as risks for the FCC regarding efficiency and revenue. The second confounding consideration is the possibility of *learning* and *communication*. In settings with substantial "common value" elements, bidders inevitably learn important information during the rounds of an auction. Yet most of the proposals for Vickrey-style designs are cast in terms of static sealed-bid auctions with no substantial opportunities for learning. The implementations of dynamic versions that allow learning but do not use the Vickrey pricing rules, such as AUSM, rely heavily on vigorous communication among bidders in order to negotiate the assembly of packages of licenses. This is the traditional role of bidding rings (Graham and Marshall 1987, McAfee and McMillan 1992), but it is fraught with danger for the FCC because it could easily be used as a mechanism for illegal collusion.

We envision two basic proposals for the experimental designs.

DESIGN 1: A PARTIAL VICKREY AUCTION

In this design, the key features are:

A well-structured environment in which the number of relevant licenses is small (up to ten) or the potential groupings of substitutes and complements can be neatly characterized in advance of the auction. The latter may be a hierarchical structure. It may be also possible to allow the potential bidders to nominate the packages they want to include, subject to the pre-determined limits on the capacity of the integer-programming algorithm.

The pricing rule is the *Vickrey rule*: based on the bids "declarations of values" received, the highest-value allocation determines the winners, and each winner pays the externality cost imposed on other bidders. The implementation will require a vigorous program to inform bidders about the workings of the Vickrey pricing rule, so that they understand thoroughly that the submitted bids represent value declarations, that the amounts paid by a winning bidder is unaffected by its value declarations, and that a package bid may be rejected even though it exceeds the payments by the winning bidders for the components of the package.

If the trial situation allows a sealed-bid format then this should be used. We also anticipate trying to develop a form of the Vickrey auction with *multiple iterations* to allow learning about common-value components of valuations and about budget requirements, but the prospect of such a design remains uncertain.

DESIGN 2: ADDING PACKAGES TO THE SIMULTANEOUS ASCENDING AUCTION

The purpose of this alternative design is to provide a full-scale test of the claims made for this class of auction designs. The key features are:

The context is chosen to be one in which there are complex "fitting" constraints or unpredictable and heterogeneous complementarities among licenses. *No prior restriction* is placed on the allowed packages for which bids are acceptable. Due to the possibility that package bids would be used strategically to gain advantages in the auction in the absence of significant complementarities or fitting constraints, it will be essential to conduct this trial auction only in a context in which it is presumed evident that there is little chance that any bidders would view packaged licenses as substitutes. The ideal context would be a contest among bidders about whether to use the licenses for one or another of several conflicting purposes, such as conflicting standards, incompatible technologies, or interfering uses.

The pricing rule is *pay-your-bid*. That is, the winning bidder or implicit consortium for each accepted package pays the amount bid for that package. The winning packages are selected by solving an integer-programming problem whose objective is to maximize the seller's revenue. A key design feature will be whether to provide discounts for individual licenses. If a discount is adopted, then guidelines must be developed about how to select its magnitude (and there will presumably be a prejudice in favor of discounts that are more likely to be too large than too small); it may be that a simulation study will be necessary to gauge the right magnitude. In this design, the education program for bidders will focus on developing understanding of what the integer programming algorithm does and how it affects whether a bid is accepted.

The auction is *dynamic*, with multiple rounds, and it is supplemented by a *bulletin board* (BB) or other means of replicating the main features of the AUSM standby queue. A key design feature will be the format of allowed postings on the BB, and in particular to devise a format for contingent offers and a means of purging information that might be used collusively.

The procedural rules must be designed along the lines of the FCC's spectrum auctions, with the addition of package bids. It would be potentially disastrous to substitute the procedures used in the laboratory experiments with AUSM. In particular, the use in the economics laboratory of short, random stopping times to force a frenetic pace on bidders will be entirely unsuitable for a trial auction by the FCC.

The form of contingent bids requires further study before explicit design proposals are submitted in our next report. In the simplest format, in which the BB is effectively integrated with the auction board, an individual bidder can offer firm bids for single licenses, and also package bids (as in a Vickrey auction) with the understanding that these are mutually exclusive: the integer program selects a feasible or consistent combination of bids for non-overlapping licenses and packages. In a more elaborate format a bid can be of the form of a package bid for one set of licenses that is contingent on winning one or another (or a sub-collection) among several possible complementary sets. Bids submitted in these and other formats can in principle all be evaluated by an integer-programming algorithm, but the complexity of the software and the solution time typically grows exponentially with the options included in the contingent bids. The software that can be obtained may dictate the allowed format of contingent bids in the auction. On the other hand, if the BB is distinct from the auction board then the burden of assembling package bids from posted offers on the BB rests with the bidders (which just pushes the software issue to a different venue).

A preliminary sketch of procedural rules

The central problem in designing procedural rules for a simultaneous ascending auction with package bids is to devise suitable activity rules. There is a fine balance to be attained. On one hand, one wants bidders to be able to post bids to the standby queue in order to facilitate the formation of well-fitting packages. On the other, one does not want all such bids to count as activity, because that would create excessive opportunities for “parking” of eligibility. To motivate a sketch of our current thinking, consider the following scenario, which illustrates the free rider/bargaining problem at the heart of the matter.

Scenario. Bidder 3 currently has the standing high bid of $30 for the package ABC of three licenses A, B, C. Neither bidder 1 nor bidder 2 foresees sufficient value from the entire package to surpass 3’s bid. However, each anticipates that there is some prospect that together they could outbid 3. Thus, 1 might post a bid on the queue of $22 for the package AB, intending this as an offer to 2 to bid in excess of $8 for C to beat 3’s standing high bid. This offer might be accepted by 2, but alternatively 2 might also post an offer of $22 for BC, thereby sending an analogous signal to 1. In the latter case, each realizes that their joint opportunity to acquire the package ABC depends on which one is willing to acquiesce to the other’s acquisition of the contested B license. One can envision a sequence of higher offers, say to prices of $23 or $24 in successive rounds, but eventually one or the other must cave in and accept the single license A or C if they are to win. During this process of implicit negotiation, additional offers might convey further information; e.g., 1 might offer $6 for license A, which could be the signal to 2 that raising its package bid for BC to $25 will suffice to beat 3’s standing high bid of $30 for the package ABC.

This scenario conveys the brightest prospects for use of the standby queue, because it illustrates how implicit negotiations can be organized via package bids to realize the most efficient combinations of licenses. Without a carefully planned activity rule, however, these negotiations could be endlessly extended, or the process of price discovery might be curtailed unnecessarily. (The laboratory approach avoids the first problem by imposing a deadline on the bidders.)

As in the current designs of simultaneous ascending auctions, we anticipate that activity credits will be carried over to the next round only for the standing high bids at the end of the round, and not for high bids on the standby queue. We also anticipate the bidders will be considered active when they make a “*serious*” bid, just as they are in the current simultaneous ascending auction. Also, a bid is certainly serious if it is high enough to defeat some current standing high bid. What will change is that certain other bids, which do not become standing high bids, will be deemed serious as well. The identification of those will rely on a “*coverage measure*.”

For the sake of illustration, let us focus on the PCS auctions, in which the *coverage measure* could be the number of MHz-Pops associated with a license. Dividing any group of standing high bids by their corresponding coverage measure creates *price index*, in this case expressed in $/MHz-Pop, for the group. The key observation is that a consistent collection of packages in the standby queue can supplant a group of standing high bids only if at least one of the new bids has a higher price index than the index for the displaced group. Negotiations are destined to be fruitless unless some bidder is willing to make a bid with a price index that exceeds that level and unless the others respond to that bid with bids that raise the total for the collection sufficiently. We use this observation to construct a rule that distinguishes potentially serious offers from non-serious ones.

We anticipate that a suitable activity rule can be constructed along the following lines.

- *The Activity Rule.* A bidder must be active on a specified fraction of its eligibility, or lose a proportionate share, exactly as in the PCS auctions. A bidder cannot make new bids that cause the total coverage of its current standing high bids and its high bids on the standby queue to exceed its eligibility. (However, a bidder need not withdraw bids from the standby queue to bring the total coverage of its bids within its eligibility limit.)

- *Standing high bids*. A bid is a "standing high bid" at the end of a round if it would be a winning bid if the auction were to end with no new bids at the next round.

- *Serious bids.* A bid is "serious" if it displaces a current standing high bid (or would displace that bid but for other new bids in the same round) or if its price index exceeds a specified percentage, say 80%, of the price index of the group of standing high bids with which it shares a license.

- *The Definition of Activity.* A bidder receives activity credit on any package for which it has the *standing high bid* from the previous round or for which it makes a new *serious* bid that also exceeds the specified minimum bid.

  (It is important to observe that a previous "out-of-the-money" offer that is not a standing high bid does not provide activity credit, even if it is the current high bid for a package in the queue. For this reason, only one bidder gets activity credit for any license in a round, and that is the owner of the standing high that includes that license.)

- *The Minimum Bid Increment Rule.* A new bid for a package must exceed the previous high bid in the auction or on the queue by the minimum bid increment, say 5% or 10%.

- *Waivers*. Each bidder will have ten waivers during the auction. (The extra waivers are needed to maintain eligibility during the "bargaining" process, for example while awaiting a response to an "offer" made at the previous round.)

This tentative formulation involves several features peculiar to package bidding and standby queues. First, the highest posted bids for packages in the queue are defined not to be "standing high bids." Unlike standing high bids, these bids can be withdrawn and they do not count as activity in the next round. Second, new bids for unselected packages can receive activity credits for the current round, but only if they meet a criterion of seriousness. Third, eligibility refers to the right to bid, *not to the right to win packages*. A bidder who does not wish to improve its high bid in the queue may allow its eligibility to expire without having to withdraw its "offer."

The suggested percentage of 80% is merely illustrative. Unless some bidder is willing to make a bid with an index of that is at least 100% of the index of the potentially displaced bids, the negotiations will fail. Under our proposed rule, new bids near 100% of the index can be counted as serious from round to round, but they continue to count as activity only if they are continually raised. This permits some scope for bargaining. Once some bidder has made a bid of more than 100%, other bidders may be able to make bids with lower indexes that become standing high bids.

In our next report we expect to undertake a further analysis of this and other formulations of the procedural rules for a simultaneous ascending auction with package bidding. We see this task as essentially one of compromising between the conflicting objectives of reliable price discovery, deterrence of collusion, and prevention of excessive parking behaviors.

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