

Monetary Policy and Financial Market Evolution

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I. INTRODUCTION

In the 1950s and 1960s, Gurley and Shaw (1955, 1960, 1967) advanced a particular view of the joint evolution of per capita income and the financial system. They observed that at low levels of development, most investment is self-financed. As per capita income rises, bilateral borrowing and lending becomes more important. With further increases in per capita income, banks and similar financial intermediaries become prominent in financing investment. Eventually, more sophisticated financial markets, such as equity markets, arise. In the Gurley and Shaw view, rising per capita income and increasing financial depth reinforce each other. Therefore, a model of the joint evolution of per capita income and the banking system must allow usage of banks to be endogenous, and the level of per capita income and usage of banks must be determined simultaneously.

Many poor countries have relatively poorly developed financial systems. Why might this be the case? One possibility is that at low levels of development, the costs of financial intermediation are too high relative to the benefits. There is ample evidence that costs of accessing the banking system are high in developing countries.¹ In developing countries, penetration of the formal banking system into rural areas is limited; the high costs to the rural poor of accessing banks, such as the costs of traveling to a town with a bank branch and foregone income, are frequently cited as a reason for low utilization rates of banks. Even in the United States, about 13 percent of families do not have a checking account, and when asked why not, about half cited high ser-

vice charges or other reasons related to the costs of banking.²

However, another possibility is that monetary policy also influences the choice between self-financed and intermediated investment. Many developing countries have relatively high nominal interest rates and relatively low measures of financial depth, as measured, for example, by the ratio of M2 to gross domestic product (GDP).³ Because the nominal interest rate represents the opportunity cost of holding currency, the relatively low rates at which banks are used in many developing countries with high nominal interest rates may seem puzzling. But banks also hold reserves of currency to provide liquidity to their depositors, and the rates of return banks offer to their depositors—the degree to which banks insure against depositors' liquidity needs—are influenced by the nominal interest rate. Therefore, a model of the joint evolution of per capita income and the banking system must also allow monetary policy to affect the benefits of financial intermediation—rates of return on deposits and to what degree banks insure depositors against the need for liquidity.

This paper considers a model in which both of these factors—the resource cost of saving through intermediaries and monetary policy, specifically the money growth rate—are important determinants of (i) whether banks are used and (ii) the level of per capita income. Our model incorporates a fixed resource cost of intermediation, similar to that in Bencivenga and Smith (1998). It also incorporates a role for monetary policy, by creating a role for government-supplied fiat money.

We consider an overlapping-generations model of capital accumulation with currency as a second primary asset. Young agents can save their wage income as currency or by investing in capital formation; or young agents can deposit their saving in banks, which hold the primary assets (currency and capital investment) on behalf of their depositors. The model generates a transactions demand for

¹ See, for example, Gine (2001), who estimates the magnitude of transactions costs of accessing banks in rural Thailand.

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² Kennickell, Starr-McCluer, and Surette (2000).

³ For example, see Levine (1996).

currency by subjecting each agent to a random shock whose realization determines whether or not the agent will be relocated across spatially separated locations. In the event of relocation, an agent needs currency to purchase consumption after relocation. Also, any investment in capital formation undertaken directly by a young agent who subsequently is relocated is lost (both to the investor and socially).

Banks insure their depositors against the risk of an adverse realization of the relocation shock, by holding currency to pay a return to depositors who end up being relocated. Also, since no capital investment undertaken by banks is ever lost, a depositor's expected return (per unit deposited) is higher than the expected return of a young agent who holds the primary assets directly. However, in this model, it is costly for agents to utilize banks; specifically, agents incur a fixed resource cost when they deposit their saving in a bank. This implies that the returns on bank deposits must be sufficiently high relative to the return on currency, and the insurance that banks provide against the need for liquidity arising from the risk of relocation must be sufficiently good, for agents to be induced to utilize banks. Agents may find it optimal to bear the fixed costs of financial intermediation, or they may find it optimal to avoid these costs by holding the primary assets directly.

In this model, the money growth rate plays an important role in agents' decisions about whether to utilize banks. The nominal interest rate represents the opportunity cost of holding currency, and for this reason, the degree of insurance optimally provided by banks against the need for liquidity falls as the nominal interest rate rises. The model simultaneously determines the capital-labor ratio, the real interest rate, per capita income, and the nominal interest rate, as well as whether or not banks will be used, as a function of the money growth rate.

The main results we obtain about the utilization of banks, and the impact of monetary policy on the utilization of banks, are as follows. Agents do not use banks—they self-insure against the risk of relocation by holding currency directly and self-finance capital formation—for low values of the nominal interest rate or (possibly, depending on parameter values) for high values of the nominal interest rate. At low nominal interest rates, capital is not much better an asset than currency; the costs of holding currency to self-insure against the risk of relocation and of losing capital in the event of relocation are relatively low, so agents avoid the fixed costs of

saving through banks. Higher money growth rates shrink the range of low nominal interest rates for which autarkic saving is optimal, but do not eliminate it. Agents may also reject the use of banks (depending on parameter values) at high nominal interest rates. The higher the nominal interest rate, the less insurance banks offer against the risk of an adverse realization of the relocation shock. For sufficiently high nominal interest rates, the value of the insurance offered by banks is less than the value of the resource cost of using banks. Higher money growth rates exacerbate this effect, in that they expand the range of high nominal interest rates for which autarkic saving is optimal. Thus, a high inflation rate can deter development of a banking system.

The remainder of the paper proceeds as follows. Sections II and III lay out the environment and trade in factor markets. Sections IV through VII derive the optimal saving behavior of agents who save autarkically, and analyze the steady-state equilibrium and stability properties of this equilibrium. In these sections it is assumed that agents do not have access to a banking system. Sections VIII through XII assume that agents incur the transactions costs associated with access to the banking system and that all saving is intermediated. The optimal behavior of banks and the laws of motion of the capital stock and nominal interest rate are derived. Existence and stability of steady-state equilibrium (or equilibria) are analyzed. The comparative static effects of an increase in the money growth rate are also analyzed. Finally, Section XIII asks when agents will find it optimal to save autarkically as opposed to utilizing banks and discusses the impact of a change in the money growth rate on that decision. Section XIV concludes.

II. THE ENVIRONMENT

We consider a discrete time model, with time indexed by $t = 1, 2, \dots$. The economy consists of an infinite sequence of two-period-lived, overlapping generations. We ignore the initial old generation. The model features two locations, or islands, across which agents are distributed. At each date a continuum of ex ante identical young agents with unit mass is born in each location. Our assumptions will guarantee that the locations are always symmetric, and the description that follows applies to each location.

In each location in each period a single final good is produced, using capital and labor as inputs

into a constant return to scale technology. Letting K_t be the aggregate time t capital stock and letting L_t be aggregate time t employment, output of the final good is $F(K_t, L_t)$. Defining $k_t \equiv K_t/L_t$ to be the time t capital-labor ratio, let $f(k_t) \equiv F(K_t/L_t, 1)$ denote the intensive production function. For simplicity, we assume that the production function has the Cobb-Douglas form $f(k) = Ak^\alpha$, with $\alpha \in (0, 1)$. In addition, we assume that capital depreciates 100 percent in the production process.

Each young agent is endowed with one unit of labor, which is supplied inelastically. Again, to attain maximum simplicity, we assume that agents derive utility from consumption only when old. Let c_t denote the second period consumption of an agent born at t . Then the agent has the lifetime utility level $u(c_t)$, where $u(c_t) = c_t^{1-\rho}/(1-\rho)$. We assume throughout that $\rho \in (0, 1)$. This assumption implies that a higher opportunity cost of holding currency induces agents to economize on their balances of currency.⁴

In order to introduce a transactions role for money, we follow Townsend (1980, 1987) and emphasize the importance of limited communication between the two locations in the economy. In particular, we assume that at each date an agent can trade only with agents who inhabit his current location and that there is no communication between islands. Communication and record keeping within any island pose no problems. However, between dates t and $t + 1$, each agent faces the probability $\pi \in (0, 1)$ that he will be relocated to the other island. When agents are relocated, they lose contact with agents in their original location. Moreover, the absence of communication between locations implies that agents in their new location do not know the asset position of relocated agents. Hence, relocated agents will require currency to purchase goods. On the other hand, agents who are not relocated can purchase goods with credit instruments; they do not require currency to make purchases. Stochastic relocation, then, is a physical story about which transactions do and do not require the use of currency.

In addition to providing a framework that requires currency to be used in some exchanges, the presence of stochastic relocation implies that agents face the risk of having to convert potentially higher-yielding assets into currency. This risk represents an analog of the liquidity preference shock in the Diamond and Dybvig (1983) model. Agents will

wish to be insured against this shock. We will describe situations under which they either self-insure or are insured by banks.

This economy has two primary assets—currency and physical capital. One unit of the final good invested at t becomes one unit of capital at $t + 1$. Capital investment cannot be transported between locations. As we have emphasized, spatial separation and limited communication imply that relocated agents require currency to consume. The economy's primary assets can be held directly by agents, or they can be held by intermediaries. We now describe the access of agents to intermediation.

We assume that each young agent can choose to deposit his saving in a bank. However, utilization of a bank is costly; saving in the form of bank deposits involves a fixed cost of $\phi > 0$ units of the final good. In other words, resources are “used up” (lost to society) each time an agent saves through a bank.⁵ Agents who use a bank will deposit all of their saving (net of the fixed costs incurred in accessing banks). Then, once agents' relocation shocks are realized, agents who must relocate contact their banks again (in a decentralized manner) and withdraw cash that is taken to the agents' new location. Agents who are not relocated will not require cash to purchase the consumption good; they can use checks, credit cards, or other credit instruments. There is, of course, an alternative to using intermediaries. Agents can hold the economy's primary assets directly, thereby avoiding the incurrence of the fixed cost, ϕ . However, avoiding the fixed cost requires that an agent invest autarkically, which prevents the sharing of relocation risk. We describe below the circumstances under which agents will and will not choose to use banks.

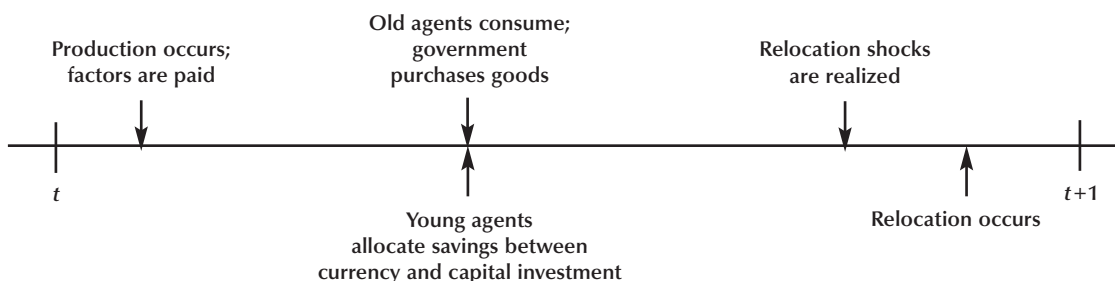
As is typical in models of spatial separation and limited communication, the timing of events within a period is of considerable importance. In this economy, the timing of events is as follows. At the beginning of a period, production of the final good occurs and factors are paid. Each young agent supplies his labor inelastically, earning the prevailing real wage. All of this wage income is saved. At this point, each young agent decides whether to hold the primary assets directly (i.e., to allocate his saving between holdings of currency and the investment technology) or to save in the form of bank deposits. If saving is

⁴ See Schreft and Smith (1998).

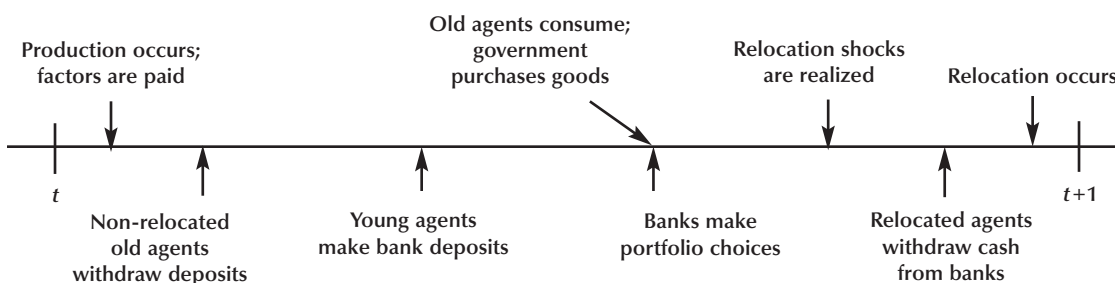
⁵ The notion that utilizing a bank involves a fixed cost has a long tradition in macroeconomics, a tradition that is reflected, for instance, in Baumol (1952) and Tobin (1956).

Figure 1

A. Timing of Economic Activity within a Period (Autarky)



B. Timing of Economic Activity within a Period (Banks)



intermediated, banks make their portfolio decisions. The goods market clears, with output of the final good going to investment in capital formation, government purchases (described below), and consumption on the part of old agents. Later in the same period, each young agent learns the realization of his relocation shock. By assumption, young agents do not meet after their relocation status is realized (unless saving is intermediated, in which case they may contact their banks). Therefore, relocated agents who save and invest autarkically lose the value of their investment in the technology to produce physical capital, because capital investment cannot be transported between locations. On the other hand, if saving is intermediated, relocated agents return to their banks and withdraw currency. Relocated agents carry currency to their new location, using it to purchase consumption when old. Agents who are not relocated take no action until the beginning of the next period, at which point they consume the gross return on their asset holdings (currency and capital, if saving and investment is autarkic, or bank deposits, if saving is intermediated). The timing

of events is depicted in Figure 1.⁶ Figure 1A describes the case of autarkic saving, and Figure 1B the case of intermediated saving.

In addition to the old and young agents and (potentially) banks in each location, this economy has a government. The government prints money and purchases the final good. Let M_t be the nominal money supply, per young agent, at t . M_t evolves according to $M_{t+1} = \sigma M_t$, with σ being chosen once and for all at the beginning of time. If p_t denotes the time t price level, then seigniorage revenue of the government at t is

$$\left(\frac{\sigma - 1}{\sigma}\right) \frac{M_t}{p_t}$$

We assume that the government uses this revenue to purchase the final good. Government purchases of the final good do not affect agents' saving behavior or portfolio allocations. Note that this way of inject-

⁶ Notice that we have compressed all the events against which agents want to be insured into one period. This convention follows Champ, Smith, and Williamson (1996) and Schreft and Smith (1997, 1998). It implies that there are no opportunities for intergenerational interactions through the banking system.

ing money restricts us to the case $\sigma \geq 1$; that is, there can be no contraction of the money supply. This restriction is not particularly important here. As we will see, with this restriction certain commonly considered contractionary policies, such as the Friedman rule, are infeasible in a nontrivial equilibrium of this model.

III. TRADE IN FACTOR MARKETS

At the beginning of period t , firms hire labor and rent capital. These trades take place in competitive factor markets in each location. Let w_t denote the time t real wage rate, and let r_t denote the time t capital rental rate. Then the following standard factor pricing relationships obtain in each location:

$$(1) \quad r_t = f'(k_t) = \alpha A k_t^{\alpha-1}$$

$$(2) \quad w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) = (1 - \alpha) A k_t^\alpha.$$

IV. BEHAVIOR OF AGENTS WHO SAVE AUTARKICALLY

From this point, we will proceed by describing (i) how agents behave if they save by holding the primary assets of the economy directly and (ii) how agents and banks behave if agents save in the form of bank deposits. Then we will discuss conditions under which young agents will choose to incur the fixed cost necessary for saving to be intermediated. We begin with the behavior of agents who save autarkically.

A young agent at t earns the real wage w_t , all of which is saved. If he saves autarkically, he divides his saving between holdings of currency and investment in physical capital. Let γ_{at} denote the fraction of saving held in the form of real money balances by a young agent at t ; then $1 - \gamma_{at}$ is the fraction held in the form of capital investment. The gross real return on holdings of currency between t and $t + 1$ is p_t/p_{t+1} , and the gross real return on capital investment held from t to $t + 1$ is r_{t+1} (since capital depreciates 100 percent). Agents behave competitively in asset markets, taking these returns as unaffected by their own saving behavior. Lifetime expected utility of a young agent is given by the expression

$$\pi u \left[\gamma_{at} \left(\frac{p_t}{p_{t+1}} \right) \right] + (1 - \pi) u \left[\gamma_{at} \left(\frac{p_t}{p_{t+1}} \right) + (1 - \gamma_{at}) r_{t+1} \right],$$

which he maximizes by choice of γ_{at} . Notice that the young agent's expected utility is the probability of being relocated multiplied by utility generated by the consumption that can be purchased when old with the agent's real balances, plus the probability of not being relocated multiplied by utility generated by consumption of the proceeds of the agent's capital, as well as purchases with his real balances, when old. To emphasize, relocated agents cannot move or trade claims to their capital investment, which is simply lost.

The solution to the problem of an autarkic young agent sets

$$\gamma_{at} = \min \left\{ \frac{I_t}{I_t - 1 + \left(\frac{1 - \pi}{\pi} \right)^{1/\rho} (I_t - 1)^{1/\rho}}, 1 \right\} \equiv \gamma_a(I_t),$$

where $I_t \equiv r_{t+1}(p_{t+1}/p_t)$ is the gross nominal rate of interest. The nominal interest rate, of course, represents the opportunity cost of holding currency. Several properties of the function $\gamma_a(I_t)$ will be useful in the subsequent analysis. These properties are stated in the following lemma. Its proof appears in Appendix A.

Lemma 1.

(a) $\gamma_a(I_t) = 1$ holds for all $I_t \in \left[1, \frac{1}{1 - \pi} \right]$. $\gamma_a(I_t) < 1$ holds for all $I_t > \frac{1}{1 - \pi}$.

(b) $\lim_{I_t \rightarrow \infty} \gamma_a(I_t) = 0$.

(c) For $I_t > \frac{1}{1 - \pi}$, $\frac{I_t \gamma'_a(I_t)}{\gamma_a(I_t)}$ satisfies

$$\frac{I_t \gamma'_a(I_t)}{\gamma_a(I_t)} = 1 - \gamma_a(I_t) \left[1 + \left(\frac{1}{\rho} \right) \left(\frac{1 - \pi}{\pi} \right)^{1/\rho} (I_t - 1)^{(1-\rho)/\rho} \right] < 0.$$

V. GENERAL EQUILIBRIUM WITH AUTARKIC SAVING

Young agents at t earn the real wage $w(k_t)$, all of which is saved. The fraction $\gamma_a(I_t)$ of their saving is held in the form of real balances, and the fraction $1 - \gamma_a(I_t)$ is held as capital investment. Hence, if $m_t \equiv (M_t/p_t)$ denotes the outstanding per capita supply of real balances at t , the money market clears if

$$(3) \quad m_t = \gamma_a(I_t) w(k_t).$$

In addition, the time $t + 1$ per capita capital stock, k_{t+1} , is given by

$$(4) \quad k_{t+1} = (1 - \pi) [1 - \gamma_a(I_t)] w(k_t).$$

Equation (4) obtains because a fraction $1 - \gamma_a(I_t)$ of saving is invested in capital formation and the fraction π of capital investment is lost due to relocation of some agents.⁷

The gross nominal rate of interest at t is given by

$$(5) \quad I_t = r_{t+1} \left(\frac{p_{t+1}}{p_t} \right) = r_{t+1} \sigma \left(\frac{m_t}{m_{t+1}} \right).$$

Substituting (1) and (3) into (5) yields the equilibrium law of motion for I_t when agents save autarkically:

$$I_t = \sigma f'(k_{t+1}) \frac{\gamma_a(I_t) w(k_t)}{\gamma_a(I_{t+1}) w(k_{t+1})} = \frac{\sigma k_{t+1} f'(k_{t+1}) \gamma_a(I_t) w(k_t)}{w(k_{t+1}) \gamma_a(I_{t+1}) k_{t+1}}.$$

Upon substituting (4) into this law of motion, we obtain

$$(6) \quad \gamma_a(I_{t+1}) = \frac{\sigma \alpha}{(1 - \alpha)(1 - \pi)} \left\{ \left(\frac{1}{I_t} \right) \left[\frac{\gamma_a(I_t)}{1 - \gamma_a(I_t)} \right] \right\}.$$

VI. STEADY STATE UNDER AUTARKIC SAVING

Imposing $I_t = I_{t+1}$ in equation (6) yields the following steady-state equilibrium condition:

$$(7) \quad I [1 - \gamma_a(I)] = \left(\frac{\sigma}{1 - \pi} \right) \left(\frac{\alpha}{1 - \alpha} \right).$$

Lemma 1 implies that (7) has a unique solution with $I > (1/1 - \pi)$. Clearly, the steady-state value of the gross nominal rate of interest is an increasing function of the money growth rate. It is also straightforward to show that, in a steady state,

$$\frac{\partial(I/\sigma)}{\partial \sigma} = \frac{1}{\sigma} \left[\frac{I \gamma'_a(I)}{1 - \gamma_a(I)} \right] < 0.$$

Therefore, as the money growth rate increases, the

steady-state real interest rate, $I/\sigma = f'(k)$, declines. To sum up, when agents save autarkically, a higher rate of money creation leads to a higher steady-state nominal rate of interest, a lower real rate of interest, and a higher steady-state capital stock. Intuitively, this occurs because a higher nominal rate of interest implies a higher opportunity cost of holding currency. As a result, young agents substitute away from real balances and into capital investment. This leads to a higher steady-state capital stock and clearly constitutes a version of the Mundell-Tobin effect.

Notice that it is impossible for this economy to have a nontrivial equilibrium under the Friedman rule (that is, setting $I_t = 1$). This is because the Friedman rule makes currency such a good asset that agents will hold it to the exclusion of any other, and no capital investment will ever occur.⁸

Example: Assume the following parameter values: $\phi = 0.1$, $\alpha = 0.35$, $\rho = 0.95$, $\sigma = 1.05$, $A = 1$, and $\pi = 0.3$. The (gross) nominal interest rate in an autarkic steady state is 1.97, and γ_a , the share of saving held as currency, is 0.59.

VII. DYNAMICS UNDER AUTARKIC SAVING

The equilibrium law of motion for I_t given in equation (6) is depicted in Figure 2. It is easy to show that (6) gives I_{t+1} as an increasing function of I_t and that the steady state is unstable. This means that the steady state is the unique equilibrium.

VIII. AN ECONOMY WITH INTERMEDIATED SAVING

We now turn our attention to an economy where saving is intermediated. For the present, we simply assume that all saving is intermediated. Later, we provide conditions under which this is the optimal choice for young agents.

When young agents save through banks, they incur the fixed transactions cost, ϕ . They then deposit all remaining saving (each young agent deposits $w_t - \phi$ at date t) in banks.⁹ Banks promise a gross real return of d_t^m to young agents who are relocated between t and $t + 1$, and a gross real return of d_t to those who are not, per unit of the final good

⁷ It is not essential to our analysis that all capital held by relocated agents is abandoned. The model easily could be modified so that ownership of capital held by relocated agents is transferred to non-relocated agents in a post-relocation shock asset market where some transactions costs are incurred. The term $1 - \pi$ in equation (4) would then be replaced by a term that reflects the transactions costs associated with transferring ownership of capital investment. This change would have little effect on our analysis. The central point is that ownership of capital investment does not need to be transferred between agents when capital investment is intermediated.

⁸ See Smith (2002), who makes a similar point in a model without capital accumulation. In his economy it is feasible to follow the Friedman rule in a nontrivial equilibrium, but it is not optimal to do so.

⁹ As in Diamond and Dybvig (1983), all savings will be deposited in banks if agents strictly prefer intermediated to autarkic savings.

deposited at t . Banks allocate deposits between reserves of currency and investment in capital, prior to realization of agents' relocation shocks. After banks allocate their portfolios, agents who must relocate contact their banks in a decentralized manner and withdraw their deposits, with interest, in the form of currency. Banks must give these agents adequate quantities of currency for them to be able to consume at the promised level in their new location at $t + 1$. Agents who are not relocated can make purchases with checks or other credit instruments at $t + 1$. Let γ_{bt} denote the fraction of a bank's assets that are held in the form of currency and $1 - \gamma_{bt}$ denote the fraction held as investment in capital.

As in Diamond and Dybvig (1983), a bank can be thought of as a coalition of ex ante identical young agents. A bank will then choose real rates of return on deposits and a reserve-to-deposit ratio (γ_{bt}) to maximize the expected utility of a representative depositor, subject to the following constraints. First, young agents who must relocate must be given enough currency to deliver the promised gross real return, d_t^m , between t and $t + 1$. Since the gross real return on currency carried between t and $t + 1$ is p_t/p_{t+1} , this constraint requires that

$$(8) \quad \pi d_t^m \leq \gamma_{bt} (p_t/p_{t+1}).$$

If currency is dominated in rate of return,¹⁰ then agents who remain in their original location will be paid out of the returns on the bank's investment in capital. This return is simply the capital rental rate in $t + 1$ (since capital depreciates 100 percent). Therefore, the second constraint requires that

$$(9) \quad (1 - \pi) d_t \leq (1 - \gamma_{bt}) r_{t+1}.$$

Banks take the gross real returns on the primary assets, p_t/p_{t+1} and r_{t+1} , as given. Then a bank at t chooses d_t^m , d_t , and γ_{bt} to maximize the expected utility of a representative depositor

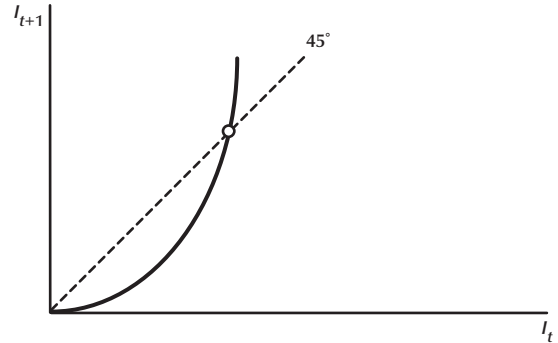
$$\frac{(w_t - \phi)^{1-\rho}}{1-\rho} \left[\pi (d_t^m)^{1-\rho} + (1-\pi)(d_t)^{1-\rho} \right],$$

subject to the constraints (8) and (9) and non-negativity.

The optimal reserve-to-deposit ratio of a bank at t is given by

Figure 2

Steady State in Autarky



$$(10) \quad \gamma_{bt} = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right) I_t^{(1-\rho)/\rho}} \equiv \gamma_b(I_t).$$

Using (10), a bank's optimal deposit return schedule can be recovered from (8) and (9). It is easy to show that $d_t = I_t^{1/\rho} d_t^m$ holds at an optimum. With positive nominal rates of interest ($I_t > 1$), banks do not provide complete insurance against the risk of relocation. This is because they must hold currency to provide insurance against the risk of relocation, and holding currency involves an opportunity cost that is reflected in the nominal rate of interest. As this opportunity cost rises, banks provide less insurance.

Various properties of the function $\gamma_b(I)$ will be important in the analysis that follows. We now state these properties.¹¹

Lemma 2.

(a) $\gamma_b(I) = \pi.$

(b) $\lim_{I \rightarrow \infty} \gamma_b(I) = 0.$

(c) $\frac{I \gamma_b'(I)}{\gamma_b(I)} = -\left(\frac{1-\rho}{\rho}\right) [1 - \gamma_b(I)] < 0.$

Notice that, when saving is intermediated, setting the gross nominal interest rate equal to unity does not induce agents to save exclusively in the form of real balances. This contrasts with the situation of autarkic saving. When saving is intermediated, withdrawal demand is completely predictable, so there is

¹⁰ As we have noted, currency will be dominated in rate of return ($I_t > 1$ will hold) in any nontrivial equilibrium. When currency is dominated in rate of return, banks will not carry reserves of currency between periods.

¹¹ For a proof of lemma 2, see Schreft and Smith (1998).

no reason for banks to hold precautionary reserves. For autarkic savers, only currency can provide complete insurance against relocation risk, and agents will hold only currency if $I_t = 1$. As we will see, if $I_t = 1$, autarkic saving will be optimal, but part (a) of lemma 2 will prove useful nonetheless. Finally, part (c) of the lemma indicates that, with $\rho < 1$, an increase in the nominal rate of interest induces banks to economize on their holdings of reserves. This is clearly the intuitively appealing (and empirically supported) case.¹²

IX. GENERAL EQUILIBRIUM WITH INTERMEDIATED SAVING

When all saving is intermediated, all beginning-of-period demand for currency derives from banks. Each young agent deposits his saving, net of the transactions cost $(w_t - \phi)$, and each bank holds the fraction $\gamma_b(I_t)$ of deposits in the form of reserves of currency. Hence, at date t , the money market clears if

$$(11) \quad m_t = \gamma_b(I_t)[w(k_t) - \phi].$$

Banks invest $1 - \gamma_{bt}$ of deposits in capital formation. In contrast to the situation of autarkic saving, capital investment is not lost when agents relocate (since capital investment is undertaken by banks). Therefore, the per capita capital stock evolves according to

$$(12) \quad k_{t+1} = [1 - \gamma_b(I_t)][w(k_t) - \phi].$$

The condition that determines the evolution of the gross nominal rate of interest remains to be stated. By definition,

$$(13) \quad I_t = r_{t+1} \left(\frac{p_{t+1}}{p_t} \right) = r_{t+1} \sigma \left(\frac{m_t}{m_{t+1}} \right).$$

Substituting (1) and (11) into (13) yields

$$(14) \quad I_t = \frac{\sigma f'(k_{t+1}) \gamma_b(I_t) [w(k_t) - \phi]}{\gamma_b(I_{t+1}) [w(k_{t+1}) - \phi]}$$

or, upon rearranging terms,

$$(15) \quad \gamma_b(I_{t+1}) = \frac{\sigma f'(k_{t+1}) \gamma_b(I_t) [w(k_t) - \phi]}{I_t [w(k_{t+1}) - \phi]} \equiv G(k_{t+1}, k_t, I_t).$$

Equations (12) and (15) constitute the equilibrium laws of motion for k_t and I_t . We begin with a consideration of steady states.

X. STEADY-STATE EQUILIBRIA UNDER INTERMEDIATED SAVING

Imposing $I_t = I_{t+1} = I$ and $k_{t+1} = k_t = k$ in equation (14) yields one of the steady-state equilibrium conditions:

$$(16) \quad I = \sigma f'(k).$$

Under our assumption of Cobb-Douglas production, equation (16) implies that

$$(17) \quad k = (\sigma \alpha A / I)^{1/(1-\alpha)}$$

and, consequently, that $w(k) - \phi = (1 - \alpha) A (\sigma \alpha A / I)^{\alpha/(1-\alpha)} - \phi$. Substituting this condition into (12), rearranging terms, and making use of (17) gives a condition that determines the steady-state value(s) of I :

$$(18) \quad \frac{1}{1 - \gamma_b(I)} = \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{I}{\sigma} \right) - \mu \left(\frac{I}{\sigma} \right)^{1/(1-\alpha)} \equiv H(I),$$

with $\mu \equiv \phi(\alpha A)^{1/(\alpha-1)}$. Some properties of the function $H(I)$ are stated in the following lemma. Its proof appears in Appendix B.

Lemma 3.

(a) $H'(I) \geq 0$ holds if and only if

$$I \leq \sigma \left[\frac{(1 - \alpha)^2}{\alpha \mu} \right]^{(1-\alpha)/\alpha} \equiv \hat{I}.$$

(b) $H(I) \geq 0$ holds if and only if $\left(\frac{1 - \alpha}{\alpha} \right)^{(1-\alpha)/\alpha} \geq I$.

(c) $H(I)$ is a concave function of I .

Lemma 3 implies that there are three possibilities concerning the existence of steady-state equilibria with intermediated saving. These are depicted in Figure 3.

Case 1. If $H(1) < \frac{1}{1 - \pi}$, $\hat{I} > 1$, and $H(\hat{I}) > 1 - \gamma_b(\hat{I})$

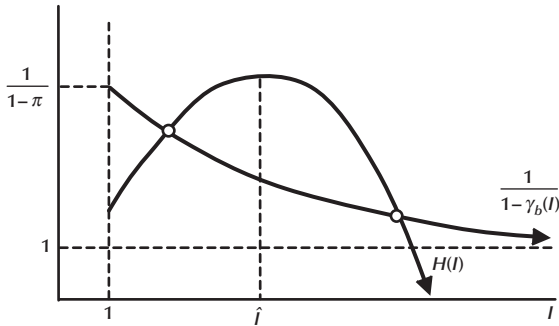
hold, we have the situation depicted in Figure 3A. Since $H(I)$ is concave, and since it is easy to verify

that $\frac{1}{1 - \gamma_b(I)}$ is a convex function of I , equation (18)

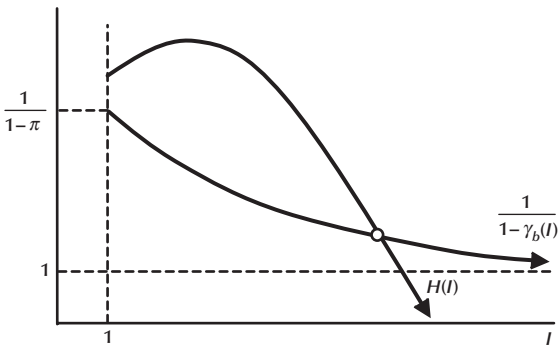
¹² Goldfeld (1966) reports an interest elasticity of excess reserves (all reserves here are excess reserves) of -0.3 . Schreft and Smith (2002) report that this figure continues to be widely used in policy circles.

Figure 3

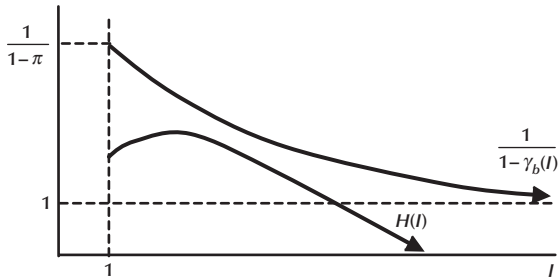
A. Two Steady States with Banks



B. One Steady State with Banks



C. No Steady State with Banks

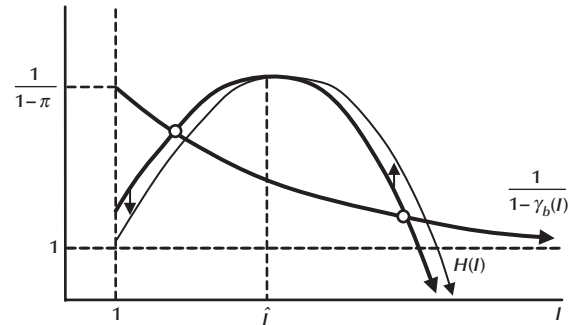


has two solutions. These are the candidate steady states when all saving is intermediated.

Case 2. If $H(1) > \frac{1}{1-\pi}$, we have the situation depicted in Figure 3B. Equation (18) has only one solution with $I > 1$, which is the candidate steady-state equilibrium.

Figure 4

Effect of an Increase in σ on Steady States with Banks



Case 3. It is possible that equation (18) has no solutions. This can happen if the fixed cost of contacting a bank is too large, for example. In this case, depicted in Figure 3C, there are no steady-state equilibria where all saving is intermediated.

XI. COMPARATIVE STATICS OF A CHANGE IN MONETARY POLICY

We now indicate how a change in the money growth rate, σ , affects the nominal rate of interest and the capital stock in a steady-state equilibrium with intermediation. We focus on case 1; from that it will be apparent what effects a change in σ has in case 2.

Figure 4 depicts the consequences of an increase in σ , which shifts the function $H(I)$ down (up) if $H'(I) > (<) 0$. Notice that, in each candidate steady state, an increase in the rate of money creation has the effect of increasing I , the gross nominal rate of interest. The consequences of higher money growth rates for the steady-state capital stock, however, depend on which of the two steady states obtains. The relevant result is reported in proposition 1, which is proved in Appendix C.

Proposition 1. $\partial k / \partial \sigma > (<) 0$ holds in the steady state with the low (high) nominal interest rate.

Proposition 1 says that at the steady state where $H'(I) > (<) 0$, an increase in the rate of money creation raises (lowers) the steady-state capital stock (as well as steady-state output). Thus, the long-run real effects of a higher rate of money growth depend on which steady state obtains.

Of course, in a case 2 economy, there is a single steady state with $H'(I) < 0$. In a case 2 economy, a higher rate of money creation (a higher steady-state rate of inflation) reduces the per capita capital stock and per capita output.

XII. DYNAMICS WITH INTERMEDIATED SAVING

The dynamic system governing the evolution of $\{k_t, I_t\}$ consists of equations (12) and (15). In this section, we analyze local dynamics in a neighborhood of a steady-state equilibrium. The stability properties of a steady-state equilibrium depend on the number of steady states and their configuration. For this reason, we consider case 1 and case 2 economies separately.

Case 1. Here there are two candidate steady-state equilibria. Appendix D establishes the following result.

Proposition 2. In a case 1 economy, the low (high) nominal interest rate steady state is a saddle (source).

Because it is a source, the high nominal interest rate steady state cannot be approached from any nearby point. There is a unique path converging to the low nominal interest rate steady state, which is a saddle. In addition, it is easy to verify that dynamics in a neighborhood of the low nominal interest rate steady state are monotone.

Case 2. In a case 2 economy, there is a unique steady state. In Appendix E we prove the following claim.

Proposition 3. The steady state is a source.

It is therefore unclear what happens in a case 2 economy asymptotically, even if initial conditions put the economy in a neighborhood of the steady state. An analysis of global dynamics would be necessary; however, such an analysis is beyond the scope of the present paper.

Example: The parameter values are the same as in the example in Section VI. With intermediation of saving, these parameter values produce a case 1 economy. The two steady-state nominal interest rates are 1.12039 and 10.2615. For both steady states, the Jacobian matrix of partial derivatives of (12) and (15), evaluated at the steady state, has real eigenvalues. The low nominal interest rate steady state is a saddle, and the high nominal interest rate steady state is a source.

XIII. WHEN IS SAVING INTERMEDIATED?

To this point we have imposed either that agents save autarkically, or that agents' saving is intermediated, and we examined the potential equilibria emerging in each case. In this section we turn our attention to conditions under which agents will find it optimal to incur the fixed cost associated with intermediated saving.

We begin by considering the lifetime expected utility of an agent who saves autarkically. This (maximized) utility level is given by the expression

$$\begin{aligned} V_a(w_t, I_t) &\equiv \left. \frac{w_t^{1-\rho}}{1-\rho} \left\{ \pi \left[\left(\frac{p_t}{p_{t+1}} \right) \gamma_a(I_t) \right]^{1-\rho} \right. \right. \\ &\quad \left. \left. + (1-\pi) \left[\left(\frac{p_t}{p_{t+1}} \right) \gamma_a(I_t) + r_{t+1}(1-\gamma_a(I_t)) \right]^{1-\rho} \right\} \right. \\ &= \frac{w_t^{1-\rho}}{(1-\rho)} \left(\frac{p_t}{p_{t+1}} \right)^{1-\rho} \\ &\quad \left. \gamma_a(I_t)^{1-\rho} \left\{ \frac{\pi + (1-\pi)}{\left[\frac{\gamma_a(I_t) + I_t(1-\gamma_a(I_t))}{\gamma_a(I_t)} \right]^{1-\rho}} \right\} \right. \end{aligned}$$

It is easily verified that

$$\begin{aligned} &\pi + (1-\pi) \left[\frac{\gamma_a(I_t) + I_t(1-\gamma_a(I_t))}{\gamma_a(I_t)} \right]^{1-\rho} \\ &= \pi \left[\frac{1}{\gamma_a(I_t)} \right] \left(\frac{I_t}{I_t - 1} \right). \end{aligned}$$

Therefore,

$$V_a(w_t, I_t) = \pi \left(\frac{w_t^{1-\rho}}{1-\rho} \right) \left(\frac{p_t}{p_{t+1}} \right)^{1-\rho} \left[\frac{1}{\gamma_a(I_t)} \right]^\rho \left(\frac{I_t}{I_t - 1} \right).$$

The (maximized) lifetime expected utility of agents whose saving is intermediated is given by

$$\begin{aligned} V_b(w_t, I_t) &\equiv \left. \left[\frac{(w_t - \phi)^{1-\rho}}{1-\rho} \right] \left\{ \pi \left[\left(\frac{p_t}{p_{t+1}} \right) \frac{\gamma_b(I_t)}{\pi} \right]^{1-\rho} \right. \right. \\ &\quad \left. \left. + (1-\pi) \left[\frac{r_{t+1}(1-\gamma_b(I_t))}{1-\pi} \right]^{1-\rho} \right\} \right. \end{aligned}$$

$$\begin{aligned} &= \left[\frac{(w_t - \phi)^{1-\rho}}{1-\rho} \right] \left(\frac{p_t}{p_{t+1}} \right)^{1-\rho} \left[\frac{\gamma_b(I_t)}{\pi} \right]^{1-\rho} \left[\pi + (1-\pi)I_t^{(1-\rho)/\rho} \right] \\ &= \pi^\rho \left[\frac{(w_t - \phi)^{1-\rho}}{1-\rho} \right] \left(\frac{p_t}{p_{t+1}} \right)^{1-\rho} \left[\frac{1}{\gamma_b(I_t)} \right]^\rho \end{aligned}$$

Agents will save autarkically at date t if and only if

$$(19) \quad V_a(w_t, I_t) \geq V_b(w_t, I_t).$$

Based on our previous observations, the condition (19) is equivalent to

$$(20) \quad \frac{w_t - \phi}{w_t} \leq \pi \left[\frac{\gamma_b(I_t)}{\gamma_a(I_t)} \right]^{\rho/(1-\rho)} \left(\frac{I_t}{I_t - 1} \right)^{1/(1-\rho)} \equiv \pi Q(I_t)^{1/(1-\rho)}.$$

If the autarkic candidate equilibrium real wage and nominal rate of interest at t satisfy (20), then there exists an equilibrium in which agents save autarkically at t .

Clearly the properties of the function $Q(I)$ will be important in the subsequent analysis. These properties are stated in the following lemma, whose proof appears in Appendix F.

Lemma 4.

(a) $\lim_{I \downarrow 1} Q(I) = \infty$.

(b) $\lim_{I \rightarrow \infty} Q(I) = \left[\frac{1-\pi}{\pi} \right]^{1-\rho}$.

(c) $\frac{IQ'(I)}{Q(I)} = (1-\rho)[\gamma_b(I) - \gamma_a(I_t)]$.

(d) $Q'(I) \leq 0$ holds for all $I \in \left(1, \frac{1}{1-\pi} \right)$. For $I > \frac{1}{1-\pi}$,

$Q'(I) < 0$ holds if $\pi \geq 1/2$. If $\pi < 1/2$, then $Q'(I) \leq (>)$ 0 if and only if $I \leq (>) \bar{I}$, where \bar{I} is the unique solution to

$$(21) \quad 1 + \left(\frac{1-\pi}{\pi} \right) I^{1/\rho} = \left(\frac{1-\pi}{\pi} \right)^{1/\rho} (I-1)^{1/\rho}$$

and $\bar{I} > \frac{1}{1-\pi}$ holds.

Equation (20) is equivalent to

$$(22) \quad 1 \leq \left[\frac{\phi}{(1-\alpha)A(\alpha A)^{\alpha/(1-\alpha)}} \right] \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} + \pi Q(I)^{1/(1-\rho)}.$$

In autarky

$$\frac{I}{\sigma} = \left[\frac{\alpha}{(1-\alpha)(1-\pi)} \right] \left[\frac{1}{1-\gamma_a(I)} \right];$$

and therefore, in autarky, (22) becomes

$$(23) \quad 1 \leq \Phi [1-\gamma_a(I)]^{\alpha/(\alpha-1)} + \pi Q(I)^{1/(1-\rho)} \equiv T(I),$$

where $\Phi \equiv \frac{\phi}{[(1-\alpha)A]^{1/(1-\alpha)}(1-\pi)^{\alpha/(1-\alpha)}}$.

The relevant properties of the function $T(I)$ are summarized in the following lemma, which is proved in Appendix G.

Lemma 5.

(a) $\lim_{I \downarrow 1} T(I) = \infty$.

(b) $\lim_{I \rightarrow \infty} T(I) = \Phi + (1-\pi)$.

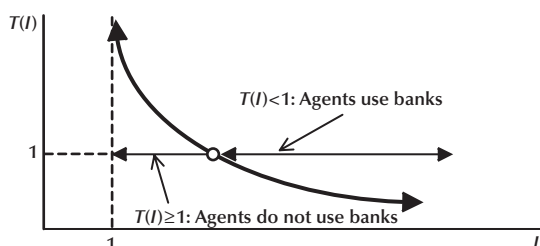
(c) If $Q'(I) \leq 0$, $T'(I) < 0$ holds.

(d) $T(I) = 1$ holds at (at most) two points.

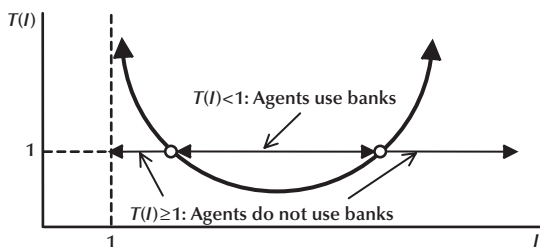
Lemma 5 implies that the locus of nominal interest rates for which agents prefer autarky, described by (23), has the possible configurations depicted in Figure 5. In Figure 5A, (23) holds only for low values of the nominal interest rate. The intuition behind Figure 5A is straightforward; when the nominal interest rate is low, currency is a relatively good asset and the resource cost to an agent of saving through a bank exceeds the value of the insurance against an adverse realization of the relocation shock provided by the bank. In Figure 5B, (23) holds for two disjoint ranges of values of the nominal interest rate, implying that agents save autarkically for either low or high values of the nominal interest rate. The intuition behind the range of low values for which (23) holds is the same as in Figure 5A. The intuition behind the possibility that (23) holds for a range of high values of the nominal interest rate is as follows. As the nominal interest rate increases, banks provide less insurance against an adverse realization of the relocation shock. (Recall that banks

Figure 5

A. Agents Choose Autarky at a Low Nominal Interest Rate



B. Agents Choose Autarky at a Low or a High Nominal Interest Rate



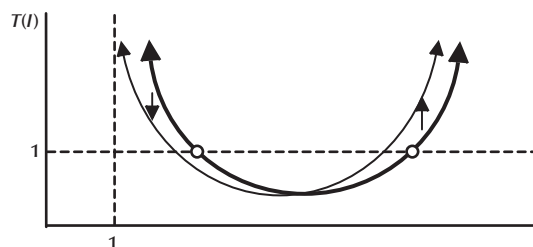
optimally offer $d = I^{1/\rho} d^m$, implying that the wedge between the rates of return offered by banks to non-relocated and relocated agents increases as I increases.) For parameter values leading to the configuration of $T(I)$ in Figure 5B, there is some value of I above which the insurance provided by banks deteriorates to the point where an agent values this insurance less than the agent values the resources it would cost to access a bank.

Note that the rate of money growth, σ , is held constant along $T(I)$; along $T(I)$, the steady-state nominal interest rate changes due to an underlying change in the capital-labor ratio, with (i) low capital-labor ratios corresponding to high values of the marginal product of capital and (ii) high nominal interest rates (given the rate of money growth). In Figure 5B, the upper range of values of I for which a steady state will be autarkic corresponds to low capital-labor ratios and low wages. For a steady state in this range, the insurance against relocation provided by banks is relatively poor, while the fixed cost of accessing banks is high relative to the wage; as a result, agents optimally engage in autarkic saving.

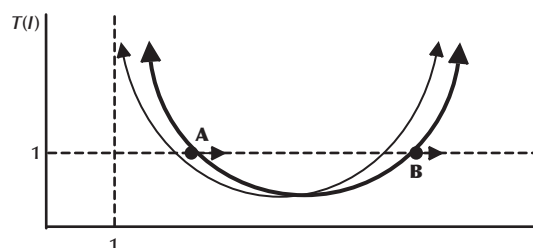
How does monetary policy—a change in the money growth rate—affect the choice between

Figure 6

A. Effect of an Increase in σ on $T(I)$



B. An Increase in σ May Change Whether Intermediation Is Optimal



autarkic saving and saving through banks? Here, too, the range of high values of the nominal interest rate for which autarky will be chosen in Figure 5B generates an interesting possibility. To see this, we must establish the comparative static effects of an increase in the money growth rate on $T(I)$, which are summarized in the following proposition and illustrated in Figure 6A.

Proposition 4. In response to an increase in σ , $T(I)$ shifts down (up) if $T'(I) < (>) 0$ at $T(I) = 1$. (Proposition 4 is proved in Appendix H.)

In Figure 6A, the economy may exhibit an autarkic steady state if the nominal interest rate is either low or high. Starting from an autarkic steady state, an increase in the money growth rate will raise the nominal interest rate (despite lowering the steady-state marginal product of capital). Proposition 4 implies the following: Starting from an autarkic steady state with a low nominal interest rate and raising that rate (by increasing the money growth rate) may move the steady state into one in which saving optimally is intermediated. This possibility is illustrated in Figure 6B: Point A, which is initially an autarkic steady state, moves to the right as a result of an increase in σ , while $T(I)$ shifts to

the left. However, starting from an autarkic steady state in which the nominal interest rate is high, an increase in σ will leave the economy with an autarkic steady state; see point B in Figure 6B.

For an economy with parameter values leading to the configuration of $T(I)$ in Figure 5B, a higher money growth rate lowers the threshold for the upper range of nominal interest rates at which agents save autarkically. A higher money growth rate therefore increases the range of nominal interest rates for which banks will be unable to provide sufficiently good insurance against agents' liquidity needs to make utilization of banks worthwhile.

There are a number of historical episodes in which a sudden, substantial reduction in the inflation rate, due to fiscal reform and correspondingly reduced reliance on seigniorage, was followed by rapid development of the banking system. For example, in Argentina, the ratio of deposit money bank assets to GDP, which is a measure of the size of the banking system, increased from an average of 0.178 for 1983-91 to an average of 0.216 for 1992-97.¹³ During most of the earlier period, Argentina's money supply expanded very rapidly due to large government budget deficits that were being monetized, and the adoption of a currency board in 1991 removed any possibility of reliance on seigniorage. Similarly, Brazil's fiscal reforms of 1994 caused an increase from an average of 0.227 for 1980-93 to an average of 0.361 for 1994-97. Bolivia's fiscal reforms at the end of 1985, which ended a period of government budget deficits and rapid money growth culminating in a short but severe hyperinflation, provides a dramatic example. The ratio of deposit money bank assets to GDP increased from an average of 0.08 for 1975-85 to an average of 0.28 for 1986-97, and it experienced a strong trend during this latter period, increasing from 0.063 in 1986 to 0.497 in 1997. These episodes are consistent with the model here, in which a reduction in the money growth rate may change the steady-state equilibrium from one without banks to one with banks, by increasing the degree to which the structure of bank deposit rates offered by banks insure depositors against liquidity shocks.

XIV. CONCLUSION

In the world, we see countries with low levels of per capita income, low utilization rates of banks,

apparently high costs of utilizing banks, and, in some cases, high inflation rates and high nominal interest rates. Here, we have explored the implications of a model in which both the resource costs associated with banking and monetary policy are important factors determining whether or not banks are utilized, and in which this decision is analyzed jointly with the determination of per capita income and the nominal interest rate. Our results suggest that monetary policy exerts an important influence over both financial and real development. By altering the opportunity cost of holding currency, a change in the money growth rate affects—in quite complex ways—the relative costs and benefits of self-financed investment and self-insurance against liquidity needs, on the one hand, and financial intermediation, on the other hand.

One result that we believe to be especially interesting is the possibility that banks will not be used for high values of the nominal interest rate. As mentioned earlier, many developing countries have relatively high nominal interest rates and relatively low utilization rates of banks. Since the nominal interest rate represents the cost of holding currency, this observation seems puzzling at first glance. Our model suggests that this observation might be explained by the negative impact of high money growth rates and high inflation rates on the degree to which banks insure depositors against liquidity shocks.

There are several directions in which this analysis could be extended. One would be to investigate a more sophisticated model of the resource costs of banking. For example, the average cost of intermediation (per unit deposited) may be a decreasing function of the volume of saving through the banking system. The impact of subsidizing the costs of banking also could be studied. Another possible line of inquiry would involve the alternative ways in which the government's purchases of the final good are used and alternative methods for injecting base money into the economy.

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¹³ These data on the ratio of deposit money bank assets to GDP are from the financial structures database accompanying Beck, Demirgüç-Kunt, and Levine (2000).

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Appendix

A. PROOF OF LEMMA 1.

Parts (a) and (b) of the lemma follow from the definition of γ_a . For part (c), differentiation yields

$$\begin{aligned} \frac{I\gamma'_a(I)}{\gamma_a(I)} &= \frac{-1 + [(1-\pi)/\pi]^{1/\rho} (I-1)^{1/\rho} [1-\rho^{-1} I/(I-1)]}{I-1 + [(1-\pi)/\pi]^{1/\rho} (I-1)^{1/\rho}} \\ &= 1 - \gamma_a(I) - \gamma_a(I) \left[\frac{(1-\pi)^{1/\rho} (I-1)^{(1-\rho)/\rho}}{\rho\pi^{1/\rho}} \right]. \end{aligned}$$

B. PROOF OF LEMMA 3.

Differentiating $H(I)$ yields

$$H'(I) = \left[\left(\frac{1-\alpha}{\alpha} \right) - \left(\frac{1}{1-\alpha} \right) \mu \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} \right] \left(\frac{1}{\sigma} \right).$$

Part (a) of the lemma follows immediately. Part (b) follows immediately from the definition of $H(I)$. For part (c), we have

$$H''(I) = - \left\{ \left[\frac{\alpha}{(1-\alpha)^2} \right] \mu \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} \right\} \left(\frac{1}{\sigma I} \right) < 0.$$

C. PROOF OF PROPOSITION 1.

Differentiation of equation (18) with respect to σ yields

$$\begin{aligned} \text{(A.1)} \quad \frac{\partial I}{\partial \sigma} &\left\{ \frac{\gamma'_b(I)}{[1-\gamma_b(I)]^2} - \left(\frac{1-\alpha}{\sigma\alpha} \right) \right\} + \left[\frac{\mu}{\sigma(1-\alpha)} \right] \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} \\ &= - \left(\frac{I}{\sigma^2} \right) \left[\left(\frac{1-\alpha}{\alpha} \right) - \left(\frac{\mu}{1-\alpha} \right) \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} \right]. \end{aligned}$$

Rearranging terms in (A.1), one obtains

$$\frac{\sigma}{I} \frac{\partial I}{\partial \sigma} = \frac{\left(\frac{1-\alpha}{\alpha} \right) - \left(\frac{\mu}{1-\alpha} \right) \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)}}{\left(\frac{1-\alpha}{\alpha} \right) - \left(\frac{\mu}{1-\alpha} \right) \left(\frac{I}{\sigma} \right)^{\alpha/(1-\alpha)} - \left(\frac{\sigma\gamma'_b(I)}{(1-\gamma_b(I))^2} \right)}.$$

From part (a) of lemma 3, it is then apparent that $\frac{\sigma}{I} \frac{\partial I}{\partial \sigma} \in (0,1)$ if $H'(I) > 0$ holds and that $\frac{\sigma}{I} \frac{\partial I}{\partial \sigma} > 1$ if $H'(I) < 0$ holds.

Now note that $f'(k) = I/\sigma$, so that $f''(k)(\partial k/\partial \sigma) = \left(\frac{I}{\sigma^2} \right) \left[\left(\frac{\sigma}{I} \right) \left(\frac{\partial I}{\partial \sigma} \right) - 1 \right]$. Thus $\partial k / \partial \sigma > (<) 0$ holds if $H'(I) > (<) 0$.

Appendix cont'd

D. PROOF OF PROPOSITION 2.

We begin by linearizing equations (12) and (15) in a neighborhood of a steady state. Doing so yields the linear approximation $(k_t - k, I_t - I)' = J(k_{t-1} - k, I_{t-1} - I)'$, where k and I denote steady-state values and where J is the standard Jacobian matrix with partial derivatives evaluated at the appropriate steady state. To derive some properties of J , we begin with the following observations.

First, differentiation of equation (12) implies that

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{k w'(k)}{w(k) - \phi} = \frac{k w'(k)}{w(k)} \frac{w(k)}{w(k) - \phi} = \alpha \frac{w(k)}{w(k) - \phi}$$

and

$$\frac{I}{k} \frac{\partial k_{t+1}}{\partial I_t} = - \left[\frac{I \gamma_b'(I)}{\gamma_b(I)} \right] \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right].$$

Second, from the definition of the function $G(k_{t+1}, k_t, I_t)$,

$$(A.2) \quad \frac{k_{t+1} G_1(k_{t+1}, k_t, I_t)}{G(k_{t+1}, k_t, I_t)} = \frac{k f''(k)}{f'(k)} - \left[\frac{k w'(k)}{w(k)} \right] \left[\frac{w(k)}{w(k) - \phi} \right] = -(1 - \alpha) - \alpha \left[\frac{w(k)}{w(k) - \phi} \right],$$

$$(A.3) \quad \frac{k_t G_2(k_{t+1}, k_t, I_t)}{G(k_{t+1}, k_t, I_t)} = \left[\frac{k_t w'(k_t)}{w(k_t)} \right] \left[\frac{w(k_t)}{w(k_t) - \phi} \right] = \alpha \left[\frac{w(k_t)}{w(k_t) - \phi} \right],$$

and

$$(A.4) \quad \frac{I_t G_3(k_{t+1}, k_t, I_t)}{G(k_{t+1}, k_t, I_t)} = \frac{I_t \gamma_b'(I_t)}{\gamma_b(I_t)} - 1.$$

Third, differentiation of equation (15) yields

$$(A.5) \quad \left[\frac{I \gamma_b'(I)}{\gamma_b(I)} \right] \left[\frac{k}{I} \frac{\partial I_{t+1}}{\partial k_t} \right] = \left[\frac{k G_1(k_{t+1}, k_t, I_t)}{G(k_{t+1}, k_t, I_t)} \right] \frac{\partial k_{t+1}}{\partial k_t} + \frac{k G_2(k_{t+1}, k_t, I_t)}{G(k_{t+1}, k_t, I_t)}$$

$$= -\phi \alpha^2 \left[\frac{\phi w(k)}{(w(k) - \phi)^2} \right],$$

where the second equality follows from (A.2), (A.3), and the expression for $\partial k_{t+1} / \partial k_t$. In addition,

$$(A.6) \quad \left[\frac{I \gamma_b'(I)}{\gamma_b(I)} \right] \left[\frac{\partial I_{t+1}}{\partial I_t} \right] = \left[\frac{k G_1}{G} \right] \left[\frac{I}{k} \frac{\partial k_{t+1}}{\partial I_t} \right] + \frac{I G_3}{G}$$

$$= \frac{I \gamma_b'(I)}{\gamma_b(I)} - 1 - \left\{ 1 - \alpha + \alpha \left[\frac{w(k)}{w(k) - \phi} \right] \frac{I}{k} \frac{\partial k_{t+1}}{\partial I_t} \right\},$$

where the second equality follows from (A.2) and (A.4).

Appendix cont'd

Now let T denote the trace of J and D denote the determinant of J . From the preceding expressions it is straightforward to verify that

$$(A.7) \quad T = 1 + (1 - \alpha) \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right] - \left[\frac{\gamma_b(I)}{I \gamma'_b(I)} \right] + \alpha \left[\frac{w(k)}{w(k) - \phi} \right] \left[\frac{1}{1 - \gamma_b(I)} \right] > 0$$

and

$$(A.8) \quad D = \alpha \left[\frac{w(k)}{w(k) - \phi} \right] \left[1 - \frac{\gamma_b(I)}{I \gamma'_b(I)} \right] + \alpha \left[\frac{w(k)}{w(k) - \phi} \right] \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right] \\ = \left(\frac{\alpha}{1 - \rho} \right) \left[\frac{w(k)}{w(k) - \phi} \right] \left[\frac{1}{1 - \gamma_b(I)} \right] = \left(\frac{1 - \alpha}{1 - \rho} \right) \frac{I}{\sigma} > 0,$$

where the last equality follows from

$$(A.9) \quad \left[\frac{w(k)}{w(k) - \phi} \right] \left[\frac{1}{1 - \gamma_b(I)} \right] = \frac{w(k)}{k} = \left(\frac{1 - \alpha}{\alpha} \right) f'(k) = \left(\frac{1 - \alpha}{\alpha} \right) \frac{I}{\sigma}.$$

Equations (A.7) and (A.8) imply that either J has two positive real eigenvalues or the eigenvalues of J are complex conjugates. Also, clearly

$$(A.10) \quad T = 1 + D + (1 - \alpha) \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right] + \alpha \left[\frac{w(k)}{w(k) - \phi} \right] \left[\frac{\gamma_b(I)}{I \gamma'_b(I)} \right] - \frac{\gamma_b(I)}{I \gamma'_b(I)}.$$

We now make two observations. One is that equation (A.9) implies that

$$\left[\frac{w(k)}{w(k) - \phi} \right] = \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{I}{\sigma} \right) [1 - \gamma_b(I)].$$

The second is that, from (18),

$$\left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{I}{\sigma} \right) [1 - \gamma_b(I)] = \frac{1}{1 - \mu \left(\frac{\alpha}{1 - \alpha} \right) \left(\frac{I}{\sigma} \right)^{\alpha/(1 - \alpha)}} \equiv \frac{\left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{I}{\sigma} \right)}{H(I)}.$$

From these observations, it follows that

$$T = 1 + D + (1 - \alpha) \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right] - \left(\frac{\rho}{1 - \rho} \right) \left[\frac{1}{1 - \gamma_b(I)} \right] \left\{ (1 - \alpha) \left[\frac{I/\sigma}{H(I)} \right] - 1 \right\}.$$

It therefore follows that $T > (<) 1 + D$ holds if and only if

$$(A.11) \quad \left(\frac{1 - \rho}{\rho} \right) (1 - \alpha) \left[\frac{\gamma_b(I)}{1 - \gamma_b(I)} \right] > (<) \left[\frac{1}{1 - \gamma_b(I)} \right] \left\{ (1 - \alpha) \left[\frac{I/\sigma}{H(I)} \right] - 1 \right\} \\ = (1 - \alpha) (I/\sigma) - H(I) = -(1 - \alpha) I H'(I).$$

Appendix cont'd

We now note that

$$\frac{\partial[1-\gamma_b(I)]^{-1}}{\partial I} = -\left(\frac{1}{I}\right)\left(\frac{1-\rho}{\rho}\right)\left[\frac{\gamma_b(I)}{1-\gamma_b(I)}\right].$$

Thus, from (A.11), $T > (<) 1 + D$ holds if and only if

$$\frac{\partial[1-\gamma_b(I)]^{-1}}{\partial I} < (>) H'(I).$$

Thus $T > (<) 1 + D$ is satisfied at steady states where $H'(I) > (<) 0$. It follows that the low nominal interest rate steady state is a saddle and that dynamics in a neighborhood of it are monotone. The high nominal interest rate steady state is a source if $D > 1$ holds (Azariadis, 1993). It is straightforward to show that the condition $T < 1 + D$ is equivalent to

$$D > \frac{(1-\alpha)\gamma_b(I)}{\rho[1-\gamma_b(I)]} + \frac{1}{(1-\rho)[1-\gamma_b(I)]}.$$

Since the second term on the right exceeds 1, then, clearly, satisfaction of the condition $T < 1 + D$ implies $D > 1$. This establishes the proposition.

E. PROOF OF PROPOSITION 3.

The proposition follows immediately from the fact that the only steady state has $H'(I) < 0$ and from the observations in the proof of proposition 2.

F. PROOF OF LEMMA 4.

Part (a) of the lemma is obvious from the definition of $Q(I)$. For part (b), L'Hopital's rule implies that $\lim_{I \rightarrow \infty} [\gamma_b(I)/\gamma_a(I)]^\rho = [(1-\pi)/\pi]^{(1-\rho)/\rho}$. Part (b) of the lemma then follows from continuity.

To establish part (c), differentiate the definition of $Q(I)$ to obtain

$$\begin{aligned} \text{(A.12)} \quad \frac{IQ'(I)}{Q(I)} &= -\left(\frac{1}{I-1}\right) + \rho \left[\frac{I\gamma_b'(I)}{\gamma_b(I)} \right] - \rho \left[\frac{I\gamma_a'(I)}{\gamma_a(I)} \right] \\ &= -\left(\frac{1}{I-1}\right) - (1-\rho)[1-\gamma_b(I)] - \rho[1-\gamma_a(I)] + \gamma_a(I) \left(\frac{1-\pi}{\pi}\right)^{1/\rho} (I-1)^{(1-\rho)/\rho}, \end{aligned}$$

where the second equality follows from applying lemmas 1 and 2. Moreover,

$$\text{(A.13)} \quad \gamma_a(I) \left(\frac{1-\pi}{\pi}\right)^{1/\rho} (I-1)^{(1-\rho)/\rho} - \left(\frac{1}{I-1}\right) = 1 - \gamma_a(I)$$

holds. Substituting (A.13) into (A.12) yields the expression in part (c) of the lemma.

Clearly $Q'(I) \leq 0$ holds if and only if $\gamma_a(I) \geq \gamma_b(I)$. For $I \leq \frac{1}{1-\pi}$, $\gamma_a(I) = 1 > \gamma_b(I)$ is satisfied. For $I > \frac{1}{1-\pi}$, $\gamma_a(I) \geq \gamma_b(I)$ is easily shown to hold if and only if

$$\text{(A.14)} \quad 1 + \left(\frac{1-\pi}{\pi}\right) I^{1/\rho} \geq \left(\frac{1-\pi}{\pi}\right)^{1/\rho} (I-1)^{1/\rho}.$$

Appendix cont'd

It is straightforward to show that (A.14) holds for all $I \geq 1$ if $\pi \geq 1/2$. If $\pi < 1/2$, then it is easily shown that (A.14) has a unique solution, \bar{I} , with $\bar{I} > \frac{1}{1-\pi}$. This completes the proof.

G. PROOF OF LEMMA 5.

Part (a) of the lemma follows from part (a) of lemma 1 and part (a) of lemma 4. Part (b) follows from part (b) of lemma 1 and part (b) of lemma 4. To obtain part (c), note that

$$T'(I) = \Phi\left(\frac{\alpha}{1-\alpha}\right) [1-\gamma_a(I)]^{1/(\alpha-1)} \gamma'_a(I) + \pi\left(\frac{1}{1-\rho}\right) Q(I)^{\rho/(1-\rho)} Q'(I).$$

Since $0 \leq \gamma_a(I) \leq 1$, $\gamma'_a(I) \leq 0$, and $Q(I) \geq 0$, it is clear that $Q'(I) < 0$ implies $T'(I) < 0$.

To prove part (d) of the lemma, suppose that $T'(I) \geq 0$ holds at some value of I that satisfies (23) at equality. At that value of I

$$(A.15) \quad \left(\frac{1}{\rho}\right) \left(\frac{1-\pi}{\pi}\right)^{1/\rho} (I-1)^{(1-\rho)/\rho} \left[\frac{\gamma_a(I)}{1-\gamma_a(I)}\right] - 1 \leq \left(\frac{1-\alpha}{\alpha}\right) \left[\frac{1}{\gamma_a(I)}\right] \left[\frac{\pi Q(I)^{1/(1-\rho)}}{1-\pi Q(I)^{1/(1-\rho)}}\right] [\gamma_b(I) - \gamma_a(I)].$$

Note that if $T'(I) \geq 0$, then necessarily $Q'(I) > 0$, $\gamma_b(I) \geq \gamma_a(I)$, and $\frac{\gamma_b(I)}{\gamma_a(I)}$ is increasing in I , so the right-hand side of (A.15) is increasing in I . Also, the left-hand side of (A.15) equals

$$\left(\frac{1}{\rho}\right) \left(\frac{I}{I-1}\right) \left[\frac{\left(\frac{1-\pi}{\pi}\right)^{1/\rho} (I-1)^{1/\rho}}{\left(\frac{1-\pi}{\pi}\right) (I-1)^{1/\rho} - 1}\right] - 1,$$

which is decreasing in I .

Therefore, if $T'(I_1) \geq 0$ for some I_1 satisfying (23) with equality, then for any $I_2 > I_1$ satisfying (23) with equality, $T'(I_2) \geq 0$ also must hold. This is a contradiction. This establishes that there is at most one value of I at which $T(I) = 1$, for which $T'(I) \geq 0$. This completes the proof.

H. PROOF OF PROPOSITION 4.

Differentiation of equation (7) yields

$$\frac{I}{\sigma} + I [1-\gamma_a(I)]^{-1} \gamma'_a(I) \frac{\partial I}{\partial \sigma} - \frac{\partial I}{\partial \sigma} = 0,$$

which implies that

$$\frac{\partial I}{\partial \sigma} = -\frac{I}{\sigma} \left\{ \frac{I \gamma'_a(I)}{1-\gamma_a(I)} - 1 \right\}^{-1} > 0$$

since $\gamma'_a(I) < 0$. Obviously $\frac{\partial T(I)}{\partial \sigma} = T'(I) \frac{\partial I}{\partial \sigma}$, and therefore the sign of $\frac{\partial T(I)}{\partial \sigma}$ depends on the sign of $T'(I)$.

This completes the proof.

