

A Troubled Asset Reverse Auction

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1 Summary

The US Treasury has proposed purchasing \$700 billion of troubled assets to restore liquidity and solve the current financial crisis, using market mechanisms such as reverse auctions where appropriate. This paper presents a high-level design for a troubled asset reverse auction and discusses the auction design issues. We assume that the key objectives of the auction are to:

- provide a quick and effective means to purchase troubled assets and increase liquidity;
- protect the taxpayer by yielding a price for assets related to their value; and
- offer a transparent rules-based process that minimizes discretion and favoritism.

We propose a two-part approach.

Part 1. Groups of related securities are purchased in simultaneous descending clock auctions. The auctions operate on a security-by-security basis to avoid adverse selection. To assure that the auction for each security is competitive, the demand for each security is capped at the total quantity offered by all but the largest three sellers. Demand bids from private buyers are also allowed. The simultaneous clock auctions protect the taxpayer by yielding a competitive price for each security and allow bidders to manage liquidity constraints and portfolio risk. The resulting price discovery also improves the liquidity of the securities that are not purchased in the auctions.

Part 2. Following Part 1, the remaining quantity is purchased in descending clock auctions in which many securities are pooled together. To minimize adverse selection, reference prices are calculated for each security from a model that includes all of the characteristics of each security including the market information revealed in the security-by-security auctions of Part 1. Bids in the pooled auctions are specified in terms of a percentage of the reference price for each security.

The two parts are complementary. Part 1 quickly adds liquidity and establishes competitive market prices for many securities. Additional assets are then purchased in Part 2, taking full advantage of the market information revealed in Part 1.

Clock auctions have been used successfully in the electricity and natural gas sectors for assets worth tens of billions of dollars over the last seven years. Related approaches have been used to auction emission allowances, radio spectrum, timber rights, and rough diamonds.

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The approach is feasible even on an extremely tight timetable. The development of the reference price model and data will require the most time to complete, but fortunately this is not needed until Part 2. The auctions of Part 1 can begin in a matter of weeks.

We limit the scope of the paper to taking the legislation as given and providing an auction design within the requirements of the legislation.

The paper is organized as follows. We begin by explaining the adverse selection problem that arises when different securities are pooled together. Then we provide an overview of the two-part reverse auction plan. Next we develop details of Part 1, the security-by-security auctions. We then present the motivation for the simultaneous descending clock auction. Next we present further details of Part 2, the pooled auctions. Then we explore how stock warrants and senior debt can be usefully incorporated into the purchases of troubled assets. Next we discuss the feasibility of successful implementation on an expedited schedule. We conclude that the design achieves the key objectives and can be implemented with minimal risk in a short period of time.

2 Pitfalls of pooling different securities

A simple but naïve approach would be to invite the holders of all troubled assets to bid in a single reverse auction. The Treasury sets an overall quantity of securities to be purchased. The auctioneer starts at a price of nearly 100 cents on the dollar. All holders of illiquid securities would presumably be happy to sell at nearly face value, so there would be excess supply. The auctioneer then progressively lowers the price—90 cents, 80 cents, etc.—and bidders indicate the securities that they are willing to sell at each lower price. Eventually, a price, perhaps 30 cents, is reached at which supply equals demand. The Treasury buys the securities offered at the clearing price, paying 30 cents on the dollar.

The approach of pooling different securities in a naïve way is fatally flawed, as it gives rise to a severe adverse selection problem. For example, suppose that the values of the troubled assets are uniformly distributed on the interval from zero cents on the dollar to 75 cents on the dollar. If the pooling auction clears at 30 cents on the dollar, then all assets valued at less than 30 cents on the dollar would remain in the auction, while all assets valued at greater than 30 cents on the dollar would exit. Consequently, the Treasury pays 30 cents on the dollar, purchasing assets that are worth only 15 cents on the dollar on average. The Treasury buys only the worst of the worst, intervening in a way that rewards the least deserving. And, as a result of overpaying drastically, the Treasury can mop up relatively few distressed securities with its limited budget.

The adverse-selection problem can be somewhat ameliorated if the Treasury is able to determine "reference prices" reflecting the true value of the various assets. The pooled auction is then conducted in terms of a markup or discount to the reference prices. For example, the auctioneer starts by naming 110% of the reference prices. Most holders of illiquid securities would presumably be happy to sell at well above the reference prices, so there would be excess supply. The auctioneer then progressively lowers the price—105%, 100%, 95%, etc.—and bidders indicate the securities that they are willing to sell at each lower percentage of the reference price. Eventually, a percentage, perhaps 90% of the reference price, is reached at which supply equals demand.

Unfortunately, it seems unlikely that the Treasury will be able to determine accurate reference prices, particularly at the beginning of this process. The secondary markets for these assets have largely shut down—recall that this is the rationale for the intervention in the first place—and the few transaction prices reported (as well as the standing bids) can be presumed to be highly misleading indicators of value. Inaccurate reference prices give rise to a similar adverse-selection problem. Without reference prices that reflect current market information and without enhancements to the pooled auction design, the Treasury would likely end up owning the assets worth the least relative to the reference prices and the taxpayer would not be protected.

3 Overview of a two-part reverse auction plan

Thus, a better approach would be for the Treasury to begin by conducting a separate auction for each security and limit itself to buying perhaps 50% of each security's face value (Ausubel and Cramton 2008). Again, the auction starts at a high price and works its way down. If the security clears at 30 cents on the dollar, this means that the median holder values it at 30 cents on the dollar. (If the median holder valued it at only 15 cents on the dollar, then most holders would offer 100% of their securities at 30 cents; there would be excess supply and the price would be pushed lower.) The auction then works as intended. The price is reasonably close to value. The "winners" are the bidders who value the asset the least and value liquidity the most.

However, a security-by-security auction will yield the desired result only if the holdings of the security are sufficiently dispersed—making the auction sufficiently competitive. To see the worst case scenario, consider a security which is entirely owned by a single party. The single owner will optimally drop its quantity offered to 50% of the security's face value, immediately ending the auction at the starting price, independent of whether the security is valuable or worthless. For this reason, while a security-by-security auction is preferable, any plan must specify an alternative market mechanism for securities where the ownership is too concentrated.

A workable approach is a two-part reverse auction plan. Broadly speaking (details are provided in Sections 4–7), the assets to be auctioned are divided into two categories: those whose ownership is sufficiently dispersed that they may be auctioned security-by-security; and those whose ownership is sufficiently concentrated that they need to be pooled.

The security-by-security auctions occur first. This has several advantages. First, the security-by-security auctions can be conducted without any external information such as reference prices. This means that the auctions can begin without delay in October 2008 and thereby have the maximum beneficial effect. Second, the results of the security-by-security auctions themselves provide one methodology by which reference prices for the remaining securities can be developed: prices from the initial auctions can be regressed on all available security characteristics, and these can then be applied to the characteristics of the remaining securities to forecast auction prices. In this way, the reference prices reflect current market information.

Third, while the security-by-security auctions purchase only some, but not all of each security, the remaining holders are not left high and dry. By determining the market clearing price, the auction increases liquidity for the remaining holders, as well as for related securities. The auction has effectively aggregated market information about each security's value. This price information is the essential ingredient needed to restore the secondary market for mortgage backed securities. Thus, the Treasury's limited resources are likely to find an assist from the private market to further enhance liquidity.

4 Part 1: Security-by-security auctions

In this section, we provide a description of the security-by-security auctions that could be held initially for those securities whose ownership is sufficiently diffuse. Attention is paid to describing a mechanism, all of whose aspects could be implemented quickly (in the next several weeks).

4.1 Preliminaries: Which securities can be auctioned; how much should be sold

As a starting point, the Treasury would need to determine which securities are suitable for security-by-security auctions, and how much to purchase of each. This decision must be made without extensive information about the distribution of private holdings.

At the outset, the Treasury would issue a notice listing the CUSIP¹ numbers of securities that it is contemplating purchasing in the initial auctions. (Optionally, the notice could also invite holders to suggest securities that are appropriate for the initial auctions.) Holders of these securities who wish to participate in the auctions would respond in the following form:

- Holders would nominate the securities that they would like to offer for government purchase and they would be required to disclose the total face value of each such security that they (or affiliated parties) hold;
- Holders would undertake not to buy or sell any of the nominated securities between the time of nomination and the completion of the initial auctions—this would be intended to be only a short period of time; and
- Holders might be required to name a firm price at which they would be willing to sell each nominated security.

One simple rule that could be used for addressing concentrated ownership is a *three pivotal seller rule*: never buy more than the total nominated quantity less the quantity nominated by the three largest bidders. This rule guarantees that there are at least four bidders competing for every share in the auction. A further advantage of the rule is that disclosing demand reveals relatively little information to the bidders about the exact distribution of ownership shares.

The rule is motivated by the "three pivotal supplier test" used as a test of competition in the largest US electricity market since 2005 (PJM 2008): an auction is viewed as competitive whenever demand can be fully satisfied by bidders other than the three largest. When demand is capped by the rule above, it automatically satisfies the three pivotal supplier test. Although any market power rule is somewhat arbitrary, this rule does guarantee a reasonable amount of competition and it has been successfully applied in repeated uniform-price auctions in an important setting for many years.

The Treasury may also wish to limit its purchases to a particular fraction, x%, of the total face value of each security. As discussed above, an illustrative value for x is 50%. The motivation is to limit the Treasury to purchasing only some and not all of a given security so that the price reflects value and so that there is economizing on government resources used for

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¹ The Committee on Uniform Securities Identification Procedures supplies a unique nine-character identification, called a CUSIP number, for each class of security approved for trading in the US.

cleaning up each security. In the case of a global limit of x%, the Treasury's demand for each security is the smaller of x% and the quantity from the three pivotal seller rule.

To be more precise, let F be the aggregate face value of a given security, let $q_1, q_2, ..., q_n$ $(q_1 \ge q_2 \ge ... \ge q_n)$ be the nominated quantities in million dollars of face value of the security listed in descending order, and let $Q = q_1 + q_2 + ... + q_n$ be the total nominated quantity of the security. Then the Treasury's demand, D, for a given security could be set at

$$D = \min \{ xF, Q - q_1 - q_2 - q_3 \}$$
.

In addition, the Treasury may wish to entirely exclude certain securities from security-by-security auctions. Those securities which are viewed as too concentrated in their ownership may be routinely excluded from the initial auctions—for example, if the largest owner holds at least 50% of the face value or if the result of the three pivotal seller rule is less than 15% of the aggregate face value. Also, problematic securities with missing or ambiguous characteristics or legal encumbrances may be excluded.

Rather than purchasing a fixed percentage of the face value of each security, it is also possible to make the amount of face value purchased depend on the price to be paid (Section 4.2). Furthermore, it is possible to invite private buyers to join the Treasury in making purchases (Section 5.5).

4.2 Mechanics of the descending clock auction

For divisible goods like securities, simultaneous clock auctions are both effective and simple. In a simultaneous clock auction, there is a price "clock" for each security indicating its tentative price per unit of quantity. Bidders express the quantities they wish to supply at the current prices. The price is decremented for each security with excess supply, and bidders again express the quantities they wish to supply at the new prices. This process repeats until supply is made equal to demand. The tentative prices and assignments then become final.

The simultaneous clock auction is implemented easily over a secure Internet connection. Bidders only require a standard web browser and access to the Internet.

In practice, clock auctions use discrete rounds for two important reasons. First, communication is rarely so reliable that bidders would be willing to be exposed to a continuous clock. A bidder would find it unsatisfactory if the price clock swept past the bidder's willingness to provide the good because of a brief communication glitch. Discrete rounds are robust to communication problems. Discrete rounds have a bidding window of significant duration, rarely shorter than ten minutes and sometimes as long as one hour. This window gives bidders time to correct any communication problems, to resort to back-up systems, or to contact the auctioneer and have the round extended. Second, a discrete round auction improves price discovery by giving the bidders an opportunity to reflect between rounds. Bidders need time to incorporate information from prior rounds into a revised bidding strategy. This updating is precisely the source of price discovery and its associated benefits.

Figure 1 shows an auction for a single security with four bidders. The auction begins at the starting price P1. Each bidder names the quantity it desires to sell, and these four quantities are added together to form the aggregate supply indicated by the horizontal line at P1. Supply exceeds demand. The price is reduced to P2, and the bidders respond with their desired quantities.

Again, there is excess supply. The process continues until round 6, where supply is equal to demand at the closing price P6.

Price (cents)

Aggregate Supply

Round 1

P2

Round 2

P3

Round 3

P4

Round 4

P5

Round 5

Round 5

Round 6

Quantity (million \$)

Figure 1. Descending clock auction

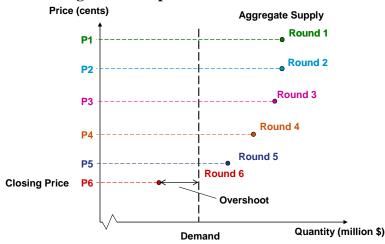
In dynamic auctions, bidders sometimes have an incentive to wait until late in the auction to bid seriously, so as to avoid revealing information to competitors. This snake-in-the-grass strategy, referred to as "bid sniping" in eBay, has the effect of limiting price discovery. To avoid bid sniping and promote price discovery, we require the quantity expressed by each bidder to satisfy an activity rule. As price decreases, the bidder's quantity can stay the same or decrease—it is not permitted to increase. Thus, a bidder's supply must be weakly upward sloping. This is a natural restriction in this setting. Bidders want to supply more, not less, at higher prices.

An important issue in discrete-round auctions is the size of the bid decrements. Larger bid decrements enable the auction to conclude in fewer rounds, but they potentially introduce inefficiency from the use of a coarse price grid. Large decrements also introduce incentives for gaming as a result of the expanded importance of ties and rationing rules. But using small decrements especially in an auction with many securities can greatly increase the number of rounds and, hence, the time required to complete the auction. Bidders generally prefer a shorter auction, since exposure to price movements and news events during the auction is reduced.

At the end of each round, the auctioneer forms the aggregate supply curve from the individual bids. Then, if there is excess supply at the current price, the auctioneer reports the aggregate supply and a new round begins. Otherwise, the auctioneer reports the clearing price and each bidder is informed of the quantity it has sold.

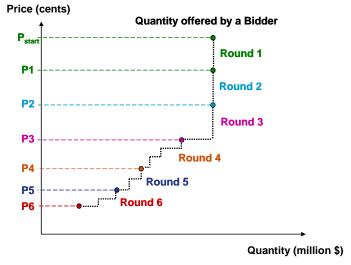
A potential difficulty with discrete rounds is the problem of overshoot, illustrated in Figure 2. When the price is reduced from P5 to P6 in round 6, the supply could go from excess supply to excess demand. Then, the auctioneer would have overshot the true clearing price.

Figure 2. The problem of overshoot



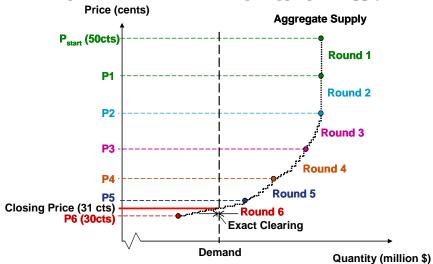
Fortunately this issue is easily addressed by letting the bidders name a price at which they wish to make each reduction in quantity. This technique, known as "intraround bidding," allows us to capture the benefits of a continuous auction and still conduct the auction in a limited number of discrete rounds. Figure 3 shows an individual supply curve using intraround bidding. In each round, the bidder specifies the exact prices at which the bidder wishes to reduce quantity.

Figure 3. Intraround bidding—individual supply



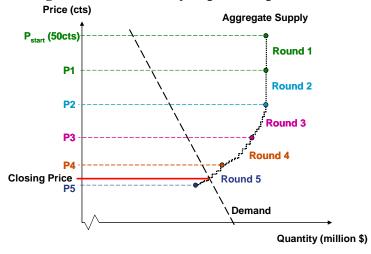
Then when the auctioneer forms the aggregate supply, as shown in Figure 4, the curve becomes smoother, and exact clearing of the market occurs at a price specified by a bidder. In the event that multiple bidders make reductions at the clearing price, then the quantity is awarded using proportionate rationing as is done in US Treasury bill auctions.

Figure 4. Intraround bidding—aggregate supply



Notice that the approach does not require a vertical demand curve. Any downward sloping demand curve is fine, as shown in Figure 5. This allows the Treasury to buy more of a security at lower prices. More importantly, demand bids from private parties can be included. Demand bids would be submitted prior to the start of the clock auction, together with financial guarantees as necessary.

Figure 5. Demand may depend on price



Handling many securities is a straightforward extension as mentioned above. Different but related securities can be grouped together in the same auction and purchased simultaneously. Each security has its own price. In each round, the bidders indicate the quantity of each security that they would like to sell at the specified prices. The price is reduced for any security with excess supply and the process repeats until a clearing price is found for each security.

Figure 6 shows the round-by-round aggregate information from an example auction with eight securities. Initially, prices are set at the lower of 98 cents on the dollar or 120% of the reference price. This is the sole use of reference prices in the security-by-security auctions. At these high prices, there is excess supply for each security. Hence in round 2, the price is decremented. The size of the decrement is larger for securities with greater excess supply. Prices

continue to decrease in subsequent rounds. In round 5, two of the securities, cleared. Demand equals supply at the price specified by the marginal bidder. Three more securities clear in round 6, and the final three securities clear in round 7.

Figure 6. An example auction with eight securities

Security-by-Security Auction

quantity in \$25,000 of face value; price in cents on the dollar

Excess supply Security clears

		Security1	Security2	Security3	Security4	Security5	Security6	Security7	Security8
Reference price		94.35	80.22	72.58	92.11	62.14	54.77	56.11	63.17
Round	Demand	1,000	1,200	2,000	1,500	800	2,500	1,000	1,200
1	Price	98.00	96.00	87.00	98.00	75.00	66.00	67.00	76.00
'	Supply	2,300	3,120	6,000	6,000	2,800	5,500	1,500	3,000
2	Price	90.00	88.00	80.00	90.00	69.00	61.00	63.00	70.00
	Supply	2,000	2,160	5,000	4,500	2,400	5,250	1,500	2,400
3	Price	83.00	82.00	74.00	83.00	63.00	56.00	60.00	64.00
١	Supply	2,000	1,920	4,400	3,300	1,680	4,000	1,400	1,920
4	Price	76.00	77.00	68.00	76.00	58.00	53.00	57.00	60.00
4	Supply	1,700	1,560	3,600	2,850	1,280	4,000	1,200	1,560
5	Price	71.00	74.29	63.00	70.00	55.00	50.00	55.63	57.00
5	Supply	1,400	1,200	2,800	2,250	1,040	3,000	1,000	1,320
6	Price	67.00		60.00	66.00	53.99	48.38		55.94
	Supply	1,200		2,600	1,650	800	2,500		1,200
7	Price	64.38		57.74	63.08				
	Supply	1,000		2,000	1,500				

When auctioning many securities, the activity rule can be enforced either on a security-by-security basis or with respect to the total quantity of securities the bidder is offering. The latter approach gives the bidder greater flexibility in arbitraging across securities as prices change. It, however, complicates the auction significantly and introduces new gaming strategies. For this reason, we recommend the much simpler approach of requiring that the bidder's supply curve for each security is weakly upward sloping. For each security, as the price descends, the bidder must offer the same quantity or a lower quantity.

The experience from high-stakes clock auctions indicates that the combination of simultaneous clock auctions and intraround bidding enables the auctioneer to conduct auctions with a large number of securities in about eight rounds, completing the auction event in a single day. This has the advantage that bidders would not need to hold open positions overnight.

5 Reasons for the design choices

We now provide more explanation for the key design choices: 1) open vs. sealed-bid, 2) uniform price vs. pay-as-bid, 3) simultaneous vs. sequential, and 4) individual bids vs. package bids. We also address the issue of limited competition.

5.1 Why open (vs. sealed bid)?

One of the initial design decisions is whether to conduct a static (sealed-bid) or dynamic (descending-bid) auction.²

A frequent motivation for the use of dynamic auctions is reducing common-value uncertainty (Milgrom and Weber 1982). In this setting there is a strong common-value element. A security's value is closely related to its "hold to maturity value," which is roughly the same for each bidder. Each bidder has an estimate of this value, but the true value is unknown. The dynamic auction, by revealing market supply as the price declines, lets the bidders condition their bids on the aggregate market information. As a result, common-value uncertainty is reduced and bidders will be comfortable bidding more aggressively without falling prey to the winner's curse—the tendency in a procurement setting of naïve sellers to sell at prices below true value.

In the context of buying many securities, the price discovery of a dynamic auction plays another important role. By seeing tentative price information, bidders are better able to make decisions about the quantity of each good to sell. This is particularly useful because the securities being procured are related. Bidding in the absence of price information makes the problem much more difficult for bidders. Furthermore, with a dynamic auction, the bidder is better able to manage both liquidity needs and portfolio risk. In contrast, managing liquidity needs in a simultaneous sealed-bid auction is almost impossible.

A final advantage of an open auction is transparency. Each bidder can see what it needs to do to win a particular quantity. If the bidder sells less, it is the result of the bidder's conscious decision to sell less at such a low price. This transparency is a main reason for the high efficiency of the descending clock auction in practice.

The experimental economics literature strongly supports the conclusion that dynamic auctions outperform sealed-bid auctions in terms of efficiency and price discovery. In sealed-bid auctions there is a tendency to consistently overbid (Kagel, Harstad, and Levin 1987; McCabe, Rassenti, and Smith 1990), often resulting in inefficient outcomes. In contrast, many laboratory and field experiments have demonstrated that the clock auction format is simple enough that even inexperienced bidders can quickly learn to behave optimally (Kagel, Harstad, and Levin 1987). Kagel (1995) finds that bidders readily transfer the experience gained in sealed-bid auctions to the clock auction format. Bidders in Levin, Kagel, and Richard (1996) appear to adopt simple strategies that incorporate dynamically changing information from the clock auction, namely the prices at which other bidders drop out, and efficient outcomes are obtained.

A principal benefit of the clock auction is the inherent price-discovery feedback mechanism that is absent in any sealed-bid auction format. Specifically, as the auction progresses, participants learn how the aggregate demand changes with price, which allows bidders to update their own strategies and avoid the winner's curse (Kagel 1995). Levin, Kagel, and Richard (1996) show that bidders suffer from a more severe winner's curse in the sealed-bid format than in a clock auction. Kagel and Levin (2001) compare a clock auction and a sealed-bid auction when bidders demand multiple units, and confirm that outcomes are much closer to optimal in the clock auction. Efficiency in the clock auction always exceeded 97%. Moreover, in the Ausubel auction (a particular type of clock auction, see Ausubel 2004, 2006) bidders achieve

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² See Ausubel and Cramton (2004, 2006), Cramton (1998), McAfee and McMillan (1987), and Milgrom (2004) for further discussion.

optimal outcomes 85.2% of the time, as compared to only 13.6% of the time in a sealed-bid auction. McCabe, Rassenti and Smith (1990) found 100% efficient outcomes in 43 of 44 auctions using a clock auction. Kagel and Levin (2008) provide further evidence of more efficient outcomes with a clock format in the multi-unit setting. Alsemgeest, Noussair, and Olson (1996) also find that clock auctions are efficient both in single and multi-unit supply scenarios, achieving better than 99.5% efficiency and 98% efficiency.

5.2 Why uniform price (vs. pay-as-bid)?

There is a general assessment that uniform pricing performs at least as well as pay-as-bid for financial securities. That was the Treasury's assessment in changing the format of Treasury bill auctions in 1998 from pay-as-bid to uniform-price.

From a bidder's perspective, pay-as-bid auctions are problematic. To bid well, bidders must attempt to guess the clearing price, since any bid below the clearing price involves selling at an unnecessarily low price, and any bid too high involves not selling at all. Such a rule favors those larger bidders who are better able to estimate or influence the clearing price.

Pay-as-bid pricing also creates an extra reason for bidders to try to collude. Bidders may attempt to reduce the risk of being paid too little or not selling by colluding with others.

Uniform-pricing is often viewed as fairer than pay-as-bid pricing, since each winner receives the same price for the securities sold.

With uniform pricing, it is important to encourage sufficient competition to avoid problems of demand reduction (Ausubel and Cramton 2002). This is addressed below.

5.3 Why simultaneous (vs. sequential)?

Different securities' values are determined, in part, by the same factors, such as systemic risk about the housing market and the economy in future years. Hence, the bidding on one security is useful information for other securities. With simultaneous bidding, bidders can condition their bids for one security on the bidding for other securities. This improves price formation.

Simultaneous bidding also lets bidders better manage their liquidity needs and portfolio risk. This may be especially important in the context of the current financial crisis. Each bank has its own liquidity needs based on the obligations it faces. Liquidity is an aggregate constraint for the bank. For example, the bank may need to generate \$5 billion in cash from the sale of its illiquid securities. In a simultaneous auction, the bank can bid more efficiently to satisfy the liquidity constraint, adjusting its bids across all securities as information is revealed.

The alternative of sequential auctions has the effect of limiting the information that is available to bidders and of limiting how bidders can respond to information. With sequential auctions, bidders must guess what prices will be in future auctions when determining bids in the current auction. Incorrect guesses may result in an inefficient assignment when security values are interdependent. A sequential auction also eliminates many strategies. A bidder cannot switch back to an earlier item if prices fall too low in a later auction. Bidders are likely to regret having sold early at low prices, or not having sold early at high prices. The guesswork about future auction outcomes makes strategies in sequential auctions complex, and the outcomes less efficient.

5.4 Why individual bids (vs. package bids)?

An important simplifying assumption of the proposed design is that a bidder's offered quantity for a security is a distinct bid, separate from its bids for other securities. Each security clears independently of the others when supply equals demand for the security. In contrast, some clock auctions are conducted as package auctions. This means that the bid in any round is the full package of securities that the bidder is bidding for at the current prices. The bidder sells the entire package or nothing.

Package bids become important when there are strong complementarities among the items.³ For example, in an auction of airport slots, a landing slot is of no value without a neighboring takeoff slot. Similarly, in some spectrum auctions, bidders may require multiple lots of adjacent spectrum in order to deploy particular wireless technologies.

Securities, on the other hand, are substitutes, not complements. With substitutable securities, we do not need the extra complexity of package bids. Bidding for individual securities works well.

5.5 Addressing limited competition

Market power is a final practical concern. For many securities, the number of bidders is small or a few bidders hold a large share of the security. In such cases, the design needs to address the potential exercise of market power. There are four main instruments:

- adjusting the quantity purchased,
- encouraging participation on both the demand and supply side,
- adopting an information policy to limit the bidders' ability to adopt collusive strategies, and
- setting reserve prices to limit the incentive for collusion and to assure reasonable prices even if competition is weak or collusion is effective.

Demand. As described earlier the Treasury's demand for each security must be adjusted based on the quantities nominated and other factors. We proposed the three pivotal seller rule as the primary method for setting demand. To get a better sense of the implications of such a rule, we look at the fraction of the total face value of each security purchased as a function of the number of bidders and the distribution from which quantity nominations are drawn. We let the number of bidders range from 4 to 16, and consider two possible distributions for nominations, the uniform distribution on the interval [0,1] and the beta distribution with parameters $\alpha=1$ and $\beta=3$, which makes it more likely that the security will have a few large bidders. (Other distributions were considered as well with similar results.) For each number of bidders and distribution, we constructed 20,000 nomination vectors, applied the three pivotal seller rule, and computed the fraction of shares purchased.

Figure 7 shows the mean percent of shares purchased as a function of the number of bidders for the two distributions. Also shown is the band ± 2 standard deviations around the mean, which includes about 95% of the observations. For securities with three or fewer bidders, the security is

³ See Cramton, Shoham, and Steinberg (2006) for recent research on package auctions.

not sold. With four bidders, about 8% is sold and that quantity increases steadily with the number of bidders. With about 10 bidders, roughly one-half of the nominated quantity is demanded.

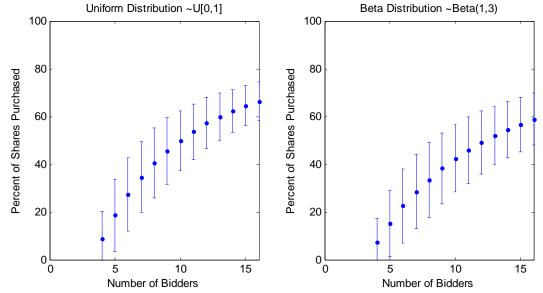


Figure 7. Percent of shares purchased by number of bidders (mean \pm 2 std. devs.)

Participation. Encouraging participation is the best method of addressing market power. Participation creates competition. For this reason, we recommend that all holders of a security can offer to sell. Small holders can participate effectively and easily with a proxy bid that is submitted at the start of the clock auction. For the small bidders the auction is effectively a sealed-bid auction. The bidder's supply curve for each security is entered once at the start of the auction. Excluding foreign participation may prove difficult as it would seem fairly easy for foreign owners to participate through a US entity.

It also makes sense to encourage participation on the demand side. As mentioned earlier, this is accomplished by allowing parties to submit demand bids before the start of the clock auction. Each demand bid is a downward-sloping demand curve, specifying the quantity demanded as a function of price. The demand bids are accepted and reported in aggregate for each security before the clock auction begins. Demand bids have three main benefits: 1) they provide additional liquidity, 2) they provide additional market information about the value of the securities, and 3) they enhance competition by rewarding sellers with additional quantity if the price is sufficiently low.

Information policy. In a competitive auction, price is used to resolve the allocation problem. Those bidders willing to sell at lower cost, get to sell more. However, when there is a lack of competition, a major concern is that bidders will agree on quantities by means other than the auction price. Certain types of information revealed in a dynamic auction can sometimes be used to adopt and enforce collusive strategies.

In clock auctions, a useful information policy for mitigating collusive strategies is to report only the aggregate supply after each round. In this setting, the aggregate supply contains most of the information needed for price discovery. If, instead, the auctioneer revealed the individual supply curves of the bidders, this detailed information could be used to facilitate a coordinated reduction of supply at high prices. For example, the bidders might cooperatively reciprocate the

quantity reductions of competitors, and punish those who do not reciprocate. In order to avoid such possibilities, in all real-world clock auctions of which we are aware, the auctioneer has reported only aggregate quantities, and not the individual bids.

Reserve price. A reserve price is another instrument to address limited competition. It does this in two ways. First, it reduces the incentive for collusion by limiting the maximum gain from collusion. Bidders must provide the good at no higher than the reserve price no matter how effective their collusion. Second, the reserve price guarantees that the price paid by the buyer is not unreasonably high, even when competition is weak. Reserve prices are easily implemented in clock auctions. Most commonly, the starting price serves as a reserve. Bidders are not permitted to express supply at prices above the reserve.

More generally, the Treasury may wish to adjust demand in response to bids (Ausubel and Cramton 2004; McAdams 2005). In a clock auction, a demand adjustment is most easily accomplished by specifying an explicit downward-sloping demand curve. This has the effect of expanding the quantity demanded when there is ample competition, but reducing the quantity demanded (and implicitly introducing a reserve-like mechanism) when there is insufficient competition.

6 Part 2: Enhanced pooled auctions

Securities found to be inappropriate for the initial security-by-security auctions would instead be purchased in enhanced pooled auctions. Recall, as observed in Section 2, that a pooled auction is subject to a significant adverse-selection problem, unless reference prices can be accurately determined. In this section, we note that the results from the security-by-security auctions can be used to help determine reference prices for the pooled auctions, we describe the pooled auction in some detail, and finally we describe some enhancements that can be utilized to improve the performance of the pooled auctions.

A basic reason why it will be difficult to construct accurate reference prices for the troubled securities is that the securities are illiquid and meaningful recent transaction prices are sparse. One of the benefits of the security-by-security auctions (Part 1) is that they will develop contemporaneous transaction prices. These can be used to help develop reference prices for the remaining securities. Data sets consisting of all relevant characteristics of all of the troubled securities would need to be assembled. The security-by-security auctions could be run for the selected securities without need for any reference prices. The resulting auction prices could be regressed on the set of characteristics. In turn, the estimated coefficients could be applied to the characteristics of the remaining securities. The resulting estimates could be used as reference prices—or as a substantial input into reference prices.

The previous paragraph is not intended to suggest that the Treasury should not also use experts with financial models to help determine reference prices. Rather, the point is that transaction data is also a needed input; illiquid markets have a dearth of meaningful transaction data, which the security-by-security auctions can help to fill.

6.1 Auction mechanics of a pooled auction

The main defining features of a pooled auction are as follows:

- Different securities (i.e. securities with different CUSIP numbers) are included within the same auction:
- The prices in each round of the descending-clock auction are the markup or discount from the reference prices for each security; and
- Clearing occurs when the cost of purchasing the securities offered at a given markup or discount equals the budget allocated for the auction.

In most other respects, the auction proceeds in similar fashion to the mechanics of the security-by-security auctions (as described in Section 4).

To examine a simple example, let us suppose that just two different securities are pooled within an auction. (The extension to any number of securities is straightforward.) Security A has a reference price of 60 cents on the dollar, while Security B has a reference price of 40 cents on the dollar. The auction might proceed as follows:

	Percentage of Reference Price	Implied Price of Security A (cents on dollar)	Implied Price of Security B (cents on dollar)
Round 1	110%	66.00	44.00
Round 2	105%	63.00	42.00
Round 3	100%	60.00	40.00
Round 4	95%	57.00	38.00
Round 5	90%	54.00	36.00

To continue the example, let us suppose that Security A has \$3 billion in face value, Security B has \$5 billion in face value, and the Treasury allocates a budget of \$900 million to spend in the auction. Then the pooled auction might proceed with the following bids (expressed in amounts of face value):

	Percentage of Reference Price	Quantity Offered of Security A (face value)	Quantity Offered of Security B (face value)
Round 1	110%	\$3 billion	\$5 billion
Round 2	105%	\$2.75 billion	\$4 billion
Round 3	100%	\$2.5 billion	\$2.5 billion
Round 4	95%	\$2 billion	\$1.5 billion
Round 5	90%	\$1 billion	\$1 billion

In this example, the pooled auction would clear at 90% of the reference prices (i.e. a 10% discount), as this is the price at which the cost of purchasing the quantity offered of Security A

 $(0.54 \times \$1 \text{ billion})$ plus that of purchasing the quantity offered of Security B $(0.36 \times \$1 \text{ billion})$ equals the allocated budget of \$900 million.

Figure 8 shows an example of the round-by-round aggregate information from a pooled auction with two pools, each with four securities. The higher-quality pool has a budget of \$120 million and the lower-quality pool has a budget of \$80 million. At the starting price of 110% of the reference price, there is excess supply for both pools in that the total dollar spend for each pool exceeds the pool's budget. Hence, both pool prices are decremented for round 2. Still there is excess supply. The process continues until in round 6 the higher-quality pool clears at the price of 98.20% of the reference price. Then in round 7 the lower-quality pool clears at 93.68%.

Figure 8. An example with two pools, each with four securities

Excess supply

Pooled Auction

quantity in \$25,000 of face value; price in % of reference price; spend in million \$								Pool clears			
quantity in \$25,000 or race value, price in % or reference price, spend in million \$								1 001 (Jicais		
Higher-Quality Pool Lower-Qu								ality Pool			
		HQ Pool	Security1	Security2	Security3	Security4	LQ Pool	Security5	Security6	Security7	Security8
Refe	rence price		90.35	84.25	81.78	89.11		78.02	54.77	68.24	72.58
Round	Budget	\$120					\$80				
1	Price	110%	99.39	92.68	89.96	98.02	110%	85.82	60.25	75.06	79.84
'	Spend	\$176	1,703	2,343	1,978	1,433	\$117	1,231	2,741	1,482	1,076
2	Price	107%	96.67	90.15	87.50	95.35	106%	82.70	58.06	72.33	76.93
	Spend	\$155	1,647	2,145	1,837	1,133	\$107	1,189	2,572	1,407	1,004
3	Price	104%	93.96	87.62	85.05	92.67	102%	79.58	55.87	69.60	74.03
3	Spend	\$146	1,603	2,121	1,801	1,023	\$98	1,100	2,422	1,367	989
4	Price	102%	92.16	85.94	83.42	90.89	100%	78.02	54.77	68.24	72.58
4	Spend	\$136	1,521	1,945	1,777	984	\$94	1,069	2,401	1,340	975
5	Price	100%	90.35	84.25	81.78	89.11	97%	75.68	53.13	66.19	70.40
5	Spend	\$131	1,489	1,922	1,733	975	\$90	1,025	2,366	1,320	962
6	Price	98.20%	88.72	82.73	80.31	87.51	94%	73.34	51.48	64.15	68.23
	Spend	\$120	1,475	1,744	1,521	945	\$84	995	2,311	1,256	940
7	Price						93.68%	73.09	51.31	63.93	67.99
	Spend						\$80	955	2,256	1,145	901

6.2 Potential enhancements to the pooled auction

The following enhancements might be considered for the pooled auction:

- Sellers could be required to bundle securities in fixed proportions before learning the reference prices;
- The Treasury's cumulative purchases of each security could be capped at a fixed percentage of face value, such as 50%;
- Self-selecting tariffs could be introduced; and
- Ex-post performance measures could be introduced into the sales contract, requiring the seller to repay the difference if the Treasury takes a loss on the securities.

The purpose of requiring sellers to bundle their securities in fixed proportions before learning the reference prices is to reduce the extent of adverse selection. (Sellers would be unable to offer selectively the securities whose reference prices are inaccurately high, but not the securities whose reference prices are inaccurately low.) However, to the extent that sellers could anticipate whether the reference prices for particular securities would be inaccurately high or low,

the requirement of bundling in fixed proportions would not help. Also the constraint reduces the potential gains from trade from the pooled auction.

The purpose of capping the Treasury's cumulative purchases of each security is two-fold. First, it would limit the expenditure of government money on securities whose reference prices are inaccurately high, thereby protecting taxpayers against the adverse-selection problem. Second, as in the security-by-security auctions, the Treasury would intentionally leave some of each security unsold, with the hope that the private market would assist in the job. Note that, in the event that the offerings of a given security at the overall clearing price exceeded the cap, the price clock would be allowed to continue to run downward on the given oversubscribed security (while the price clock would have stopped for any security that was within the cap).

By a self-selecting tariff, we mean for example that a seller could be offered a choice of selling one-half of its holdings of a security at 40 cents on dollar or all of its holdings of the security at 30 cents on dollar. Such pricing schemes may be useful tools for extracting asymmetric information from sellers. They also reduce the adverse selection problem.

A seller's obligations to repay according to ex post performance measures would be backed by stock warrants or senior debt instruments; this is discussed in the next section.

7 The role for stock warrants or senior debt instruments

Section 113(d) of the current draft of the Emergency Economic Stabilization Act of 2008 requires that the Treasury receive warrants for nonvoting common or preferred stock or senior debt instruments of financial institutions from which it purchases troubled assets. In this section, we outline some thoughts on the most useful role for such stock warrants or senior debt instruments.

As noted above, it can be helpful to include an ex-post performance measure into the sales contract, requiring the seller to repay the difference if the Treasury takes a loss on a given troubled asset. Stock warrants and senior debt instruments provide a vehicle that can back any repayments required by such ex-post performance measures. The motivation for such ex-post performance measures is to protect the Treasury in situations where it faces an adverse-selection problem. Thus, the extent of the protections should depend on whether the assets are purchased in security-by-security auctions (Part 1) or pooled auctions (Part 2). In the former event, the quantity of stock warrants or senior debt instruments could safely be kept comparatively small; while in the latter event, the quantity of warrants or debt would need to be comparatively large. Obviously, in situations where direct purchases are made in lieu of a market mechanism, the reliance on stock warrants or senior debt instruments would need to be the greatest.

The standard sales contract for troubled assets would include a calculation to be made of the Treasury's net profit or loss from holding each given asset. This calculation would incorporate cash flows arising from the Treasury's cash outlay in purchasing the security, all revenues or costs associated with holding the asset, and all revenues or costs associated with the eventual disposal of the asset. An interest rate appropriate to the risk category of the asset would be applied.

For all monies spent on asset purchases, the Treasury would receive warrants or senior debt instruments valued at percentages (to be specified) of the money spent. To give a sense of our intention (but somewhat arbitrary numbers), the value of the warrants or debt might range from

10% to 90% of the monies spent for a particular asset. The base amount (10%) would be suitable for security-by-security auctions where there is confidence that the market mechanism would yield a competitive price. The high amount (90%) would be suitable for direct purchases outside of a market mechanism and for pooled auctions of the lowest-quality assets.

The base amount would simply be held by the Treasury to provide "reasonable participation by the Secretary, for the benefit of taxpayers, in equity appreciation." Such warrants would not be sold until such time that, in Treasury's judgment, there is no longer a capital shortage among the relevant financial institutions. Amounts above the base amount would be used to back the ex-post performance measures. For example, in a situation where the Treasury required a 70% figure, the amount above the base amount (i.e. 60%) would be used toward covering potential losses on the Treasury's purchases of the relevant security. However, to the extent that the Treasury subsequently recovers its outlay and makes a net profit (or breaks even) on the security, the warrants above the base amount would be eliminated.

The amount of warrants or senior debt instruments, above the base amount, would mark the upper bound on the Treasury's recourse against the seller. The purpose of limiting the exposure is to assist in giving financial institutions relatively unencumbered balance sheets, and so to encourage private injections of capital. Investors could be confident that the security was now fully off the institution's balance sheet, with the only lurking liability being the stock warrants or senior debt instruments held by the Treasury. Meanwhile, the base amount (i.e. the 10%) of warrants or debt would be the taxpayers' to keep, regardless of whether the ex-post performance measures are met, so that taxpayers share in the upside potential in all scenarios, as required by the draft legislation.

8 Feasibility

While the previous sections may have convinced the reader that an effective auction design is provided, it remains to overcome the hurdle of showing that it can be implemented in a short time. Thus, the purpose of this section is to demonstrate that implementing the auctions so as to begin in October 2008 is fully feasible, without sacrificing any of the essential features of the design.

The prototypical example of a dynamic clock auction is the Electricité de France (EDF) Virtual Power Plant Auctions, which have operated quarterly since 2001 and were the first practical implementation of simultaneous clock auctions. In a typical EDF auction, some 20 products are auctioned simultaneously. There are generally about 40 bidders and 15 − 20 winners. The electricity auctioned in a given quarterly auction represents approximately €300 million in value, and the auction is run in approximately 8 − 10 rounds that are completed in a single day. The EDF auctions have been followed by a variety of other dynamic clock auctions for electricity, natural gas, telecommunications spectrum, environmental allowances, rough diamonds, and other products. Table 1 provides examples of large auctions that have been successfully implemented as descending or ascending clock auctions; most auctions listed below have exceeded \$100 million in value transacted in each auction event.

Table 1. Examples of large implementations of clock auctions

Name of Auction	Sector	Dates	Number of Auctions to Date	Annual Volume Transacted
Electricité de France Virtual Power Plant (VPP) Auctions	Electricity	2001-present	29	5.7 GW
Public Utility Commission of Texas Capacity Auctions	Electricity	2001-2005	18	8 GW
New Jersey Basic Generation Service Auctions	Electricity	2002-present	7	18 GW
Electrabel VPP Auctions (Belgium)	Electricity	2003-2005	7	1.2 GW
Endesa-Iberdrola VPP Auctions (Spain)	Electricity	2007-present	6	1.8 GW
E.ON VPP Auction (Germany)	Electricity	2007	1	0.25 GW
ISO-New England Forward Capacity Auction (US)	Electricity	2008-present	1	\$1.75 billion of generating capacity procured
Electricité de France Long-Term Contract Auctions	Electricity	2008-present	1	1.5 GW
E.ON Ruhrgas (Germany)	Natural Gas	2003-2008	6	25.1 TWh energy
E.ON Földgáz Trading (Hungary)	Natural Gas	2006-2010	3	7.5 TWh energy
Danish Oil and Natural Gas	Natural Gas	2006-2011	3	2 TWh energy
Gaz de France	Natural Gas	2004	1	6 TWh energy
Total Gas and Power (France/UK)	Natural Gas	2004	1	0.5 TWh energy
Gaz de France Storage	Natural Gas	2006	1	400 GWh storage capacity
Office of Communications (Ofcom) (UK)	Telecom spectrum	2008	2	10-40 GHz and L-band spectrum
Telecommunications Authority of Trinidad and Tobago	Telecom Spectrum	2005	1	80 MHz of GSM spectrum
Emissions Trading Scheme (Greenhouse Gas Emissions) (UK)	Environmental	2002	1	£215 million of emission reductions
BHP Billiton (Belgium)	Rough Diamonds	2008-present	2	\$100 million of diamonds

Commercial-grade auction software is available for running dynamic clock auctions. As an example, the following is a description of the software that is used for operating the EDF Virtual Power Plant Auctions. The auction software operates as a web application on a Linux server located in the United Kingdom, with backup servers located in the United States. Bidders submit bids in the auction and follow the auction's progress using standard web browsers (Internet Explorer or Mozilla Firefox). The software enforces all of the rules of the auction. For example, if a bidder attempts to submit a bid that violates the auction's activity rule, the software identifies this in real time, preventing the bidder from submitting the bid and giving the bidder guidance as to how to correct the bid. After bidding, the software allows the bidder to generate a bid confirmation. The software also announces the start-of-round and end-of-round prices before each round, and calculates the aggregate supply after each round. It provides the relevant

⁴ The authors designed the EDF Virtual Power Plant Auctions in 2001 and have a continuing role in their implementation.

information to bidders and to the auction manager as screen displays and as downloads. It also provides a secure messaging capability between the auction manager and bidders. The auction software includes the same security features used by financial institutions on the Internet, including SSL encryption, a digital certificate for authentication, hardware and software firewalls, and login identities and passwords distributed to bidders by non-electronic means. In the event that an individual bidder suffers failure of its own computer system or Internet connection, the bidder is able to contact the auction manager and submit its bid by fax, with signature verification. The software then permits the auction manager to enter the bid on behalf of the bidder. All transactions on the auction system are recorded in an audit log. The audit log notes the time and amount of all bid submissions, and also records all other relevant events such as the generation of a bid confirmation by a bidder or the submission of a bid by the auction manager on behalf of a bidder.

During the last week, the software team that wrote the main software platform took as a challenge to modify the EDF auction software to implement the Troubled Asset Reverse Auction described in this paper. A software demo is now available to illustrate the technical feasibility of running the initial auctions in October.

9 Conclusion

The US is embarking on the greatest public intervention into financial markets since the Great Depression. The ultimate success or failure of the intervention will depend, in part, on the fine details of the auction design.

The basic auction approach suggested here is neither new nor untested. It has been used successfully in many countries in recent years to auction tens of billions of dollars in electricity and natural gas contracts, as detailed in Section 8. Moreover, it is quite similar to the approach that has been used to auction more than \$100 billion in mobile telephone spectrum worldwide. It is a dynamic version of the approach that financial markets use for share repurchases. If implemented correctly, each auction can be completed in less than one day. And the same software used for implementing electricity and gas auctions could be used to initiate these auctions in October.

Thus, the auction approach meets the three main requirements of the rescue plan: 1) provide a quick and effective means for the Treasury to purchase troubled assets and increase liquidity; 2) protect the taxpayer by yielding prices that are closely related to value; and 3) offer a transparent rules-based process that minimizes discretion and favoritism.

Indeed, the second and third requirements may be decisive for obtaining broad political acceptance of the intervention. The main alternative to auctions is direct purchases by professional asset managers. To the extent that direct purchases or other individualized trading arrangements are used, the public will be rightfully wary that favoritism may be exerted and that some security holders will be offered sweetheart deals. By contrast, a transparent auction process is readily subjected to oversight.

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