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MŰHELYTANULMÁNYOK

DISCUSSION PAPERS

MT–**DP.** 2004/18

OPTIMAL INCENTIVE MIX OF PERFORMANCE PAY AND EFFICIENCY WAGE

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Institute of Economics
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Optimal incentive mix of performance pay and efficiency wage

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OPTIMAL INCENTIVE MIX OF PERFORMANCE PAY AND EFFICIENCY WAGE

BY ANDERS FREDERIKSEN AND ELŐD TAKÁTS

Abstract

Firms use a rich set of incentives including fixed wages, bonuses, threat of firing and promise of promotion. Yet, we do not have a theoretical understanding of how such a mix of incentives can arise. This paper aims to build a theoretical model which describes the incentive mix as the solution to an optimal contracting problem and provides broader testable implications. The basic model has a principal-agent relationship with unobservable effort. The integrative model includes the basic model and three building blocks: job-assignment, learning and human capital. The derived incentive mix is a consequence of the dual role of firing. It is both an incentive and a sorting decive. The model's predictions are tested on firm-level data from a large pharmaceutical company. The broader testable implications beyond the incentive mix are also confirmed by the data.

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ANDERS FREDERIKSEN – TAKÁTS ELŐD

OPTIMÁLIS ÖSZTÖNZÉSI CSOMAG

Összefoglalás

A gazdaságban, a cégeknél széles skálájú ösztönzőket látunk: fix fizetés, bonusz, az állás elvesztésének a félelme és az előléptetés lehetősége egyaránt szerepet játszik a munkavállalók ösztönzésében. Ugyanakkor még nem teljesen értjük a közgazdasági mozgatórugóit a fent vázolt többelemű ösztönzési csomagnak. A cikkben bemutatott modellben ez a csomag az optimális ösztönző, az optimális szerződés. A modell továbbá egyéb, szélesebb körű következtetéseket is nyújt, így érvényessége empirikusan is tesztelhető. Az alap-modell egy megbízó-ügynök kapcsolatot elemez nem megfigyelhető erőfeszítéssel az ügynök (munkavállaló) részéről. Az integrált modell az alapmodellt három további munkagazdaságban használatos elméleti építőelemmel egészíti ki: munkaerő-kiválasztás, tanulás és humán tőke. Az ösztönzési mix alapvetően az elbocsátások kettős szerepén múlik: ez egyfelől ösztönző, másfelől lehetőséget biztosít a cégnek a megfelelő alkalmazottak kiválogatására. A modell következtetéseit egy nagy gyógyszercég humánpolitikai adatbázisán teszteljük. Az empirikus elemzés nemcsak a modell főbb előrejelzéseit, hanem az egyéb, szélesebb körű következtetéseit is alátámasztja.

CONTENS

1 Motivation

Firms offer highly complex contracts, which involve a rich set of incentives, to their employees. This paper asks the question how such an incentive mix can arise. The goal of the paper is twofold. First, it aims at building a model which is able to reproduce the optimal incentive mix observed. Second, following Gibbons and Waldman (1999a) it aims at deriving and testing the broader implications of the model. The empirical testing of the model's conjectures is done using the personnel records from a large pharmaceutical company.

Detailed information on firm's compensations systems are not easily obtained. Nevertheless the evidence that firms use a rich set of incentives is accumulating, see Medoff and Abraham (1980, 1981), Baker, Gibbs and Holmström (1994a, 1994b) and Lazear (1992, 2000). In the dataset studied in this paper we observe a contract similar to the one described in Lazear (2000) where the firm is offering a combination of efficiency wages and performance pay to the workers. In particular, we see four incentive parameters: fixed wages, bonuses, firing and promotion.

Yet, economic theory cannot explain the mix of incentives. The performance pay and efficiency literature contribute separately with a partial understanding of how firms use incentives. First, the performance pay literature, which originates from Holmström (1979, 1982) and Mirrlees (1974, 1976), explains bonuses. According to this theory, wages should include variable elements (like bonuses or piece rate) to reward employee effort. Second, the efficiency wage literature originating from Shapiro and Stiglitz (1984) explains firing (and promotion). If employment involves rent (due to higher than market clearing wages), then the fear of loosing the job might provide incentives. Similarly, if there are rents in higher ranks, promise of promotion can yield incentives.[1]

MacLeod and Malcolmson (1998) build a model in which performance pay and efficiency wages are possible choices. The model rests on implicit contracting: firms cannot be trusted to pay bonuses. The outcome of the model is that efficiency wage, though in general more expensive than performance pay, might arise because it gives compensation up-front and solves the firm's commitment problem. They find, however, that firms choose either efficiency wage or performance pay. Thus, their results do not allow for the incentive mix.

Our basic model, which is a traditional one-shot principal-agent model with hidden effort, replicates the findings in MacLeod and Malcolmson (1998). To address the aspects of worker heterogeneity the basic model is restated in an infinitely repeated setting and three additional building blocks are added: job assignment, learning and human capital acquisition. The most important component of the integrated model in the present context is job assignment. Job assignment implies that the purpose of firing becomes twofold. First, it creates incentives for the employees as in the basic model. Second, it works as a sorting device where the firm can adjust the composition of the workforce to a profit maximizing level. The non-linearities introduced into the profit function by the sorting mechanism establishes that the incentive mix observed in the data can be an optimal contract.

In the next section the basic model is presented. In section 3 the basic model is extended to an

[1] Tournament theory, the third major branch of the incentive is disregarded here. Originating from Lazear and Rosen (1981) tournaments put emphasis on competition between workers. If the number of workers rewarded, fired or promoted is pre-set, then workers have an incentive to exert effort – and to compete with each other. The reason to disregard the theory is twofold. First, tournament incentives are the same as under performance pay (like bonuses) or under efficiency wage (like firing and promotion). Thus, the empirical implication are not sharply different from the first two theories. Second, tournament games give incentives to sabotage and undermine team effort as shown in Lazear (1989). In industries where team work is important (and sabotage is potentially costly) tournament games can be extremely counterproductive. Our data provides no such information.

integrative model by incorporating three additional building blocks. Furthermore the conjectures of the integrated theoretical model are stated here. The data are presented section 4 and the close relation between the empirical contract and the integrative model's assumptions is emphasized. In section 5 the conjectures of the model are tested. Finally, section 6 summarizes and concludes.

2 Basic model

The basic model entails the contracting relationship between a risk neutral firm and a risk neutral employee. The firm and the agent form a principal-agent relationship where the firm maximizes profit, while the employee maximizes utility.

The employee produces probabilistic output which is normalized to 0 (low) and C (high). The probability that the output is C, is θ and naturally $\theta \in [0,1]$. The employee, once accepted the job, can influence the probability of high output by exerting effort. The utility cost of effort exertion is e, and effort increases the probability of success by δ. (We assume that $\theta + \delta < 1$)

The employees alternative job option is denoted by $\bar{U}$. The additional utility of promotion is U_P, and the utility cost from firing is U_F. (Note that both parameters are positive.) The utility loss from firing assumption reflects that if the employee is laid off then he must search for a new job, which is a costly procedure as there are frictions on the labor market. On the other hand, the utility gains from promotion assumption implies that promotion into higher ranks entails some rent. A possible reason for this rent is that firms have strong incentives to retain managers as disruption at the managerial level is very costly. Thus voluntary turnover can be reduced by paying some rents to managers.[2]

The firm sets the contract through the following possible variables. She offers a fixed wage w, which is paid unconditionally to the employee who accepts the job. Three variables can be conditioned on the output: bonus (b), which is assumed to be paid to well-performing agent. Conditional promotion probability (π_P), and conditional firing probability (π_F). The probabilities are conditioned on observable output realization. It is assumed (without loss of generality) that the firm only considers promoting employees with high observable output and only consider firing employees with low observed output. The variables are (realistically) constrained as follows: $w, b \geq 0$; $\pi_F, \pi_P \in [0,1]$.

Turnover is costly as the firm has to replace, and train workers who are fired. These costs are summarized in the firing cost parameter K.

The firm, however, cannot promote unlimited number of employees (if we take the hierarchical structure as given). The number of employees promoted is constrained by the number of positions opening at the higher hierarchical level. (Of course, this reasoning implicitly acknowledges that the firm's hierarchy has more than a single employee.) The volume of promotions takes into account the pyramid shape of hierarchies. As - quite naturally - higher level positions are less numerous than lower level ones, the number of promotions $(\theta + \delta)\pi_P$ has to be multiplies by a hierarchy size factor ($H > 1$) to represent this discrepancy. Openings have two sources: First, the natural turnover in the higher ranks and growth of the firm opens up new positions. This is captured by parameter g. Second, the firm's firing decision in the higher ranks of the hierarchy also opens up new positions. The model assumes that the firing rate is a firm-wide policy. Thus the volume of firm initiated separations at the higher hierarchical level are given by $(1 - \theta - \delta)\pi_F$. Summarizing the above argumentation, equation

[2] This observation is also underlined by the casual observation saying, that workers prefer promotion over non-promotion. Thus, wages in managerial level seem to be over the compensating level for increased workload or responsibility.

(1) shows the promotion constraint.

$$\overbrace{H(\theta+\delta)\pi_P}^{\text{promotion volume}} \leq \underbrace{g}_{\text{natural turnover}} + \overbrace{(1-\theta-\delta)\pi_F}^{\text{firing one step-above}} \tag{1}$$

Finally, we assume that the firm has all the bargaining power. In order to rule out indeterminacies we also assume that when indifferent the firm prefers to fire as little as possible. This can be understood as a weak unwillingness on the part of managers to engage in conflicts.

2.1 Contracting problem

The contracting problem can be summarized in the following equations:

$$\max_{w,b,\pi_F,\pi_P} \Pi(w,b,\pi_F,\pi_P) = \max_{w,b,\pi_F,\pi_P} (\theta+\delta)C - w - (1-\theta-\delta)\pi_F K - (\theta+\delta)b \tag{2}$$

subject to

$$\begin{array}{lrcl}
(IC) & w - e + (\theta+\delta)[b+\pi_P U_P] - (1-\theta-\delta)\pi_F U_F & \geq & w + \theta[b+\pi_P U_P] - (1-\theta)\pi_F U_F \\
(IR) & w - e + (\theta+\delta)[b+\pi_P U_P] - (1-\theta-\delta)\pi_F U_F & \geq & \bar{U} \\
(F) & H(\theta+\delta)\pi_P & \leq & (1-\theta-\delta)\pi_F + g \\
\text{nonnegativity} & w, b & \geq & 0 \\
\text{probability} & 0 & \leq & \pi_F, \pi_P \leq 1
\end{array}$$

Intuitively, the program above states that the firm maximizes profit. The IC constraints implies that the employee accepting the contract exerts effort and the IR constraint states that the employee is better off accepting the contract. The other constraints follow from the assumptions directly.

Note that the IC constraint can be simplified:

$$(IC) \qquad -e + \delta[b+\pi_P U_P] + \delta\pi_F U_F \geq 0$$

2.2 Solving the basic model

The model is first solved by assuming that the incentive constraints are binding. This is not an unreasonable assumption, as non-binding constraints would imply wages below zero or other anomalies. However, it is shown that the basic conclusions are not changed even without this assumption.

No slack condition: There is no slack in the IR, IC and F constraints.

The no slack condition implies that:

$$b = \frac{e}{\delta} - \pi_F U_F - \pi_P U_P \tag{3}$$

$$w = e - (\theta+\delta)[b+\pi_P U_P] + (1-\theta-\delta)\pi_F U_F + \bar{U} \tag{4}$$

$$\pi_P = \frac{(1-\theta-\delta)\pi_F + g}{H(\theta+\delta)} \tag{5}$$

As we had only four parameters to set the contract, choosing a single parameter (let's say the firing parameter) determines the contract through these three equations. Consequently, the original four variable incentive problem is reduced to a single variable optimization. Expressing the profit as a function of π_F, the first order condition reveals that the objective function is maximized at the

extreme values.[3] The two extreme solutions are referred to as "efficiency wage" and "performance pay". Efficiency wage is characterized by a threat of firing and promise of promotion as a way to motivate workers. The performance pay equilibrium has positive bonus payments and no firing. Lemma (1) summarizes the results:

Lemma 1 *In the risk neutral case under the no slack condition the solution of the incentive problem is efficiency wages if:*

$$2(1-\theta-\delta)\frac{U_P}{H} > U_F + (1-\theta-\delta)K$$

and if the inequality is not satisfies, then the performance pay solution prevails.

Lemma (1) tells, that the more profitable solution prevails. It also allows for some comparative statics. On the left hand side, the higher the utility of promotion (U_P), the more likely that the firm will prefer the efficiency wage solution. The reason is that promotion rents represent a "free lunch" in incentives. Workers prefer promotion, which is not costly to the firm if positions are open. The higher the hierarchical size differences (H) are, the weaker the promotion effect is, thus the more likely that the performance pay solution prevails.

On the right hand side, increasing the cost of turnover (K) makes the performance pay solution more profitable as the required firing decision for the efficiency wage solution becomes more expensive. Increasing disutility from firing (U_F) has a similar effect. The intuition is that in equilibrium workers are compensated for this disutitiliy, thus for the firm it acts similarly as the cost of firing parameter. Finally, changes in the outside option ($\bar{U}$) and the cost of effort (e) do not change the relative profitability of the two competing solutions. The other parameter's (θ,δ) effect are ambiguous.

Remark 1 *If the optimal solution involves incentive slack, then either the efficiency wage solution or zero fixed wage prevails. Thus even with slack incentive conditions no incentive mix of fixed wages, bonuses, firing and promotion can arise.*

Consequently, the no slack condition is not pivotal in the result. The intuition for the result follows simply from profit maximization. The no slack condition on promotion probability (5) cannot be violated in a profit maximizing context. Firms promote as often as they have open slots as it motivates workers for free. If the condition on bonuses (3) is satisfied with slack, then by definition the bonus must be zero. (Else it could be reduced.) So, the efficiency wage solution prevails. Finally, if the condition on wage (4) is satisfied with a slack, then the wage must be zero. (Else again it could be profitably reduced.)

2.3 Comments

The basic model reproduces the two competing theoretical solutions: The performance pay and the efficiency wage hypothesis. In the basic model the choice between the two models depends on the parameter values (Lemma 1). In this sense the model is close in spirit to MacLeod and Malcolmson (1998) where the market conditions determine the optimal form of the compensation scheme. The model also reproduces their result as either the efficiency wage or performance pay is optimal.

The rent feature of the efficiency wage contract is disguised in the model. The individual rationality constraint (IR) is binding - consequently there is no ex-ante rent. However, the fact that firing causes

[3] The proof is provided in the appendix.

disutilities (U_F) shows that interim there is rent. The interpretation is, that even though the worker was initially indifferent between the firm's offer and other offers, going back to the labor market and continue searching is costly. Thus, interim there is rent from retaining the job. The firm can exploit this feature to motivate the worker.

3 The integrative model

The integrative model extends the basic model in a way that will make it conform both with the observed incentive mix and with other broader findings in the data. Hence, the aims of the model are the same as in Gibbons and Waldmann (1999). Two major technical modifications are done. First, the problem is recast in an infinitely repeated setting. Thus allows for investigation of dynamic aspects. Second, the integrative model incorporated three different building block theories: job assignment, learning and, human capital. The introduction of job assignment is to create sorting considerations. This will, as it is shown below, lead to the optimal incentive mix. Learning and human capital are used to establish a closer link between the data and the theoretical model.

There are two main differences between the infinitely repeated game and the one-period game.[4] First, a technical assumption about the number of employees. Now we assume a continuum of employees (with unit volume) instead of a single one. This allows for an interpretation of the firing and promotion probabilities as measures of workers dismissed or promoted with probability one. Second, we have to be explicit about the timing of the events for the model to conform with the repeated structure:

1. The period starts.
2. The firm offers a contract (w,b,π_P,π_F).
3. The employees inside the firm accept or decline the contract. (IR)
4. Potential employees outside of the firm decide on the contract (IR)
5. The employees decide about the effort level. (IC)
6. Output is realized.
7. Bonuses are paid, employees are promoted or fired.
8. The period starts all over again.

The timing setup implies competitive wage setting through two channels. First, the firm and the worker renegotiate the contract every period. The renegotiation creates a formal channel to introduce competitive market mechanisms in wage setting of the employees. It is similar in intent to the Gibbons and Waldman (1999) assumption of competitive, symmetrically informed firms. Also the assumption is in line with our one-period basic model setup.

The second channel is, that the timing requires the firm to set the same contract for new and incumbent employees. As was discussed, market frictions lock the workers into the firm. Thus the firm can abuse its power interim, for instance, by reducing the wage of the employee. Of course, abusing the contract interim is no free-lunch for the firm. In equilibrium employees foresee capture and abuse and set their demands accordingly. The firm thus needs the commitment device not to abuse this capture

[4] There is a third change in interpretation. To work with steady state values g is reinterpreted to mean only natural turnover (and growth is normalized to zero).

and not to lower the wages of old employees. The no-discrimination requirement embedded into the contract yields such a formal commitment device. Concluding, the timing structure of the infinitely repeated game sets the formal conditions for competitive wage setting.

The formal modeling restrict attention to stationary contracts at this stage. Firms handle all workers the same, when deciding about bonuses and promotions: only the current period output matters. Stationarity implies that the firm can maximize profit by maximizing the period profit. (Thus, it is not necessary to formally define a discount rate for the firm.)

The infinitely repeated structure is introduced through the homogenous employee setting. The competitive wage setting means that workers care only with the current period's incentives. (This also renders specifying their discount rate unnecessary.) The firm also concentrates on the period profit level. Thus, in each period the contracting problem displayed in the profit function (2) is solved. Consequently, the results of Lemma (1) and Remark (1) apply to the contract set. Lemma (2) summarizes the result.

Lemma 2 *In the infinitely repeated game with homogenous employees, the firm offers the same contract (in terms of w,b,π_P,π_F), as in the one-period game outlined in Lemma (1), in every period. Thus, if the fixed wage is non-zero then either the performance pay or the efficiency wage solution prevails. Again, there is no optimal mix of incentives.*

In the following the model is expanded to allow for a richer and more realistic setting. For the sake of tractability, the three building blocks are introduced sequentially. First, employee sorting is analyzed by introducing heterogenous employees. Second, learning relaxes the very restrictive assumption on history invariance. Third, human capital acquisition is introduced to capture employee heterogeneity across observable characteristics.

3.1 Job assignment

In this section we extend the infinitely repeated model to accommodate employee heterogeneity. Employee heterogeneity will introduce an additional reason to fire non-performing employees. As non-performing employees are more likely to be of bad quality (as it will be shown), the firm can improve on the equilibrium workforce composition by increasing the firing rate. As this increase is non-linear, the optimal incentive mix, in which the firm simultaneously use efficiency wages and performance pay, can arise.

The extension to heterogenous employees is trivial if a separating equilibrium can be obtained - it only involves adding an constraint to the optimization problem that will discourage bad workers to apply for the job. The interesting case, which will be studied below, arises in the more realistic situation where the firm can not force the potential employees to self select ex-ante. In this case selection issues become important for the firm since the employee composition affects the firm's overall profitability.

Potential employees are heterogenous with good (G) and bad (B) ability, respectively. Good workers are more likely to produce a high outcome than bad workers i.e. $\theta_G > \theta_B$. The proportion of good types at a given hierarchical level in the beginning of the period is given by μ_F. The proportion of good types at a one step higher hierarchical level is given by μ_F^H. Given that we assumed strict history independence, the outside firms can not discriminate between the two kinds of employees. However, the incentive scheme might imply different utilities for the different types. Thus good employees outside option is $\bar{U}_G$, and bad employees is $\bar{U}_B$.

3.1.1 The period problem

In each period the same contracting problem is repeated. If separating equilibrium can arise, then the problem is essentially the same as in the homogenous case. However, if separation is impossible, then the firm is unable to separate the workers by the contract offered.

Remark 2 *Separation is impossible ex-ante and the pooling equilibrium prevails, if the bad worker's outside option is sufficiently low (lower than $\bar{U}_B^*$ specified in the appendix is sufficient).*

The intuition is that the punishment that the firm can impose on non-performing workers is limited. It can deny bonuses and also it can fire employees. If, however, bad type workers have worse outside options than good types, then these disincentives might not be strong enough to elicit self selection of good types. Moreover, the firm can not offer arbitrarily unfavorable contracts (with very little bonuses or promotion possibilities) because, then the good types would not apply.

Remark (2) implies that the individual rationality constraint of the bad employees is always satisfied in the pooling equilibrium. Further more, note that although the profit depends on μ_F it is taken as given since the firm cannot control the composition of the workers within the period. Consequently, the contracting problem of the pooling equilibrium can be summarized as follows (**??**):

$$\begin{aligned}\max_{w,b,\pi_F,\pi_P} \Pi(w,b,\pi_F,\pi_P,\mu_F) = \max_{w,b,\pi_F,\pi_P} \mu_F(\theta_G+\delta)(C-b) + (1-\mu_F)(\theta_B+\delta)(C-b) \\ -w - \mu_F(1-\theta_G-\delta)\pi_F K - (1-\mu_F)(1-\theta_B-\delta)\pi_F K \end{aligned} \tag{6}$$

subject to

$$\begin{array}{lrcl}
(IC_G) & w-e+(\theta_G+\delta)[b+\pi_P U_P]-(1-\theta_G-\delta)\pi_F U_F & \geq & w+\theta_G[b+\pi_P U_P]-(1-\theta_G)\pi_F U_F \\
(IC_B) & w-e+(\theta_B+\delta)[b+\pi_P U_P]-(1-\theta_B-\delta)\pi_F U_F & \geq & w+\theta_B[b+\pi_P U_P]-(1-\theta_B)\pi_F U_F \\
(IR_G) & w-e+(\theta_G+\delta)[b+\pi_P U_P]-(1-\theta_G-\delta)\pi_F U_F & \geq & \bar{U}_G \\
(F) & H(\mu_F\theta_G+(1-\mu_F)\theta_B+\delta)\pi_P & \leq & (1-\delta-\mu_F^H\theta_G-(1-\mu_F^H)\theta_B)\pi_F+g \\
\text{nonnegativity} & b & \geq & 0 \\
\text{pooling condition} & \bar{U}_B & \leq & \bar{U}_B^* \\
\text{probability} & 0 & \leq & \pi_F,\pi_P \leq 1
\end{array}$$

The two IC constraint can be simplified to a single equation:

$$(IC) \qquad \delta[b+\pi_P U_P]+\delta\pi_F U_F \geq e$$

The no slack condition means again that the IC, IR and F constraints are binding. Consequently, the following equalities will hold.

$$\begin{aligned}
b &= \frac{e}{\delta}-\pi_F U_F-\pi_P U_P \\
w &= e-(\theta_G+\delta)[b+\pi_P U_P]+(1-\theta_G-\delta)\pi_F U_F+\bar{U}_G \\
\pi_P &= \frac{(1-\delta-\mu_F^H\theta_G-(1-\mu_F^H)\theta_B)\pi_F+g}{H(\mu_F\theta_G+(1-\mu_F)\theta_B+\delta)}
\end{aligned}$$

3.1.2 Dynamic game

The one-period game alone does not solve the infinitely repeated problem (quite differently from the homogenous case). The problem is that in the one-period game the equilibrium employee composition is undetermined. In order to endogenously solve for this equilibrium composition steady state equilibria are explored. If the firm applies a universal firing policy, then the workforce composition will converge to a steady state value. This steady state value is determined by Lemma (3).

Lemma 3 *In steady state the equilibrium proportion of good workers (μ_F^*) in the workforce depends on the firing values:*

$$\mu_F^*(\pi_F) = \frac{\mu(1-\theta_B-\delta)\pi_F + \mu g}{(1-\theta_G-\delta)\pi_F + \mu(\theta_G-\theta_B)\pi_F + g}$$

Moreover, the proportion of good quality workers in steady state equilibrium at the higher hierarchical level can be determined as well:

$$\mu_F^{H*} = \frac{\mu_F^*(\theta_G+\delta)}{\mu_F^*(\theta_G+\delta) + (1-\mu_F^*)(\theta_B+\delta)}$$

With these results at hand the equilibrium steady state profit levels can be computed as a function of the firing parameter alone by substituting back into (**??**).[5] Then the steady state equilibrium profit can be expressed as a function of the firing parameter: Expressing all the other variables in terms of the firing decision variable as in the previous subsection. The solution is summarized in lemma (4).

Lemma 4 *Under sorting an optimal incentive mix of fixed wages, bonuses, firing and promotion can arise in equilibrium.*

The result is graphically illustrated in Figure (1).[6]

3.1.3 Characterization

In the model it is established that in a firm where the optimal contract involves paying a combination of performance pay and efficiency wages the composition of the workforce is important for profit maximization. Lemma (3) established the relation between the firm's choice of the firing probability and the steady stage composition of the workforce. Lemma (3) is, however, silent about the implications for individual workers. This section investigates these implications.

In each period the firm is laying off a proportion of the low performing workers. So, for the high ability worker the probability of being fired in any given period is $(1-\theta_G-\delta)\pi_F$ and for the low ability worker the probability is $(1-\theta_B-\delta)\pi_F$. Hence it is more likely that the bad worker looses the job. Moreover, by the same logic good workers are more likely to be promoted, $(\theta_G+\delta)\pi_P > (\theta_B+\delta)\pi_P$. The relation between the firing and promotion probabilities will determine the firms selection scheme and the results are summarized in Lemma (5).

Lemma 5 *The selection process in the firm is always positive in rank. Selection on tenure is negative if*

$$\pi_P > \pi_F$$

and positive if

$$\pi_P < \pi_F$$

[5] We are consciously abstracting away from the potential effects of increasing quality at the management level. It could be captured by adding $M\mu_F^{H*}$ to equation (**??**), where M is fixed parameter. This would only make the profit function more concave in the firing parameter c.f. Figure (1). Thus, it is left for the sake of simplicity.

[6] The parameters used: $\mu = .8, \theta_G = .5, \theta_B = .2, \delta = .2, C = 6, H = 25,\ K = 1, e = .9, g = 4\%,\ U_F = 1, U_P = 1.5, \bar{U}_G = 3.9,\ w^* = .7, \bar{U}_B^* = 0.40$

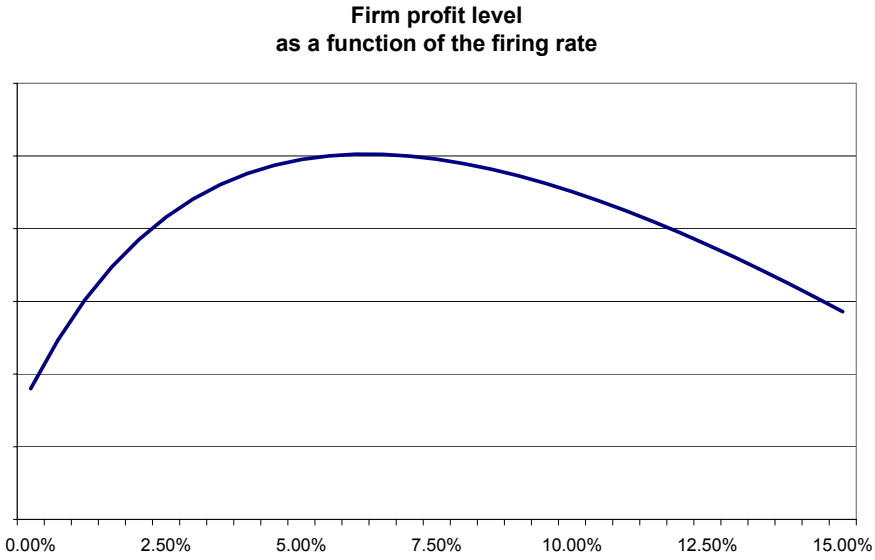

Figure 1: Optimal incentive mix in case of sorting

It is important to note that given the firm uses the performance of the workers in the decision on who to promote and fire, which is a key assumption of the model, the firm will always have positive selection in rank. The condition for positive selection in tenure on the other hand requires that the firing probability exceeds the promotion probability. The intuition behind this result is that high ability workers are more likely to be promoted so unless the firm is firing low ability workers accordingly a disproportionately large fraction of the workers remaining in the rank will be of low ability. The positive selection on tenure is secured by laying off a sufficiently large proportion of the low performers who are more likely to be of low ability.

A direct implication of positive selection is that the individual workers probability of receiving bonus increase in tenure and rank. This result arises because high ability workers are more likely to be high performers.

Lemma 6 *The probability of receiving bonus payments is increasing in tenure conditional on rank and increasing in rank conditional on tenure if and only if the firm has positive selection on rank and tenure.*

Lemma (6) has implications for the firing and promotion probabilities.

Corollary 1 *The probability of promotion is increasing in the probability of receiving bonus payments, whereas the probability of firing decreases with it.*

Since there is no learning in the model and the individuals external option ($\bar{U}_B$ or $\bar{U}_G$) is constant the sorting has no implications for the wage. The combination of an increasing likelihood of receiving bonuses and the flat wage profile, however, leads to an increasing earnings profile. Corollary (2) summarizes the result.

Corollary 2 *The fixed wage (w) paid does not respond to sorting. Earnings, however, rise with tenure, if the firm has positive selection on tenure.*

3.2 Learning

The sorting problem was investigated without learning. However, realistically the firm can identify good employees on the basis of signals. Also, outside firms can expect to base offers on verifiable signals. Signals can include bonus payments, survival or promotion and, wage history. Thus learning has a profound impact on the contract offered. This subsection investigates the consequences of learning and explores to which extent the results are changed.

Learning is introduced formally as follows. The firm fixes the bonus paid, the promotion probability and the firing probability.[7] The firm can use the wage to offer better employees a more favorable contract. Outside firms' offer captured by $\bar{U}_B$ and $\bar{U}_G$, is also assumed to depend on the signals.

Note, that the firm and outside firms have the same information about the employee. Two factors assure that. First, the firm and the outside firms have the same prior about potential employees distribution. Second, the firm does not condition bonuses, firing and promotion on signals - thus can not retain private information. These assumptions mirror those of Gibbons and Waldman (1999).

The sufficient statistic for the signal history is the perceived type, which is denoted by $\hat{\mu}$. Perceived type can be deduced by Bayesian updating. Perceived type can be interpreted as a monotone transformation of the updated probability that the worker is of good quality. Note, that perceived quality is the function of bonus history: bonuses are linked to high output, and exactly pin down output history. Hence, promotion and firings provides no additional information. Formally,

$$\hat{\mu} = \hat{\mu}(b_0, b_1, b_2...) \tag{7}$$

As perceived type is a sufficient statistic, both the firm's and the outside firms' offers can be written in terms of perceived type. Given that bonus payments, firing and promoting probabilities are not altered along with $\hat{\mu}$, only the relevant individual rationality constraint (IR_G) is changed. This constraint is binding under the usual no slack conditions.

$$(IR_G) \qquad w(\hat{\mu}) - e + (\theta_G + \delta)[b + \pi_P U_P] - (1 - \theta_G - \delta)\pi_F U_F = \bar{U}_G(\hat{\mu})$$

Trivially, better workers have better outside options and $\bar{U}$ is increasing in perceived type.[8] This implies, however, through the IR constraint, that better perceived workers also receive higher wages in the firm. Formally, if these variables are differentiable:

$$\frac{\partial \bar{U}_G(\hat{\mu})}{\partial \hat{\mu}} = \frac{\partial w(\hat{\mu})}{\partial \hat{\mu}} > 0 \tag{8}$$

Thus, the above discussion implies that wages are increasing in terms of past bonuses. Lemma (7) summarize. In other aspects, however, the earlier results do not change.

Lemma 7 *Fixed wage under learning becomes an increasing function of past bonuses. Further more the results of Lemmas (3), (4), (5) and (6) apply.*

3.3 Human capital

Employee heterogeneity has been restricted to two, unobservable types. However, there are other observable differences between individual employees. Most important is education which we denote by E. The education literature (Becker 1964) has established, that educational achievement is positively

[7] This is a necessary assumption to preserve the stationary contract and the steady state equilibria. As it will be shown later, the data does not repudiate this assumption.

[8] Remember, $\hat{\mu}$ is a monotone transformation of probability of good type.

correlated with ability, which was modeled here by type. Thus, both the likelihood of receiving bonuses and the outside offers are altered by the level of education. These results are stated in Lemma (8).

Lemma 8 *Education is monotonically increasing in ability. Hence education increases the likelihood of receiving bonus payments thus it increases the probability of promotions and reduces the probability of firing. In addition, education improves on alternative offers hence it has a positive effect on wages.*

3.4 Empirical predictions

The integrative model presented above establishes that the incentive mix can be an optimal contract. Besides producing the optimal incentive mix the model provides a broad range of additional empirical predictions that are outlined in the conjectures below. In the spirit of Gibbons and Waldman (1999) these will be tested empirically in the next section. The conjectures follow directly from the derived Lemmas, so no proof is duplicated.

Conjecture 1 (Optimal mix) *Sorting is a necessary condition for an incentive mix of performance pay and efficiency wages to be an optimal contract. Thus, strategic employee sorting is present in the firm if a mix of incentives is observed.*

Conjecture 2 (Bonus) *The likelihood of receiving bonuses depends on the employees ability. Since the level of education is monotonically increasing in ability the likelihood of receiving bonus payments increases in education. Furthermore, the probability of receiving bonus payments is increasing in tenure conditional on rank and increasing in rank conditional on tenure if and only if the firm has positive selection on rank and tenure.*

Conjecture 3 (Wage) *Wages are positively correlated with signals. Thus the wage depends positively on education. Furthermore, if the firm has positive selection on tenure and rank the wages increase in these variables due to their positive signalling value. Furthermore, since past bonus payments convey information about the employees type, past bonus payments will increase current wages.*

Conjecture 4 (Promotion) *The likelihood of promotion depends exclusively on past bonuses. Hence, conditional on the full history of past bonuses, the likelihood of promotion does not depend on tenure.*

Conjecture 5 (Firing) *The likelihood of firing depends exclusively on the lack of past bonuses. Hence, conditional on the full history of past bonuses, the likelihood of firing does not depend on tenure.*

4 The Data

Four years of monthly personnel records from the main production size of an international pharmaceutical company are used in the analysis. Average full time employment in the plant over the period 1997 to 2000 is 5055 persons.[9] These workers are distributed across four hierarchical levels ranging from the CEO to non-management, see Figure 2. The share of management workers in the firm is 4.75 percent on average over the four years.

The analysis below use only those individuals who participate in the performance pay system i.e. those employees who besides their base salary can get a bonus given sufficiently high performance evaluations. The distinction between strictly fixed paid employees and employees facing an incentive

[9] The analysis is focused on permanent full time employment which corresponds to 92.66 percent of all individuals employed on the production site.

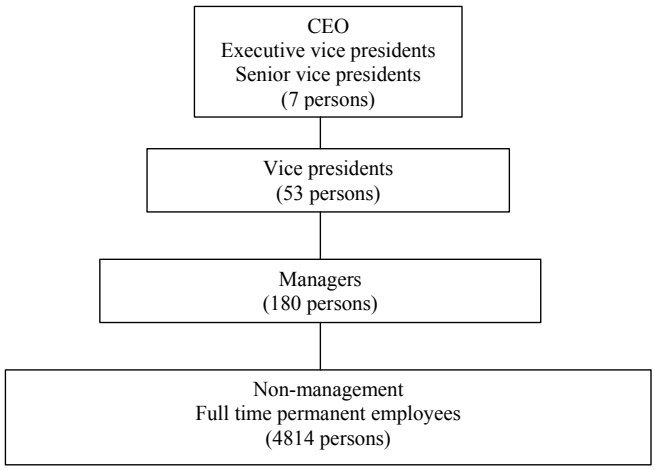

Figure 2: The hierarchy of the firm, 1997 to 2000.

mix can be made by looking at the worker's job category. This exercise reveals that the group of fixed paid employees constitute 36.96 percent of the workforce and that it mainly consists of production workers.[10]

The characteristics of the employees calculated from the monthly employee-based observations are presented in the second and third columns of Table 3. On average the employees included in the analysis (column 3) have 8.91 years of tenure. The same group of individuals consists of 60.6 percent women and the average age is 39.51 years.

The firm is operating in an industry where product development is essential for survival, hence a large proportion of the workforce is engaged in research and development. These activities are reflected by a relatively high education level in the firm.[11] It is remarkable that 21.9 percent of the employees have at least a master degree and 11.7 percent have a degree that in duration corresponds to a bachelor degree. Only 12.5 percent of the labour force is unskilled. The level of education is also relatively high in production. A possible explanation is that production requires a minimum of manual labor as there is a high degree of automation. Furthermore, in order to comply with the demands of the Food and Drug Administration (FDA) extensive quality control programmes are implemented which requires skilled labor.

The firms yearly separation rate is 10.96 percent. The turnover is costly for the firm but never the less 12.64 percent of all separations are initiated by the firm though layoffs.[12] There are two motivations for laying off workers. First, the firms official wage strategy is to *"offer attractive salary and employment conditions"* in order to *"attract, develop and retain qualified employees"*. Given the "attractive" wages the firm can use the threat of a layoff to motivate the workers i.e. the combination of "attractive" wages and layoffs will make the workers exert higher effort. Second, the layoffs serve as a sorting device where underperformers are forced to separate from the firm in each period. Hence in a pooling equilibrium where both high and low ability workers are employed by the firm layoffs can

[10] The worker choose between fixed wage and the incentive mix contracts ex-ante hence the analysis is performed conditional on this initial sorting.

[11] Information on education is missing for 8.21 percent of the employees. However, for the group of employees receiving performance pay only 2.84 percent have missing information on education.

[12] The sepration rate for the employees participating in the performance pay system is 6.64 percent. Of these 19.46 percent are initiated by the firm. The institutional settings impose no restriction on who to fire.

	Mean (standard deviations)	
	All employees Observations = 242,041	Employees participating in the performance pay system Observations = 147,602
Age	39.749	39.508
	(9.125)	(8.543)
Gender (women = 1)	0.566	0.606
Unskilled	0.125	0.035
Skilled worker	0.266	0.159
Short theoretical education	0.190	0.280
Bachelor degree	0.117	0.174
Masters or PhD degree	0.219	0.352
Tenure less than 2 years	0.126	0.119
Tenure 2 to 5 years	0.302	0.289
Tenure 6 to 10 years	0.273	0.257
Tenure above 10 years	0.299	0.334

Figure 3: Descriptive statistics

be used to control the worker composition.

The base salary paid by the firm is highly predictable given the characteristics of the individuals and the job category $\left(R^2 = 0.62\right)$. This finding is in line with expectations since the firms official wage policy explicitly states that: "*Base salary is a competitive pay for job function, responsibilities and competencies. The base salary is driven by the market and increases should not be driven by individual performance*". The firms's wage structure (unconditional on human capital and signals) is such that the wage premium for moving from non-management and into lower and middle management are 52.12 percent and 83.70 percent, respectively. Wages are not observable for higher level management.

Individuals are rewarded for performance through a firm-wide bonus system. The allocation of bonuses will, according to the firms remuneration principles, fulfill the criteria that: *"The principles, criteria and targets that will lead to bonus payments should be known by the relevant employee subgroup"*. The size of the bonus-pool varies across the different employee subgroups and constitute 2.5-4 percent of the wage sum for non-management workers and 8 percent for non-executive managers. As for wages the performance related compensation package for higher level managers is not observed in the data.[13] The widespread use of bonuses in the firm is confirmed by the data which shows that 24 percent of workers in non-management will receive bonus payments in a given year. The numbers are closer to 75 and 80 percent for managers and vice presidents, respectively.

4.1 The empirical contract and the theoretical assumptions

In the presentation of the firm it becomes clear that the contract offered by the firm to the workers is highly complex in nature. First, the firm pays fixed wages to all workers who accept the contract unconditional on performance. The fixed wages are argued to be driven by the market. Second, even

[13] The subsequent analysis is based on non-management employees, managers and, vice presidents. The Excecutive management is omitted due to lack of data. An analysis of excecutive management compensation is conducted in Murphy (1999) and Hall and Murphy (2003).

though turnover is costly for the firm, a significant part of separations are firm initiated i.e. layoffs. The presence of interim rents and the threat of firing creates incentives for the workers to exert effort. Third, 63 percent of the employees are rewarded for performance through a bonus system. Pay for performance is an additional source that encourage the employees to exert effort. Finally, the firm has ports of entry (i.e. 98.22 percent of the employees are hired into the lowest hierarchical level). This implies that promotions to higher level jobs take place, mainly, from the pool of incumbent employees creating additional incentives. In sum, the data describe a contract that contains four parameters: w, b, π_P, and π_F. These parameters are the key parameters of our theoretical model.

The description of the empirical contract reveals that the firm is using a combination of performance pay and efficiency wages. The theoretical motivation for using both policy devices is to create incentives for workers but also to be able to control the composition of the workforce. In the next section we examine how the incentive mix is implemented in practice and investigate to what extent employee sorting is important for the firm.

5 Empirical analysis

Conjecture 1 stated that employee sorting is a necessary condition for the incentive mix to be an optimal contract. Preliminary evidence for sorting is found in the description of the empirical contract where it is established that layoffs are common in the firm i.e. $\pi_F > 0$. The presence of layoffs is a necessary condition for sorting but not sufficient in the sense that random firing would produce no sorting. Random firing, however, would at all times be inefficient for the firm due to the costs associated with turnover. Thus a first test for the presence of sorting in the firm it to investigate whether the firm has random sorting or whether it is using information about the employee's ability (such as revealed performance) in the layoff decision.

To establish that the firm is sorting when it comes to layoffs a multinomial logit model, where the individual is facing the three options: Stay within rank, promotion and layoff, is estimated.[14] The point estimates of a multinomial logit are difficult to interpret hence the results of the model are evaluated using predictions.[15] The effect of a bonus payment on the transition probabilities for the average sample member is presented in row four of Table 4. A person who receive a bonus payment has a 1.553 per mille chance of being promoted in a given month while the layoff probability is as low as 0.237 per mille. In contrast, a person who did not receive a bonus payment has almost no chance of being promoted and face a 2.204 per mille risk of being laid off from the firm in any given month.

The empirical evidence presented in Table 4 shows that the firm tend to layoff low performing workers hence as stated in Conjecture 1 there is clear evidence for sorting in the firm. Further more it reveals that the layoff probability exceeds the promotion probability which according to lemma (5) would imply negative selection in tenure. However, there may be measurement errors in the variable that indicates the reason for the job separation. An obvious bias arise in the cases where the firm is signalling to the worker that the employment relation will end in the near future. This signal will make the worker look for alternative employment which may be obtained before the firm terminates the match. We have reason to believe that this procedure is common. The implication is that the layoffs observed in the firm only constitute a lower bound on the separations that in reality are layoffs. Hence, alternative evidence for the type of selection scheme used by the firm should be obtained from

[14] It should be noted that the individuals who separate for natural causes such as retirement or death and the individuals who leave the firm for a new job have been deleted from the sample.

[15] The full regression results of the multinomial logit can be seen in the appendix.

	Destinations		
	Promotion	**Layoff**	**Stay within rank**
Probability of average sample member being in category [sample means in brackets]	1.239 ‰ [1.239 ‰]	1.107 ‰ [1.107 ‰]	99.765 % [99.765 %]
Probability of average sample member being in category given bonus is received.	1.553 ‰	0.237 ‰	
Probability of average sample member being in category given no bonus is received.	0.869 ‰	2.204 ‰	
Change in probability due to bonus	0.684 ‰	- 1.967 ‰	

Note: The predictions are based on a regression with 128,233 observations.

Figure 4: Predicted monthly transitions

the data.

Conjecture 2 focuses on bonus payments. Since the likelihood of receiving bonuses depends on employee ability, the selection scheme used by the firm and the probability of receiving bonus are closely linked (Lemma 6). A consequence of positive selection in tenure and rank is that more able workers in general will have longer tenure and be assigned to higher rank jobs. Hence, if the firm has positive selection employees with longer tenure and higher ranks will be more likely to receive bonus payments. This provides an additional test for the type of selection scheme used by the firm.

Table 5 presents logit regressions for the probability of having received a bonus payment during the preceding year. In the first model only tenure and rank are included in the regression and we find that these variables increase the likelihood of receiving bonus payments. The results are robust to the inclusion of information on demography (age-splines and gender), see model 2. In model 3 information on education is added. The positive relation between educational attainment and ability implies that workers with higher levels of schooling will be more likely to receive bonus payments (Lemma 8) - a result that is confirmed by the data. The positive effect of tenure and rank on the probability of receiving bonuses remains in model 3 which is indicating that ability is signalled only partially through education.

The effect of tenure on the likelihood of receiving bonus payments is the main source to identification of the firms selection scheme. For this reason, an additional specification for the likelihood of receiving a bonus payment is provided. We estimate a model similar to model (3) in Table 5 but substitute the tenure-splines for yearly tenure dummies (detailed results are not shown).[16] Figure 6 illustrates the effect of tenure on the likelihood of receiving bonus for the first 25 years of tenure.[17] In accordance with the predictions from positive sorting the effect is increasing, but so is the variance as we get to higher tenure levels (tenure above 16 years). However, the steady increase in the likelihood of receiving bonuses during the first many years of employment reflects that sorting is an integrated part of the firms policy.

Based on the evidence from Table 5 and Figure 6 there is clear evidence for positive selection in the firm. This observation support the assumption that sorting is important for the firm which is

[16] The maximum level of tenure in the firm is 44 years but less than 3.25 percent of the employees have tenure above 25 years causing large standard errors on the point estimates of the tenure dummies exceeding the 25th year.

[17] It should be noted that it is very uncommon to receive bonus payments during the first year of employment (occur in less than 3.5 of the cases) which explains the level for the tenure effect.

	(1)	(2)	(3)
Constant	-1.957	-2.038	-2.648
	(0.082)	(0.097)	(0.174)
Tenure 0 to 2 years	-	-	-
Tenure 3 to 5 years	0.728	0.728	0.730
	(0.078)	(0.078)	(0.078)
Tenure 6 to 10 years	0.941	0.907	0.917
	(0.086)	(0.088)	(0.088)
Tenure above 10 years	1.122	1.095	1.115
	(0.086)	(0.091)	(0.091)
Non-management	-	-	-
Managers	0.577	0.559	0.604
	(0.113)	(0.114)	(0.112)
Vice presidents	0.800	0.772	0.759
	(0.157)	(0.157)	(0.157)
Unskilled			-
Skilled worker			0.287
			(0.135)
Short theoretical			0.369
education			(0.135)
Bachelor degree			0.497
			(0.138)
Masters or PhD degree			0.663
			(0.154)
Demographic variables	NO	YES	YES
Job category	YES	YES	YES
Time dummies	YES	YES	YES
Log likelihood	-73,228	-73,084	-72,953
# observations	138,703	138,703	138,703

Note: Standard errors are clustered with respect to individuals.

Figure 5: Logit regression for bonus payments

used to establish that the incentive mix of efficiency wages and performance pay can be an optimal contract. Furthermore, the positive selection has, through its effect on the likelihood of receiving bonus, implications for the individuals earnings growth which will be studied below.

Conjecture 3 describes the firm's wage setting. The firms official wage policy is to pay employees competitive wages that do not depend on the employees performance (recall that high performance is rewarded through the bonus system). In the context of the model this implies that once the optimal contract is set, i.e. π_P and π_F and b are chosen, the wage offered to the worker depends on the good type's outside valuation $\bar{U}_G$. Absent learning the fixed wage does not change over time as shown in Corollary (2). However, the firms sorting scheme described above reveals information about the workers quality that may be impossible to hide for the individuals potential employers hence the market may learn about the workers quality through observable credentials such as the workers tenure, current rank in the firm and received bonus payments. The consequence of learning is that when the perceived quality of the individual goes up it causes an increase in wages as shown in Lemma (7).

A direct test of the effect of learning on wages can be conducted using standard Mincer wage regressions, see Table 7. Model 1 in Table 7 shows increasing wages in education, tenure and rank - a result that was highly expected. Model 2 explores the effect of a bonus payment last period on current wages. The point estimate show that a bonus payment last period (which can also be interpreted at high output) increases current wages by 4.43 percent. An extension of this analysis is conducted in model 3 where information of up to 4 years of lagged bonus payments is included. The remarkable result reveal that lagged bonus payments are highly significant. Further more they seem to have similar effects on current wages hence the timing of bonus does not seem to have big effect. To investigate this issue further, model 4, estimate the effect of the last two years bonus payments on the wage together

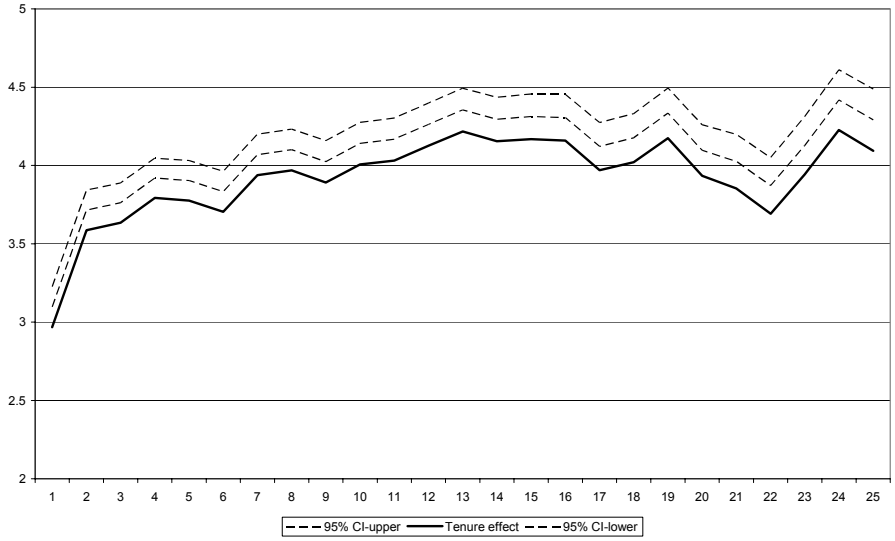

Figure 6: Tenure effect on the likelihood of receiving bonus payments.

with an interaction effect capturing the consequences for current wages of receiving bonuses in both years.[18] This regression reveals that workers are rewarded for bonus payments in both periods but the value of the signal is deflated slightly when the employee's performance is rewarded with bonuses in both periods (due to the negative interaction term). This may simply be a consequence of the Bayesian updating. In sum, there is strong empirical evidence for the learning model and the signals related to education, rank, tenure and lagged bonus payments are all important for the determination of wages.

Conjecture 4 and 5 address the models prediction related to promotions and layoffs. In the discussion of Conjecture 1 it was established that past performance, i.e. bonus payments, were a key determinant for the firms decision on whom to fire and whom to promote. These results are naturally confirmed by table 8 which presents logit regressions for the probabilities of being promoted and laid off.

An interesting aspect of the results presented in Table 8 is that not all workers have equal promotion probabilities conditional on performance. An example is that none of the employees with a short theoretical education are promoted in the four year period studied. In contrast, the employees with a masters or Ph.D. degree are significantly more likely to be promoted than employees with less than a bachelors degree (column 3). When we look within educational groups tenure is insignificant. These observations suggests that certain skills are required in order to be promoted i.e. skills that are correlated with the level of schooling. Fascinating is it to combine the results of the promotion regression with the results of the layoff regression. In the layoff regression there is a tendency that the employee subgroups who are less likely to be promoted, i.e. skill workers and employees with a short theoretical education, are "compensated" through significantly lower layoff probabilities. This combined result suggests that the firm accommodates that some employee subgroups lack incentives through promotions and adjust the parameters of the contract such that the incentive and participation constraints of the optimal contract are satisfied. The consequence of this behavior is that the firm offers different contracts to its different employee subgroups; subgroups that are defined by their educational

[18] The focus on the last two years bonus payments is only for expositional reasons and could easily be extended to include all four years and their interaction terms.

	(1)	(2)	(3)	(4)
Constant	10.182	10.180	10.225	10.219
	(0.012)	(0.012)	(0.012)	(0.012)
Bonus payment last year		0.041	0.038	0.039
		(0.007)	(0.007)	(0.007)
Bonus payment two			0.028	0.046
years ago			(0.006)	(0.009)
Bonus payment three			0.030	
years ago			(0.006)	
Bonus payment four			0.037	
years ago			(0.007)	
Bonus payment last year				-0.012
and two years ago				(0.006)
Tenure less than 2 years	-	-		
Tenure 2 to 5 years	0.023	0.032	-	-
	(0.004)	(0.004)		
Tenure 5 to 10 years	0.085	0.087	0.044	0.056
	(0.005)	(0.005)	(0.003)	(0.003)
Tenure above 10 years	0.122	0.124	0.084	0.093
	(0.006)	(0.007)	(0.006)	(0.005)
Unskilled	-	-	-	-
Skilled worker	0.052	0.050	0.052	0.051
	(0.010)	(0.010)	(0.010)	(0.010)
Short theoretical	0.060	0.059	0.062	0.060
education	(0.010)	(0.010)	(0.010)	(0.010)
Bachelor degree	0.072	0.069	0.070	0.069
	(0.010)	(0.010)	(0.011)	(0.010)
Masters or PhD degree	0.105	0.101	0.099	0.099
	(0.011)	(0.012)	(0.014)	(0.012)
Non-management	0.028	0.026	0.026	0.026
	(0.014)	(0.014)	(0.014)	(0.014)
Managers	0.290	0.286	0.293	0.287
	(0.030)	(0.030)	(0.031)	(0.030)
Demographic variables	YES	YES	YES	YES
Job category	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES
R-squared	0.6184	0.6187	0.5922	0.6138
# observations	147,602	138,703	109,440	130,068

Note: Standard errors are clustered with respect to individuals. The numbers of observations vary across the models and are determined by the number of the lagged variables included in the regression.

Figure 7: Mincer wage equations.

attainment.

In sum, the empirical evidence presented above provide strong evidence for the models conjectures. First, using the description of the empirical contract and the bonus regression we conclude that the firm has positive selection on tenure and rank. This is a conclusion that is stronger than required in order to establish that the incentive mix can be an optimal contract for the firm. Second, there is strong evidence for learning in the sense that wages are found to be positively correlated with signals i.e. tenure, rank, education and past bonus payments. Finally, bonus is found to be a strong predictor for the firms decision on who to promote and layoff. There is evidence for the fact, however, that there are skill requirements when workers are promoted. This implies that some employee subgroups are unlikely to be promoted which will reduce the incentives for the employees to exert effort. The firm accommodates this by offering heterogenous contracts to the different employee subgroups.

	Layoff	Promotion
Constant	-9.132	-7.968
	(0.759)	(1.066)
Bonus payment last year	-2.281	0.669
	(0.370)	(0.228)
Bonus payment two years ago	-1.205	0.761
	(0.356)	(0.400)
Bonus payment last year and	0.735	-0.390
two years ago	(0.611)	(0.412)
Tenure 2 to 5 years	-	-
Tenure 5 to 10 years	-0.343	0.046
	(0.263)	(0.194)
Tenure above 10 years	0.218	-0.284
	(0.218)	(0.226)
Unskilled	-	-
Skilled worker	-1.275	-0.185
	(0.475)	(1.118)
Short theoretical education	-0.644	-
	(0.382)	(-)
Bachelor degree	0.794	1.513
	(0.371)	(1.022)
Masters or PhD degree	0.476	2.305
	(0.383)	(1.008)
Demographic variables	YES	YES
Time dummies	YES	YES
Log likelihood	-866.50	-1014.29
# observations	119,762	81,956

Note: Individuals with a short theoretical education are never promoted hence they are omitted from the promotion regression.

Figure 8: Layoff and promotion regressions

6 Conclusion

Firms are known to offer highly complex contracts to their employees. In this paper we study how incentive are used in practice and provide a theoretical motivation for how the incentive mix observed in firms can be seen as an optimal contract.

A basic model with homogenous employees and the four most often used incentive parameters (base wage, bonuses, firing and promotion) is presented. This model replicates the findings in MacLeod and Malcolmson (1998) where the firm's use of either performance pay or efficiency wages is an optimal responses to the given market conditions. The basic model is transformed into an integrative model in the spirit of Gibbons and Waldman (1999) and extended to an infinitely repeated game with heterogenous employees. In this context three building blocks: job assignment, learning and human capital, are sequentially added.

Job assignment provides the critical result: the optimal incentive mix. The reason is that the heterogeneity of employees provides a dual role for firing. First, it creates incentives for the employees as in the basic model. Second, it is a sorting device that allows the firm to adjust the composition of the workforce to the profit maximizing level. The sorting mechanism introduces non-linearities into the maximization problem that creates the basis for a incentive mix of performance pay and efficiency wages to be an optimal contract.

The remaining two building blocks creates a closer link between the theoretical model and the observed data. Learning implies that wages increase in the employees perceived ability. Human capital link education to the parameters of the model through is close relation to employee ability.

The integrative model provides conjectures with respect to the contract's four incentive parameters: base wage, bonus, firing and promotion. The conjectures are tested in the empirical section of the paper

using four years of monthly personnel records from an international pharmaceutical company.

A first result established is that the firm has positive sorting in tenure and rank that is individuals with higher ability will in general be more likely to have longer tenure and be assigned to jobs at a higher rank in the firm. The positive sorting implies that the likelihood of receiving bonuses is increasing in tenure conditional on rank and in rank conditional on tenure - a result that is confirmed by the data.

Secondly, learning is established to be important for wage determination as employees use signals in the bargaining with firms to increase the base wage. In particular we find that education, tenure, current rank and past bonus payments have a positive effect on the level of current wages.

Finally, it is established that the probability of promotion is increasing in past bonus payments whereas bonus payments reduce the risk of being fired.

7 Appendix

	Layoff	Promotion
Constant	-9.650	-8.925
	(0.816)	(1.108)
Bonus payment received	-2.239	0.580
during the preceding year	(0.267)	(0.185)
Tenure less than 2 years	-	-
Tenure 2 to 5 years	0.318	0.648
	(0.490)	(0.378)
Tenure 6 to 10 years	0.618	0.704
	(0.487)	(0.389)
Tenure above 10 years	0.834	0.367
	(0.483)	(0.407)
Unskilled	-	-
Skilled worker	-1.409	-0.193
	(0.462)	(1.118)
Short theoretical education	-0.701	-
	(0.365)	(-)
Bachelor degree	0.604	1.602
	(0.355)	(1.020)
Masters or PhD degree	0.220	2.383
	(0.362)	(1.007)
Demographic variables	YES	YES
Time dummies	YES	YES
Log likelihood	-1992.38	
# observations	128,233	

Note: Reference category is: "Stay within rank". Individual observations with tenure less than one year are omitted from the regression due to the inclusion of the lagged bonus payment variable.

Figure 9: Multinomial logit.

7.1 Proofs

Proof of Lemma (1). Notice first, that the profit is a linear function of the firing probability:

$$\Pi(\pi_F) = (\theta+\delta)C - e + (\theta+\delta)\overbrace{\left[b + \underbrace{\frac{(1-\theta-\delta)\pi_F + g}{H(\theta+\delta)}}_{\pi_P} U_P\right]}^{w} - (1-\theta-\delta)\pi_F U_F - \bar{U}$$
$$-(1-\theta-\delta)\pi_F K - (\theta+\delta)\underbrace{\left[\frac{e}{\delta} + \pi_F U_F - \frac{(1-\theta-\delta)\pi_F + g}{H(\theta+\delta)} U_P\right]}_{b}$$

The first order derivative is:

$$2(1-\theta-\delta)\frac{U_P}{H} - U_F - (1-\theta-\delta)K$$

Consequently, the maximum is obtained when the firing probability is maximal or minimal such that the regularity conditions are not violated. The two extremes correspond to the efficiency wage ($\pi_F = 0$) and the performance pay ($\pi_F > 0$) solution. For the sake of completeness the two possible solutions are characterized below:

Performance pay:

$b = \frac{e}{\delta} - \frac{g}{H(\theta+\delta)} U_P$

$\pi_F = 0$

$\pi_P = \frac{g}{H(\theta+\delta)}$

$w = e - (\theta+\delta)\frac{e}{\delta} + \bar{U}$

$\Pi = (\theta+\delta)C - e - \bar{U} + gU_P$

Efficiency wage:

$b = 0$

$\pi_F = \frac{(\theta+\delta)\frac{e}{\delta} - gU_P}{(1-\theta-\delta)U_P + H(\theta+\delta)U_F}$

$\pi_P = \frac{(1-\theta-\delta)\frac{e}{\delta} + gU_F}{(1-\theta-\delta)U_P + H(\theta+\delta)U_F}$

$w = e - (\theta+\delta)\left[\frac{(1-\theta-\delta)\frac{e}{\delta} + gU_F}{(1-\theta-\delta)U_P + (\theta+\delta)U_F}\right] U_P + (1-\theta-\delta)\left[\frac{(\theta+\delta)\frac{e}{\delta} - gU_P}{(1-\theta-\delta)U_P + H(\theta+\delta)U_F}\right] U_F + \bar{U}$

$$\Pi = \begin{array}{c}(\theta+\delta)C - e - \bar{U} + \left[\frac{(1-\theta-\delta)\frac{e}{\delta} + gU_F}{(1-\theta-\delta)U_P + H(\theta+\delta)U_F}\right](\theta+\delta)U_P \\ + (1-\theta-\delta)\left[\frac{H(\theta+\delta)\frac{e}{\delta} - gU_P}{(1-\theta-\delta)U_P + H(\theta+\delta)U_F}\right] K\end{array}$$

If the FOC is undetermined, then the firm chooses the performance pay solution as it involves the least firing. ■

Proof of Remark (1). The proof follows directly from the intuition provided after stating the lemma. Slack is either inconsistent with profit maximizing (for promotion slack) or implies efficiency wage (for bonus slack) or zero wage (for wage slack). ■

Proof of Lemma (2). The lemma is proven by the discussion preceding it. Obtaining the solution to the stationary contract boils down to sloving the period problem. This is what was done in Lemma (1). ■

Proof of Remark (2). The pooling equilibrium arrise if the firm cannot force the workers to self-select ex-ante. That is, the bad type will accept the contract even if the firm sets the parameters of the contract such that they create the strongest possible incentives. For the sufficient condition consider the following: The strongest incentives trivially includes zero fixed wages and unit firing probability. Then the promotion probability is determined through the feasibility (F) constraint of problem (**??**)

and the bonus payment is determined through the individual rationality constraint of the good type in the same problem. Thus:

$$\begin{aligned}\pi_P^* &= \frac{(1-\delta-\mu_F^H\theta_G-(1-\mu_F^H)\theta_B)\pi_F+g}{H(\mu_F\theta_G+(1-\mu_F)\theta_B+\delta)}\\ b^* &= \frac{\bar{U}_G+e+(1-\theta_G-\delta)U_F}{\theta_G+\delta}-\frac{(1-\theta_G-\delta)+g}{H(\theta_G+\delta)}U_P\end{aligned}$$

Consequently, a sufficient condition for the pooling equilibrium is as follows:

$$-e+(\theta_B+\delta)[b^*+\pi_P^*U_P]-(1-\theta_B-\delta)U_F=\bar{U}_B^*\geq\bar{U}_B$$

■

Proof of Lemma (3). Given a fixed firing probability, the percentage of good workers employed by the firm can be determined. The measure of bad employees fired in steady state is given as:

$$(1-\mu_F^*)(1-\theta_B-\delta)\pi_F$$

The measure of good employees fired:

$$\mu_F^*(1-\theta_G-\delta)\pi_F$$

The steady state condition is that the composition of the workforce does not change any further. This can be captured as the measure of good employees leaving the firm in any period equals to the measure of good employees entering the firm. It is summarized in the following equation:

$$\overbrace{\mu[\underbrace{(1-\mu_F^*)(1-\theta_B-\delta)\pi_F+\mu_F^*(1-\theta_G-\delta)\pi_F}_{\text{measure of all fired}}+g]}^{\text{measure of good type newly hired}}=\overbrace{\mu_F^*(1-\theta_G-\delta)\pi_F}^{\text{measure of good type fired}}+\mu_F^*g$$

The statement on μ_F^* follows directly from here.

For the higher rank the steady state composition is given by the following logic. In the steady state only the equilibrium proportion of promoted workers matters. All initial differences - if any - are deflated to zero by natural turnover g. The volume of good quality workers promoted is given by $\mu_F^*(\theta_G+\delta)\pi_P$ and the volume of bad quality workers promoted is $(1-\mu_F^*)(\theta_B+\delta)\pi_P$. Hence the proprotion is as is given in the lamma. ■

Proof of Lemma (4). It is straightforward to create an example of an interior solution. Take, for instance, the parameter choice used for Figure (1). Expressing all the other variables in terms of the firing decision variable as in the previous subsection yields the profit function as the function of the

firing parameter (π_F).

$$\Pi(\pi_F) = \frac{\mu(1-\theta_B-\delta)\pi_F(\theta_G+\delta)}{(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)+\mu(1-\theta_B-\delta)\pi_F} *$$
$$\left(\begin{array}{c} C+\frac{e}{\delta}-\pi_F U_F \\ -\frac{\left[1-\delta-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\theta_G-\left(1-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\right)\theta_B\right]\pi_F+g}{\left[\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\theta_G+\left(1-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\right)\theta_B+\delta\right]H\pi_P}U_P \end{array} \right)$$
$$+\frac{\left[(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)\right](\theta_B+\delta)}{(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)+\mu(1-\theta_B-\delta)\pi_F} *$$
$$\left(\begin{array}{c} C+\frac{e}{\delta}-\pi_F U_F \\ -\frac{(1-\delta-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\theta_G-(1-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g})\theta_B)\pi_F+g}{H(\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g}\theta_G+(1-\frac{\mu(1-\theta_B-\delta)\pi_F+\mu g}{(1-\theta_G-\delta)\pi_F+\mu(\theta_G-\theta_B)\pi_F+g})\theta_B+\delta)\pi_P}U_P \end{array} \right)$$
$$-(e-(\theta_G+\delta)[\frac{e}{\delta}-\pi_F U_F]+(1-\theta_G-\delta)\pi_F U_F+\bar{U}_G)$$
$$-\frac{\mu(1-\theta_B-\delta)\pi_F}{(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)+\mu(1-\theta_B-\delta)\pi_F}(1-\theta_G-\delta)\pi_F K$$
$$-\frac{(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)}{(1-\theta_G-\delta)\pi_F-\mu(1-\theta_G-\delta)+\mu(1-\theta_B-\delta)\pi_F}(1-\theta_B-\delta)\pi_F K$$

This is shown on Figure (1). ■

Proof of Lemma (5). Given the separation and promotion probabilities we have

$$\begin{aligned} \Pr(tenure &= j, rank=r|G) \\ &= k_{(j,r)}\left[(\theta_G+\delta)\,\pi_P\right]^r\left[1-(\theta_G+\delta)\pi_P-(1-\theta_G-\delta)\pi_F\right]^{j-r} \\ &= k_{(j,r)}\left[(\theta_G+\delta)\,\pi_P\right]^r\left[1-\pi_{S|G}\right]^{j-r}, \end{aligned}$$

$$\begin{aligned} \Pr(tenure &= j, rank=r|B) \\ &= k_{(j,r)}\left[(\theta_B+\delta)\,\pi_P\right]^r\left[1-(\theta_B+\delta)\pi_P-(1-\theta_B-\delta)\pi_F\right]^{j-r} \\ &= k_{(j,r)}\left[(\theta_B+\delta)\,\pi_P\right]^r\left[1-\pi_{S|B}\right]^{j-r}, \end{aligned}$$

for $j \geq 0, 0 \leq r \leq j$. Where $\pi_{S|B}$ and $\pi_{S|B}$ are the probabilities of leaving a given rank for the good and bad workers, respectively and $k_{(j,r)}$ are coefficients depending on j and r.

Recalling that the initial proportion of good types hired by the firm is μ then the probability of being of a good type conditional on tenure and rank becomes

$$\begin{aligned} P_G &= \Pr(G|tenure=j, rank=r) \\ &= \frac{\mu k_{(j,r)}\left[(\theta_G+\delta)\,\pi_P\right]^r\left[1-\pi_{S|G}\right]^{j-r}}{\mu k_{(j,r)}\left[(\theta_G+\delta)\,\pi_P\right]^r\left[1-\pi_{S|G}\right]^{j-r}+(1-\mu)k_{(j,r)}\left[(\theta_B+\delta)\,\pi_P\right]^r\left[1-\pi_{S|B}\right]^{j-r}} \end{aligned}$$

and the probability of being a bad type given tenure and rank is

$$P_B = \Pr(B|tenure=t, rank=r) = 1-P_G$$

using the fact that good workers are more productive than bad workers i.e. $\theta_G = \theta_B + \epsilon$ with $\epsilon > 0$ and the notation that $\pi = \pi_P = \pi_F + \gamma$ we can calculate the ratio of the proportion of good workers

relative to the proportion of bad workers in the firm at a given tenure and rank

$$\begin{aligned}
M_G &= \frac{P_G}{P_B} \\
&= \frac{\mu k_{(j,r)} \left[(\theta_G+\delta)\,\pi_P\right]^r \left[1-(\theta_G+\delta)\pi_P-(1-\theta_G-\delta)\pi_F\right]^{j-r}}{(1-\mu)k_{(j,r)} \left[(\theta_B+\delta)\,\pi_P\right]^r \left[1-(\theta_B+\delta)\pi_P-(1-\theta_B-\delta)\pi_F\right]^{j-r}} \\
&= \frac{\mu \left[(\theta_B+\epsilon+\delta)\,\pi\right]^r \left[1-(\theta_B+\epsilon+\delta)\pi-(1-\theta_B-\epsilon-\delta)(\pi-\gamma)\right]^{j-r}}{(1-\mu) \left[(\theta_B+\delta)\,\pi\right]^r \left[1-(\theta_B+\delta)\pi-(1-\theta_B-\delta)(\pi-\gamma)\right]^{j-r}} \\
&= \frac{\mu \left[(\theta_B+\epsilon+\delta)\,\pi\right]^r \left[(1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma\right]^{j-r}}{(1-\mu) \left[(\theta_B+\delta)\,\pi\right]^r \left[(1-\pi)+(1-\theta_B-\delta)\gamma\right]^{j-r}} \\
&= \frac{\mu}{(1-\mu)} \left(\frac{(\theta_B+\epsilon+\delta)}{(\theta_B+\delta)}\right)^r \frac{\left[(1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma\right]^{j-r}}{\left[(1-\pi)+(1-\theta_B-\delta)\gamma\right]^{j-r}} \\
&= \frac{\mu}{(1-\mu)} \left(\frac{\frac{(\theta_B+\epsilon+\delta)}{(\theta_B+\delta)}}{\frac{(1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma}{(1-\pi)+(1-\theta_B-\delta)\gamma}}\right)^r \left(\frac{\left[(1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma\right]}{\left[(1-\pi)+(1-\theta_B-\delta)\gamma\right]}\right)^j \\
&= \frac{\mu}{(1-\mu)} \left(\frac{(\theta_B+\epsilon+\delta)\left((1-\pi)+(1-\theta_B-\delta)\gamma\right)}{(\theta_B+\delta)\left((1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma\right)}\right)^r \left(\frac{\left[(1-\pi)+(1-\theta_B-\epsilon-\delta)\gamma\right]}{\left[(1-\pi)+(1-\theta_B-\delta)\gamma\right]}\right)^j \\
&= \frac{\mu}{(1-\mu)} (A)^r (B)^j
\end{aligned}$$

When $\gamma = 0 => \pi_P = \pi_F, \epsilon > 0$

$$M_G = \frac{\mu}{(1-\mu)} \left(\frac{(\theta_B+\epsilon+\delta)}{(\theta_B+\delta)}\right)^r (1)^j$$

hence

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} = 0$$

In the case where $\gamma > 0 => \pi_P > \pi_F$ and $\epsilon > 0$ we have that $A > 1, B < 1$ hence

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} < 0$$

and when $\gamma < 0 => \pi_P < \pi_F$ and $\epsilon > 0$ we have that $A > 1, B > 1$ hence[19]

$$\frac{dM_G}{d(r)} > 0, \frac{dM_G}{d(j)} > 0$$

■

Proof of Lemma (6). Lemma (5) has direct implications for the bonus payments since

$$\begin{aligned}
\Pr(bonus|tenure &= j, rank = r) = P_G(\theta_G+\delta) + P_B(\theta_B+\delta) \\
&= P_G(\theta_G+\delta) + (1-P_G)(\theta_B+\delta) \\
&= P_G(\theta_G+\delta) + (1-P_G)(\theta_G-\epsilon+\delta) \\
&= \theta_G+\delta-\epsilon(1-P_G)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{d\Pr(bonus)}{d(j)} &= \epsilon\frac{dP_G}{d(j)} \\
\frac{d\Pr(bonus)}{d(r)} &= \epsilon\frac{dP_G}{d(r)}
\end{aligned}$$

[19] In general $A > 0$ for all feasible values of γ if $\epsilon > 0$.

using

$$\frac{dM_G(x)}{dx} = \frac{d}{dx}\frac{P_G(x)}{1-P_G(x)} > 0 => \frac{dP_G(x)}{dx} > 0,$$

and

$$\frac{dM_G(x)}{dx} = \frac{d}{dx}\frac{P_G(x)}{1-P_G(x)} < 0 => \frac{dP_G(x)}{dx} < 0$$

the lemma arrises. ■

Proof of Corollary (1). Follows directly from Lemma (6). ■

Proof of Corollary (2). Follows directly from Lemma (6). ■

Proof of Lemma (7). Note that as bonus, firing and promotion variables are history independent, the proofs of lemmas (5) and (6) are directly applicable. The proof on wage profile follows trivially from equations (7) and (8) and observing that a past bonus payment (as opposed to the lack of it), always increases the probability that the employee is of good type. Of course, the results are valid, even if the terms in equation (8) are not differentiable. ■

Proof of Lemma (8). The first statement follows trivially from the education bias. The second statement follows from understanding that education signals higher ability to all potential employees. ■

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